The renormalized iso-vector quasi-PDF from Lattice QCD

Yi-Bo Yang

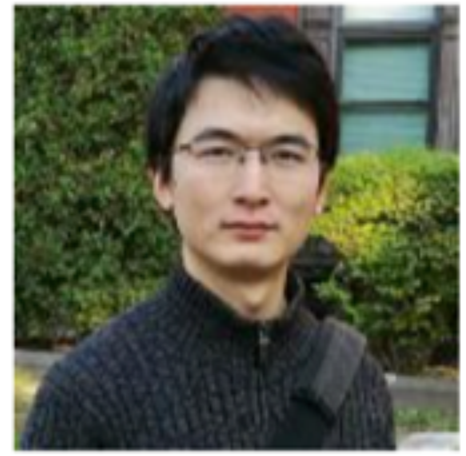
Lattice Parton Physics Project (LP3)

2017.05.22

https://www.pa.msu.edu/~hwlin/LP3/

Huey-Wen Lin (PI)
(MSU)

Xiangdong Ji
(UMD)



Luchang Jin
(BNL)

Peng Sun
(MSU)

Yi-Bo Yang
(MSU)

Yong Zhao
(MIT)

International collaborators

Jiunn-Wei Chen
(NTU)

Tomomi Ishikawa
(SJTU)

Jian-Hui Zhang
(Regensburg)

The parton distribution function (PDF)

The original quark PDF defined in the light front frame is,

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P|\overline{\psi}(\xi^-)\gamma^+ \times \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\psi(0)|P\rangle$$

- PDF is the universal distribution in nucleon;
- Many ongoing/planned experiments (BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, …);
- Important inputs to discern new physics at LHC, and currently dominate errors in Higgs production.
- The real time dependence in the PDF definition makes the direct lattice simulation to be impossible.

The parton distribution function (PDF)

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The adaptive solutions on the lattice include:

- The moments of the PDF which are matrix elements of local operators;
  See for example, J. W. Negele, Nucl. Phys. A 711, 281 (2002)
- The moments of the hadronic tensor with two heavy-light vector currents;
  W. Detmold, C.J. David Lin, Phys.Rev.D76:014501,2007
- **The quasi-PDF with a spacial wilson link.** **X.D. Ji, Phys.Rev.Lett. 110 (2013) 262002**
- The hadronic tensor with two vector currents on different t; **K. F. Liu, arXiv:1603.07352**
- The pseudo-PDF as the ratio of two operators **A.V. Radyushkin, 1705.01488**
- etc.

Definition of the quasi-PDF

The original quark PDF defined in the light front frame is,

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P|\overline{\psi}(\xi^-)\gamma^+ \times \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\psi(0)|P\rangle$$

The quasi-PDF is defined by

$$q(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P|\overline{\psi}(z)\gamma^z \times \exp\left(-ig\int_0^{z} dz' A^z(z')\right)\psi(0)|P\rangle + \mathcal{O}\left(\Lambda^2/(P^z)^2, M^2/(P^z)^2\right) \ ,$$

X.D. Ji, Phys.Rev.Lett. 110 (2013) 262002

From the quasi-PDF

to the real PDF

1- loop matching from quasi-PDF to PDF

Tree level

$$q(x,\mu) = \int_{-\infty}^{+\infty} \frac{dy}{|y|} [\delta(1-\frac{x}{y})\{1 + \frac{\alpha_s C_F}{2\pi} \int_{-\infty}^{+\infty} d\xi\, C^{OM}\left(\xi, \frac{\mu_R}{P^z}, \frac{\mu}{P^z}\right)\} - \frac{\alpha_s C_F}{2\pi} C^{OM}\left(\frac{x}{y}, \frac{\mu_R}{P^z}, \frac{\mu}{P^z}\right)]$$
$$\int_{-\infty}^{\infty} \frac{dz}{2\pi}\, e^{iyP^z z} \langle P|\bar{\psi}(z)\gamma^z W_z(z,0)\psi(0)|P\rangle_R$$
$$+ \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{P_z^2}\right) + \mathcal{O}\left((\frac{\alpha_s}{\pi})^2\right)$$

Renormalized Lattice quasi-PDF matrix elements with the mass correction which can be calculated analytically

Residual higher-twist contribution

Residual higher-loop matching

X. Xiong, X. Ji, J. Zhang, Y. Zhao, Phys. Rev. D 90, 014051 (2014)
LP3, 1603.06664

From *the bare quasi-PDF*

to the real PDF

Y-Q. Ma, J-W. Qiu, 1404.6860
C. Alexandrou et. al., Phys. Rev. D92 014502
J.-W. Chen, X. Ji, J. Zhang, Nucl.Phys. B915 (2017) 1

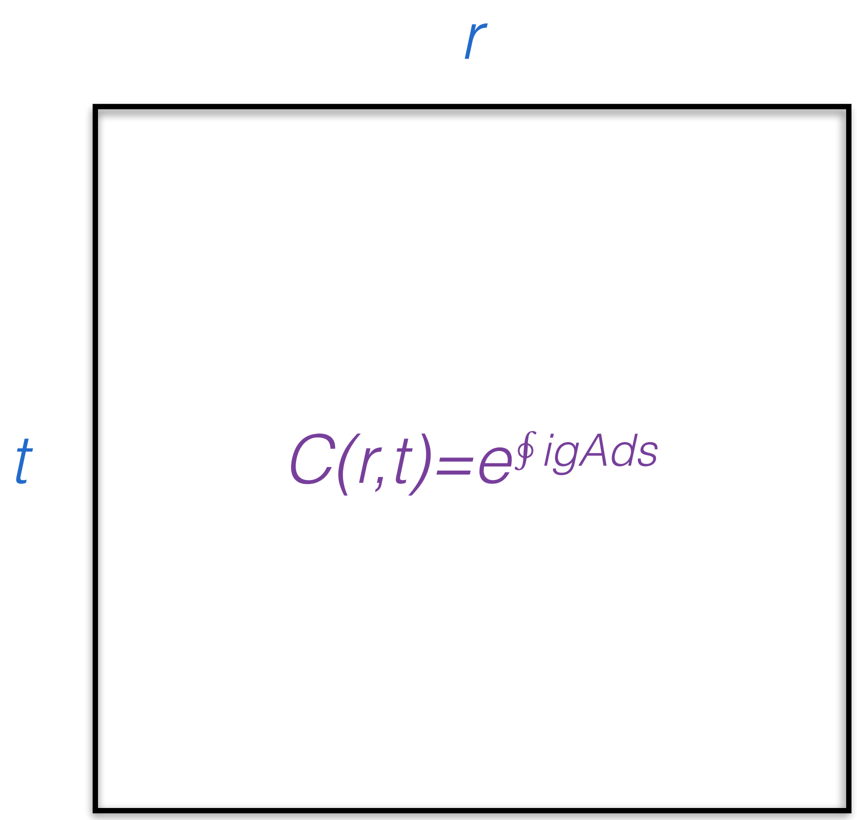
$$q(x,\mu) = \int_{-\infty}^{+\infty} \frac{dy}{|y|} \left[\delta(1-\frac{x}{y})\left\{1 + \frac{\alpha_s C_F}{2\pi} \int_{-\infty}^{+\infty} d\xi \; C^{OM}\left(\xi, \frac{\mu_R}{P^z}, \frac{\mu}{P^z}\right)\right\} - \frac{\alpha_s C_F}{2\pi} C^{OM}\left(\frac{x}{y}, \frac{\mu_R}{P^z}, \frac{\mu}{P^z}\right)\right]$$
$$\int_{-\infty}^{\infty} \frac{dz}{2\pi} \; e^{iyP^z z} \langle P|\bar{\psi}(z)\gamma^z W_z(z,0)\psi(0)|P\rangle_R$$
$$+ \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{P_z^2}\right) + \mathcal{O}\left((\frac{\alpha_s}{\pi})^2\right)$$

The linear divergence under the lattice regularization can break the convergence of the perturbative series!

$$\langle P|\bar{\psi}(z)\gamma^z W_z(z,0)\psi(0)|P\rangle_R = (1 + \frac{\alpha_S}{4\pi}(\frac{C}{a} + \mathrm{Log}(p^2 a^2) + \ldots) + \mathcal{O}((\frac{\alpha_S}{4\pi})^2))\langle P|\bar{\psi}(z)\gamma^z W_z(z,0)\psi(0)|P\rangle_{bare}$$

Linear divergence

in the wilson loop



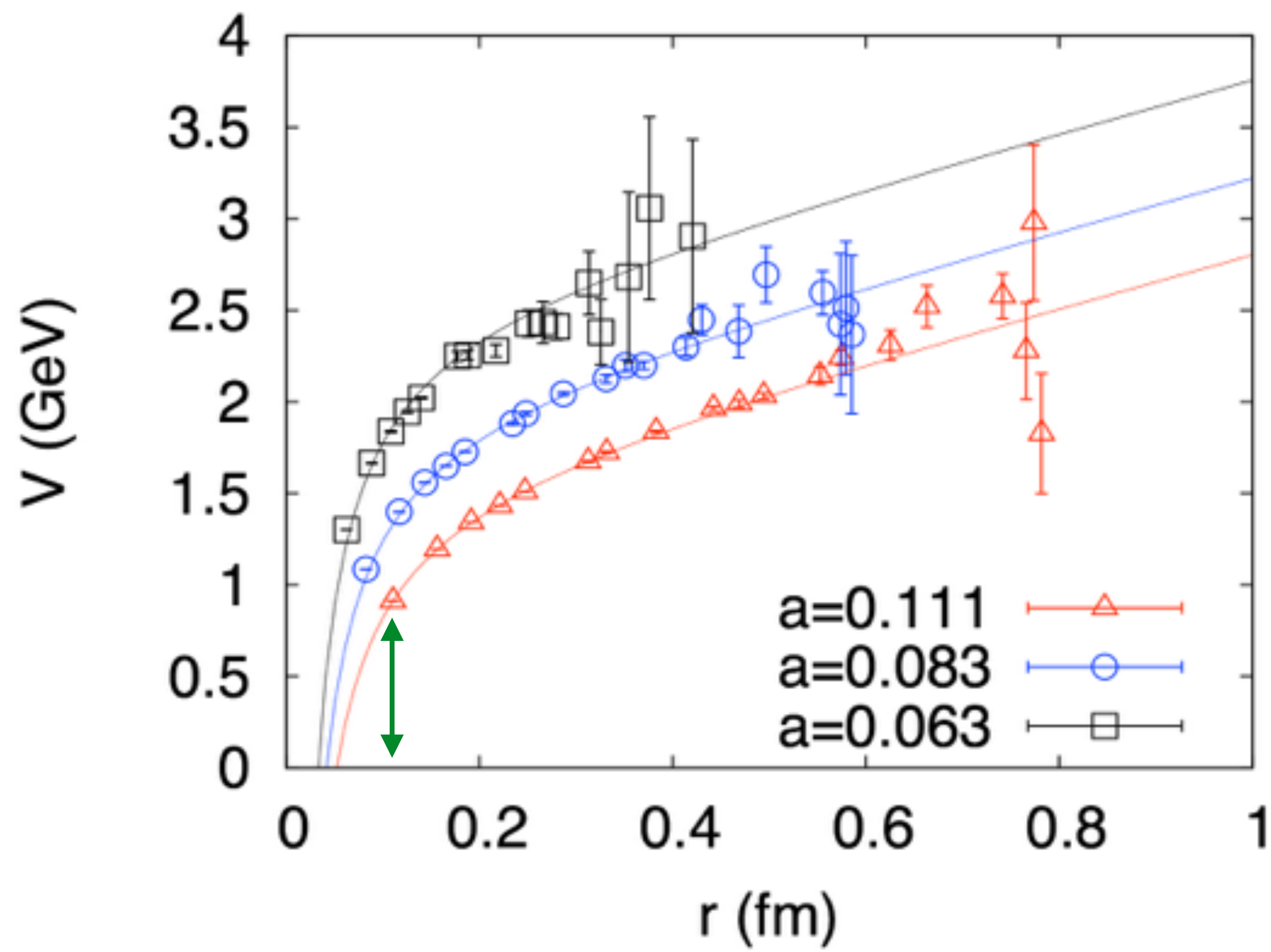
The statical potential is defined by,

$$V(R)=Log[\langle C(r,t)\rangle/\langle C(r,t+1)\rangle]\big|_{t\to\infty,\ r\to\infty}$$

$$=\alpha/r+2\mathbf{A}+\mathbf{B}r,$$

with

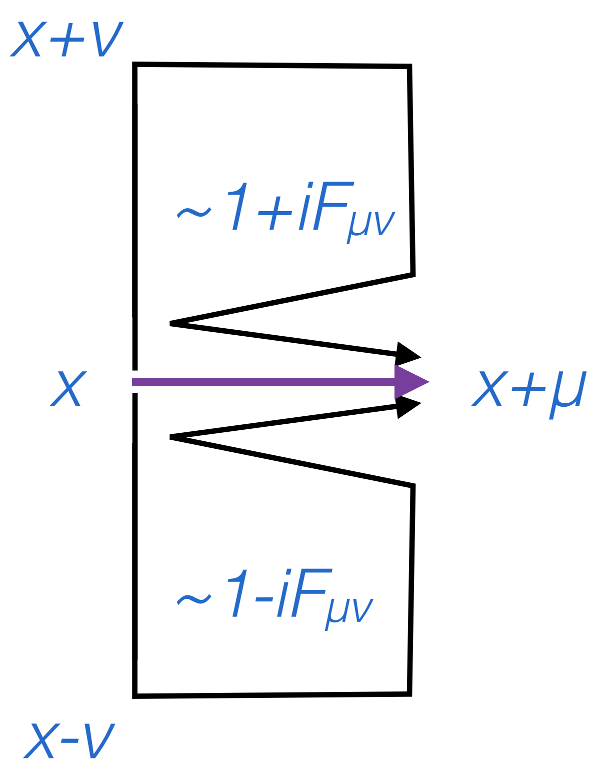
$$\mathbf{A}\sim \Delta m\ \mathbf{a_0/a} + \mathbf{A_0}$$



We can see an obvious jump at smallest r.

Smearing

on the gauge links



Step 1: $$U_\mu(x) \longrightarrow U'_\mu(x) = (1-\alpha)U_\mu(x) + \frac{\alpha}{6}\sum_{\substack{\nu \\ \nu\neq\mu}} \Sigma^\dagger_{\mu\nu}(x)\,,$$

Step 2: $$U_\mu(x) = \mathcal{P}U'_\mu(x)$$

where

$$\Sigma_{\mu\nu}(x) = U_\nu(x+\hat{\mu})U^\dagger_\mu(x+\hat{\nu})U^\dagger_\nu(x) + U^\dagger_\nu(x+\hat{\mu}-\hat{\nu})U^\dagger_\mu(x-\hat{\nu})U_\nu(x-\hat{\nu})$$

$$A'_\mu = A_\mu + \alpha/3\, D_\nu F_{\mu\nu} + O(a^2)$$

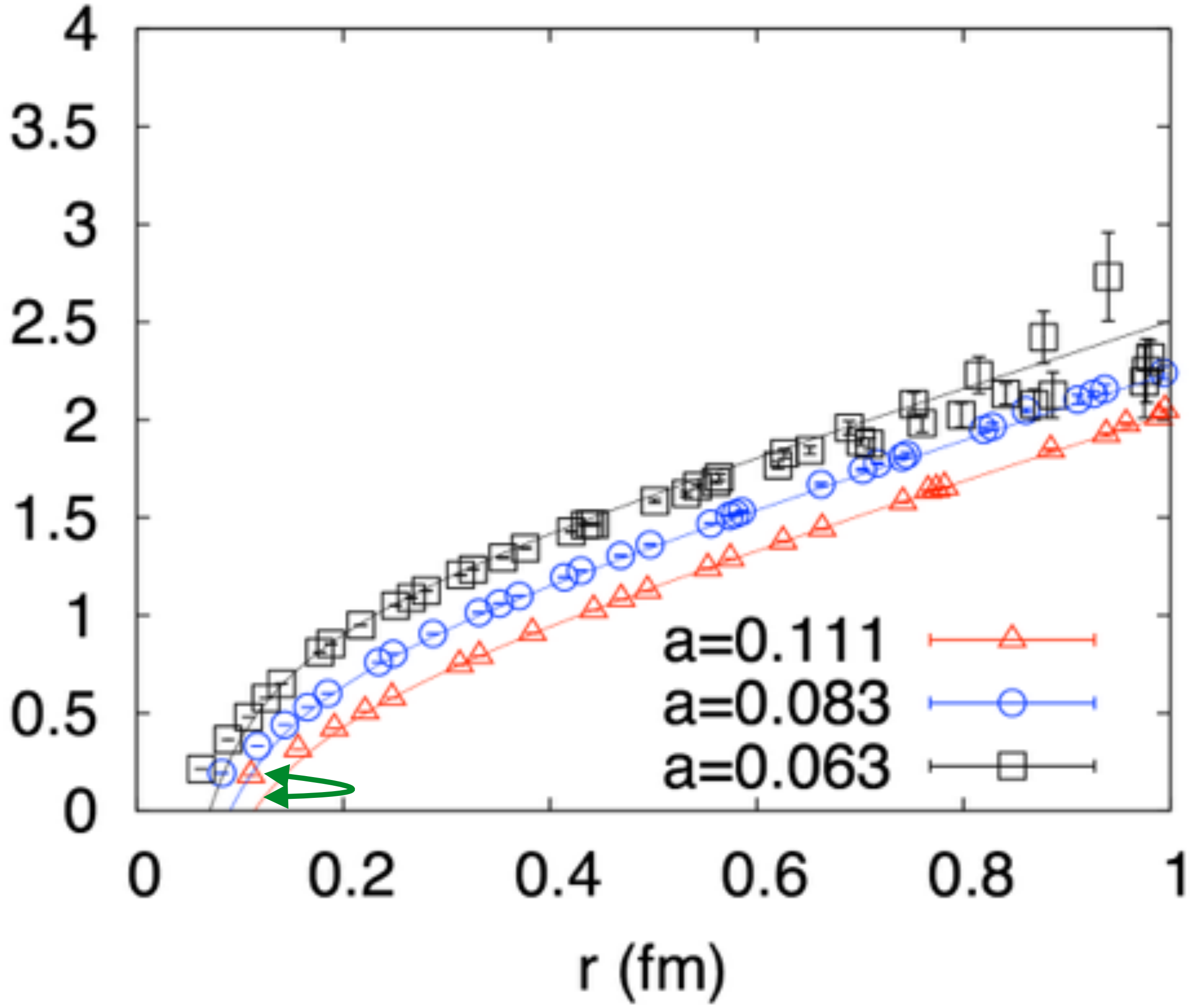
A. Hasenfratz, F. Knechtli, Phys.Rev. D64 (2001) 034504
A. Hasenfratz, R. Hoffmann, F. Knechtli, Nucl.Phys.Proc.Suppl. 106 (2002) 418

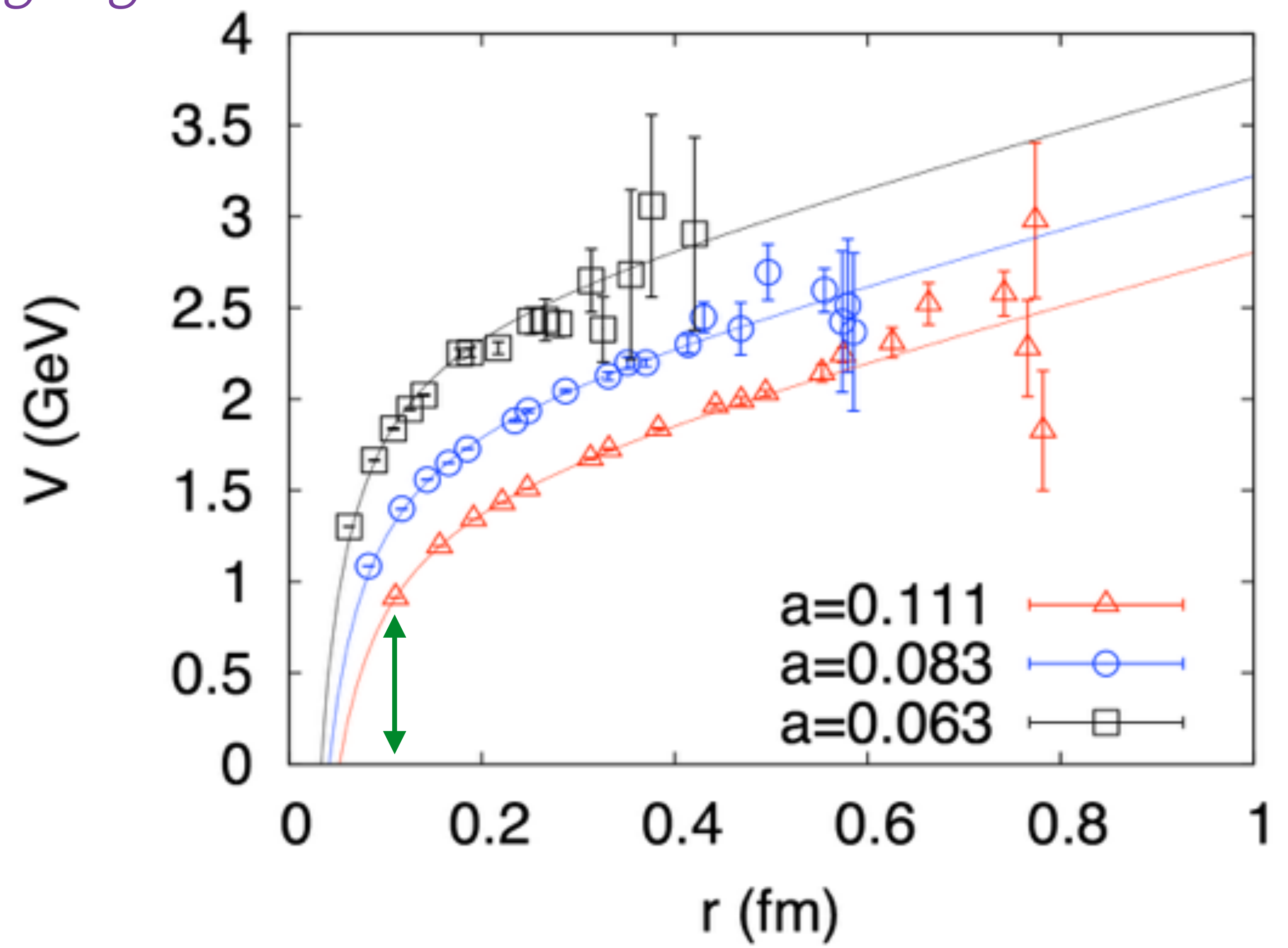
Based this idea, many smearing scheme are developed:
APE smearing, HYP smearing, gradient flow…

After HYP smearing…

Δm(a=0.1 fm) ~ 0.3 GeV ← *Δm*(a=0.1 fm) ~ 0.7 GeV

HYP smearing on the gauge links





The jump at the smallest r is much smaller

We can see an obvious jump at smallest r.

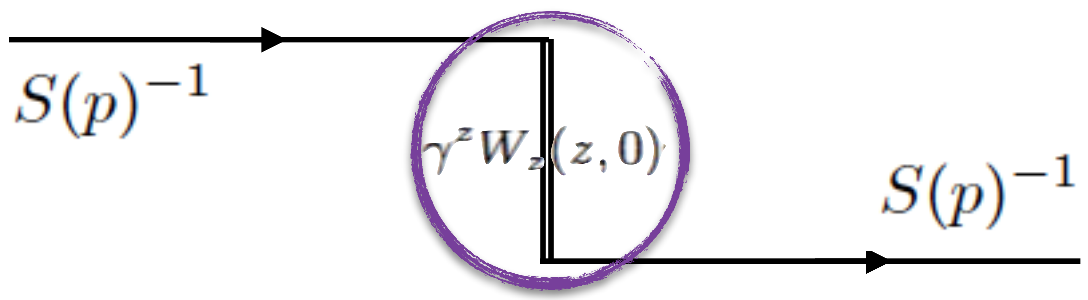
The renormalization of the quasi-PDF operator

T. Ishikawa, Y.-Q. Ma, J.-W. Qiu, S. Yoshida, 1609.02018

The authors proved that:

- The quasi-PDF operator $\langle P|\bar{\psi}(z)\gamma^z W_z(z,0)\psi(0)|P\rangle_{bare}$ with different z will NOT mix with each other.
- The linear divergence of the wilson link can be removed by multiply a factor $\sim e^{\Delta m z}$ *with* Δm from the wilson loop.
- **Or, both the linear and logarithmic UV divergence can be removed with non-perturbative renormalization (NPR) under RI/MOM scheme.**

The RI/MOM renormalization of the quasi-PDF operator



The dressed vertex function, $$\Lambda_{\mathcal{O}}(p) = S^{-1}(p) G_{\mathcal{O}}(p) S^{-1}(p)$$

can be obtained with the forward Green function, $$G_{\mathcal{O}}(p) = \sum_{x,y} e^{-ip\cdot(x-y)} \langle \psi(x) \mathcal{O}(0) \bar{\psi}(y) \rangle$$

The renormalization condition matches the dress vertex function to its tree-level value as,

$$Z_q^{-1} Z_{\mathcal{O}} \frac{1}{12} \mathrm{Tr} \left[\Lambda_{\mathcal{O}}(p) \Lambda_{\mathcal{O}}^{tree}(p)^{-1} \right]_{p^2=\mu^2} = 1$$

with the renormalization of the quark self energy defined by the charge conservation.

G. Martinelli, et. al., Nucl. Phys. B445, 81 (1995)

Would be ill-defined when the Wilson link length is large!

The setup

of the simulation

- 24^3x64 2+1+1 flavor HISQ ensemble with a=0.12 fm,
- Clover fermion with m_π~300 MeV, 1-step HYP smearing
- ~500 configurations, 3 measurements per configuration,
- Two-state fit with 0.96/1.2 fm separations to control the excited state contaminations,
- Use the data up to $z=L_S/2=12$.

LP3, in preparation

The mixing of the quasi-PDF operator

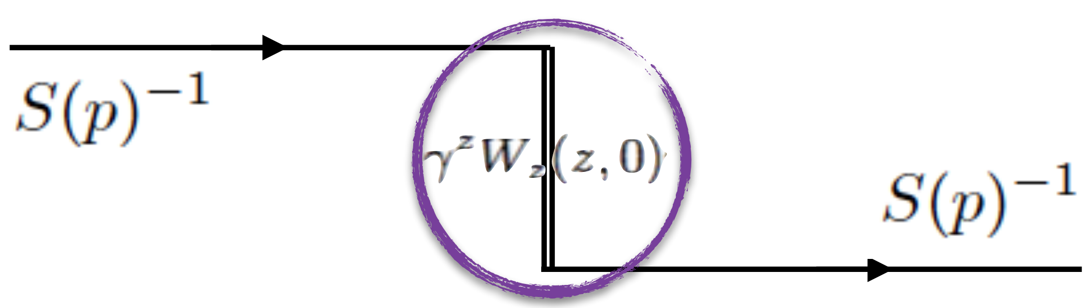
- The quasi-PDF operator can mixed with $\overline{\psi}(z)\, W_z(z,0)\psi(0)$ under the lattice regularization due to the explicit breaking of chiral symmetry.

M. Constantinou, talk in APS GHP workshop 2017

- First observed in the 1-loop Lattice perturbative calculation;

Can also be calculated with non-perturbative renormalization:

We can project the dressed vertex function,



by p-slash:
→ the renormalization constant

by the unitary matrix:
→ the mixing from the scalar matrix operator with a Wilson link.

LP3, in preparation

The mixing

of the quasi-PDF operator

- The quasi-PDF operator can mixed with $\overline{\psi}(z)\, W_z(z,0)\psi(0)$ under the lattice regularization due to the explicit breaking of chiral symmetry.

M. Constantinou, talk in APS GHP workshop 2017

- First observed in the 1-loop Lattice perturbative calculation;

- And confirmed by the non-perturbative renormalization.

- *Z_{VV}: The renormalization of the quasi-PDF operator;*
- *Z_{VS}: The mixing from the scalar operator with a Wilson link.*

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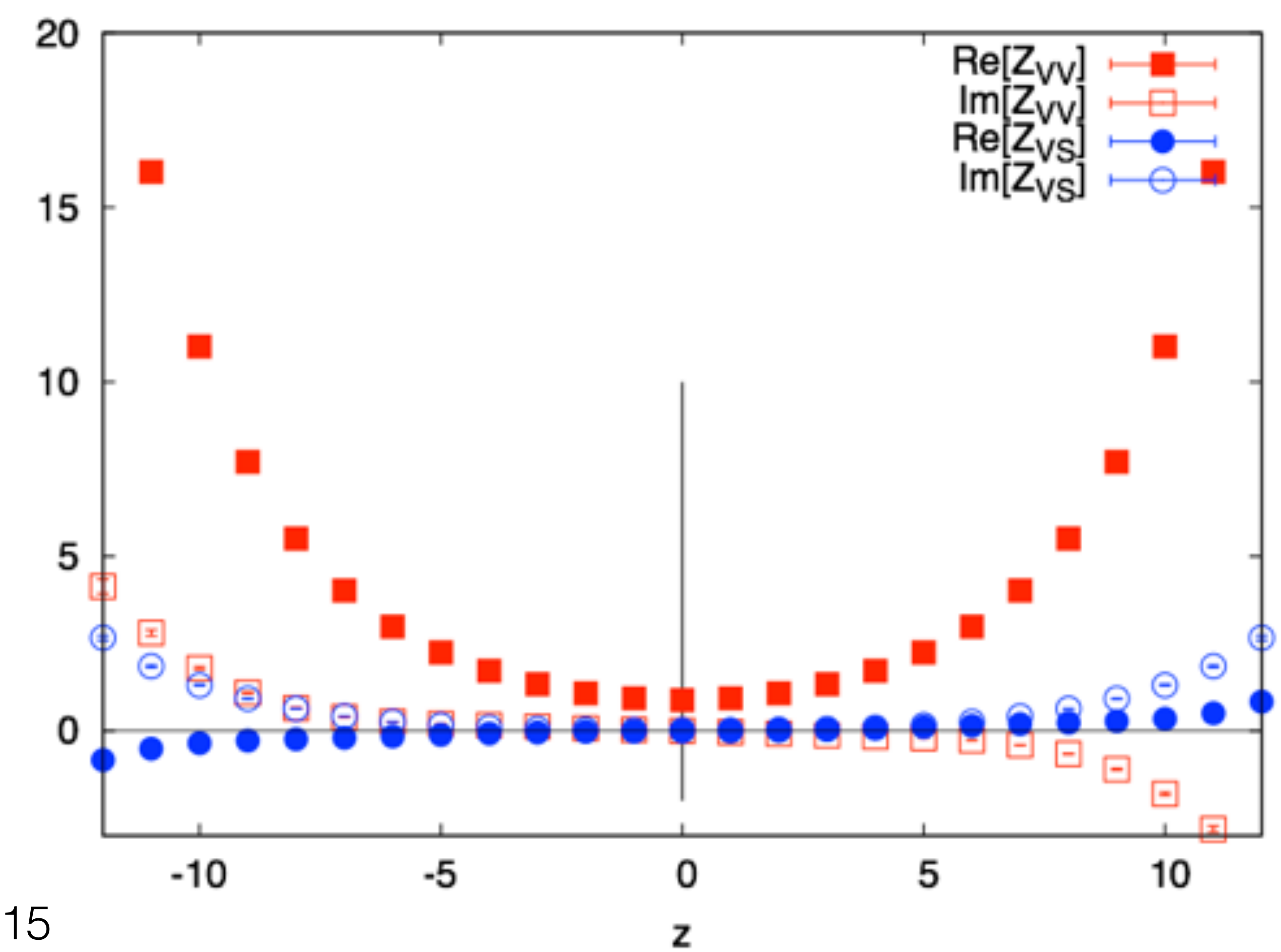
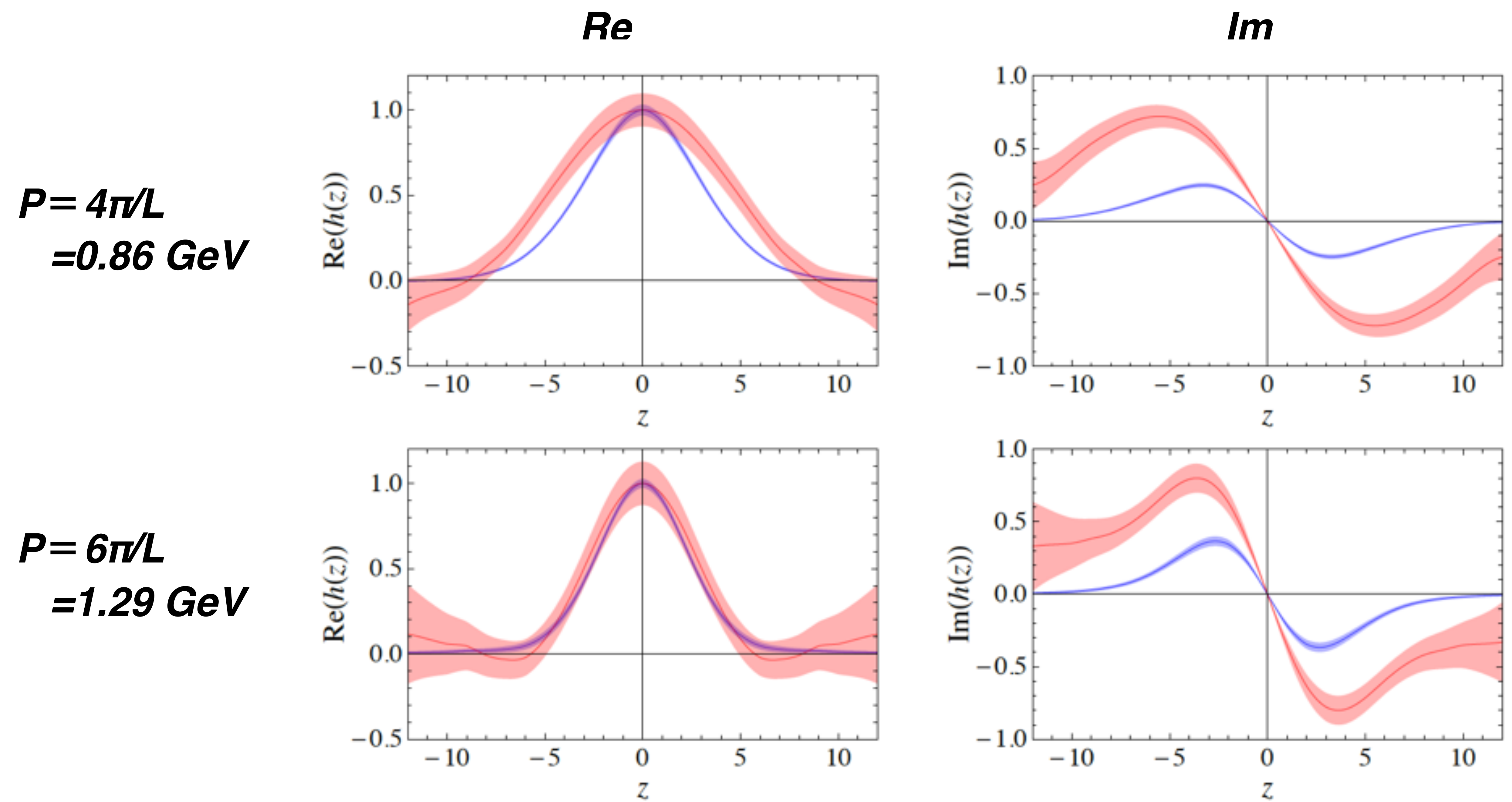


The renormalized quasi-PDF with the 1-loop matching

LP3, in preparation



Bare ***RI/MOM renormalized***

The mixing with the scalar operators are ignored in this preliminary result

From the quasi-PDF

to the real PDF

1-loop matching from RI/MOM quasi-PDF to MS-bar PDF

Tree level

$$q(x,\mu) = \int_{-\infty}^{+\infty} \frac{dy}{|y|} \left[\delta(1-\frac{x}{y})\{1 + \frac{\alpha_s C_F}{2\pi} \int_{-\infty}^{+\infty} d\xi \, C^{OM}\left(\xi, \frac{\mu_R}{P^z}, \frac{\mu}{P^z}\right)\} - \frac{\alpha_s C_F}{2\pi} C^{OM}\left(\frac{x}{y}, \frac{\mu_R}{P^z}, \frac{\mu}{P^z}\right)\right]$$
$$\int_{-\infty}^{\infty} \frac{dz}{2\pi} \, e^{iyP^z z} \langle P|\bar{\psi}(z)\gamma^z W_z(z,0)\psi(0)|P\rangle_R$$
$$+ \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{P_z^2}\right) + \mathcal{O}\left(\left(\frac{\alpha_s}{\pi}\right)^2\right)$$

The renormalized quasi-PDF under RI/MOM scheme.
We are finally here now!
But the distribution is noisy..

Residual higher-twist contribution

Residual higher-loop matching

X. Xiong, X. Ji, J. Zhang, Y. Zhao, Phys. Rev. D 90, 014051 (2014)
LP3, 1603.06664

Definition of the off-axis quasi-PDF

The quasi-PDF is defined by

X.D. Ji, Phys.Rev.Lett. 110 (2013) 262002

$$
\begin{aligned}
q(x,\mu^2,P^z) \; = & \int \frac{dz}{4\pi} e^{izk^z} \langle P|\overline{\psi}(z)\gamma^z \\
& \times \exp\left(-ig\int_0^z dz' A^z(z')\right)\psi(0)|P\rangle \\
& + \mathcal{O}\left(\Lambda^2/(P^z)^2, M^2/(P^z)^2\right) \;,
\end{aligned}
$$

The off-axis quasi-PDF is defined by

Y.B. Yang, et.al, in preparation

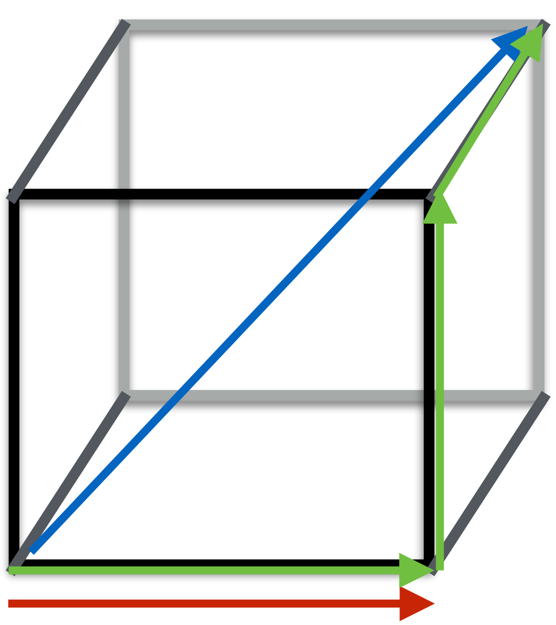
$$
\begin{aligned}
q(x,\mu^2,P^z) \; = & \int \frac{dz}{4\pi} e^{izk^z} \langle P|\overline{\psi}(z)\cancel{\gamma^z} \\
& \times \exp\left(-ig\int_0^z dz' A^z(z')\right)\psi(0)|P\rangle \\
& + \mathcal{O}\left(\Lambda^2/(P^z)^2, M^2/(P^z)^2\right) \;,
\end{aligned}
$$

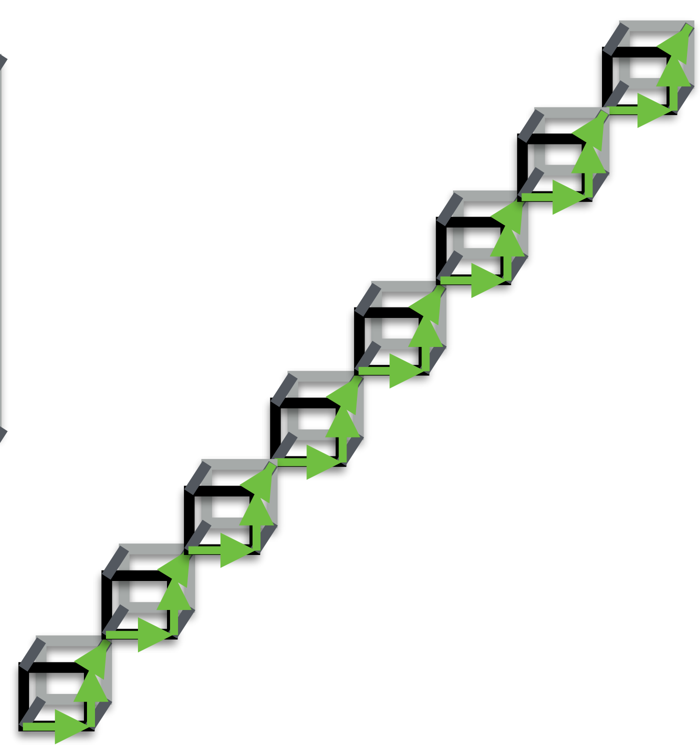
γ^t

Both of them become the original PDF in the large momentum limit, with different matchings.

The advantage of the off-axis quasi-PDF

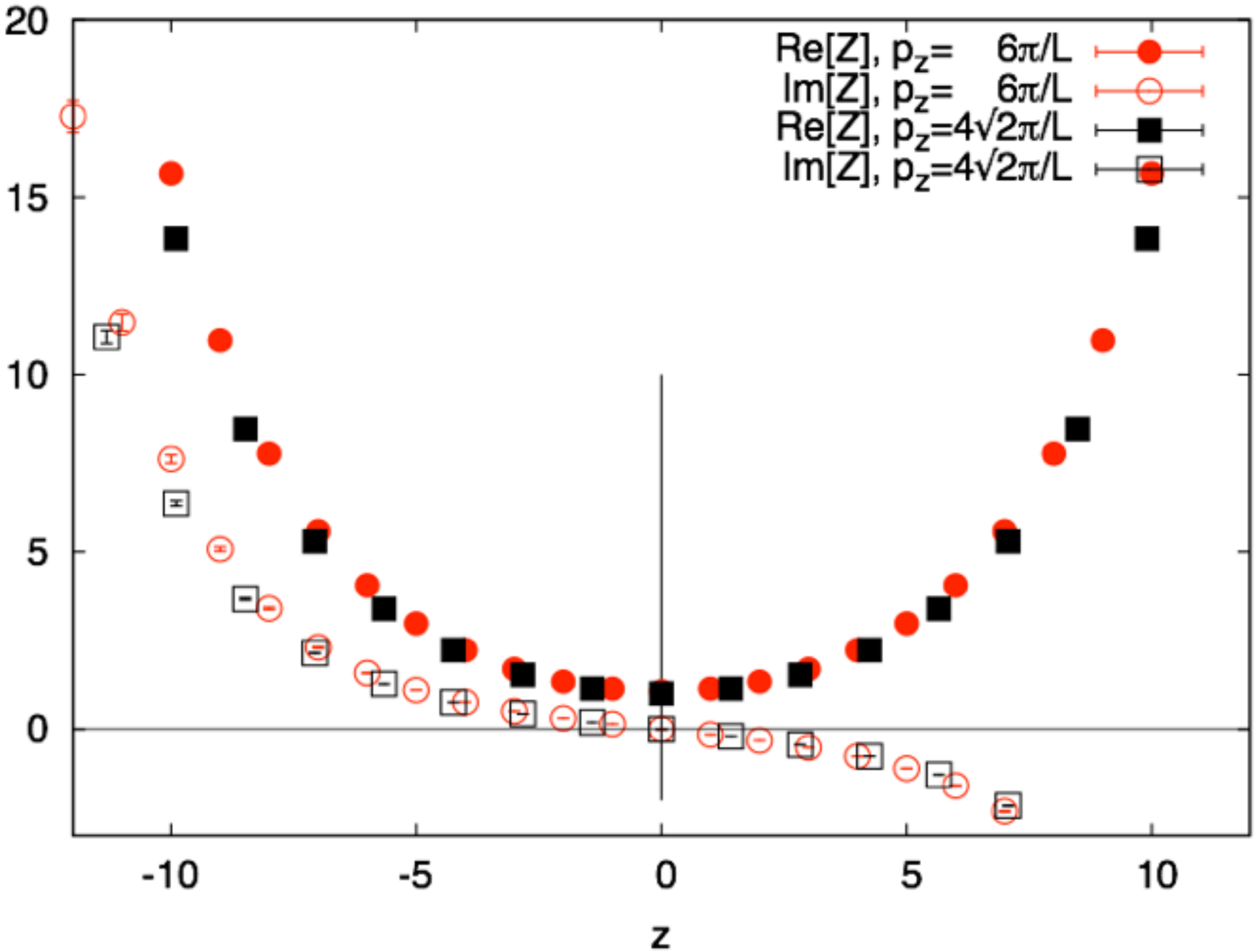
- Will not mix with the scalar matrix elements based on the discrete symmetries analysis;
- The γ_t matrix elements can have better signal on the lattice.
- The momentum of proton can be $\sqrt{3}$ times larger in a given lattice. since it doesn't have to be on-axis any more.



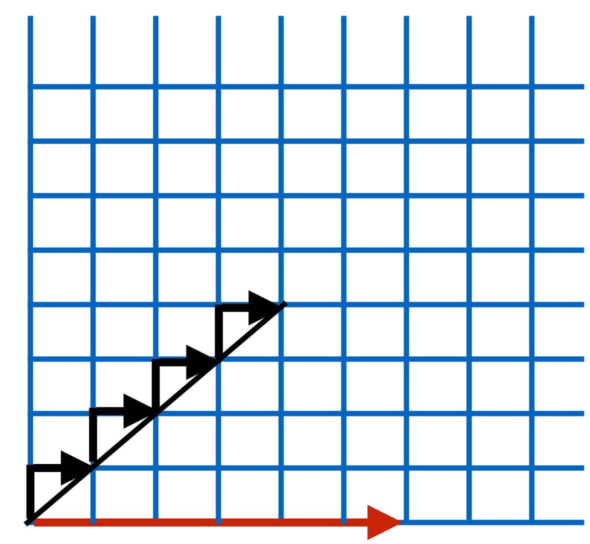


- The taxi-drive gauge links can be used to reach the off-axis position.
- Can use the NPR with the same wilson loop to renormalized the corresponding off-axis quasi-PDF operators.

The rotation symmetry in the renormalization constants



- Some tiny differences on **Z** due to a 6% difference on p_z;
- The rotation symmetry restores well with the taxi-drive gauge link.



Y.B. Yang, et.al, in preparation

Summary

- The linear and logarithmic UV divergence can be removed by the RI/MOM renormalization.
- The renormalization enhances the contribution from large |z|.
- The calculation of the mixing and a^2p^2 effects are in progress.
- The off-axis quasi-PDF has many advantages and we are making progress on that.