Jet correlators and transversity in inclusive DIS

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In collaboration with A. Bacchetta – paper to appear soon
Why jet correlators?

- Quarks are not asymptotic states
  - More mass than $m_q$ produced in the current region!
    - “Jet mass corrections” → Accardi, Qiu, JHEP 2008
    - Novel contributions to inclusive DIS structure functions → Accardi, Bacchetta, arXiv:very.soon

- Collinear factorization with “jet correlators”
  - Jet correlators, “jet mass” $M_q$, and new TMD sum rules
  - Transversity accessible in LT inclusive asymmetries:
    - New, large contribution to $g_2(x)$
    - Non-perturbative extension of BC sum rule

- Some phenomenological consequences

- Outlook
Collinear Factorization with Jet Correlators
TMDs in spin $\frac{1}{2}$ targets

<table>
<thead>
<tr>
<th>QUARKS</th>
<th>$\gamma^+$</th>
<th>$\gamma^+\gamma_5$</th>
<th>$\gamma^+\gamma^\alpha\gamma_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>U</strong></td>
<td>$f_1$</td>
<td></td>
<td>$h_1^\perp$</td>
</tr>
<tr>
<td><strong>L</strong></td>
<td></td>
<td>$g_1$</td>
<td>$h_{1L}^\perp$</td>
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<tr>
<td><strong>T</strong></td>
<td>$f_{1T}^\perp$</td>
<td>$g_{1T}$</td>
<td>$h_1^\perp h_{1T}^\perp$</td>
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</tbody>
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- Integrated (collinear) correlator: only circled ones survive
- Christ-Lee theorem (1970): $h_1$ not observable in inclusive DIS
- Not quite true:
  - Vacuum fluctuations can flip the spin of the struck quark
  - Large contribution prop to $h_1$ to the $g_2 - g_2^{ww}$ structure function

$\rightarrow$ P. Mulders’ talk
$g_2$ structure function - standard analysis

\[ W_{\mu\nu} = i_{\mu\nu\lambda\sigma} \frac{q^\lambda}{p \cdot q} \left[ g_1 S^\sigma + g_2 \left( S^\sigma - p^\sigma \frac{q \cdot S}{q \cdot p} \right) \right] \]

Wandzura-Wilczek term

"pure twist-3" (qgq correlations)

\[ g_2(x_B) - g_2^{WW}(x_B) = g_2^{tw3}(x_B) + \frac{m_q}{\Lambda} \left( \frac{h_1}{x} \right)^* (x_B) \]

quark mass term (negligible for light quarks)

\[ f^*(x) = -f(x) + \int_{x}^{1} \frac{dy}{y} f(y) \]
g₂ moments - standard analysis

- Burkhardt-Cottingham sum rule *

\[ \int_0^1 dx \ g_2(x) = 0 \]

Unless \( g_2^{tw3} \) pathological at large distances, or J=0 pole contributions \( \sim \delta(x) \) → Jaffe, Ji '91
... or large spin-flip contributions → Burkhardt, Cottingham '70

- “pure twist-3” effects, e.g.,
  Color force experienced by struck quark → talk by M. Burkardt

\[
d_2 \equiv \int_0^1 dx \ x^2 [g_2(x) - g_2^{WW}(x)] \\
= 3g_2[2] + 2g_1[1] \sim \langle P| \bar{\psi} \gamma^+ F^{+\alpha} \psi |P \rangle
\]
Inclusive DIS with jet correlators

Jet correlators

\[ \Xi_{ij}(l, n_+) = F.T. \langle 0 | U^{n+}_{(+\infty, \eta)} \psi_i(\eta) \bar{\psi}_j(0) U^{n+}_{(0, +\infty)} | 0 \rangle \]

\[ (\Xi^\mu_A)_{ij} = F.T. \langle 0 | U^{n+}_{(+\infty, \eta)} g A^\mu(\eta) \psi_i(\eta) \bar{\psi}_j(0) U^{n+}_{(0, +\infty)} | 0 \rangle \]
Factorization

- At order $1/Q$, neglect $k^-$ compared to $q^-$
  - The cross section depends only on the integrated jet correlator

\[ \Xi(l^-, l_T) \equiv \int \frac{dl^2}{2l^-} \Xi(l) = \frac{\Lambda}{2l^-} \xi_1 1 + \xi_2 \frac{\hat{n}^-}{2} + \text{h.t. terms} \]

- Coefficients can be interpreted in terms of quark spectral functions:
  \[ \xi_1 = \int d\mu^2 \frac{\mu}{\Lambda} J_1(\mu^2) \equiv \frac{M_q}{\Lambda} \]
  \[ \xi_2 = \int d\mu^2 J_2(\mu^2) = 1 \]

- Positivity constraints imply

\[ 0 < M_q < \int d\mu^2 \mu J_2(\mu^2) \implies M_q = O(100 \text{ MeV}) \]

Spin-flip average “jet” mass

\[ \text{can couple to transversity!} \]

Exactly, due to CPT invariance

Much larger than $m_q$!
Full twist-3 analysis

- Convenient and instructive to integrate the SIDIS tensor

\[ W^{\mu\nu}(x_B) = \sum_h \int dz \, d^2 p_{hT} \, z \, W_h^{\mu\nu}(z, p_{hT}, x_B) \]

- The piece of the SIDIS tensor with jet mass contributions is

\[
2\Lambda W^{\mu\nu} = i \frac{2\Lambda}{Q} \hat{t}^{[\mu} \epsilon_{\nu]}^\rho S_{\perp \rho} \times \sum_q e_q^2 \left[ 2x_B g_T^q(x_B) D_1^{q,h}(z, p_{hT}) + 2h_1^q(x_B) \tilde{E}^{q,h}(z, p_{hT}) \right] + \ldots
\]

Where e.o.m. relate the twist-3 TMD-FF \( \tilde{E} \) to twist-2 TMD-FFs:

\[
\tilde{E} = E - \frac{m_q}{\Lambda} z D_1
\]

Current quark mass \( \sim \) few MeV
Jet and TMD sum rules

- Utilize the following jet correlator sum rule:

\[
\sum_h \int \, d^2 p_{hT} \, \frac{d p^-_h}{2 p^-_h} \, p^-_h \, \Delta^h (l, p_h) = l^- \, \Xi (l)
\]

- At TMD level, this implies:

\[
\sum_h \int \, d z \, z \, = \, =
\]

\[
\sum_h \int \, d z \, d^2 p_{hT} \, z D^h_1 (z, p_{hT}) = \xi_2 = 1
\]

\[
\sum_h \int \, d z \, d^2 p_{hT} \, E^h (z, p_{hT}) = \xi_1 = \frac{M_q}{\Lambda}
\]

\[
\sum_h \int \, d z \, d^2 p_{hT} \, \tilde{E}^{q, h} (z, p_{hT}) = \frac{M_q - m_q}{\Lambda}
\]

Novel TMD sum rules

\[M_{q_{\text{pert}}} = m_q \Rightarrow \text{Old ones}\]
Finally, the DIS cross section

- Inclusive DIS

\[ \frac{d\sigma}{dx_B \, dy \, d\phi_S} \propto \left\{ F_T + \varepsilon F_L + S_{\parallel} \lambda e \sqrt{1 - \varepsilon^2} \, F_{LL} \right. \\
\left. + \left| S_\perp \right| \lambda e \sqrt{2 \varepsilon (1 - \varepsilon)} \cos \phi_S \, F_{LT}^{\cos \phi_S} \right\} \]

- Structure functions:

\[ F_T = x_B \sum_q e_q^2 f_1^q(x_B) \]

\[ F_L = 0 \]

\[ F_{LL} = x_B \sum_q e_q^2 g_1^q(x_B) \]

\[ F_{LT}^{\cos \phi_S} = -x_B \sum_q e_q^2 \frac{2\Lambda}{Q} \left( x_B g_T^q(x_B) + \frac{M_q - m_q}{\Lambda} h_1^q(x_B) \right) \]

Transversity in inclusive DIS!
Phenomenological consequences
g_2 structure function revisited

- Using EOM, Lorentz Invariance Relations, can show that

\[ g_2(x_B) - g_2^{WW}(x_B) = \frac{1}{2} \sum_a e_a^2 \left( g_2^{q,tw^3}(x_B) + \frac{m_q}{\Lambda} \left( \frac{h_1^q}{x} \right)^* (x_B) + \frac{M_q - m_q}{\Lambda} \frac{h_1^q(x_B)}{x_B} \right) \]

\( \equiv g_2^{quark} \)
\( \equiv g_2^{jet} \)

- Consequences:
  - Interplay of transverse degrees of freedom
  - New background to qGq effects extraction
  - But... h1 accessible in inclusive DIS!
    - Potentially large signal

\[ f^*(x) = -f(x) + \int_x^1 \frac{dy}{y} f(y) \]
Size of mass effects

\[ g_2(x_B) - g_2^{WW}(x_B) = \frac{1}{2} \sum_a e_a^2 \left( g_2^{q,tw3}(x_B) + \frac{m_q}{\Lambda} \left( \frac{h_1^q}{x} \right)^* (x_B) \right) + \frac{M_q - m_q}{\Lambda} \frac{h_1^q(x_B)}{x_B} \]
Size of mass effects

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g2 structure function revisited

Using EOM, Lorentz Invariance Relations, can show that

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g_2(x_B) - g_2^{WW}(x_B) = \frac{1}{2} \sum_a e_a^2 g_2^{q,tw^3}(x_B) + \frac{m_q}{\Lambda} \left( \frac{h_1^q}{x} \right)^* (x_B) + \frac{M_q - m_q}{\Lambda} \frac{h_1^q(x_B)}{x_B}
\]

\[
\equiv g_2^{m_q}
\]

\[
\equiv g_2^{jet}
\]

\[
m_q = 5 \text{ MeV}
\]

\[
M_q = 100 \text{ MeV}
\]
Novel non-perturbative sum rules

- Taking moments of $g_2$ with $M_u \approx M_d \equiv M_{\text{jet}}$

Burkardt-Cottingham

$$\int_0^1 g_2(x) = M_{\text{"jet"}} \int_0^1 dx \frac{h_1(x)}{x}$$

→ unlikely to still be zero!

→ if BC broken by finite amount, constrains:

$$h_1^q(x) \propto x^\epsilon \quad \epsilon > 0$$
Novel non-perturbative sum rules

- Taking moments of $g_2$ with $M_u \approx M_d \equiv M_{jet}$

Burkardt-Cottingham

$$\int_0^1 g_2(x) = M_{\text{\textquotedblleft jet\textquotedblright}} \int_0^1 dx \frac{h_1(x)}{x}$$

Efremov-Teryaev-Leader

$$\int_0^1 x g_2^{q-\bar{q}}(x) = 2 M_{\text{\textquotedblleft jet\textquotedblright}} \int_0^1 dx h_1^{\bar{q}-q}(x)$$

Tensor charge $\delta_T$

→ Novel way to measure the tensor charge!
Novel non-perturbative sum rules

Taking moments of $g_2$ with $M_u \approx M_d \equiv M_{\text{jet}}$

**Burkardt-Cottingham**

$$\int_0^1 g_2(x) = M_{\text{jet}} \frac{\int_0^1 dx \frac{h_1(x)}{x}}{x}$$

**Efremov-Teryaev-Leader**

$$\int_0^1 x g_2^{q-\bar{q}}(x) = 2 M_{\text{jet}} \int_0^1 dx \frac{h_1^{q-\bar{q}}(x)}{x}$$

Tensor charge $\delta_T$

**Color polarizability**

$$\int_0^1 \left[ 3x^2 g_2(x) - 2x^2 g_1(x) \right] = d_2 + 3 M_{\text{jet}} \int_0^1 x h_1(x) + O(m_q)$$

"pure twist-3"
Example: color polarizability

- Need to subtract jet term to obtain “pure twist-3” \( d_2 \sim \langle \bar{q} \gamma^+ F^{+y} q \rangle \)

\[
d_2 = \int_0^1 dx \left[ 3x^2 g_2(x) - 2x^2 g_1(x) \right] - 3 M_{"jet"} \int_0^1 dx \ x h_1(x)
\]

Data \(\rightarrow\) global fits (e.g. JAM15)
(in future also from lattice:
Chambers et al., arXiv:1703.01153)

- Experiments
(what precision now, expected?)
Global fits (Pavia, Torino)
(Can use constraints from new sum rules)
Lattice ?

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{example_graph.png}
\caption{Lattice:
Goekele et al., 2005 $Q^2 \text{ (GeV}^2\text{)}$}
\end{figure}
Measuring the jet correlator

- **e- e+ collisions**: semi-inclusive $\Lambda$ and di-hadron production

- **Global-global-global fits**: → see also talk by Ethier, Melnitchouk
  - Interplay of $q$, $\Delta q$, $\delta q$
  - Leave $M_q$ as a free parameter

- **Lattice QCD**:
  - How to measure $M_q$?
  - Any relation to the quark condensate?

→ see also G.Schnell’s talk
Outlook
Where are we going?

- **Jet correlators open a novel and rich phenomenology**
  - New terms in old observables
  - Signal enhancement in less studied channels
    - e.g. spin flip in single transverse target spin asymmetry
  - New observables?

- **Exploit new sum rules:**
  - Constraints in QCD fits
  - Measure the tensor charge
  - Proper background subtraction for twist-3 matrix elements

- **Need new data on g2, and esp. on transversity**
  - Large x: Jlab → e.g., T.Liu’s talk
  - Small x: EIC
  - ....

→ Afanasiev et al., PRD77(2008)
→ Schelegel, PRD87(2013)