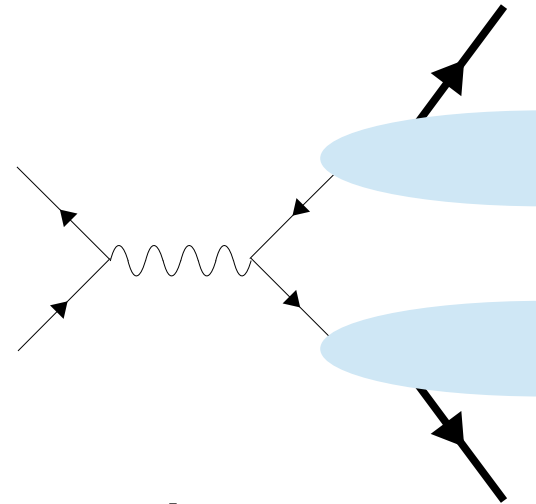
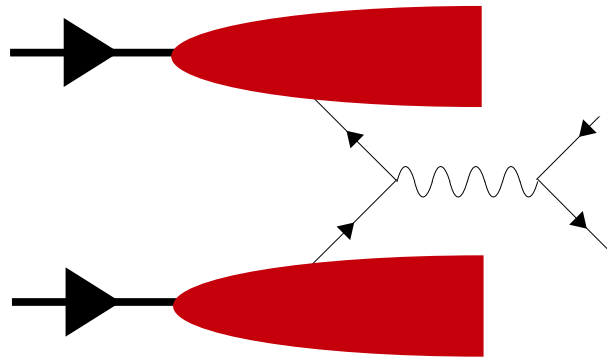


Decoupling TMD FF Effects from Data.

J. Osvaldo Gonzalez-Hernandez

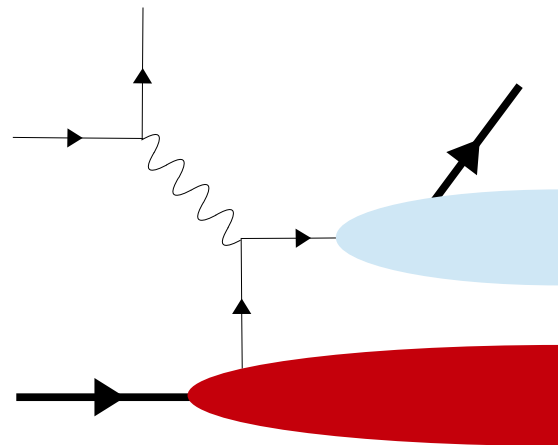
University of Turin

Drell Yan



e^+e^-

PDFs

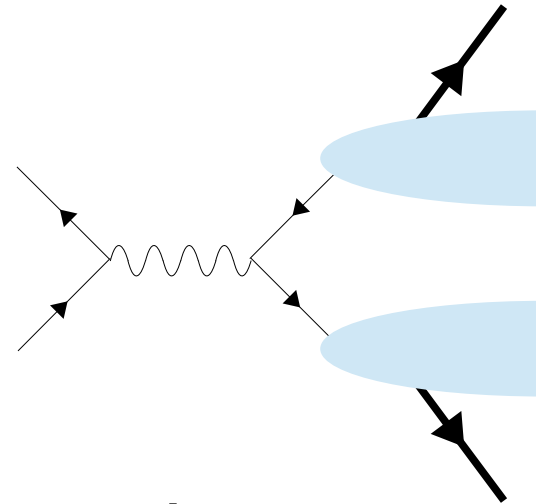
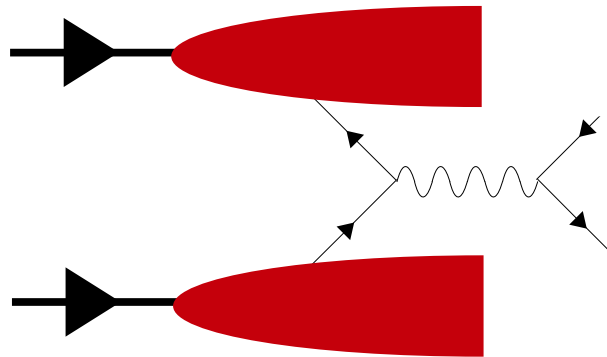


Fragmentation Functions

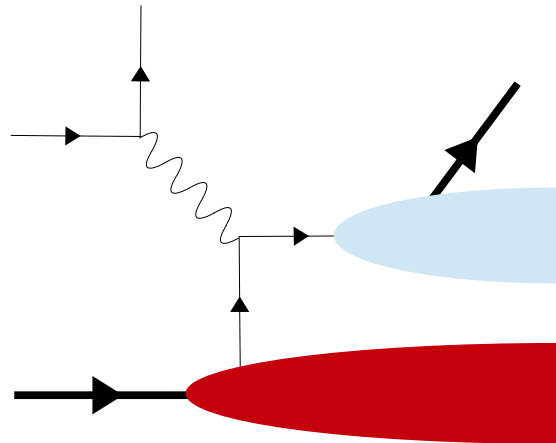
SIDIS

Extraction from data?

Drell Yan



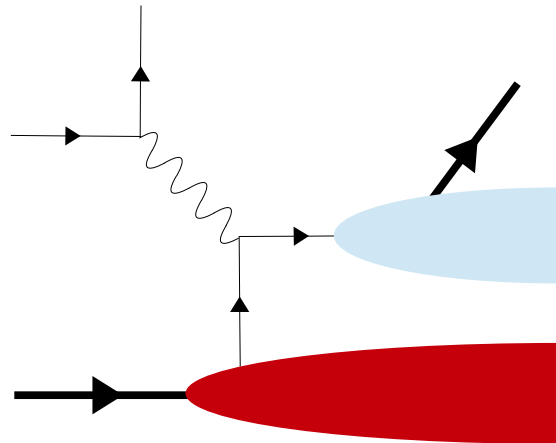
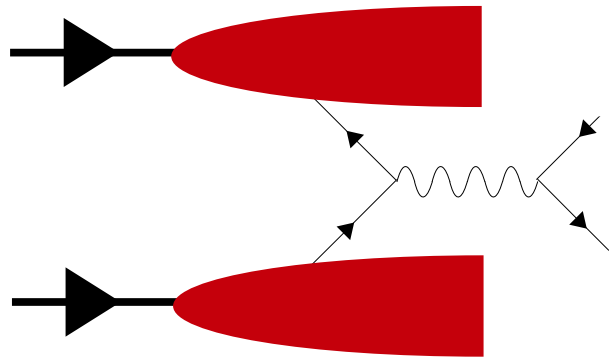
e^+e^-



SIDIS

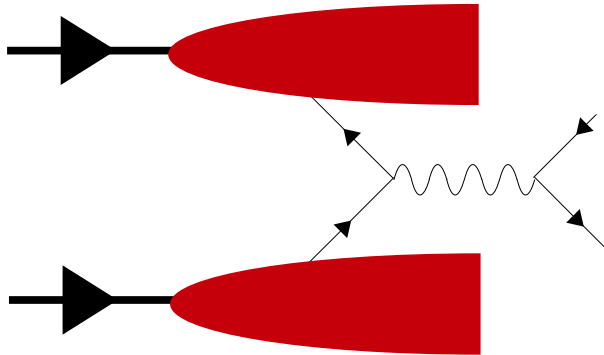
**Fragmentation
Functions**

Drell Yan



SIDIS

Drell Yan



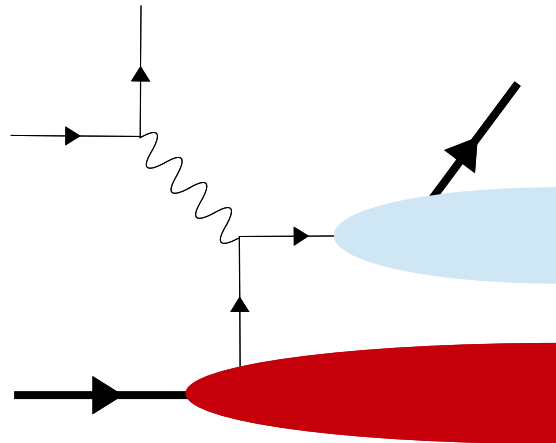
**Under control, e.g.
full implementation of CSS available**

F. Landry, R. Brock, P. M.
Nadolsky and C. P. Yuan, Phys.
Rev. D67, 073016 (2003)

Must still address some issues.

Delicate kinematics of available
multidimensional data

The matching between low and large
transverse momentum

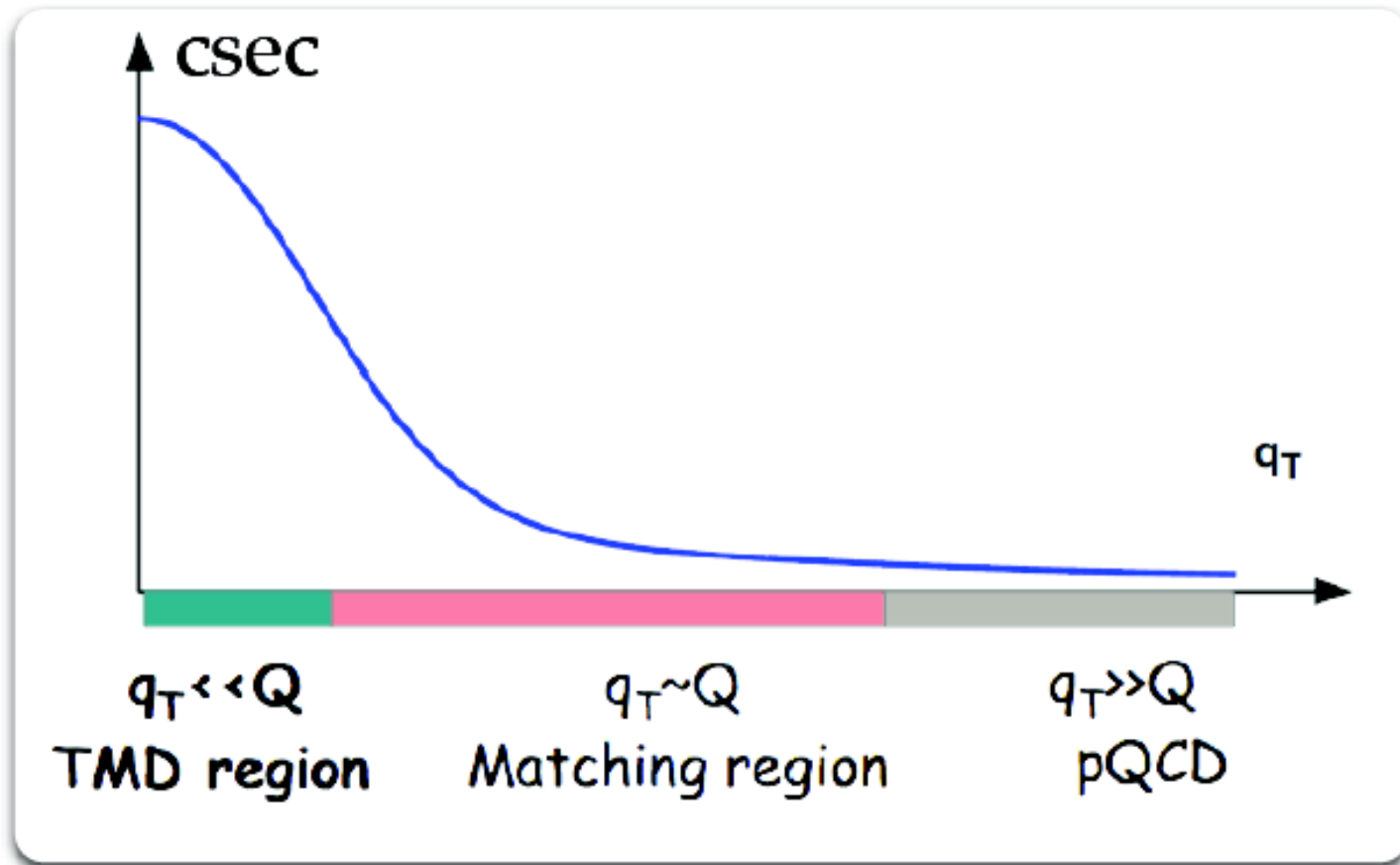


SIDIS

The Matching Problem in SIDIS

$$\{Q^2, x_B, P_{hT}, z_h\}$$

$$q_T = P_{hT}/z_h$$



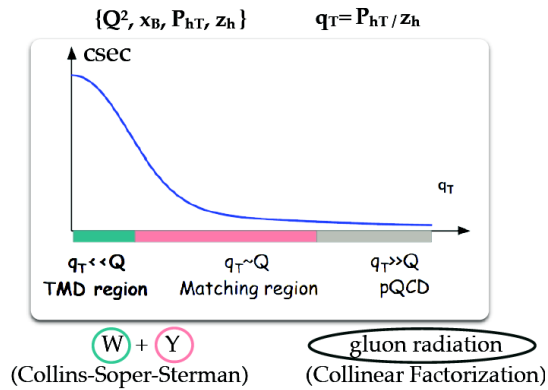
$$\textcircled{W} + \textcircled{Y}$$

(Collins-Soper-Sterman)

gluon radiation

(Collinear Factorization)

The Matching Problem in SIDIS



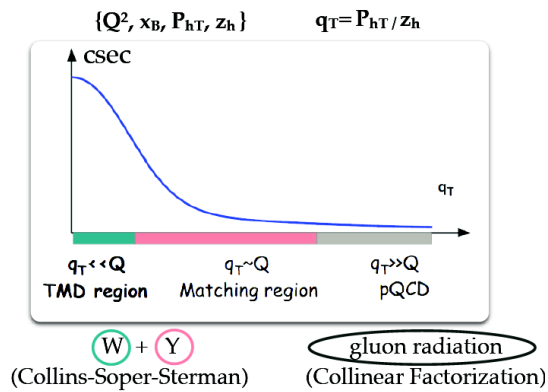
Works for SIDIS at high enough, $Q^2 > 10 \text{ GeV}^2$,
 energy flow (**integration over z_h**)

Nadolsky, Stump, Yuan

DOI: [10.1103/PhysRevD.64.059903](https://doi.org/10.1103/PhysRevD.64.059903)

However, information about z -dependence gets washed out. Also, integration over z mixes TMD and collinear factorization effects.

The Matching Problem in SIDIS



Works for SIDIS at high enough, $Q^2 > 10 \text{ GeV}^2$,
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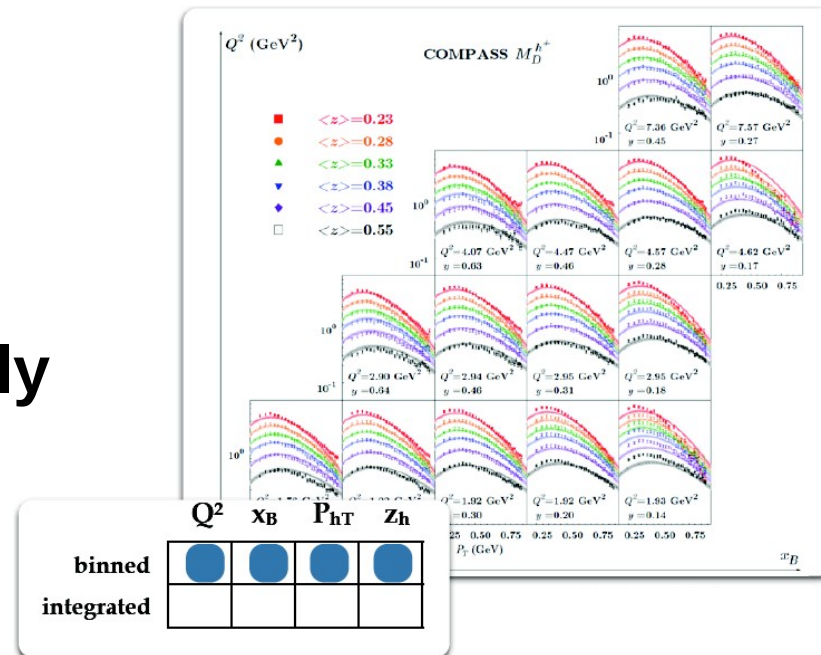
Nadolsky, Stump, Yuan

DOI: [10.1103/PhysRevD.64.059903](https://doi.org/10.1103/PhysRevD.64.059903)

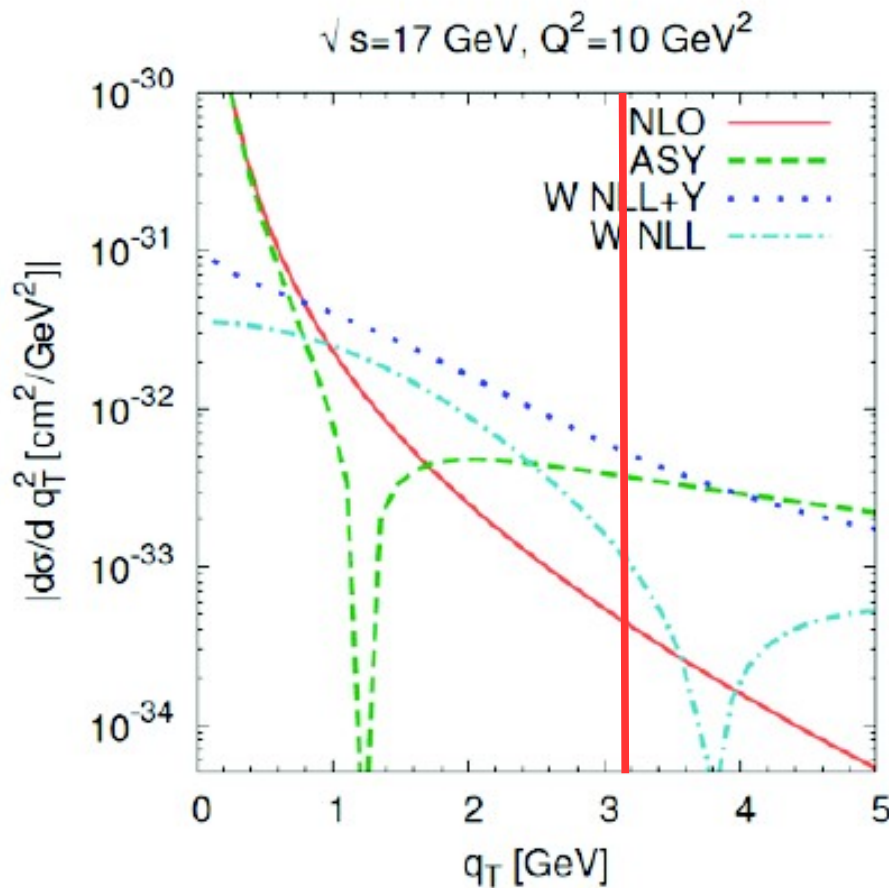
However, information about z -dependence gets washed out. Also, integration over z mixes TMD and collinear factorization effects.

Multidimensional data are ideal.

Can CSS be successfully Implemented?

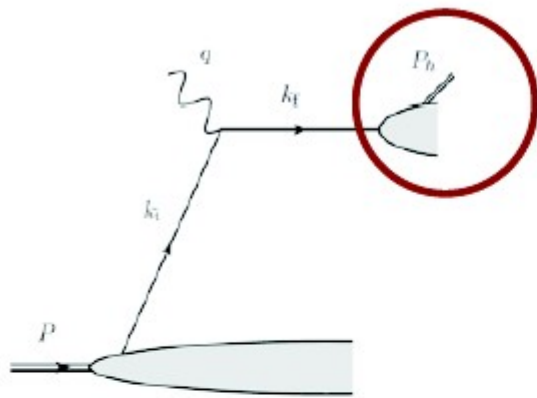


Large q_T corrections are hard to implement.

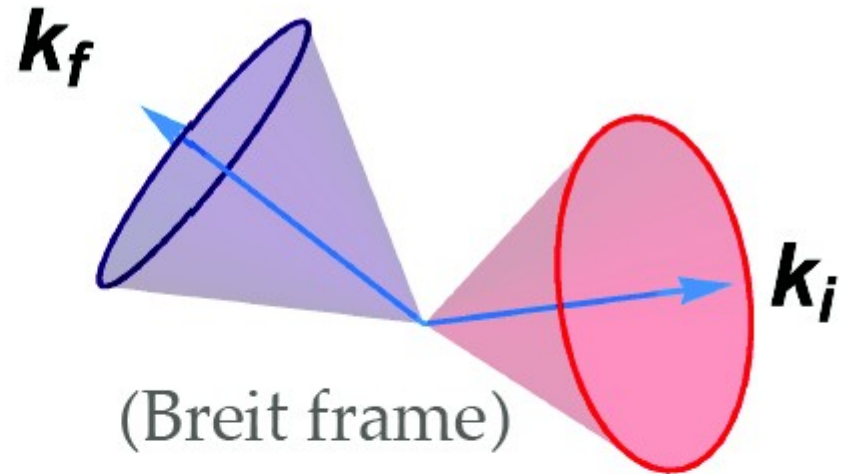


- Large Y -term at small q_T
- Small cross section at large q_T
- No smooth matching

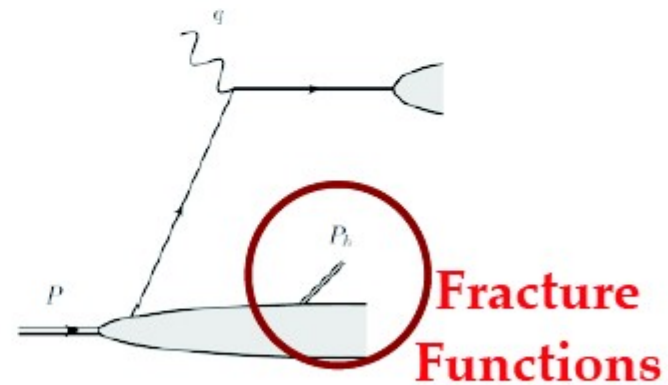
Delicate kinematics



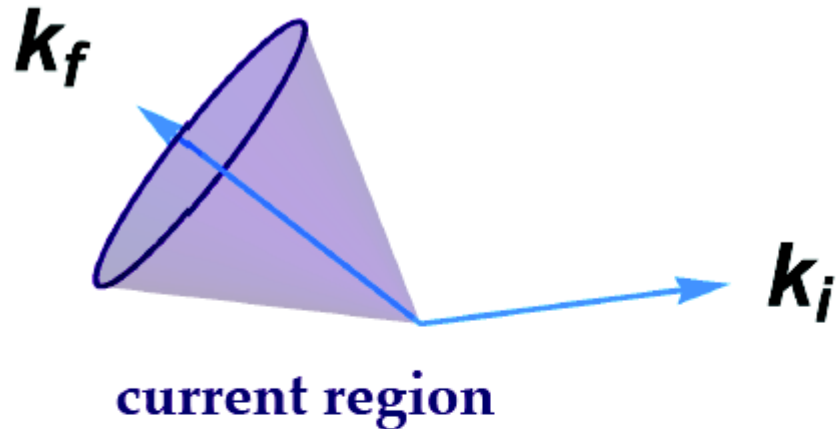
TMDs



**factorization theorems for
different leading regions**



Power counting and kinematics of the current region



small masses

$$P_h \cdot k_f = O(m^2)$$

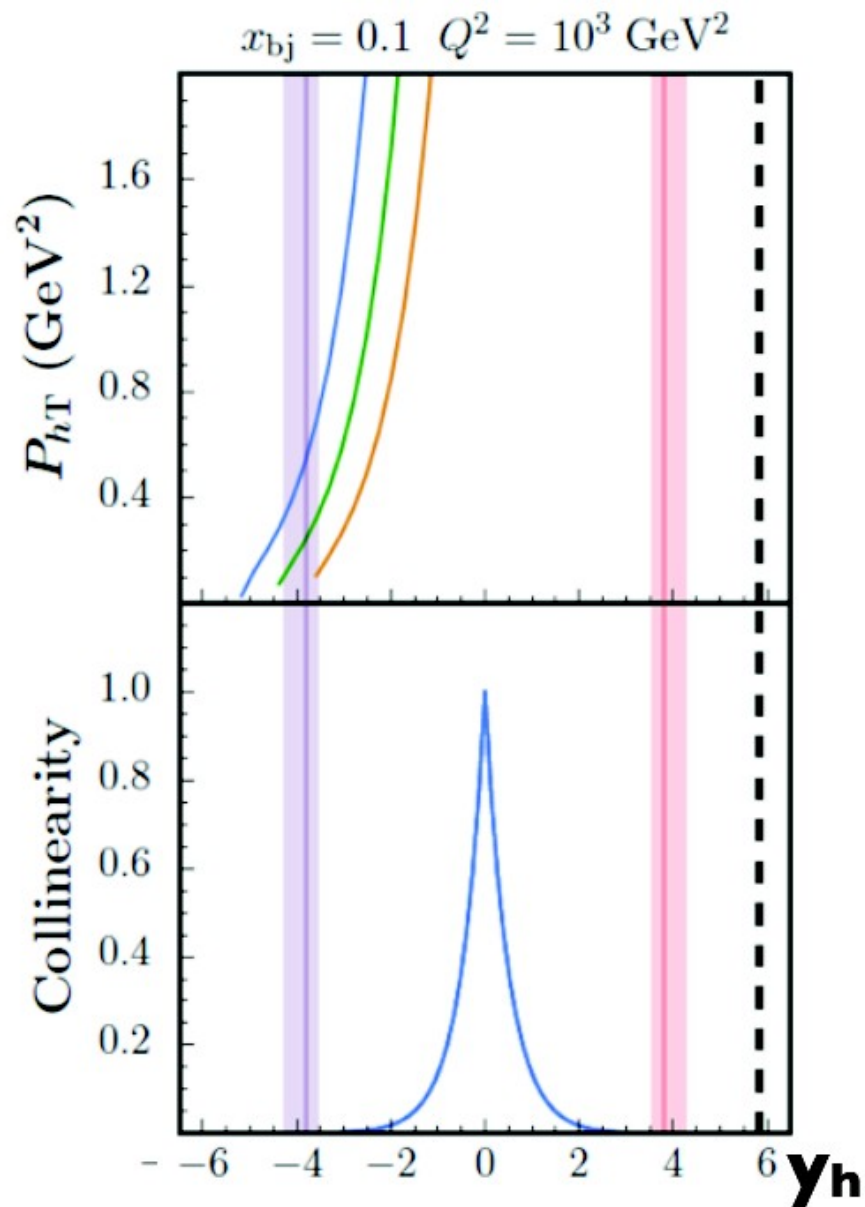
$$P_h \cdot k_i = O(Q^2)$$

hard scale

require small values for

$$R \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i}$$

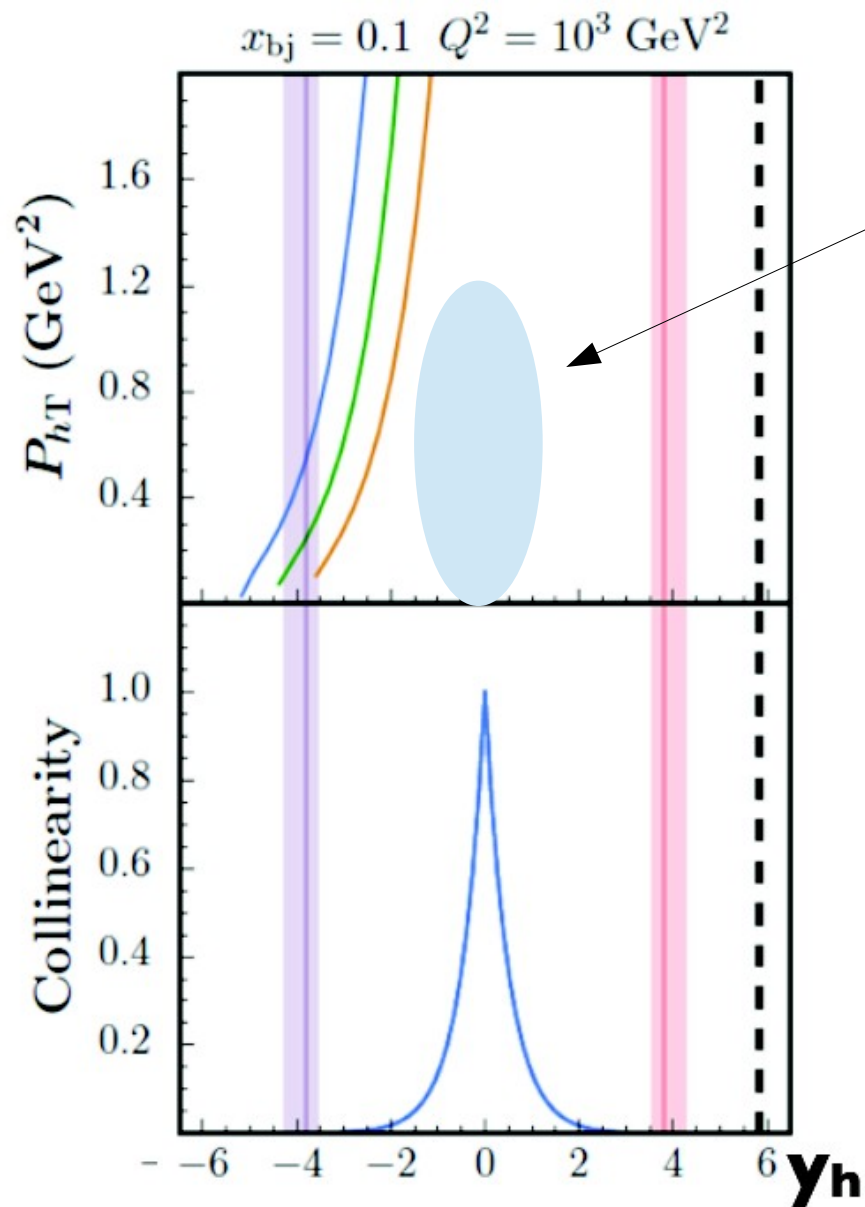
notice quark momenta have to be estimated



$$y_h \equiv \frac{1}{2} \log \frac{P_h^+}{P_h^-}$$

$$R \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i}$$

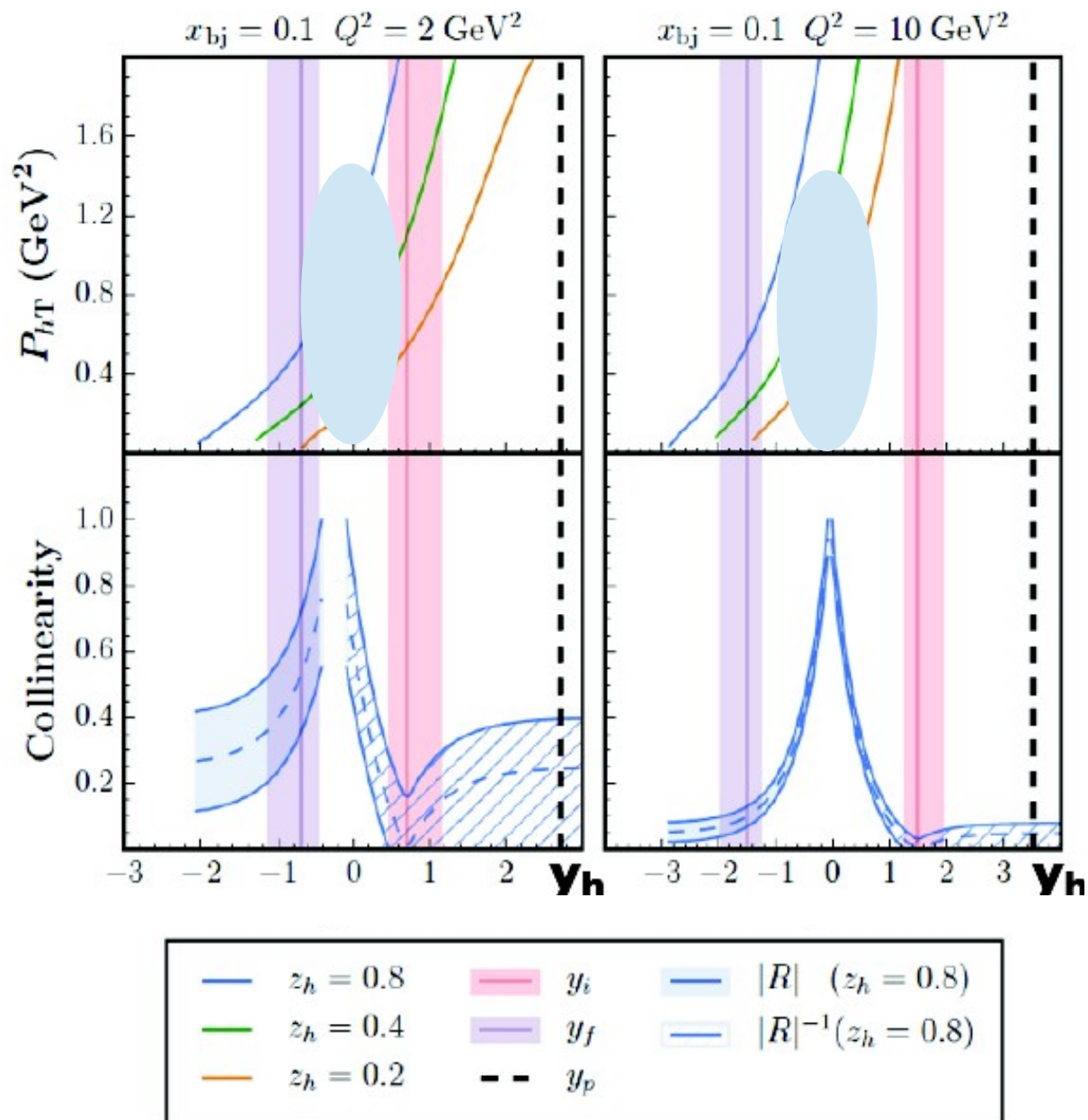




Avoid region of central rapidity, soft non-TMD effects

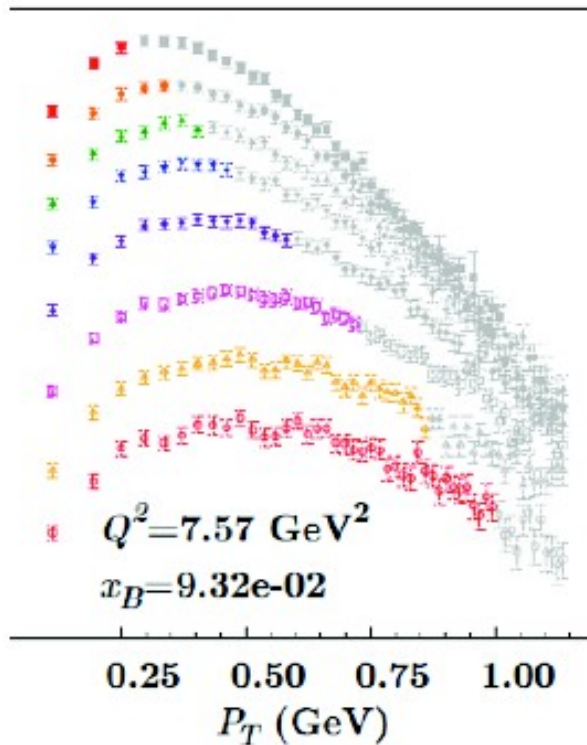
$$R \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i}$$





Available data is likely to receive contributions from non-TMD physics.

$$R \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i}$$



precise implementation of
the R criterion on data is
work in progress

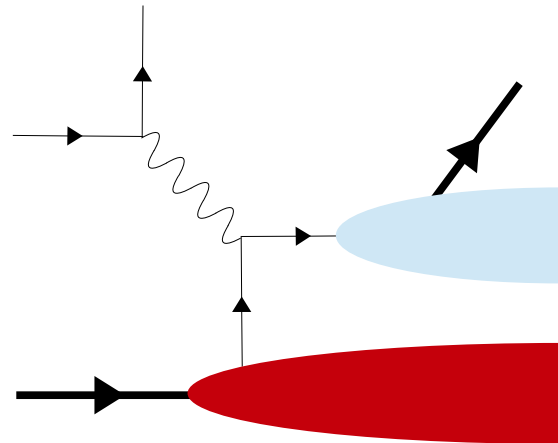
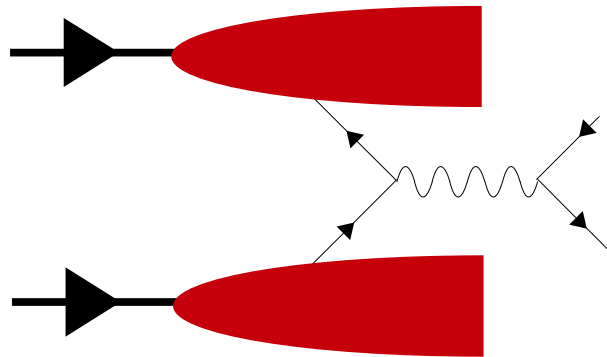
a better set of variables?

$$\{Q^2, x_B, P_{hT}, z_h\}$$

$$q_T = P_{hT} / z_h \quad y_h$$

***ONLY AN
EXAMPLE**

Drell Yan

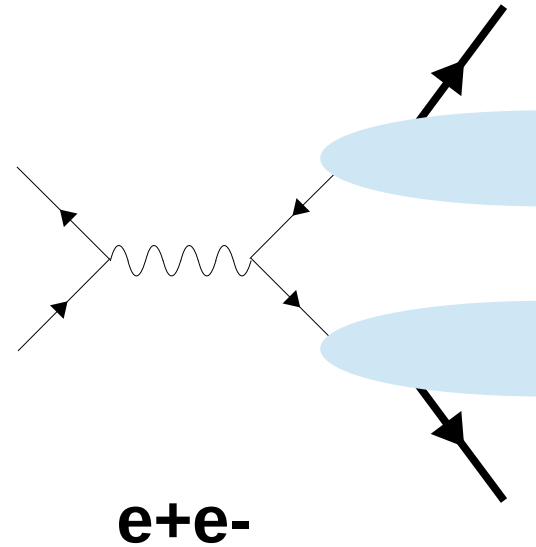


?

SIDIS

Recently, BELLE, BaBar, BES III
Collins asymmetries.

No modern unpolarized
measurements are available.

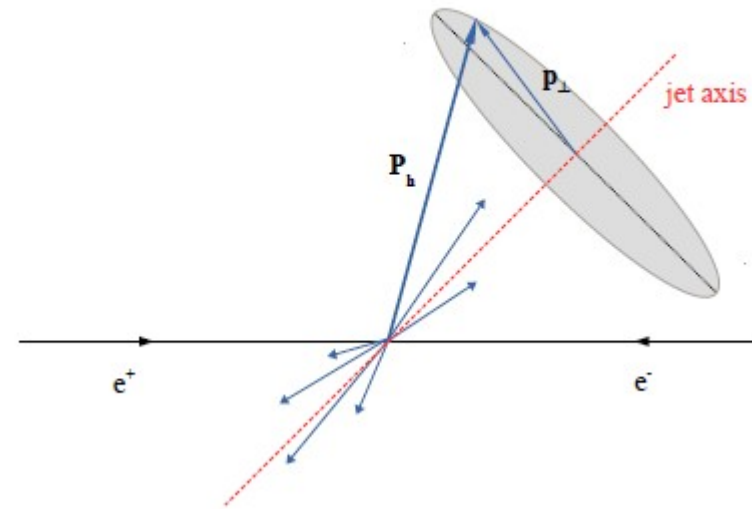


Recently, BELLE, BaBar, BES I
Collins asymmetries.

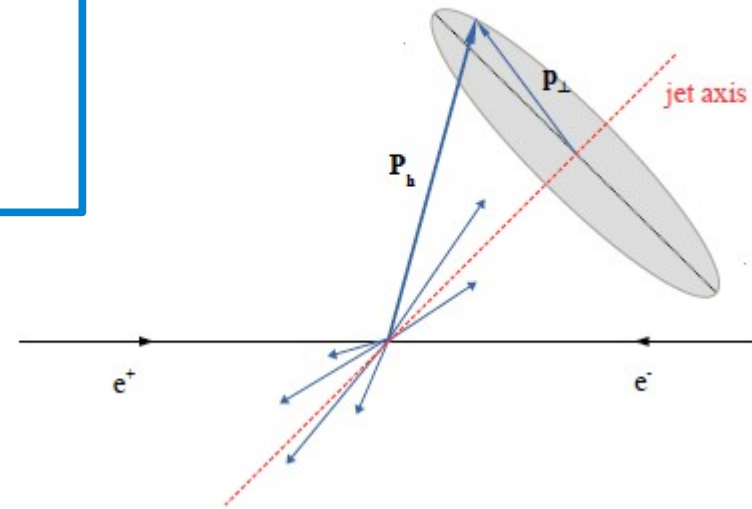
No modern unpolarized
measurements are available.

TASSO, MARK II available for
 $e^+e^- \rightarrow X h$

- **p_T** distributions
- different energies
- integrated over **z**



Boglione, JOGH, R. Taghavi
arXiv:1704.08882v1



TASSO, MARK II available for
 $e^+e^- \rightarrow X h$

- p_T distributions
- different energies
- integrated over z

New analysis:

how much information about the **unpolarized TMD FF**
can we get from these data sets?

Use this...



$$D_{h/q}(z, p_{\perp}) = d_{h/q}(z) h_d(p_{\perp})$$

To get information
about this

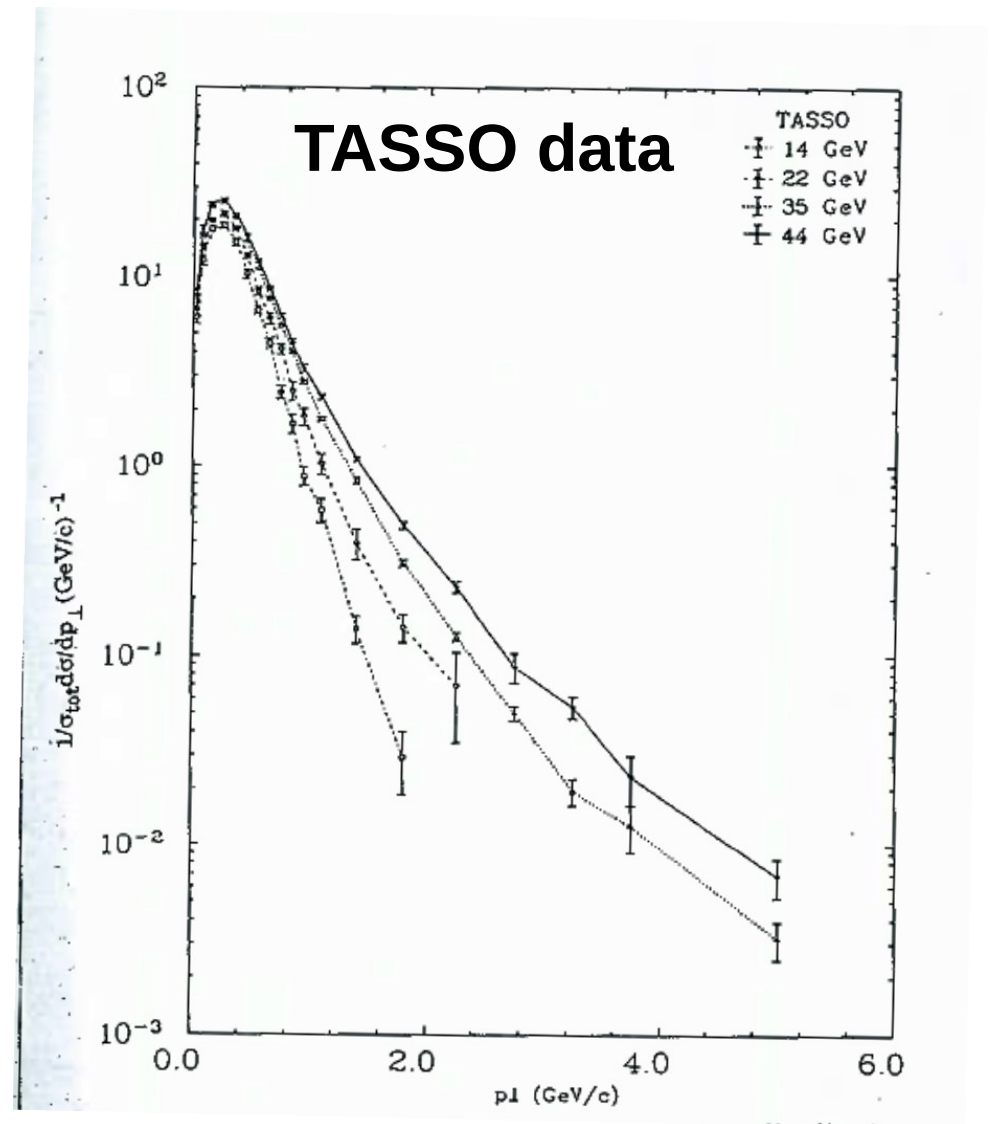


QCD picture

$$\tilde{D}_{h/q}(z, \mathbf{b}_{\perp}; Q) = \sum_j \left[\left(\tilde{C}_{j/q} \otimes \frac{d_{h/j}}{z^2} \right) e^{\Gamma_D(Q)} \right] \exp \left\{ g_{j/P}(x, b_{\perp}) + g_K(b_{\perp}) \log \left(\frac{Q}{Q_0} \right) \right\}$$

Things to investigate:

- appropriate functional form for $\mathbf{g}_{j/P}$
- scale evolution regulated by \mathbf{g}_K



$$\tilde{D}_{h/q}(z, \mathbf{b}_{\perp}; Q) = \sum_j \left[\left(\tilde{C}_{j/q} \otimes \frac{d_{h/j}}{z^2} \right) e^{\Gamma_D(Q)} \right] \exp \left\{ g_{j/P}(x, b_{\perp}) + g_K(b_{\perp}) \log \left(\frac{Q}{Q_0} \right) \right\}$$

Identify region where TMD Effects dominate:

For fully differential cross sections, matching region is Expected to be at

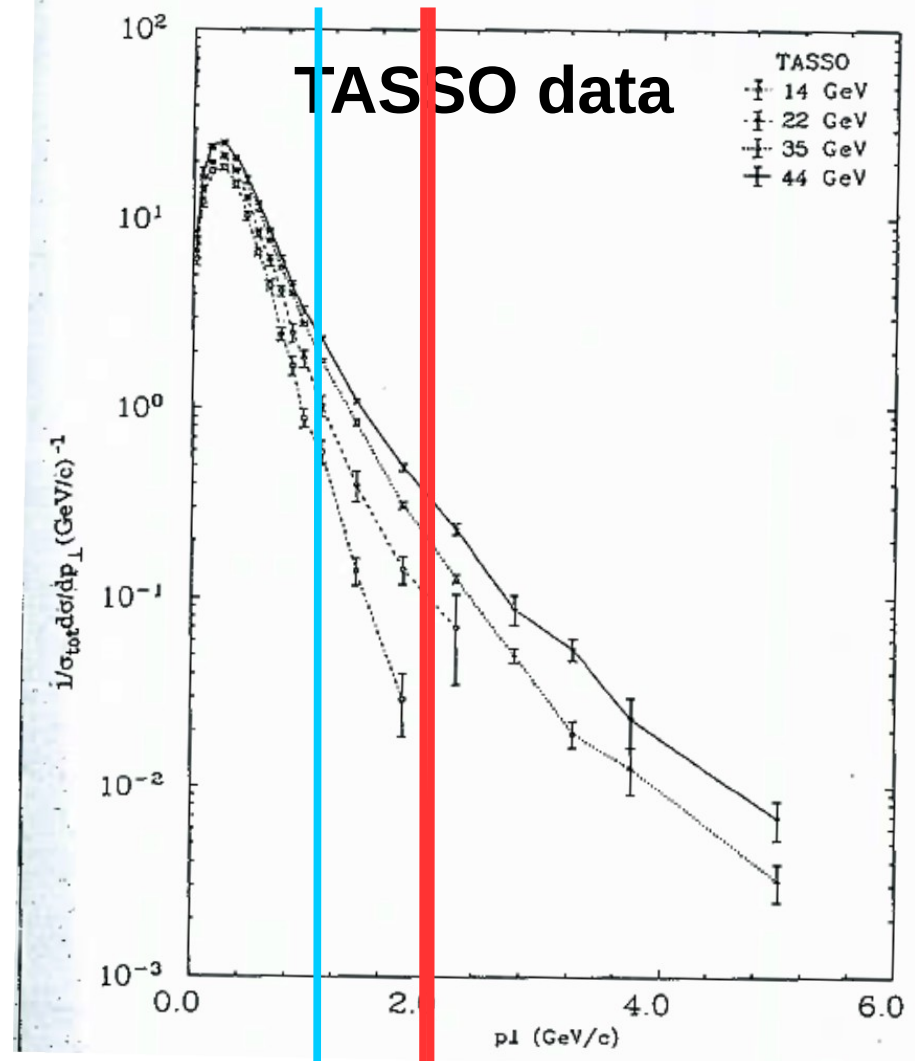
$$p_{\perp} \sim zQ$$

Use experimental $\langle z \rangle$ to make an estimate

$$p_{\perp} \sim 2 \text{ GeV}$$

We looked at a restricted range:

$$p_{\perp} < 1 \text{ GeV}$$

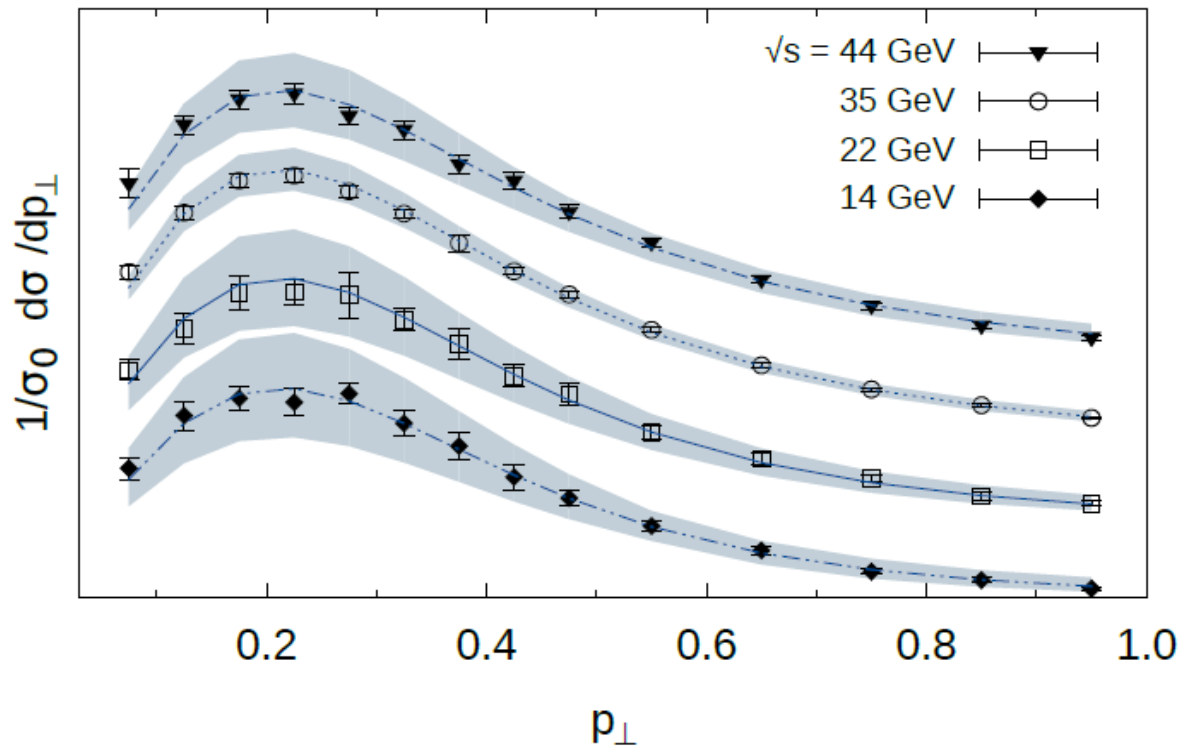


Power law to model transverse momentum dependence

$$D_{h/q}(z, p_{\perp}) = d_{h/q}(z) h_d(p_{\perp})$$

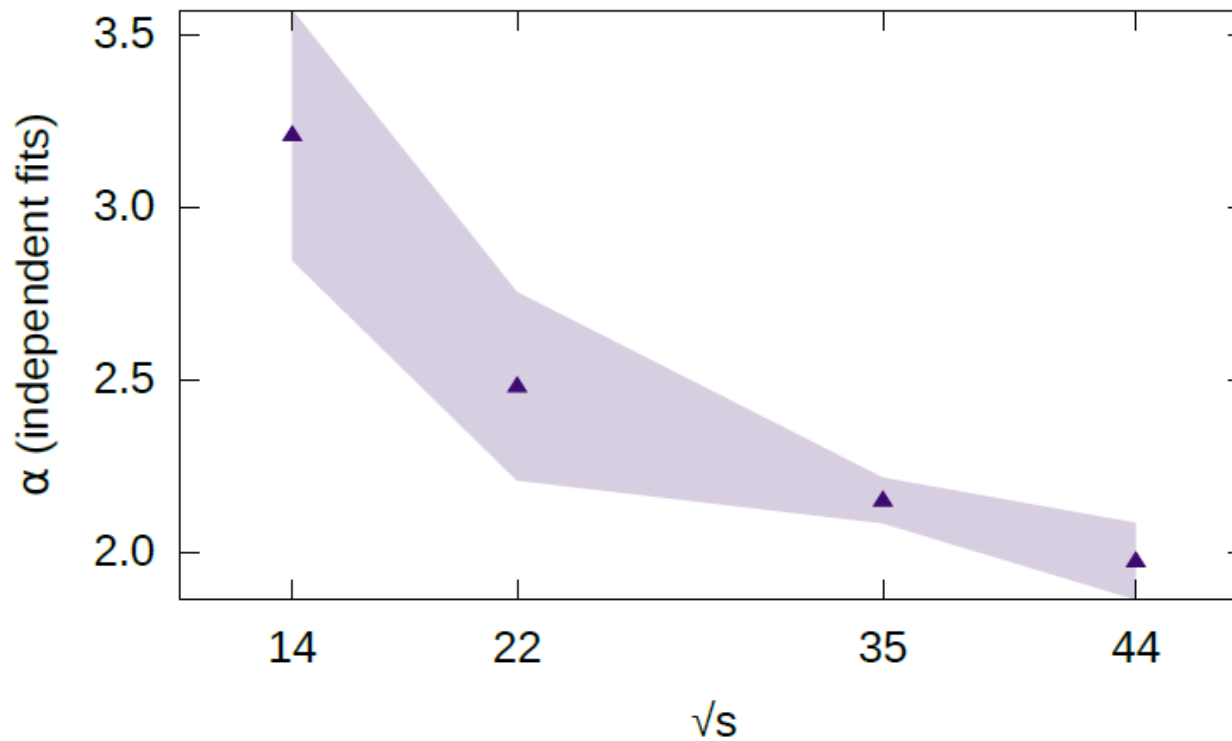
$$h(p_{\perp}) = 2(\alpha - 1)M^{2(\alpha-1)} \frac{1}{(p_{\perp}^2 + M^2)^{\alpha}}$$

FIT TO TASSO



Power law parameters follow a logarithmic trend

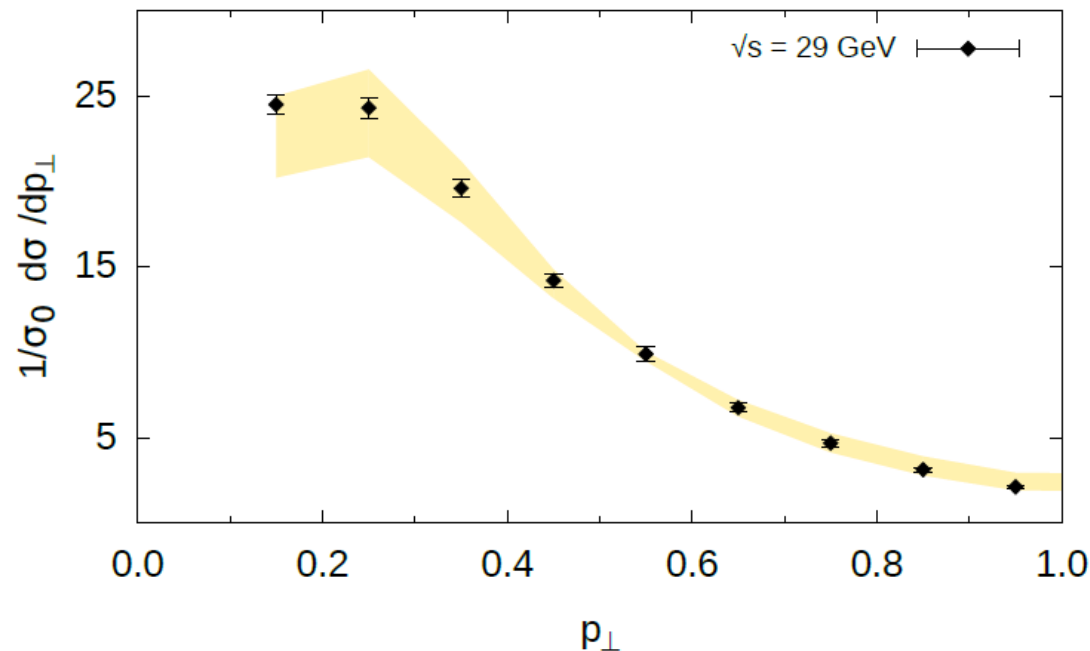
$$h(p_{\perp}) = 2(\alpha - 1)M^{2(\alpha-1)} \frac{1}{(p_{\perp}^2 + M^2)^{\alpha}}$$



Power law parameters follow a logarithmic trend
Consistent with MARK II data.

$$h(p_{\perp}) = 2(\alpha - 1)M^{2(\alpha-1)} \frac{1}{(p_{\perp}^2 + M^2)^{\alpha}}$$

COMPARISON TO MARK II



TMD

$$\mathcal{F}^{-1} \left\{ \frac{d\sigma^h}{dz d^2\mathbf{p}_\perp} \right\} \propto \exp \left\{ \left(\lambda_\Gamma(b_*) + g_K(b_\perp) \right) \log \left(\frac{Q}{Q_0} \right) \right\} \Big|_{b_\perp \rightarrow z b_\perp}$$

$$\lambda_\Gamma(b_*) \equiv \frac{32}{27} \log \left(\log \frac{2e^{-\gamma_E}}{\Lambda_{QCD} b_*} \right)$$

MODEL $h(p_\perp) = 2(\alpha - 1)M^{2(\alpha-1)} \frac{1}{(p_\perp^2 + M^2)^\alpha}$

$$\mathcal{F}^{-1} \left\{ \frac{1}{(p_\perp^2 + M^2)^\alpha} \right\} \xrightarrow{\text{large } b_\perp} \frac{1}{2^\alpha \pi \Gamma(\alpha)} \left(\frac{b_\perp}{M} \right)^{\alpha-1} \sqrt{\frac{\pi}{2}} \frac{e^{-b_\perp M}}{\sqrt{b_\perp M}} \left[1 + \mathcal{O} \left(\frac{1}{b_\perp M} \right) \right]$$

TMD

$$\mathcal{F}^{-1} \left\{ \frac{d\sigma^h}{dz d^2\mathbf{p}_\perp} \right\} \propto \exp \left\{ \left(\lambda_\Gamma(b_*) + g_K(b_\perp) \right) \log \left(\frac{Q}{Q_0} \right) \right\} \Big|_{b_\perp \rightarrow z b_\perp}$$

$$\lambda_\Gamma(b_*) \equiv \frac{32}{27} \log \left(\log \frac{2e^{-\gamma_E}}{\Lambda_{QCD} b_*} \right)$$

Logarithmic behavior of alpha may be interpreted as a consequence of the **Log** in the definition of the **TMD FF**.

$$\alpha = \alpha_0 + \tilde{\alpha} \log \left(\frac{Q}{Q_0} \right)$$

$$g_K(b_\perp) \xrightarrow{\text{large } b_\perp} \tilde{\alpha} \log(\nu b_\perp)$$

TMD

There are caveats on this interpretation, while consistent with theoretical expectations, its not the only possibility.

(loss of information through z-integration)

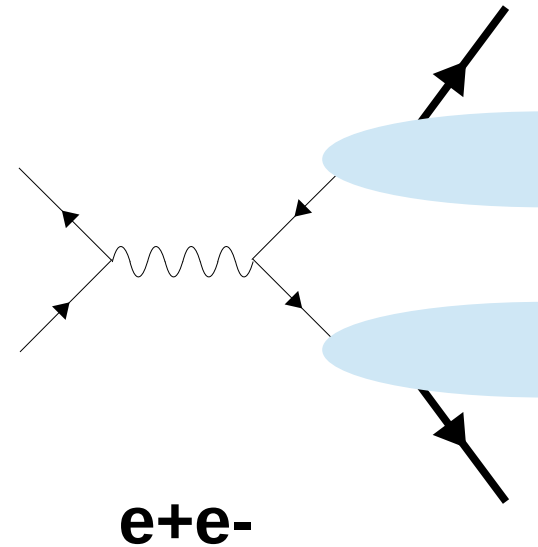
Logarithmic behavior of alpha may be interpreted as a consequence of the **Log** in the definition of the **TMD FF**.

$$\alpha = \alpha_0 + \tilde{\alpha} \log \left(\frac{Q}{Q_0} \right)$$

$$g_K(b_{\perp}) \xrightarrow{\text{large } b_{\perp}} \tilde{\alpha} \log(v b_{\perp})$$

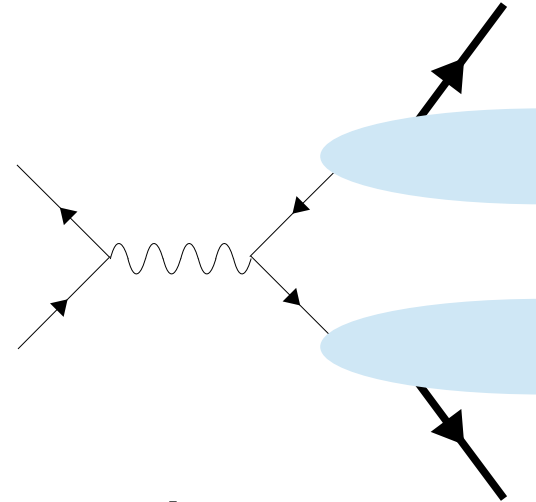
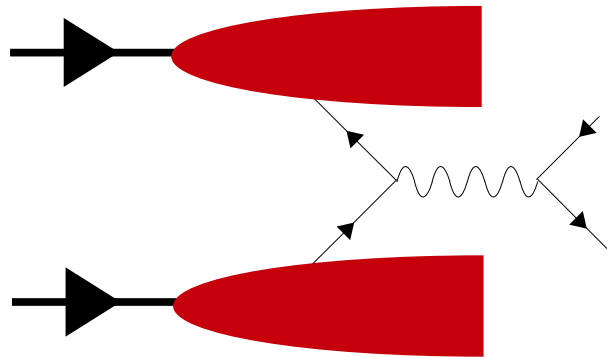
The lack of information about \mathbf{z} hinders a full TMD extraction of the FF.

Future upcoming data by BELLE on unpolarized one-hadron production may allow for a combined analysis with TASSO and MARK II data.

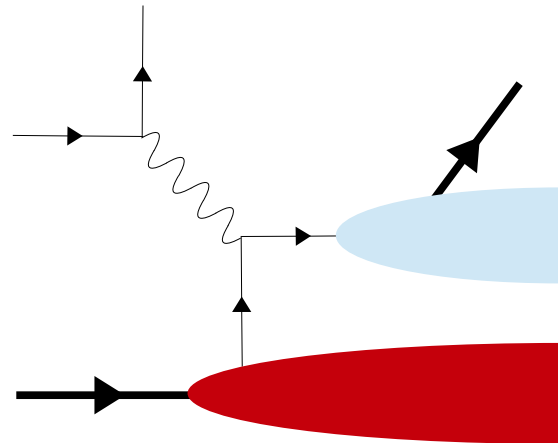


Extraction from data?

Drell Yan



e^+e^-



SIDIS

Let's keep working,
but let us
be careful with
interpretations

**Fragmentation
Functions**

Final Remarks

Currently, we are attempting to do phenomenology within **full QCD picture**.

Need to describe regions of **low and large transverse momenta** simultaneously.

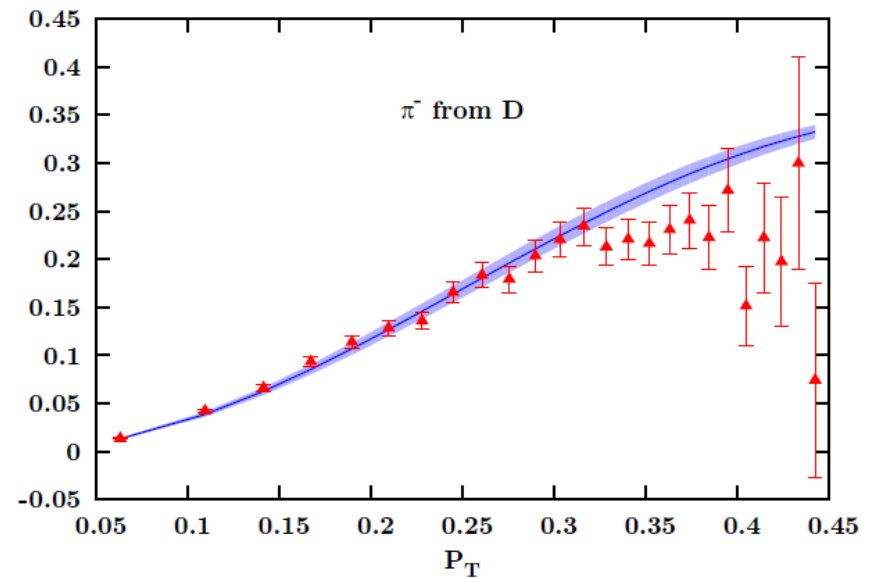
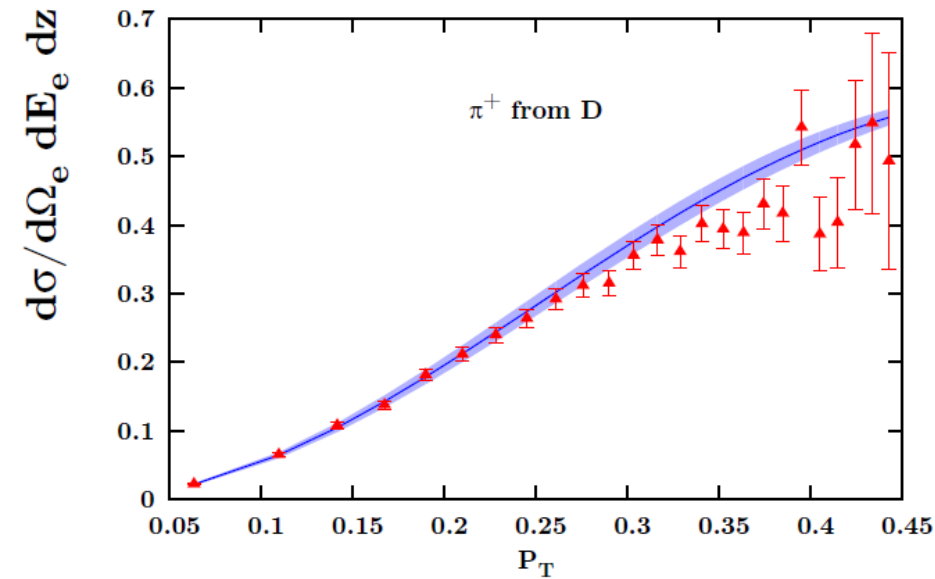
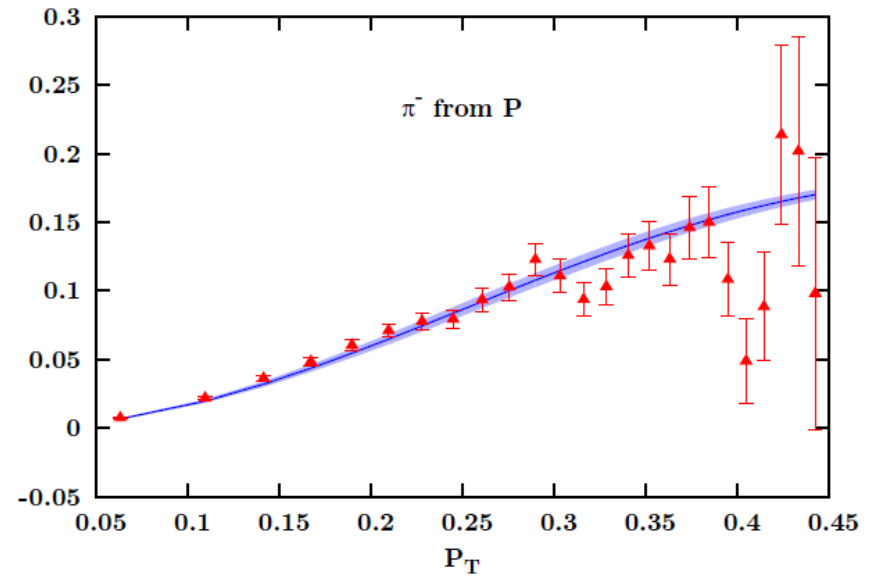
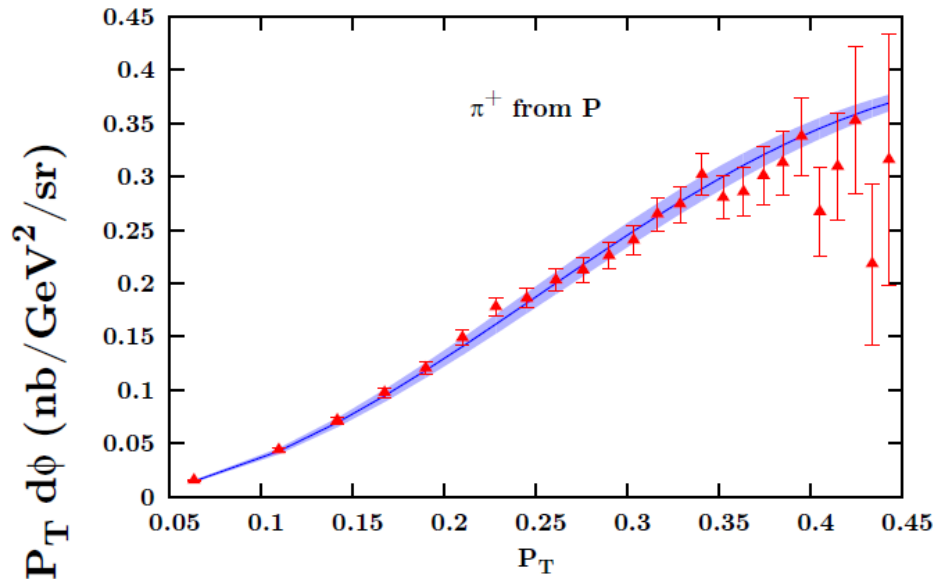
More work to be done, it's important to take a step back and think of the **theoretical issues** (solving the “matching problem” at delicate kinematics).

In SIDIS, while we have great data sets, some work need to be done to understand theoretical errors of factorization. This is needed for a correct interpretation of phenomenological analyses.

On the side of e^+e^- one hadron production, in the near future unpolarized cross sections by BELLE may allow for an analysis of the older sets, TASSO MARKII within a full TMD picture.

Thank you.

Jlab SIDIS data (2012) (Parameters from HERMES extraction).



Ingredients for extraction of Collins function.

$e^+e^- \rightarrow \pi\pi X$

SIDIS

Unpolarized TMDFF

Collins TMDFF

$$\frac{d\sigma^{e^+e^- \rightarrow h_1 h_2 X}}{dz_1 dz_2 d^2\mathbf{P}_{1T} d\cos\theta_2} = \frac{3\pi\alpha^2}{2s} \left\{ \boxed{D_{h_1 h_2}} + \boxed{N_{h_1 h_2}} \cos 2\phi_1 \right\}$$

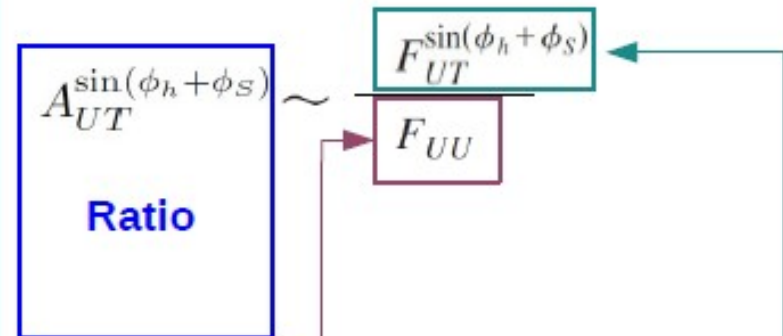
$$P_0^{U,L,C} = \frac{N^{U,L,C}}{D^{U,L,C}}$$

Ratio

$$\begin{aligned} D^U &= D_{\pi^+\pi^-} + D_{\pi^-\pi^+} & N^U &= N_{\pi^+\pi^-} + N_{\pi^-\pi^+} \\ D^L &= D_{\pi^+\pi^+} + D_{\pi^-\pi^-} & N^L &= N_{\pi^+\pi^+} + N_{\pi^-\pi^-} \\ D^C &= D^U + D^L & N^C &= N^U + N^L, \end{aligned}$$

$$\frac{A_0^U}{A_0^{L(C)}} \equiv 1 + \cos(2\phi_1) \boxed{A_0^{UL(C)}} \quad \text{Double Ratio}$$

$$\begin{aligned} \frac{d\sigma^{\ell(S_c)+p(S) \rightarrow \ell' h X}}{dx_B dQ^2 dz_h d^2\mathbf{P}_T d\phi_S} = \\ \frac{2\alpha^2}{Q^4} \left\{ \frac{1 + (1-y)^2}{2} F_{UU} + \dots \right. \\ \left. + S_T(1-y)(\sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)}) \right\}. \end{aligned}$$



Unpolarized
TMDFF
& TMDPDF

TMD Transversity
& Collins function

Unpolarized SIDIS cross section (current region)

$$\frac{d\sigma^{\ell+p \rightarrow \ell' h X}}{dx_B dQ^2 dz_h dP_T^2} = \frac{2\pi^2\alpha^2}{(x_B s)^2} \frac{[1 + (1-y)^2]}{y^2} F_{UU}$$

$$F_{UU} = \sum_q \mathcal{H}_q \text{F.T.} \left\{ \tilde{D}_{h/q}(z, z \mathbf{b}_\perp; Q) \tilde{f}_{q/P}(x, \mathbf{b}_\perp; Q) \right\}$$

+ large q_T corrections + power suppressed terms

