Connecting Different TMD Factorization Formalisms in QCD

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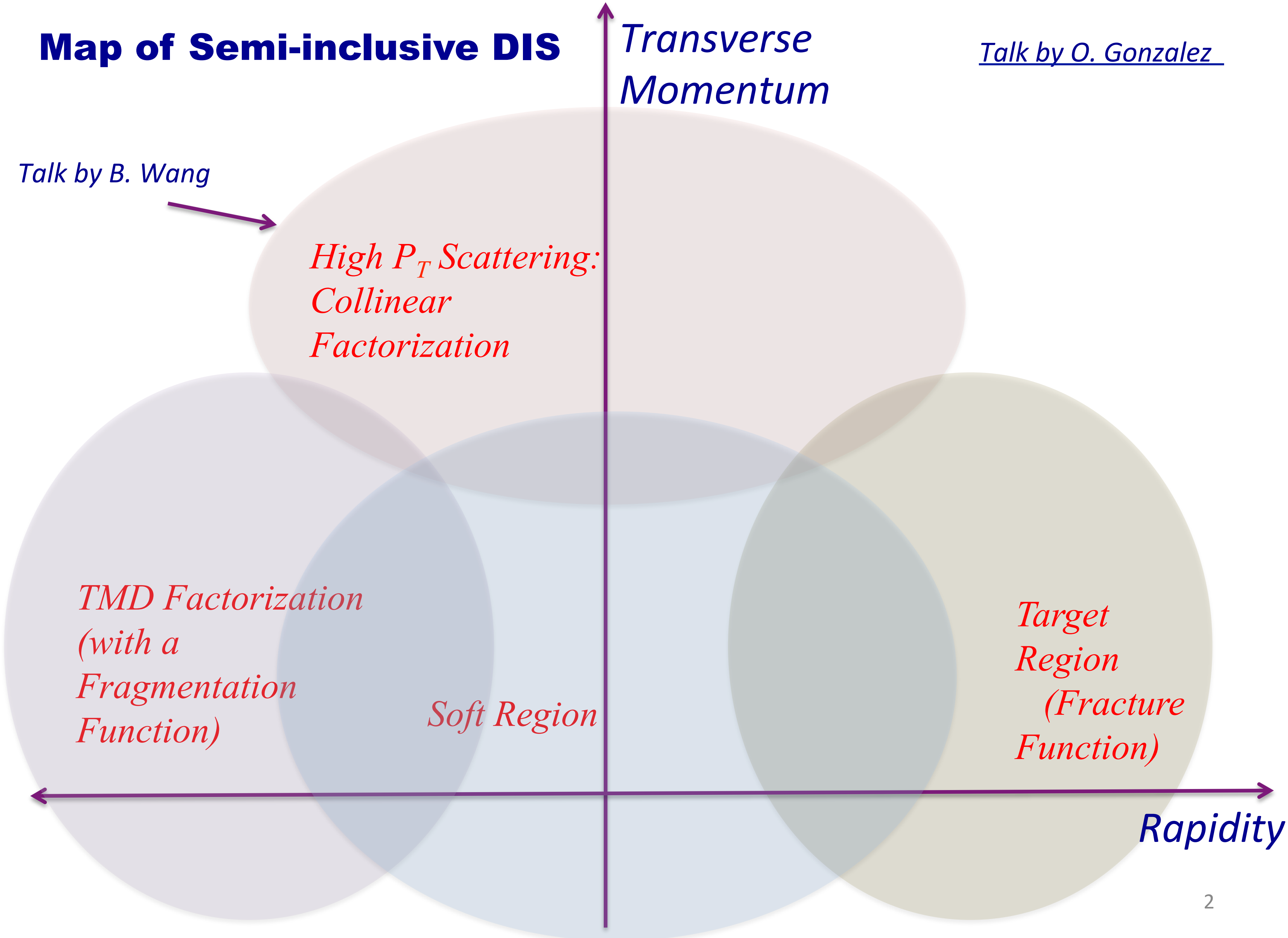
- Based on:

  J. Collins, T. Rogers: arXiv:1750.07167 (2017)

QCD Evolution Workshop, 2017

Map of Semi-inclusive DIS



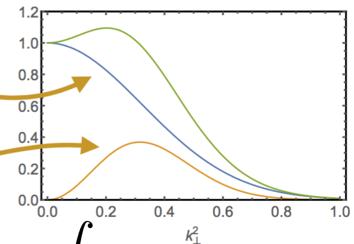
TMD Factorization

- Non-perturbative transverse momentum
  - Relationship to hadron structure, spin, power corrections etc…
    - Important to have explicit TMD definitions
    - Extract from data and compare across processes to test universality
    - Predict with non-perturbative techniques: lattice, etc…

Phenomenology and Partons

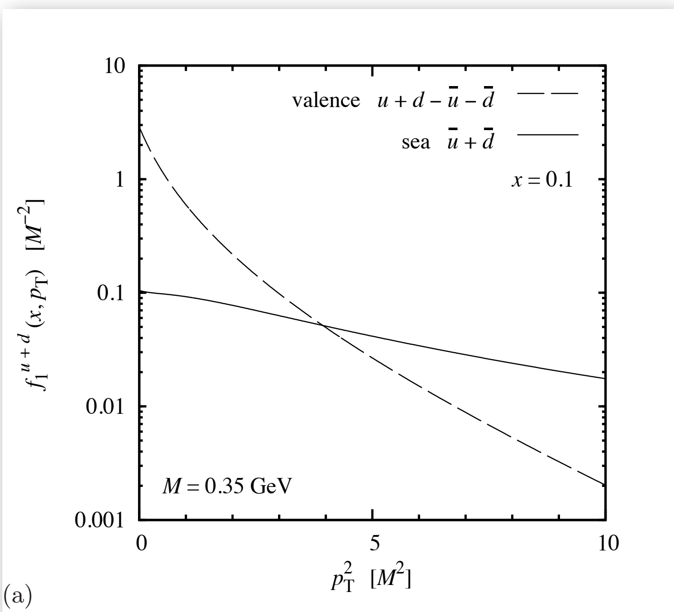
Talk by C. Pisano

$$\hat{f}^a_{\rm NP} = \text{F.T. of } \left(e^{-\frac{k_T^2}{g_{1a}}} + \lambda k_T^2 e^{-\frac{k_T^2}{g_{1a}}} \right) \frac{1}{g_{1a} + \lambda\, g_{1a}^2}$$

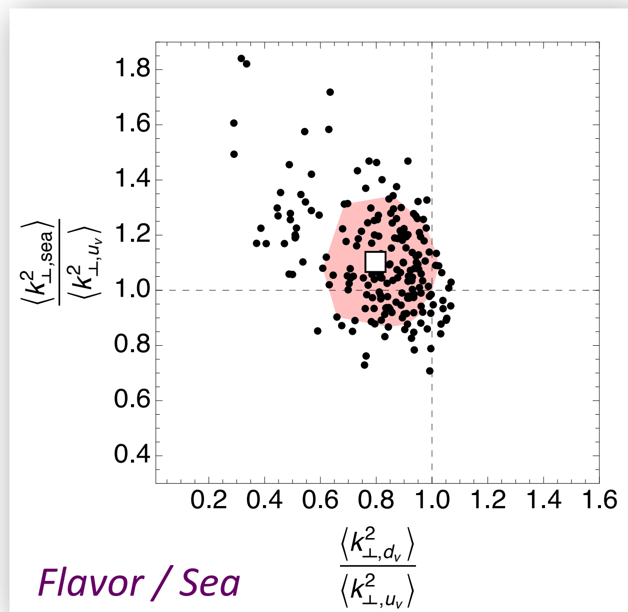


Cross Section $\approx H_f \int \mathrm{d}^2\boldsymbol{k}_\mathrm{T}\, F_{f/p}(x, \boldsymbol{k}_\mathrm{T} - \boldsymbol{q}_\mathrm{T})\, D_{h/f}(z, z\boldsymbol{k}_\mathrm{T})$

(Schweitzer, Strikman, Weiss, (2013))



(Signori, Bacchetta, Radici, Schnell (2013))



TMD Factorization

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- Issues in semi-inclusive deeply inelastic scattering
  - Smallish Q is typical
  - Match to central & target regions
  - Limited range of rapidity and transverse momentum at small Q and/or large x
  - Nature of power corrections from hadron masses?

Talks by E. Moffat and J. Guerrero

TMD Factorization

- Many results exist.
  - Resummation in collinear factorization
  - CSS
  - SCET
  - Sudakov Factors

- Formalisms often appear different on the surface.

- Goals:

  - Map old styles to new.
  - Is there convergence toward a standardized set of definitions (and results) for TMD definitions?
  - Bring diverse results together with consistent conventions (notation, etc)?

Older Formalisms…

- CSS1 - Multiple redefinitions of factors (starting from TMD definitions) No explicit hard part.

  (Collins, Soper, Sterman (1981-1985))

  – Match to collinear for $\Lambda_{QCD} << q_T << Q$ and $q_T \approx Q$.

- Catani, de Florian, Grazzini et al.

  – Factorization takes a simple form.

  – Large transverse momentum (e.g., Y-term) results are automatic.

  – Ressumation scheme dependence; shown there is no uniquely defined hard part.

  (Catani, de Florian, Grazzini (2001))

Drell-Yan

$$\mu_Q \equiv C_2 Q$$
$$\mu_b \equiv C_1/b_{\mathrm{T}}$$
$$\mu_{b_*} \equiv C_1/b_*$$

$$\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_{\mathrm{T}}^2} = {} & \frac{4\pi^2\alpha^2}{9Q^2 s}\sum_{j,j_A,j_B} e_j^2 \int \frac{\mathrm{d}^2\boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2} e^{i\boldsymbol{q}_{\mathrm{T}}\cdot\boldsymbol{b}_{\mathrm{T}}} \\
& \times \int_{x_A}^1 \frac{\mathrm{d}\xi_A}{\xi_A} f_{j_A/A}(\xi_A;\mu_{b_*})\ \tilde{C}^{\mathrm{CSS1,\ DY}}_{j/j_A}\left(\frac{x_A}{\xi_A}, b_*; \mu_{b_*}^2, \mu_{b_*}, C_2, a_s(\mu_{b_*})\right) \\
& \times \int_{x_B}^1 \frac{\mathrm{d}\xi_B}{\xi_B} f_{j_B/B}(\xi_B;\mu_{b_*})\ \tilde{C}^{\mathrm{CSS1,\ DY}}_{\bar{j}/j_B}\left(\frac{x_B}{\xi_B}, b_*; \mu_{b_*}^2, \mu_{b_*}, C_2, a_s(\mu_{b_*})\right) \\
& \times \exp\left\{-\int_{\mu_{b_*}^2}^{\mu_Q^2} \frac{\mathrm{d}\mu'^2}{\mu'^2}\left[A_{\mathrm{CSS1}}(a_s(\mu');C_1)\ln\left(\frac{\mu_Q^2}{\mu'^2}\right) + B_{\mathrm{CSS1,\ DY}}(a_s(\mu');C_1,C_2)\right]\right\} \\
& \times \exp\left[-g^{\mathrm{CSS1}}_{j/A}(x_A, b_{\mathrm{T}}; b_{\max}) - g^{\mathrm{CSS1}}_{\bar{j}/B}(x_B, b_{\mathrm{T}}; b_{\max}) - g^{\mathrm{CSS1}}_K(b_{\mathrm{T}}; b_{\max})\ln(Q^2/Q_0^2)\right] \\
& + \text{suppressed corrections.}
\end{aligned}$$

No explicit hard part here

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& \times \exp\left[-g^{\text{CSS1}}_{j/A}(x_A, b_T; b_{\max}) - g^{\text{CSS1}}_{\bar{j}/B}(x_B, b_T; b_{\max}) - g^{\text{CSS1}}_K(b_T; b_{\max}) \ln(Q^2/Q_0^2) \right] \\
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& \times \exp\left\{ -\int_{\mu_{b_*}^2}^{\mu_Q^2} \frac{\mathrm{d}\mu'^2}{\mu'^2} \left[\underline{A_{\mathrm{CSS1}}(a_s(\mu');C_1)} \ln\left(\frac{\mu_Q^2}{\mu'^2}\right) + \underline{B_{\mathrm{CSS1,\ DY}}(a_s(\mu');C_1,C_2)} \right] \right\} \\
& \times \exp\left[\underline{-g^{\mathrm{CSS1}}_{j/A}(x_A, b_\mathrm{T}; b_{\max}) - g^{\mathrm{CSS1}}_{\bar{\jmath}/B}(x_B, b_\mathrm{T}; b_{\max}) - g^{\mathrm{CSS1}}_K(b_\mathrm{T}; b_{\max})} \ln(Q^2/Q_0^2) \right] \\
& + \text{suppressed corrections.}
\end{aligned}$$

No explicit hard part here

b_{max} method

- Replacement

$$\boldsymbol{b}_*(\boldsymbol{b}_T) \to \begin{cases} \boldsymbol{b}_T & b_T \ll b_{\mathrm{max}} \\ \boldsymbol{b}_{\mathrm{max}} & b_T \gg b_{\mathrm{max}} \end{cases}$$

- One commonly used functional form.

$$\mathbf{b}_*(\mathbf{b}_\mathrm{T}) \equiv \frac{\mathbf{b}_\mathrm{T}}{\sqrt{1 + b_T^2/b_{\mathrm{max}}^2}}$$

$$\mu_Q \equiv C_2 Q$$
$$\mu_b \equiv C_1/b_\mathrm{T}$$
$$\mu_{b_*} \equiv C_1/b_*$$

New TMD methods

- Improved TMD function definitions (e.g., CSS2).

  (J. Collins textbook, (2011))

  - SCET-based approaches
    - Main differences from CSS2: Implementation of regulators.
    - At least two are equivalent to CSS2
      (Echevarria, Idilbi, Scimemi (2012); Collins, TCR (2013))
      (Li, Neill, Zhu, (2016); Collins, TCR (2017) App. B)

      (See talks by V. Vaidya and Esp. I. Scimemi)
  - Better oriented for hadron structure studies (e.g. lattice QCD)
  - Structurally familiar from phenomenology.

    $$H_f \int \mathrm{d}^2\boldsymbol{k}_\mathrm{T}\, F_{f/p}(x, \boldsymbol{k}_\mathrm{T} - \boldsymbol{q}_\mathrm{T})\, D_{h/f}(z, z\boldsymbol{k}_\mathrm{T})$$

  - Hard parts are fixed by factorization of operator structures.

    $$\frac{\text{Cross Section}}{\int \mathrm{d}^2\boldsymbol{k}_\mathrm{T}\, F_{f/p}(x, \boldsymbol{k}_\mathrm{T} - \boldsymbol{q}_\mathrm{T})\, D_{h/f}(z, z\boldsymbol{k}_\mathrm{T})} = H_f$$

New TMD methods

- TMD parton model structure + evolution equations.

Ex: CSS2

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_\mathrm{T}^2} = \frac{4\pi^2\alpha^2}{9Q^2 s} \sum_j \underline{H_{j\bar{j}}^{\mathrm{DY}}(Q,\mu_Q,a_s(\mu_Q))} \int \frac{\mathrm{d}^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2}\, e^{i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}}\; \underline{\tilde{f}_{j/A}(x_A,b_\mathrm{T};Q^2,\mu_Q)}\; \underline{\tilde{f}_{\bar{j}/B}(x_B,b_\mathrm{T};Q^2,\mu_Q)}$$

$$+ \text{suppressed corrections,}$$

$$\frac{\partial \ln \tilde{f}(x,b_T;\mu,\zeta)}{\partial \ln\sqrt{\zeta}} = \tilde{K}(b_T;\mu)$$

$$\tilde{f}_{j/H}(x,b_\mathrm{T};\zeta;\mu) = \sum_k \int_{x-}^{1+} \frac{\mathrm{d}\xi}{\xi}\, \tilde{C}_{j/k}^{\mathrm{PDF}}(x/\xi,b_\mathrm{T};\zeta,\mu,a_s(\mu))\, f_{k/H}(\xi;\mu) \;+\; O[(mb_\mathrm{T})^p]$$

$$\frac{\mathrm{d}\tilde{K}(b_T;\mu)}{\mathrm{d}\ln\mu} = -\gamma_K(a_s(\mu))$$

$$\frac{\mathrm{d}\ln \tilde{f}(x,b_T;\mu,\zeta)}{\mathrm{d}\ln\mu} = \gamma_j(a_s(\mu)) - \frac{1}{2}\gamma_K(a_s(\mu))\ln\frac{\zeta}{\mu^2}$$

$$\mu_Q \equiv C_2 Q$$
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Perturbation-Theory-Optimized Solution

$$
\begin{aligned}
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& \times e^{-g_{j/A}(x_A,b_\mathrm{T};b_{\max})} \int_{x_A}^{1} \frac{\mathrm{d}\xi_A}{\xi_A} f_{j_A/A}(\xi_A;\mu_{b_*})\ \tilde{C}^{\mathrm{PDF}}_{j/j_A}\left(\frac{x_A}{\xi_A}, b_*;\mu_{b_*}^2,\mu_{b_*},a_s(\mu_{b_*})\right) \\
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& \times e^{\underline{-g_{j/A}(x_A,b_\mathrm{T};b_\mathrm{max})}} \int_{x_A}^{1} \frac{\mathrm{d}\xi_A}{\xi_A} \underline{f_{j_A/A}(\xi_A;\mu_{b_*})}\, \underline{\tilde{C}^{\mathrm{PDF}}_{j/j_A}\left(\frac{x_A}{\xi_A}, b_*;\mu^2_{b_*},\mu_{b_*},a_s(\mu_{b_*})\right)} \\
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& \times e^{-g_{j/A}(x_A, b_\mathrm{T}; b_\mathrm{max})} \int_{x_A}^{1} \frac{\mathrm{d}\xi_A}{\xi_A} f_{j_A/A}(\xi_A; \mu_{b_*})\, \tilde{C}^{\mathrm{PDF}}_{j/j_A}\left(\frac{x_A}{\xi_A}, b_*; \mu_{b_*}^2, \mu_{b_*}, a_s(\mu_{b_*}) \right) \\
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\end{aligned}
$$

Compare CSS1 vs. CSS2

$$\mu_Q \equiv C_2 Q$$
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$$\frac{d\sigma}{dQ^2\, dy\, dq_{\rm T}^2} = \frac{4\pi^2\alpha^2}{9Q^2 s} \sum_{j,j_A,j_B} e_j^2 \int \frac{d^2\boldsymbol{b}_{\rm T}}{(2\pi)^2} e^{i\boldsymbol{q}_{\rm T}\cdot\boldsymbol{b}_{\rm T}}$$
$$\times \int_{x_A}^1 \frac{d\xi_A}{\xi_A} \underline{f_{j_A/A}(\xi_A;\mu_{b_*})}\ \underline{\tilde{C}^{\rm CSS1,\ DY}_{j/j_A}\left(\frac{x_A}{\xi_A}, b_*; \mu_{b_*}^2, \mu_{b_*}, C_2, a_s(\mu_{b_*})\right)}$$
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$$\times \exp\left[\underline{-g^{\rm CSS1}_{j/A}(x_A,b_{\rm T};b_{\rm max}) - g^{\rm CSS1}_{\bar{\jmath}/B}(x_B,b_{\rm T};b_{\rm max}) - g^{\rm CSS1}_K(b_{\rm T};b_{\rm max})} \ln(Q^2/Q_0^2)\right]$$
$$+ \text{suppressed corrections.}$$

No explicit hard part

Old Schemes and New Schemes

- Questions:
  - CSS1 involves “A” and “B” functions not explicit in CSS2.
  - Non-perturbative parts in CSS1 and in TMD functions?
  - Anomalous dimension of PDFs vs. FFs?
  - Many high order calculations in old resummation, SCET, etc… how to utilize in, for example, CSS2?

Fast translation to new TMD methods

- CSS1 and CSS2 drop same subleading powers:

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_{\mathrm{T}}^2}\right|_{\mathrm{DY}}^{\mathrm{CSS1}} = \left.\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_{\mathrm{T}}^2}\right|_{\mathrm{DY}}^{\mathrm{CSS2}} \quad ; \quad \left.\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_{\mathrm{T}}^2}\right|_{\mathrm{SIDIS}}^{\mathrm{CSS1}} = \left.\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_{\mathrm{T}}^2}\right|_{\mathrm{SIDIS}}^{\mathrm{CSS2}}$$

- Derivatives given by evolution equations.
  (anomalous dimensions)

- b_{max} independence.

- Charge conjugation invariance.

Fast translation to new TMD methods

- First equate ln(Q) and ln(b_T) derivatives

- Use b_{max} independence

$$A_{\rm CSS1}(a_s(\mu_{b_*}); C_1) = -\frac{\mathrm{d}\tilde{K}(b_*;\mu_{b_*})}{\mathrm{d}\ln b_*^2} + \frac{1}{2}\gamma_K(a_s(\mu_{b_*})) = -\left.\frac{\partial\tilde{K}(b_*;\mu)}{\partial\ln b_*^2}\right|_{\mu\mapsto\mu_{b_*}}$$

$$B_{\rm CSS1,\ DY}(a_s(\mu_Q); C_1, C_2) = -\tilde{K}(C_1/\mu_Q;\mu_Q) - \frac{\partial\ln H_{j\bar{\jmath}}^{\rm DY}(Q,\mu_Q,a_s(\mu_Q))}{\partial\ln Q^2}$$

$$g_K^{\rm CSS1}(b_{\rm T}; b_{\rm max}) = g_K(b_{\rm T}; b_{\rm max})$$

Fast translation to new TMD methods

- Use result and repeat for undifferentiated cross section

$$e_j^2 \tilde{C}^{\text{CSS1, DY}}_{j/j_A}\left(\frac{x_A}{\xi_A}, b_*; \mu_{b_*}^2, \mu_{b_*}, C_2, a_s(\mu_{b_*})\right) \times \tilde{C}^{\text{CSS1, DY}}_{\bar{\jmath}/j_B}\left(\frac{x_B}{\xi_B}, b_*; \mu_{b_*}^2, \mu_{b_*}, C_2, a_s(\mu_{b_*})\right)$$

$$= \tilde{C}^{\text{PDF}}_{j/j_A}\left(\frac{x_A}{\xi_A}, b_*; \mu_{b_*}^2, \mu_{b_*}, a_s(\mu_{b_*})\right) \times \tilde{C}^{\text{PDF}}_{\bar{\jmath}/j_B}\left(\frac{x_B}{\xi_B}, b_*; \mu_{b_*}^2, \mu_{b_*}, a_s(\mu_{b_*})\right) \times H^{\text{DY}}_{j\bar{\jmath}}(\mu_{b_*}/C_2, \mu_{b_*}, a_s(\mu_{b_*})) \exp\left[-2\tilde{K}(b_*; \mu_{b_*}) \ln C_2\right]$$

$$g_{j/A}(x_A, b_{\text{T}}; b_{\max}) + g_{\bar{\jmath}/B}(x_B, b_{\text{T}}; b_{\max}) = g^{\text{CSS1}}_{j/A}(x_A, b_{\text{T}}; b_{\max}) + g^{\text{CSS1}}_{\bar{\jmath}/B}(x_B, b_{\text{T}}; b_{\max})$$

Fast translation to new TMD methods

- Charge conjugation invariance:

$$|e_j|\tilde{C}^{\text{CSS1, DY}}_{j/k}\left(\frac{x}{\xi}, b_*; \mu_{b_*}^2, \mu_{b_*}, C_2, a_s(\mu_{b_*})\right)$$
$$= \tilde{C}^{\text{PDF}}_{j/k}\left(\frac{x}{\xi}, b_*; \mu_{b_*}^2, \mu_{b_*}, a_s(\mu_{b_*})\right)\sqrt{H^{\text{DY}}_{j\bar{j}}(\mu_{b_*}/C_2, \mu_{b_*}, a_s(\mu_{b_*}))}\exp\left[-\tilde{K}(b_*;\mu_{b_*})\ln C_2\right]$$

$$g^{\text{CSS1}}_{j/H}(x, b_{\text{T}}; b_{\max}) = g_{j/H}(x, b_{\text{T}}; b_{\max})$$

- “Non-perturbative” g-functions are exactly equal in CSS1 and CSS2.

Fast translation to new TMD methods

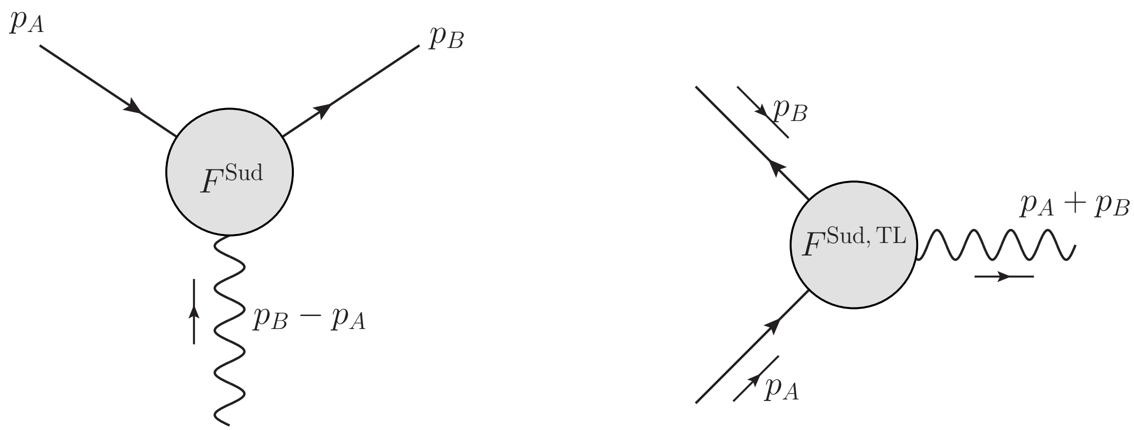
- Fitted functions are same in new approaches (CSS2 and SCET) as in CSS1.
  - *Ex: (Nadolsky, Stump, Yuan (1999), Landry, Brock, Nadolsky, Yuan (2003); Konychev, Nadolsky (2006))*

- CSS1 "A," "B," "C" fixed by TMD-based expressions.

- TMDs have $\gamma_j(\mu),\ \gamma_K(\mu),\ K(b_{\mathrm{T}};\mu),\ H(\alpha_s(\mu);\mu/Q)$

  CSS1 has "A" and "B"
  - Need independent information on hard parts and anomalous dimensions

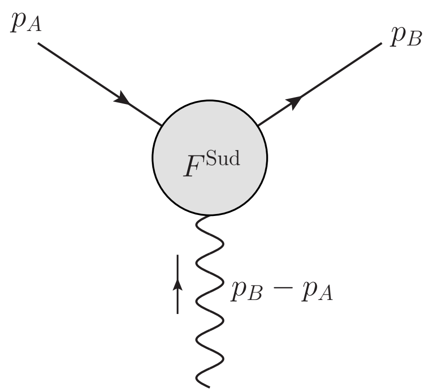
Sudakov Factors and Hard Parts

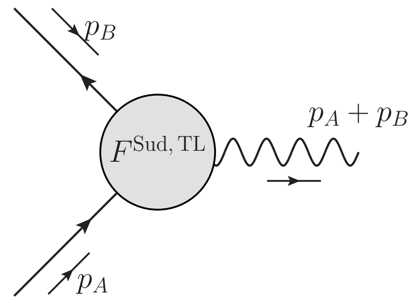


$$H^{\rm DY}_{j\bar{j}}(Q,\mu;a_s(\mu)) = e_j^2 \left| H_j^{\rm Sud,\ TL}(Q^2) \right|^2 = e_j^2 \left| H_j^{\rm Sud}(-Q^2 - i\epsilon) \right|^2$$

- TMD factorization hard part determined from Sudakov factor.
- Known for some time:
  (Moch, Vermaseren, Vogt (2005): Three-loop poles)
  (Idilbi, Ji, Yuan (2006))
- Complete three loop result
  (Gehrmann, Glover, Huber, N. Ikizleri, Studerus (2010))

Sudakov Factors and Hard Parts





$$H^{\mathrm{DY}}_{j\bar{j}}(Q,\mu;a_s(\mu)) = e_j^2 \left| H_j^{\mathrm{Sud,\ TL}}(Q^2) \right|^2 = e_j^2 \left| H_j^{\mathrm{Sud}}(-Q^2 - i\epsilon) \right|^2$$

- Anomalous dimensions of collinear factors: γ_j and γ_K

$$\frac{\mathrm{d} \ln H^{\mathrm{Sud,\ TL}}}{\mathrm{d} \ln \mu} = -\gamma_j(a_s(\mu)) - i\frac{\pi}{2}\gamma_K(a_s(\mu)) + \frac{1}{2}\gamma_K(a_s(\mu)) \ln \frac{Q^2}{\mu^2},$$

;

$$\ln F^{\mathrm{Sud,\ TL}}(Q^2) = \ln H^{\mathrm{Sud,\ TL}} + 2 \ln C_j^{\mathrm{bare}} + D(a_s,\epsilon) - i\pi E(a_s,\epsilon) + \ln \frac{Q^2}{\mu^2} E(a_s,\epsilon)$$

  – Equality of γ_j^{PDF} and γ_j^{FF} to all orders: TP invariance

(Collins, TCR (2017), Appendix A)

Rapidity Evolution Kernels

- Knowledge of B: CSS1 to CSS2, two loops

$$B_{\mathrm{CSS1,\ DY}}(a_s(\mu_Q); C_1, C_2) = -\tilde{K}(C_1/\mu_Q; \mu_Q) - \frac{\partial \ln H^{\mathrm{DY}}_{j\bar{\jmath}}(Q, \mu_Q, a_s(\mu_Q))}{\partial \ln Q^2}$$

(B calculated in Davies and Stirling (1984))

- Direct calculation from operators (using SCET): Three loops

(Li, Neill, Zhu; Li, Zhu (2016, 2017))
(Vladimirov (2017))

Fast translation to new TMD methods

- Charge conjugation invariance:

$$
|e_j|\tilde{C}^{\text{CSS1, DY}}_{j/k}\left(\frac{x}{\xi}, b_*; \mu^2_{b_*}, \mu_{b_*}, C_2, a_s(\mu_{b_*})\right)
$$

$$
= \tilde{C}^{\text{PDF}}_{j/k}\left(\frac{x}{\xi}, b_*; \mu^2_{b_*}, \mu_{b_*}, a_s(\mu_{b_*})\right)\sqrt{H^{\text{DY}}_{j\bar{j}}(\mu_{b_*}/C_2, \mu_{b_*}, a_s(\mu_{b_*}))}\,\exp\left[-\tilde{K}(b_*;\mu_{b_*})\ln C_2\right]
$$

$$
g^{\text{CSS1}}_{j/H}(x, b_{\text{T}}; b_{\text{max}}) = g_{j/H}(x, b_{\text{T}}; b_{\text{max}})
$$

- "Non-perturbative" g-functions are exactly equal in CSS1 and CSS2.

Wilson Coefficients

- CSS1 to CSS2: order α_s^2

$$\tilde{C}^{\rm PDF}_{j/k}\left(\frac{x}{\xi}, b_*; \mu_{b_*}^2, \mu_{b_*}, a_s(\mu_{b_*})\right) = \frac{\tilde{C}^{\rm CSS1,\ DY}_{j/k}\left(\frac{x}{\xi}, b_*; \mu_{b_*}^2, \mu_{b_*}, C_2, a_s(\mu_{b_*})\right)}{\sqrt{(1/e_j^2) H^{\rm DY}_{j\bar{j}}(\mu_{b_*}/C_2, \mu_{b_*}, a_s(\mu_{b_*}))}} \exp\left[\tilde{K}(b_*; \mu_{b_*}) \ln C_2\right]$$

$C^{CSS1,\ DY}$ taken from (Catani, Cieri, de Florian, Ferrera, Grazzini (2012))

- Directly from operators (using SCET)
  (Echevarria, Scimemi, Vladimirov (2016))

$\mathbf{b_{max}}$ method

$$
\begin{aligned}
W(q_{\mathrm{T}}, Q) &= \int \frac{\mathrm{d}^2 \boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2} e^{i \boldsymbol{q}_{\mathrm{T}} \cdot \boldsymbol{b}_{\mathrm{T}}} \, \tilde{W}(b_{\mathrm{T}}, Q) \\
&= \int \frac{\mathrm{d}^2 \boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2} e^{i \boldsymbol{q}_{\mathrm{T}} \cdot \boldsymbol{b}_{\mathrm{T}}} \tilde{W}^{\mathrm{OPE}}(b_*(b_{\mathrm{T}}), Q) \tilde{W}_{\mathrm{NP}}(b_{\mathrm{T}}, Q; b_{\mathrm{max}})
\end{aligned}
$$

$\mathbf{b_{max}}$ method

$$W(q_\mathrm{T}, Q) = \int \frac{\mathrm{d}^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2} e^{i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}}\, \tilde{W}(b_\mathrm{T}, Q)$$

$$= \int \frac{\mathrm{d}^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2} e^{i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}}\, \tilde{W}^\mathrm{OPE}(b_*(b_\mathrm{T}), Q) \underbrace{\tilde{W}_\mathrm{NP}(b_\mathrm{T}, Q; b_\mathrm{max})}$$

Interesting part.

(Esp. for JLab)

$$\frac{\tilde{W}(b_\mathrm{T}, Q)}{\tilde{W}^\mathrm{OPE}(b_*(b_\mathrm{T}), Q)}$$

$\mathbf{b_{max}}$ method

$$W(q_T, Q) = \int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2} e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}\, \tilde{W}(b_T, Q)$$

$$= \int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2} e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}\, \tilde{W}^{\mathrm{OPE}}(b_*(b_T), Q)\, \underbrace{\tilde{W}_{\mathrm{NP}}(b_T, Q; b_{\max})}$$

Interesting part.

(Esp. for JLab)

$$\frac{\tilde{W}(b_T, Q)}{\tilde{W}^{\mathrm{OPE}}(b_*(b_T), Q)}$$

$$\boxed{\tilde{W}(b_T, Q) = \tilde{W}^{\mathrm{OPE}}(b_*(b_T), Q) + O\left((b_T m)^p\right)}$$

$$\tilde{f}_{j/H}(x, b_T; \zeta; \mu) = \sum_k \int_{x-}^{1+} \frac{d\xi}{\xi}\, \tilde{C}^{\mathrm{PDF}}_{j/k}(x/\xi, b_T; \zeta, \mu, a_s(\mu))\, f_{k/H}(\xi; \mu) \;+\; O[(mb_T)^p]$$

Wanted

$$W(q_{\mathrm{T}}, Q) = \int \frac{\mathrm{d}^2 \boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2} e^{i \boldsymbol{q}_{\mathrm{T}} \cdot \boldsymbol{b}_{\mathrm{T}}} \, \tilde{W}(b_{\mathrm{T}}, Q)$$

$$= \int \frac{\mathrm{d}^2 \boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2} e^{i \boldsymbol{q}_{\mathrm{T}} \cdot \boldsymbol{b}_{\mathrm{T}}} \tilde{W}^{\mathrm{OPE}}(b_*(b_{\mathrm{T}}), Q) \tilde{W}_{\mathrm{NP}}(b_{\mathrm{T}}, Q; b_{\max})$$

- Banish any explicit non-perturbative calculations here.
- Calculated from operator definitions.
- Allow scale dependence.
- Large contribution all the way to $b_T \approx 1/Q$.

Wanted

$$W(q_\mathrm{T},Q) = \int \frac{\mathrm{d}^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2} e^{i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}}\, \tilde{W}(b_\mathrm{T},Q)$$

$$= \int \frac{\mathrm{d}^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2} e^{i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}}\, \tilde{W}^\mathrm{OPE}(b_*(b_\mathrm{T}),Q)\,\tilde{W}_\mathrm{NP}(b_\mathrm{T},Q;b_\mathrm{max})$$

- Banish any explicit non-perturbative calculations here.
- Calculated from operator definitions.
- Allow scale dependence.
- Large contribution all the way to $b_T \approx 1/Q$.

- Banish any scale dependence here (strictly universal)
- Allow for non-perturbative b_T-dependence (don't necessarily require)
- Power suppression at small b_T

b_{max} method

- Combine b_T argument substitution with evolution.

- Maintains exact operator definitions for all factors and for all b_T. (Including any possible non-perturbative b_T-dependence)

b_{max} method

- Evolution

$$\tilde{W}(b_T, Q) = \tilde{W}(b_T, Q_0) \exp\left\{ \tilde{K}(b_T; \mu_0) \ln\left(\frac{Q^2}{Q_0^2}\right) + \int_{\mu_0}^{\mu_Q} \frac{d\mu'}{\mu'} \left[2\gamma_j(\alpha_s(\mu'); 1) - \ln\frac{Q^2}{(\mu')^2} \gamma_K(\alpha_s(\mu')) \right] \right\}$$

$$\tilde{W}(b_*(b_T), Q) = \tilde{W}(b_*(b_T), Q_0) \exp\left\{ \tilde{K}(b_*(b_T); \mu_0) \ln\left(\frac{Q^2}{Q_0^2}\right) + \int_{\mu_0}^{\mu_Q} \frac{d\mu'}{\mu'} \left[2\gamma_j(\alpha_s(\mu'); 1) - \ln\frac{Q^2}{(\mu')^2} \gamma_K(\alpha_s(\mu')) \right] \right\}$$

- At a reference scale Q_0

$$\frac{\tilde{W}(b_T, Q_0)}{\tilde{W}(b_*(b_T), Q_0)} \equiv \tilde{W}_{NP}(b_T, Q_0; b_{max}) = e^{-g_A(x_A, b_T; b_{max}) - g_B(z_B, b_T; b_{max})}$$

- Evolution

$$\frac{\partial \ln \tilde{W}_j(\boldsymbol{b}_T, Q, x_A, x_B)}{\partial \ln Q^2} = \tilde{K}(b_T; \mu) + b_T \text{ independent parts} \left. \vphantom{\frac{a}{b}} \right\}$$

✓ Exact for all b_T.

b_{max} method

- Evolution

$$\frac{\partial \ln \tilde{W}_j(\boldsymbol{b}_{\rm T}, Q, x_A, x_B)}{\partial \ln Q^2} = \tilde{K}(b_{\rm T}; \mu)$$ + b_T independent parts

These equations are valid for all b_T

- RG for K:

$$\frac{{\rm d}\tilde{K}(b_{\rm T}; \mu)}{{\rm d}\ln \mu} = -\gamma_K(\alpha_s(\mu))$$

- At any scale

$$\tilde{K}(b_{\rm T}; \mu_0; \alpha_s(\mu_0)) = \tilde{K}(b_{\rm T}; \mu_{b_*}; \alpha_s(\mu_{b_*})) - \int_{\mu_{b_*}}^{\mu_0} \frac{d\mu'}{\mu'} \gamma_K(\alpha_s(\mu'))$$

$$\begin{aligned}
\frac{\tilde{W}(b_{\rm T}, Q)}{\tilde{W}(b_*(b_{\rm T}), Q)} &= \frac{\tilde{W}(b_{\rm T}, Q_0)}{\tilde{W}(b_*(b_{\rm T}), Q_0)} e^{-\left[-\tilde{K}(b_{\rm T};\mu_0)+\tilde{K}(b_*(b_{\rm T});\mu_0)\right] \ln\left(\frac{Q^2}{Q_0^2}\right)} \\
&= \frac{\tilde{W}(b_{\rm T}, Q_0)}{\tilde{W}(b_*(b_{\rm T}), Q_0)} e^{-g_K(b_{\rm T};b_{\max}) \ln\left(\frac{Q^2}{Q_0^2}\right)} \\
&= e^{-g_A(x_A,b_{\rm T};b_{\max})-g_B(x_B,b_{\rm T};b_{\max})-2g_K(b_{\rm T};b_{\max}) \ln(Q/Q_0)}
\end{aligned}$$

$\mathbf{b_{max}}$ method

$$W(q_\mathrm{T}, Q) = \int \frac{\mathrm{d}^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2} e^{i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}}\, \tilde{W}(b_\mathrm{T}, Q)$$

$$= \int \frac{\mathrm{d}^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2} e^{i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}} \tilde{W}^\mathrm{OPE}(b_*(b_\mathrm{T}), Q) \tilde{W}_\mathrm{NP}(b_\mathrm{T}, Q; b_\mathrm{max})$$

$$= \int \frac{\mathrm{d}^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2} e^{i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}} \tilde{W}^\mathrm{OPE}(b_*(b_\mathrm{T}), Q) e^{-g_A(x_A, b_\mathrm{T}; b_\mathrm{max}) - g_B(x_B, b_\mathrm{T}; b_\mathrm{max}) - 2g_K(b_\mathrm{T}; b_\mathrm{max}) \ln(Q/Q_0)}$$

- Note that still:

$$\frac{\mathrm{d}}{\mathrm{d}b_\mathrm{max}} W(q_\mathrm{T}, Q) = 0$$

b_{max} method

- The large b_T functions are scale independent. g_K is independent of everything except b_T
- Everything is written in terms of the $W(b_T,Q)$
- Can directly calculate with the definitions,
  - $$\frac{\tilde{W}(b_{\mathrm{T}}, Q)}{\tilde{W}(b_*(b_{\mathrm{T}}), Q)}$$
  - $$g_K(b_{\mathrm{T}}; b_{\max}) \equiv -\tilde{K}(b_{\mathrm{T}}; \mu_0) + \tilde{K}(b_*(b_{\mathrm{T}}); \mu_0)$$

in perturbation theory, if b_T is small.
using non-perturbative theory and
operator defs if b_T is large

The b_{min} mechanism

$$W(q_{\rm T},Q) = \int \frac{{\rm d}^2\boldsymbol{b}_{\rm T}}{(2\pi)^2} e^{i\boldsymbol{q}_{\rm T}\cdot\boldsymbol{b}_{\rm T}}\, \tilde{W}(b_{\rm T},Q)$$

$$\int \frac{{\rm d}^2\boldsymbol{b}_{\rm T}}{(2\pi)^2} e^{i\boldsymbol{q}_{\rm T}\cdot\boldsymbol{b}_{\rm T}}\, \tilde{W}^{\rm OPE}(b_*(b_{\rm T}),Q)\underbrace{\tilde{W}_{\rm NP}(b_{\rm T},Q)}_{e^{-ab_{\rm T}^2}}$$

Ex: ResBos Generator

- Simple ansatz will tend to introduce b_{max} dependence.
- Other options remove b_{max} power corrections, e.g

$$g_K(b_{\rm T};b_{\rm max}) = \frac{C_F}{\pi}\frac{b_{\rm T}^2}{b_{\rm max}^2}\alpha_s(\mu_{b_*}) + O\left(\frac{b_{\rm T}^4 C_F^2 \alpha_s(\mu_{b_*})^2}{b_{\rm max}^4 \pi^2 g_0(b_{\rm max})}\right)$$

J. Collins, T. Rogers: arXiv:1750.07167 (2017)

Summary

- Many calculations now exist in old CSS, pure collinear factorization, SCET.
- Different methods of calculation produce same results.
- Different approaches to derivation converging on standardized TMD definitions.
- Altogether these give necessary ingredients for new operator-based TMD factorization up to to order α_s^3 (except in Wilson coefficient).