Twist Three GPDs

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why twist 3 GPDs:
- ‘force’ distribution
- OAM sum rule
- $\delta(x)$-terms in twist-3 PDFs
→ get ‘smeared out’ in GPDs, but contribution discontinuous at $x = \pm \xi$
→ potential issue for twist-3 factorization
Motivation

**Twist two**
- Twist-2 PDFs $\rightarrow$ long. momentum distribution
- Twist-2 GPDs $\leftrightarrow$ $\perp$ imaging of PDFs

**Twist two**
- Twist-3 PDFs $\rightarrow$ average $\perp$ force due to FSI
- Twist-3 GPDs $\rightarrow$ $\perp$ force distribution
- Twist-3 GPD $\int dx \ x G_2^q(x, 0, 0) = -L_q$ (Polyakov)

**Potential issues at twist three**
- Twist-3 PDFs known to contain terms $\propto \delta(x)$
- Related to contributions to twist-3 GPDs that
  - Contribute only in ERBL region $-\xi < x < \xi$
  - Diverge as $\xi \rightarrow 0$
- Factorization?
Deeply Virtual Compton Scattering (DVCS)

**form factor**
- electron hits nucleon & nucleon remains intact
- form factor $F(q^2)$
- position information from Fourier trafo
- no sensitivity to quark momentum
- $F(q^2) = \int dx GPD(x, \xi, q^2)$
- GPDs provide momentum dissected form factors

**Compton scattering**
- electron hits nucleon, nucleon remains intact & photon gets emitted
- additional quark propagator
- additional information about momentum fraction $x$ of active quark
- generalized parton distributions $GPD(x, q^2)$
- info about both position and momentum of active quark
Twist Two GPDs

transverse imaging

- \[ q(x, \mathbf{b}_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} H(x, 0, -\Delta_\perp^2) e^{-i \mathbf{b}_\perp \cdot \Delta_\perp} \]
- \[ F_1(\Delta_\perp^2) = \int dx H(x, \xi, -\Delta_\perp^2) \]
- \[ x = \text{average momentum fraction of active quark} \]
- \[ \mathbf{b}_\perp \text{ relative to } \perp \text{ center of momentum} \]
- \[ E(x, 0, t) \perp \text{ deformation for } \perp \text{ pol. target} \]
- \[ F_2(\Delta_\perp^2) = \int dx E(x, \xi, -\Delta_\perp^2) \]

experimental determination

- \[ \Im \mathcal{A}_{DVCS} \rightarrow GPD(\xi, \xi, t) \]
- \[ \Re \mathcal{A}_{DVCS} \rightarrow \int dx \frac{GPD(x, \xi, t)}{x \pm \xi} \]
  \[ \rightarrow \text{ requires relevant GPDs to be continuous at } x = \xi \]
- \[ \text{no issues known for twist two GPDs} \]

momentum fractions

- \[ x = \frac{x_i + x_f}{2} \text{ average long. mom. fraction of active quark} \]
- \[ \xi = \frac{x_f - x_i}{2} \text{ long. mom. transfer on active quark} \]
Digression: Average \perp Force on Quarks in DIS

**d_2 \leftrightarrow average \perp force on quark in DIS from \perp pol target**

polarized DIS:

- \( \sigma_{LL} \propto g_1 - \frac{2Mx}{\nu} g_2 \)
- \( \sigma_{LT} \propto g_T \equiv g_1 + g_2 \)

\( \rightarrow \) 'clean' separation between \( g_2 \) and \( \frac{1}{Q^2} \) corrections to \( g_1 \)

- \( g_2 = g_{WW}^W + \bar{g}_2 \) with \( g_{WW}^W(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y) \)

\[ d_2 \equiv 3 \int dx \, x^2 \bar{g}_2(x) = \frac{1}{2MP^2Sx} \langle P, S \mid \bar{q}(0)\gamma^+ gF^{+y}(0)q(0) \mid P, S \rangle \]

**color Lorentz Force on ejected quark**

\[ \sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = - \left( \vec{E} + \vec{v} \times \vec{B} \right)^y \text{ for } \vec{v} = (0, 0, -1) \]

matrix element defining \( d_2 \leftrightarrow 1^{st} \) integration point in QS-integral

\( d_2 \Rightarrow \perp force \leftrightarrow QS\text{-integral} \Rightarrow \perp impulse \)

**sign of \( d_2 \)**

- \( \perp \) deformation of \( q(x, b_\perp) \)

\( \leftrightarrow \) sign of \( d_2^q \): opposite Sivers

**magnitude of \( d_2 \)**

- \( \langle F^y \rangle = -2M^2d_2 = -10\frac{GeV}{fm}d_2 \)
- \( |\langle F^y \rangle| \ll \sigma \approx 1\frac{GeV}{fm} \Rightarrow d_2 = O(0.01) \)
Digression: Average $\perp$ Force on Quarks in DIS

$d_2 \leftrightarrow$ average $\perp$ force on quark in DIS from $\perp$ pol target

polarized DIS:

- $\sigma_{LL} \propto g_1 - \frac{2Mx}{\nu} g_2$
- $\sigma_{LT} \propto g_T \equiv g_1 + g_2$
- $\to$ 'clean' separation between $g_2$ and $\frac{1}{Q^2}$ corrections to $g_1$
- $g_2 = g_2^{WW} + \bar{g}_2$ with $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$

$$d_2 \equiv 3 \int dx \, x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x} \langle P, S \mid \bar{q}(0) \gamma^+ gF^{+y}(0)q(0) \mid P, S \rangle$$

color Lorentz Force on ejected quark

$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{\nu} \times \vec{B}\right)^y$ for $\vec{\nu} = (0, 0, -1)$

**sign of $d_2$**
- $\perp$ deformation of $q(x, b_\perp)$
- $\to$ sign of $d_2^q$: opposite Sivers

**magnitude of $d_2$**
- $\langle F^y \rangle = -2M^2d_2 = -10 \frac{GeV}{fm} d_2$
- $|\langle F^y \rangle| \ll \sigma \approx 1 \frac{GeV}{fm} \Rightarrow d_2 = O(0.01)$

consistent with experiment (JLab, SLAC), model calculations (Weiss), and lattice QCD calculations (Göckeler et al., 2005)
Digression: Average $\perp$ Force on Quarks in DIS

\[ g_2(x) \]

\[ \int dx \, x^2 g_2(x) \Rightarrow \perp \text{ force on unpolarized quark in } \perp \text{ polarized target} \]

\[ \Rightarrow \text{‘Sivers force’} \]

\[ e(x) \]

\[ \int dx \, x^2 e(x) \Rightarrow \perp \text{ force on } \perp \text{ polarized quark in unpolarized target} \]

\[ \Rightarrow \text{‘Boer-Mulders force’} \]

\[ h_2(x) \]

\[ \int dx \, x^2 h_2(x) = 0 \]

\[ \Rightarrow \perp \text{ force on } \perp \text{ pol. quark in long. pol. target vanishes due to parity} \]

\[ \int dx \, x^3 h_2(x) \Rightarrow \text{long. gradient of } \perp \text{ force on } \perp \text{ polarized quark in } \perp \text{ polarized target} \]

\[ \Rightarrow \text{‘Boer-Mulders force’} \]

\[ \text{force distributions} \]

\[ \text{use FT of twist-3 GPDs to map these forces in the } \perp \text{ plane} \]
example: $h_L(x)$

- Dirac structure $\sigma^{+-}$
- Relevant for LT double-spin asymmetry in DY
- $h_L(x) = h_{WW}^L(x) + h_m^L(x) + h_3^L(x)$
- $h_{WW}^L(x) = 2x \int_x^1 dy \frac{h_{11}(y)}{y^2}$
- $h_m^L(x)_{\text{reg}} = \frac{m_q}{M} \left[ \frac{g_1(x)}{x} - 2x \int_x^1 dy \frac{g_1(y)}{y^3} \right] \quad (x > 0)$
- $h_3^L(x)$ involves quark-gluon correlations

δ-function piece

- $h_m^L(x) = h_m^L(x)_{\text{reg}} - \frac{m_q}{2M} [g_1(0+) - g_1(0-)] \delta(x)$
- Confirmed for quark target model (1-loop QCD)

$\rightarrow h_m^{(1)}(x) = \frac{m_q}{M} \left[ \frac{2}{(1-x)_+} - 4x \ln \frac{1-x}{x} - 3 + 3x - \frac{1}{2} \delta(x) \right]$ 

Connection to twist three GPDs

For corresponding GPDs, δ-function gets ‘smeared out’ to interval $-\xi < x < \xi$
\( \delta(x) \) crucial for Lorentz invariance relations

- \( \int dx h_L(x) = \int dx h_1(x) \)
- \( \int dx g_T(x) = \int dx g_1(x) \)

Origin:

\[
\int \frac{dk^-}{(k^2 - m^2 + i\varepsilon)^2} = \int \frac{dk^-}{(2k^+ k^- - k^2_{\perp} - m^2 + i\varepsilon)^2} \propto \delta(k^+) \]

- \( \int dk^- \frac{1}{(k^2 - m^2 + i\varepsilon)^2} = 0 \) for \( k^+ \neq 0 \)
- \( \int d^2k \frac{1}{(k^2 - m^2 + i\varepsilon)^2} = \text{const} \frac{1}{k^2_{\perp} + m^2} \neq 0 \)

\( \xi \neq 0: \)

\[
\frac{1}{(2k^+ k^- - k^2_{\perp} - m^2 + i\varepsilon)^2} \rightarrow \frac{1}{(k - \Delta^+)^2 - m^2 + i\varepsilon} \frac{1}{(k + \Delta^+)^2 - m^2 + i\varepsilon}
\]

\( \leftrightarrow \) nonzero for \( -\frac{\Delta^+}{2} < k^+ < \frac{\Delta^+}{2} \) only

- integral becomes representation of \( \delta \) function as \( \Delta^+ \rightarrow 0 \)
OAM from twist 3 GPDs

**Lorentz invariance relations**

- $\int dx G_1^q(x, \xi, t) = 0$
- $\int dx G_2^q(x, \xi, t) = 0$
- $\int dx G_3^q(x, \xi, t) = 0$
- $\int dx G_4^q(x, \xi, t) = 0$

**Tests**

- test above relations in scalar diquark model & QED
- compare $\mathcal{L}(x)$ with $-xG_2(x)$

**QCD Eqs. of motion**

- $\int dx xG_2^q(x, 0, 0) = -L^q$

**Results**

- $-xG_2(x, 0, 0) \propto 2x(1-x) \ln \Lambda^2$
- $\mathcal{L}_{JM}(x) \propto (1-x)^2 \ln \Lambda^2$
- $'L_Ji(x)' \equiv \frac{1}{2} \left[H + E - \tilde{H}\right]$
- $\propto \frac{1}{2} \left[1 - x^2\right] \ln \Lambda^2$
- integrals equal
- $x$-dependence not same
- how about $\int dx G_2(x) \equiv 0$???
OAM from twist 3 GPDs

Lorentz invariance relations

\[ \int dx G_2^q(x, \xi, t) = 0 \]
\[ -xG_2(x, 0, 0) \propto 2x(1-x) \ln \Lambda^2 \]

\( \delta(x) \) in \( G_2(x, 0, t) \) (QED/QCD)

- \( G_2(x, \xi, t) \) has term \( \propto \frac{\Theta(-\xi < x < \xi)}{\xi} \)
- rep. of \( \delta(x) \) for \( \xi \to 0 \)
- without \( \delta(x) \): \( \int dx G_2(x, 0, t) \neq 0 \)
- with \( \delta(x) \): \( \int dx G_2(x, 0, t) = 0 \)

\( \delta(x) \) and twist 3

- \( \delta(x) \) terms possible in twist 3 PDFs
- twist 3 GPDs allow studying those contributions in detail
- cannot use Lorentz invariance relation to constrain \( G_2(x, 0, 0) \)

Twist-3 PDFs (MB&Y.Koike ‘02)

Lorentz invariance relations:

- \( \int dx g_1(x) = \int dx g_{T}(x) \) (probably satisfied in QCD)
- \( \int dx h_1(x) = \int dx h_L(x) \) (violated in QCD)
  \( \rightarrow \) \( h_L \) contains term \( \propto \delta(x) \)
  \( \rightarrow \) missed in \( \lim_{\varepsilon \to 0} \int_{\varepsilon}^{1} dx \ h_L(x) \)
  similar for \( e(x) \) (scalar twist 3)
relevant DVCS amplitudes involve $G_2(x, \xi, t) \pm \tilde{G}_2(x, \xi, t)$

$$
\tilde{G}_2
$$

$$
\int dz^- e^{ixz^- \not p^+} \langle p' | \bar{q}(z^- / 2) \gamma^x \gamma_5 \gamma_{5} q(-z^- / 2) | p \rangle = \frac{1}{2p^+} \bar{u}(p') \left[ \gamma^x \gamma_5 \tilde{G}_2 + \ldots \right] u(p)
$$

forward limit

- $\tilde{G}_2(x, 0, 0) = g_2(x)$
- $g_2(x)$ no $\delta$ function in QCD

→ expect no cancellation of singularities between $G_2$ and $\tilde{G}_2$

$G_2$ vs. $\tilde{G}_2$ for Quark Target (1 Loop)

\[ G_2(x, \xi, t) = \begin{cases} 
0 & x < -\xi \\
\frac{(1+x)}{\xi(1-x)} & -\xi < x < \xi \\
\frac{1+x}{2} & \xi < x < 1 
\end{cases} \]

$\delta(x)$ contribution as $\xi \to 0$

\[ \tilde{G}_2(x, \xi, t) = \begin{cases} 
0 & x < -\xi \\
\frac{4x}{(1-\xi^2)} & -\xi < x < \xi \\
-\frac{2x}{\xi(1+\xi)} & \xi < x < 1 
\end{cases} \]

no $\delta(x)$ contribution as $\xi \to 0$

$G_2(x, \xi, t)$ (full) vs. $\tilde{G}_2(x, \xi, t)$ (dashed)
relevant DVCS amplitudes involve $G_2 \pm \tilde{G}_2$
both $G_2$ & $\tilde{G}_2$ discontinuous at $x \pm \xi$
but discontinuity does **NOT** cancel in $G_2 \pm \tilde{G}_2$
→ ... twist three factorization...?
verified OAM sum rule for twist-3 GPD $G_2$
(Scalar diquark as well as QED/QCD)
$G_2(x, \xi, t)$ discontinuous at $x \pm \xi$
→ arises from contribution to $G_2(x, \xi, t)$ that only exists between $-\xi < x < \xi$
→ that contribution diverges as $\xi \to 0$
→ $G_2(x, 0, t)$ contains $\delta(x)$
→ QCD eqs. of motion: $\frac{m_q}{M}$ effect
→ very common for twist-3 PDFs/GPDs!
→ crucial for Lorentz invariance condition
→ $\tilde{G}_2(x, \xi, t)$ also discontinuous at $x \pm \xi$, but no cancellation in $G_2 \pm \tilde{G}_2$
→ implications for twist 3 factorization?