Twist Three GPDs

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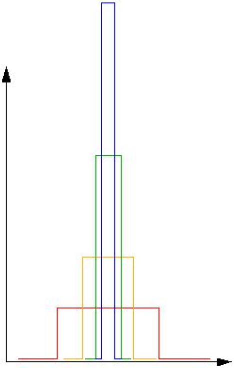
- why twist 3 GPDs:
  - ‘force’ distribution
  - OAM sum rule
- $\delta(x)$-terms in twist-3 PDFs

$\hookrightarrow$ get ‘smeared out’ in GPDs, but contribution discontinuous at $x = \pm\xi$

$\hookrightarrow$ potential issue for twist-3 factorization



twist two

- twist-2 PDFs $\longrightarrow$ long. momentum distribution
- twist-2 GPDs $\leftrightarrow$ $\perp$ imaging of PDFs

twist two

- twist-3 PDFs $\longrightarrow$ average $\perp$ force due to FSI

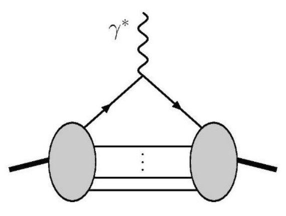
$\hookrightarrow$ twist-3 GPDs $\longrightarrow$ $\perp$ force distribution

- twist-3 GPD $\int dx\, xG_2^q(x,0,0) = -L_q$ (Polyakov)

potential issues at twist three

- twist-3 PDFs known to contain terms $\propto \delta(x)$
- related to contributions to twist-3 GPDs that
  - contribute only in ERBL region $-\xi < x < \xi$
  - diverge as $\xi \to 0$
- factorization?

form factor



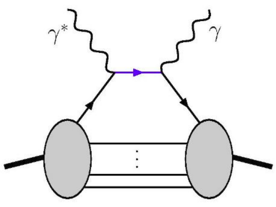
- electron hits nucleon & nucleon remains intact

↪ form factor $F(q^2)$

- position information from Fourier trafo
- no sensitivity to quark momentum
- $F(q^2) = \int dx GPD(x, \xi, q^2)$

↪ GPDs provide momentum disected form factors

Compton scattering



- electron hits nucleon, nucleon remains intact & photon gets emitted
- additional quark propagator

↪ additional information about momentum fraction x of active quark

↪ generalized parton distributions $GPD(x, q^2)$

- info about both position and momentum of active quark

transverse imaging

MB,PRD 62, 071503 (2000)

- $q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H(x, 0, -\boldsymbol{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \boldsymbol{\Delta}_\perp}$
- $F_1(-\boldsymbol{\Delta}_\perp^2) = \int dx H(x, \xi, -\boldsymbol{\Delta}_\perp^2)$
- $x =$ average momentum fraction of active quark
- $\mathbf{b}_\perp$ relative to $\perp$ center of momentum
- $E(x, 0, t)$ $\perp$ deformation for $\perp$ pol. target
- $F_2(-\boldsymbol{\Delta}_\perp^2) = \int dx E(x, \xi, -\boldsymbol{\Delta}_\perp^2)$

experimental determination

- $\Im \mathcal{A}_{DVCS} \rightarrow GPD(\xi, \xi, t)$
- $\Re \mathcal{A}_{DVCS} \rightarrow \int dx \frac{GPD(x,\xi,t)}{x \pm \xi}$

$\hookrightarrow$ requires relevant GPDs to be continuous at $x = \xi$

- no issues known for twist two GPDs

momentum fractions

- $x = \frac{x_i + x_f}{2}$ average long. mom. fraction of active quark
- $\xi = \frac{x_f - x_i}{2}$ long. mom. transfer on active quark

$d_2 \leftrightarrow$ average $\perp$ force on quark in DIS from $\perp$ pol target

polarized DIS:

- $\sigma_{LL} \propto g_1 - \frac{2Mx}{\nu} g_2$
- $\sigma_{LT} \propto g_T \equiv g_1 + g_2$

$\hookrightarrow$ 'clean' separation between g_2 and $\frac{1}{Q^2}$ corrections to g_1

- $g_2 = g_2^{WW} + \bar{g}_2$ with $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$

$$d_2 \equiv 3 \int dx\, x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x} \left\langle P,S \left| \bar{q}(0)\gamma^+ gF^{+y}(0)q(0) \right| P,S \right\rangle$$

color Lorentz Force on ejected quark

MB, PRD 88 (2013) 114502

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v} \times \vec{B}\right)^y \text{ for } \vec{v} = (0,0,-1)$$

matrix element defining $d_2 \leftrightarrow 1^{st}$ integration point in QS-integral

$d_2 \Rightarrow \perp$ force $\leftrightarrow$ QS-integral $\Rightarrow \perp$ impulse

sign of d_2

- $\perp$ deformation of $q(x, \mathbf{b}_\perp)$

$\hookrightarrow$ sign of d_2^q: opposite Sivers

magnitude of d_2

- $\langle F^y \rangle = -2M^2 d_2 = -10 \frac{GeV}{fm} d_2$
- $|\langle F^y \rangle| \ll \sigma \approx 1 \frac{GeV}{fm} \Rightarrow d_2 = \mathcal{O}(0.01)$

$d_2 \leftrightarrow$ average ⊥ force on quark in DIS from ⊥ pol target

polarized DIS:

- $\sigma_{LL} \propto g_1 - \frac{2Mx}{\nu} g_2$
- $\sigma_{LT} \propto g_T \equiv g_1 + g_2$

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consitent with experiment (JLab,SLAC), model calculations (Weiss), and lattice QCD calculations (Göckeler et al., 2005)

$g_2(x)$

MB, PRD 88 (2013) 114502

- $\int dx\, x^2 g_2(x) \Rightarrow \perp$ force on unpolarized quark in ⊥ polarized target

↪ 'Sivers force'

$e(x)$

MB, PRD 88 (2013) 114502

- $\int dx\, x^2 e(x) \Rightarrow \perp$ force on ⊥ polarized quark in unpolarized target

↪ 'Boer-Mulders force'

$h_2(x)$

M.Abdallah & MB, PRD94 (2016) 094040

- $\int dx\, x^2 h_2(x) = 0$

↪ ⊥ force on ⊥ pol. quark in long. pol. target vanishes due to parity

- $\int dx\, x^3 h_2(x) \Rightarrow$ long. gradient of ⊥ force on ⊥ polarized quark in ⊥ polarized target

↪ 'Boer-Mulders force'

force distributions

M.Abdallah & MB: *work in progress*

- use FT of twist-3 GPDs to map these forces in the ⊥ plane

δ-functions in twist 3 PDFs

example: $h_L(x)$

- Dirac structure σ^{+-}
- relevant for LT double-spin asymmetry in DY
- $h_L(x) = h_L^{WW}(x) + h_L^m(x) + h_L^3(x)$
- $h_L^{WW}(x) = 2x \int_x^1 dy \frac{h_1(y)}{y^2}$
- $h_L^m(x)_{reg} = \frac{m_q}{M} \left[\frac{g_1(x)}{x} - 2x \int_x^1 dy \frac{g_1(y)}{y^3} \right] \qquad (x > 0)$
- $h_L^3(x)$ involves quark-gluon correlations

δ-function piece

- $h_L^m(x) = h_L^m(x)_{reg} - \frac{m_q}{2M} \left[g_1(0+) - g_1(0-) \right] \delta(x)$
- confirmed for quark target model (1-loop QCD)

$\hookrightarrow$ $h_L^{m(1)}(x) = \frac{m_q}{M} \left[\frac{2}{(1-x)_+} - 4x \ln \frac{1-x}{x} - 3 + 3x - \frac{1}{2}\delta(x) \right]$

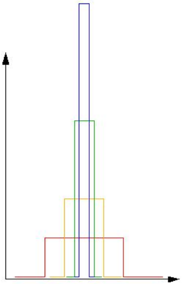
connection to twist three GPDs

for corresponding GPDs, δ-function gets ‘smeared out’ to interval $-\xi < x < \xi$

- $\delta(x)$ crucial for Lorentz invariance relations
  - $\int dx h_L(x) = \int dx h_1(x)$
  - $\int dx g_T(x) = \int dx g_1(x)$
- origin: $\int dk^- \frac{1}{(k^2-m^2+i\varepsilon)^2} = \int dk^- \frac{1}{(2k^+k^- - k_\perp^2 - m^2+i\varepsilon)^2} \propto \delta(k^+)$
  - $\int dk^- \frac{1}{(k^2-m^2+i\varepsilon)^2} = 0$ for $k^+ \neq 0$
  - $\int d^2k \frac{1}{(k^2-m^2+i\varepsilon)^2} = \frac{const}{k_\perp^2+m^2} \neq 0$
- $\xi \neq 0$: $\frac{1}{(2k^+k^- - k_\perp^2 - m^2+i\varepsilon)^2} \longrightarrow \frac{1}{\left(k-\frac{\Delta}{2}\right)^2 - m^2+i\varepsilon} \frac{1}{\left(k+\frac{\Delta}{2}\right)^2 - m^2+i\varepsilon}$

$\hookrightarrow$ nonzero for $-\frac{\Delta^+}{2} < k^+ < \frac{\Delta^+}{2}$ only

- integral becomes representation of δ function as $\Delta^+ \longrightarrow 0$



twist-3 GPDs (Polyakov & Kitpily)

$$\int dz^- e^{ixz^-\bar{p}^+} \langle p'|\bar{q}(z^-/2)\gamma^x q(-z^-/2)|p\rangle$$

$$= \frac{1}{2\bar{p}^+}\bar{u}(p')\left[\frac{\Delta^x}{2M}G_1 + \gamma^x(H+E+G_2) + \frac{\Delta^x\gamma^+}{\bar{p}^+}G_3 + \frac{i\Delta^y\gamma^+\gamma_5}{\bar{p}^+}\right]u(p)$$

Lorentz invariance relations

- $\int dx G_1^q(x,\xi,t) = 0$
- $\int dx G_2^q(x,\xi,t) = 0$
- $\int dx G_3^q(x,\xi,t) = 0$
- $\int dx G_4^q(x,\xi,t) = 0$

Tests

- test above relations in scalar diquark model & QED
- compare $\mathcal{L}(x)$ with $-xG_2(x)$

QCD Eqs. of motion

- $\int dx\, xG_2^q(x,0,0) = -L^q$

results

- $-xG_2(x,0,0) \propto 2x(1-x) \ln \Lambda^2$
- $\mathcal{L}_{JM}(x) \propto (1-x)^2 \ln \Lambda^2$
- $`L_{Ji}(x)' \equiv \frac{1}{2}\left[H+E-\tilde{H}\right]$
  $\propto \frac{1}{2}\left[1-x^2\right] \ln \Lambda^2$
- integrals equal
- x-dependence not same
- how about $\int dx G_2(x) \stackrel{?}{=} 0$???

Lorentz invariance relations

- $\int dx G_2^q(x,\xi,t) = 0$
- $-xG_2(x,0,0) \propto 2x(1-x) \ln \Lambda^2$

$\delta(x)$ in $G_2(x,0,t)$ (QED/QCD)

- $G_2(x,\xi,t)$ has term $\propto \frac{\Theta(-\xi<x<\xi)}{\xi}$

$\hookrightarrow$ rep. of $\delta(x)$ for $\xi \to 0$

- without $\delta(x)$: $\int dx G_2(x,0,t) \neq 0$
- with $\delta(x)$: $\int dx G_2(x,0,t) = 0$

$\delta(x)$ and twist 3

- $\delta(x)$ terms possible in twist 3 PDFs

$\hookrightarrow$ twist 3 GPDs allow studying those contributions in detail

- cannot use Lorentz invariance relation to constrain $G_2(x,0,0)$

twist-3 PDFs (MB&Y.Koike '02)

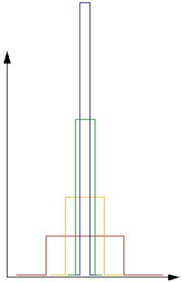
Lorentz invariance relations:

- $\int dx g_1(x) = \int dx g_T(x)$ (probably satisfied in QCD)
- $\int dx h_1(x) = \int dx h_L(x)$ (violated in QCD)

$\hookrightarrow$ h_L contains term $\propto \delta(x)$

$\hookrightarrow$ missed in $\lim_{\varepsilon\to 0} \int_\varepsilon^1 dx\, h_L(x)$

- similar for $e(x)$ (scalar twist 3)



relevant DVCS amplitudes involve $G_2(x,\xi,t) \pm \tilde{G}_2(x,\xi,t)$

$\tilde{G}_2$

$$\int dz^- e^{ixz^-\bar{p}^+} \langle p'|\bar{q}(z^-/2)\gamma^x\gamma_5 q(-z^-/2)|p\rangle$$
$$= \frac{1}{2\bar{p}^+}\bar{u}(p')\left[\gamma^x\gamma_5\tilde{G}_2 + ...\right]u(p)$$

forward limit

- $\tilde{G}_2(x,0,0) = g_2(x)$
- $g_2(x)$ <u>no</u> δ function in QCD MB & Y.Koike, NPB632 (2002) 311

$\hookrightarrow$ expect <u>no</u> cancellation of singularities between G_2 and $\tilde{G}_2$

G_2 vs. $\tilde{G}_2$ for Quark Target (1 Loop)

$G_2(x,\xi,t)$

$$G_2 = \begin{cases} 0 & x < -\xi \\ \frac{(1+x)}{\xi(1-x)} & -\xi < x < \xi \\ 2\frac{1+x}{1-\xi^2} & \xi < x < 1 \end{cases}$$

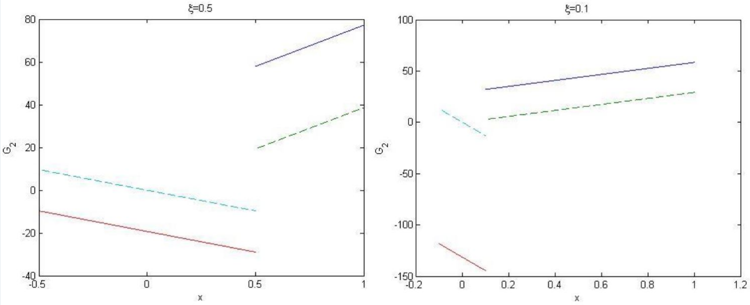
$\delta(x)$ contribution as $\xi \to 0$

$\tilde{G}_2(x,\xi,t)$

$$\tilde{G}_2 = \begin{cases} 0 & x < -\xi \\ \frac{4x}{(1-\xi^2)} & -\xi < x < \xi \\ -\frac{2x}{\xi(1+\xi)} & \xi < x < 1 \end{cases}$$

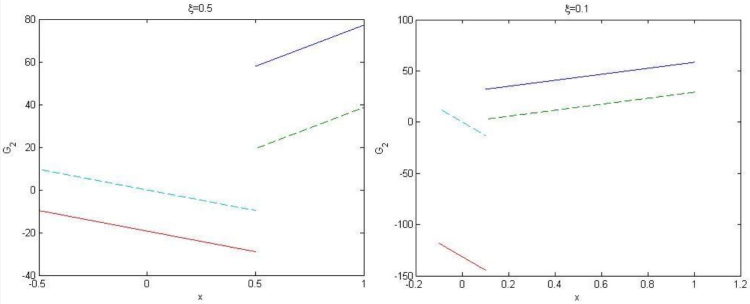
no $\delta(x)$ contribution as $\xi \to 0$

$G_2(x,\xi,t)$ (full) vs. $\tilde{G}_2(x,\xi,t)$ (dashed)



G_2 vs. $\tilde{G}_2$ for Quark Target (1 Loop)

G_2 (full) vs. $\tilde{G}_2$ (dashed)



- relevant DVCS amplitudes involve $G_2 \pm \tilde{G}_2$
- both G_2 & $\tilde{G}_2$ discontinuous at $x \pm \xi$
- but discontinuity does <u>NOT</u> cancel in $G_2 \pm \tilde{G}_2$

$\hookrightarrow$ ... twist three factorization...?

- verfied OAM sum rule for twist-3 GPD G_2 (scalar diquark as well as QED/QCD)
- $G_2(x,\xi,t)$ discontinuous at $x \pm \xi$

$\hookrightarrow$ arises from contribution to $G_2(x,\xi,t)$ that only exists between $-\xi < x < \xi$

- that contribution diverges as $\xi \to 0$

$\hookrightarrow$ $G_2(x,0,t)$ contains $\delta(x)$

- QCD eqs. of motion: $\frac{m_q}{M}$ effect
- very common for twist-3 PDFs/GPDs!
- crucial for Lorentz invariance condition)
- $\tilde{G}_2(x,\xi,t)$ also discontinuous at $x \pm \xi$, but no cancellation in $G_2 \pm \tilde{G}_2$

$\hookrightarrow$ implications for twist 3 factorization?