Matching the Quasi Parton Distribution in a Momentum Subtraction Scheme to PDF

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I. Stewart and YZ, to be published soon.

Outline

- PDF from lattice QCD through LaMET
- Quasi PDF in the RI/MOM scheme
- Match quasi PDF(RI/MOM) to PDF(MSbar)

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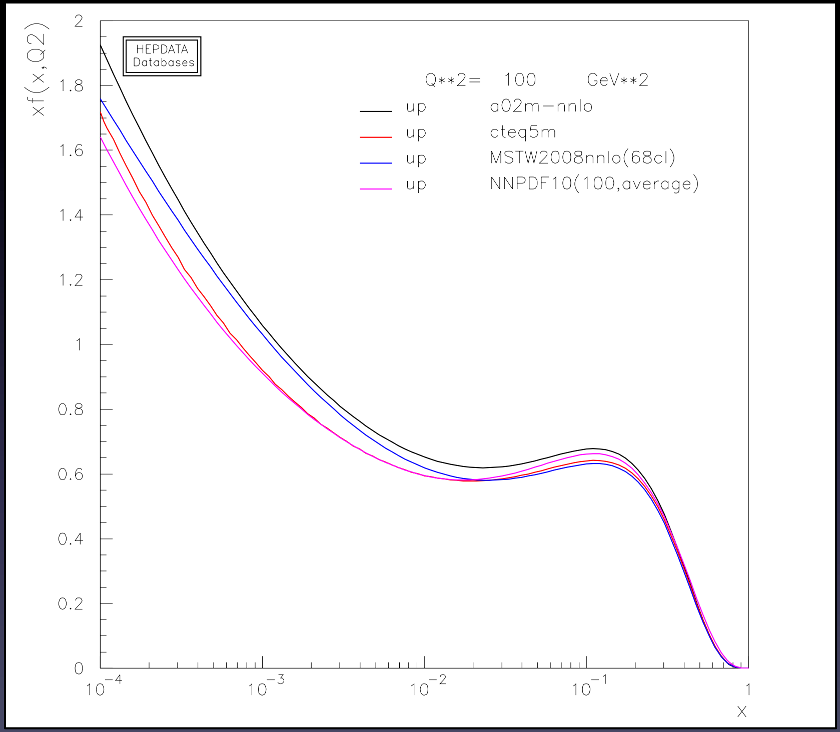
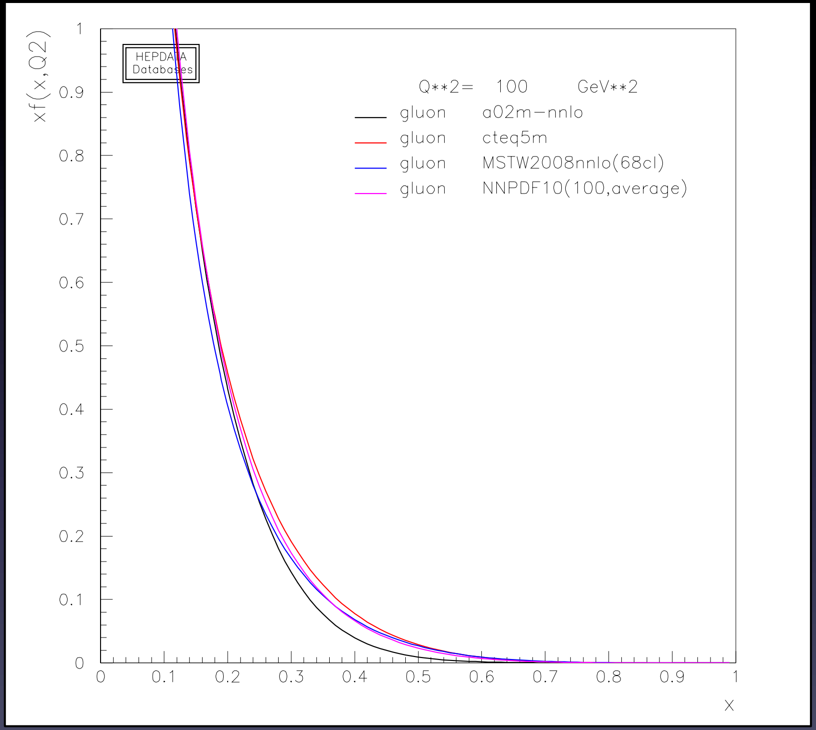
Parton distribution function

Our main knowledge of PDFs comes from the experimental data:





HEPDATA databases, http://hepdata.cedar.ac.uk/pdf/pdf3.html

While the phenomenological PDFs can vary, there is only one theory——QCD——that has a unique solution for PDF.

Parton distribution function:

$$q^B(x,\epsilon) \equiv \int \frac{d\xi^-}{4\pi}\ e^{-ixP^+\xi^-}\left\langle P\middle|\bar{\psi}(\xi^-)\gamma^+ \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\psi(0)\middle|P\right\rangle\ ,$$

$$q_i^B(x,\epsilon) = \sum_j \int_0^1 \frac{dy}{y} Z_{ij}^{\overline{\mathrm{MS}}}\left(\frac{x}{y},\epsilon,\mu\right) q_j(y,\mu)\ ,$$

- Light-cone coordinates $\xi^{\pm}=(x^0\pm x^3)/\sqrt{2}$;
- Clear interpretation as parton number density in the light-cone quantization ($A^+=0$);
- ☹ Not directly calculable in lattice QCD due to light-cone dependence. In Euclidean space, $z^2=0$ -> $z=(0,0,0,0)$.

A method in practice

- Lattice QCD can calculate the moments of PDFs which are matrix elements of local gauge-invariant operators;

$$\int x^{n-1} q(x) dx \quad \sim \quad n_{\mu_1} ... n_{\mu_n} \langle P | \overline{\psi}(0) \gamma^{\mu_1} i D^{\mu_2} ... i D^{\mu_n} \psi(0) | P \rangle$$

- The more moments we can calculate, the better we know about the PDF;

W. Detmold et al., 2003

😞 Number of calculable moments is limited (<4). Mixing with operators of lower dimensions.

Ji's Proposal

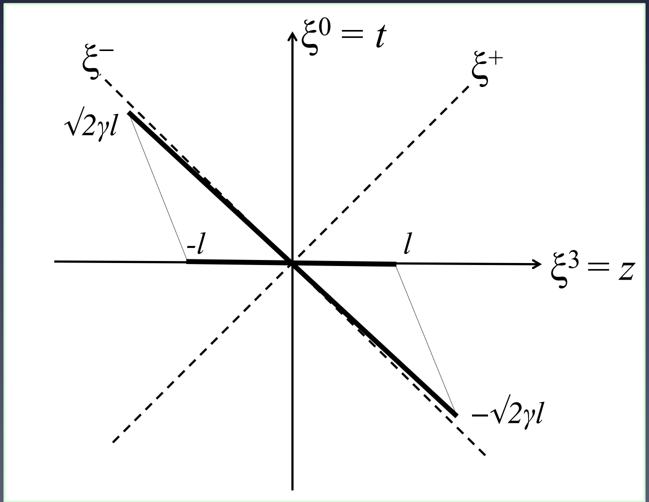
Quasi Parton Distribution:

Ji, 2013, 2014

$$\tilde{q}^B(x, P^z, \epsilon) \equiv \int_{-\infty}^{\infty} \frac{dz}{2\pi}\, e^{ixP^z z} \tilde{q}_i^B(z, P^z, \epsilon)\,, \qquad \tilde{q}_i^B(z, P^z, \epsilon) \equiv \frac{1}{2}\Big\langle P\Big|\bar{\psi}(z)\gamma^z \exp\left(ig\int_0^z dz' A^z(z')\right)\psi(0)\Big|P\Big\rangle\,.$$

$$\tilde{q}_i^B(z, P^z, \epsilon) = \sum_j \tilde{Z}_{ij}(z, \epsilon, \tilde{\mu})\ \tilde{q}_j(z, P^z, \tilde{\mu})\,.$$

- Spatial correlation along the z direction, calculable in lattice QCD;
- Approaches the collinear PDF under an infinite Lorentz boost along the z direction;



Ji's Proposal

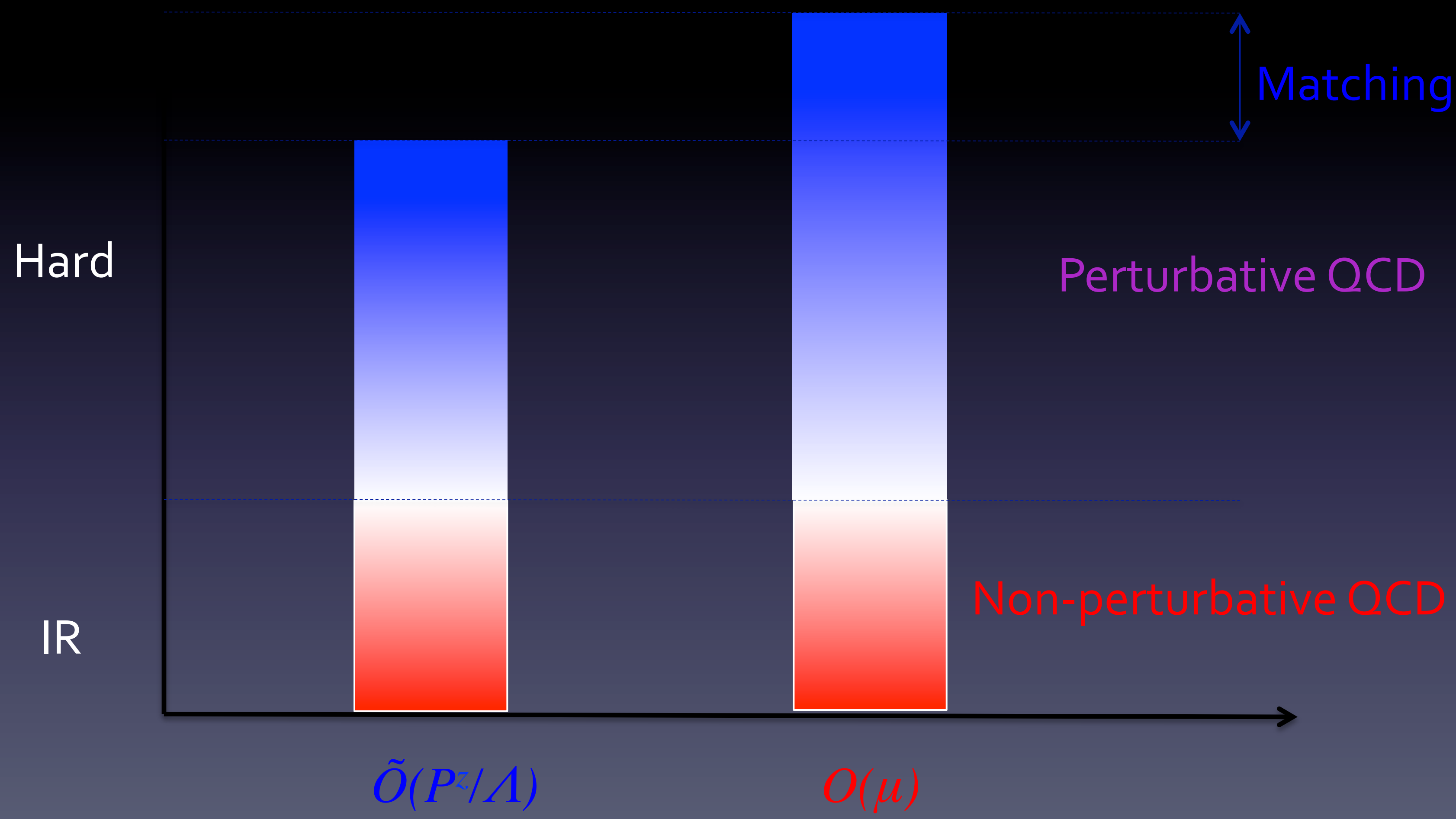
- Hierarchy of scales
  - Quasi PDF $\Lambda >> P^z >> M, \Lambda_{\text{QCD}}$
  - (Light-cone) PDF $P^z >> \Lambda >> M, \Lambda_{\text{QCD}}$

UV cut-off of the theory

Soft scales, related to IR physics

- Taking the infinite momentum limit changes the UV physics, but not the IR physics;
- Quasi PDF can be perturbatively matched to PDF!

How matching works?



Large Momentum Effective Theory (LaMET)

- The quasi PDF is related to the PDF through a factorization formula:

Ji, 2014

$$\tilde{q}_i(x, P^z, \tilde{\mu}) = \int_{-1}^{+1} \frac{dy}{|y|}\, C_{ij}\left(\frac{x}{y}, \frac{\tilde{\mu}}{P^z}, \frac{\mu}{P^z}\right) q_j(y, \mu) + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{P_z^2}\right) ,$$

  - They have the same IR divergences;
  - C factor matches the difference in their UV divergence, and can be calculated in perturbative QCD.

Procedure of Calculating PDF

Lattice quasi PDF

The continuum limit (renormalization)

Quasi PDF in the continuum

Subtracting higher twist corrections; matching from quasi PDF in its particular scheme to MSbar PDF.

PDF in the MSbar scheme

Current status of calculation

Lattice simulation of the bare quasi PDF	✔: Iso-vector quark distributions H. W. Lin et al., 2015; J.-W. Chen et al., 2016; C. Alexandrou et al., 2015, 2016
Renormalization of the quasi PDF on the lattice	? Ishikawa et al., 2016; J.-W. Chen et al., 2016
Subtraction of the higher twist corrections	✔: All orders of mass correction M^2/P_z^2 exactly calculated; $O(\Lambda^2_{QCD}/P_z^2)$ correction fitted. H. W. Lin et al., 2015; J.-W. Chen et al., 2016; C. Alexandrou et al., 2015, 2016
Matching the quasi PDF to PDF in the MSbar scheme.	✔: One-loop matching coefficient obtained in the continuum theory Xiong, Ji, Zhang and Y.Z., 2014; Y. Ma and J. Qiu, 2014.

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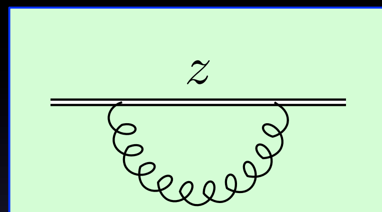
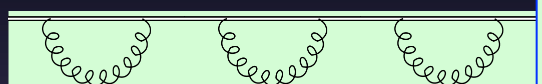
Divergences in the quasi PDF

For an open smooth Wilson line *W(z,0)*, its self energy includes a linear divergence:



$$= \frac{\alpha_S}{2\pi} c_1 \Lambda |z|$$

J.W. Chen et al., 2016



$$+\ldots\ldots = e^{\delta m * |z|}, \qquad \delta m \sim \Lambda$$

In coordinate space, it can be multiplicatively renormalized as:

$$W^B(z,0) = Z_z e^{\delta m|z|} W^R(z,0)$$

- $e^{\delta m * |z|}$ introduces counterterms that cancel the linear divergences in the Wilson line self energy;
- Z_z depends on the end points and only includes logarithmic divergences.

V. S. Dotsenko and S. N. Vergeles, 1980
N. S. Craigie and H. Dorn, 1981
H. Dorn, 1986

This relation should also apply to gauge-invariant nonlocal quark bilinears:

$$(\bar{\psi}(z)W(z,0)\psi(0))^B = Z_{\psi,z} e^{\delta m^* |z|} (\bar{\psi}(z)W(z,0)\psi(0))^R$$
$$= Z(z,\Lambda,\mu_R)(\bar{\psi}(z)W(z,0)\psi(0))^R$$

T. Ishikawa et al., 2016
J.W. Chen et al., 2016

- The separated quark fields do not generate extra power divergences;
- The linear divergence in δm is the same as that for the Wilson line;
- $Z_{\psi,z}$ depends on the end points and only includes logarithmic divergences.

RI/MOM scheme

- For a nonperturbative renormalization, we can use the so called regularization-invariant momentum subtraction (RI/MOM) scheme that has been widely used in lattice QCD.

G. Martinelli et al., 1994

- A momentum subtraction scheme:

$$Z_O^{-1}\langle p|O_B|p\rangle\Big|_{p^2=-\mu_R^2}=\langle p|O_B|p\rangle_{\text{tree}}$$

$$Z_O = Z_O(\Lambda,\mu_R)$$

Regularization invariance (RI)

- In dimensional regularization ($d=4-2\varepsilon$):

Bare: $\langle p|O_B|p\rangle = \left[1+\frac{g^2}{16\pi^2}\left(\gamma\left(\frac{1}{\varepsilon}-\gamma_E+\ln 4\pi+\ln\frac{\mu^2}{-p^2}\right)+R\right)+O(g^4)\right]\langle p|O_B|p\rangle_{\text{tree}}$

MSbar: $\langle p|O^{\overline{MS}}{}_R\,|p\rangle = \left[1+\frac{g^2}{16\pi^2}\left(\gamma\ln\frac{\mu^2}{-p^2}+R\right)+O(g^4)\right]\langle p|O_B|p\rangle_{\text{tree}}$

MOM: $\langle p|O^{\text{MOM}}{}_R\,|p\rangle = \left[1+\frac{g^2}{16\pi^2}\left(\gamma\ln\frac{\mu_R{}^2}{-p^2}\right)+O(g^4)\right]\langle p|O_B|p\rangle_{\text{tree}}$

★ RI applies to multiplicatively renormalizable operators

- In UV-cut off regularization:

Bare: $\langle p|O_B|p\rangle = \left[1+\frac{g^2}{16\pi^2}\left(\gamma\ln(\frac{\Lambda^2}{-p^2})+P(-\frac{p^2}{\Lambda^2})\right)+O(g^4)\right]\langle p|O_B|p\rangle_{\text{tree}}$

MOM:
$$\langle p|O^{\text{MOM}}{}_R\,|p\rangle = \left[1+\frac{g^2}{16\pi^2}\left(\gamma\ln(\frac{\mu_R{}^2}{-p^2})+P(-\frac{p^2}{\Lambda^2})-P(\frac{\mu_R{}^2}{\Lambda^2})\right)+O(g^4)\right]\langle p|O_B|p\rangle_{\text{tree}}$$
$$=\left[1+\frac{g^2}{16\pi^2}\left(\gamma\ln(\frac{\mu_R{}^2}{-p^2})+O(-\frac{p^2}{\Lambda^2},\frac{\mu_R{}^2}{\Lambda^2})\right)+O(g^4)\right]\langle p|O_B|p\rangle_{\text{tree}}$$

If $-p^2, \mu_R{}^2 << \Lambda^2$, then the renormalized matrix element should be the same in dim reg and cut-off regularizations!

RI/MOM Scheme

- We can impose the RI/MOM condition on the off-shell quark matrix element:

$$Z(z,p^z,a^{-1},\mu_R)^{-1}\left\langle p\left|(\bar{\psi}(z)W(z,0)\psi(0))^B\right|p\right\rangle\Big|_{p_E^2=\mu_R^2}$$
$$=\left\langle p\left|(\bar{\psi}(z)W(z,0)\psi(0))^B\right|p\right\rangle_{\text{tree}}$$
$$=2p_E^z e^{-ip^{z}*z}$$

On the lattice, $p_E{}^2=\mu_R{}^2$, Z is a complex number!

RI/MOM Scheme

The renormalized matrix element

$$Z(z, p^z, a^{-1}, \mu_R)^{-1} \langle p | (\bar{\psi}(z) W(z,0) \psi(0))^B | p \rangle$$

should be independent of the UV cut-off $1/a$ for arbitrary ${p_E}^2$ and p^z, except for lattice discretization effects of $O(ap^z, a\sqrt{p_E^2}, a\mu_R).$

Working region: $\Lambda_{\text{QCD}} \ll p^z, \sqrt{p_E^2}, \mu_R \ll 1/a$

RI/MOM Scheme

★ The renormalized matrix element is independent of UV regulator, so it should be the same even in the continuum with dimensional regularization;

- To obtain PDF in the MSbar scheme, one just need to match from quasi PDF in the RI/MOM in the continuum with dimensional regularization!
- In dimensional regularization, the linear divergence vanishes for $d=4-2\varepsilon$, and the calculation becomes much easier!

We can briefly summarize the procedure as:

- For each z, determine the nonperturbative renormalization factor;
- Renormalize the lattice nucleon matrix element of the quasi PDF with the same factor, and then Fourier transform it to momentum space $k^z = xp^z$;
- Calculate the matching coefficient between the quasi PDF in the RI/MOM and PDF in the MSbar in the continuum with dim-reg;

See also M. Constantinou and Y. Yang's talks.

- PDF from lattice QCD through LaMET
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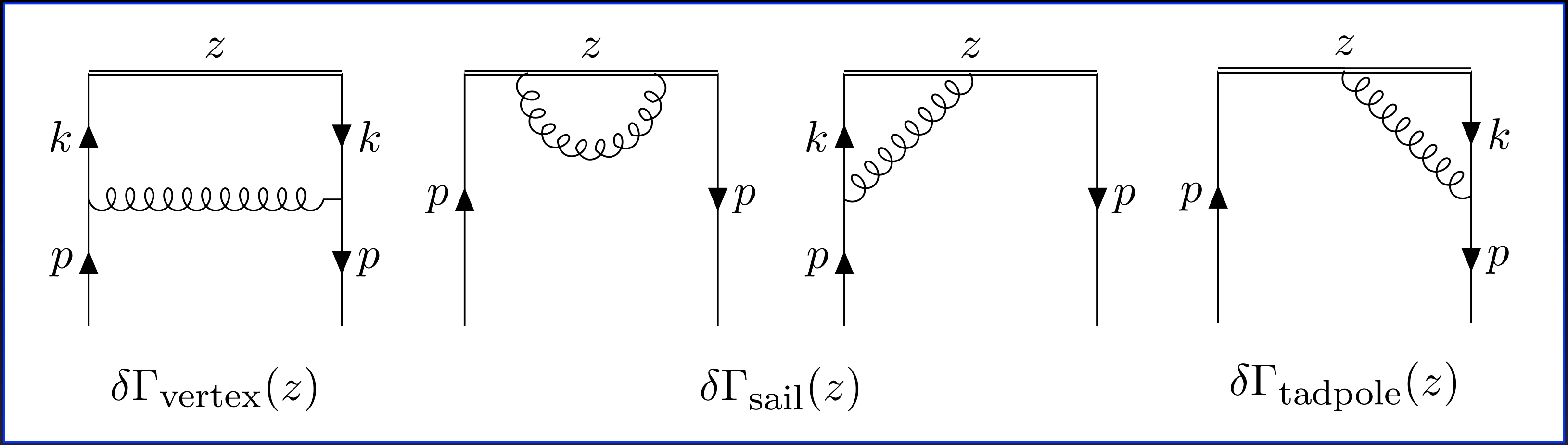
One-loop diagrams



- Dimensional regularization $d=4-2\varepsilon$; Off-shell quarks with $p=(p^0,0,0,p^z)$ and $p^2<0$.
- Fourier transform to the momentum space;
- Virtual contribution expressed as the integration of the real contribution times $\delta(x-1)$;

One-loop diagrams

$$
\begin{aligned}
\delta\Gamma(z) &= \delta\Gamma_{\rm vertex}(z) + \delta\Gamma_{\rm sail}(z) + \delta\Gamma_{\rm tadpole}(z) + \text{quark self-energy correction} \\
&= \frac{g^2 C_F}{4\pi} \int_{-\infty}^{\infty} \frac{dx}{2\pi} e^{-ixp^z z} \\
&\quad \times \begin{cases}
\frac{1}{\sqrt{1-\rho+i\varepsilon}} \left[\frac{1+x^2}{1-x} - \frac{\rho}{2(1-x)}\right] \ln \frac{2x-1+\sqrt{1-\rho+i\varepsilon}}{2x-1-\sqrt{1-\rho+i\varepsilon}} - \frac{\rho}{4x(x-1)+\rho} + 1 \,, & x > 1 \\
\frac{1}{\sqrt{1-\rho+i\varepsilon}} \left[\frac{1+x^2}{1-x} - \frac{\rho}{2(1-x)}\right] \ln \frac{1+\sqrt{1-\rho+i\varepsilon}}{1-\sqrt{1-\rho+i\varepsilon}} - \frac{2x}{1-x} \,, & 0 < x < 1 \\
\frac{1}{\sqrt{1-\rho+i\varepsilon}} \left[\frac{1+x^2}{1-x} - \frac{\rho}{2(1-x)}\right] \ln \frac{2x-1-\sqrt{1-\rho+i\varepsilon}}{2x-1+\sqrt{1-\rho+i\varepsilon}} + \frac{\rho}{4x(x-1)+\rho} - 1 \,, & x < 0
\end{cases} \\
&\quad - \frac{g^2 C_F}{4\pi} \int_{-\infty}^{\infty} \frac{dx}{2\pi} e^{-ixp^z z} D(\rho)\delta(x-1) \,,
\end{aligned}
$$

Preliminary

- Where $\rho = -p^2/p_z^2 < 1$ and $D(\varrho)$ stands for integration of the second line over x.
- The real part is UV finite, but its integration over x is not, which is cancelled by the virtual part.

Counter-term in the RI/MOM Scheme

$$\delta\Gamma_{CT}(z, \rho(\mu_R)) = -\,\delta\Gamma(z)\big|_{p^2=-\mu_R^2} \,,$$

Since on the lattice, $\mu_R^2 = p_E^2 \geq p_z^2$

If we do renormalization in lattice QCD, $\rho(\mu_R) = \frac{\mu_R^2}{p_z^2} \geq 1$

We need to keep $\varrho(\mu_R)$ finite, and analytically continue it to $\varrho(\mu_R) \geq 1$.

- For the unrenormalized matrix element, the physical scale $-p^2$ serves as the collinear divergence regulator $ln(-p^2)$ here;
- To compare to the collinear divergence of PDF, we need to do an expansion for the onshell limit $\varrho<<1\ (-p^2<<p_z^2)$ to separate out the collinear divergence:

Renormalized quasi PDF

Preliminary

$$
\begin{aligned}
&\delta\Gamma(z,\rho,\rho(\mu_R)) \\
&= \frac{g^2 C_F}{4\pi}\int_{-\infty}^{\infty}\frac{dx}{2\pi}e^{-ixp^z z} \\
&\times\begin{cases}
\frac{1+x^2}{1-x}\ln\frac{x}{x-1} - \frac{2}{\sqrt{\rho(\mu_R)-1}}\left[\frac{1+x^2}{1-x}-\frac{\rho(\mu_R)}{2(1-x)}\right]\arctan\frac{\sqrt{\rho(\mu_R)-1}}{2x-1} + \frac{\rho(\mu_R)}{4x(x-1)+\rho(\mu_R)}\,, & x>1 \\
\frac{1+x^2}{1-x}\ln\frac{4}{\rho} - \frac{2}{\sqrt{\rho(\mu_R)-1}}\left[\frac{1+x^2}{1-x}-\frac{\rho(\mu_R)}{2(1-x)}\right]\arctan\sqrt{\rho(\mu_R)-1}\,, & 0<x<1 \\
\frac{1+x^2}{1-x}\ln\frac{x-1}{x} + \frac{2}{\sqrt{\rho(\mu_R)-1}}\left[\frac{1+x^2}{1-x}-\frac{\rho(\mu_R)}{2(1-x)}\right]\arctan\frac{\sqrt{\rho(\mu_R)-1}}{2x-1} - \frac{\rho(\mu_R)}{4x(x-1)+\rho(\mu_R)}\,, & x<0
\end{cases} \\
&-\frac{g^2 C_F}{4\pi}\int_{-\infty}^{\infty}\frac{dx}{2\pi}e^{-ixp^z z}\left[\lim_{\rho\to 0}D(\rho)-D(\rho(\mu_R))\right]\delta(x-1)\,, \qquad (20)
\end{aligned}
$$

Compared to normal PDF renormalized in the MSbar scheme

$$
q^{(1)}(x,\mu) = \frac{\alpha_s C_F}{2\pi}\begin{cases}
0\,, & x>1 \\
\frac{1+x^2}{(1-x)_+}\ln\frac{\rho(\mu)}{\rho} - \frac{1+x^2}{(1-x)_+}\ln x(1-x) - 2(1-x) + \left(\frac{3}{2}\ln\frac{\rho(\mu)}{\rho}+4\right)\delta(x-1)\,, & 0<x<1 \\
0\,, & x<0
\end{cases}
$$

Matching

$$\tilde{q}_i(x, P^z, \tilde{\mu}) = \int_{-1}^{+1} \frac{dy}{|y|}\, C_{ij}\left(\frac{x}{y}, \frac{\tilde{\mu}}{P^z}, \frac{\mu}{P^z}\right) q_j(y, \mu) + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{P_z^2}\right) ,$$

$$C^{\rm RI/MOM}(\xi, \frac{\mu_R}{p^z}, \frac{\mu}{p^z}) \qquad \xi = \frac{x}{y}$$

$$= \tilde{q}(\xi, \frac{\mu_R}{p^z}, -p^2) - q(\xi, \frac{\mu}{p^z}, -p^2)$$

$$= \frac{\alpha_s C_F}{2\pi}\begin{cases} (1+\xi^2)\left(\frac{1}{1-\xi}\ln\frac{\xi}{\xi-1}\right)_{\oplus} - \frac{2(1+\xi^2)-\rho(\mu_R)}{\sqrt{\rho(\mu_R)-1}}\left(\frac{1}{(1-\xi)}\arctan\frac{\sqrt{\rho(\mu_R)-1}}{2\xi-1}\right)_{\oplus} \\ +\frac{\rho(\mu_R)}{4\xi(\xi-1)+\rho(\mu_R)}\,, & \xi > 1 \\ \frac{1+\xi^2}{(1-\xi)_+}\ln\frac{4}{\rho(\mu)} + \frac{1+\xi^2}{(1-\xi)_+}\ln\xi(1-\xi) + 2(1-\xi) \\ -\frac{2\arctan\sqrt{\rho(\mu_R)-1}}{\sqrt{\rho(\mu_R)-1}}\left[\frac{1+\xi^2}{(1-\xi)_+} - \frac{\rho(\mu_R)}{2(1-\xi)_+}\right]\,, & 0 < \xi < 1 \\ \frac{1+\xi^2}{1-\xi}\ln\frac{\xi-1}{\xi} + \frac{2}{\sqrt{\rho(\mu_R)-1}}\left[\frac{1+\xi^2}{1-\xi} - \frac{\rho(\mu_R)}{2(1-\xi)}\right]\arctan\frac{\sqrt{\rho(\mu_R)-1}}{2\xi-1} \\ -\frac{\rho(\mu_R)}{4\xi(\xi-1)+\rho(\mu_R)}\,, & \xi < 0 \end{cases}$$

$$-\frac{\alpha_s C_F}{2\pi}\left[D_2(\rho, \rho(\mu_R)) + \frac{3}{2}\ln\frac{\rho(\mu)}{\rho} + 4\right]\delta(\xi - 1)\,.$$

Preliminary

Other schemes

Transverse momentum cut-off scheme (Xiong, Ji, Zhang, and Y.Z. (2014)):

Linear divergence

$$C^{\text{cut-off}}(\xi, \frac{\mu}{p^z}) = \frac{\alpha_s C_F}{2\pi} \begin{cases} \frac{1+\xi^2}{1-\xi} \ln\frac{\xi}{\xi-1} + 1 + \frac{1}{(1-\xi)^2}\frac{\Lambda}{P^z}\,, & \xi > 1 \\ \frac{1+\xi^2}{1-\xi} \ln\frac{4}{\rho(\mu)} + \frac{1+\xi^2}{1-\xi}\ln\xi(1-\xi) + 1 - \frac{2\xi}{1-\xi} + \frac{1}{(1-\xi)^2}\frac{\Lambda}{P^z}\,, & 0 < \xi < 1 \\ \frac{1+\xi^2}{1-\xi} \ln\frac{\xi-1}{\xi} - 1 + \frac{1}{(1-\xi)^2}\frac{\Lambda}{P^z}\,, & \xi < 0 \end{cases}$$
$$-\frac{\alpha_s C_F}{2\pi} D_{\text{cut-off}}(\rho(\mu))\delta(\xi - 1)\,,$$

MSbar scheme (M. Constantinou, 2016)

No μ_R dependence in either schemes!

$$C^{\overline{\text{MS}}}(\xi, \frac{\mu}{p^z}) = \frac{\alpha_s C_F}{2\pi} \begin{cases} \frac{1+\xi^2}{1-\xi} \ln\frac{\xi}{\xi-1} + 1\,, & \xi > 1 \\ \frac{1+\xi^2}{1-\xi} \ln\frac{4}{\rho(\mu)} + \frac{1+\xi^2}{1-\xi}\ln\xi(1-\xi) + 2(1-\xi) - \frac{2\xi}{1-\xi}\,, & 0 < \xi < 1 \\ \frac{1+\xi^2}{1-\xi} \ln\frac{\xi-1}{\xi} - 1\,, & \xi < 0 \end{cases}$$
$$-\frac{\alpha_s C_F}{2\pi} D_{\overline{\text{MS}}}(\rho(\mu))\delta(\xi - 1)\,,$$

Preliminary

Convolution

- Take the iso-vector parton distribution $f_{u\text{-}d}$ as example:

$$f_{u-d}(x,\mu) = f_u(x,\mu) - f_d(x,\mu) - f_{\bar{u}}(-x,\mu) + f_{\bar{d}}(-x,\mu) \ ,$$

$$f_{\bar{u}}(-x,\mu) = -f_{\bar{u}}(x,\mu) \ , \qquad f_{\bar{d}}(-x,\mu) = -f_{\bar{d}}(x,\mu) \ .$$

- Input:
  - “MSTW 2008” PDF
  - NLO $\alpha_s(\mu)$

Convolution

$$C^{\mathrm{RI/MOM}}(\xi) = C_r^{\mathrm{RI/MOM}}(\xi) - \delta(1-\xi)\int d\xi' C_r^{\mathrm{RI/MOM}}(\xi')\ ,$$

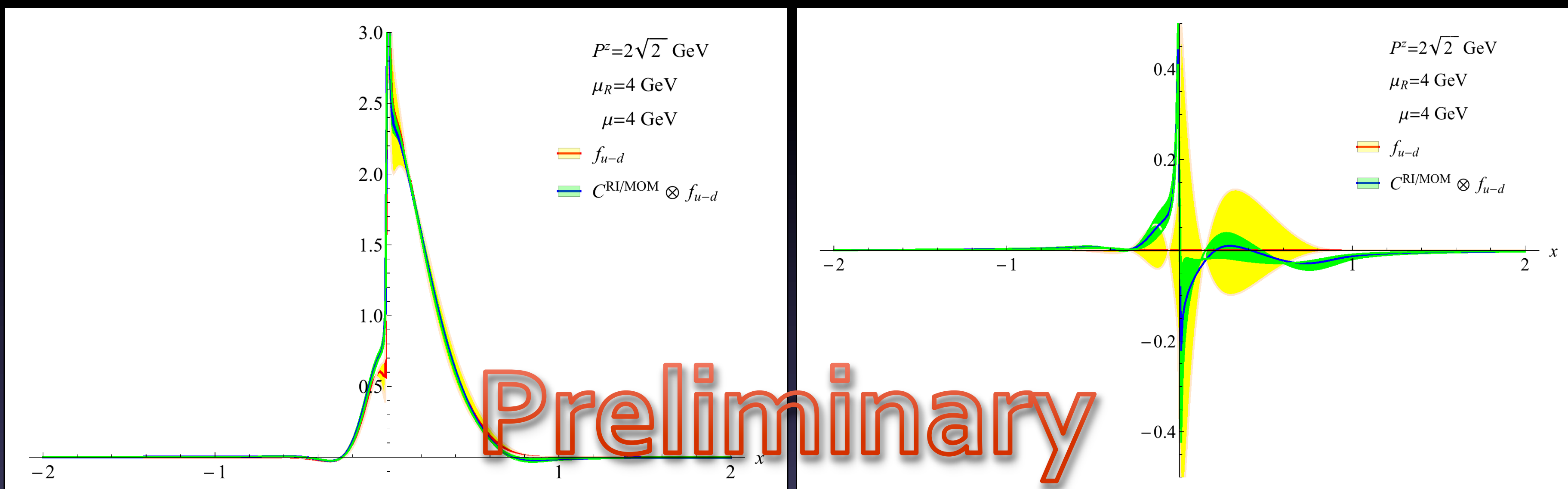
$$\begin{aligned}
&\int_{-1}^{1}\frac{dy}{|y|}C_r^{\mathrm{RI/MOM}}\left(\frac{x}{y},\frac{\mu}{P^z},\frac{\mu_R}{P^z}\right)f_{u-d}(y,\mu)\\
&=\int_{-1}^{1}\frac{dy}{|y|}\left[C_r^{\mathrm{RI/MOM}}\left(\frac{x}{y},\frac{\mu}{P^z},\frac{\mu_R}{P^z}\right)-\delta(1-\frac{x}{y})\int d\xi\ C_r^{\mathrm{RI/MOM}}\left(\xi,\frac{\mu}{P^z},\frac{\mu_R}{P^z}\right)\right]f_{u-d}(y,\mu)\\
&=\int_{-\infty}^{\infty}dy\left[\frac{1}{|y|}C_r^{\mathrm{RI/MOM}}\left(\frac{x}{y},\frac{\mu}{P^z},\frac{\mu_R}{P^z}\right)f_{u-d}(y,\mu)-\frac{1}{|x|}C_r^{\mathrm{RI/MOM}}\left(\frac{y}{x},\frac{\mu}{P^z},\frac{\mu_R}{P^z}\right)f_{u-d}(x,\mu)\right]\ .
\end{aligned}$$

Singularities and divergecnes:

1. Soft singularity at $x/y=1$, use a soft cut-off $1\pm10^{-\varepsilon}$;
2. UV divergence when $x/y\to\infty$, and $y/x\to\infty$, use a hard cut-off $x_{cut}=10^{\pm n}$. $x/y\to\infty$ corresponds to $y\to0$, which is related to the smallest momentum fraction in the PDF (or quasi PDF if we reverse the factorization formula);

RI/MOM

Insensitive to soft cut-off $10^{-\varepsilon}$ or UV cut-off 10^{n}



(a) (b)

FIG. 2: Comparison between the convoluted result $C^{\mathrm{RI/MOM}} \otimes f_{u-d}$ and f_{u-d}. The yellow and green bands indicate the uncertainties in the factorization scale μ. (a) $C^{\mathrm{RI/MOM}} \otimes f_{u-d}$ and f_{u-d}; (b) Differences between $C^{\mathrm{RI/MOM}} \otimes f_{u-d}$, f_{u-d}, and $f_{u-d}(x, 4\ \mathrm{GeV})$;

Other schemes

Insensitive to soft cut-off $10^{-\varepsilon}$, but sensitive to UV cut-off 10^{n}

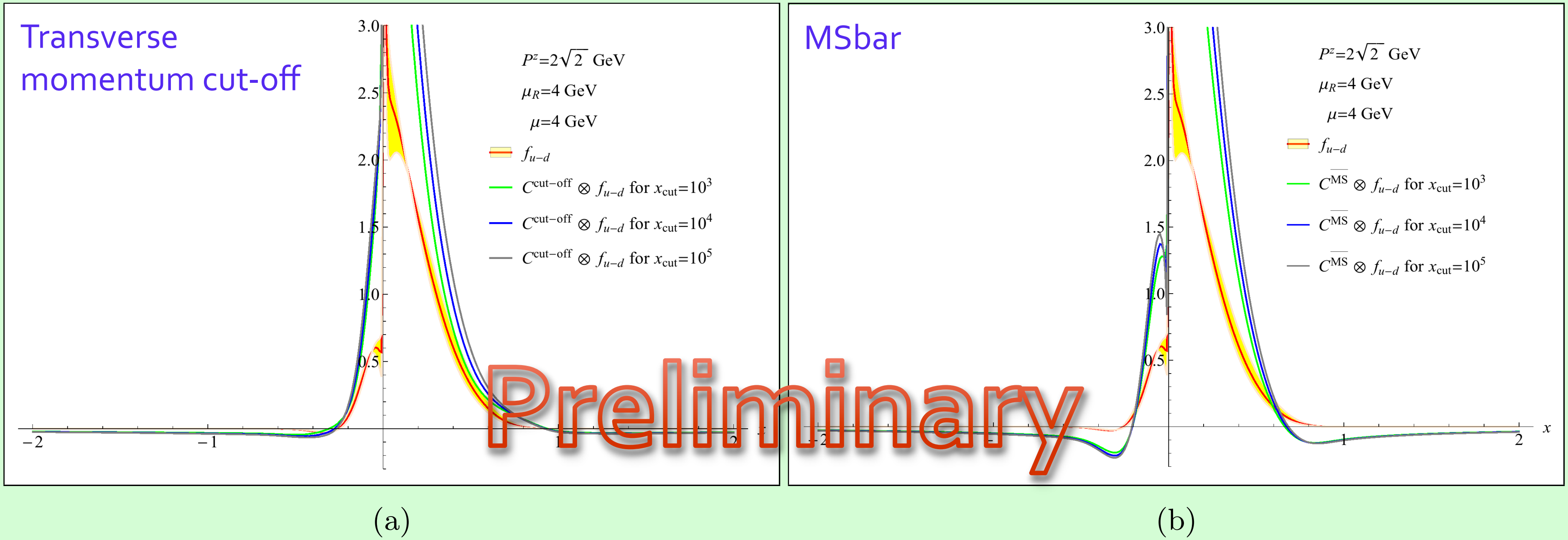


FIG. 4: Convoluted results for other schemes. The yellow and green bands indicate the uncertainties in the renormalization scale of PDF. (a) $C^{\text{cut-off}} \otimes f_{u-d}$ and f_{u-d} with different x_{cut}'s; (b) $C^{\overline{\text{MS}}} \otimes f_{u-d}$ and f_{u-d} with different x_{cut}'s.

- The counterterm in RI/MOM guarantees that the UV divergence in the integration over x is subtracted, so the convolution integral is insensitive to the UV cut-off;
- So far lattice results only come from quasi PDF in the transverse momentum cut-off scheme. We should test the sensitive of the factorization formula to the smallest momentum fraction in the quasi PDF.

Summary

- The implementation of the RI/MOM scheme on the nonperturbative renormalization of the quasi PDF in lattice QCD is discussed;
- Matching between quasi PDF in the RI/MOM scheme and PDF in the MSbar scheme is calculated at one-loop level.
- Matching between RI/MOM quasi PDF and PDF is a small effect, and has nice convergence.

Back up slides

Perturbative Calculation of $Z_{\psi,z}$

- The calculation is done for off-shell quark (with momentum p, $p^2<0$) matrix element of the operator;
- In order to calculate the matching coefficient, we need expansion around $p^2=0$ to identify the IR divergence $ln(p^2)$ and compare to PDF;
- However, on the Euclidean lattice, $p_E{}^2 \geq p_z{}^2$, so this expansion cannot be done.

Perturbative Calculation of $Z_{\psi,z}$

What should we do next for the matching?

Renormalized quasi PDF in lattice QCD

Numerical matching, the matching is independent of p^2 and p^z.

Renormalized quasi PDF in the continuum

Analytical matching, already done.

1. Fit the $ln(p_E^2)$ dependence with the Altarelli-Parisi kernel in the numerical value of $Z_{\psi,z}$

2. Analytically continue $ln(p_E^2)$ to $ln(p^2)$, so that we can compare to PDF in the MSbar scheme to do the matching

Renormalized PDF in the MSbar scheme

Renormalization of Quasi PDF on the Lattice

One of the standard methods is the lattice perturbation theory (S. Capitani, 2003):

- ☹ Feynman rules complicated by fermion and gluon actions on the lattice;
- ☹ Calculation of lattice Feynman diagrams is cumbersome and difficult, limiting our ability to go to higher loop orders.

Therefore, we focus on a nonperturbative method, the regularization-invariant momentum subtraction (RI/MOM) scheme, that has been widely used in lattice theory.

G. Martinelli et al., 1994

Other nonperturbative methods such as Yang-Mills gradient flow (K. Orginos and C. Monohan, 2016)

Open questions

- Rigorous proof of the renormalization relation of the quasi PDF . ✔
- Determination of δm from lattice QCD.
- If δm is absorbed into the RI/MOM renormalization factor, whether the result is still regularization-invariant.