NLO($O(\alpha_s^2)$) SIDIS at large transverse momentum

Bowen Wang

Collaborators:
J. Gonzalez, T. Rogers, A. Signori, N. Sato

JLab
May 22 2017

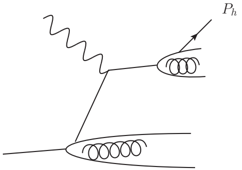
Outline

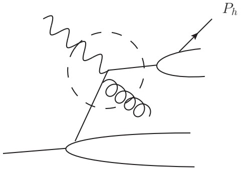
Role of perturbative term

Some details on NLO calculation

SIDIS at small and large q_T

$$q_T \sim m \qquad\qquad q_T \gtrsim Q$$

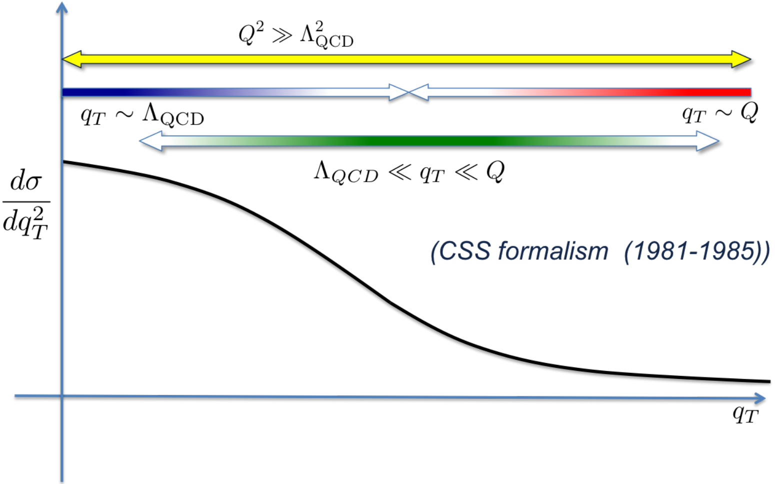




TMD Factorization

Coll. Factorization

Unified: Transverse Momentum:



Why collinear term is important

TMD term

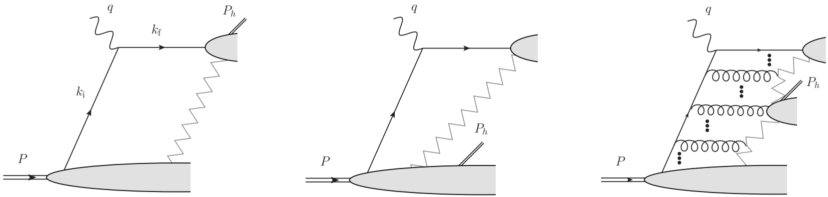
- contains TMD PDFs and TMD FFs
- describes intrinsic structure of nucleons
- process independent

Collinear term

- Calculable in pQCD
- process dependent

Role of collinear term

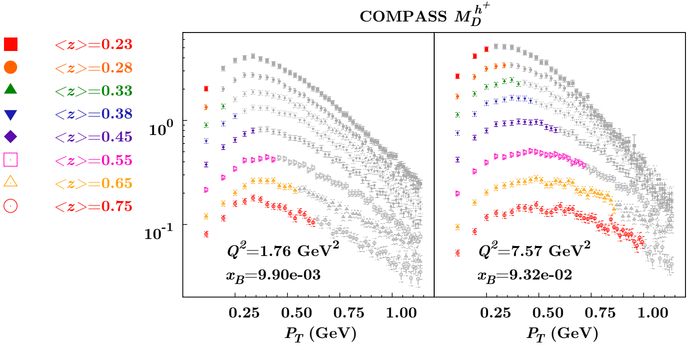
Theory designed for fragmentation of the struck quark (current fragmentation)



M. Boglion, J.Collins, L.Gamberg, J.O.Gonzalez, T.C.Rogers, N.Sato, arXiv:1611.10329

Are all the data in the current region?

- Need to isolate a current region to apply TMD factorization.



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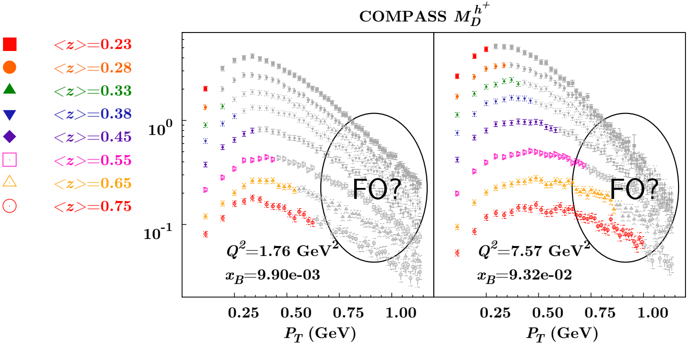
What to do with data not in the current region?

- No problem for large enough q_T — collinear factorization



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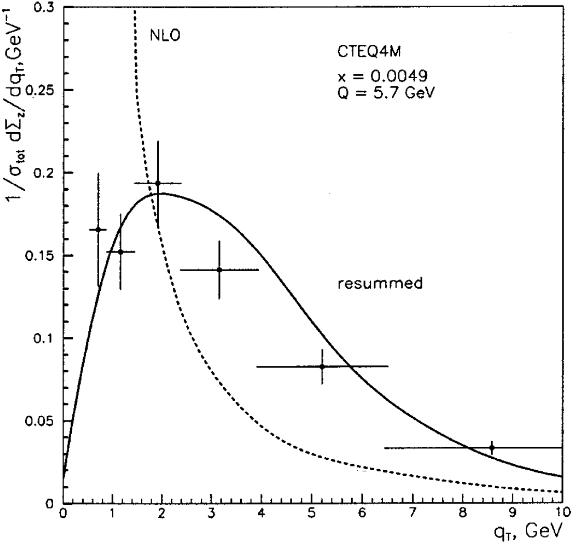
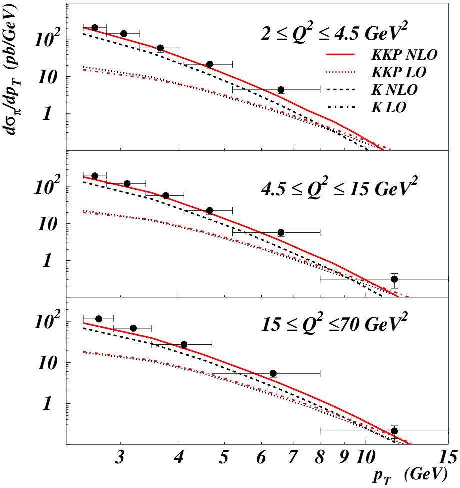
FO at $O(\alpha_s)$



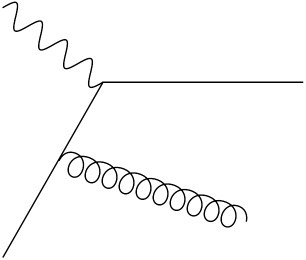
TMD and FO cross section in SIDIS. P. Nadolsky, *etal* (1999)

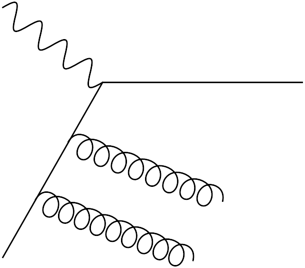
$O(\alpha_s)$ and $O(\alpha_s^2)$

Perturbative cross section in SIDIS. A. Daleo, *etal* (2004)



$O(\alpha_s)$ and $O(\alpha_s^2)$





Kinematical reason: with extra radiations, it is more likely to produce low energy jets at the central rapidity region.

Calculation of hard scattering coefficients

$$\frac{d\Gamma}{dxdydzdP_T^2} = \frac{2\pi^2\alpha_{EM}^2}{Q^4 y(1-\epsilon)}\Big[Q^2(y^2-2y+2)g^{\mu\nu}W_{\mu\nu} + 4x^2(y^2-6y+6)p^\mu p^\nu W_{\mu\nu}\Big] \quad (y=\frac{Q^2}{xs})$$

$$g^{\mu\nu}W_{\mu\nu} = \int |M|^2\, dPS_3(\overline{z, P_T^2}) \equiv \frac{d\sigma}{dzdP_T^2}$$

or change to Mandelstam's variables

$$\frac{d\sigma}{dudt} \equiv \int |M|^2\, dPS_3(\overline{u,t}), \quad \frac{d\sigma}{dzdP_T^2} = \frac{d\sigma}{dudt}\left|\frac{\partial(u,t)}{\partial(z,P_T^2)}\right|$$

similar definitions are used for $p^\mu p^\nu W_{\mu\nu}$

Calculation of hard scattering coefficients

$$\frac{d\sigma}{dudt} = \int d\xi d\zeta \frac{d\hat{\sigma}}{dudt} f(\xi) d(\zeta)$$

$$\begin{aligned}
\frac{d\sigma^{(1)}}{dudt} + \frac{d\sigma^{(2)}}{dudt} + \ldots &= \int d\xi d\zeta \left(\frac{d\hat{\sigma}^{(1)}}{dudt} + \frac{d\hat{\sigma}^{(2)}}{dudt} + \ldots \right) \\
&\quad \times (f^{(0)}(\xi) + f^{(1)}(\xi) + \ldots)(d^{(0)}(\zeta) + d^{(1)}(\zeta) + \ldots) \\
\frac{d\hat{\sigma}^{(1)}}{dudt} &= \frac{d\sigma^{(1)}}{dudt} \\
\frac{d\hat{\sigma}^{(2)}}{dudt} &= \frac{d\sigma^{(2)}}{dudt} - \int d\xi \frac{d\sigma^{(1)}}{dudt} f^{(1)}(\xi) - \int d\zeta \frac{d\sigma^{(1)}}{dudt} d^{(1)}(\zeta) \\
&\ldots
\end{aligned}$$

Calculation of hard scattering coefficients

$$f^{(0)}(\xi) = \delta(1-\xi)$$

$$d^{(0)}(\zeta) = \delta(1-\zeta)$$

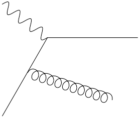
$$f_{qq}^{(1)\overline{MS}}(\xi) = C_F\left(\frac{1+\xi^2}{(1-\xi)_+} + \frac{3}{2}\delta(1-\xi)\right)\left(-\frac{1}{\epsilon} - \ln 4\pi + \gamma_E + \ln\left(\frac{M^2}{\mu^2}\right)\right)$$

$$d_{qq}^{(1)\overline{MS}}(\zeta) = f_{qq}^{(1)\overline{MS}}(\zeta)$$

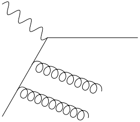
$$\dots$$

Calculation of hard scattering coefficients

$\frac{d\hat{\sigma}^{(1)}}{dudt}$:

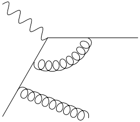


$\frac{d\hat{\sigma}^{(2)}}{dudt}$:



$+ \ldots$

$+$



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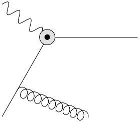

$+ \ldots$

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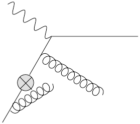

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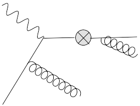
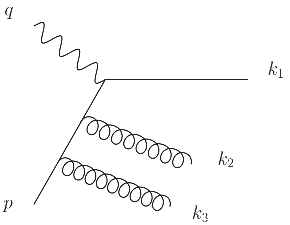


$+ \ldots$

Three particle phase space



$$s \equiv (p+q)^2$$
$$t_i \equiv (q-k_i)^2$$
$$u_i \equiv (p-k_i)^2$$
$$s_{ij} \equiv (k_i+k_j)^2$$
$$i,j = 1,2,3$$
$$u_1 \equiv u$$
$$t_1 \equiv t$$

$$PS_3 = \int \frac{d^nk_1}{(2\pi)^{n-1}2E_1}\frac{d^nk_2}{(2\pi)^{n-1}2E_2}\frac{d^nk_3}{(2\pi)^{n-1}2E_3} \times (2\pi)^n\delta^{(n)}(p+q-k1-k2-k3)$$
$$n = 4-2\epsilon$$

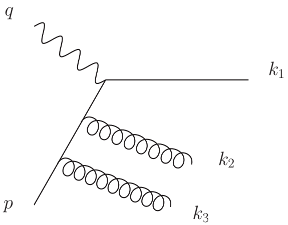
In the CM frame of $k_2 + k_3$,

$$k_2 = E_2(1,\ldots,\cos\theta_2\sin\theta_1,\cos\theta_1)$$
$$k_3 = E_2(1,\ldots,-\cos\theta_2\sin\theta_1,-\cos\theta_1)$$

the above phase space simplifies to

$$\int \frac{-1}{128(s+Q^2)\Gamma(1-2\epsilon)}\left(\frac{16\pi^2(s+Q^2)^2}{s_{23}u(st+Q^2s_{23})}\right)^{\epsilon} \times \sin^{1-2\epsilon}\theta_1\sin^{-2\epsilon}\theta_2 d\theta_1 d\theta_2 du dt$$

Partial fraction



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$$i,j = 1,2,3$$
$$u_1 \equiv u$$
$$t_1 \equiv t$$

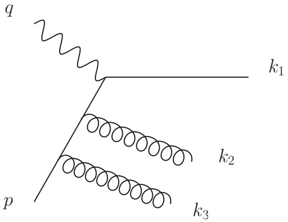
Need to reduce the number of Mandelstam variables in each term in the squared matrix element to less than two. Example:

$$\frac{u_2}{u_3 t_2^2 s_{13}}$$

Apply the relation $u_1 = t_2 - u_3 - s_{13}$ to get

$$\frac{u_2}{u_3 t_2^2 s_{13}} = \frac{u_2}{u_3 t_2^2 s_{13}} \frac{t_2 - u_3 - s_{13}}{u_1}$$
$$= -\frac{u_2}{s_{13} t_2^2 u_1} - \frac{u_2}{t_2^2 u_3 u_1} + \frac{u_2}{s_{13} t_2 u_3 u_1}$$
$$\frac{u_2}{s_{13} t_2 u_3 u_1} = -\frac{u_2}{s_{13} t_2 u_1^2} + \frac{u_2}{s_{13} u_3 u_1^2} - \frac{u_2}{t_2 u_3 u_1^2}$$

Partial fraction



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$$i,j = 1,2,3$$
$$u_1 \equiv u$$
$$t_1 \equiv t$$

There are six relevant variables: t_2,t_3, u_2,u_3,s_{12},s_{13}. (s_{23} is constrained by the momentum conservation $s+u+t = -Q^2 + s_{23}$). Four relations between them

$$t_1 + t_2 + t_3 + s + 2Q^2 = 0$$
$$u_1 + u_2 + u_3 + s + Q^2 = 0$$
$$s_{12} = s + t_3 + u_3 + Q^2$$
$$s_{13} = s + t_2 + u_2 + Q^2$$

reduce the number of independent variables to two. Therefore any variable in numerator can be expressed in terms of the two variables in denominator.

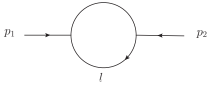
Phase space integration

$$\int_0^\pi d\theta_1 \sin^{1-2\epsilon}\theta_1 \int_0^\pi d\theta_2 \sin^{-2\epsilon}\theta_2$$
$$\times \frac{1}{(1-\cos\theta_1)^j(1-\cos\theta_1\cos\chi-\sin\theta_1\cos\theta_2\sin\chi)^l}$$
$$= 2\pi\frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)^2}2^{-j-l}B(1-\epsilon-j,1-\epsilon-l)\,{}_2F_1\left(j,l,1-\epsilon,\cos^2\frac{\chi}{2}\right)$$

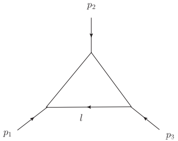
(Van Neervan 1985)

If the variables in integrand depend on momentum of massive particles, e.g. t_2 or t_3, the above identity cannot be used.

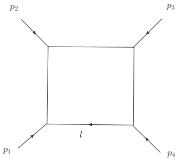
Virtual corrections: Passarino-Veltman decomposition



$$B_0; B^{\mu}; B^{\mu\nu} = \int d^n l \frac{1; l^{\mu}; l^{\mu} l^{\nu}}{d_1 d_2}, \quad d_i = (l + \sum_{k=1}^{i-1} p_k)^2 - m_i^2$$



$$C_0; C^{\mu}; C^{\mu\nu}; C^{\mu\nu\alpha} = \int d^n l \frac{1; l^{\mu}; l^{\mu} l^{\nu}; l^{\mu} l^{\nu} l^{\alpha}}{d_1 d_2 d_3}$$



$$D_0; D^{\mu}; D^{\mu\nu}; D^{\mu\nu\alpha}; D^{\mu\nu\alpha\beta} = \int d^n l \frac{1; l^{\mu}; l^{\mu} l^{\nu}; l^{\mu} l^{\nu} l^{\alpha}; l^{\mu} l^{\nu} l^{\alpha} l^{\beta}}{d_1 d_2 d_3 d_4}$$

Virtual corrections: Passarino-Veltman decomposition

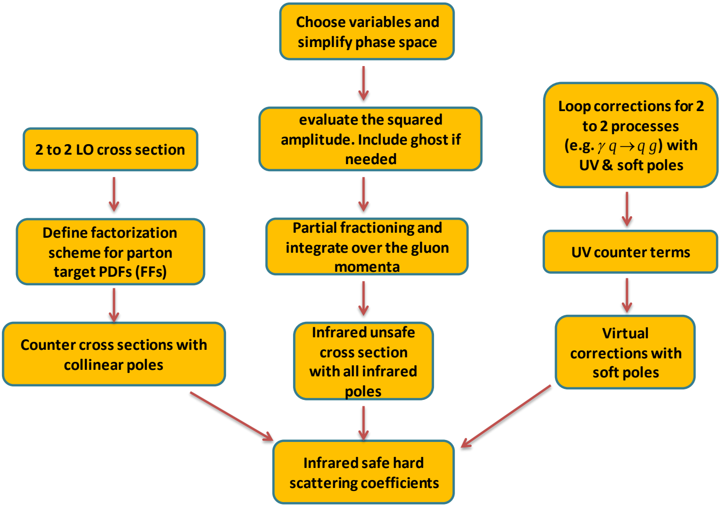
D_{ijkl}	$\rightarrow$	$D_{00ij}, D_{ijk}, C_{ijk}, C_{ij}, C_i, C_0$
D_{00ij}	$\rightarrow$	$D_{ijk}, D_{ij}, C_{ij}, C_i$
D_{0000}	$\rightarrow$	D_{00i}, D_{00}, C_{00}
D_{ijk}	$\rightarrow$	$D_{00i}, D_{ij}, C_{ij}, C_i$
D_{00i}	$\rightarrow$	D_{ij}, D_i, C_i, C_0
D_{ij}	$\rightarrow$	D_{00}, D_i, C_i, C_0
D_{00}	$\rightarrow$	D_i, D_0, C_0
D_i	$\rightarrow$	D_0, C_0
C_{ijk}	$\rightarrow$	$C_{00i}, C_{ij}, B_{ij}, B_i$
C_{00i}	$\rightarrow$	C_{ii}, C_i, B_i, B_0
C_{ij}	$\rightarrow$	C_{00}, C_i, B_i, B_0
C_{00}	$\rightarrow$	C_i, C_0, B_0
C_i	$\rightarrow$	C_0, B_0
B_{ii}	$\rightarrow$	B_{00}, B_i, A_0
B_{00}	$\rightarrow$	B_i, B_0, A_0
B_i	$\rightarrow$	B_0, A_0

$$B^\mu = p_1^\mu B_1$$
$$B^{\mu\nu} = g^{\mu\nu} B_{00} + p_1^\mu p_1^\nu B_{11}$$
$$\dots$$

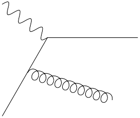
Reduction chains R. Ellis, *etal* (2012)

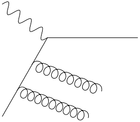
Procedure

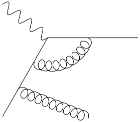
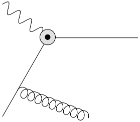
Calculation of FO term at NLO (e.g. $\gamma q \to qgg$)

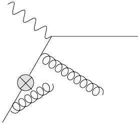
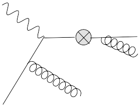


Status

$\frac{d\hat{\sigma}^{(1)}}{dudt}$:  ✓

$\frac{d\hat{\sigma}^{(2)}}{dudt}$:  ✓ $+ \dots$

$+$  almost $+$  $+ \dots$

$-$  ✓ $-$  ✓ $+ \dots$