QCDevolution 2017 (JLab)

Unifying the 3D structure of gluons at small-x

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in collaboration(s) with a.o.:

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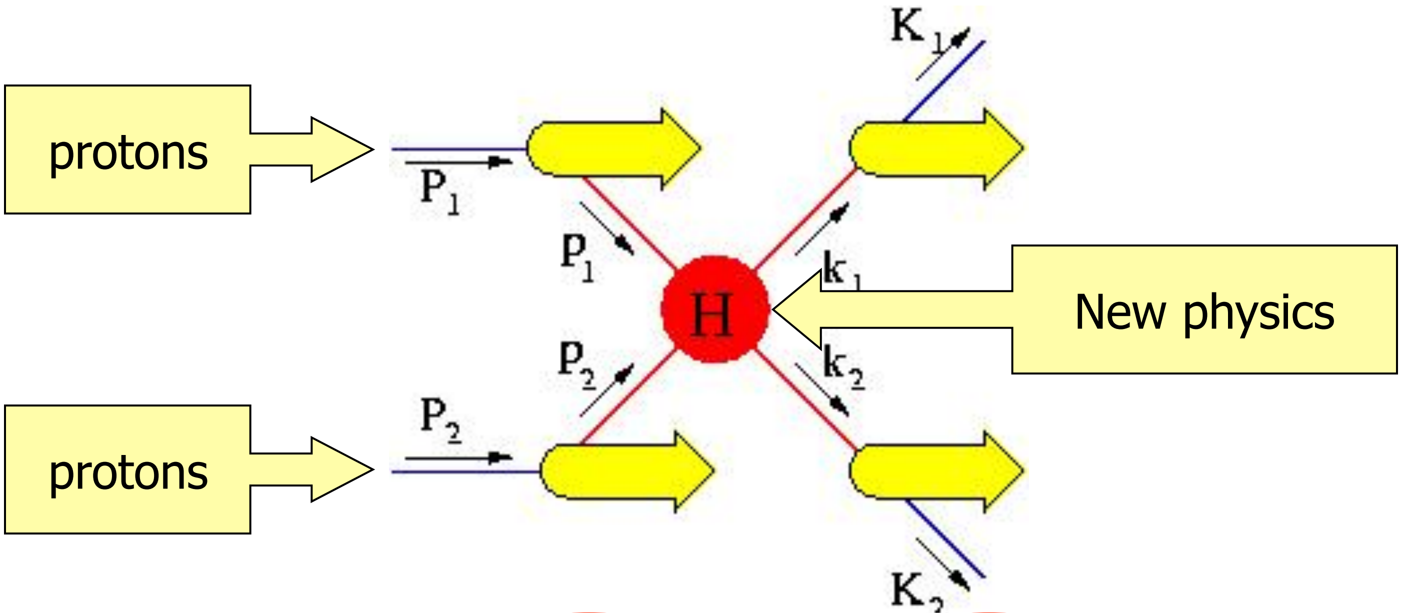
Abstract

We link the structure of the expectation value of Wilson loops in unpolarized and polarized hadronic states to gluon distribution functions depending on both fractional (longitudinal) momentum x and transverse momentum pT, referred to as the gluon TMDs. It provides ways to unify various descriptions at small-x including the dipole picture and the notions of pomeron and odderon exchange. We study the structure of gluon TMDs for unpolarized, vector polarized and tensor polarized targets, bounds on these functions and in the limit of small x the relation to diffractive processes.

Hard QCD processes



PDF's

$q(x) \quad g(x)$

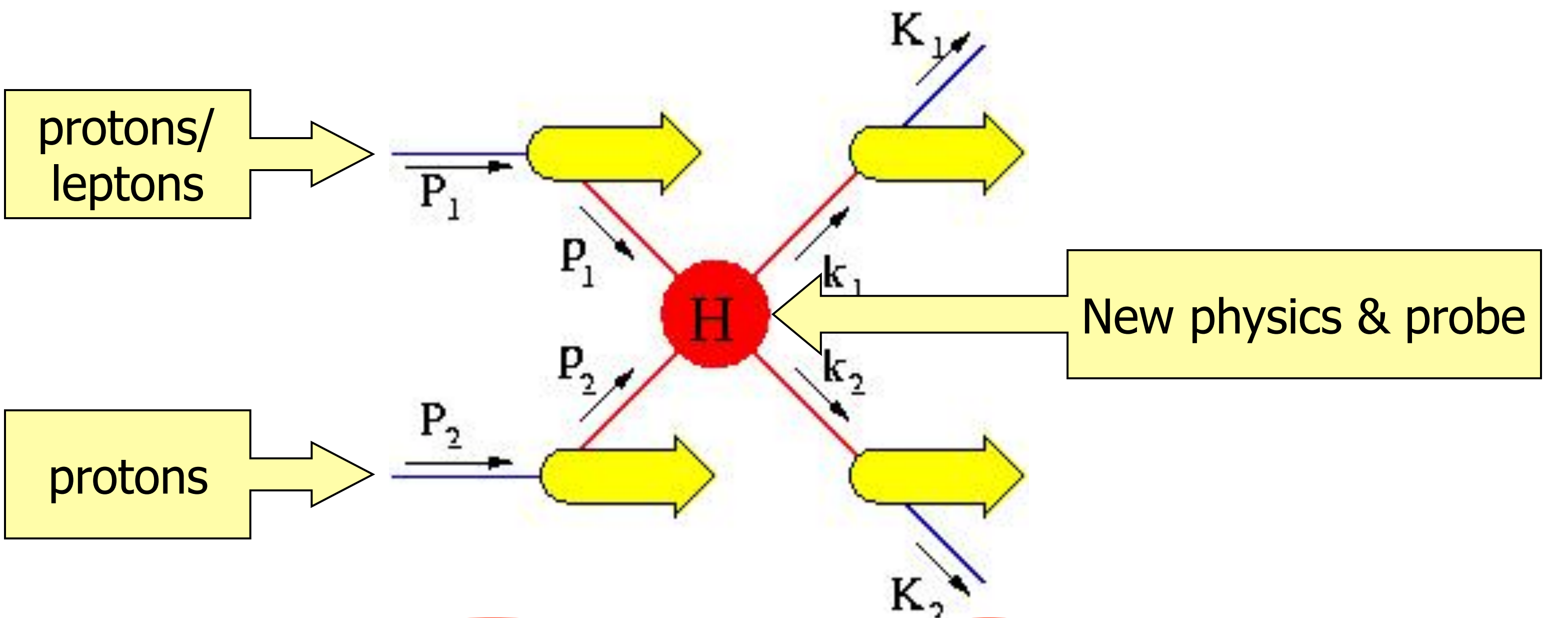
FF's (MC)

$D(z)$

$$p_i = x_i P_i \qquad K_i = z_i k_i$$

collinear treatment

Hard QCD processes and their evolution ....



Scales/evolution
Quarks & gluons
TMDs: x, p_T
Spin & flavor
GPDs
GTMDs
Hadrons

PDF's

$q(x, p_T) \quad g(x, p_T)$

FF's (MC)

$D(z, k_T)$

$p_i = x_i P_i + p_{iT}$

$K_i = z_i k_i$

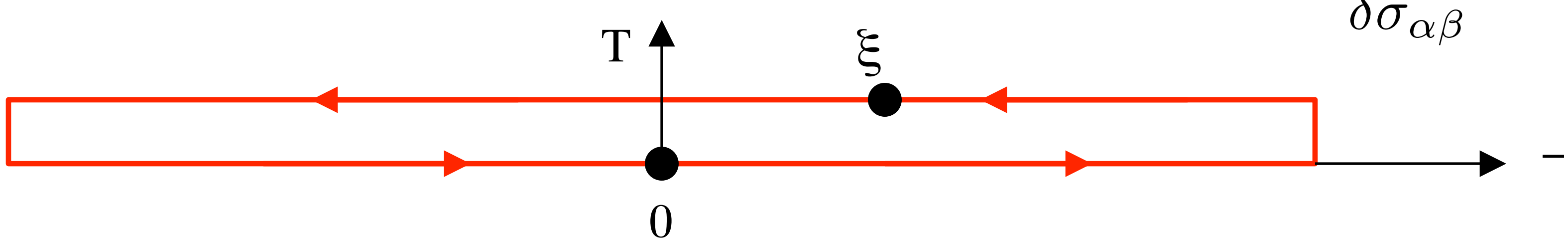
Some speculative ideas: duality color – space

- QCD is part of the standard model
- Symmetry of leptons/gauge bosons and Higgs governed by algebra [P(1,3), SU(2) x U(1)]
- Symmetry of fast/collinear (good/1-dim) quarks governed by algebra [P(1,1), SU(3)]
- But SU(3) = [SO(3), SU(2) x U(1)] and P(1,3) = [P(1,1), SO(3)]

?????? Problems and !!!!!! Opportunities

- But it would naturally link gluons to (collinear → TMD) transition
- Central role for Wilson loop linking transverse spatial structure and transverse gluons

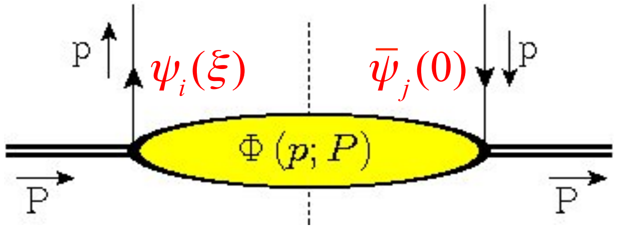
$$F^{\alpha\beta} = \frac{\delta W[C]}{\delta \sigma_{\alpha\beta}}$$



Matrix elements for TMDs (with gauge links)

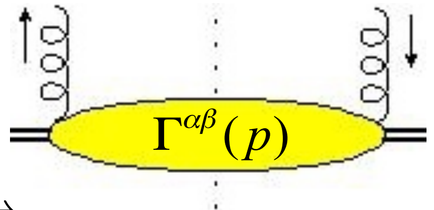
- quark-quark: $u_i(k)\overline{u}_j(k) \Longrightarrow$

$$\Phi_{ij}^{[U]}(x,p_T;n) = \int \frac{d\,\xi{\cdot}P\,d^2\xi_T}{(2\pi)^3}\,e^{ip\cdot\xi}\langle P,S|\overline{\psi}_j(0)\,U_{[0,\xi]}\psi_i(\xi)|P,S\rangle\,\big|_{\xi\cdot n=0}$$



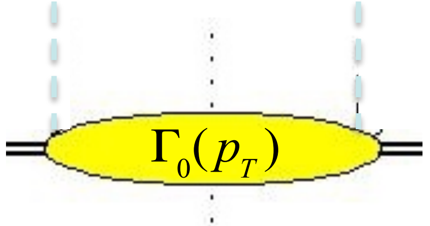
- gluon-gluon: $\epsilon^{\alpha}(k)\epsilon^{\beta *}(k) \Longrightarrow$

$$\Gamma^{[U,U']\,\mu\nu}(x,p_T;n) = \int \frac{d\,\xi{\cdot}P\,d^2\xi_T}{(2\pi)^3}\,e^{ip\cdot\xi}\,\langle P,S|\,F^{n\mu}(0)\,U_{[0,\xi]}\,F^{n\nu}(\xi)\,U'_{[\xi,0]}\,|P,S\rangle\big|_{\xi\cdot n=0}$$



- … and even single Wilson loop correlator: $\delta(0) \Longrightarrow$

$$\delta(x)\,\Gamma_0^{[U,U']}(p_T;n) = \int \frac{d\,\xi{\cdot}P\,d^2\xi_T}{(2\pi)^3}\,e^{ip\cdot\xi}\,\langle P,S|U_{[0,\xi]}\,U'_{[\xi,0]}\,|P,S\rangle\big|_{\xi\cdot n=0}$$



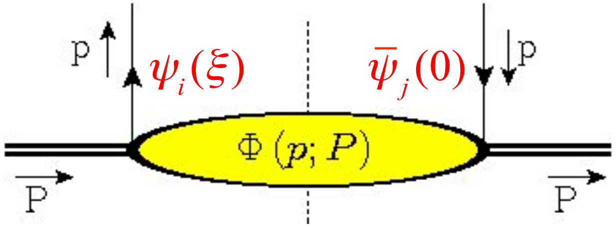
Matrix elements for TMDs (omitting gauge links)

- quark-quark

(Relevant in diagrammatic expansion)

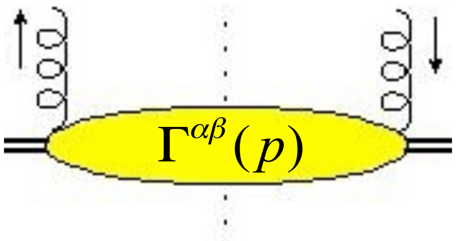
$$\Phi_{ij}(x,p_T;n) = \int \frac{d\,\xi{\cdot}P\,d^2\xi_T}{(2\pi)^3}\, e^{ip\cdot\xi}\langle P,S|\overline{\psi}_j(0)\,\psi_i(\xi)|P,S\rangle\,\Big|_{\xi\cdot n=0}$$

(after ξ.n integration T-ordering irrelevant)

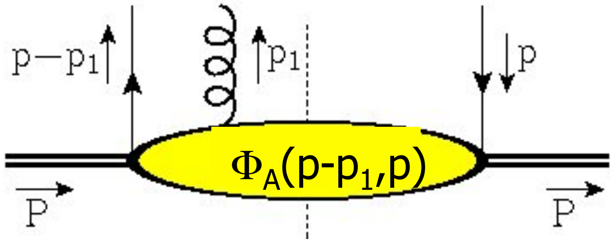


- gluon-gluon

$$\Gamma^{\mu\nu}(x,p_T;n) = \int \frac{d\,\xi{\cdot}P\,d^2\xi_T}{(2\pi)^3}\, e^{ip\cdot\xi}\,\langle P,S|\,F^{n\mu}(0)\,F^{n\nu}(\xi)\,|P,S\rangle\Big|_{\xi\cdot n=0}$$



- quark-gluon-quark



$$\Phi^{\alpha}_{D;ij}(p-p_1,p_1\,|\,p) = \int \frac{d^4\xi\,d^4\eta}{(2\pi)^8}\, e^{i(p-p_1).\xi+ip_1.\eta}\left\langle P\left|\overline{\psi}_j(0)D^{\alpha}(\eta)\,\psi_i(\xi)\right|P\right\rangle$$

$$\Phi^{\alpha}_{F;ij}(p-p_1,p_1\,|\,p) = \int \frac{d^4\xi\,d^4\eta}{(2\pi)^8}\, e^{i(p-p_1).\xi+ip_1.\eta}\left\langle P\left|\overline{\psi}_j(0)F^{n\alpha}(\eta)\,\psi_i(\xi)\right|P\right\rangle$$

TMDs and color gauge invariance (gauge links)

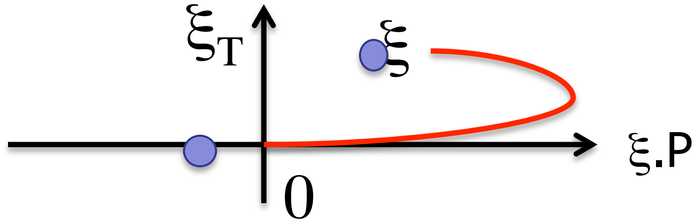
- Gauge invariance in a non-local situation requires a gauge link U(0,ξ)

$$\bar{\psi}(0)\psi(\xi) = \sum_n \frac{1}{n!}\xi^{\mu_1}\ldots\xi^{\mu_N}\bar{\psi}(0)\partial_{\mu_1}\ldots\partial_{\mu_N}\psi(0)$$

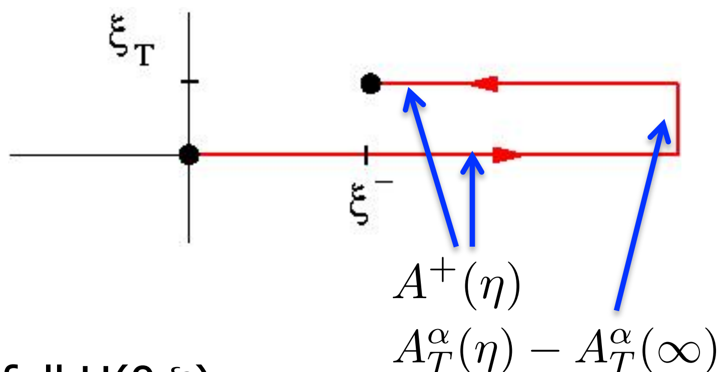
$$U(0,\xi) = \mathcal{P}\exp\left(-ig\int_0^\xi ds^\mu A_\mu\right)$$

$$\bar{\psi}(0)U(0,\xi)\psi(\xi) = \sum_n \frac{1}{n!}\xi^{\mu_1}\ldots\xi^{\mu_N}\bar{\psi}(0)D_{\mu_1}\ldots D_{\mu_N}\psi(0)$$

- Introduces path dependence in $\Phi^{[U]}(x,p_T)$



- 'Dominant' paths: along lightcone connected at lightcone infinity (staples)



- Reduces to 'straight line' for Φ(x)
  $\Phi^{[U]}(x,p_T) \;\Rightarrow\; \Phi(x)$
  (no gluon dynamics)

- Be aware that one needs all orders in g to obtain full U(0,ξ)

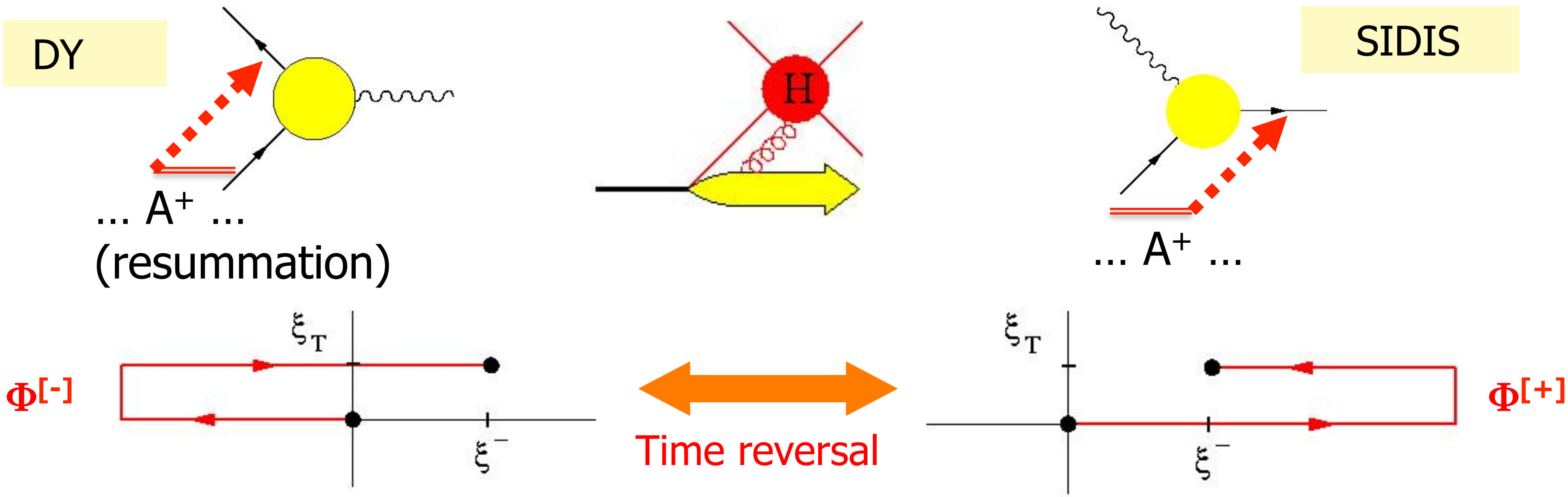
Non-universality because of process dependent gauge links

$$\Phi_{ij}^{q[C]}(x,p_T;n) = \int \frac{d(\xi.P)\,d^2\xi_T}{(2\pi)^3}\, e^{i\,p.\xi} \langle P|\,\overline{\psi}_j(0)\,U_{[0,\xi]}^{[C]}\,\psi_i(\xi)\,|P\rangle_{\xi.n=0}$$

TMD

path dependent gauge link

- Gauge links associated with dimension zero (not suppressed!) collinear $A^n = A^+$ gluons, leading for TMD correlators to process-dependence:



Belitsky, Ji, Yuan, 2003; Boer, M, Pijlman, 2003

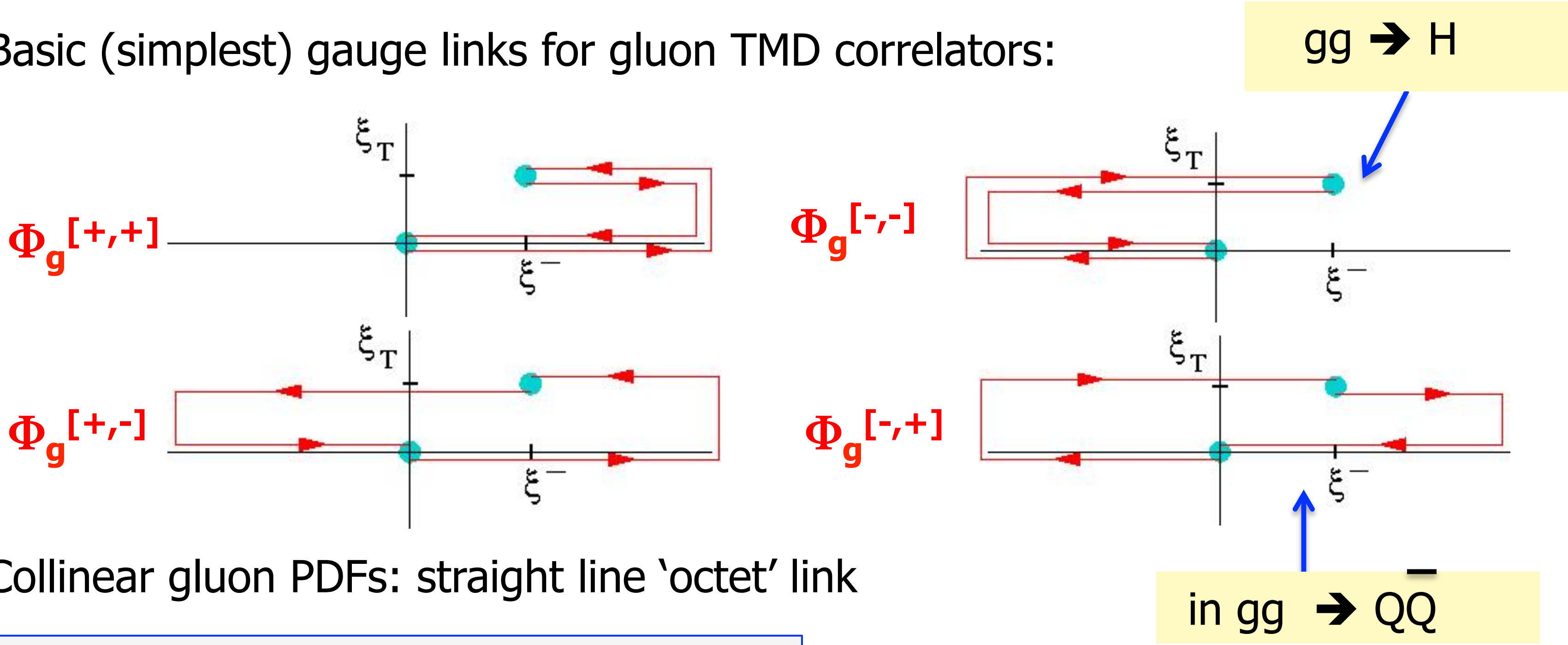
Non-universality because of process dependent gauge links

$$\Phi_g^{\alpha\beta[C,C']}(x,p_T;n)=\int\frac{d(\xi.P)d^2\xi_T}{(2\pi)^3}e^{ip.\xi}\left\langle P\left|U_{[\xi,0]}^{[C]}F^{n\alpha}(0)U_{[0,\xi]}^{[C']}F^{n\beta}(\xi)\right|P\right\rangle_{\xi.n=0}$$

- The TMD gluon correlators contain two links, which can have different paths. Note that standard field displacement involves C = C'

$$F^{\alpha\beta}(\xi)\rightarrow U_{[\eta,\xi]}^{[C]}F^{\alpha\beta}(\xi)U_{[\xi,\eta]}^{[C]}$$

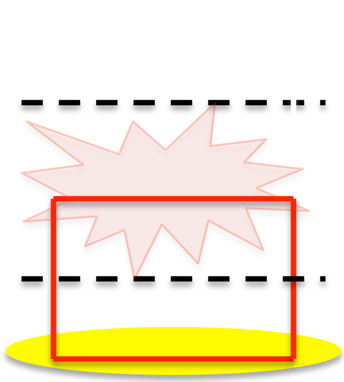
- Basic (simplest) gauge links for gluon TMD correlators:



- Collinear gluon PDFs: straight line 'octet' link

Bomhof, M, Pijlman, 2006; Dominguez, Xiao, Yuan, 2011

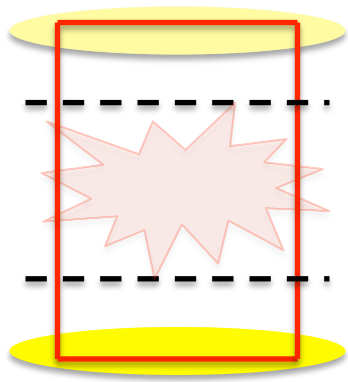
Simplest color flow classes for quarks (in lower hadron)



$\gamma q \to q$

$\langle |\overline{\psi}(0)\, U^{[+]}_{[0,\xi]}\, \psi(\xi)| \rangle$

$\Phi^{[+]}(x, k_T)$

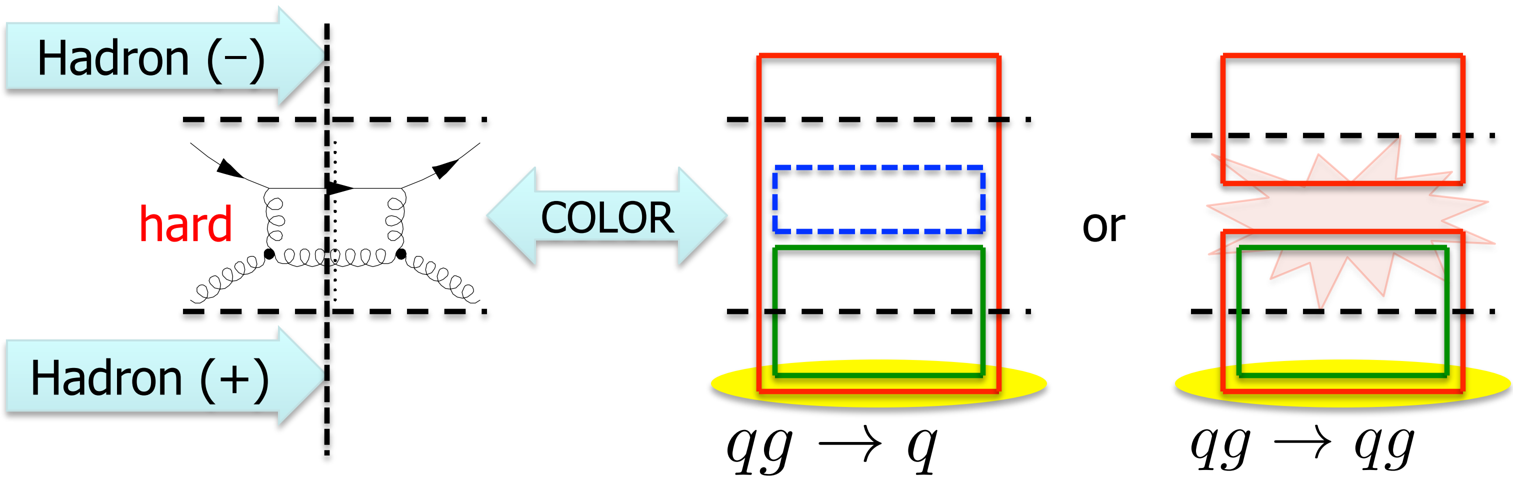


$\bar{q} q \to \gamma$

$\langle |\overline{\psi}(0)\, U^{[-]}_{[0,\xi]}\, \psi(\xi)| \rangle$

$\Phi^{[-]}(x, k_T)$

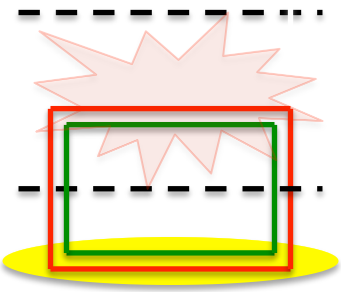
Color flow and gauge-link



$$\langle | F(0)\, U^{[+]}_{[0,\xi]}\, F(\xi)\, U^{[-]}_{[\xi,0]} | \rangle$$

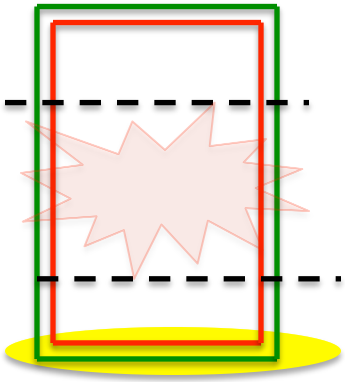
$$\langle | F(0)\, U^{[+]}_{[0,\xi]}\, F(\xi)\, U^{[+]}_{[\xi,0]} U^{(\Box)} | \rangle$$

Color flow classes for gluons (in lower hadron)



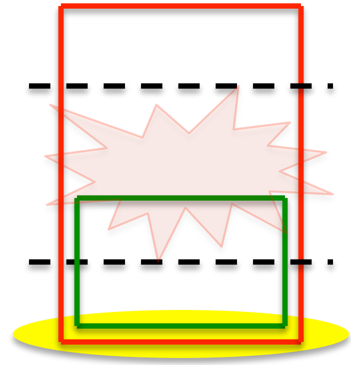
$\gamma g \to g$

$\Gamma^{ij[+,+]]}(x,k_T)$



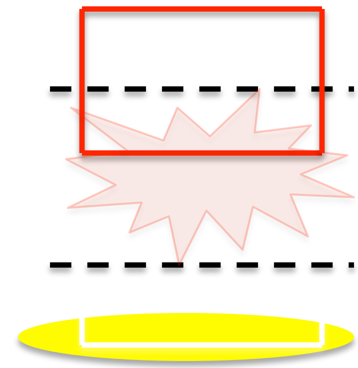
$gg \to H$

$\Gamma^{ij[-,-]]}(x,k_T)$



$qg \to qg$

$\Gamma^{ij[+,-]]}(x,k_T)$



$q(gg) \to q$

$\Gamma_0^{[+,-]}(x,k_T)$

Gluon correlators at small x related to Wilson loop correlator

$$\delta(x)\,\Gamma_0^{[U,U']}(k_T;n) = \int \frac{d\,\xi{\cdot}P\,d^2\xi_T}{(2\pi)^3}\;e^{ik\cdot\xi}\,\langle P,S|U_{[0,\xi]}\,U'_{[\xi,0]}\,|P,S\rangle\big|_{\xi\cdot n=0}$$

Wilson loop correlator linked to dipole picture and diffraction at small x

Quark correlator (in practice): replacing polarization sums

- Unpolarized target: leading part $\not{p} = x\,\not{P}$ becomes:

$$\Phi^{[U]}(x,k_T) = \left\{ f_1^{[U]}(x,k_T^2) + i\,h_1^{\perp\,[U]}(x,k_T^2)\,\frac{\not{k}_T}{M} \right\} \frac{\not{P}}{2}$$

- Vector polarized target:

$$\Phi_L^{[U]}(x,k_T) = \left\{ S_L g_1^{[U]}(x,k_T^2)\gamma_5 + S_L h_{1L}^{\perp\,[U]}(x,k_T^2)\,\frac{\gamma_5 \not{k}_T}{M} \right\} \frac{\not{P}}{2}$$

$$\Phi_T^{[U]}(x,k_T) = \Big\{ g_{1T}^{[U]}(x,k_T^2)\,\frac{k_T\cdot S_T}{M}\gamma_5 + f_{1T}^{\perp[U]}(x,k_T^2)\,\frac{k_T\times S_T}{M}$$
$$+ h_1^{[U]}(x,k_T^2)\gamma_5 \not{S}_T + h_{1T}^{\perp\,[U]}(x,k_T^2)\,\frac{k_T^{\alpha\beta}S_{T\alpha}\gamma_\beta\gamma_5}{M^2} \Big\} \frac{\not{P}}{2}$$

- Surviving in collinear correlators Φ(x) and including flavor index

$$f_1^q(x) \equiv q(x) \quad g_1^q(x) = \Delta q(x) \quad h_1^q(x) = \delta q(x)$$

- In case of TMDs there are T-odd functions
- Note: be careful with use of h_{1T} and non-traceless tensor with $k_T.S_T$ since h_{1T} is not a TMD of definite rank!

Structure of quark (8) **TMD PDF**s in spin ½ target

- 8 TMDs F…(x,k_T^2)

QUARKS	PARTON SPIN: γ^+	$\gamma^+\gamma_5$	$\gamma^+\gamma^\alpha\gamma_5$
TARGET SPIN: U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

- Integrated (collinear) correlator: only circled ones survive
- Collinear functions are spin-spin correlations
- TMDs also momentum-spin correlations (spin-orbit) including also T-odd (single-spin) functions (appearing in single-spin asymmetries)
- Existence of T-odd functions because of gauge link dependence!

Structure of quark **TMD PDFs** in spin 1 target

PARTON SPIN / TARGET SPIN

QUARKS	γ^+	$\gamma^+\gamma_5$	$\gamma^+\gamma^\alpha\gamma_5$
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$
LL	f_{1LL}		$h_{1LL}^\perp$
LT	f_{1LT}	g_{1LT}	h_{1LT} $h_{1LT}^\perp$
TT	f_{1TT}	g_{1TT}	h_{1TT} $h_{1TT}^\perp$

Bacchetta function (h_{1LT})

Hoodbhoy, Jaffe & Manohar, NP B312 (1988) 571: introduction of $f_{1LL} = b_1$

Bacchetta & M, PRD 62 (2000) 114004; h_{1LT} first introduced as T-odd PDF

X. Ji, PRD 49 (1994) 114; introduction of $H_{1LT} \equiv \hat{h}_{\bar{1}}$ (PFF)

Definite rank TMDs

- Expansion in constant tensors in transverse momentum space

$$g_T^{\mu\nu} = g^{\mu\nu} - P^{\{\mu}n^{\nu\}} \qquad \epsilon_T^{\mu\nu} = \epsilon^{Pn\mu\nu} = \epsilon^{-+\mu\nu}$$

- ... or traceless symmetric tensors (of definite rank)

$$k_T^i$$

$$k_T^{ij} = k_T^i k_T^j - \tfrac{1}{2} k_T^2\, g_T^{ij}$$

$$k_T^{ijk} = k_T^i k_T^j k_T^k - \tfrac{1}{4} k_T^2 \left(g_T^{ij} k_T^k + g_T^{ik} k_T^j + g_T^{jk} k_T^i \right)$$

- Simple azimuthal behavior: $k_T^{i_1 \ldots i_m} \longleftrightarrow |k_T|\, e^{\pm i m \varphi}$

functions showing up in cos(mϕ) or sin(mϕ) asymmetries (wrt e.g. ϕ_T)

- Simple Bessel transform to b-space (relevant for evolution):

$$F_m(x, k_T) = \int_0^\infty b db\; J_m(k_T b)\, F_m(x, b)$$

$$F_m(x, b) = \int_0^\infty k_T\, dk_T\; J_m(k_T b)\, F_m(x, k_T)$$

Gluon correlators

- Unpolarized target

$$\Gamma^{ij[U]}(x,k_T) = \frac{x}{2}\left\{-g_T^{ij}\, f_1^{[U]}(x,k_T^2) + \frac{k_T^{ij}}{M^2}\, h_1^{\perp\,[U]}(x,k_T^2)\right\}$$

- Vector polarized target

$$\Gamma_L^{ij[U]}(x,k_T) = \frac{x}{2}\left\{i\epsilon_T^{ij}\, S_L g_1^{[U]}(x,k_T^2) + \frac{\epsilon_{T\,\alpha}^{\{i}k_T^{j\}\alpha}}{M^2}\, S_L h_{1L}^{\perp\,[U]}(x,k_T^2)\right\}$$

$$\Gamma_T^{ij[U]}(x,k_T) = \frac{x}{2}\left\{\frac{g_T^{ij}\epsilon_T^{kS_T}}{M}\, f_{1T}^{\perp[U]}(x,k_T^2) - \frac{i\epsilon_T^{ij}k_T\cdot S_T}{M}\, g_{1T}^{[U]}(x,k_T^2)\right.$$
$$\left. - \frac{\epsilon_T^{k\{i}S_T^{j\}} + \epsilon_T^{S_T\{i}k_T^{j\}}}{4M}\, h_1(x,k_T^2) - \frac{\epsilon_{T\,\alpha}^{\{i}k_T^{j\}\alpha S_T}}{2M^3}\, h_{1T}^{\perp}(x,k_T^2)\right\}$$

Gluon correlators

- Tensor polarized target

$$\Gamma^{ij[U]]}_{LL}(x,k_T) = \frac{x}{2}\Big\{-\,g_T^{ij}S_{LL}\,f^{[U]}_{1LL}(x,k_T^2) + \frac{k_T^{ij}}{M^2}\,S_{LL}h^{\perp[U]}_{1LL}(x,k_T^2)\Big\}$$

$$\Gamma^{ij[U]}_{LT}(x,k_T) = \frac{x}{2}\Big\{-g_T^{ij}\,\frac{k_T{\cdot}S_{LT}}{M}\,f^{[U]}_{1LT}(x,k_T^2) + i\epsilon_T^{ij}\,\frac{\epsilon_T^{S_{LT}k}}{M}\,g^{[U]}_{1LT}(x,k_T^2)$$

$$+\,\frac{S_{LT}^{\{i}k_T^{j\}}}{M}\,h^{[U]}_{1LT}(x,k_T^2) + \frac{k_T^{ij\alpha}S_{LT\alpha}}{M^3}\,h^{\perp[U]}_{1LT}(x,k_T^2)\Big\}$$

$$\Gamma^{ij[U]}_{TT}(x,k_T) = \frac{x}{2}\Big\{-g_T^{ij}\,\frac{k_T^{\alpha\beta}S_{TT\alpha\beta}}{M^2}\,f^{[U]}_{1TT}(x,k_T^2) + i\epsilon_T^{ij}\,\frac{\epsilon_{T}{}^{\beta}{}_{\gamma}k_T^{\gamma\alpha}S_{TT\alpha\beta}}{M^2}\,g^{[U]}_{1TT}(x,k_T^2)$$

$$+\,S_{TT}^{ij}\,h^{[U]}_{1TT}(x,k_T^2) + \frac{S_{TT\alpha}^{\{i}k_T^{j\}\alpha}}{M^2}\,h^{\perp[U]}_{1TT}(x,k_T^2)$$

$$+\,\frac{k_T^{ij\alpha\beta}S_{TT\alpha\beta}}{M^4}\,h^{\perp\perp[U]}_{1TT}(x,k_T^2)\Big\}$$

D Boer, S Cotogno, T van Daal, PJM, Y Zhou, JHEP 1610 (2016) 013, ArXiv 1607.01654

Structure of gluon **TMD PDFs** in spin 1 target

		PARTON SPIN	
GLUONS	$-g_T^{\alpha\beta}$	$\varepsilon_T^{\alpha\beta}$	$p_T^{\alpha\beta}, \ldots$
U	f_1^g		$h_1^{\perp g}$
L		g_1^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_1^g \quad h_{1T}^{\perp g}$
LL	f_{1LL}^g		$h_{1LL}^{\perp g}$
LT	f_{1LT}^g	g_{1LT}^g	$h_{1LT}^g \quad h_{1LT}^{\perp g}$
TT	f_{1TT}^g	g_{1TT}^g	$h_{1TT}^g \quad h_{1TT}^{\perp g} \quad h_{1TT}^{\perp\perp g}$

TARGET SPIN

Jaffe & Manohar, Nuclear gluonometry, PL B223 (1989) 218

PJM & Rodrigues, PR D63 (2001) 094021

Meissner, Metz and Goeke, PR D76 (2007) 034002

D Boer, S Cotogno, T van Daal, PJM, Y Zhou, JHEP 1610 (2016) 013, ArXiv 1607.01654

Untangling operator structure in collinear case (reminder)

- Collinear functions and x-moments

$$\Phi^q(x) = \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \left\langle P \right| \bar{\psi}(0) U_{[0,\xi]}^{[n]} \psi(\xi) \left| P \right\rangle_{\xi.n=\xi_T=0}$$

$$x^{N-1}\Phi^q(x) = \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \left\langle P \right| \bar{\psi}(0) (\partial_\xi^n)^{N-1} U_{[0,\xi]}^{[n]} \psi(\xi) \left| P \right\rangle_{\xi.n=\xi_T=0}$$

x = p.n

$$= \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \left\langle P \right| \bar{\psi}(0) U_{[0,\xi]}^{[n]} (D_\xi^n)^{N-1} \psi(\xi) \left| P \right\rangle_{\xi.n=\xi_T=0}$$

- Moments correspond to local matrix elements of operators that all have the same twist since dim(D^n) = 0

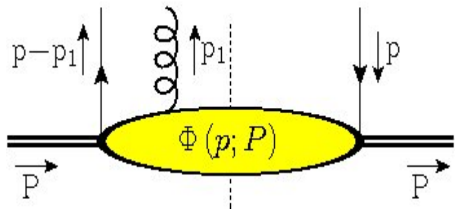
$$\Phi^{(N)} = \left\langle P \right| \bar{\psi}(0) (D^n)^{N-1} \psi(0) \left| P \right\rangle$$

- Moments are particularly useful because their anomalous dimensions can be rigorously calculated and these can be Mellin transformed into the splitting functions that govern the QCD evolution.

Transverse moments → operator structure of TMD PDFs

- Operator analysis for [U] dependence (e.g. [+] or [-]) TMD functions: in analogy to Mellin moments consider transverse moments → role for quark-gluon m.e.

$$p_T^\alpha \Phi^{[\pm]}(x, p_T; n) = \int \frac{d(\xi.P)\,d^2\xi_T}{(2\pi)^3}\, e^{i\,p.\xi} \left\langle P \middle| \bar{\psi}(0)\, U_{[0,\pm\infty]}\, iD_T^\alpha U_{[\pm\infty,\xi]}\, \psi(\xi) \middle| P \right\rangle_{\xi.n=0}$$



- $$\int dp_T\; p_T^\alpha \Phi^{[U]}(x, p_T; n) = \tilde{\Phi}_\partial^\alpha(x) + C_G^{[U]}\Phi_G^\alpha(x)$$

calculable

T-even

T-odd

$$\tilde{\Phi}_\partial^\alpha(x) = \Phi_D^\alpha(x) - \Phi_A^\alpha(x)$$

T-even (gauge-invariant derivative)

$$\Phi_A^\alpha(x) = PV \int \frac{dx_1}{x_1}\, \Phi_F^{n\alpha}(x - x_1, x_1 \,|\, x)$$

$$\Phi_D^\alpha(x) = \int dx_1\; \Phi_D^\alpha(x - x_1, x_1 \,|\, x)$$

$$\Phi_G^\alpha(x) = \pi\, \Phi_F^{n\alpha}(x, 0 \,|\, x)$$

T-odd (soft-gluon or gluonic pole, ETQS m.e.)

$$\Phi_F^{n\alpha}(x, 0 \,|\, x) = -\,\Phi_F^{n\alpha *}(x \,|\, 0, x)$$

Efremov, Teryaev; Qiu, Sterman;
Brodsky, Hwang, Schmidt; Boer, Teryaev; Bomhof, Pijlman, M

Gluonic pole factors are calculable

- $C_G^{[U]}$ calculable gluonic pole factors (quarks)

U	$U^{[\pm]}$	$U^{[+]}\,U^{[\Box]}$	$\frac{1}{N_c}\,\mathrm{Tr}_c(U^{[\Box]})\,U^{[+]}$
$\Phi^{[U]}$	$\Phi^{[\pm]}$	$\Phi^{[+\Box]}$	$\Phi^{[(\Box)+]}$
$C_G^{[U]}$	± 1	3	1
$C_{GG,1}^{[U]}$	1	9	1
$C_{GG,2}^{[U]}$	0	0	4

- Similarly for gluons with many color possibilities

Buffing, Mukherjee, M, PRD86 (2012) 074030, ArXiv 1207.3221

Buffing, Mukherjee, M, PRD88 (2013) 054027, ArXiv 1306.5897

Buffing, M, PRL 112 (2014), 092002

Operator classification of quark TMDs (polarized nucleon)

factor	QUARK TMD RANK FOR VECTOR POL. (SPIN ½) HADRON			
	0	1	2	3
1	$f_1 \quad g_1 \quad h_1$	$g_{1T}^{[\partial]} \quad h_{1L}^{\perp[\partial]}$	$h_{1T}^{\perp[\partial\partial]}$	
$C_{G,c}^{[U]}$		$h_1^{\perp[G]} \quad f_{1T}^{\perp[G]}$		
$C_{GG,c}^{[U]}$			$h_{1T}^{\perp[GG1]} \quad h_{1T}^{\perp[GG2]}$	
$C_{GGG,c}^{[U]}$				

Three pretzelocities:

$$[\partial\partial]: \quad \bar{\psi}\partial\partial\psi = Tr_c\left[\partial\partial\psi\bar{\psi}\right]$$

$$[GG\ 1]: Tr_c\left[GG\psi\bar{\psi}\right]$$

$$[GG\ 2]: Tr_c\left[GG\right]Tr_c\left[\psi\bar{\psi}\right]$$

Process dependence also for (T-even) pretzelocity,

$$h_{1T}^{\perp[U]} = h_{1T}^{\perp[\partial\partial]} + C_{GG,1}^{[U]} h_{1T}^{\perp[GG1]} + C_{GG,2}^{[U]} h_{1T}^{\perp[GG2]}$$

Buffing, Mukherjee, M, PRD86 (2012) 074030, ArXiv 1207.3221

Operator classification of quark TMDs (including trace terms)

factor	QUARK TMD RANK FOR VECTOR POL. (SPIN ½) HADRON			
	0	1	2	3
1	$f_1 \quad g_1 \quad h_1$	$g_{1T}^{[\partial]} \quad h_{1L}^{\perp[\partial]}$	$h_{1T}^{\perp[\partial\partial]}$	
$C_{G,c}^{[U]}$		$h_1^{\perp[G]} \quad f_{1T}^{\perp[G]}$		
$C_{GG,c}^{[U]}$	$\delta f_1^{[GGc]} \ldots$	…	$h_{1T}^{\perp[GG1]} \quad h_{1T}^{\perp[GG2]}$	
$C_{GGG,c}^{[U]}$		…		

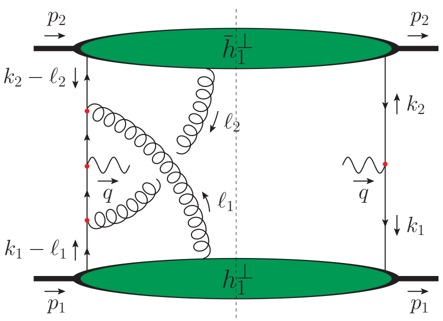
Process dependence in p_T dependence of TMDs due to gluonic pole operators (e.g. affecting $\langle p_T^2 \rangle$)

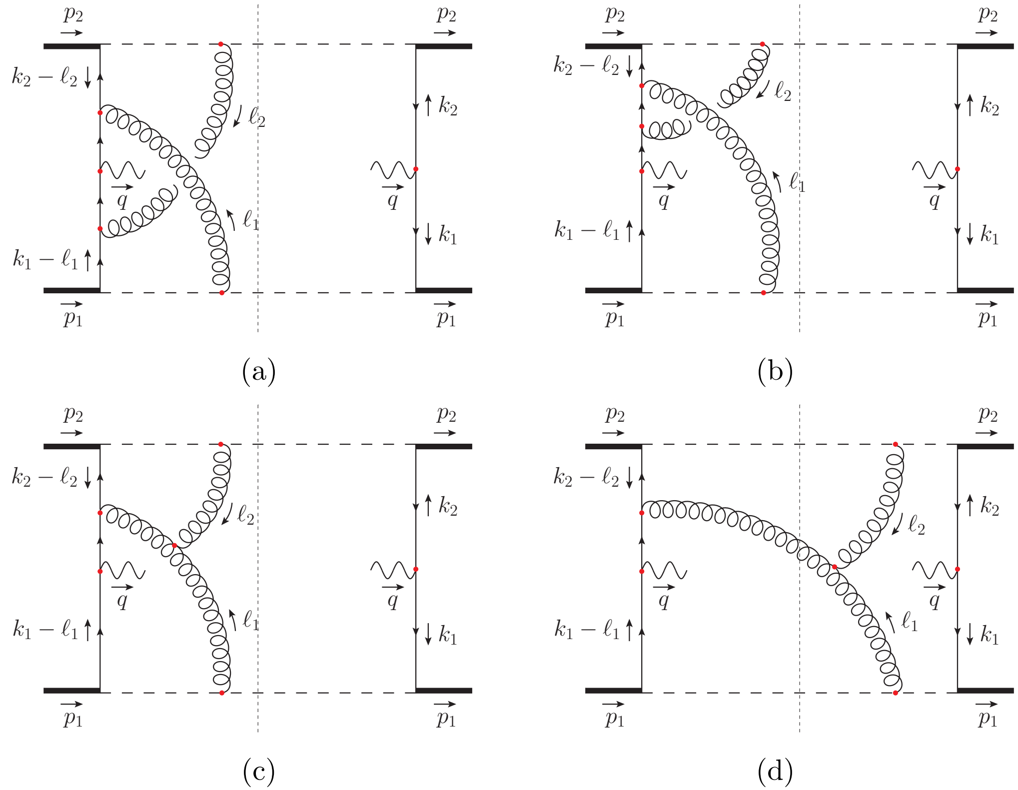
$$f_1^{[U]}(x, p_T^2) = f_1 + C_{GG,c}^{[U]} \delta f_1^{[GGc]} \quad \text{with } \delta f_1^{[GG\,c]}(x) = 0$$

Boer, Buffing, M, JHEP08 (2015) 053, arXiv:1503.03760

Color structure for double T-odd

- GLs complicate life for 'double p_T' situation such as Sivers-Sivers or BM-BM in DY





- $(a) \Longrightarrow -\dfrac{1}{N_c^2 - 1}\dfrac{1}{N_c}$

- $(a) + (c) + (d) \Longrightarrow \dfrac{1}{N_c}$

- Which one is applicable to B-term in

$$\sigma_{UU}(x_1, x_2, q_T) = A\, f_1(x_1)\overline{f}_1(x_2)\hat{\sigma}_{UU} + B\cos(2\phi)\, h_1^\perp(x_1)\overline{h}_1^\perp(x_2)\hat{\sigma}_{TT}$$

Buffing, M, PRL 112 (2014), 092002

Boer, Van Daal, Gaunt, Kasemets, M, under study

Classifying Polarized Quark TMDs (including tensor pol)

factor	QUARK TMD RANK FOR VECTOR POL. (SPIN ½) HADRON			
	0	1	2	3
1	$f_1 \quad g_1 \quad h_1$	$g_{1T}^{[\partial]} \quad h_{1L}^{\perp[\partial]}$	$h_{1T}^{\perp[\partial\partial]}$	
$C_{G,c}^{[U]}$		$h_1^{\perp[G]} \quad f_{1T}^{\perp[G]}$		
$C_{GG,c}^{[U]}$	$\delta f_1^{[GGc]} \ldots$	…	$h_{1T}^{\perp[GG1]} \quad h_{1T}^{\perp[GG2]}$	
$C_{GGG,c}^{[U]}$		…		

factor	QUARK TMD RANK FOR TENSOR POL. (SPIN 1) HADRON			
	0	1	2	3
1	f_{1LL}	$f_{1LT}^{[\partial]}$	$f_{1TT}^{[\partial\partial]}$	
$C_G^{[U]}$	$\delta h_{1LT}^{[\partial.G]}$	$h_{1LL}^{\perp[G]} \quad g_{1LT}^{[G]} \quad h_{1TT}^{[G]}$	$h_{1LT}^{\perp[\partial G]} \quad g_{1TT}^{[\partial G]}$	$h_{1TT}^{\perp[\partial\partial G]}$
$C_{GG,c}^{[U]}$	…	…	$f_{1TT}^{[GGc]}$	
$C_{GGG,c}^{[U]}$	…	…	…	$h_{1TT}^{\perp[GGGc]}$

$$h_{1LT}^{[U]}(x, p_T^2) = C_G^{[U]}\, \delta h_{1LT}^{[\partial.G]}(x, p_T^2) \quad with \quad \delta h_{1LT}^{[\partial.G]}(x) = 0$$

Operator classification of gluon TMDs

factor	GLUON TMD PDF RANK FOR SPIN ½ HADRON				
	0	1	2	3	
1	f_1 g_1	$g_{1T}^{[\partial]}$	$h_1^{\perp[\partial\partial]}$		
$C_{G,c}^{[U]}$		$f_{1T}^{\perp[Gc]}$ $h_1^{[Gc]}$	$h_{1L}^{\perp[\partial Gc]}$	$h_{1T}^{\perp[\partial\partial Gc]}$	
$C_{GG,c}^{[U]}$	$\delta f_1^{[GGc]}$ ...	...	$h_1^{\perp[GGc]}$		
$C_{GGG,c}^{[U]}$		...	...	$h_{1T}^{\perp[GGGc]}$	

factor	ADDITIONAL PDFs FOR TENSOR POL. SPIN 1 HADRON				
	0	1	2	3	4
1	f_{1LL} h_{1TT}	$f_{1LT}^{[\partial]}$ $h_{1LT}^{[\partial]}$	$f_{1TT}^{[\partial\partial]}$ $h_{1LL}^{\perp[\partial\partial]}$ $h_{1TT}^{\perp[\partial\partial]}$	$h_{1LT}^{\perp[\partial\partial\partial]}$	$h_{1TT}^{\perp\perp[\partial\partial\partial\partial]}$
$C_{G,c}^{[U]}$		$g_{1LT}^{[Gc]}$	$g_{1TT}^{\perp[\partial Gc]}$		
$C_{GG,c}^{[U]}$	...	...	$f_{1TT}^{\perp[GGc]}$ $h_{1LL}^{\perp[GGc]}$ $h_{1TT}^{\perp[GGc]}$	$h_{1LT}^{\perp[\partial GG]}$	$h_{1TT}^{\perp\perp[\partial\partial GGc]}$
$C_{GGG,c}^{[U]}$		...	...		
$C^{[U]}$	...	...	...	...	$h_{1TT}^{\perp\perp[GGGGc]}$

D Boer, S Cotogno, T van Daal, PJM, Y Zhou, JHEP 1610 (2016) 013, ArXiv 1607.01654

Small x physics in terms of TMDs

- The single Wilson-loop correlator Γ_0

$$\Gamma_0(k_T) = \frac{1}{2M^2}\left\{e(k_T^2) - \frac{\epsilon^{kS_T}}{M}\,e_T(k_T^2)\right\}$$

factor	GLUON TMD PDF RANK FOR UNPOL. AND SPIN ½ HADRON			
	0	1	2	3
1	e			
$C_{G,c}^{[U]}$		$e_T^{[Gc]}$		
$C_{GG,c}^{[U]}$	$\delta e^{[GGc]}$			
$C_{GGG,c}^{[U]}$		$\delta e_T^{[G.GGc]}$		

- Note limit x → 0 for gluon TMDs linked to gluonic pole m.e. of Γ_0

$$(2\pi)^2\,\Gamma^{ij\,[U,U']}(0,k_T) \sim C_{GG}^{[U,U']} M^2 \Gamma_{0\,GG}^{\,ij[U,U']}(k_T) \sim C_{GG}^{[U,U']}\frac{k_T^i\,k_T^j}{M^2}\Gamma_0^{[U,U']}(k_T)$$

- RHS depends on t, which for x = 0 becomes p_T^2 → off-forward studies

Small x physics in terms of TMDs

- Note limit x → 0 for gluon TMDs linked to gluonic pole m.e. of Γ_0

$$\pi^2\,\Gamma^{\alpha\beta\,[U,U']}(0,p_T) = C_{GG}^{[U,U']}\Gamma_{0\,GG}^{\alpha\beta}(p_T)$$

- Dipole correlators: at small x only two structures for unpolarized and transversely polarized nucleons: pomeron & odderon structure

$$x\,f_1^{[+,-]}(x,k_T^2) \longrightarrow \frac{k_T^2}{2M^2}\,e^{[+,-]}(k_T^2)$$
$$x\,h_1^{\perp[+,-]}(x,k_T^2) \longrightarrow e^{[+,-]}(k_T^2)$$

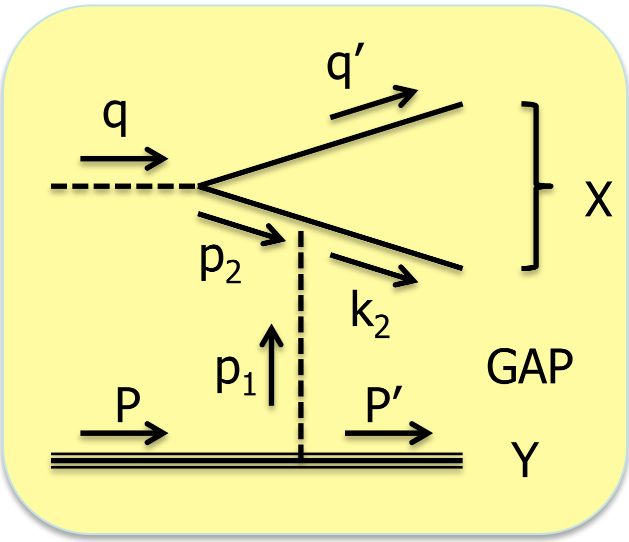
$$x\,f_{1T}^{\perp[+,-]}(x,k_T^2) \longrightarrow \frac{k_T^2}{2M^2}\,e_T^{[+,-]}(k_T^2)$$
$$x\,h_1^{[+,-]}(x,k_T^2) \longrightarrow \frac{k_T^2}{2M^2}\,e_T^{[+,-]}(k_T^2)$$
$$x\,h_{1T}^{\perp[+,-]}(x,k_T^2) \longrightarrow e_T^{[+,-]}(k_T^2)$$

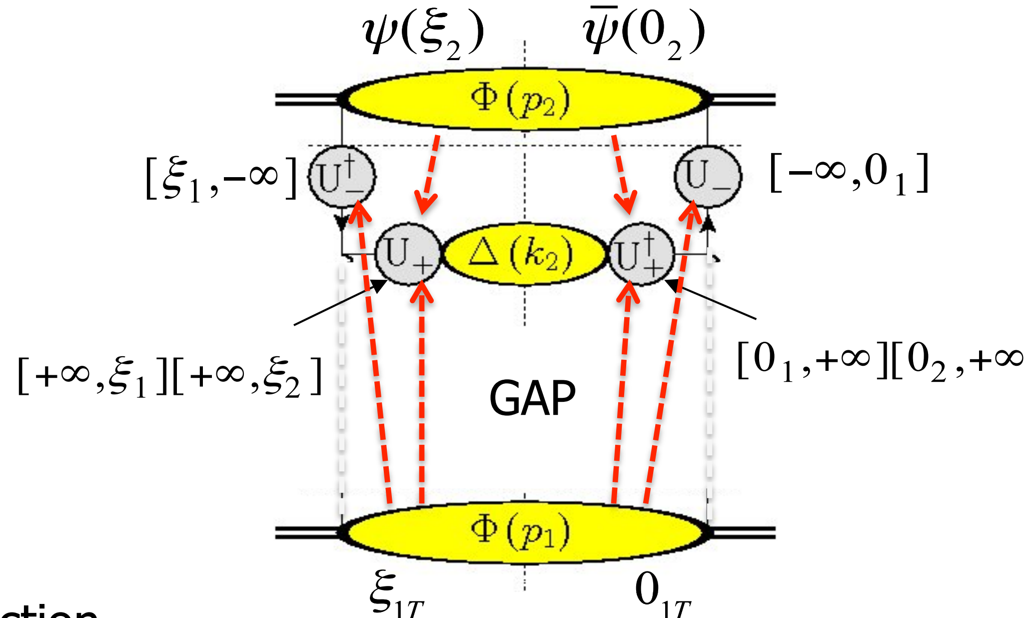
Dominguez, Xiao, Yuan 2011
D Boer, MG Echevarria, PJM, J Zhou, PRL 116 (2016) 122001, ArXiv 1511.03485
D Boer, S Cotogno, T van Daal, PJM, Y Zhou, JHEP 1610 (2016) 013, ArXiv 1607.01654

Conclusions and outlook

- (Generalized) universality of TMDs studied via operator product expansion, extending the well-known collinear distributions (including polarization 3 for quarks and 2 for gluons) to TMD PDF and PFF functions, ordered into functions of definite rank with rich momentum-spin structure (available results include spin 1)
- Nonlocal operator structure allows derivations of positivity bounds.
- Multiple operator possibilities for pretzelocity/transversity
- Non-universality for p_T-widths of TMDs
- Color entanglement for double T-odd functions (?)
- Wilson loops simplify gluonic TMD-structure at small x
- Wilson loops offer applications in diffractive processes (pomeron/odderon structure)

A TMD picture for diffractive scattering

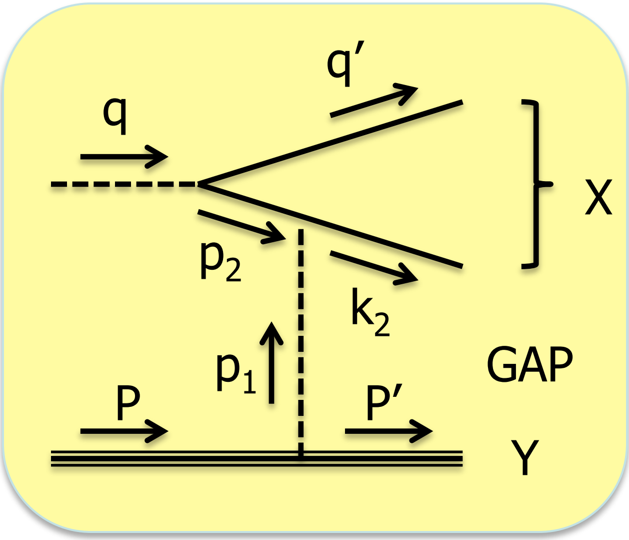


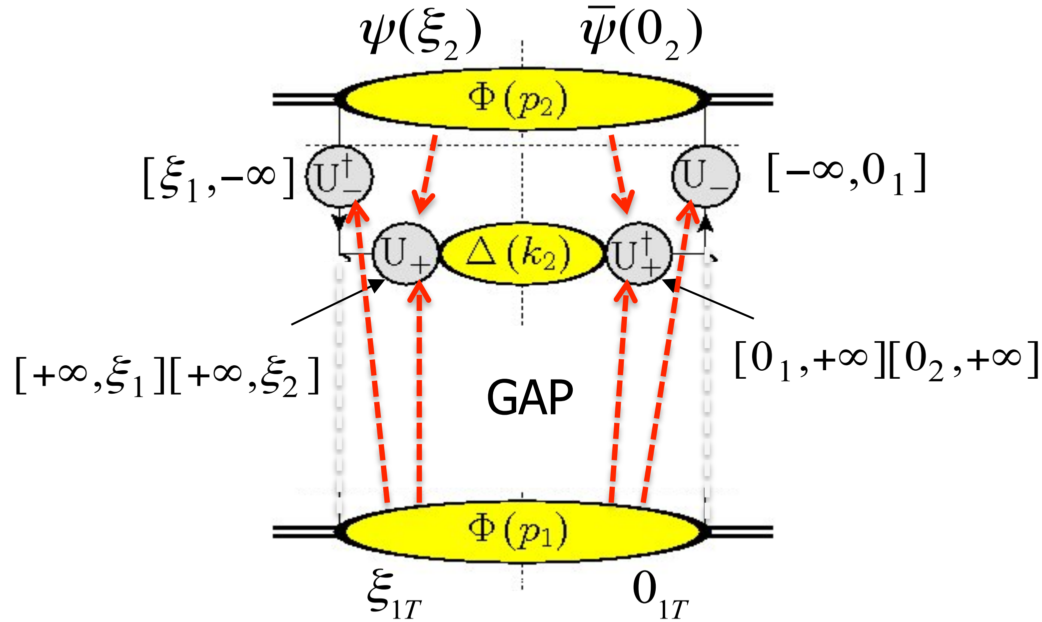


- Momentum flow in case of diffraction
  $x_1 \rightarrow M_X^2/W^2 \rightarrow 0$ and $t \rightarrow p_{1T}^2$
- Picture in terms of TMD and inclusion of gauge links
  (including gauge links/collinear gluons in M ~ S – 1)

- (Another way of looking at diffraction, cf Dominguez, Xiao, Yuan 2011 or older work of Gieseke, Qiao, Bartels 2000)

A TMD picture for diffractive scattering





- Cross section

$$d\sigma = \Phi(p_{1T};P)\,\mathrm{Tr}_c\left[U_-^\dagger[p_1]U_+[p_1,p_2]\,U_+^\dagger[p_1,p_2]U_-[p_1]\Phi(x_2,p_{2T};q)\right]$$

- involving correlators for proton and photon

$$\Phi^{q/\gamma[+]}(x_2,p_{2T};q) = \int \frac{d(\xi.q)d^2\xi_T}{(2\pi)^3}e^{i\,p_2.\xi_2}\left\langle \gamma^*(q)\middle|\bar{\psi}(0)U_{[0,\xi]}^{[+]}\psi(\xi)\middle|\gamma^*(q)\right\rangle_{\xi.n=0}$$

$$\Phi_{DIF}^{[loop]}(x_1,p_{1T};P) = \delta(x_1)\int \frac{d^2\xi_T}{(2\pi)^2}e^{i\,p_{1T}.\xi}\left\langle P\middle|U^{[loop]}-1\middle|P\right\rangle_{\xi.n=0}$$