

Perturbative QCD from the LHC to the EIC

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QCD Evolution 2017

May 22, 2017

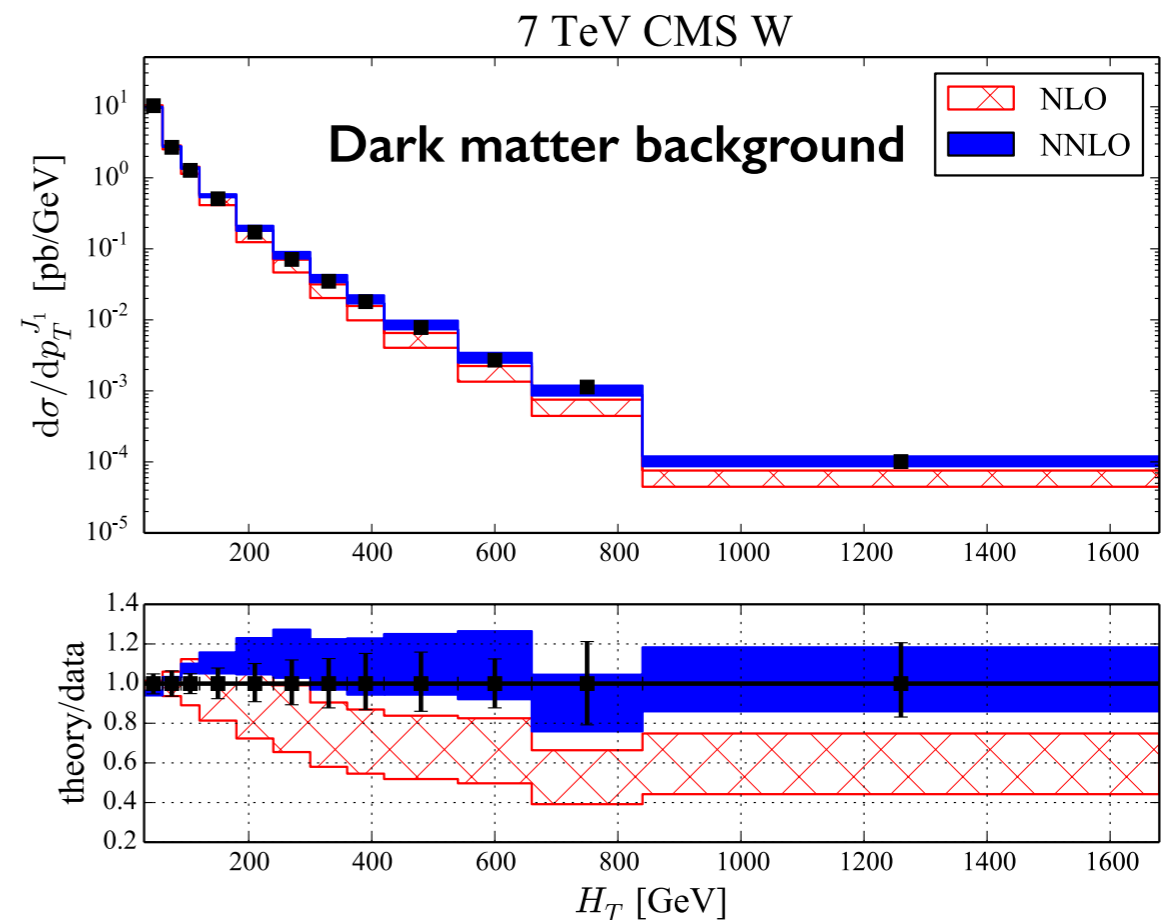
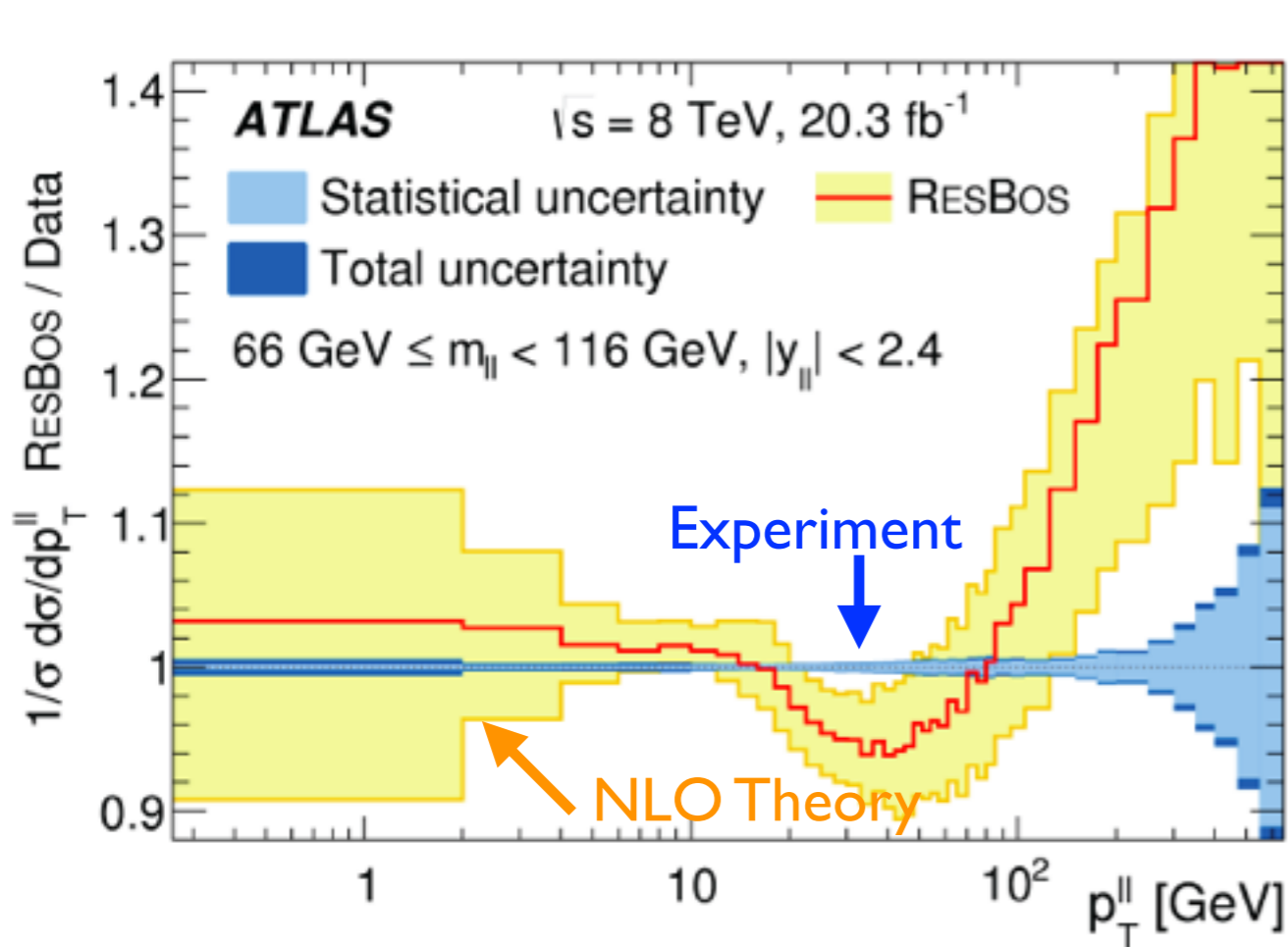


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Precision pQCD at the LHC

- Higher-order QCD to the next-to-next-to-leading order (NNLO) is recognized as a critical tool for the high energy physics program. Precision of the experimental data demands this matching theoretical precision.



Boughezal, Liu, FP (2016)

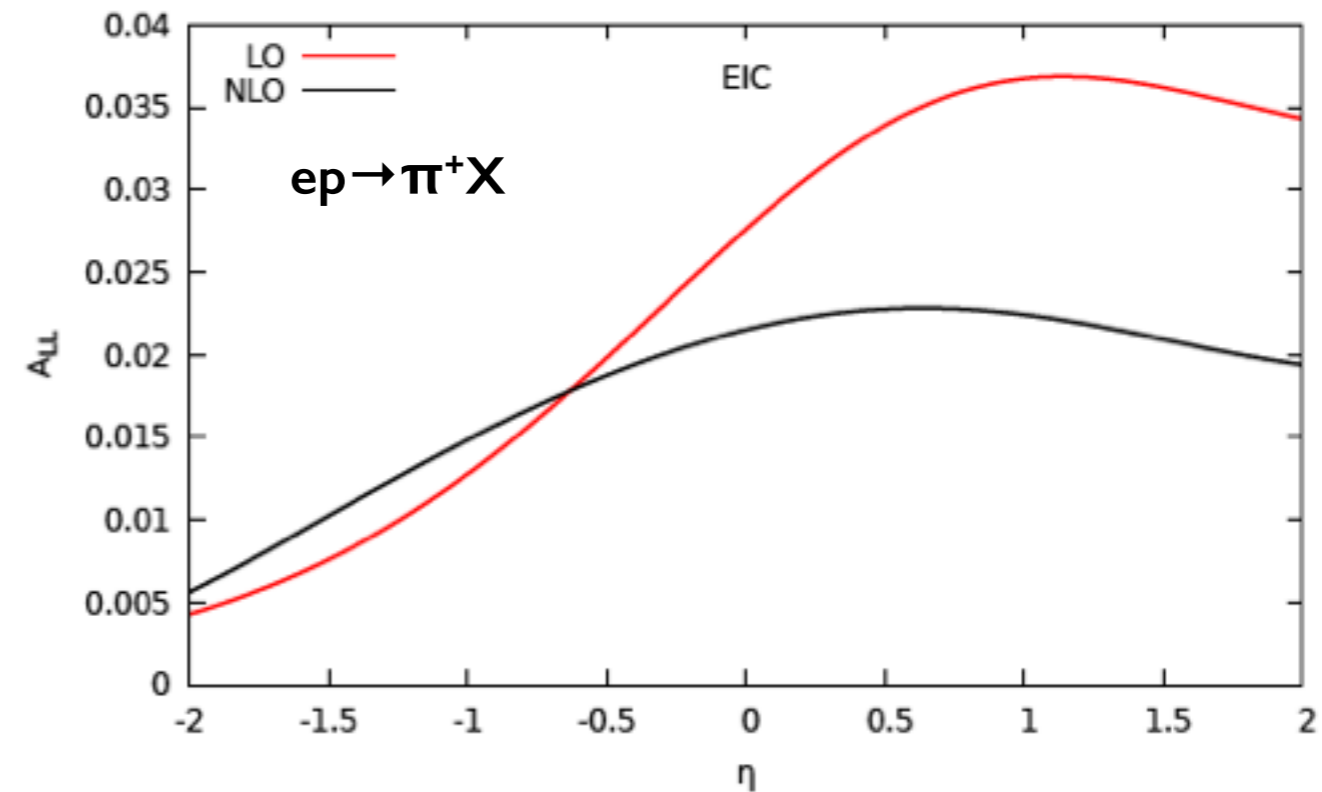
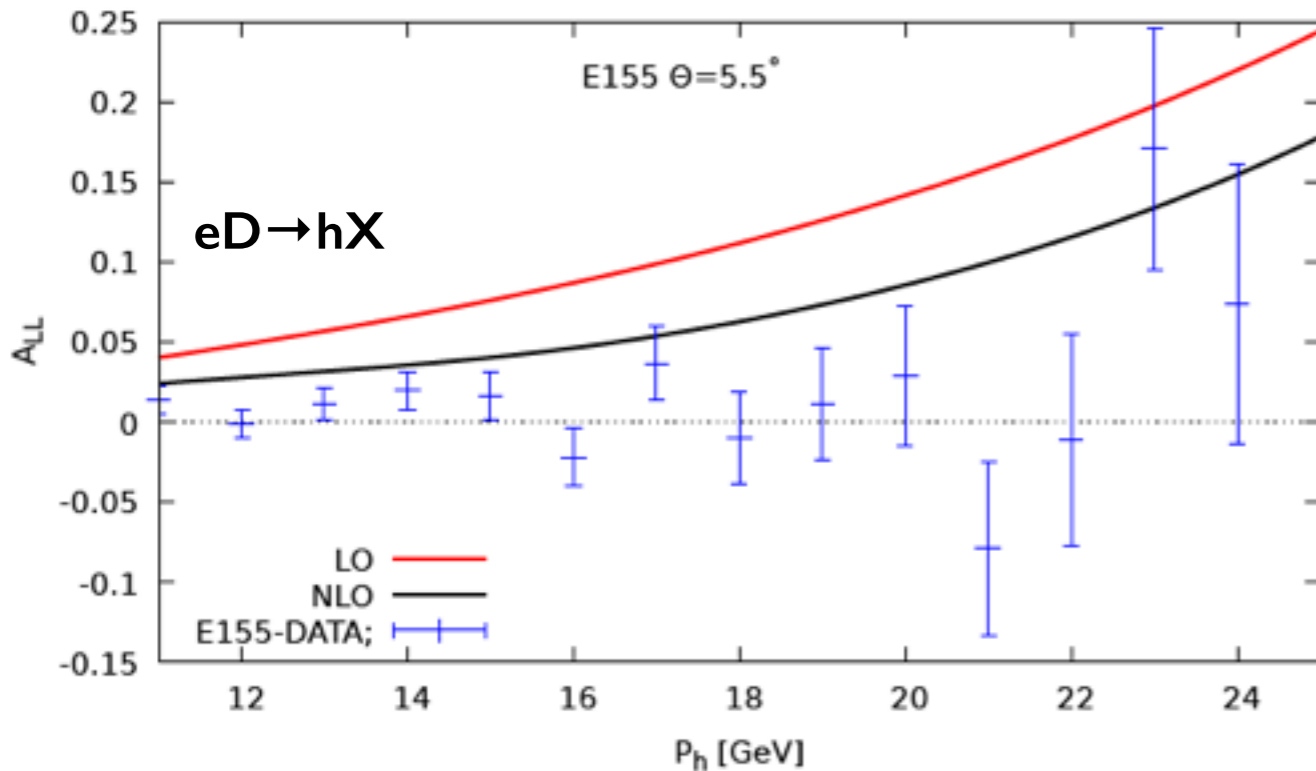
Can't describe increasingly precise LHC data without NNLO!

pQCD in lower-energy DIS

- Double longitudinal spin asymmetry:

$$A_{LL} = \frac{\sigma(l^+N^+ \rightarrow hX) - \sigma(l^-N^+ \rightarrow hX)}{\sigma(l^+N^+ \rightarrow hX) + \sigma(l^-N^+ \rightarrow hX)}$$

Hinderer, Schlegel, Vogelsang 1703.10872



(a)

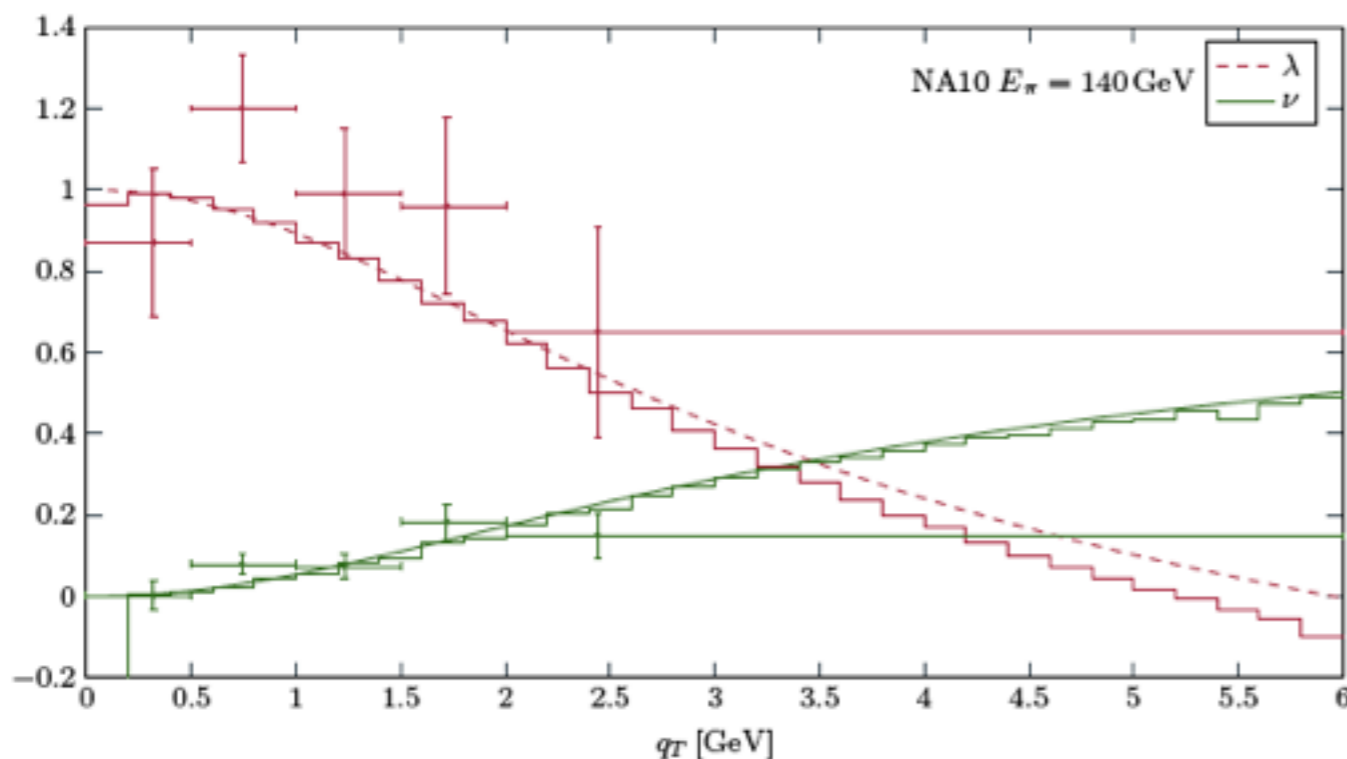
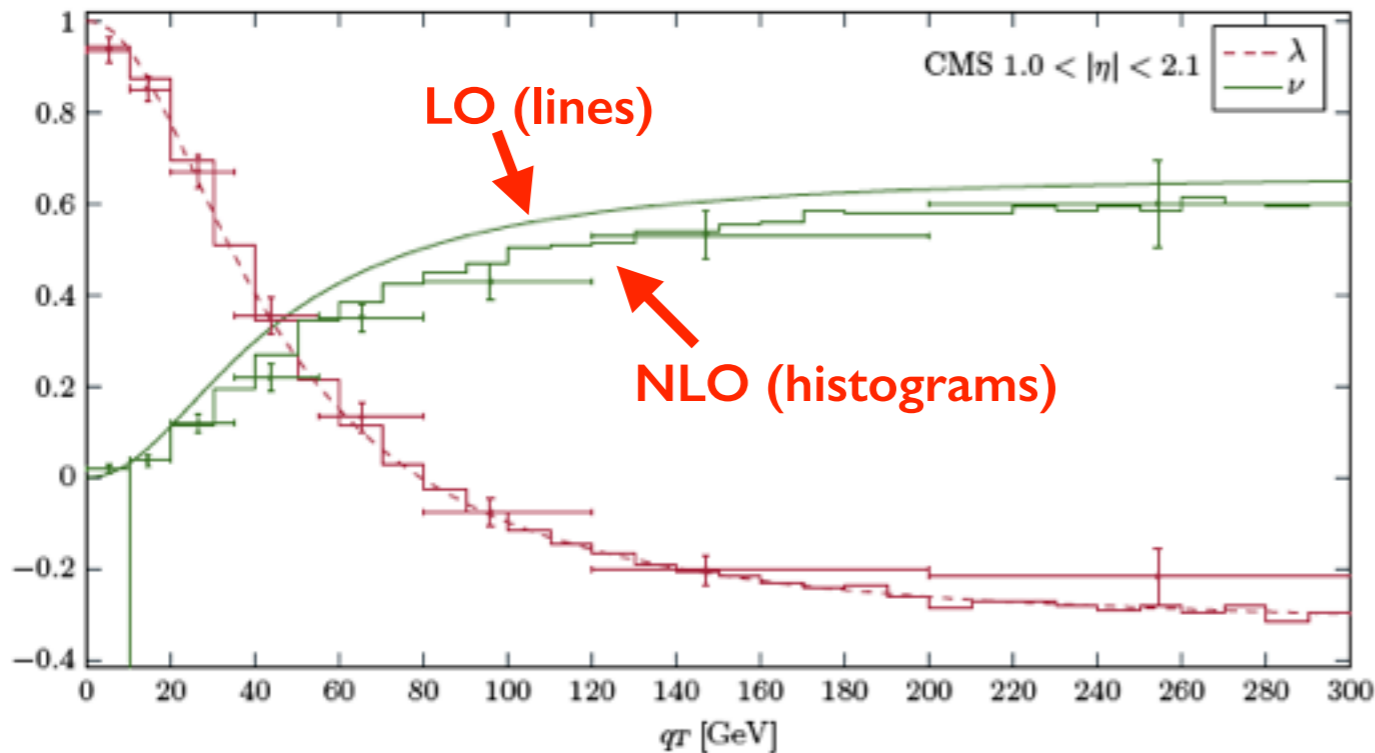
Large corrections predicted for future EIC kinematics!

Significant NLO corrections with important impact on theory/data agreement; higher orders still needed?

see Thursday talk by M. Schlegel

Drell-Yan from high to low energies

Lambertsen, Vogelsang 1605.02625



- Drell-Yan angular coefficients:

$$\frac{dN}{d\Omega} = \frac{3}{4\pi} \frac{1}{\lambda + 3} \left[1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi \right]$$

- pQCD framework describes the DY angular coefficients from p_T of a few GeV to hundreds of GeV

- Same code used to describe all data!
FEWZ: Melnikov, FP (2006); Gavin et al. (2010)

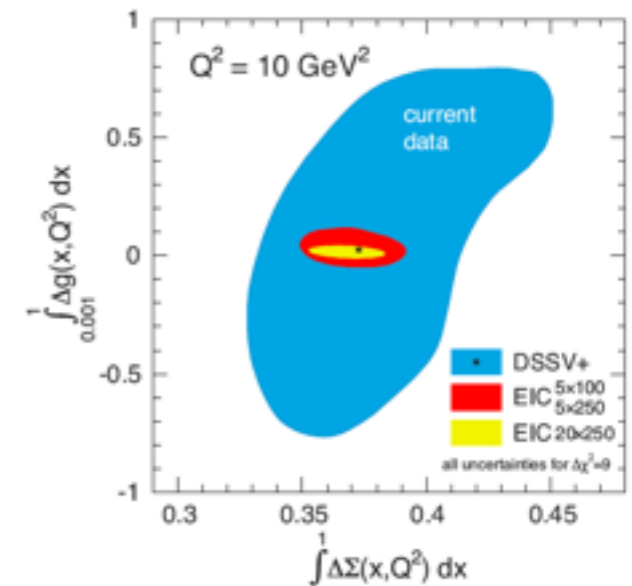
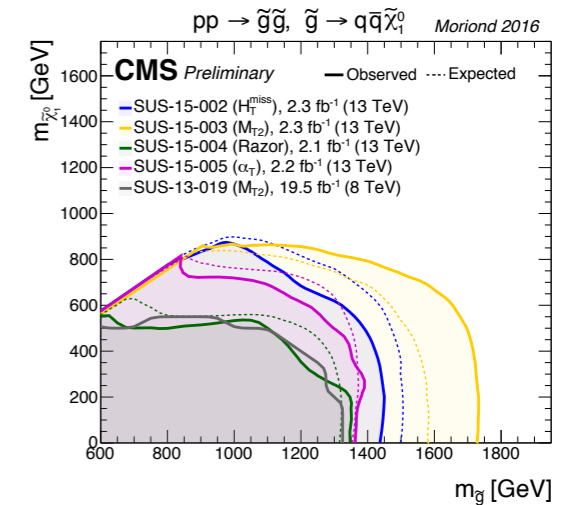
Tools and techniques developed for LHC can help at lower-energy machines such as the EIC!

The role of pQCD

- Similar roles of pQCD at the LHC and at a future EIC:

LHC: disentangle pQCD effects from new beyond-the-SM phenomena (SUSY, dark matter, extra dimensions, ...)

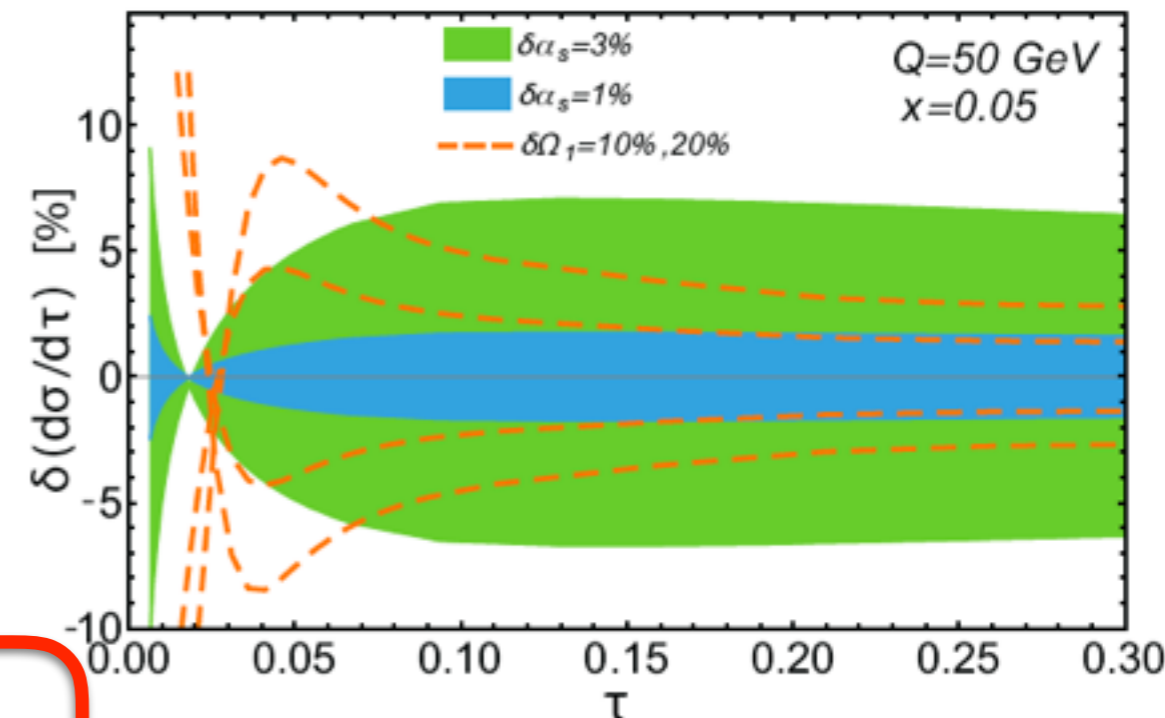
EIC: disentangle pQCD effects from the measurements of proton structure (helicity PDFs, higher-twist, ...)



Extracted TMDs depend on the scheme (UV) and perturbative order (LO, NLO, NNLL, ...), at which the hard part and the TMD evolution kernels, were calculated!

An example: jet physics at the EIC

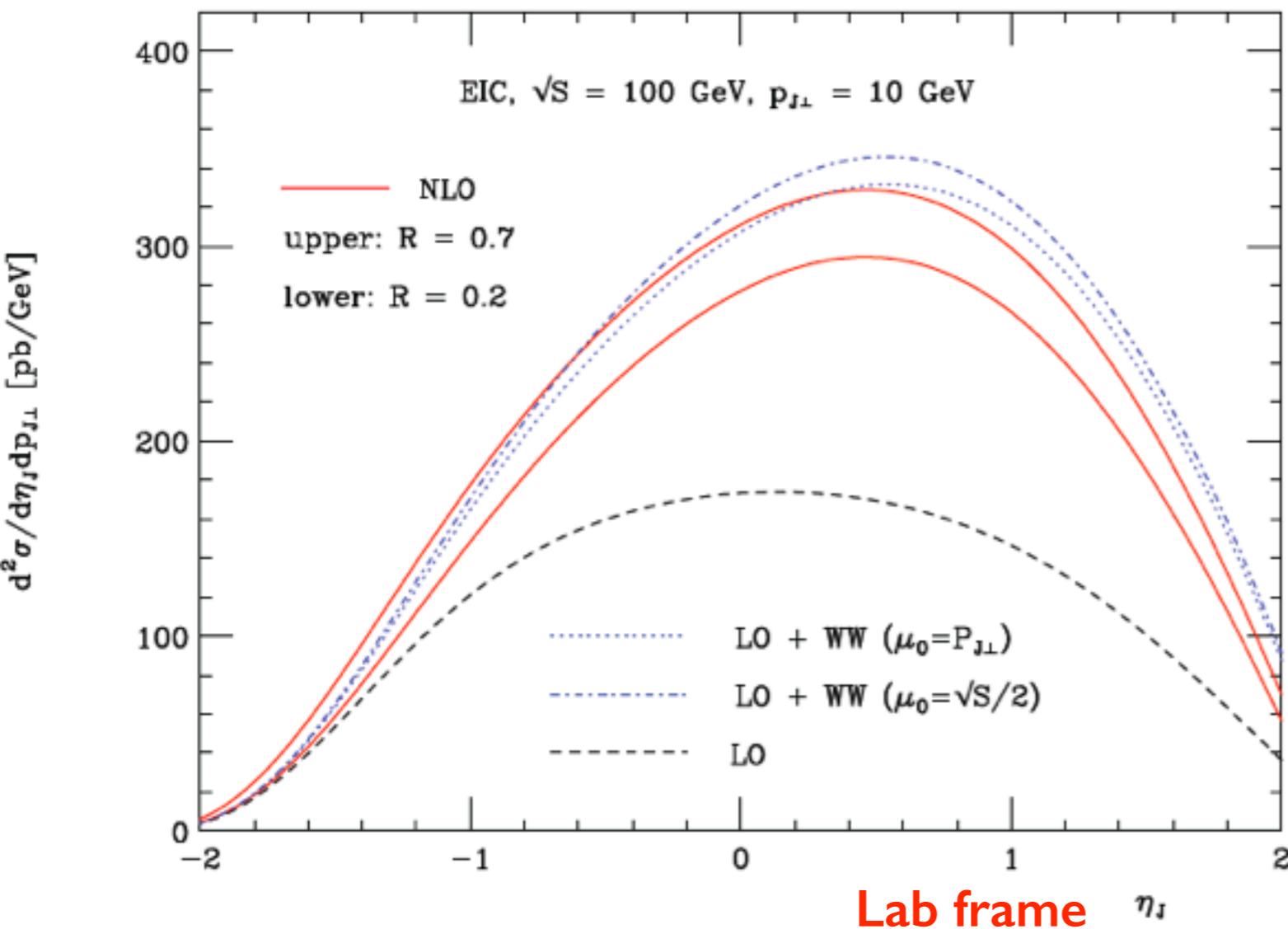
- Numerous physics motivations for studying jet production at a future EIC
- Measurement of the strong coupling constant [D. Kang, Lee, Stewart \(2013\)](#)
- Determination of higher-twist properties of the proton [Z. Kang, Metz, Qiu, Zhou \(2011\)](#)
- Determination of parton distribution functions
- Measure properties of the nuclear medium with event shapes [Z. Kang, Liu, Mantry, Qiu \(2012\)](#)



The precision of an EIC plays a critical role in all of these measurements!

The challenge: large corrections

Hinderer, Schlegel, Vogelsang 1505.06415



- Large NLO perturbative corrections, $O(100\%)$
- Important, but not dominant, corrections from photon-initiated processes
- Does the perturbative series converge at NNLO?
- Are the NNLO corrections dominated by a single channel?

Goals:

- Investigate the NNLO corrections to EIC jet production for its intrinsic interest
- Show that new techniques from LHC can also enable precision EIC studies

Definition of the process

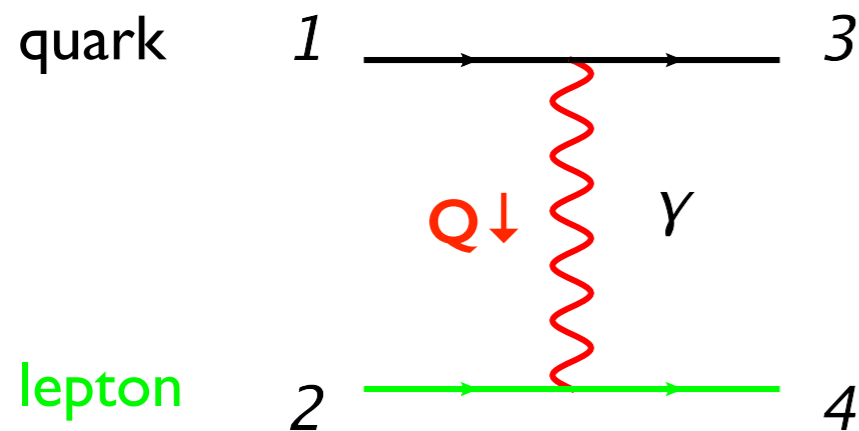
DIS: $eN \rightarrow eN$

- lepton tagged
- Cut on Q^2
- hard scale: Q

Inclusive jet production: $eN \rightarrow jX$

- lepton *not* tagged
- Cut on p_{Tjet}
- hard scale: p_{Tjet}

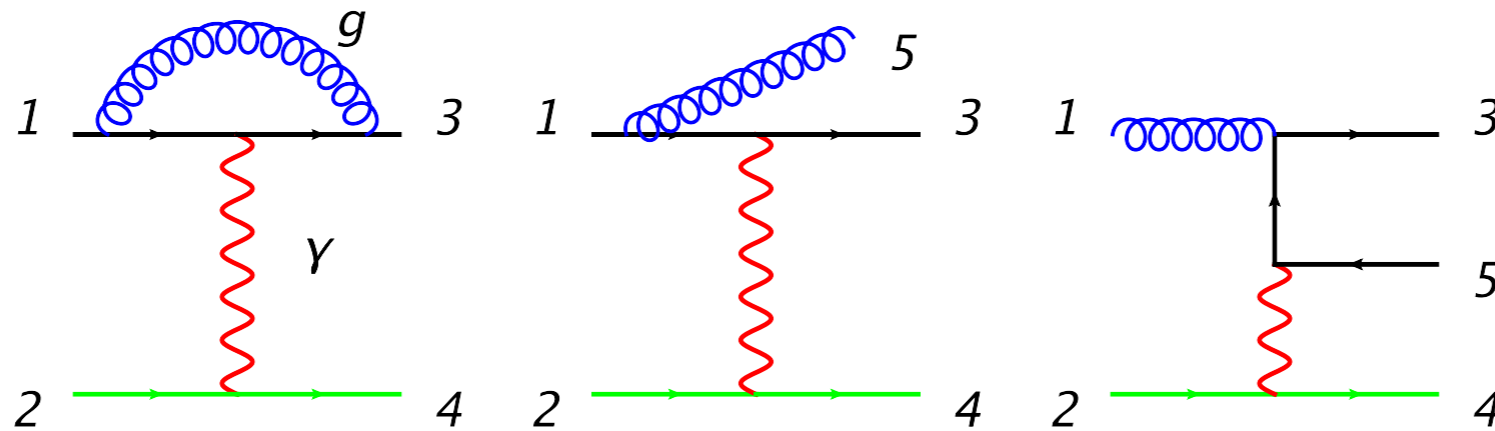
$$q(p_1) + l(p_2) \rightarrow q(p_3) + l(p_4)$$



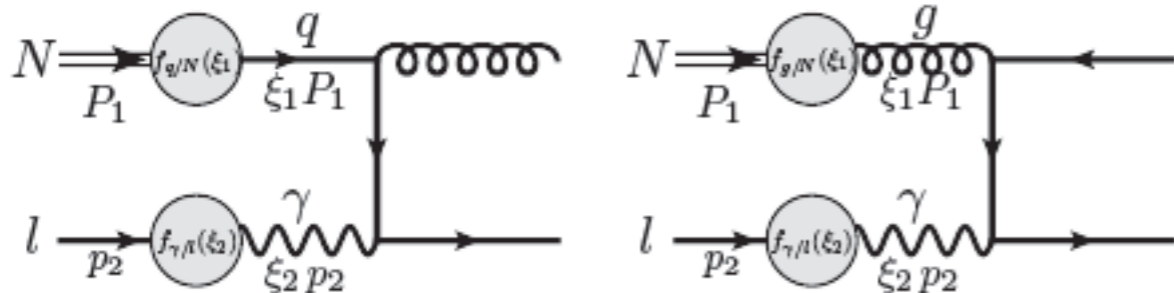
- **Leading order:** identical for both processes, lepton recoils against a jet

NLO $\mathcal{O}(\alpha^2\alpha_s)$ corrections

- Typical real and virtual corrections to the quark-lepton scattering processes; new contribution from gluon-lepton scattering \rightarrow calculation amenable to standard techniques



- **New configuration:** lepton collinear to the beam ($Q^2 \sim 0$), with two jets balancing in the transverse plane; on-shell photon scattering with quark \rightarrow differentiates DIS and inclusive jet production

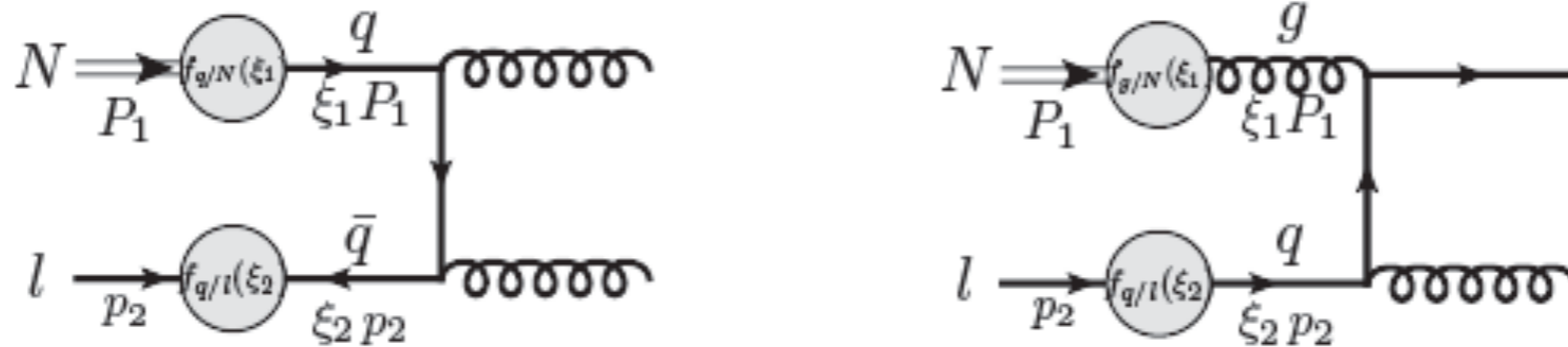


$$f_{\gamma/l}(\xi) = \frac{\alpha}{2\pi} P_{\gamma l}(\xi) \left[\ln \left(\frac{\mu^2}{\xi^2 m_l^2} \right) - 1 \right] + \mathcal{O}(\alpha^2)$$

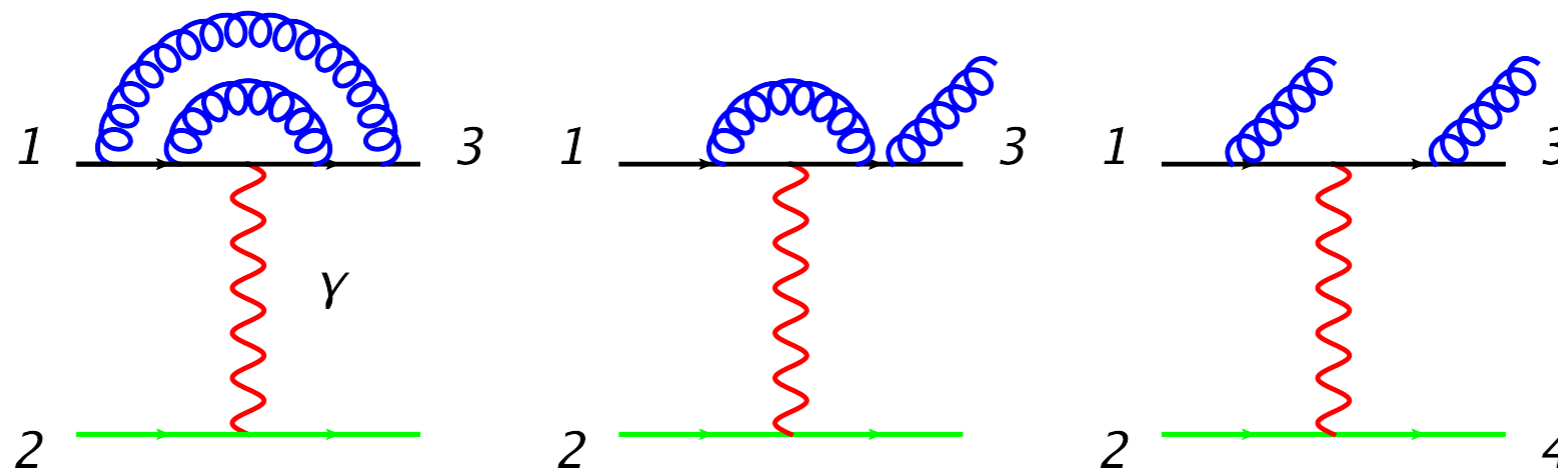
$$P_{\gamma l}(\xi) = \frac{1 + (1 - \xi)^2}{\xi}$$

NNLO $O(\alpha^2\alpha_s^2)$ corrections

- **New configuration:** incoming lepton can split into a quark, leading to parton-parton scattering channels. They first appear at this order, and are therefore effectively leading order in our treatment.



- Standard NLO corrections to quark-photon scattering
- Double-virtual, real-virtual, and double-real corrections to quark-lepton scattering



NNLO $O(\alpha^2\alpha_s^2)$ corrections

- **New configuration:** incoming lepton can split into a quark, leading to parton-parton scattering channels. They first appear at this order, and are therefore effectively leading order in our treatment.

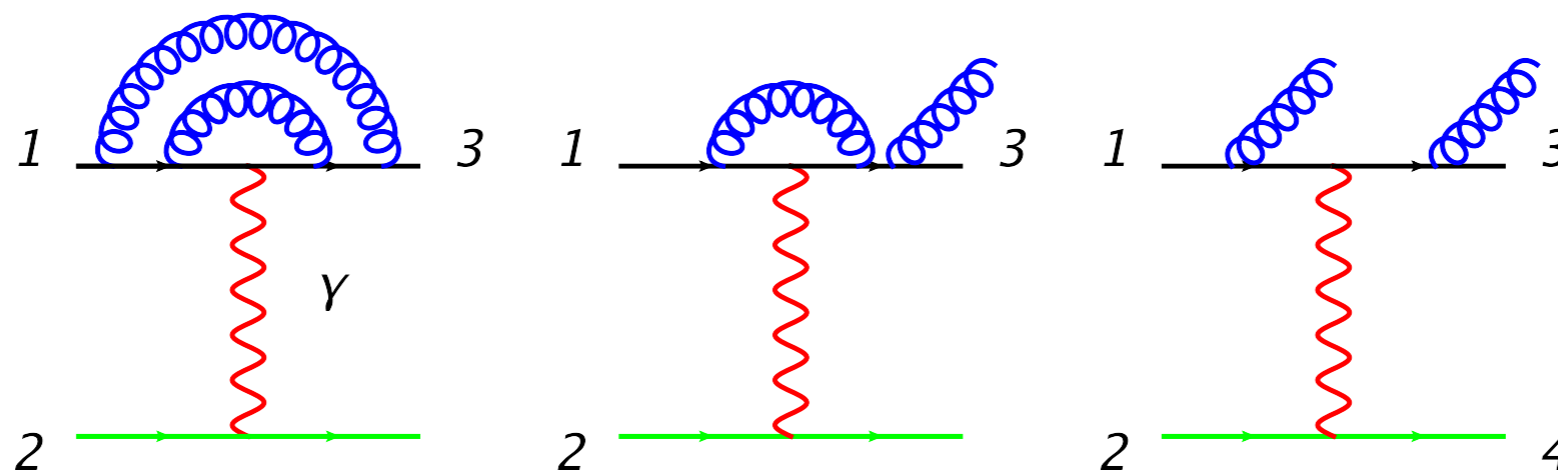
All contributions separately divergent, with numerous singular configurations (triple-collinear, double-soft, soft+collinear, etc.)
 How do we regularize and cancel to arrive at a finite result?

- Standard

- Double

lepton scattering

quark-



NNLO subtraction

Enormous progress solving this problem for LHC physics!

- First complete predictions for LHC V+jet, Higgs+jet production at NNLO; partial results for di-jet production
- **N-jettiness subtraction**: (Boughezal, Focke, Liu, FP (2015); Gaunt, Stahlhofen, Tackmann, Walsh (2015))

$$\mathcal{T}_1 = \frac{2}{Q^2} \sum_i \min \{ p_B \cdot q_i, p_J \cdot q_i \}$$

Stewart, Tackmann, Waalewijn (2010);
D. Kang, Lee, Stewart (2013)

p_B : beam axis

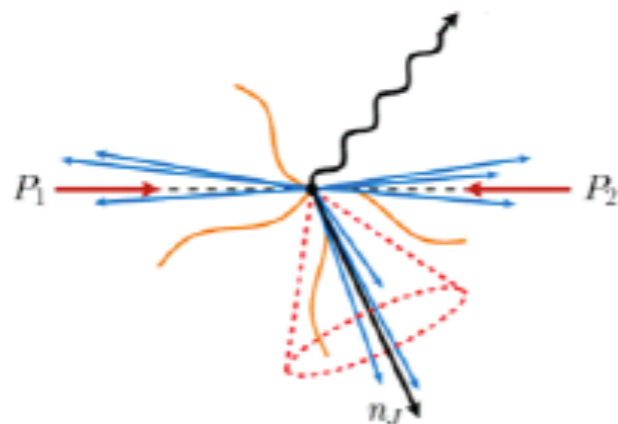
p_J : leading-jet axis

q_i : outgoing parton momenta

1 jet ← Small \mathcal{T}_1 → Large → At least 2 jets

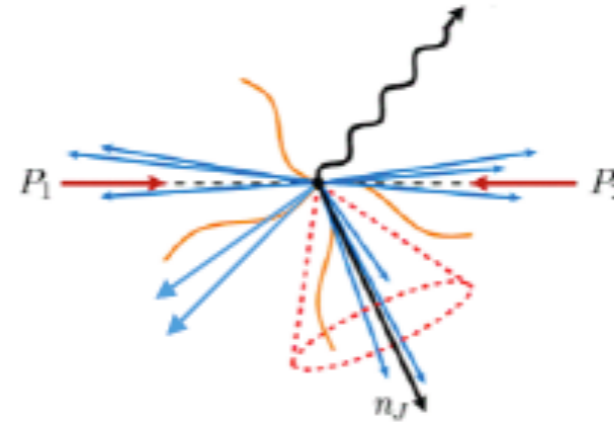
Contributions from

- Two-loop
- Soft and collinear radiation



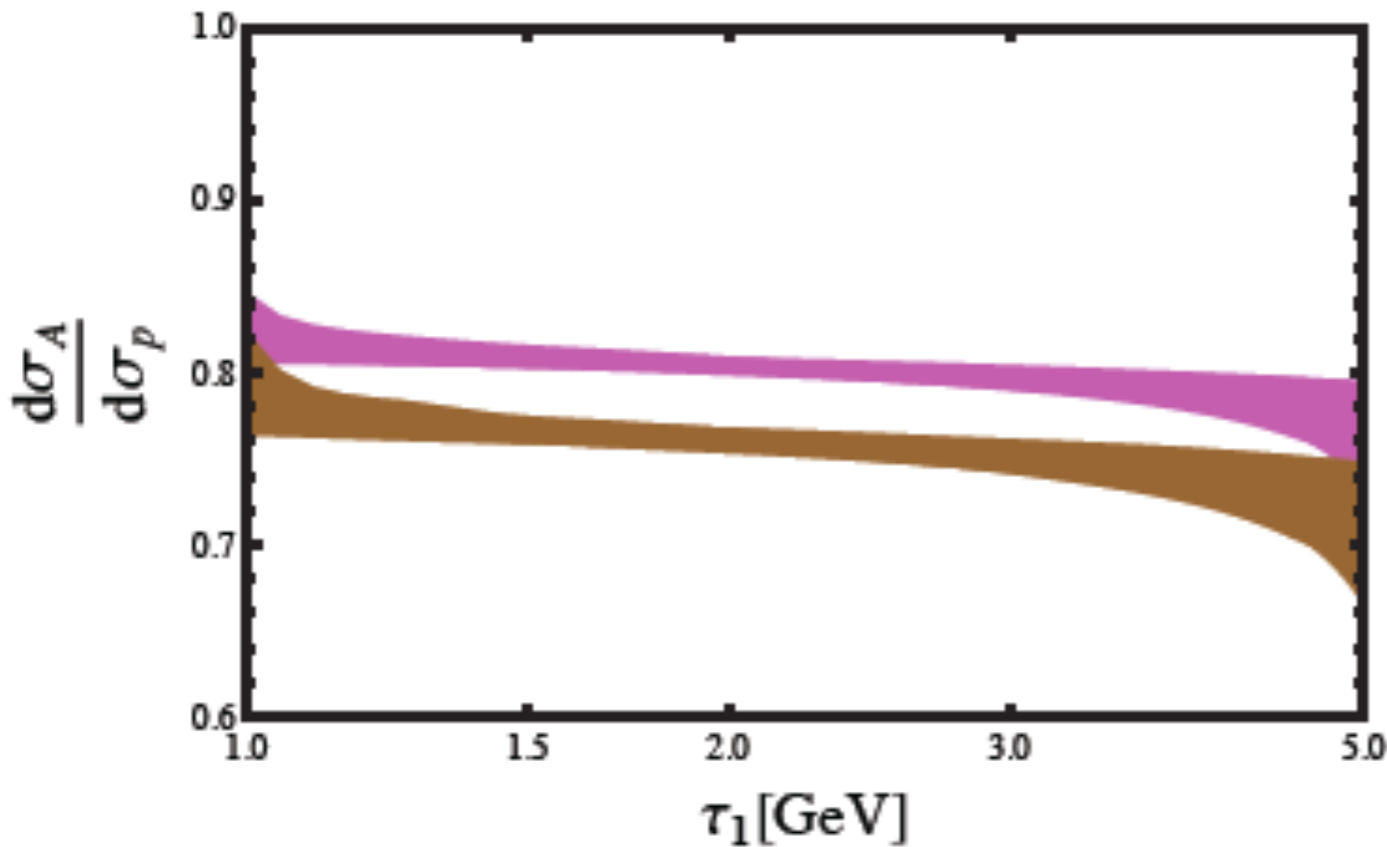
Contributions with at least 2 hard jets

- NLO two-jet calculation. Use known results/tools



Other N-jettiness applications

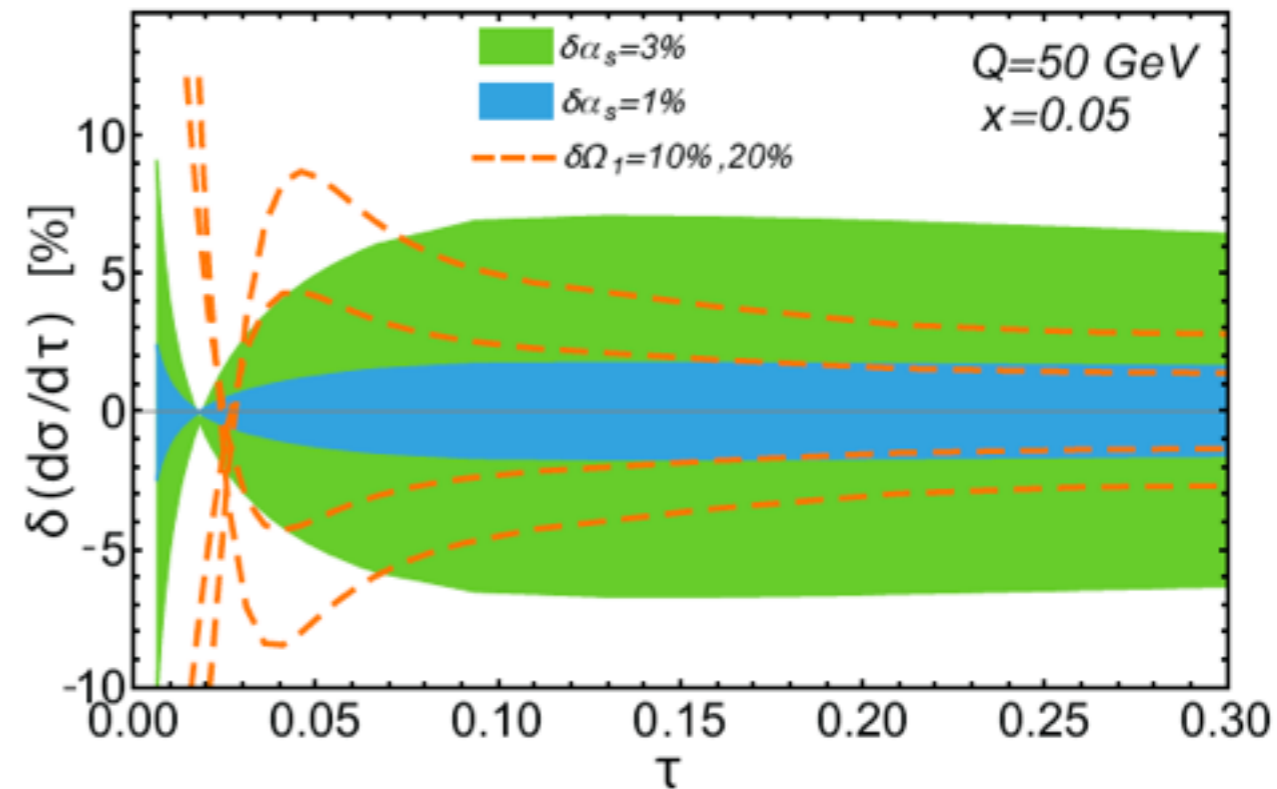
- N-jettiness measurements are of intrinsic interest at a future EIC



(f) C and Ur

As a probe of nuclear modifications of PDFs

Kang, Mantry, Qiu I204.5469;
Kang, Liu, Mantry, Qiu I303.3063



As a handle on the strong coupling constant

D. Kang, Lee, Stewart I303.6952

N-jettiness subtraction

- N-jettiness can be applied to obtain exact NNLO cross sections
- Introduce τ_N^{cut} that separates the $\tau_N=0$ doubly-unresolved limit of phase space from the single-unresolved and hard regions

$$\begin{aligned}\sigma_{NNLO} &= \int d\Phi_N |\mathcal{M}_N|^2 + \int d\Phi_{N+1} |\mathcal{M}_{N+1}|^2 \theta_N^< \\ &+ \int d\Phi_{N+2} |\mathcal{M}_{N+2}|^2 \theta_N^< + \int d\Phi_{N+1} |\mathcal{M}_{N+1}|^2 \theta_N^> \\ &+ \int d\Phi_{N+2} |\mathcal{M}_{N+2}|^2 \theta_N^> \\ &\equiv \sigma_{NNLO}(\mathcal{T}_N < \tau_N^{\text{cut}}) + \sigma_{NNLO}(\mathcal{T}_N > \tau_N^{\text{cut}})\end{aligned}$$

$$\theta_N^< = \theta(\tau_N^{\text{cut}} - \tau_N) \quad \text{and} \quad \theta_N^> = \theta(\tau_N - \tau_N^{\text{cut}})$$

N-jettiness subtraction

- For $\tau_N > \tau_N^{\text{cut}}$, at least one of the two additional radiations that appear at NNLO is resolved; this region of phase space contains the NLO correction to the N+1 jet process. **A solved problem!**
- For $\tau_N < \tau_N^{\text{cut}}$, both additional radiations are unresolved. A factorization theorem giving the all-orders result for small N-jettiness was derived
[Stewart, Tackmann, Waalewijn 0910.0467](#)

$$\frac{d\sigma}{d\mathcal{T}_1} = \int d\Phi_B \int dt_J dt_B dk_S \delta \left(\mathcal{T}_1 - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q} \right) \times \sum_q J_q(t_J, \mu) S(k_S, \mu) H_q(\Phi_2, \mu) B_q(t_B, x, \mu) + \dots$$

H: describes hard radiation; in dim-reg, coincides with the 2-loop virtual corrections

B: describes radiation collinear to an initial-state beam

S: describes soft radiation

J: describes radiation collinear to a final-state jet

- The ellipses denote power corrections that become negligible for small τ_N^{cut}

Ingredients for the factorization theorem

$$\frac{d\sigma}{d\mathcal{T}_1} = \int d\Phi_B \int dt_J dt_B dk_S \delta \left(\mathcal{T}_1 - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q} \right) \\ \times \sum_q J_q(t_J, \mu) S(k_S, \mu) H_q(\Phi_2, \mu) B_q(t_B, x, \mu) + \dots$$

- Expand this formula to $\mathcal{O}(\alpha_s^2)$, and turn off all resummation, to get the NNLO cross section below the cut. Need each of these separate functions to NNLO.
- The beam and jet functions depend only on the flavor of the parton (quark, gluon); the soft function depends only on the parton flavors and the external hard directions; the hard function is the only process-dependent piece.
 - $H@NNLO$: Matsuura, van der Merck, van Nerven (1988)
 - $B@NNLO$: Gaunt, Stahlhofen, Tackmann (2014)
 - $S@NNLO$: Boughezal, Liu, FP (2015)
 - $J@NNLO$: Becher, Neubert (2006); Becher, Bell (2011)

Within the past two years all ingredients have become available to apply this idea to jet production at the EIC!

Unpolarized versus polarized collisions

- Schematic form of factorization theorem for unpolarized and longitudinally polarized collisions:

unpolarized: $d\sigma/d\tau \sim H \otimes B \otimes J \otimes S$

polarized: $d\Delta\sigma/d\tau \sim \Delta H \otimes \Delta B \otimes J \otimes S$

known helicity-dependent 2-loop virtual corrections

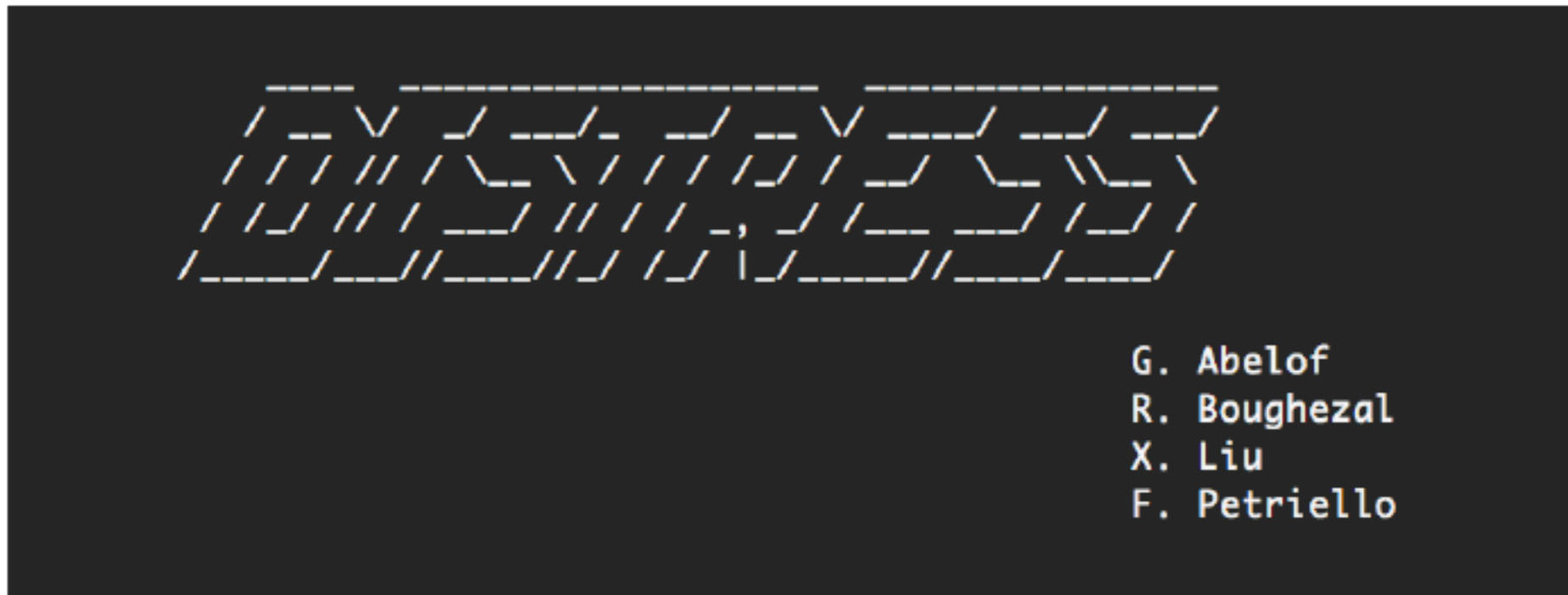
two-loop helicity-dependent beam function now known!

Boughezal, FP, Schubert, Xing | 704.05457

see Wednesday talk by H.Xing

DISTRESS

- **DISTRESS: DIS** Through a **R**obust **E**nabling **S**ubtraction **S**cheme



- Parton-level integrator for inclusive jet production in eN collisions
- Fully differential, allowing for arbitrary cuts on final-state jets/leptons
- Parallelized Monte Carlo integration
- Flexible framework allows for future extension to other processes

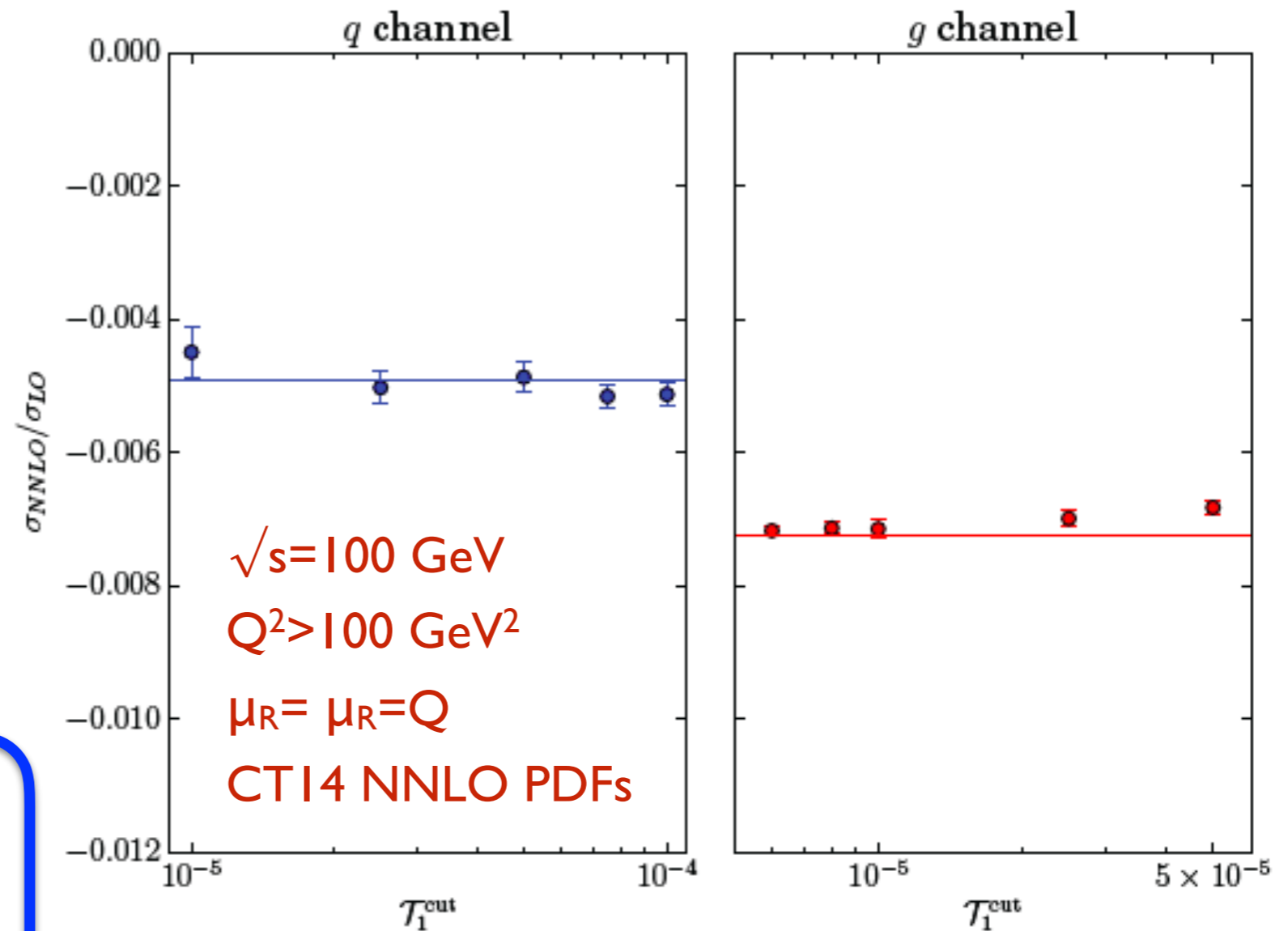
Validation

- We have two primary checks of our result at NNLO:

1. Independence of the full result from τ_1^{cut} ; also determines when power corrections are negligible
2. Upon integration over final-state radiation, must reproduce inclusive structure function

**Agreement with NNLO
structure function**

Zijlstra, van Neerven (1992);
Moch, Vermaseren (1999)



Numerics: setup and Q^2 distribution

- Study the predictions from DISTRESS for possible future EIC parameters:

$$\sqrt{s} = 100 \text{ GeV}$$

$$p_{T\text{jet}} > 5 \text{ GeV}$$

$$|\eta_{\text{jet}}| < 2.0$$

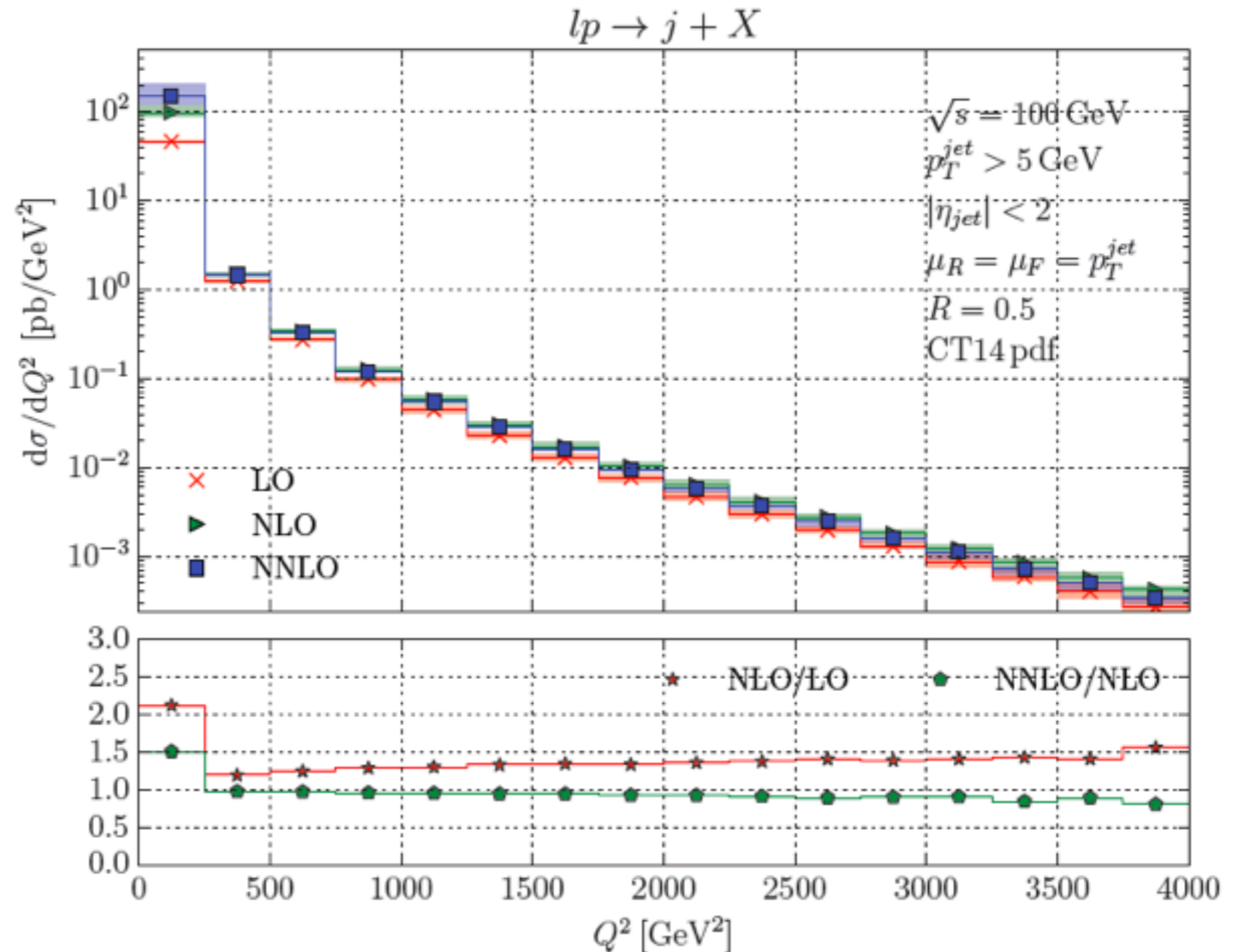
$$\text{Anti-}k_T, R=0.5$$

$$\mu_R = \mu_F = p_{T\text{jet}}$$

$$\alpha = 1/137.036$$

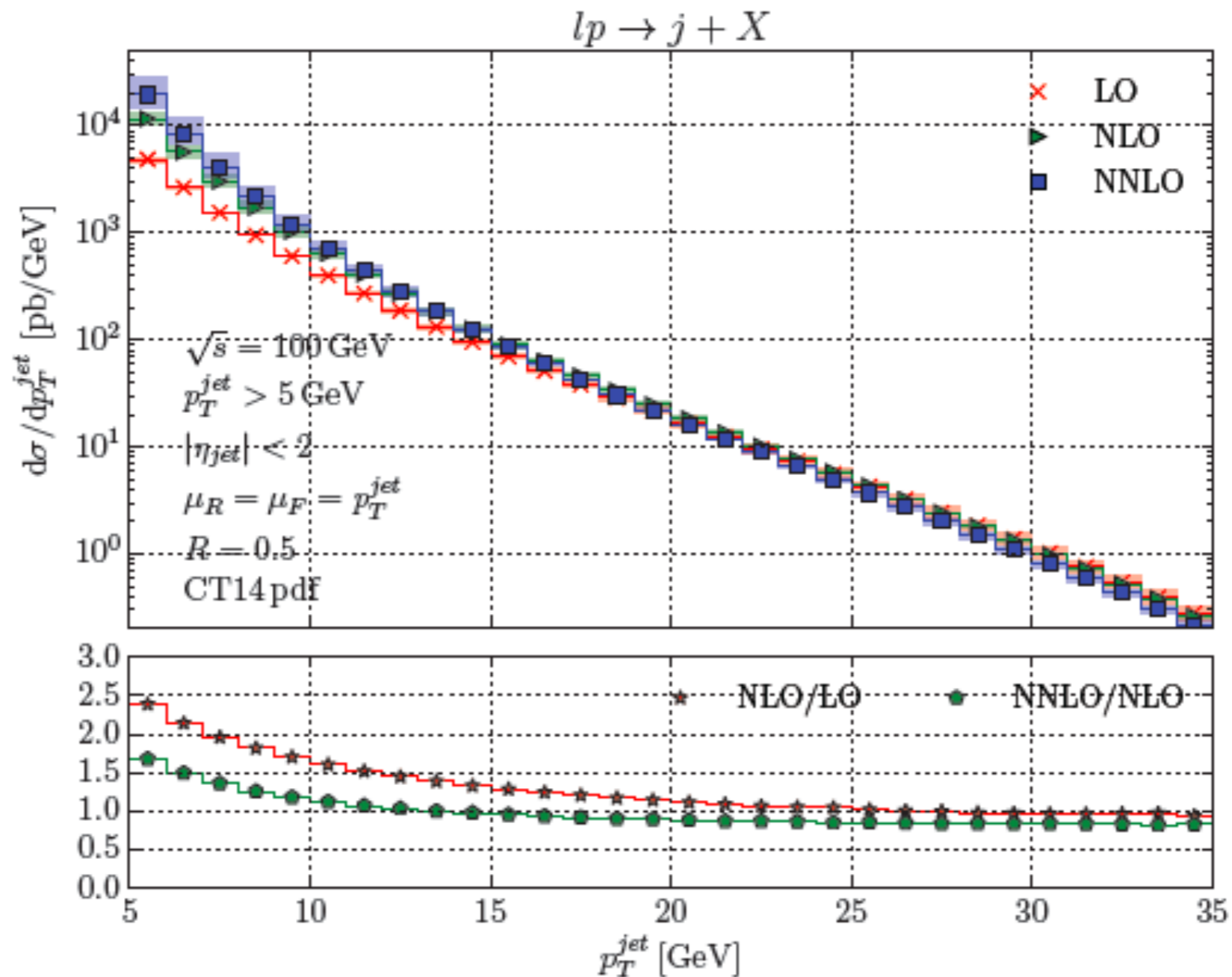
$$m_e = 0.511 \text{ MeV}$$

CT14 PDFs



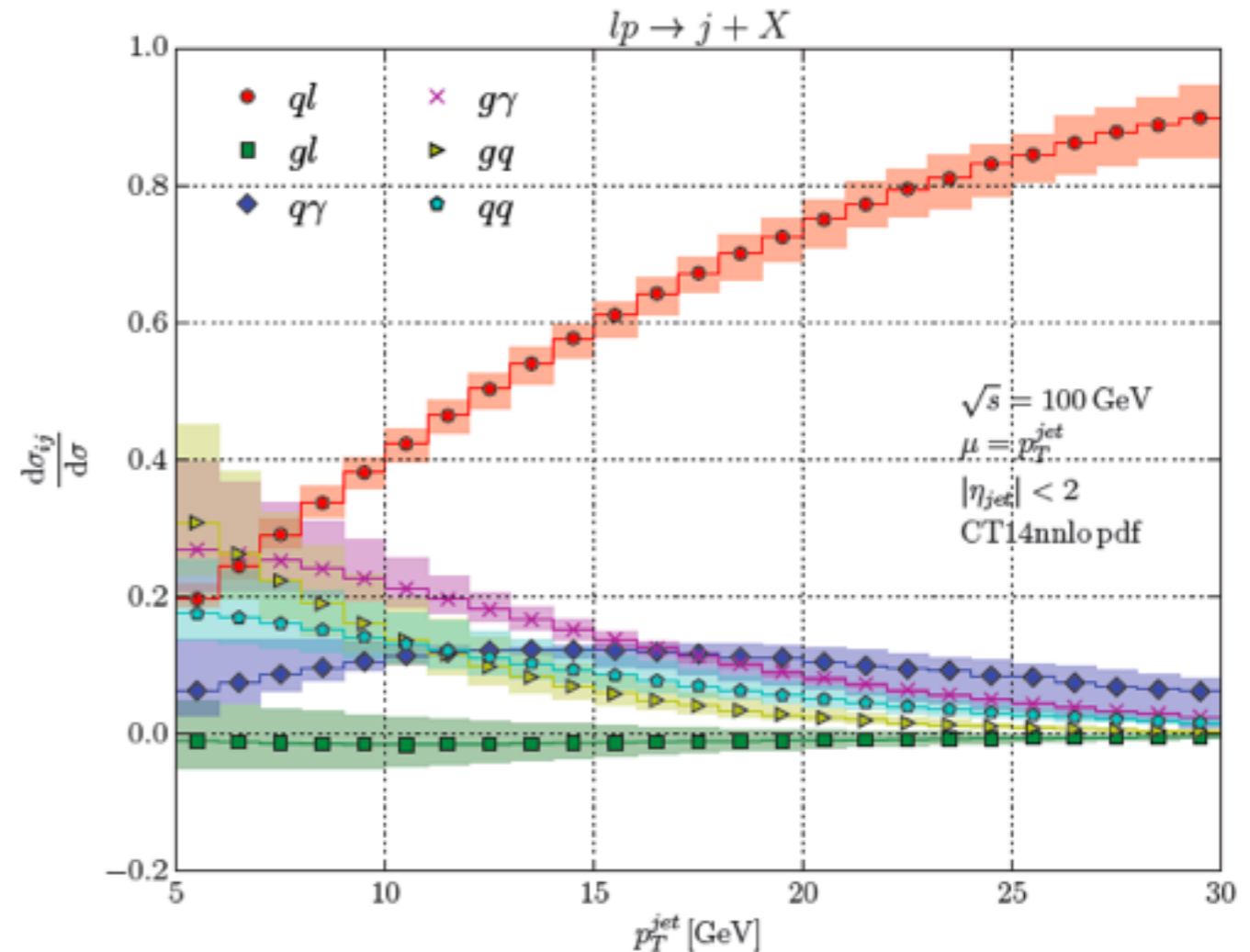
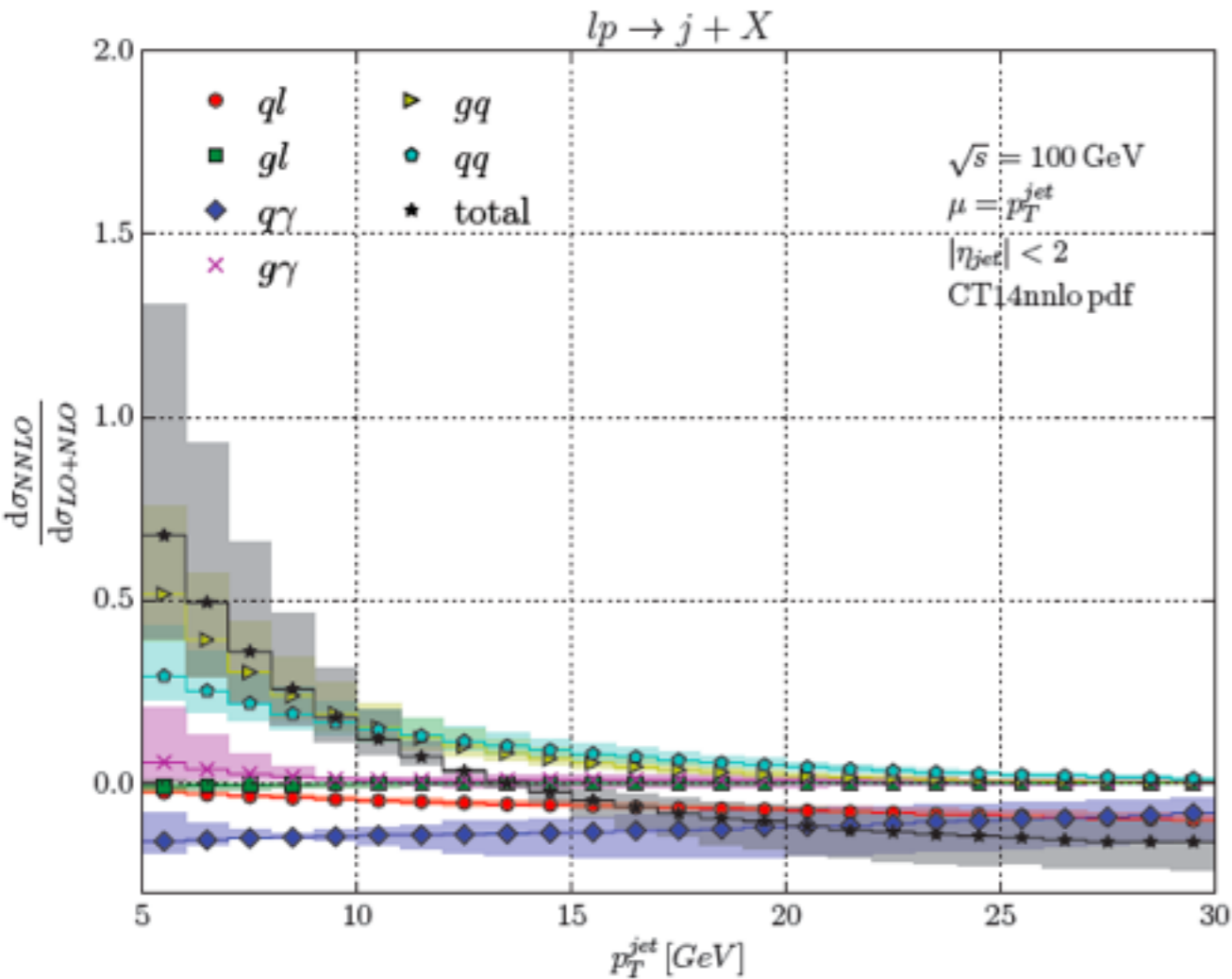
Large corrections at low Q^2 (photon-initiated processes)

Numerics: p_{Tjet} distribution



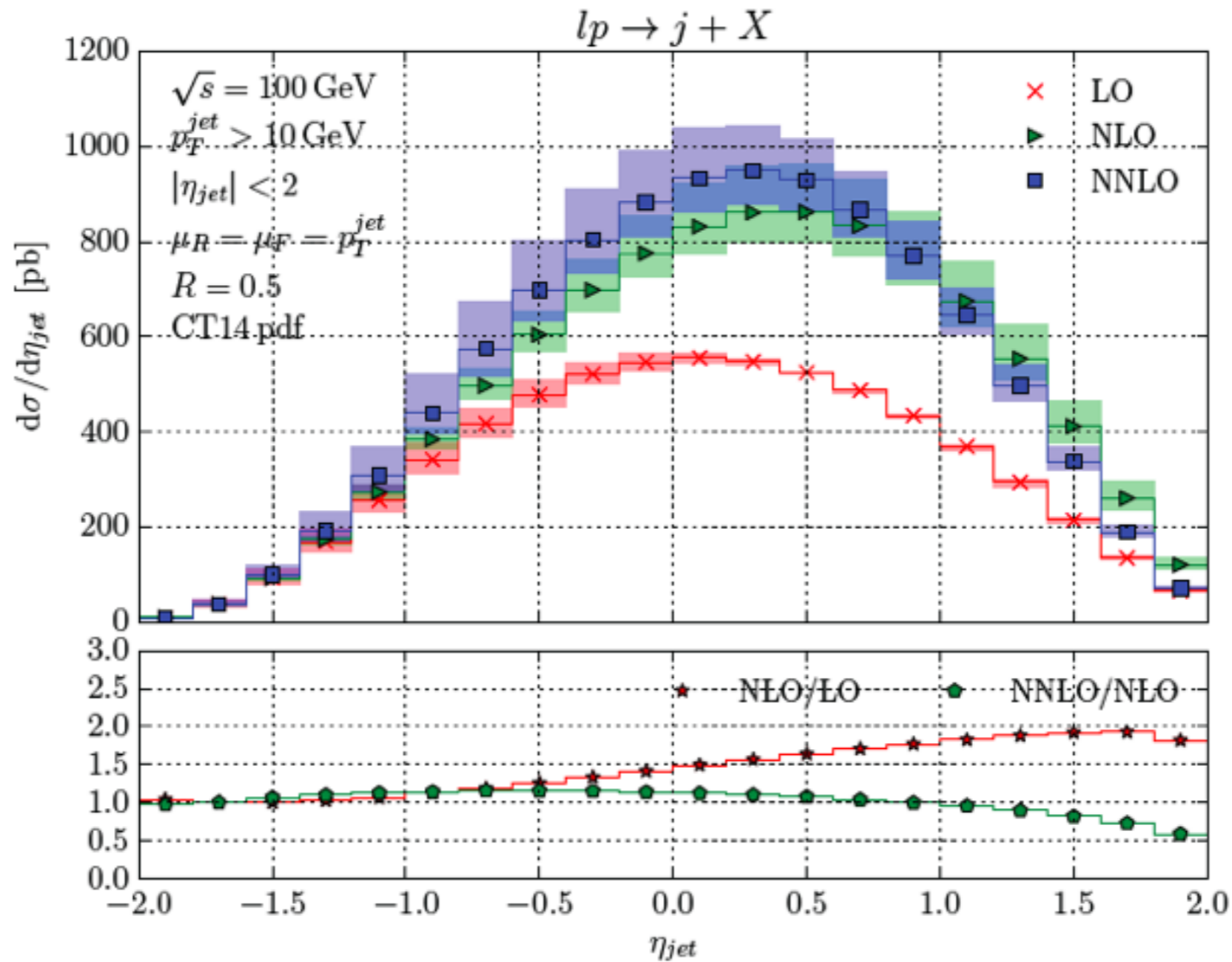
- NNLO large and positive for $p_{Tjet} < 10 \text{ GeV}$; near unity for large momenta
- Scale dependence *increases* at NNLO for $p_{Tjet} < 10 \text{ GeV}$

$p_{T\text{jet}}$ distribution: partonic channels



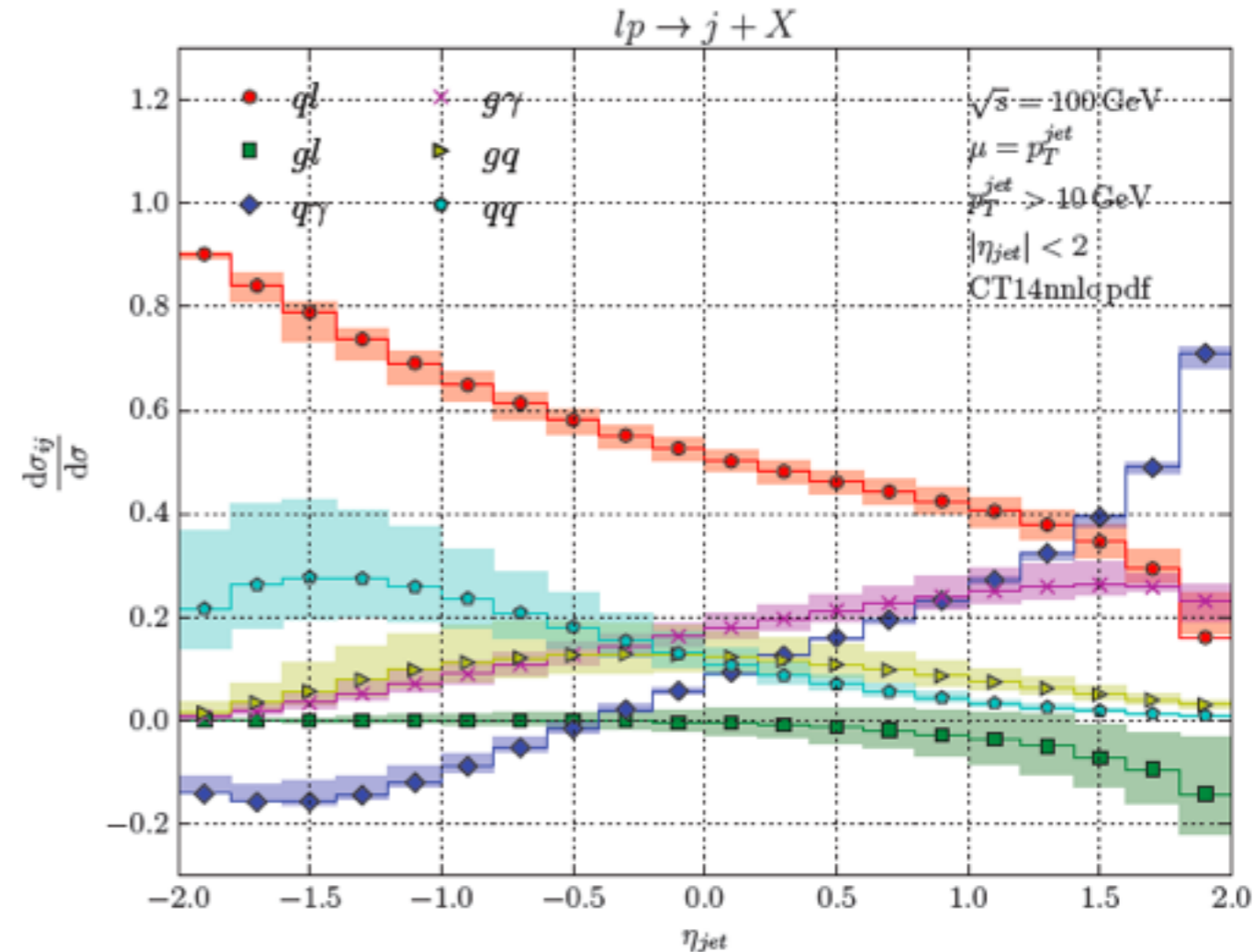
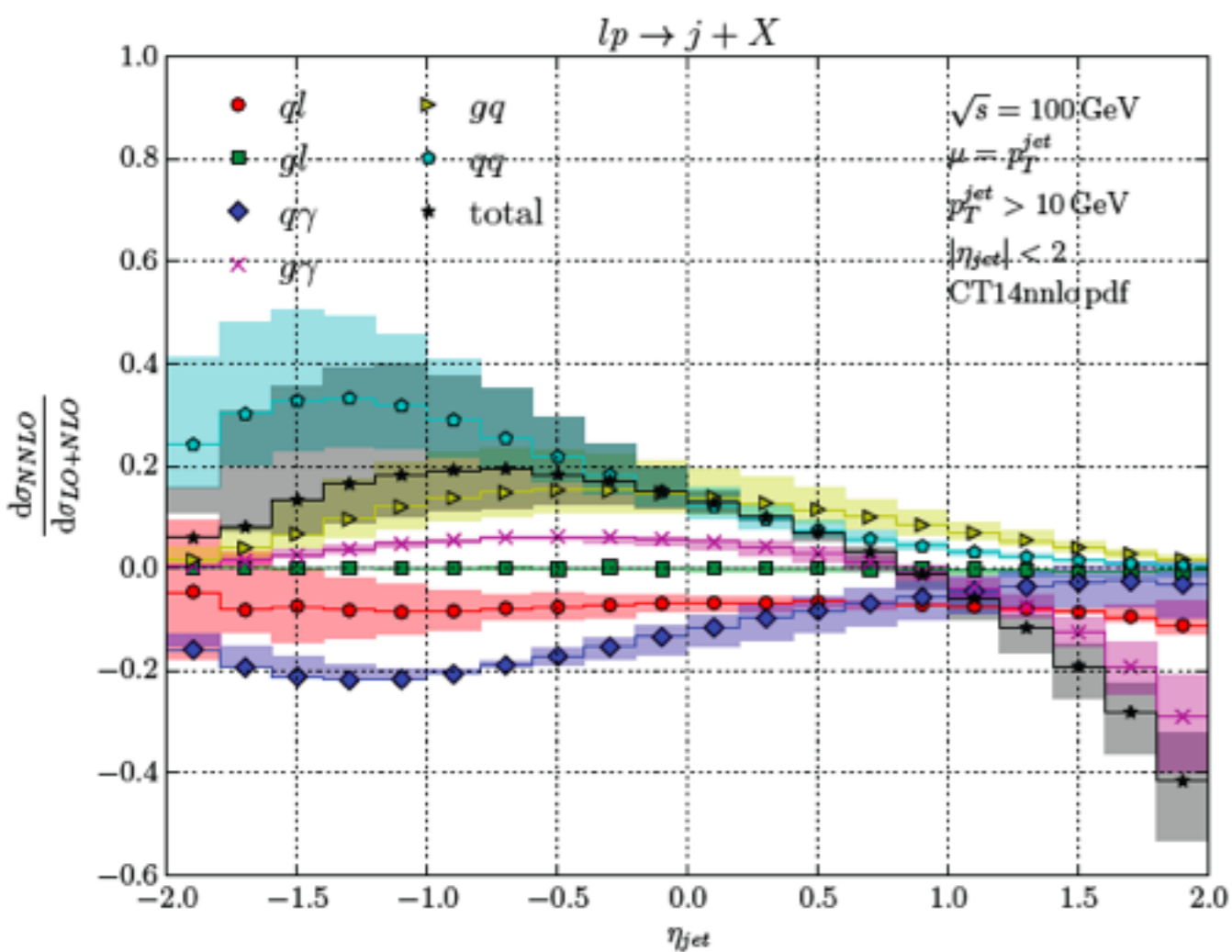
- qq and qg dominate the NNLO correction for low $p_{T\text{jet}}$
- These channels begin at $O(\alpha^2\alpha_s^2)$, are effectively leading-order in this result, and drive the increased scale dependence at NNLO
- ql channel dominates for high $p_{T\text{jet}}$
- **No single channel** furnishes a good approximation to the full result

η_{jet} distribution



- NNLO corrections small for $\eta_{\text{jet}} < 1$, but increase as $\eta_{\text{jet}} \rightarrow 2$
- Scale dependence *increases* at NNLO for $\eta_{\text{jet}} < 0$

η_{jet} distribution: partonic channels



- qq channel drives the large scale uncertainty for $\eta_{\text{jet}} < 0$; it begins at $O(\alpha^2\alpha_s^2)$, and is effectively leading-order in this result,
- ql channel dominates for low η_{jet} ; $q\gamma$ channel dominates at high η_{jet}
- **No single channel** furnishes a good approximation to the full result

Conclusions

- We have presented a calculation of the full $O(\alpha^2\alpha_s^2)$ corrections to inclusive jet production at a future EIC
- Our calculation allows for arbitrary final state cuts as is implemented in the pardon-level program **DISTRESS**
- The magnitude of the corrections indicate that higher-order corrections will play an important role in the future EIC program
- Many additional EIC applications are possible using the techniques developed here; **stay tuned!**