Jefferson Lab

Quasi parton distributions and the gradient flow

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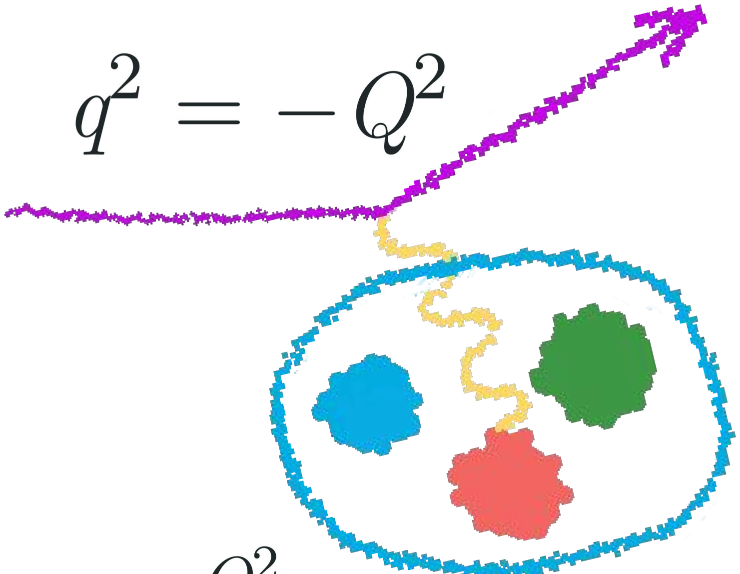
With Kostas Orginos

HOW FAST DO PARTONS TRAVEL?

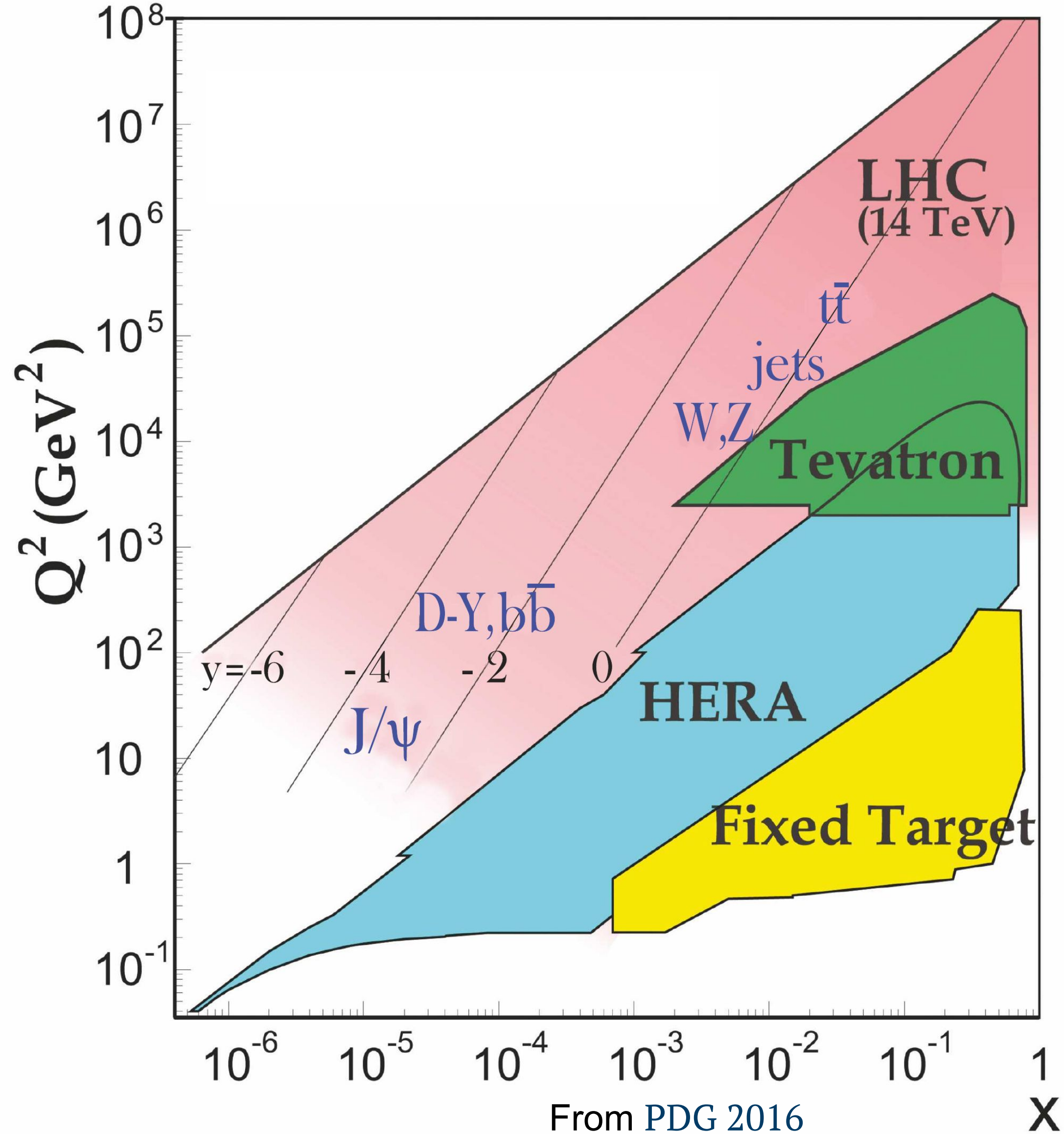
How is the momentum of a fast-moving nucleon distributed amongst its constituents?

EXPERIMENTAL EXTRACTION

$$q^2 = -Q^2$$



$$x = \frac{Q^2}{2P \cdot q}$$



From PDG 2016

PDFs FROM FIRST PRINCIPLES: QUASI DISTRIBUTIONS

THE GRADIENT FLOW

SMEARED QUASI DISTRIBUTIONS

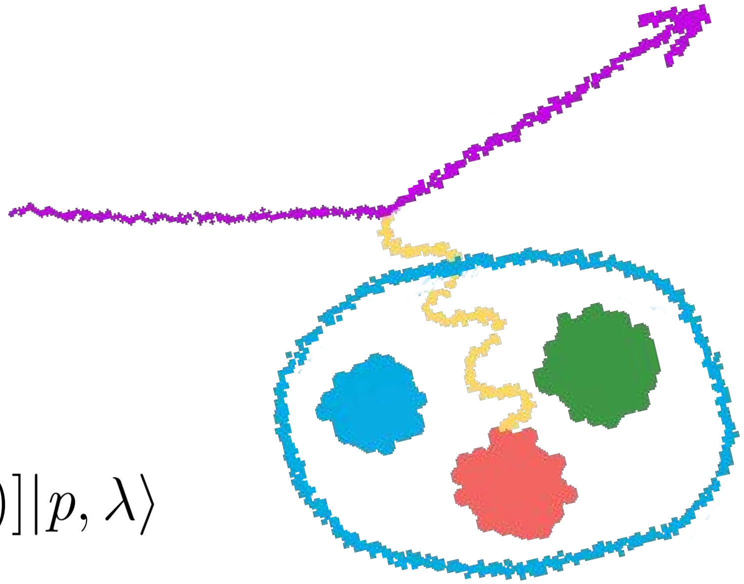
PDFS FROM FIRST PRINCIPLES

An unsolved challenge

DIS

Decompose cross-section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega\mathrm{d}E} = \frac{e^4}{16\pi^2 Q^4}\ell^{\mu\nu} W_{\mu\nu}$$



Hadronic contribution

$$W_{\mu\nu}(p,q) = \frac{1}{4\pi}\int \mathrm{d}^x e^{iq\cdot x}\langle p,\lambda'|[j_\mu(x), j_\nu(x)]|p,\lambda\rangle$$

in turn, expressed in terms of structure functions

$$F(x,Q^2) = \int \mathrm{d}y\, C\left(\frac{x}{y},\frac{Q^2}{\mu^2}\right) f(x,\mu^2)$$

parton distribution functions (PDFs)

PDFs

Defined as

$$f^{(0)}(\xi) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega^-}{4\pi} e^{-i\xi P^+\omega^-} \left\langle P \middle| T\, \overline{\psi}(0,\omega^-,\mathbf{0}_\mathrm{T}) W(\omega^-,0)\gamma^+ \frac{\lambda^a}{2}\psi(0) \middle| P \right\rangle_\mathrm{C}$$

where

$$W(\omega^-,0) = \mathcal{P}\exp\left[-ig_0 \int_0^{\omega^-} \mathrm{d}y^- A^+_\alpha(0,y^-,\mathbf{0}_\mathrm{T}) T_\alpha\right]$$

and

$$\langle P'|P\rangle = (2\pi)^3 2P^+ \delta\left(P^+ - P'^+\right)\delta^{(2)}\left(\mathbf{P}_\mathrm{T} - \mathbf{P}'_\mathrm{T}\right)$$

Renormalised PDFs

$$f(\xi,\mu) = \int_x^1 \frac{\mathrm{d}\zeta}{\zeta} \mathcal{Z}\left(\frac{\zeta}{\xi},\mu\right) f^{(0)}(\zeta)$$

PDFs

Mellin moments of PDFs

$$a^{(n)}(\mu) = \int_0^1 \mathrm{d}\xi\, \xi^{n-1} \left[f(\xi,\mu) + (-1)^n \overline{f}(\xi,\mu)\right] = \int_{-1}^1 \mathrm{d}\xi\, \xi^{n-1} f(\xi,\mu)$$

related to matrix elements

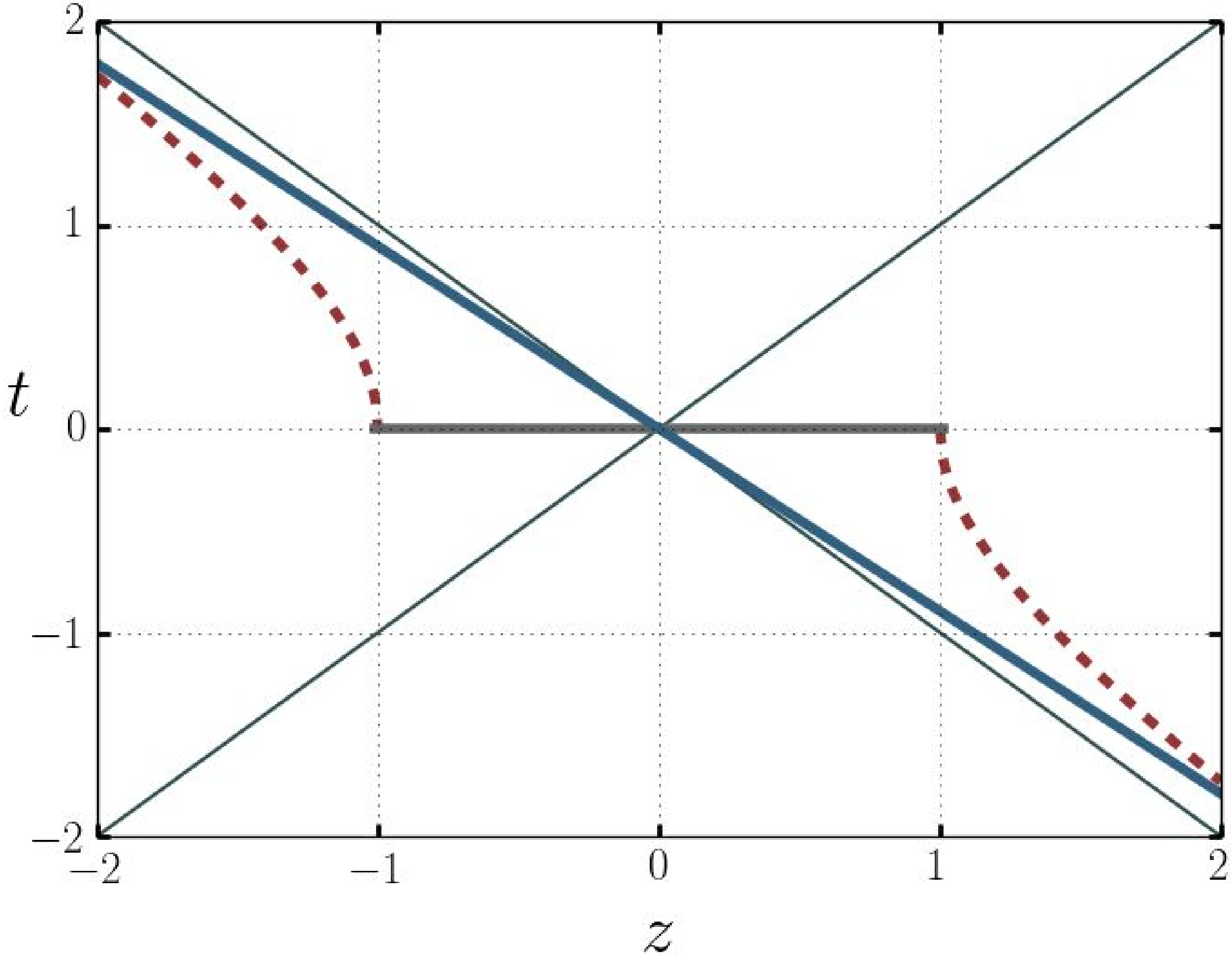
$$\left\langle P \middle| \mathcal{O}^{\{\nu_1 \ldots \nu_n\}}(\mu) \middle| P \right\rangle = 2a^{(n)}(\mu)\left(P^{\nu_1} \cdots P^{\nu_n} - \mathrm{traces}\right)$$

of local twist-two operators

$$\mathcal{O}^{\{\nu_1 \ldots \nu_n\}}(\mu) = Z_{\mathcal{O}}(\mu)\left[i^{n-1}\overline{\psi}(0)\gamma^{\{\mu_1} D^{\mu_2} \cdots D^{\mu_n\}} \frac{\lambda^a}{2}\psi(0) - \mathrm{traces}\right]$$

QUASI DISTRIBUTIONS

X. Ji, PRL 110 (2013) 262002
X. Ji, Sci.Ch. PMA 57 (2014) 1407



QUASI DISTRIBUTIONS

X. Ji, PRL 110 (2013) 262002
X. Ji, Sci.Ch. PMA 57 (2014) 1407

Defined as

$$q(x,\mu^2,P^z) = \int \frac{\mathrm{d}z}{4\pi} e^{i\,x\,z\,k^z} \langle P|\overline{\psi}(z)\gamma^z e^{-ig\int_0^z \mathrm{d}z' A^z(z')}\psi(0)|P\rangle_C$$

Recall

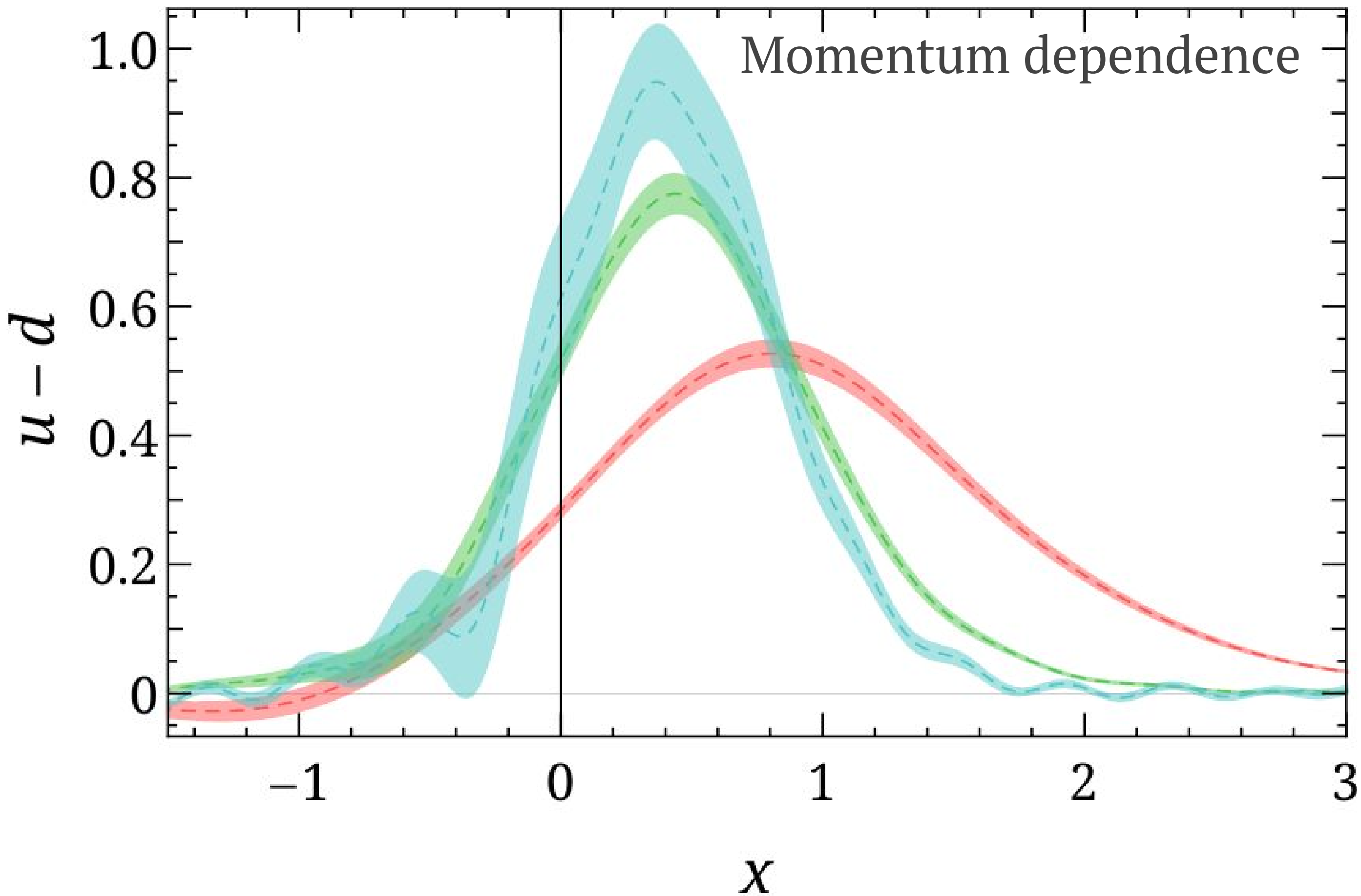
$$f^{(0)}(\xi) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega^-}{4\pi} e^{-i\xi P^+\omega^-} \left\langle P \left| T\,\overline{\psi}(0,\omega^-,\mathbf{0}_\mathrm{T}) W(\omega^-,0)\gamma^+ \frac{\lambda^a}{2}\psi(0)\right| P \right\rangle_\mathrm{C}$$

Related to light-front PDFs via

$$q(x,\mu^2,P^z) = \int_x^1 \frac{\mathrm{d}y}{y} Z\left(\frac{x}{y},\frac{\mu}{P^z}\right) f(y,\mu^2) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{(P^z)^2},\frac{M^2}{(P^z)^2}\right)$$

QUASI DISTRIBUTIONS

From J.-W. Chen et al., NPB 911 (2016) 246



See also:

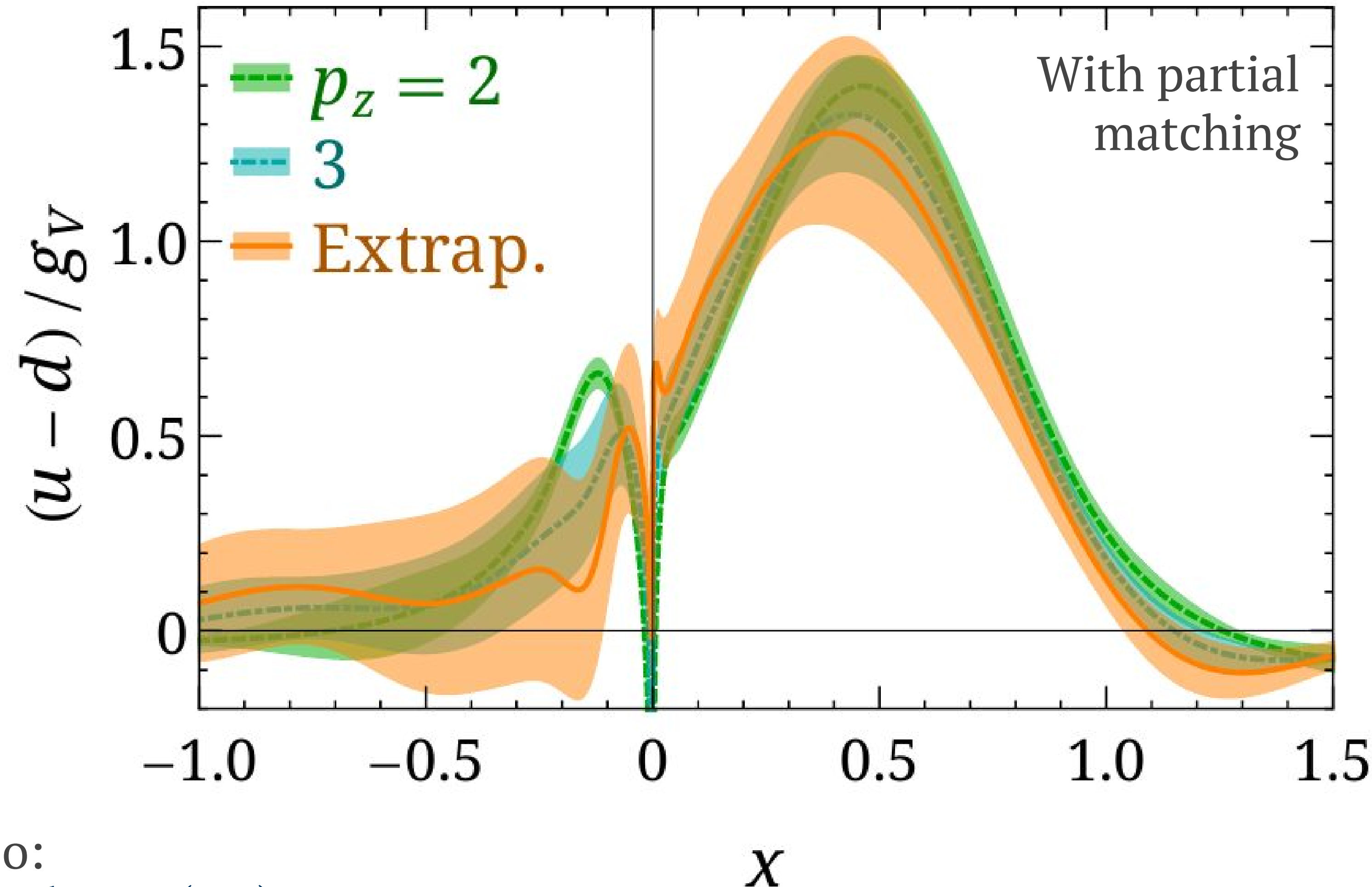
H.-W. Lin et al, PRD 91 (2015) 054510

C. Alexandrou et al., PRD 92 (2015) 014502

J.-H. Zhang et al., arXiv:1702.00008

QUASI DISTRIBUTIONS

From J.-W. Chen et al., NPB 911 (2016) 246



See also:

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C. Alexandrou et al., PRD 92 (2015) 014502

J.-H. Zhang et al., arXiv:1702.00008

A MORE GENERAL FRAMEWORK

Define lattice "cross-sections"

Y.-Q. Ma & J.-W. Qiu, arXiv:1404.6860

$$\lim_{a\to 0} \sigma\left(x, a, P^z\right) = \widetilde{\sigma}\left(x, \widetilde{\mu}, P^z\right)$$

Quasi distributions - lattice "cross-section" from which one can extract PDFs

$$\widetilde{\sigma}\left(x, \widetilde{\mu}, P^z\right) = \sum_{\alpha} H_{\alpha}\left(x, \frac{\widetilde{\mu}}{P^z}, \frac{\widetilde{\mu}}{\mu}\right) \otimes f_{\alpha}(x, \mu^2) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{\widetilde{\mu}^2}\right)$$

See also:

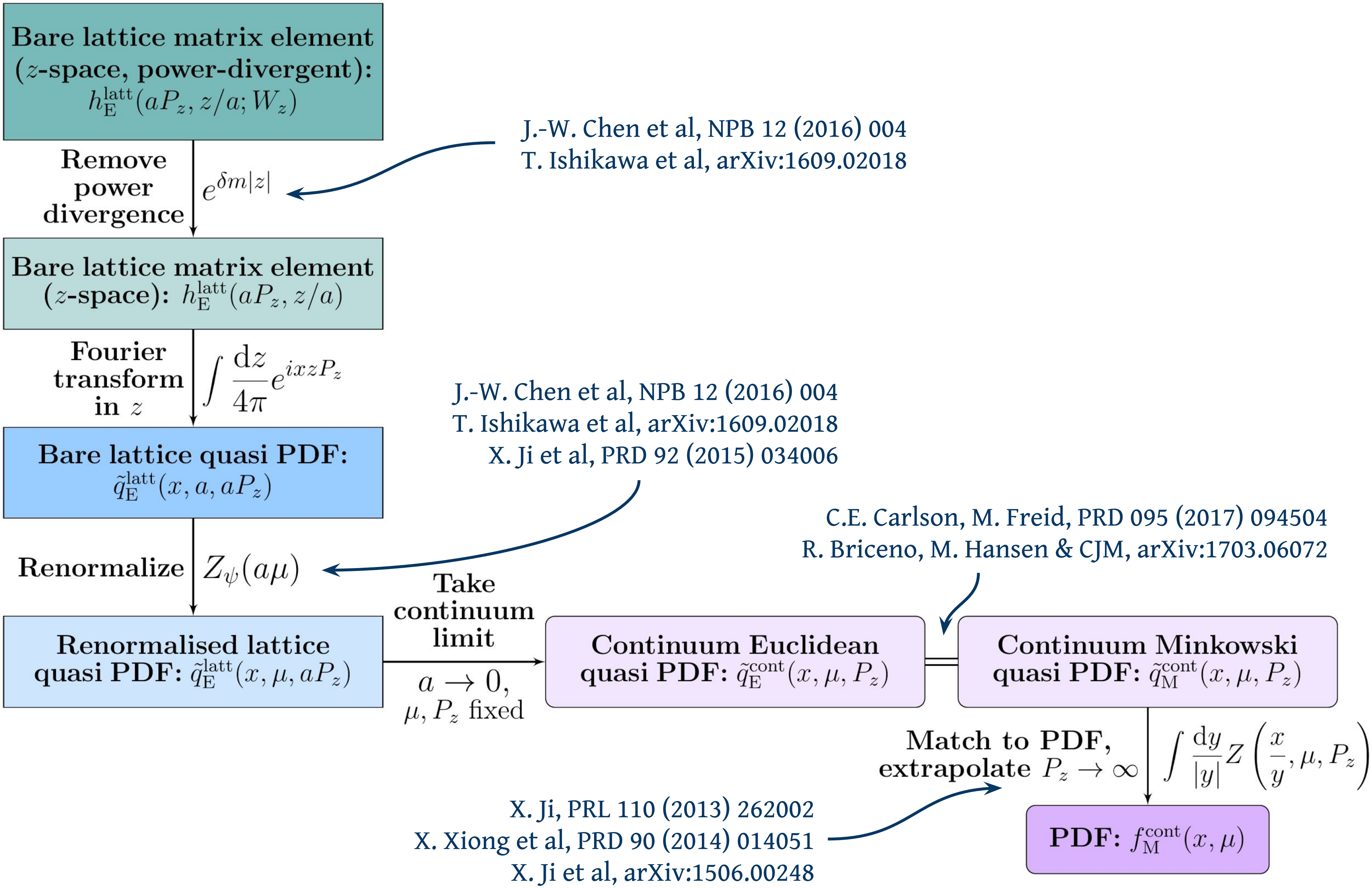
K.-F. Liu, PRD 62 (2000) 074501
W. Detmold & C.J.D. Lin, PRD 73 (2006) 014501
A. Radyushkin, PLB (2017) 02 019

GENERAL PROCEDURE

Bare lattice matrix element
(z-space, power-divergent):
$h_{\mathrm{E}}^{\mathrm{latt}}(aP_z, z/a; W_z)$

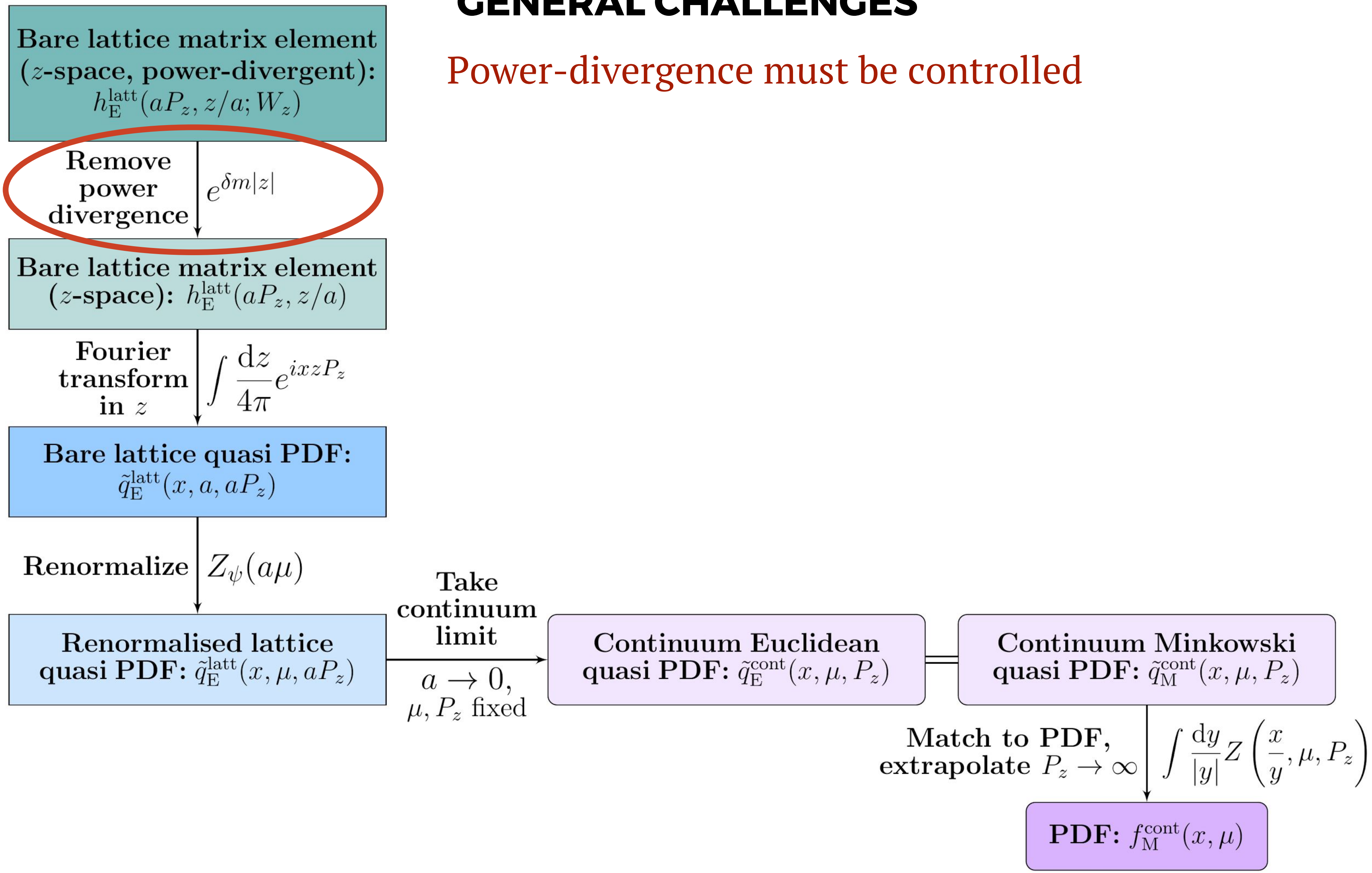
PDF: $f_{\mathrm{M}}^{\mathrm{cont}}(x, \mu)$

GENERAL PROCEDURE



GENERAL PROCEDURE: GENERAL CHALLENGES

Power-divergence must be controlled

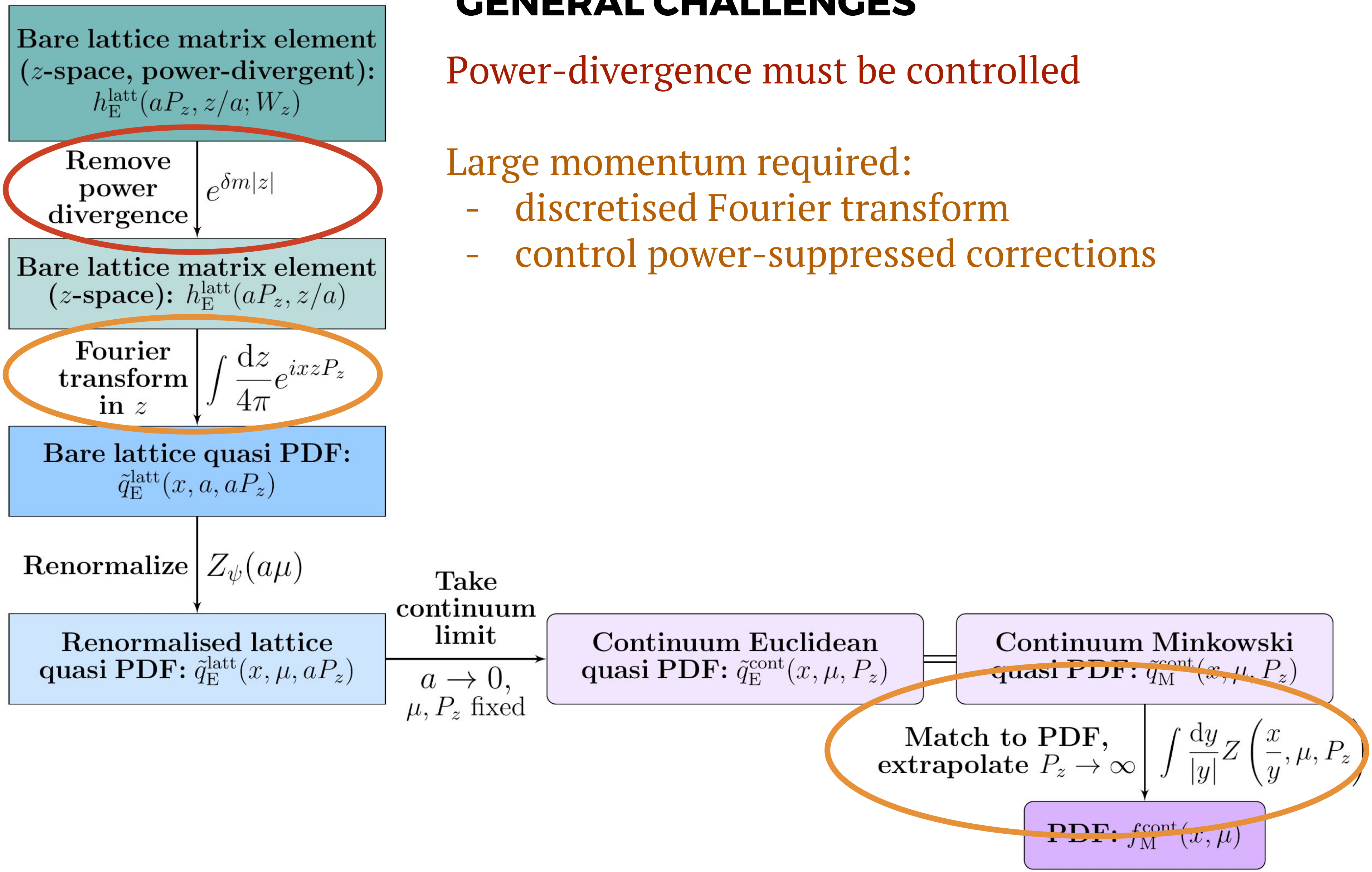


GENERAL PROCEDURE: GENERAL CHALLENGES

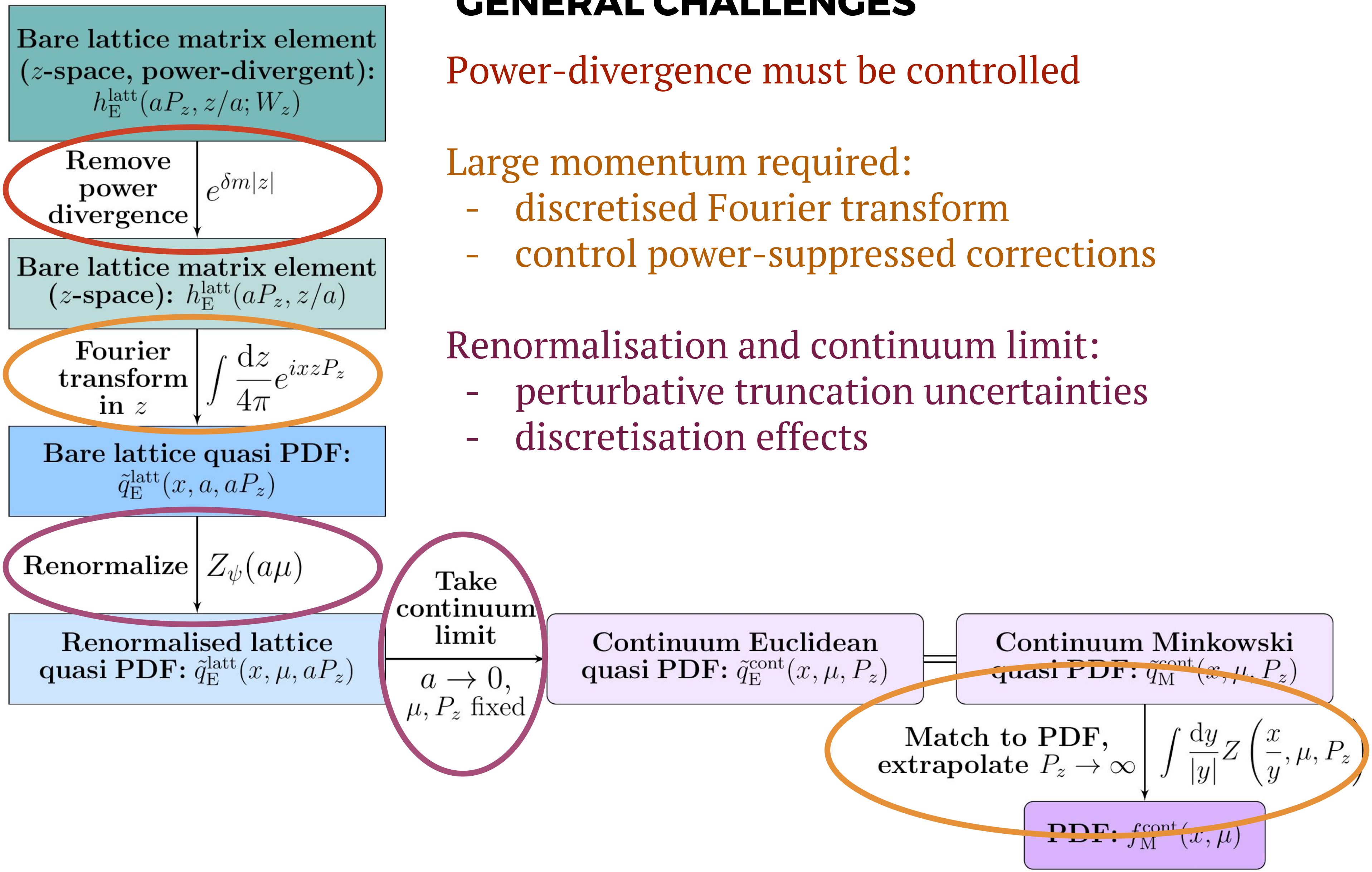
Power-divergence must be controlled

Large momentum required:

- discretised Fourier transform
- control power-suppressed corrections



GENERAL PROCEDURE: GENERAL CHALLENGES



Power-divergence must be controlled

Large momentum required:
- discretised Fourier transform
- control power-suppressed corrections

Renormalisation and continuum limit:
- perturbative truncation uncertainties
- discretisation effects

GENERAL PROCEDURE: GENERAL CHALLENGES

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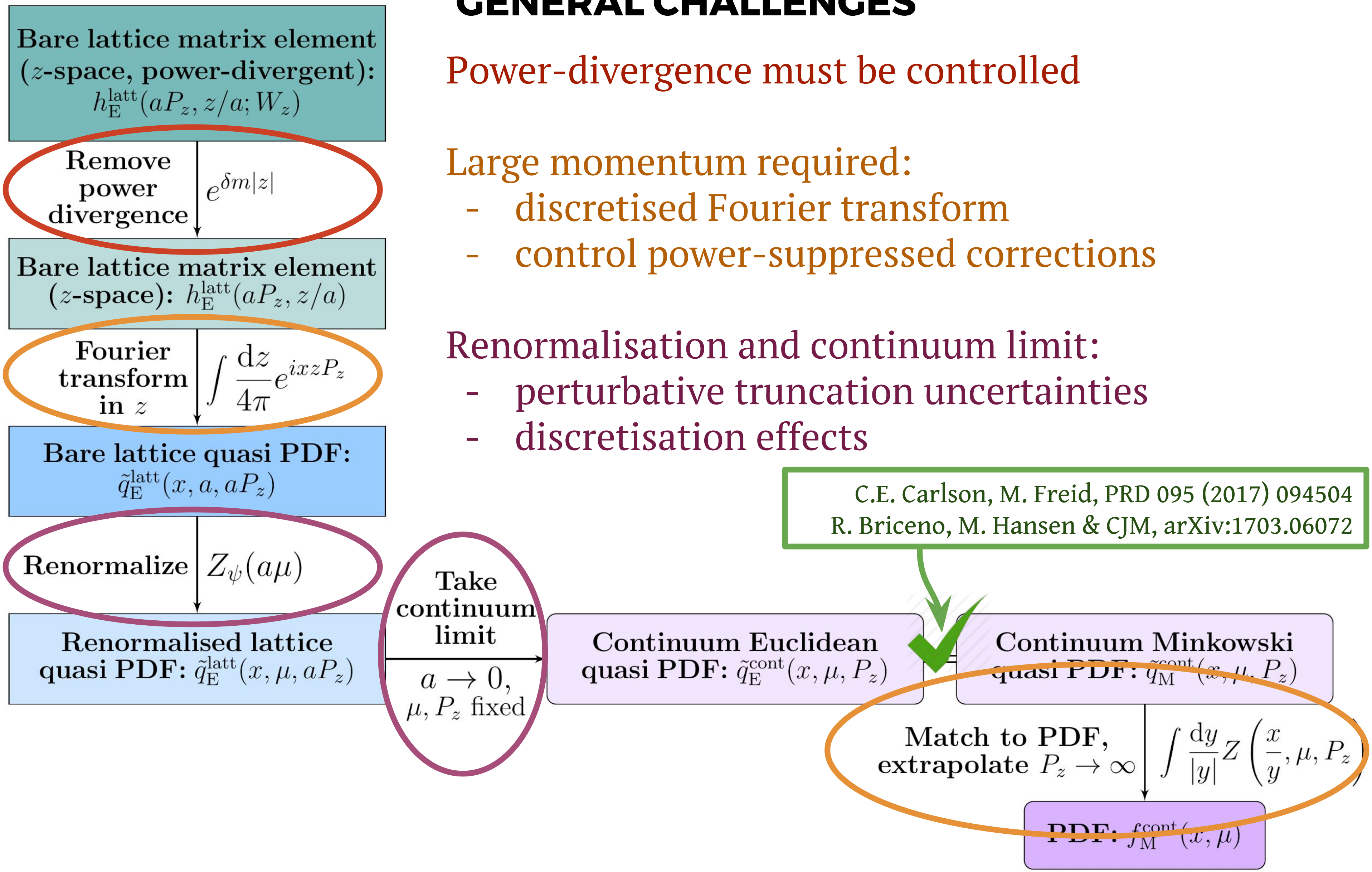


THE GRADIENT FLOW

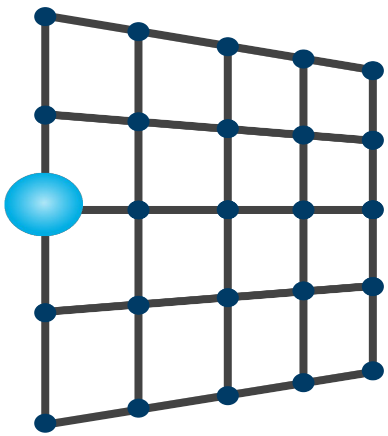
A continuum hammer for lattice nails

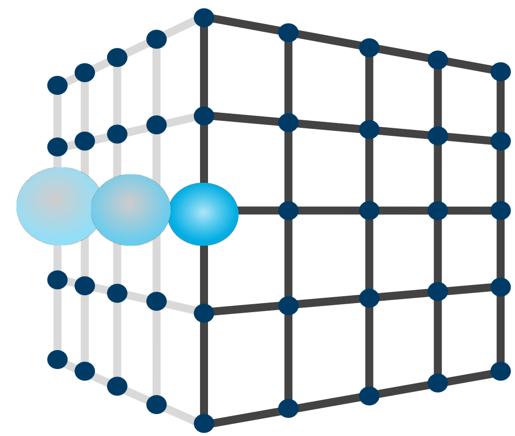
GRADIENT FLOW

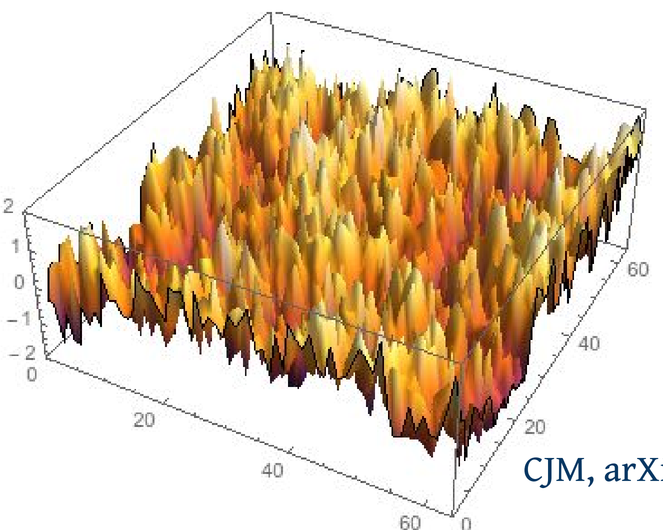
Deterministic evolution in new parameter - flow time

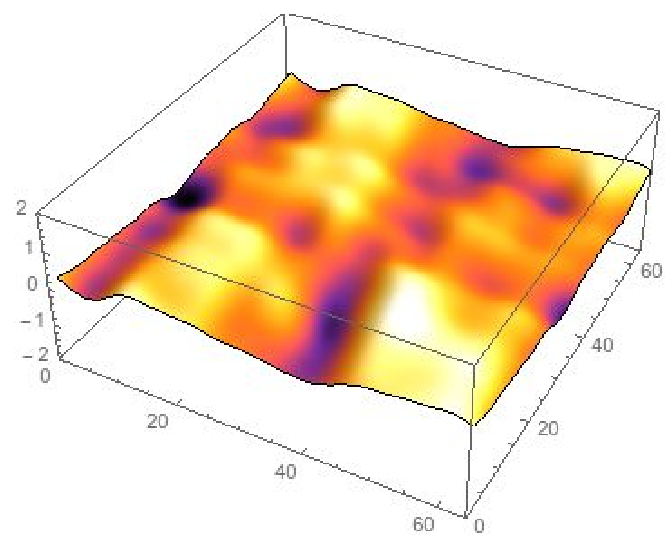
Drives fields to minimise action - removes UV fluctuations

Correlation functions of “bulk” fields provide probe of underlying field theory









CJM, arXiv:1512.00294

FINITE CORRELATION FUNCTIONS

Renormalised correlation functions remain finite at non-zero flow time.

Lüscher & Weisz, JHEP 1102 (2011) 51
Luscher, JHEP 04 (2013) 123

GRADIENT FLOW

Gradient flow is a smearing (smoothing) tool that:

- generates more continuum-like operators
- provides a method to fix smearing length scale

Flow time serves as a nonperturbative, rotationally-invariant cutoff

Matrix elements of operators at fixed flow time are finite

Fixing the flow time (physical units) allows a continuum limit

Roughly speaking: exchange lattice for gradient flow regulator

CJM & K. Orginos, PRD 91 (2015) 074513

SMEARED QUASI DISTRIBUTIONS

Provides continuum limit

SMEARED QUASI DISTRIBUTIONS

CJM & K. Orginos, JHEP 03 (2017) 116

Defined as

$$q(x, \sqrt{\tau}P^z, \sqrt{\tau}\Lambda_{\rm QCD}, \sqrt{\tau}M_N) = \int \frac{{\rm d}z}{4\pi} e^{ixz\,k^z} \left\langle P|\overline{\chi}(z,\tau)\gamma^z e^{-ig\int_0^z {\rm d}z' B^z(z',\tau)}\chi(0,\tau)|P\right\rangle_C$$

Related to light-front PDFs via

$$q(x,\mu^2,P^z) = \int_x^1 \frac{{\rm d}y}{y} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) f(y,\mu^2) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{(P^z)^2}, \frac{M^2}{(P^z)^2}\right)$$

Here “ringed” fermions remove need for wavefunction renormalisation

H. Makino and H. Suzuki, PTEP (2014) 063B02
K. Hieda and H. Suzuki, MPLA 31 (2016) 1650214

$$\chi(x,\tau) = \sqrt{\frac{-2\dim(R)N_f}{(4\pi)^2\tau^2\left\langle\overline{\psi}(x,\tau)\overleftrightarrow{\not{D}}\psi(x,\tau)\right\rangle}}\psi(x,\tau)$$

SMEARED QUASI DISTRIBUTIONS

CJM & K. Orginos, JHEP 03 (2017) 116

Defined as

$$q(x, \sqrt{\tau}P^z, \sqrt{\tau}\Lambda_{\mathrm{QCD}}, \sqrt{\tau}M_N) = \int \frac{\mathrm{d}z}{4\pi} e^{ixz\,k^z} \langle P|\overline{\chi}(z,\tau)\gamma^z e^{-ig\int_0^z \mathrm{d}z' B^z(z',\tau)}\chi(0,\tau)|P\rangle_C$$

If we write

$$q^{(s)}\left(\xi, \sqrt{\tau}P_z, \sqrt{\tau}\Lambda_{\mathrm{QCD}}, \sqrt{\tau}M_{\mathrm{N}}\right) = \int_{-\infty}^{\infty} \frac{\mathrm{d}z}{2\pi} e^{i\xi z P_z} P_z\, h^{(s)}(\sqrt{\tau}z, \sqrt{\tau}P_z, \sqrt{\tau}\Lambda_{\mathrm{QCD}}, \sqrt{\tau}M_{\mathrm{N}})$$

then, in practice,

$$h^{(s)}\left(\frac{z}{\sqrt{\tau}}, \sqrt{\tau}P_z, \sqrt{\tau}\Lambda_{\mathrm{QCD}}, \sqrt{\tau}M_{\mathrm{N}}\right) = \lim_{a\to 0} h\left(\frac{z}{a}, \frac{\sqrt{\tau}}{a}, aP_z, a\Lambda_{\mathrm{QCD}}, aM_{\mathrm{N}}\right)$$

where

$$h\left(\frac{z}{a}, \frac{\sqrt{\tau}}{a}, aP_z, a\Lambda_{\mathrm{QCD}}, aM_{\mathrm{N}}\right) =$$

$$\frac{1}{2aP_z}\left\langle aP_z \middle| \overline{\chi}\left(\frac{z}{a}; \frac{\sqrt{\tau}}{a}\right) W\left(0, \frac{z}{a}; \frac{\sqrt{\tau}}{a}\right)\gamma_z \frac{\lambda^a}{2}\chi\left(0; \frac{\sqrt{\tau}}{a}\right) \middle| aP_z\right\rangle_{\mathrm{C}}$$

RELATION TO PDFs

CJM & K. Orginos, JHEP 03 (2017) 116

Introduce Mellin moments

$$b_n\left(\sqrt{\tau}P^z, \frac{\Lambda_{\mathrm{QCD}}}{P^z}, \frac{M_N}{P^z}\right) = \int_{-\infty}^{\infty} \mathrm{d}x\, x^{n-1} q(x, \sqrt{\tau}P^z, \sqrt{\tau}\Lambda_{\mathrm{QCD}}, \sqrt{\tau}M_N)$$

In limit of vanishing separation z

$$b_n\left(\sqrt{\tau}P^z, \frac{\Lambda_{\mathrm{QCD}}}{P^z}, \frac{M_N}{P^z}\right) = \frac{c_n(\sqrt{\tau}P^z)}{2P^z}\left\langle P^z \middle| \chi(z,\tau)\gamma_z(iD_z)^{n-1}\frac{\lambda^a}{2}\chi(0,\tau) \middle| P^z \right\rangle_C$$

Unlike PDFs these matrix elements are not twist-2, but related via

$$b_n\left(\sqrt{\tau}P^z, \frac{\Lambda_{\mathrm{QCD}}}{P^z}, \frac{M_N}{P^z}\right) = C(\sqrt{\tau}\mu, \sqrt{\tau}P^z)\, a_n(\mu) + \mathcal{O}\left(\sqrt{\tau}\Lambda_{\mathrm{QCD}}, \frac{\Lambda_{\mathrm{QCD}}^2}{(P^z)^2}\right)$$

RELATION TO PDFs

CJM & K. Orginos, JHEP 03 (2017) 116

Introduce a kernel with Mellin moments

$$\left[C_n^{(0)}(\sqrt{\tau}\mu, \sqrt{\tau}P_z)\right]^{-1} = \int_{-\infty}^{\infty} dx\, x^{n-1} Z(x, \sqrt{\tau}\mu, \sqrt{\tau}P_z)$$

Then, assuming one corrects for target mass effects and that

$$\Lambda_{\mathrm{QCD}}, M_N \ll P_z \ll \tau^{-1/2}$$

This leads to

$$q(x, \sqrt{\tau}\Lambda_{\mathrm{QCD}}, \sqrt{\tau}P^z) = \int_{-1}^{1} \frac{\mathrm{d}y}{y} Z\left(\frac{x}{y}, \sqrt{\tau}\mu, \sqrt{\tau}P^z\right) f(y, \mu^2) + \mathcal{O}\left(\sqrt{\tau}\Lambda_{\mathrm{QCD}}, \frac{\Lambda_{\mathrm{QCD}}^2}{(P^z)^2}\right)$$

DGLAP EQUATION

CJM & K. Orginos, JHEP 03 (2017) 116

PDFs obey

$$\mu \frac{\mathrm{d}\, f(x,\mu)}{\mathrm{d}\mu} = \frac{\alpha_s(\mu)}{\pi} \int_x^1 \frac{\mathrm{d}y}{y} f(y,\mu) P\left(\frac{x}{y}\right)$$

and (renormalised) moments obey

$$\left[\mu \frac{\mathrm{d}}{\mathrm{d}\mu} - \frac{\alpha_s(\mu)}{\pi} \gamma^{(n)}\right] a^{(n)}(\mu) = 0$$

with

$$\int_0^1 \mathrm{d}x\, x^{n-1} P(x) = \gamma^{(n)}$$

DGLAP EQUATION

CJM & K. Orginos, JHEP 03 (2017) 116

Introducing a small-flow time expansion

$$b_n^{(s)}\left(\sqrt{\tau}\Lambda_{\mathrm{QCD}}\right) = C_n^{(0)}(\sqrt{\tau}\mu, \sqrt{\tau}P_z)a^{(n)}(\mu) + \mathcal{O}\left(\sqrt{\tau}\Lambda_{\mathrm{QCD}}, \frac{\Lambda_{\mathrm{QCD}}^2}{P_z^2}\right)$$

such that

$$\left[\mu\frac{\mathrm{d}}{\mathrm{d}\mu} + \frac{\alpha_s(\mu)}{\pi}\gamma^{(n)}\right]C_n^{(0)}(\sqrt{\tau}\mu, \sqrt{\tau}P_z) = 0 + \mathcal{O}(\sqrt{\tau}\Lambda_{\mathrm{QCD}})$$

The matching kernel satisfies

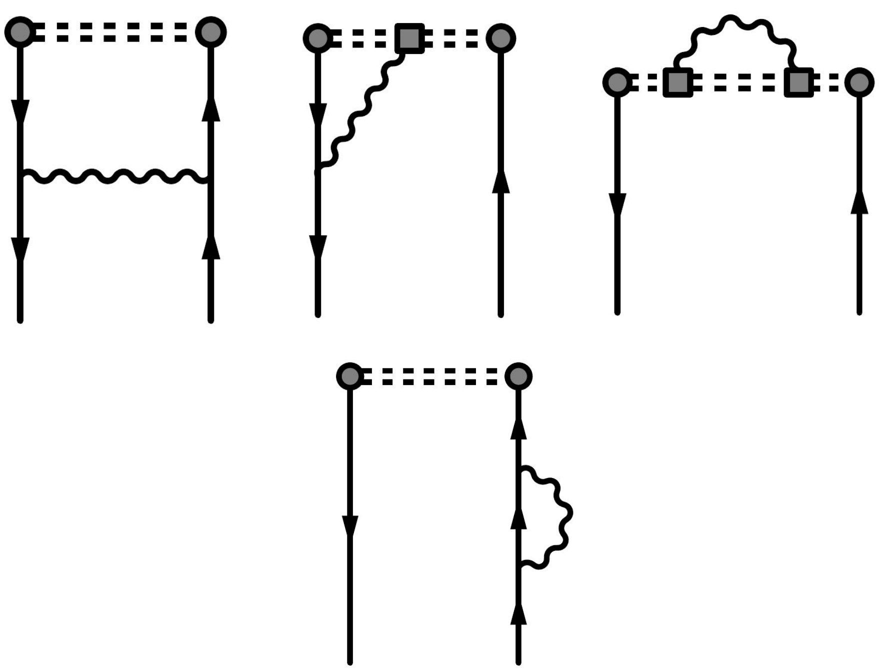
$$\mu\frac{d}{d\mu}Z\left(x, \sqrt{\tau}\mu, \sqrt{\tau}P_z\right) = \frac{\alpha_s(\mu)}{\pi}\int_x^\infty \frac{dy}{y}Z\left(y, \sqrt{\tau}\mu, \sqrt{\tau}P_z\right)P\left(\frac{x}{y}\right)$$

MATCHING IN PERTURBATION THEORY

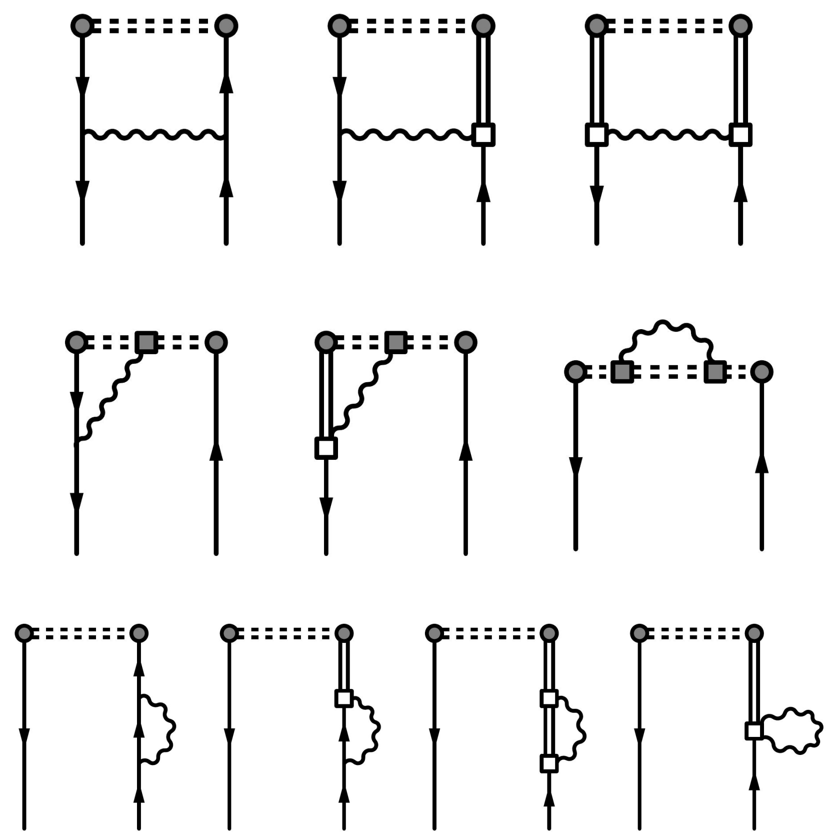
MATCHING IN PERTURBATION THEORY

Axial gauge simplest. But gradient flow requires generalised Feynman gauge.

Quasi distribution



Smeared quasi distribution



X. Xiong *et al.*, PRD 90 (2014) 014051
Ji et al., PRD 92 (2015) 014039

MATCHING IN PERTURBATION THEORY

In principle, one can calculate these diagrams directly.

In practice, integrals not solvable analytically

- usual techniques (or at least, all the ones I can think of) fail

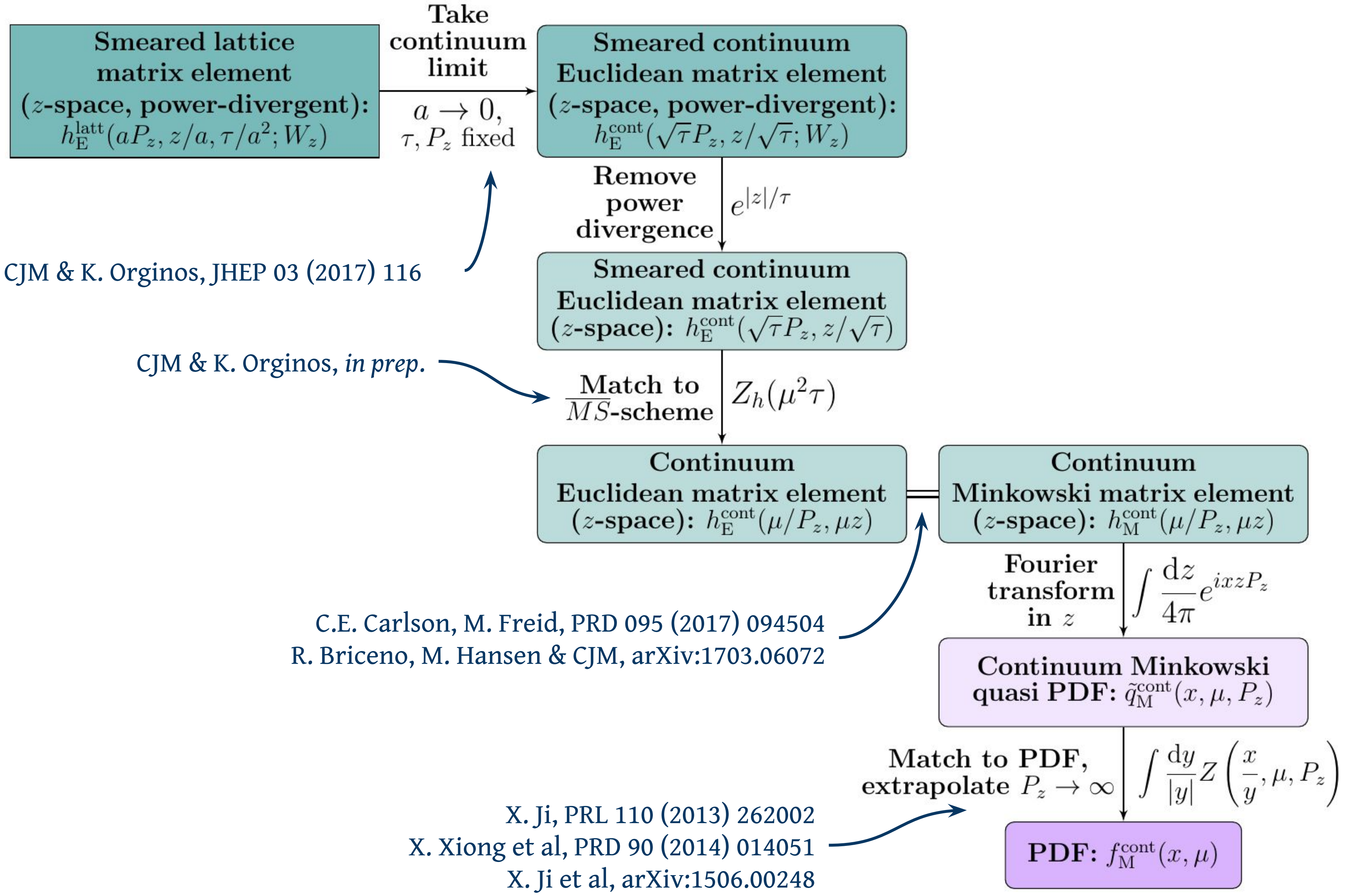
Matching procedure considerably simplified in coordinate space:

- infrared behaviour identical
- set external mass and momentum to zero
- reduces number of diagrams and simplifies remaining integrals

One-loop calculation complete - but not checked

CJM & K. Orginos, *in prep*.

GRADIENT FLOW PROCEDURE



SUMMARY

PDFs FROM FIRST PRINCIPLES: QUASI DISTRIBUTIONS

Quasi distributions

Current issues - the continuum limit

THE GRADIENT FLOW

Matrix elements finite at fixed flow time

SMEARED QUASI DISTRIBUTIONS

Finite continuum distributions

Matching in perturbation theory ~ complete

Nonperturbative study of systematics underway

THANK YOU

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QCD

QCD

$$\frac{\partial}{\partial \tau} B_\mu(\tau,x) = D_\nu\Big(\partial_\nu B_\mu - \partial_\mu B_\nu + [B_\nu, B_\mu]\Big) \qquad D_\mu = \partial_\mu + [B_\mu, \cdot\,]$$

$$\frac{\partial}{\partial \tau} \chi(\tau,x) = D^F_\mu D^F_\mu \chi(\tau,x) \qquad D^F_\mu = \partial_\mu + B_\mu$$

Exact solution not possible (even with Dirichlet boundary conditions)

$$B_\mu(\tau,x) = \int \mathrm{d}^4 y \Big\{ K_\tau(x-y)_{\mu\nu} A_\nu(y) + \int_0^\tau \mathrm{d}\sigma K_{\tau-\sigma}(x-y)_{\mu\nu} R_\nu(\sigma,y) \Big\}$$

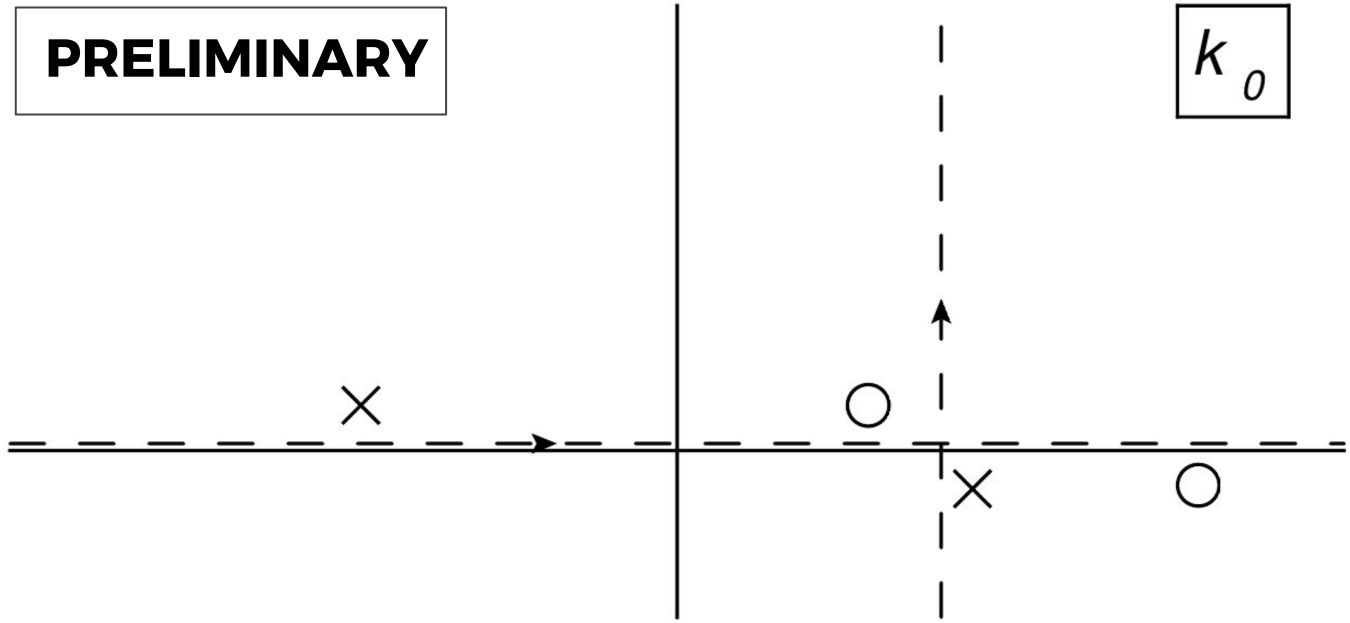
$$K_\tau(x)_{\mu\nu} = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{e^{ipx}}{p^2} \Big\{ (\delta_{\mu\nu} p^2 - p_\mu p_\nu) e^{-\tau p^2} + p_\mu p_\nu \Big\}$$

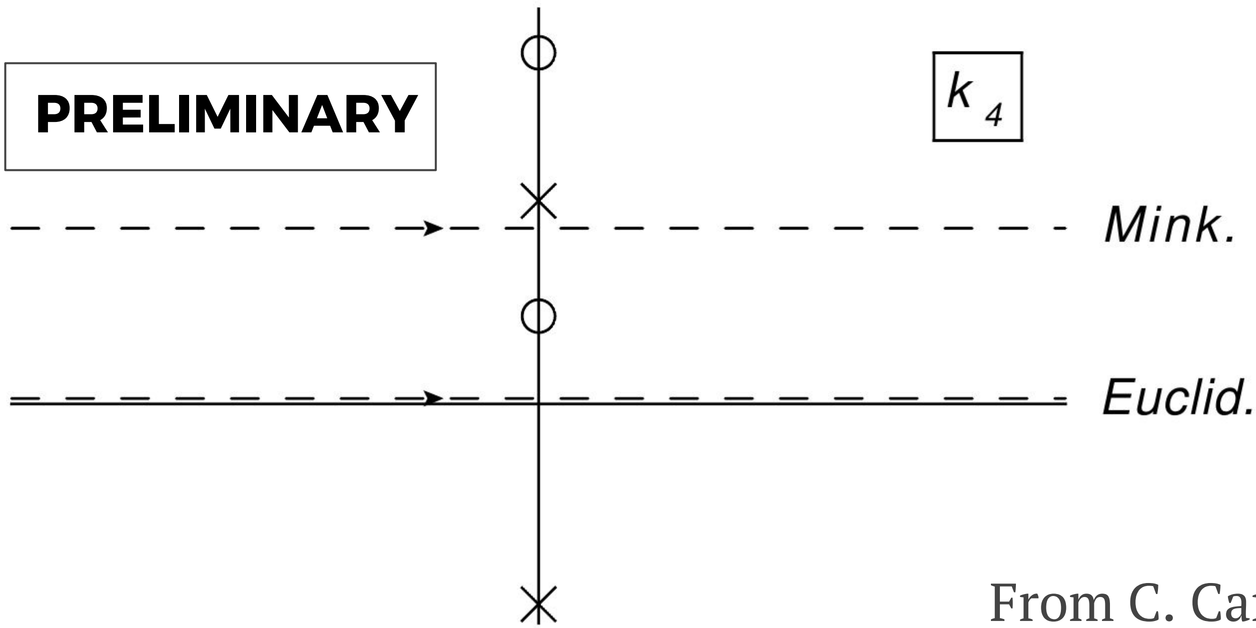
$$R_\mu(\tau,x) = 2[B_\nu, \partial_\nu B_\mu] - [B_\nu, \partial_\mu B_\nu] - [B_\mu, \partial_\nu B_\nu] + [B_\nu, [B_\nu, B_\mu]]$$

Smearing radius $s_{\mathrm{rms}} = \sqrt{8\tau}$

Interactions occur at non-zero flow time: generalised BRST symmetry guarantees renormalised correlation functions remain finite.

Lüscher & Weisz, JHEP 1102 (2011) 51
Luscher, JHEP 04 (2013) 123





From C. Carlson & M. Freid (2017)
"Lattice corrections to the quark quasidistribution at one-loop"

EXPERIMENTAL EXTRACTION

Process	Subprocess	Partons	x range
$\ell^{\pm}\,\{p,n\} \to \ell^{\pm}\,X$	$\gamma^* q \to q$	$q, \bar{q}, g$	$x \gtrsim 0.01$
$\ell^{\pm}\,n/p \to \ell^{\pm}\,X$	$\gamma^*\,d/u \to d/u$	d/u	$x \gtrsim 0.01$
$pp \to \mu^+\mu^-\,X$	$u\bar{u}, d\bar{d} \to \gamma^*$	$\bar{q}$	$0.015 \lesssim x \lesssim 0.35$
$pn/pp \to \mu^+\mu^-\,X$	$(u\bar{d})/(u\bar{u}) \to \gamma^*$	$\bar{d}/\bar{u}$	$0.015 \lesssim x \lesssim 0.35$
$\nu(\bar{\nu})\,N \to \mu^-(\mu^+)\,X$	$W^* q \to q'$	$q, \bar{q}$	$0.01 \lesssim x \lesssim 0.5$
$\nu\,N \to \mu^-\mu^+\,X$	$W^* s \to c$	s	$0.01 \lesssim x \lesssim 0.2$
$\bar{\nu}\,N \to \mu^+\mu^-\,X$	$W^* \bar{s} \to \bar{c}$	$\bar{s}$	$0.01 \lesssim x \lesssim 0.2$
$e^{\pm}\,p \to e^{\pm}\,X$	$\gamma^* q \to q$	$g, q, \bar{q}$	$10^{-4} \lesssim x \lesssim 0.1$
$e^+\,p \to \bar{\nu}\,X$	$W^+\,\{d,s\} \to \{u,c\}$	d, s	$x \gtrsim 0.01$
$e^{\pm}p \to e^{\pm}\,c\bar{c}X, e^{\pm}\,b\bar{b}X$	$\gamma^* c \to c,\ \gamma^* g \to c\bar{c}$	c, b, g	$10^{-4} \lesssim x \lesssim 0.01$
$e^{\pm}p \to$ jet$+X$	$\gamma^* g \to q\bar{q}$	g	$0.01 \lesssim x \lesssim 0.1$
$p\bar{p}, pp \to$ jet$+X$	$gg, qg, qq \to 2j$	g, q	$0.00005 \lesssim x \lesssim 0.5$
$p\bar{p} \to (W^{\pm} \to \ell^{\pm}\nu)\,X$	$ud \to W^+, \bar{u}\bar{d} \to W^-$	$u, d, \bar{u}, \bar{d}$	$x \gtrsim 0.05$
$pp \to (W^{\pm} \to \ell^{\pm}\nu)\,X$	$u\bar{d} \to W^+, d\bar{u} \to W^-$	$u, d, \bar{u}, \bar{d}, g$	$x \gtrsim 0.001$
$p\bar{p}(pp) \to (Z \to \ell^+\ell^-)X$	$uu, dd, ..(u\bar{u}, ..) \to Z$	$u, d, ..(g)$	$x \gtrsim 0.001$
$pp \to W^- c,\ \ W^+\bar{c}$	$gs \to W^- c$	$s, \bar{s}$	$x \sim 0.01$
$pp \to (\gamma^* \to \ell^+\ell^-)X$	$u\bar{u}, d\bar{d}, .. \to \gamma^*$	$\bar{q}, g$	$x \gtrsim 10^{-5}$
$pp \to b\bar{b}\,X,\ \ t\bar{t}X$	$gg \to b\bar{b},\ \ t\bar{t}$	g	$x \gtrsim 10^{-5}, 10^{-2}$
$pp \to$ exclusive $J/\psi,\ \Upsilon$	$\gamma^*(gg) \to J/\psi,\ \Upsilon$	g	$x \gtrsim 10^{-5}, 10^{-4}$
$pp \to \gamma\,X$	$gq \to \gamma q,\ g\bar{q} \to \gamma\bar{q}$	g	$x \gtrsim 0.005$

From PDG 2016