Small-x Evolution of Quark Helicity

Matthew D. Sievert

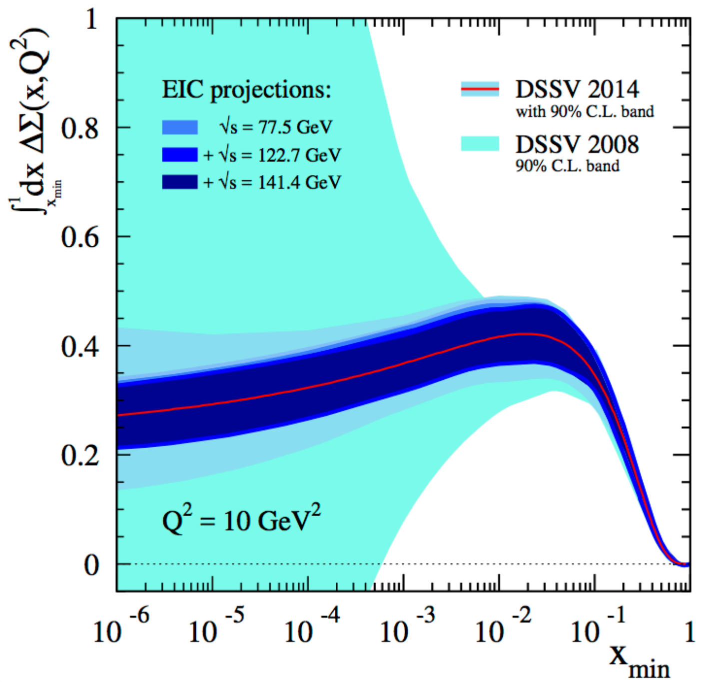
with Yuri Kovchegov

and Daniel Pitonyak

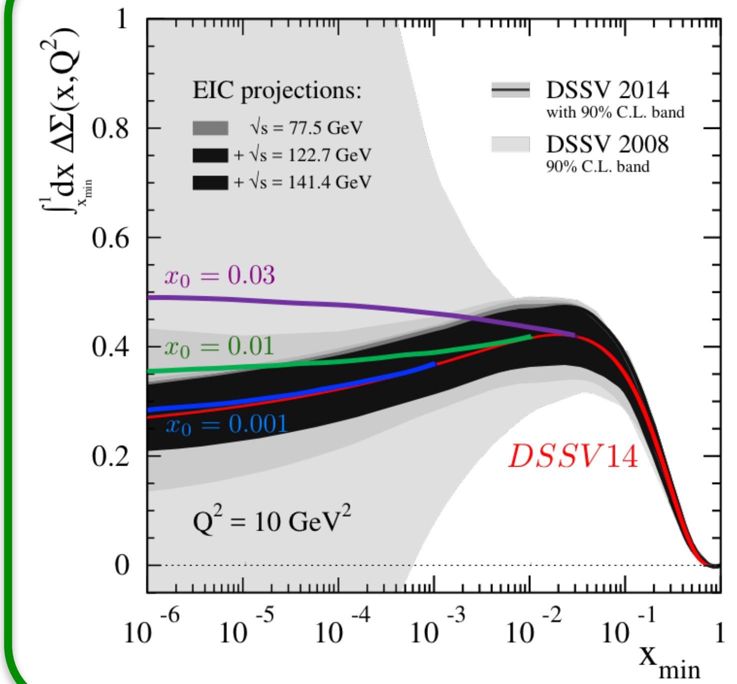
QCD Evolution 2017

Fri. May 26, 2017

Without Small-x Evolution



With Small-x Evolution



adapted from Aschenauer et al., Phys. Rev. ***D92*** *(2015) no.9 094030*

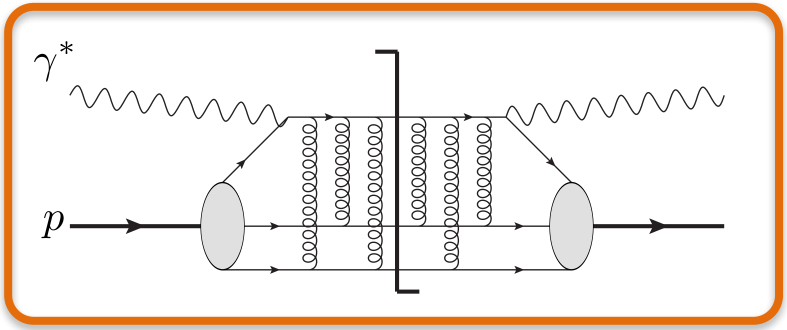
- Small-x helicity PDFs are enhanced by “polarized BFKL evolution”
- Exotic polarized evolution intimately related to DGLAP physics

Small-x Evolution: The Unpolarized Case

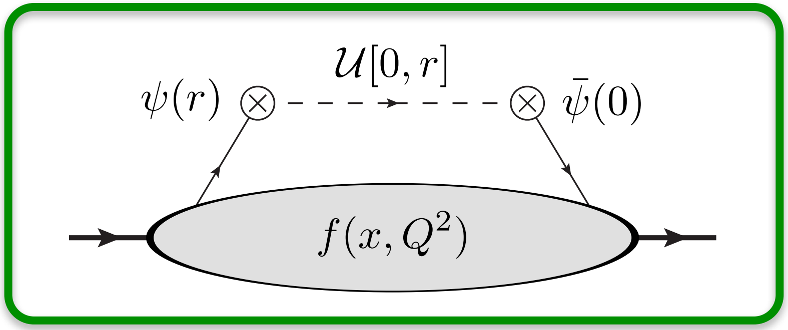
- *Kuraev, Lipatov, and Fadin, Sov. Phys. JETP **45** (1977) 199*
- *Balitsky and Lipatov, Sov. J. Nucl. Phys. **28** (1978) 822*
- *Balitsky, Nucl. Phys. **B463** (1996) 99 [hep-ph/9509348]*
- *Balitsky, Phys. Rev. **D60** (1999) 014020 [hep-ph/9812311]*
- *Kovchegov, Phys. Rev. **D60** (1999) 034008 [hep-ph/9901281]*
- *Kovchegov, Phys. Rev. **D61** (2000) 074018 [hep-ph/9905214]*
- *Jalilian-Marian, Kovner, and Weigert, Phys. Rev. **D59** (1998) 014015 [hep-ph/9709432]*
- *Jalilian-Marian, Kovner, Leonidov, and Weigert, Phys. Rev. **D59** (1998) 014014 [hep-ph/9706377]*
- *Iancu, Leonidov, and McLerran, Phys. Lett. **B510** (2001) 133*
- *Iancu, Leonidov, and McLerran, Nucl. Phys. **A692** (2001) 583 [hep-ph/0011241]*

Factorization at Moderate x

"Knockout" DIS Cross-Section



"Handbag" PDF



$$q(x,Q^2) = \int \frac{dr^-}{2\pi} \, e^{ixp^+r^-} \, \langle p | \, \bar{\psi}(0) \, \mathcal{U}[0,r^-] \, \frac{\gamma^+}{2} \, \psi(r^-) \, | p \rangle$$

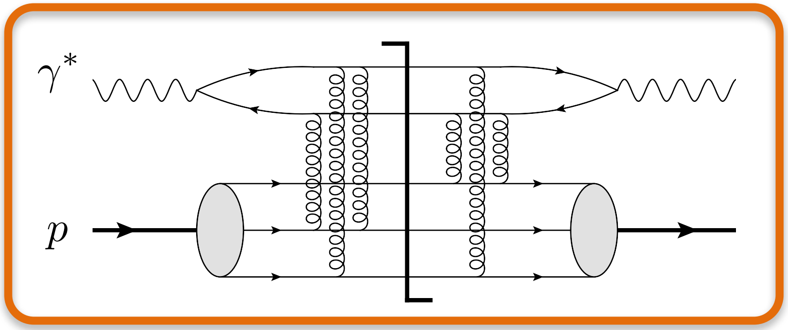
Factorization:

A one-to-one correspondence between the DIS cross-section and PDFs

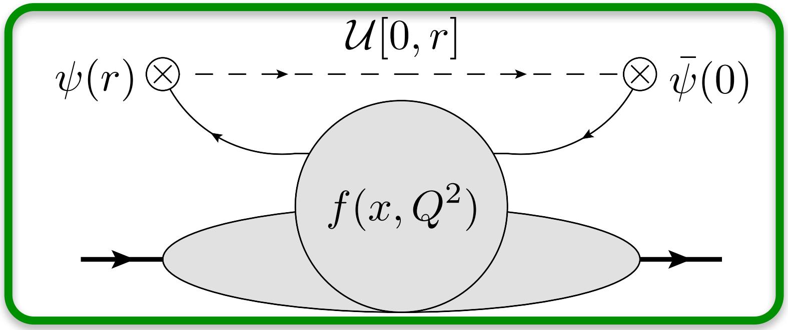
$$\frac{Q^2}{4\pi^2 \alpha_{EM}} \frac{d\sigma^{(\gamma^* p)}}{dx \, dQ^2} = F_2(x,Q^2) \overset{L.O.}{=} \sum_f e_f^2 \, x q_f(x,Q^2)$$

PDFs and Factorization

“Dipole” DIS Cross-Section



“Annihilation” PDF



At small x, the process looks different, but the relationship still holds

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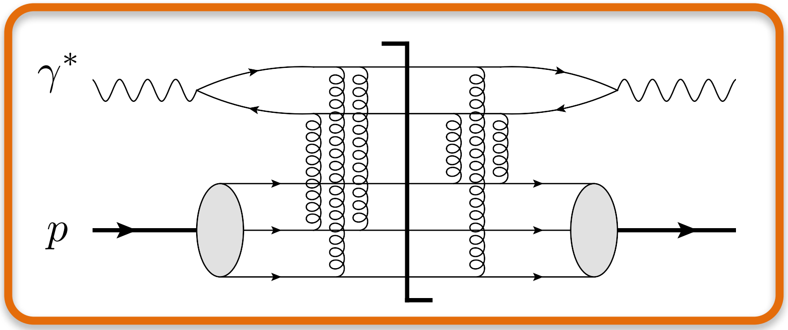
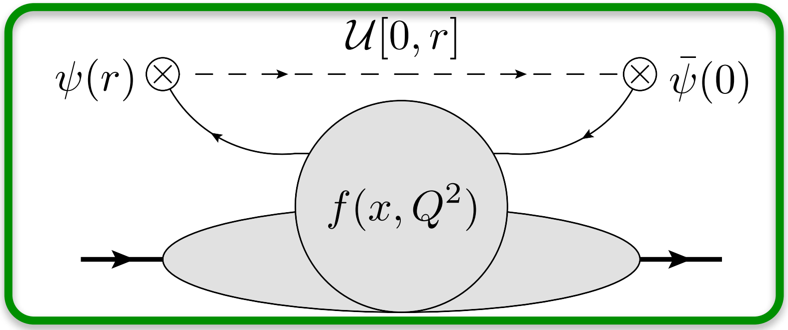


"Annihilation" PDF



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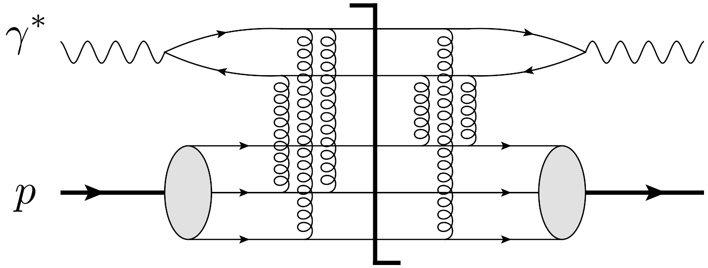
The high-energy limit of the cross-section is the small-x limit of the PDF

$$\frac{1}{x} \approx \frac{s}{Q^2} \qquad \frac{d\sigma^{(\gamma^* p)}}{dx\, dQ^2} \sim \left(\frac{1}{x}\right)^{\alpha_P - 1} \sim x\, q_f(x, Q^2)$$

At small x, the virtual photon fluctuates into a $(q\bar{q})$ dipole



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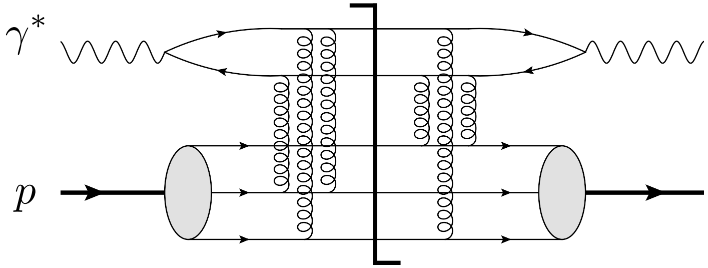

The PDF is expressed in terms of a dipole scattering amplitude

$$x q_f(x, Q^2) \overset{L.O.}{=} \frac{Q^2 N_c}{2\pi^2 \alpha_{EM}} \int \frac{d^2 x_{10}\, dz}{4\pi z(1-z)} \sum_{L,T} \left|\Psi_f(x_{10}^2, z)\right|^2 \int d^2 b_{10}\, (1 - S_{10}(zs))$$

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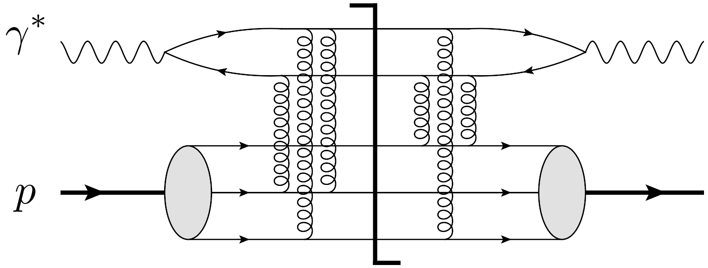

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Dipole S-matrix (cross-section) resums multiple unpolarized scattering

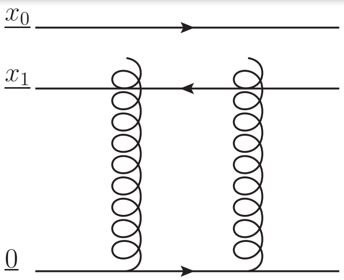
$$V_{\underline{x}} = \mathcal{P} \exp\left[ig \int dz^- \, \hat{A}^+(0^+, z^-, \underline{x}) \right]$$

$$S_{10}(zs) \equiv \left\langle \frac{1}{N_c} \mathrm{tr}[V_{\underline{x}_0} V_{\underline{x}_1}^\dagger]_{(zs)} \right\rangle = 1 - \frac{1}{2} \frac{d\sigma^{(q^{unp}_{\underline{x}_0} \, \bar{q}^{unp}_{\underline{x}_1})}}{d^2 b_{10}}(zs)$$

Energy Dependence of the Dipole Amplitude

Initial conditions from, e.g., quark target model.

$$S_{10}^{(0)}(zs) = \frac{2\alpha_s^2 C_F}{N_c} \ln^2 \frac{x_{0T}}{x_{1T}}$$



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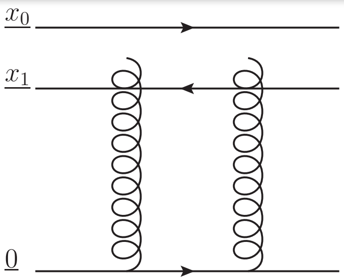
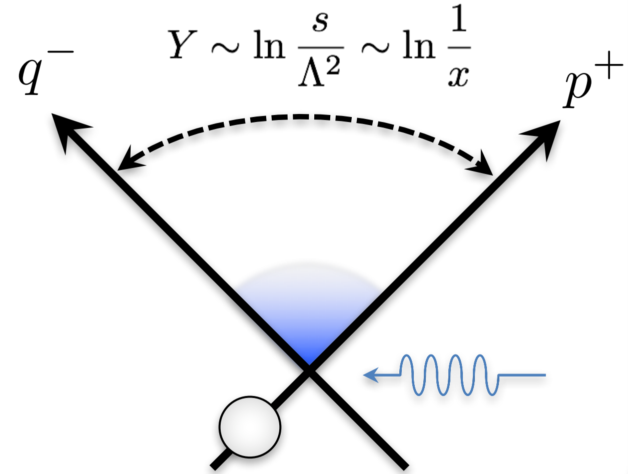


Soft gluons are radiated uniformly over the full rapidity interval



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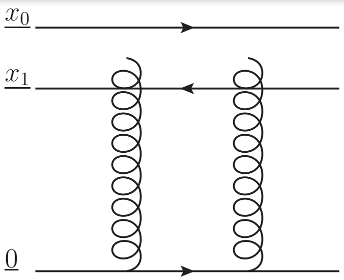

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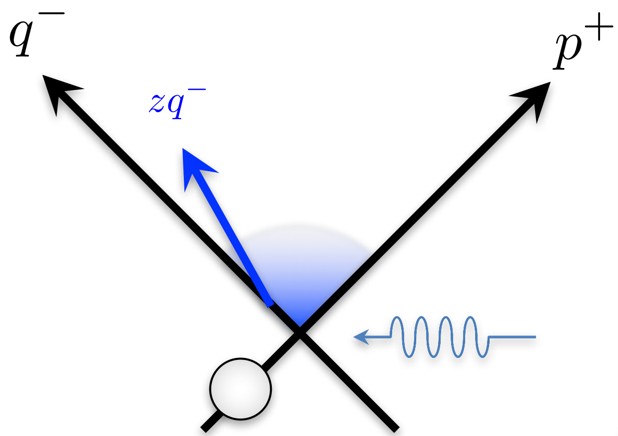

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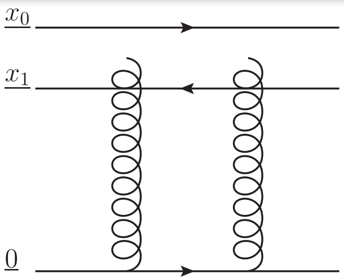

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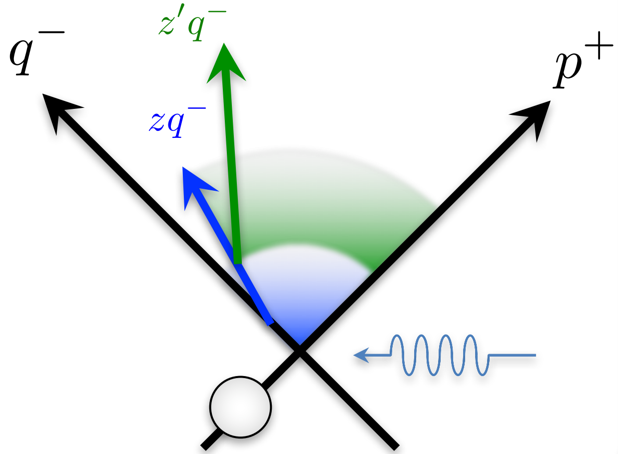

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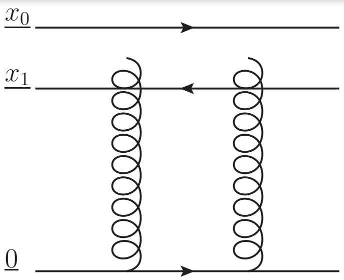
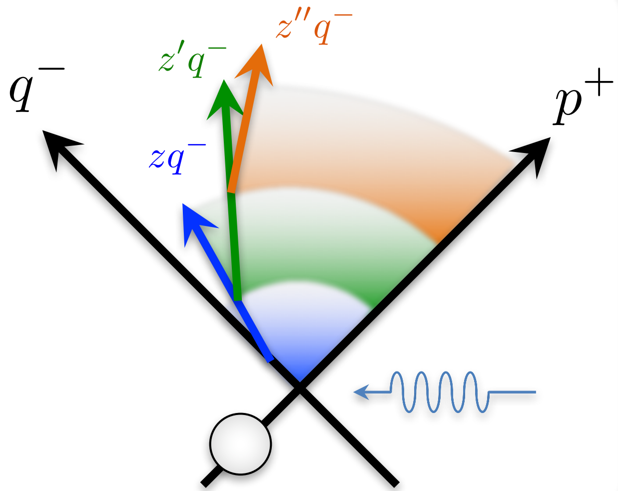


Soft gluons are radiated uniformly over the full rapidity interval

Successive emissions with longitudinal ordering are systematically enhanced

$$1 \gg z \gg z' \gg z'' \gg \cdots$$



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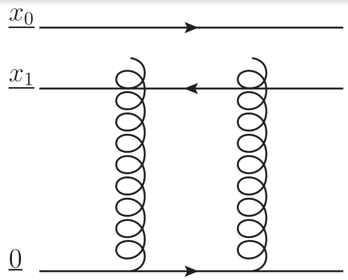

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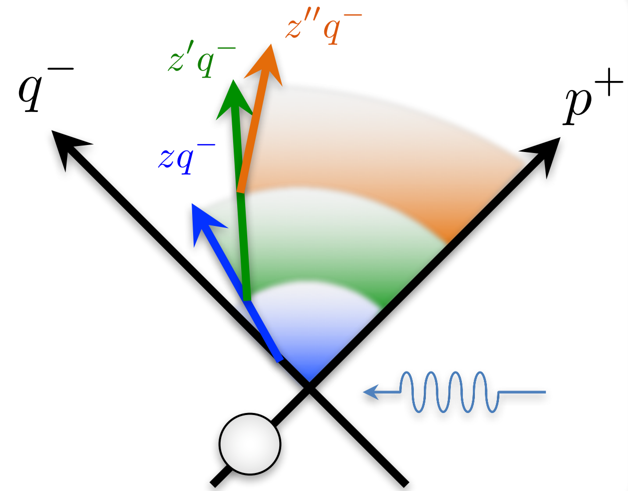


Leading-log resummation of the unpolarized gluon radiation drives the high-energy limit

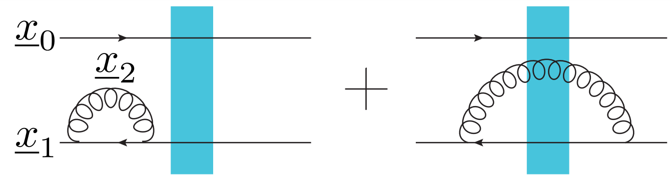
$$\alpha_s \ln \frac{1}{x} \sim 1$$

Unpolarized BFKL / B-JIMWLK Evolution

$$S_{10}(zs) = S_{10}^{(0)}(zs) + \frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int d^2 x_2 \left(\frac{1}{x_{21}^2} - 2 \frac{\underline{x}_{21} \cdot \underline{x}_{20}}{x_{21}^2 \, x_{20}^2} + \frac{1}{x_{20}^2} \right)$$

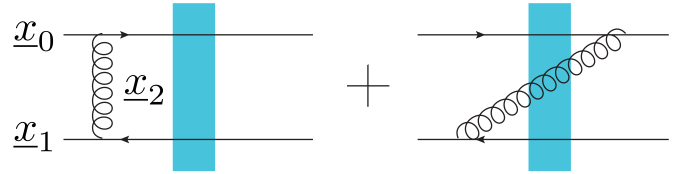
$$\times \left[\frac{1}{N_c^2} \left\langle \mathrm{tr}[V_{\underline{x}_2} V_{\underline{x}_1}^\dagger] \, \mathrm{tr}[V_{\underline{x}_0} V_{\underline{x}_2}^\dagger] \right\rangle_{(z's)} - S_{10}(z's) \right]$$

Ladder emissions from line 1



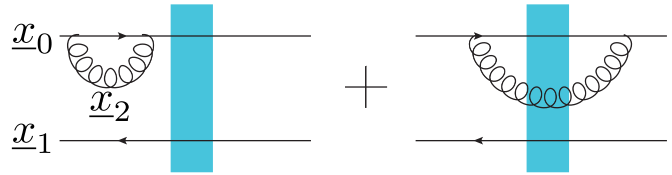
+ "c.c."

Non-ladder emissions



+ "c.c."

Ladder emissions from line 2



+ "c.c."

Unpolarized BFKL / B-JIMWLK Evolution

$$S_{10}(zs) = S_{10}^{(0)}(zs) + \frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int d^2 x_2 \, \frac{x_{01}^2}{x_{21}^2 \, x_{20}^2}$$

$$\times \left[\frac{1}{N_c^2} \left\langle \text{tr}[V_{\underline{x}_2} V_{\underline{x}_1}^\dagger] \, \text{tr}[V_{\underline{x}_0} V_{\underline{x}_2}^\dagger] \right\rangle_{(z's)} - S_{10}(z's) \right]$$

BFKL kernel

Unpolarized BFKL / B-JIMWLK Evolution

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Longitudinal “soft” logarithm

BFKL kernel

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Longitudinal “soft” logarithm

Operator hierarchy

BFKL kernel

Unpolarized Small-x Asymptotics

Operator hierarchy closes in the large-Nc limit:

$$S_{10}(zs) = S_{10}^{(0)}(zs) + \frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int d^2x_2 \, \frac{x_{10}^2}{x_{12}^2 \, x_{20}^2} \, [S_{12}(z's) \, S_{20}(z's) - S_{10}(z's)]$$

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BFKL: The dilute limit $1 - S_{10} \ll 1$

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Solve by Laplace/Mellin transform: the Pomeron intercept

$$x \, q_f(x, Q^2) \sim S_{10}\left(s = \frac{Q^2}{x}\right) \sim \left(\frac{1}{x}\right)^{\alpha_P - 1} \qquad \alpha_P - 1 = \frac{4\alpha_s N_c}{\pi} \ln 2$$

Small-x Evolution: The Quark Helicity Case

- *Kirschner and Lipatov, Nucl.Phys.* ***B213*** *(1983) 122*
- *Kirschner, Z.Phys.* ***C65*** *(1995) 505 [hep-th/9407085]*
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- *Bartels, Ermolaev, and Ryskin, Z.Phys.* ***C70*** *(1996) 273 [hep-ph/9507271]*
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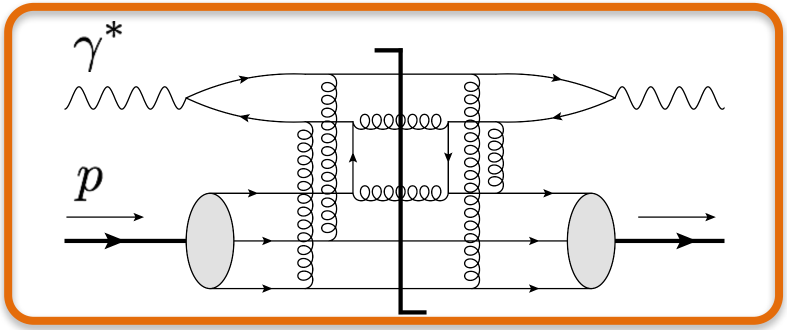
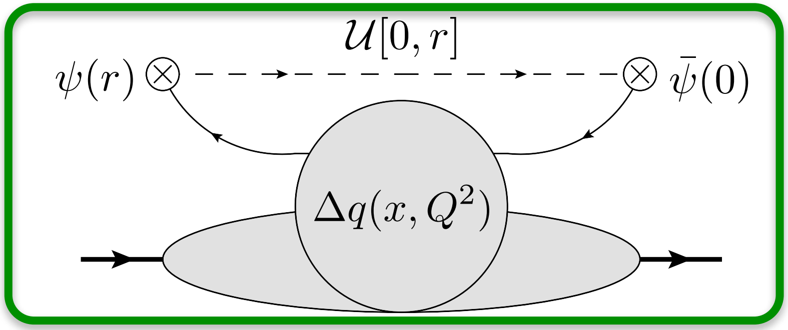
- *Kovchegov, Pitonyak, and M. S., Phys. Rev.* ***D95*** *(2017) 014033 [arXiv:1610.0619]*

Polarized DIS and Quark Helicity

DIS Longitudinal Spin Asymmetry



Quark Helicity PDF



$$\Delta q^S(x,Q^2) = \sum_f \int \frac{dr^-}{2\pi} e^{ixp^+r^-} \langle p|\, \bar{\psi}(0)\, \mathcal{U}[0,r^-]\, \frac{\gamma^+\gamma^5}{2}\, \psi(r^-)\, |p\rangle$$

Factorization:

$$\frac{Q^2}{4\pi^2\alpha_{EM}} \frac{d\,\Delta\sigma^{(\gamma^*\,p)}}{dx\,dQ^2} = 2x\, g_1(x,Q^2) \overset{L.O.}{=} \sum_f e_f^2\, x\Delta q_f(x,Q^2)$$

What is the "quark helicity intercept" at small x?

$$\frac{1}{x} \approx \frac{s}{Q^2}$$

$$\frac{d\,\Delta\sigma^{(\gamma^*\,p)}}{dx\,dQ^2} \sim \left(\frac{1}{x}\right)^{\alpha_h^q - 1} \sim x\,\Delta q(x,Q^2)$$

What Makes Helicity Special?

Helicity sensitivity is power suppressed at small x *(scale it out)*

$$V_{\underline{x}} = \mathcal{P} \exp \left[ig \int dz^- \, \hat{A}^+(0^+, z^-, \underline{x}) \right]$$

$$\frac{d\ \Delta\sigma}{d^2 b}^{Born} \sim x \sim \frac{1}{s}$$

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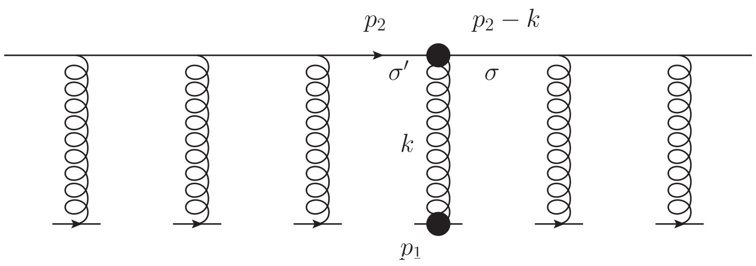
Exactly one spin-dependent scattering dominates at high energy



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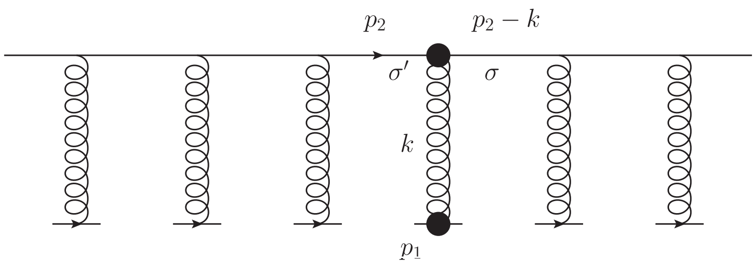
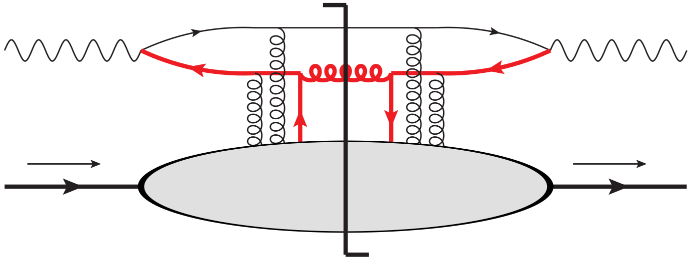


Especially sensitive to fluctuations about the distinct polarized line.



Polarized Dipoles and Wilson Lines

Helicity PDF is expressed in terms of a polarized dipole amplitude

$$x\,\Delta q^S(x,Q^2) \overset{L.O.}{=} \frac{Q^2\,N_c}{2\pi^2\,\alpha_{EM}} \sum_f \int \frac{d^2x_{10}\,dz}{4\pi\,z(1-z)} \sum_{L,T} \left|\Delta\Psi_f(x_{10}^2,z)\right|^2 \int d^2b_{10}\,\left[\frac{1}{zs} G_{10}(zs)\right]$$

Polarized Dipoles and Wilson Lines

Helicity PDF is expressed in terms of a polarized dipole amplitude

$$x\,\Delta q^S(x, Q^2) \overset{L.O.}{=} \frac{Q^2\,N_c}{2\pi^2\,\alpha_{EM}} \sum_f \int \frac{d^2 x_{10}\,dz}{4\pi\,z(1-z)} \sum_{L,T} \left|\Delta\Psi_f(x_{10}^2, z)\right|^2 \int d^2 b_{10} \left[\frac{1}{zs} G_{10}(zs)\right]$$

Polarized dipole: spin-dependent dipole S-matrix (cross-section)

$$\frac{1}{zs} G_{10}(zs) \equiv \left\langle \frac{1}{2N_c} \mathrm{tr}[V_{\underline{x}_0} V_{\underline{x}_1}^{pol\,\dagger}] + c.c. \right\rangle_{(zs)} = -\frac{1}{4} \left(\frac{d\,\Delta\sigma^{(q_{\underline{x}_0}^{unp}\,\bar{q}_{\underline{x}_1}^{pol})}}{d^2 b_{10}}(zs) + ch.c. \right)$$

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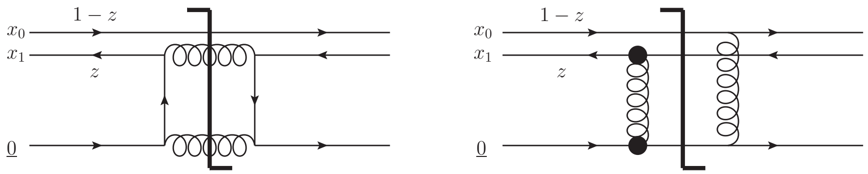
Formally define a polarized Wilson line; for gluon exchange only:

$$(V_{\underline{x}}^{pol})^g = \frac{1}{s}\int_{-\infty}^{+\infty} dx^-\,V_{\underline{x}}[+\infty,x^-]\left(\left(-ig\,p^+\right)\epsilon_T^{ij}\,\frac{\partial}{\partial x_\perp^i}A_\perp^j(0^+,x^-,\underline{x})\right)V_{\underline{x}}[x^-,-\infty]$$

Initial conditions from, e.g., quark target model

$$G_{10}^{(0)}(zs) = \frac{\alpha_s^2 C_F}{N_c}\left[\frac{C_F}{x_{1T}^2} - 2\pi\delta^2(\underline{x}_1)\,\ln(zs\,x_{10}^2)\right]$$



+ perm's

Origins of Helicity Evolution

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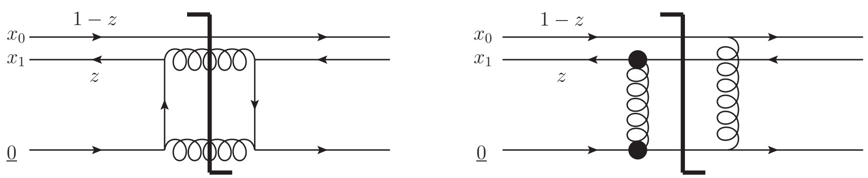

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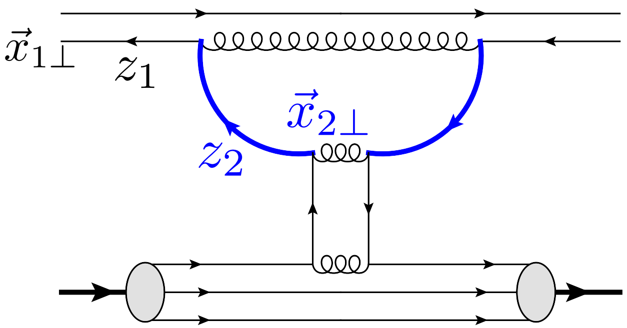

$$\frac{1}{z_1 s}G_{10}(z_1 s) \sim \int\limits_{\frac{\Lambda^2}{s}}^{z_1}\frac{dz_2}{z_2}\int d^2x_{21}\left(\frac{\alpha_s C_F}{2\pi^2}\,\frac{z_2}{z_1}\,\frac{1}{x_{21}^2}\right)\frac{1}{z_2 s}G_{21}(z_2 s)$$

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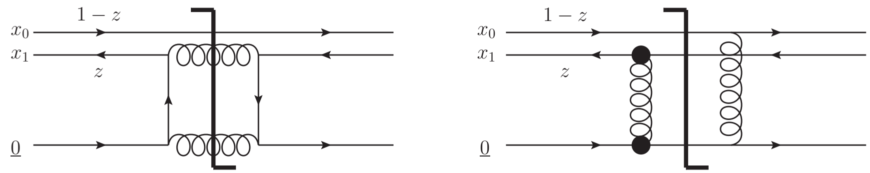

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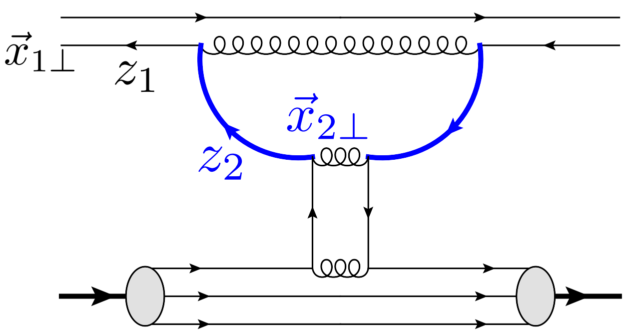

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$$G_{10}(z_1 s) \sim \frac{\alpha_s C_F}{2\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z_1} \frac{dz_2}{z_2} \int\limits_{\frac{1}{z_2 s}}^{x_{10}^2 \frac{z_1}{z_2}} \frac{dx_{21}^2}{x_{21}^2} G_{21}(z_2 s)$$

Double logs: $\ln^2 \frac{z_1 s}{\Lambda^2}$ soft + (anti)collinear

Origins of Helicity Evolution

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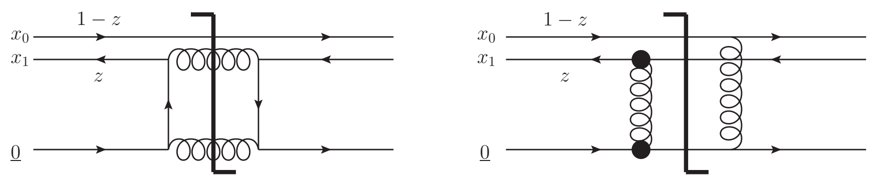

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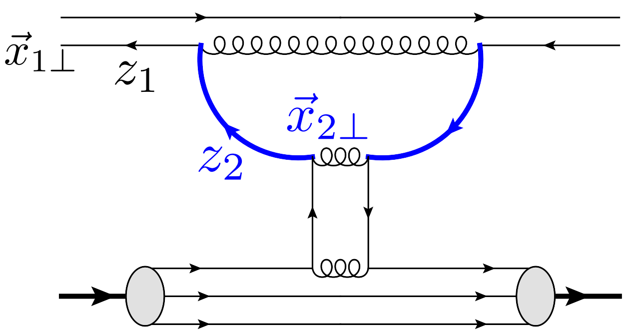

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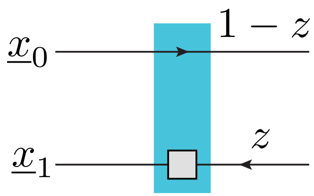
Double logs: $\ln^2\frac{z_1 s}{\Lambda^2}$ soft + (anti)collinear

Polarized splittings are sensitive to an additional transverse log

$$\alpha_s \ln^2\frac{1}{x} \sim 1$$

Example: the (Anti)Collinear BFKL Sector

$$G_{10}(zs) \sim$$



Consider unpolarized BFKL-type corrections:

$$\int d^2x_2 \left(\frac{1}{x_{21}^2} - 2\frac{\underline{x}_{21}\cdot\underline{x}_{20}}{x_{21}^2\, x_{20}^2} + \frac{1}{x_{20}^2} \right) \left[\frac{1}{N_c^2} \left\langle\!\!\left\langle \mathrm{tr}[V_{\underline{x}_2} V_{\underline{x}_1}^{pol\,\dagger}]\,\mathrm{tr}[V_{\underline{x}_0} V_{\underline{x}_2}^{\dagger}] \right\rangle\!\!\right\rangle_{(z's)} - \frac{1}{N_c} \left\langle\!\!\left\langle \mathrm{tr}[V_{\underline{x}_0} V_{\underline{x}_1}^{pol\,\dagger}] \right\rangle\!\!\right\rangle_{(z's)} \right]$$

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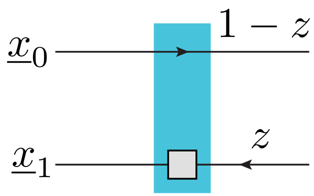

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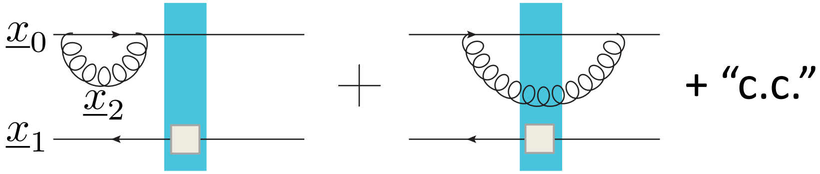
(Anti)collinear log as $\underline{x}_2 \to \underline{x}_0$



… but real/virtual cancellation.

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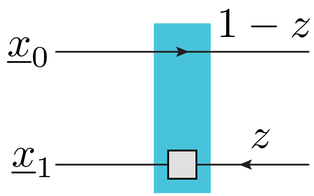

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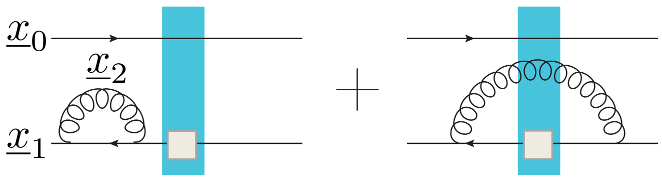
(Anti)collinear log as $\underline{x}_2 \to \underline{x}_1$



+ "c.c."

Survives for polarized line!

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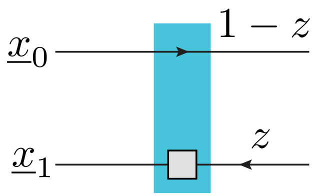
$G_{10}(zs) \sim$ 

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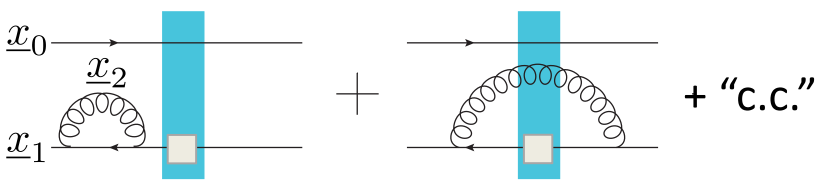

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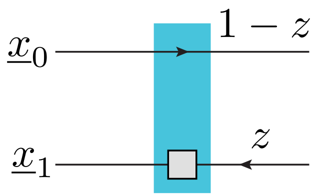

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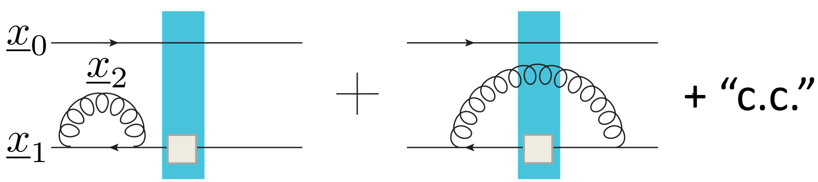
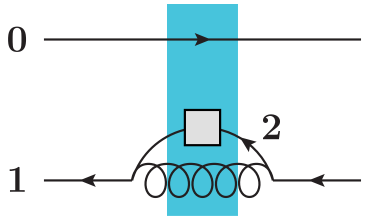


Survives for polarized line!

Emissions about the polarized line acquire an anti-collinear logarithm and become double-logarithmic

Soft polarized quark emission



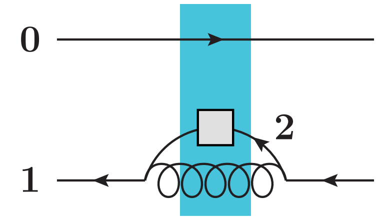
- Double logarithmic up to lifetime ordering

$$\delta G_{10}(zs) = \frac{\alpha_s N_c}{4\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z's}}^{x_{10}^2 \frac{z}{z'}} \frac{d^2 x_{21}}{x_{21}^2} \left[\frac{1}{N_c^2} \left\langle\!\!\left\langle \mathrm{tr}[V_{\underline{x}_1} V_{\underline{x}_2}^{pol\,\dagger}]\, \mathrm{tr}[V_{\underline{x}_0} V_{\underline{x}_1}^{\dagger}] \right\rangle\!\!\right\rangle_{(z's)} - \frac{1}{N_c^3} \left\langle\!\!\left\langle \mathrm{tr}[V_{\underline{x}_0} V_{\underline{x}_2}^{pol\,\dagger}] \right\rangle\!\!\right\rangle_{(z's)} \right]$$

Polarized Soft Parton Radiation

Soft polarized quark emission

- Double logarithmic up to lifetime ordering



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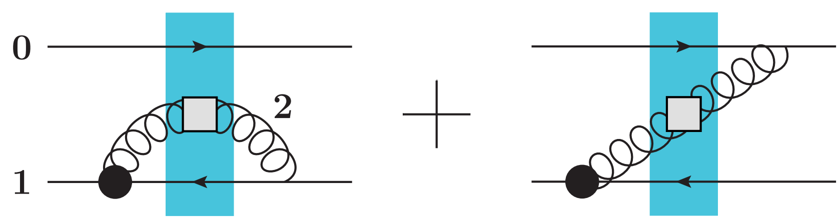
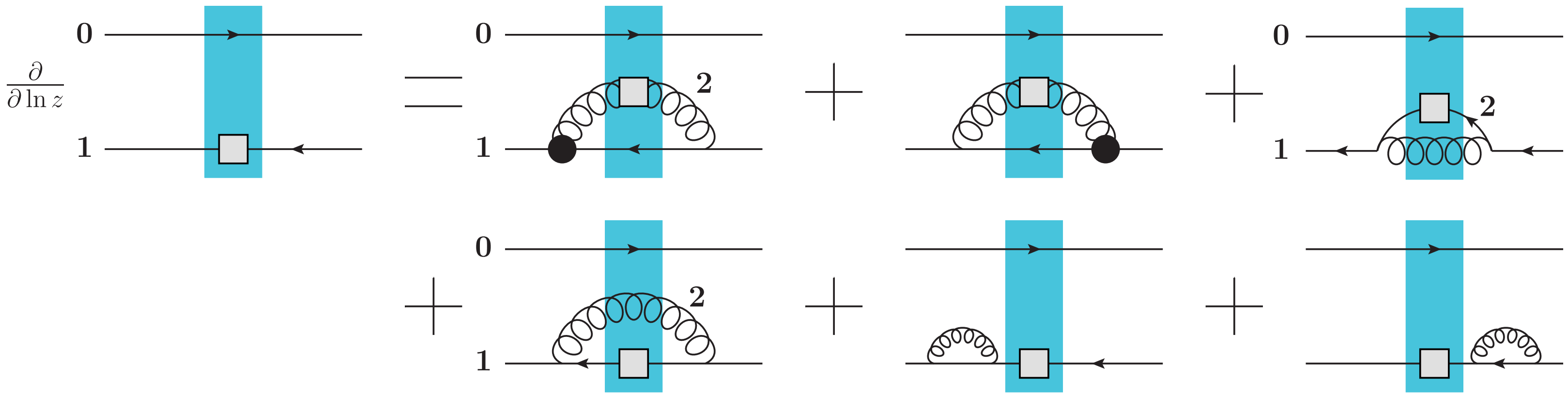
Soft polarized gluon emission

- Ladder + Non-ladder



$$\delta G_{10}(zs) = + \frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z's}}^{x_{10}^2} \frac{d^2 x_{21}}{x_{21}^2} \left[\frac{1}{N_c^2} \left\langle\!\!\left\langle \mathrm{tr}[t^b V_{\underline{x}_0} t^a V_{\underline{x}_1}^{\dagger}] \left(U_{\underline{x}_2}^{pol} \right)^{ba} \right\rangle\!\!\right\rangle_{(z's)} \right]$$

One Step of Helicity Evolution



Soft gluon radiation: Only anticollinear region is double logarithmic

$$x_{21}^2 \ll x_{10}^2$$

- Soft quark splitting includes IR region

$$x_{10}^2 \leq x_{21}^2 \ll x_{10}^2 \frac{z}{z'}$$

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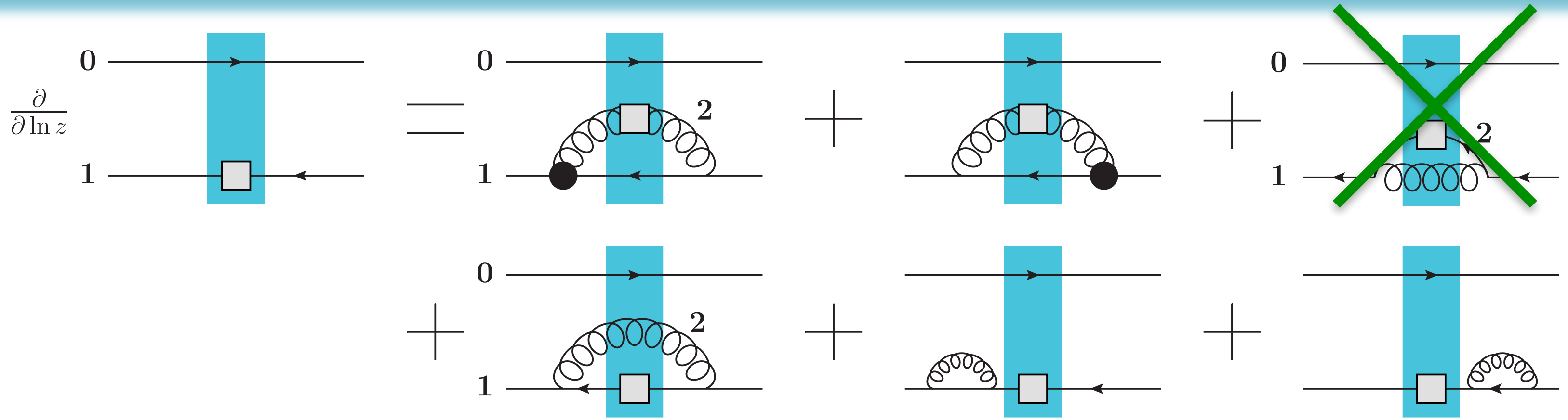

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Operator hierarchy closes in the large-Nc limit *(also large-Nc, Nf)*

The strict double-log regime is linearized *("polarized BFKL")*

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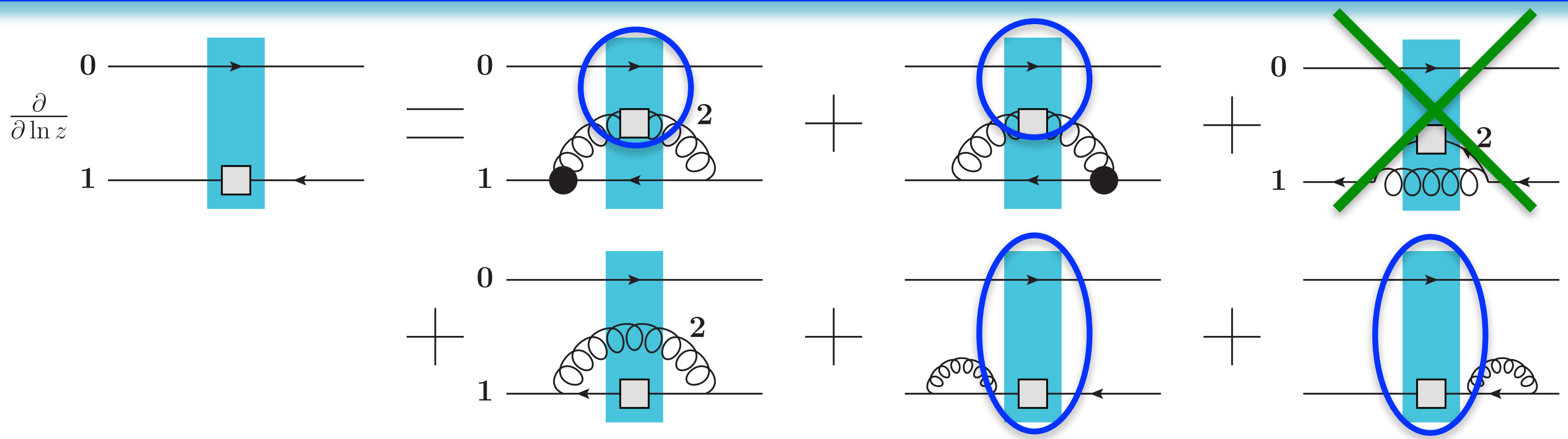

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Operator hierarchy closes in the large-Nc limit *(also large-Nc, Nf)*

The strict double-log regime is linearized *("polarized BFKL")*

The lifetime of large dipoles may be more constrained by smaller "neighbor dipoles" than the anticollinear phase space

Large-Nc Evolution Equations

$$G(x_{10}^2, z) = G^{(0)}(x_{10}^2, z) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{x_{10}^2 s}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z' s}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left[\Gamma(x_{10}^2, x_{21}^2, z') + 3G(x_{21}^2, z')\right]$$

$$\Gamma(x_{10}^2, x_{21}^2, z') = G^{(0)}(x_{10}^2, z') + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{x_{10}^2 s}}^{z'} \frac{dz''}{z''} \int\limits_{\frac{1}{z'' s}}^{\min[x_{10}^2, x_{21}^2 \frac{z'}{z''}]} \frac{dx_{32}^2}{x_{32}^2} \left[\Gamma(x_{10}^2, x_{32}^2, z'') + 3G(x_{32}^2, z'')\right]$$

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Initial Conditions

Large-Nc Evolution Equations

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Neighbor dipole amplitude

Large-Nc Evolution Equations

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Need to solve these coupled equations for the high-energy asymptotics

$$G(x_{10}^2, zs) \sim (zs)^{\alpha_h}$$

Solution and Implications

- *Kovchegov, Pitonyak, and M. S., Phys. Rev. Lett.* ***118*** *(2017) 052001 [arXiv:1610.0618]*
- *Kovchegov, Pitonyak, and M. S., accepted to Phys. Lett. B (2017) [arXiv:1703.0580]*

Choosing the Right Variables

Choose logarithmic variables *(scale out the coupling)*

$$s_{ij} \equiv \sqrt{\frac{\alpha_s N_c}{2\pi}} \ln \frac{1}{x_{ij}^2 \Lambda^2} \qquad \eta^{(\prime,\prime\prime)} \equiv \sqrt{\frac{\alpha_s N_c}{2\pi}} \ln \frac{z^{(\prime,\prime\prime)}}{\Lambda^2/s}$$

$$G(s_{10},\eta) = G^{(0)}(s_{10},\eta) + \int\limits_{s_{10}}^{\eta} d\eta' \int\limits_{s_{10}}^{\eta'} ds_{21} [\Gamma(s_{10},s_{21},\eta') + 3G(s_{21},\eta')]$$

$$\Gamma(s_{10},s_{21},\eta') = G^{(0)}(s_{10},\eta') + \int\limits_{s_{10}}^{\eta'} d\eta'' \int\limits_{\max[s_{10},s_{21}+\eta''-\eta']}^{\eta''} ds_{32} [\Gamma(s_{10},s_{32},\eta'') + 3G(s_{32},\eta'')]$$

Numerics: The Brute Force Method

Discretize on a grid and solve iteratively

$$G_{ij} = G_{ij}^{(0)} + \Delta\eta^2 \sum_{j'=i}^{j-1} \sum_{i'=i}^{j'} [\Gamma_{ii'j'} + 3G_{i'j'}]$$

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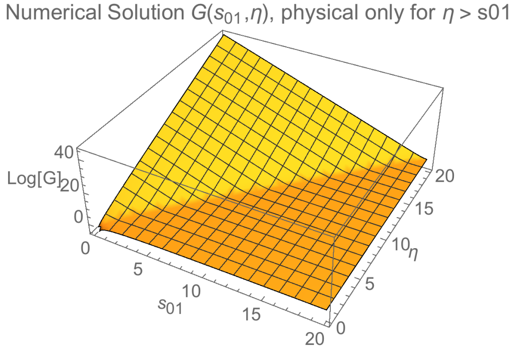
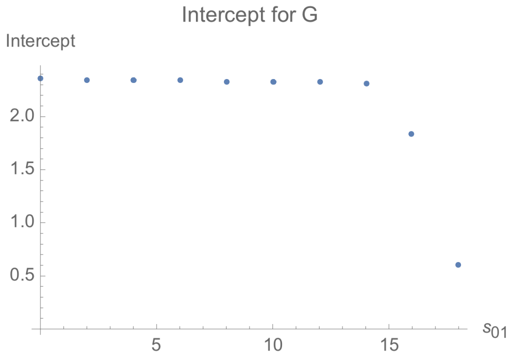
Evolve to exponential asymptotics and fit the helicity intercept





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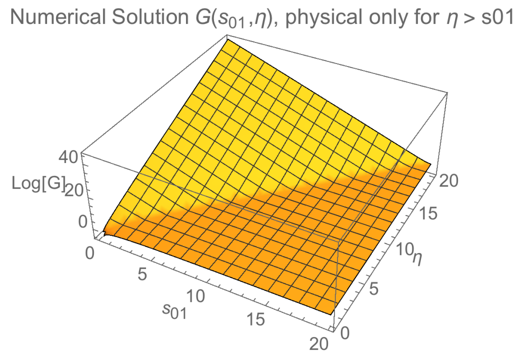
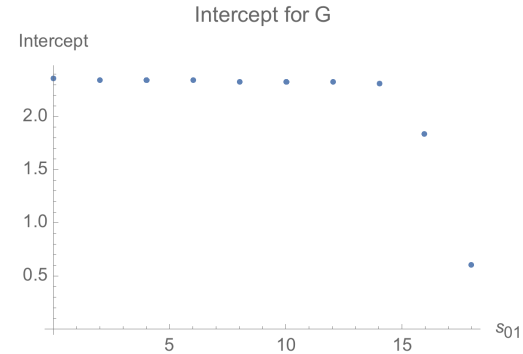




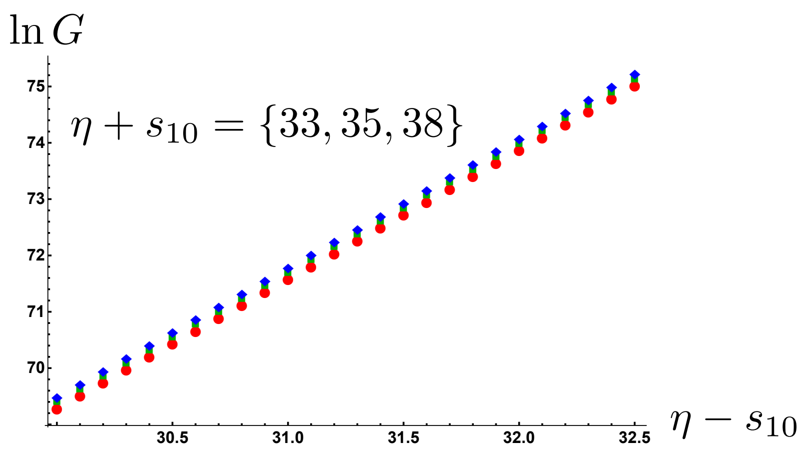
Emergent Features:

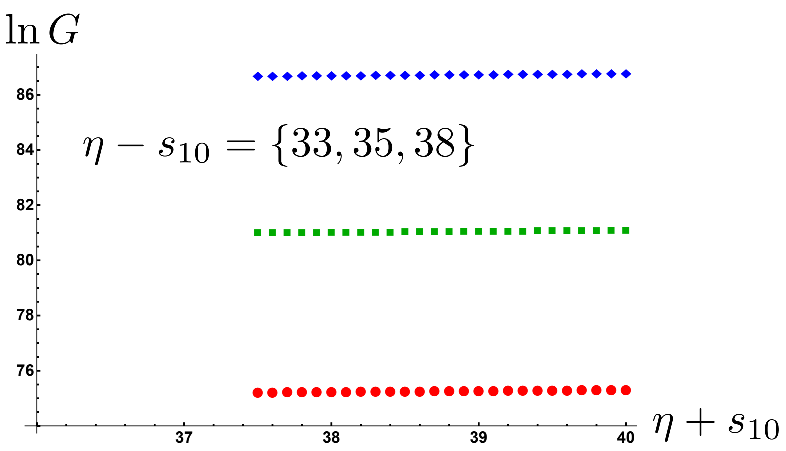
- Scaling behavior
- Insensitive to initial conditions

$$G(s_{10}, \eta) = G(\eta - s_{10})$$

$$\Gamma(s_{10}, s_{21}, \eta) = \Gamma(\eta - s_{10}, \eta - s_{21})$$

Choosing Better Variables





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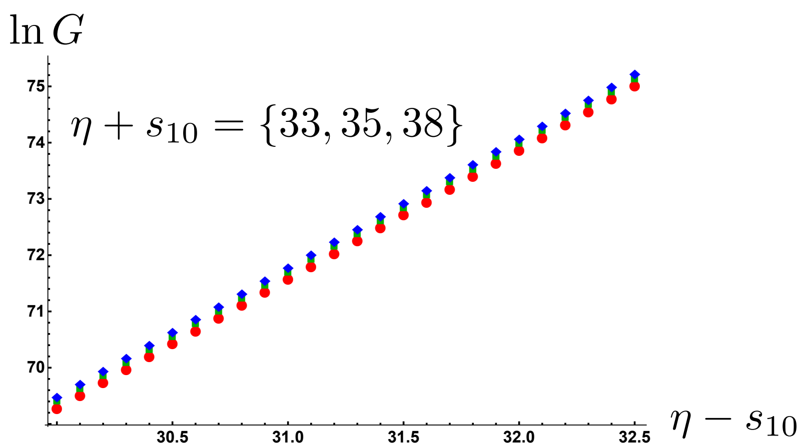

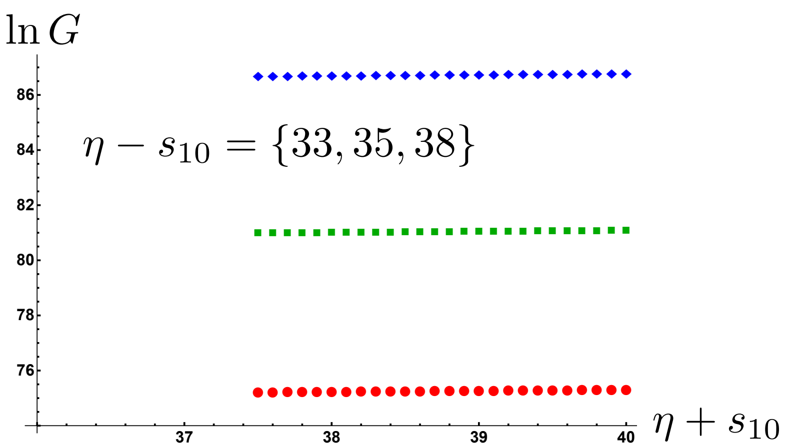

- Define scaling variables
- Neglect initial conditions
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$$\zeta \equiv \eta - s_{10} = \sqrt{\frac{\alpha_s N_c}{2\pi}} \ln(zs\, x_{10}^2)$$

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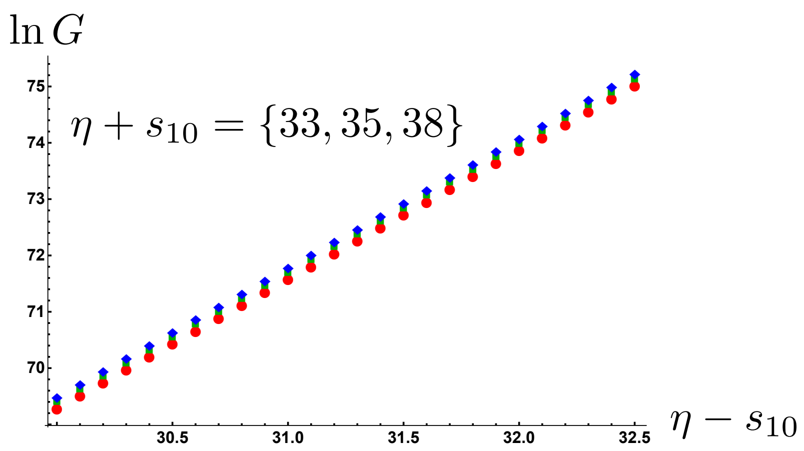

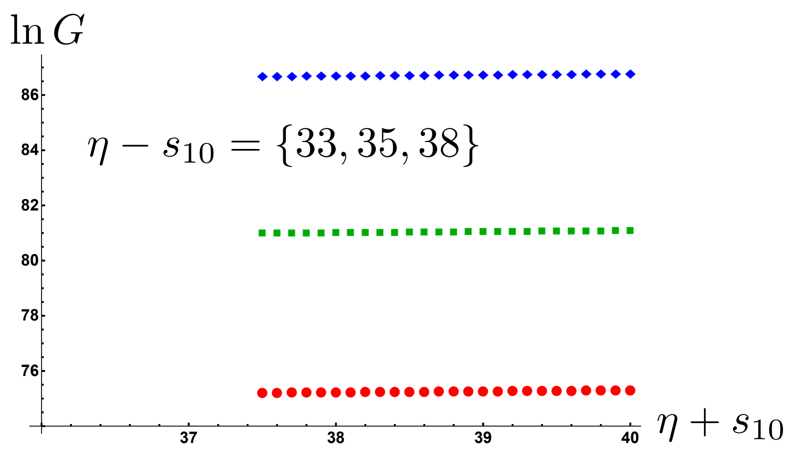

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$$\frac{\partial}{\partial \zeta} G(\zeta) = \int_0^{\zeta} d\xi' \left[\Gamma(\zeta, \xi') + 3G(\xi')\right] \qquad \frac{\partial}{\partial \zeta} \Gamma(\zeta, \zeta') = \int_0^{\zeta'} d\xi' \left[\Gamma(\zeta, \xi') + 3G(\xi')\right]$$

Analytics: The Elegant Method

Solve differential equations by Laplace-Mellin transform

$$G(\zeta) = \frac{1}{4} \int \frac{d\omega}{2\pi i} e^{(\omega + \frac{1}{\omega})\zeta} H_\omega$$

$$\Gamma(\zeta, \zeta') = \int \frac{d\omega}{2\pi i} \left[e^{\omega \zeta' + \frac{1}{\omega}\zeta} - \frac{3}{4} e^{(\omega + \frac{1}{\omega})\zeta'} \right] H_\omega$$

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Back-substitute into the differential equations to determine the poles

$$H_\omega = 4 \, \frac{\omega^2 - 1}{\omega \left(\omega^2 - 3 \right)}$$

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High-energy asymptotics are dominated by the rightmost pole

$$G(\zeta) = \int \frac{d\omega}{2\pi i} e^{(\omega + \frac{1}{\omega})\zeta} \frac{\omega^2 - 1}{\omega(\omega^2 - 3)}$$

$$\omega \to +\sqrt{3}$$

Solution: Numerical vs. Analytic

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$$\ln \left[\frac{\Gamma(s_{10}, s_{21}, \eta)}{G(s_{21}, \eta)} + 3 \right] = \ln 4 + \frac{1}{\sqrt{3}} (s_{21} - s_{10}) \approx (1.386) + (0.577)(s_{21} - s_{10})$$

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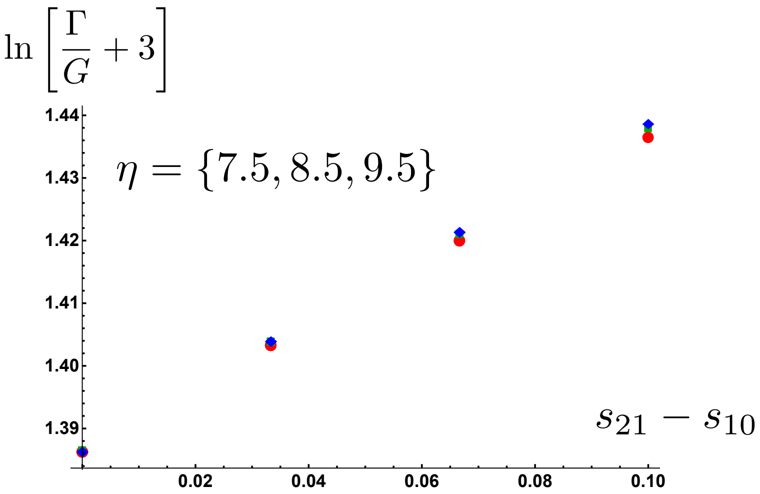

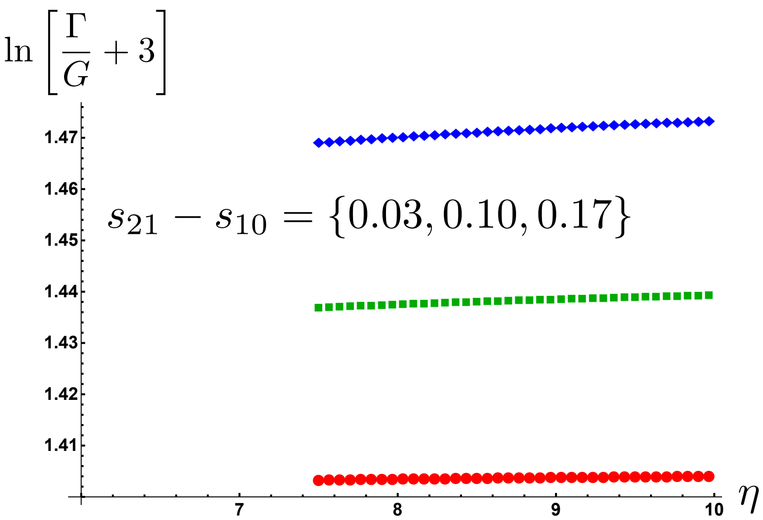

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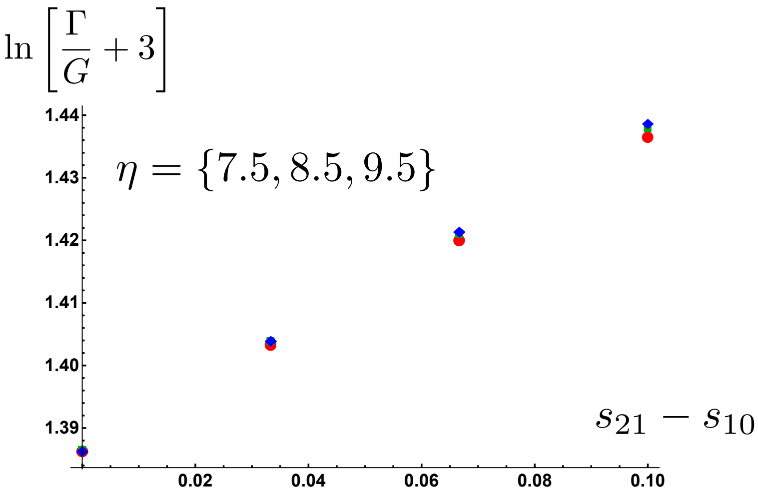

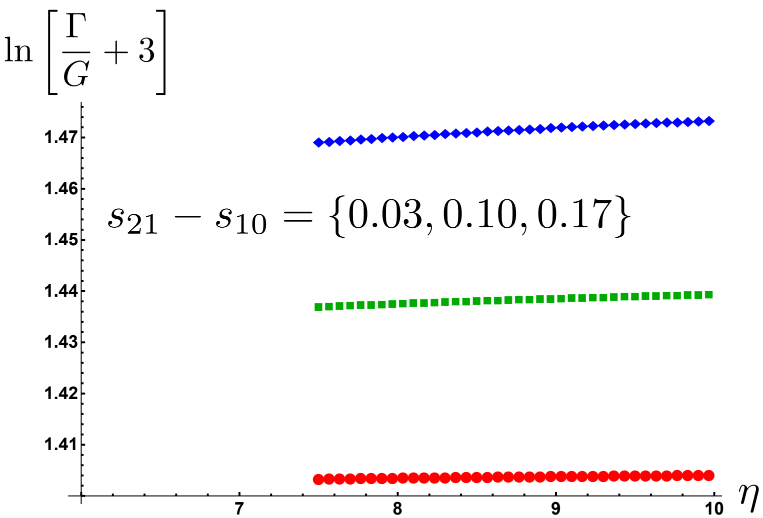

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$$\ln\left[\frac{\Gamma(s_{10}, s_{21}, \eta)}{G(s_{21}, \eta)} + 3\right] \approx (1.384) + (0.517)(s_{21} - s_{10})$$

Theoretical Implications

Quark helicity evolution is sensitive to the anticollinear DGLAP region

$$x_{10}^2 \gg x_{21}^2 \gg x_{32}^2 \gg \cdots$$

$$G(x_{10}^2, z) = G^{(0)}(x_{10}^2, z) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{x_{10}^2 s}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z' s}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left[\Gamma(x_{10}^2, x_{21}^2, z') + 3G(x_{21}^2, z')\right]$$

Double logs can reproduce the DGLAP NLO anomalous dimensions

$$\gamma_{S,GG}^{NLO}(\omega)\Big|_{\text{pure glue},\omega\to 0} = \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{8N_c^2}{\omega^3}$$

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Speculation: onset of scaling reflects convergence to the collinear limit…?

$$zs > \frac{1}{x_{10}^2} e^{\hat{\zeta}_0} \qquad \hat{\zeta}_0 \approx (1 \div 2)\sqrt{\frac{2\pi}{\alpha_s N_c}}$$

Phenomenological Implications

Robust QCD prediction for the small-x tail of the quark helicity PDF!

$$\Delta q^S(x, Q^2) \sim \left(\frac{1}{x}\right)^{\alpha_h^q} \qquad \alpha_h^q = \frac{4}{\sqrt{3}}\sqrt{\frac{\alpha_s N_c}{2\pi}}$$

Q^2	3 GeV^2	10 GeV^2
α_s (Q^2)	0.343	0.249
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Generally integrable as x → 0: guide for extrapolation of ΔΣ

$$\Delta\Sigma(Q^2) = \int_0^{x_{min}} dx\, \Delta q^S(x, Q^2) + \int_{x_{min}}^1 dx\, \Delta q^S(x, Q^2)$$

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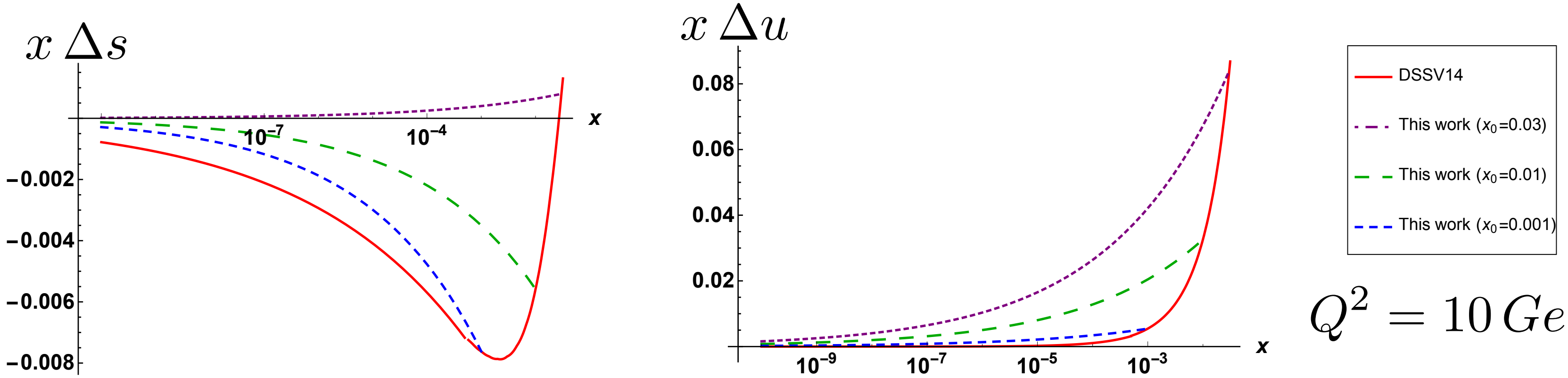
Small-x asymptotics are flavor-blind. All helicity PDFs have the same small-x power law?

$$\Delta q_i(x, Q_0^2) = N_i x^{a_i} (1-x)^{b_i}$$

Implications for the Proton Spin Budget



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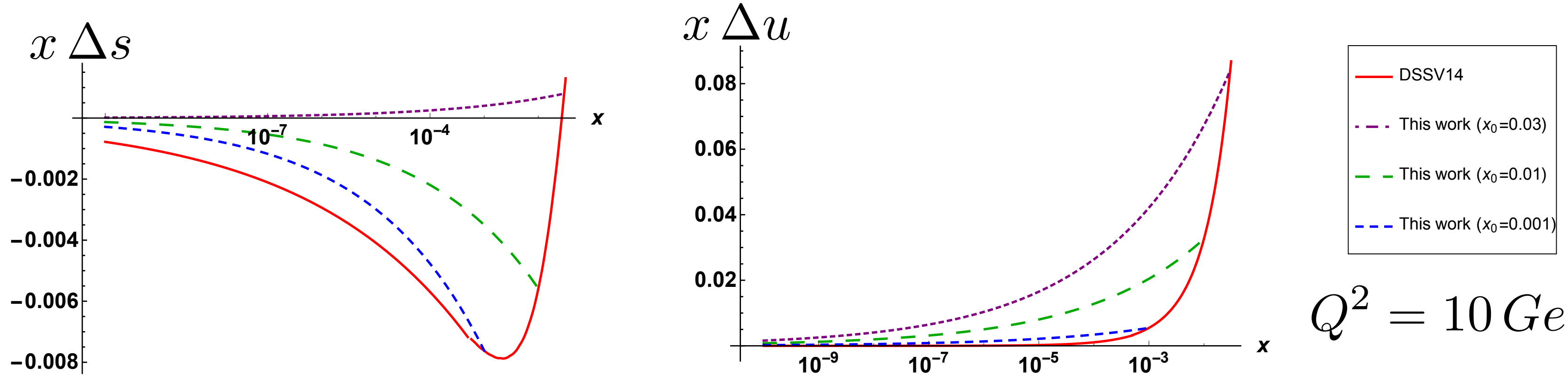

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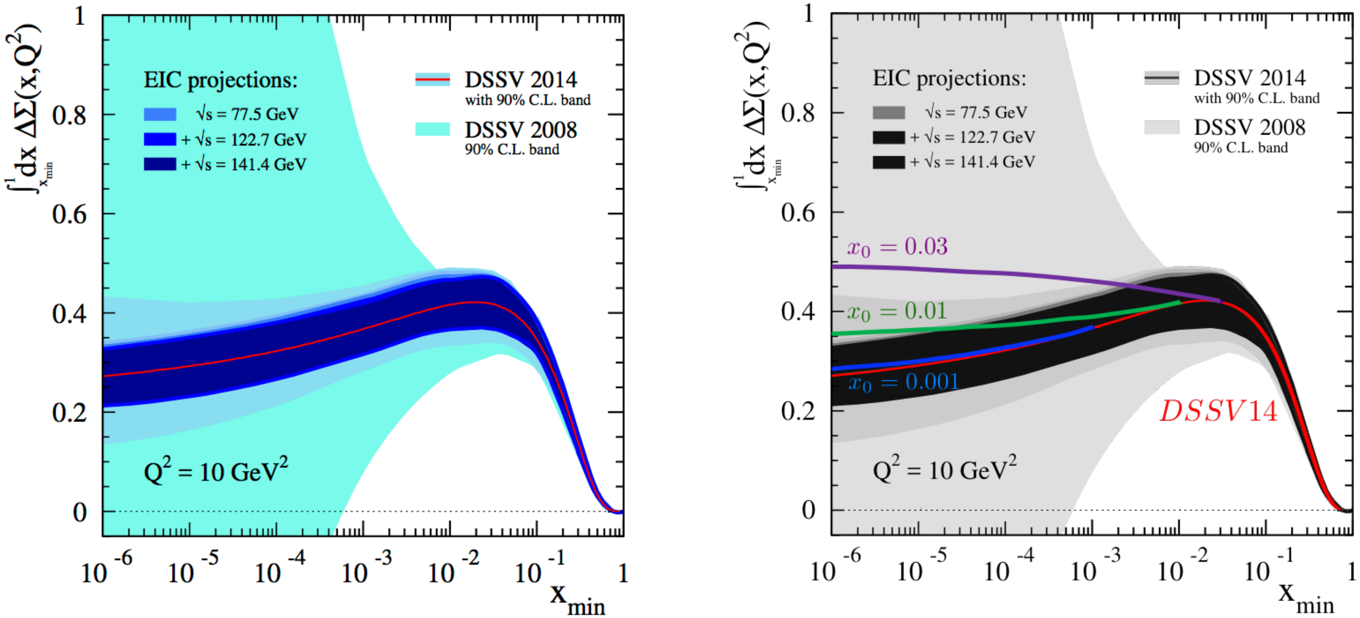

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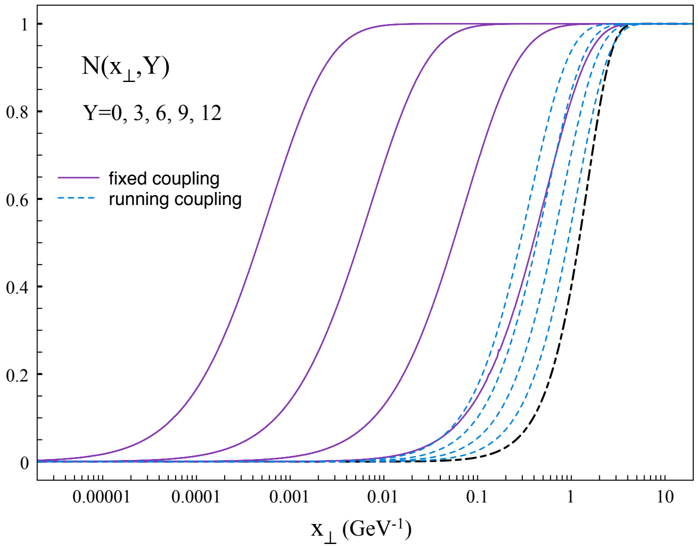

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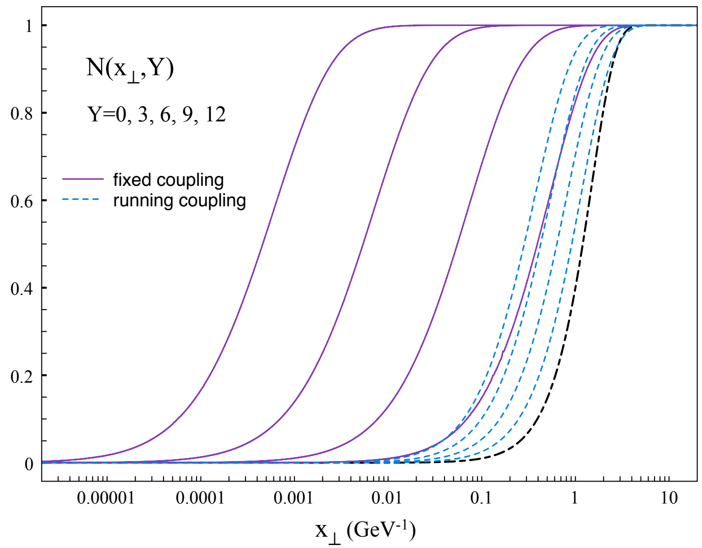

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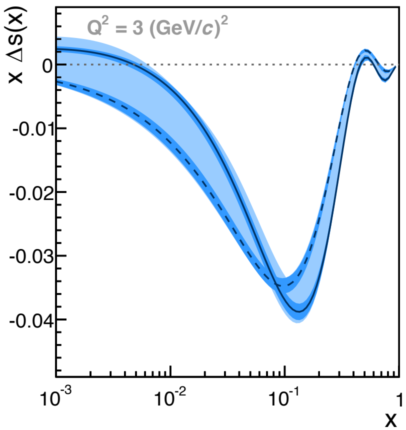

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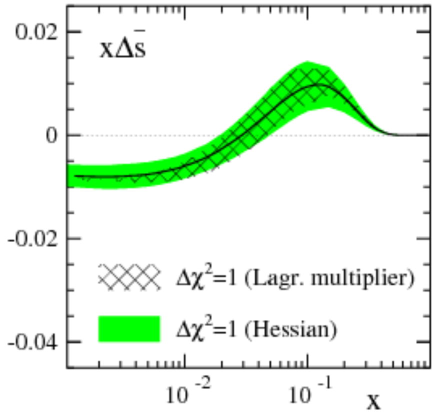

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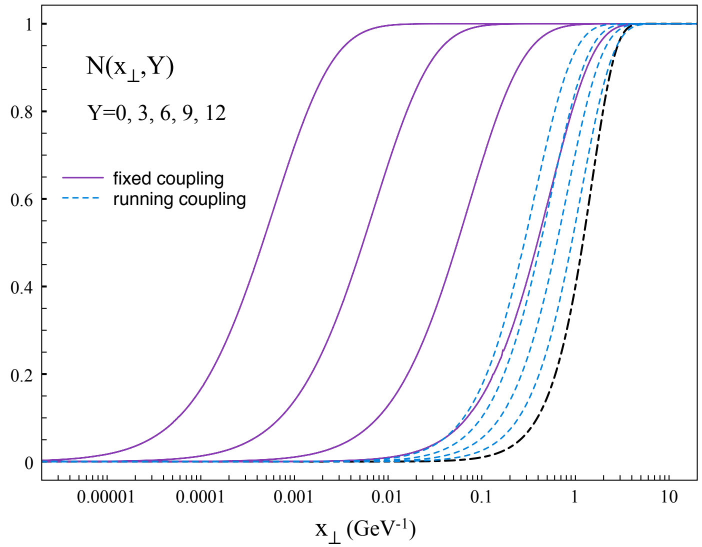

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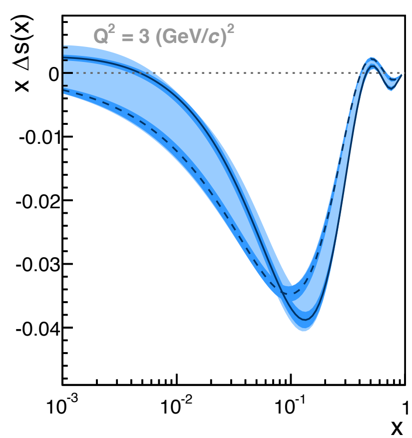

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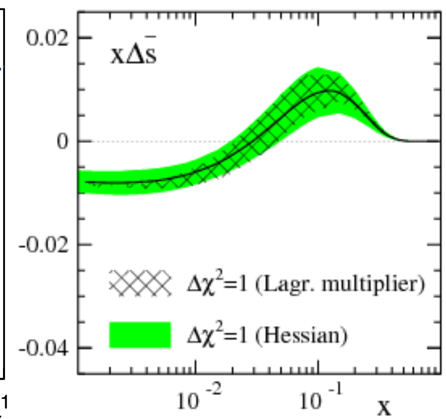

A Line-Up, and a Wanted List

In Custody:

- Singlet quark helicity
- Nonsinglet quark helicity
- Dipole gluon helicity

Kovchegov, Pitonyak, and M. S., in prepration

$$\alpha_h^{q,S} = \frac{4}{\sqrt{3}}\sqrt{\frac{\alpha_s N_c}{2\pi}} \approx 2.31\sqrt{\frac{\alpha_s N_c}{2\pi}}$$

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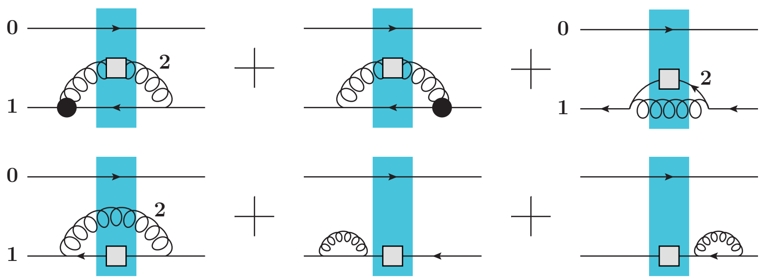
Persons of Interest:

- Large Nc + Nf limit
- Single logarithmic + saturation corrections

Conclusions

We have derived the "polarized BFKL" equations for the small-x asymptotics of the quark helicity



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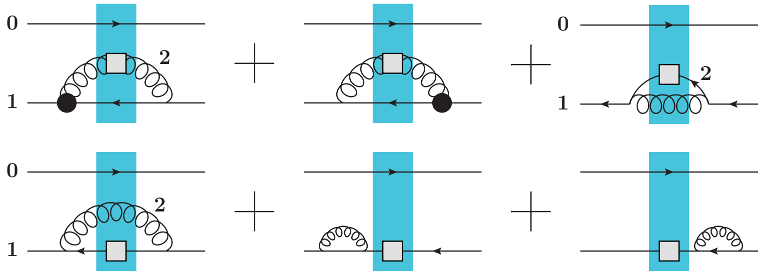

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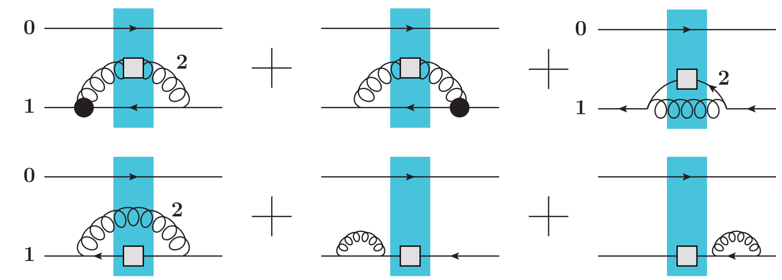
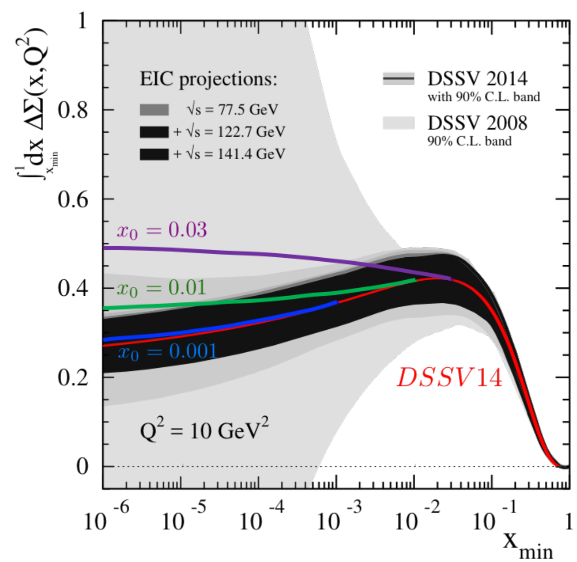


We have solved these evolution equations for the small-x quark helicity intercept α_h^q

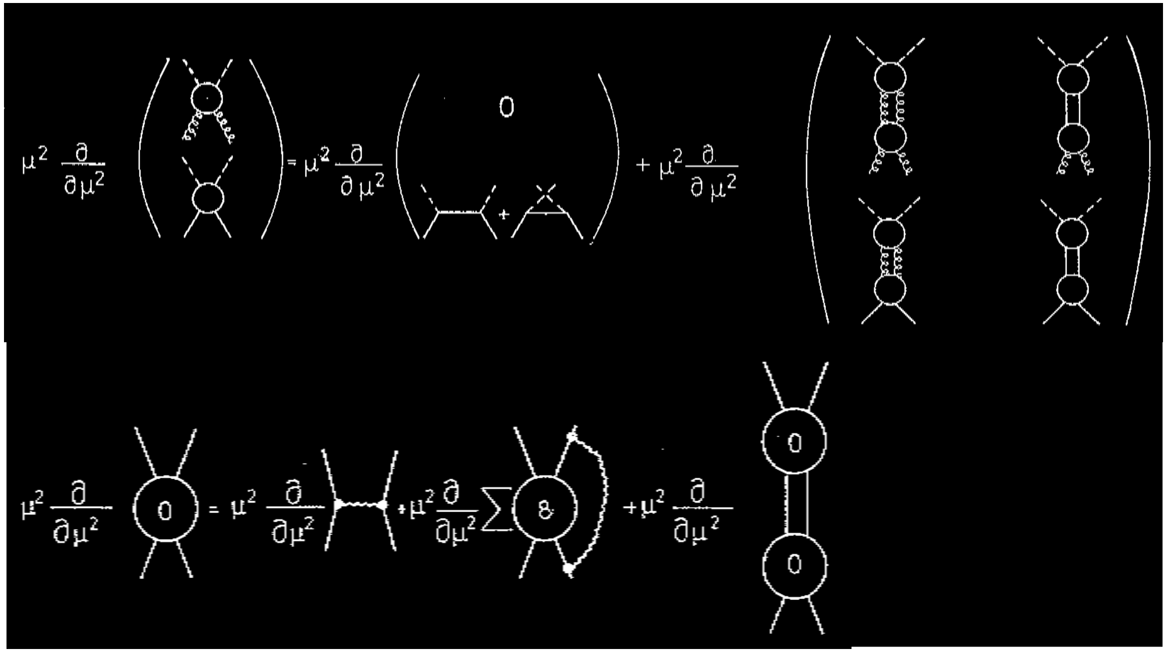
$$\Delta q^S(x, Q^2) \sim \left(\frac{1}{x}\right)^{\alpha_h^q} \qquad \alpha_h^q = \frac{4}{\sqrt{3}}\sqrt{\frac{\alpha_s N_c}{2\pi}}$$

The result leads to a potentially significant enhancement of small-x quark polarization, but needs mature phenomenology to assess.



Backup Slides

What do BER do?



- Attempt to re-sum mixed logarithms of x and Q^2.

$$(\alpha_s)^n[b_n(\ln(1/x))^{2n} + b_{n-1}(\ln(1/x))^{2n-1}\ln(Q^2/\mu^2) + \ldots + b_0(\ln(1/x))^n(\ln(Q^2/\mu^2))^n$$

- They also have both ladder and non-ladder gluons (the primary source of our complexity)

- Their calculation uses Feynman gauge (we use light-cone gauge).

What are BER's Equations?

- Transform the spin-dependent part of the hadronic tensor to Mellin space:

$$T_3 = \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} (\frac{s}{\mu^2})^{\omega} \xi_(\omega) R(\omega, y)$$

- Write down "infrared evolution equations" in Mellin space:

$$(\omega + \frac{\partial}{\partial y})R = \frac{1}{8\pi^2} F_0 R \qquad y = \ln(\frac{Q^2}{\mu^2})$$

- Obtained coupled matrix equations which can be solved analytically

$$F_0 = \begin{pmatrix} F_{gg} & F_{qg} \\ F_{gq} & F_{qq} \end{pmatrix} \quad M_0 = \begin{pmatrix} 4C_A & -2T_f \\ 2C_F & C_F \end{pmatrix}$$

$$G_0 = \begin{pmatrix} C_A & 0 \\ 0 & C_F \end{pmatrix} \quad M_8 = \begin{pmatrix} 2C_A & -T_f \\ C_A & -1/2N \end{pmatrix}$$

$$F_0(\omega) = \frac{g^2}{\omega} M_0 - \frac{g^2}{2\pi^2 \omega^2} G_0 F_8(\omega) + \frac{1}{8\pi^2 \omega} F_0(\omega)^2$$

$$F_8 = \frac{g^2}{\omega} M_8 + \frac{g^2 C_A}{8\pi^2 \omega} \frac{d}{d\omega} F_8(\omega) + \frac{1}{8\pi^2 \omega} F_8(\omega)^2$$

BER's Solution

- They obtain an analytic expression, with the intercept determined by the eigenvalues of their matrices.

$$g_1(x,Q^2) = \frac{\omega_s^{3/2}}{8\sqrt{2\pi}} \frac{\frac{2}{\omega_s} + \ln Q^2/\mu^2}{(\ln(1/x))^{3/2}} (\Delta g, \Delta\Sigma) R(\omega_s, y) \left(\frac{1}{x}\right)^{\omega_s} \left(1 + O\left(\frac{\ln^2 Q^2/\mu^2}{\ln 1/x}\right)\right)$$

$$\omega_s = z_s \sqrt{\alpha_s N_c / 2\pi}$$

$z_s = 3.45$ $(n_f = 4)$

$z = 3.66$ pure glue

- But all the complexity actually only leads to a small effect compared to the ladder graphs.

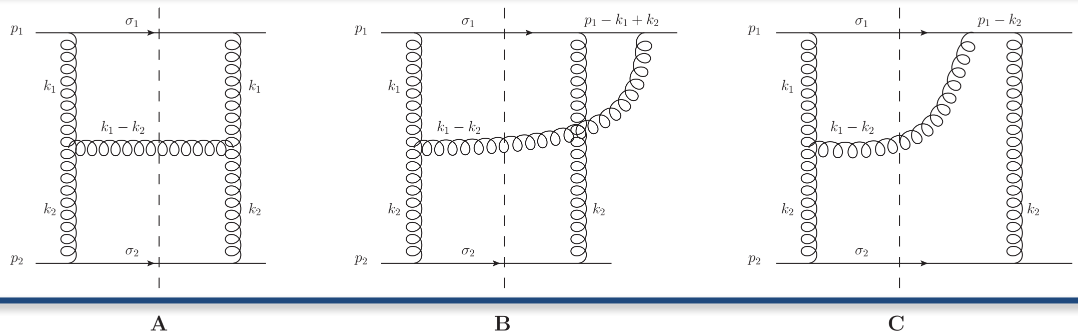
Ladder only:

$z_s = 3.81$ $(n_f = 4)$

$z_s = 4$ pure glue

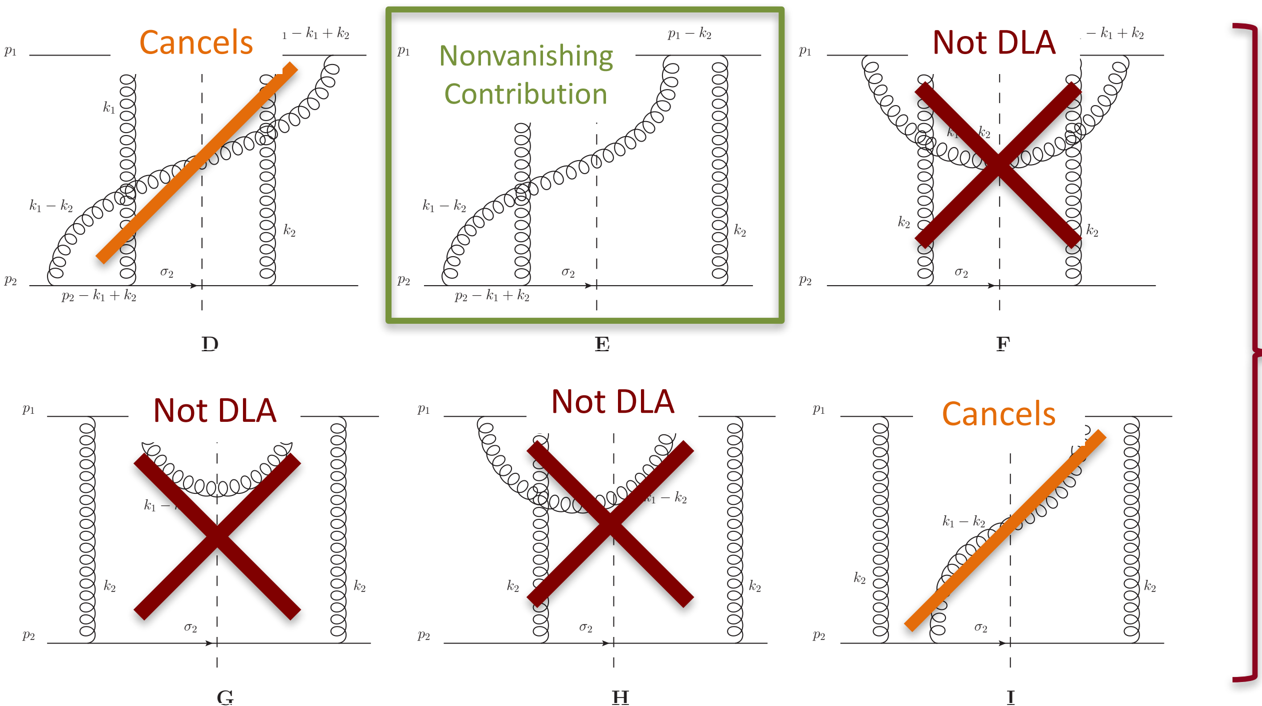
- We agree on the ladder part, but we seem to include additional diagrams which lead to a larger effect.

Diagrammatic Discrepancies



Us: All Cancel

BER: B+C = 0 for $k_{2T}^2 \gg k_{1T}^2$,
A = nonvanishing contribution



Not considered by BER?

Anomalous Dimensions

- They reproduce the DGLAP anomalous dimensions to NLO (and beyond)…

$$\gamma_S^{(1)} = (\frac{\alpha_s}{4\pi})^2 \frac{1}{\omega^3} \begin{pmatrix} \boxed{32C_A^2} - 16C_F T_f & -16C_A T_f - 8C_F T_f \\ 16C_A C_F + 8C_F^2 & 4C_F^2 - 16C_F T_f + \frac{8C_F}{N} \end{pmatrix}$$

- We also reproduce the G/G anomalous dimension in the large-Nc limit…

$$\gamma_{S,GG}^{(1)}(\omega) = \left(\frac{\alpha_s}{2\pi}\right)^2 8N_c^2 \frac{1}{\omega^3}$$

- Whatever diagrams they exclude do not miss any leading logarithms of Q^2…

- Perhaps our disagreement is over higher-twist corrections? That would explain our 35% smaller intercept….

Gluon Helicity Operators

- Dipole and Weizsacker-Williams Operators:

$$g_1^{G\,dip}(x,k_T^2) = \frac{-1}{\alpha_s\,8\pi^4}\int d^2x_0\,d^2x_1\,e^{i\underline{k}\cdot\underline{x}_{10}}\,\epsilon_T^{ij}\left\langle \mathrm{tr}\left[(V_{\underline{1}}^{pol})_\perp^i\left(\frac{\partial}{\partial(x_0)_\perp^j}V_{\underline{0}}^\dagger\right)\right] + \mathrm{c.c.}\right\rangle$$

$$g_1^{G\,WW}(x,k_T^2) = \frac{-4S_L}{g^2(2\pi)^3}\int d^2x_{10}\,d^2b_{10}\,e^{+i\underline{k}\cdot\underline{x}_{10}}\,\epsilon_T^{ij}\left\langle \mathrm{tr}\left[(V_{\underline{1}}^{pol})_\perp^i\,V_{\underline{1}}^\dagger\;V_{\underline{0}}\left(\frac{\partial}{\partial(x_0)_\perp^j}V_{\underline{0}}^\dagger\right)\right] + \mathrm{c.c.}\right\rangle$$

- Different Polarized Wilson Lines:

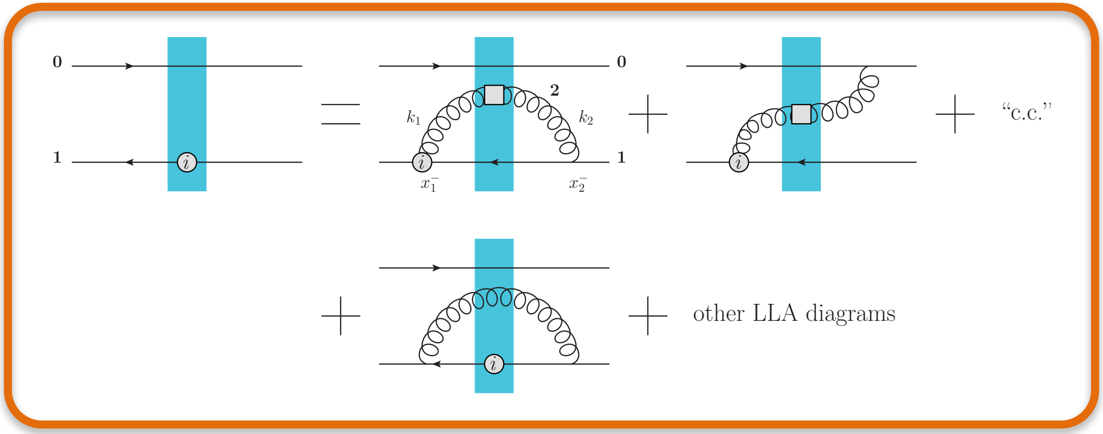
➢ For gluons:

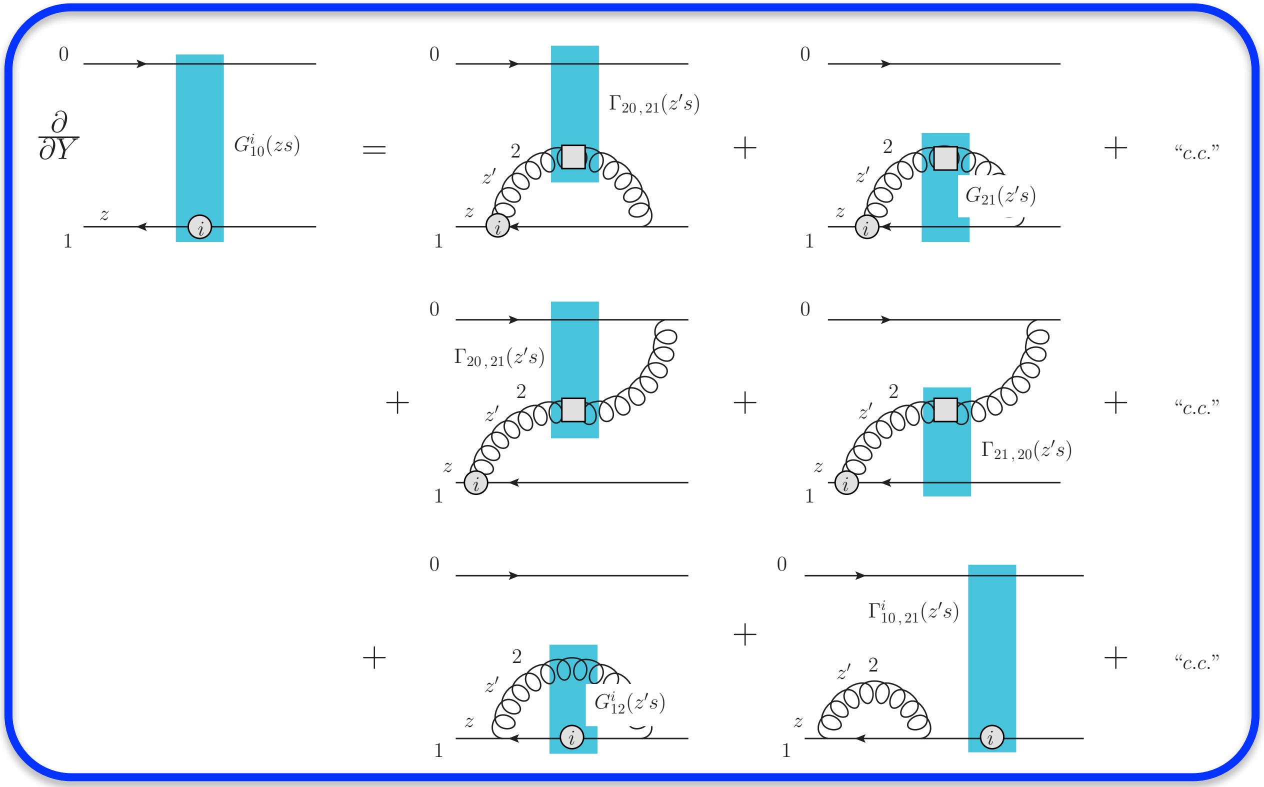
$$(V_{\underline{x}}^{pol})_\perp^i \equiv \int\limits_{-\infty}^{+\infty} dx^-\,V_{\underline{x}}[+\infty,x^-]\,\left(ig\,P^+\,\hat{A}_\perp^i(x)\right)\,V_{\underline{x}}[x^-,-\infty]$$

➢ For quarks:

$$(V_{\underline{x}}^{pol})^g = \frac{1}{s}\int\limits_{-\infty}^{+\infty} dx^-\,V_{\underline{x}}[+\infty,x^-]\,\left((-ig\,p^+)\,\epsilon_T^{ij}\,\frac{\partial}{\partial x_\perp^i}A_\perp^j(0^+,x^-,\underline{x})\right)\,V_{\underline{x}}[x^-,-\infty]$$

Gluon Helicity Evolution: Diagrams





Gluon Helicity Evolution: Equations

$$
\begin{aligned}
G^i_{10}(zs) &= G^{i\,(0)}_{10}(zs) + \frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int d^2 x_2 \, \theta\left(x_{21}^2 - \frac{1}{z' s} \right) \theta\left(x_{10}^2 \frac{z}{z'} - x_{21}^2 \right) \\
&\quad \times \, \ln \frac{1}{x_{21}\Lambda} \, \frac{\epsilon_T^{ij} \, (x_{21})_\perp^j}{x_{21}^2} \left[\Gamma_{20\,,\,21}(z's) + G_{21}(z's) \right] \\
&\quad - \frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int d^2 x_2 \, \theta\left(\min[x_{21}^2\,,\,x_{20}^2] - \frac{1}{z' s} \right) \theta\left(x_{10}^2 \frac{z}{z'} - \max[x_{21}^2\,,\,x_{20}^2] \right) \\
&\quad \times \, \ln \frac{1}{x_{21}\Lambda} \, \frac{\epsilon_T^{ij} \, (x_{20})_\perp^j}{x_{20}^2} \left[\Gamma_{20\,,\,21}(z's) + \Gamma_{21\,,\,20}(z's) \right] \\
&\quad + \frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{1}{x_{10}^2 s}}^{z} \frac{dz'}{z'} \int d^2 x_2 \frac{x_{10}^2}{x_{21}^2 \, x_{20}^2} \, \theta\left(x_{10}^2 - x_{21}^2 \right) \theta\left(x_{21}^2 - \frac{1}{z' s} \right) \left[G^i_{12}(z's) - \Gamma^i_{10\,,\,21}(z's) \right]
\end{aligned}
\tag{61a}
$$

$$
\begin{aligned}
\Gamma^i_{10\,21}(z's) &= G^{i\,(0)}_{10}(z's) + \frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z'} \frac{dz''}{z''} \int d^2 x_3 \, \theta\left(x_{31}^2 - \frac{1}{z'' s} \right) \theta\left(x_{21}^2 \frac{z'}{z''} - x_{31}^2 \right) \\
&\quad \times \, \ln \frac{1}{x_{31}\Lambda} \, \frac{\epsilon_T^{ij} \, (x_{31})_\perp^j}{x_{31}^2} \left[\Gamma_{30\,,\,31}(z''s) + G_{31}(z''s) \right] \\
&\quad - \frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z'} \frac{dz''}{z''} \int d^2 x_3 \, \theta\left(\min[x_{31}^2\,,\,x_{30}^2] - \frac{1}{z'' s} \right) \theta\left(x_{21}^2 \frac{z'}{z''} - \max[x_{31}^2\,,\,x_{30}^2] \right) \\
&\quad \times \, \ln \frac{1}{x_{31}\Lambda} \, \frac{\epsilon_T^{ij} \, (x_{30})_\perp^j}{x_{30}^2} \left[\Gamma_{30\,,\,31}(z''s) + \Gamma_{31\,,\,30}(z''s) \right] \\
&\quad + \frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{1}{x_{10}^2 s}}^{z'} \frac{dz''}{z''} \int d^2 x_3 \frac{x_{10}^2}{x_{31}^2 \, x_{30}^2} \, \theta\left(\min\left[x_{10}^2\,,\, x_{21}^2 \tfrac{z'}{z''} \right] - x_{31}^2 \right) \theta\left(x_{31}^2 - \frac{1}{z'' s} \right) \left[G^i_{13}(z''s) - \Gamma^i_{10\,,\,31}(z''s) \right].
\end{aligned}
\tag{61b}
$$

Gluon Helicity vs Quark Helicity

$$\Delta q^S(x, Q^2) = \sum_f \int \frac{dr^-}{2\pi}\, e^{ixp^+r^-}\, \langle p|\, \bar{\psi}(0)\, \mathcal{U}[0, r^-]\, \frac{\gamma^+\gamma^5}{2}\, \psi(r^-)\, |p\rangle$$

$$g_1^G(x, k_T^2) = \frac{-2i\, S_L}{x\, P^+} \int \frac{d\xi^-\, d^2\xi}{(2\pi)^3}\, e^{ixP^+\,\xi^- - i\underline{k}\cdot\underline{\xi}}$$

$$\langle P, S_L|\, \epsilon_T^{ij}\, \mathrm{tr}\left[\hat{F}^{+i}(0)\, \mathcal{U}[0, \xi]\, \hat{F}^{+j}(\xi)\, \mathcal{U}'[\xi, 0]\right] |P, S_L\rangle_{\xi^+=0}\, .$$