TMDs of a Spin-1 Target

Ian Cloët

Argonne National Laboratory

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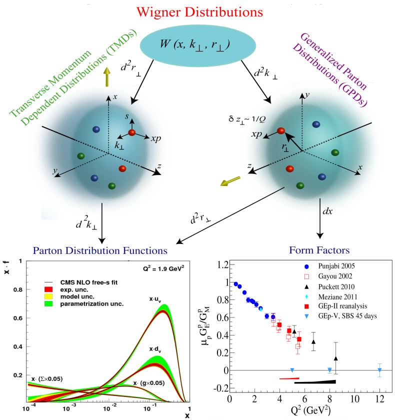
Momentum Tomography

leading twist		quark operator		
		unpolarized [U]	longitudinal [L]	transverse [T]
target polarization	U	$f_1 =$ unpolarized		$h_1^{\perp} =$ Boer-Mulders
	L		$g_1 =$ helicity	$h_{1L}^{\perp} =$ worm gear 1
	T	$f_{1T}^{\perp} =$ Sivers	$g_{1T} =$ worm gear 2	$h_1 =$ transversity $h_{1T}^{\perp} =$ pretzelosity
	TENSOR	$\theta_{LL}(x,\boldsymbol{k}_T^2)$ $\theta_{TT}(x,\boldsymbol{k}_T^2)$ $\theta_{LT}(x,\boldsymbol{k}_T^2)$	$g_{1TT}(x,\boldsymbol{k}_T^2)$ $g_{1LT}(x,\boldsymbol{k}_T^2)$	$h_{1LL}^{\perp}(x,\boldsymbol{k}_T^2)$ $h_{1TT},\ h_{1TT}^{\perp}$ $h_{1LT},\ h_{1LT}^{\perp}$



- A spin-1 target can have tensor polarization [associated with $\lambda = 0$]
  - 3 additional T-even and 7 additional T-odd quark TMDs compared to nucleon
    [A. Bacchetta and P. J. Mulders, Phys. Rev. D **62**, 114004 (2000)]
- Analogous situation for gluon TMDs [See talk of Mulders & Shanahan]
  - to fully expose role of gluons in nuclei need polarized nuclear targets [e.g. D, ^{6}Li]

TMDs of Spin-1 Targets

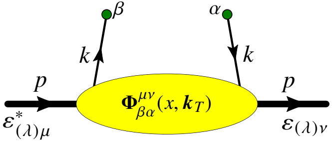
- Spin 4-vector of a spin-1 particle moving in z-direction – with spin quantization axis $\boldsymbol{S} = (\boldsymbol{S}_T, S_L)$ reads:

$$S^\mu(p) = \left(\frac{p_z}{m_h} S_L, \boldsymbol{S}_T, \frac{p_0}{m_h} S_L \right)$$

  - for given direction $\boldsymbol{S}$ the particle has the three possible spin projections $\lambda = \pm 1, 0$
  - longitudinal polarization $\Longrightarrow \boldsymbol{S}_T = 0, S_L = 1$; transverse $\Longrightarrow |\boldsymbol{S}_T| = 1, S_L = 0$

- Define quark TMDs of a spin-1 target with respect to the $\boldsymbol{k}_T$ dependent quark correlation function:

$$\boldsymbol{\Phi}^{(\lambda)s}_{\beta\alpha}(x, \boldsymbol{k}_T) =$$



- At leading-twist:

$$\left\langle \gamma^+ \right\rangle^{(\lambda)}_{\boldsymbol{S}} (x, \boldsymbol{k}_T) = f(x, \boldsymbol{k}_T^2) - \frac{3\lambda^2 - 2}{2} \left[\left(S_L^2 - \frac{1}{3} \right) \theta_{LL}(x, \boldsymbol{k}_T^2) + \frac{(\boldsymbol{k}_T \cdot \boldsymbol{S}_T)^2 - \frac{1}{3}\boldsymbol{k}_T^2}{m_h^2} \theta_{TT}(x, \boldsymbol{k}_T^2) + S_L \frac{\boldsymbol{k}_T \cdot \boldsymbol{S}_T}{m_h} \theta_{LT}(x, \boldsymbol{k}_T^2) \right]$$

$$\left\langle \gamma^+ \gamma_5 \right\rangle^{(\lambda)}_{\boldsymbol{S}} (x, \boldsymbol{k}_T) = \ldots, \qquad \left\langle \gamma^+ \gamma^i \gamma_5 \right\rangle^{(\lambda)}_{\boldsymbol{S}} (x, \boldsymbol{k}_T) = \ldots$$

PDFs of Spin-1 Targets

- Integrating over $\boldsymbol{k}_T^2$ gives 4 leading-twist quark PDFs for a spin-1 target

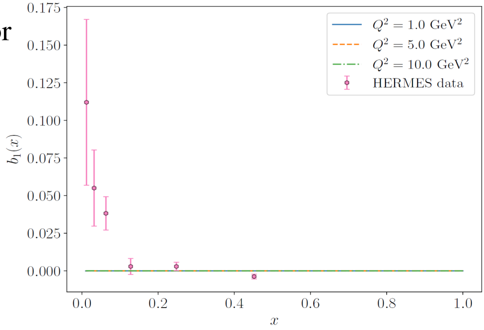
$$f(x) = \int d\boldsymbol{k}_T \; f(x, \boldsymbol{k}_T^2), \quad \theta(x) = \int d\boldsymbol{k}_T \left[\theta_{LL}(x, \boldsymbol{k}_T^2) - \frac{\boldsymbol{k}_T^2}{2\, m_h^2}\, \theta_{TT}(x, \boldsymbol{k}_T^2)\right], \ldots$$

- For DIS on spin-1 target 4 additional structure functions $b_{1\ldots4}(x)$ appear; in Bjorken limit just one $b_1(x)$ [Hoodbhoy, Jaffe and Manohar, Nucl. Phys. B **312**, 571 (1989)]

$$b_1(x) = \sum_q e_q^2 \left[b_1^q(x) + b_1^{\bar{q}}(x)\right], \quad b_1^q = \frac{1}{2}\,\theta_q = \frac{1}{4}\left[2\, q_{\boldsymbol{S}}^{(\lambda=0)} - q_{\boldsymbol{S}}^{(\lambda=1)} - q_{\boldsymbol{S}}^{(\lambda=-1)}\right]$$

- To measure $b_1(x)$ in DIS need tensor polarized target; HERMES has ^{2}H data, experiment planned at JLab

- Seems impossible to explain HERMES data with only bound nucleons degrees of freedom
  - need exotic QCD states: $6q$ bags, etc
  - JLab experiment is needed

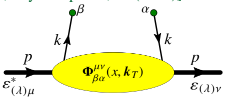


TMD Positivity Constraints

- Positivity conditions must be imposed on [Bourrely, Soffer and Leader, Phys. Rept. **59**, 95 (1980)]

$$M^{(\lambda)s}(x,\boldsymbol{k}_T) = \left[\Phi^{(\lambda)s}(x,\boldsymbol{k}_T)\gamma^+\right]^T$$

$\Phi^{(\lambda)s}_{\beta\alpha}(x,\boldsymbol{k}_T) =$



  - the matrix M is the antiquark–hadron forward scattering matrix
  - in hadron rest-frame M is a 6×6 matrix in quark and hadron spin space

- Positivity implies that eigenvalues of M must be semi-positive for all x & $\boldsymbol{k}_T$
  - imposes 6 sufficient conditions on the 9 spin-1 quark TMDs (very complicated)
  - also sub-minors of M must be semi-positive – imposes 63 necessarily conditions

- For quark PDFs of a spin-1 target this gives 3 sufficient conditions:

$$f(x) \geqslant 0, \qquad |g(x)| \leqslant f(x) - \frac{1}{3}\theta(x)$$

$$2\,h(x)^2 \leqslant \left(f(x) + \frac{2}{3}\theta(x)\right)\left(f(x) + g(x) - \frac{1}{3}\theta(x)\right)$$ spin-1 Soffer bound

[A. Bacchetta and P. J. Mulders, Phys. Lett. B **518**, 85 (2001)]

- *Positivity conditions place tight constraints on experiment and calculations*

Measuring TMDs of Spin-1 Targets

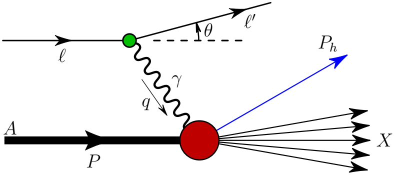
- Need longitudinal and tensor polarized spin-1 targets, e.g., deuteron and ^{6}Li
- For SIDIS there are 41 structure functions; 18 for U+L which also appear for spin-1/2 and 23 associated with tensor polarization

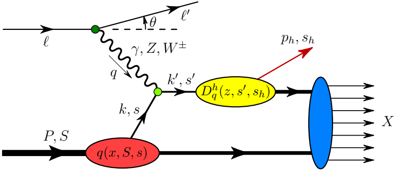
  [W. Cosyn, M. Sargsian and C. Weiss, PoS DIS **2016**, 210 (2016)]

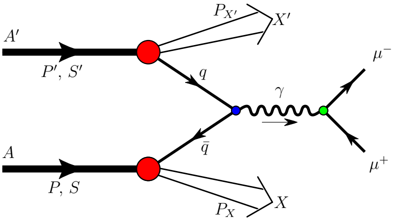
- For proton + deuteron Drell-Yan there are 108 structure functions; 60 associated with tensor structure of deuteron

  [S. Kumano, J. Phys. Conf. Ser. **543**, no. 1, 012001 (2014)]

- Very challenging experimentally
  - need solid physics motivation and likely an EIC

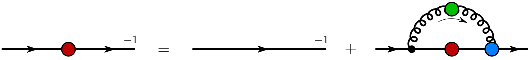






QCD's Dyson-Schwinger Equations

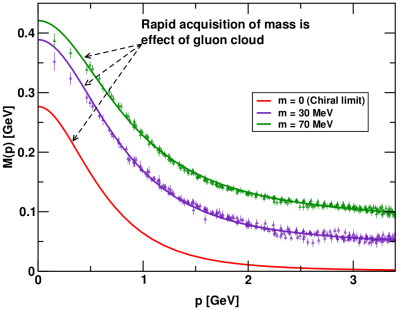
- The equations of motion of QCD $\Longleftrightarrow$ QCD's Dyson–Schwinger equations
  - an infinite tower of coupled integral equations
  - must implement a symmetry preserving truncation
- Most important DSE is QCD's gap equation $\Longrightarrow$ *dressed quark propagator*



  - ingredients – *dressed gluon propagator & dressed quark-gluon vertex*

$$S(p) = \frac{Z(p^2)}{i\not{p} + M(p^2)}$$

[M. S. Bhagwat *et al.*, Phys. Rev. **C 68**, 015203 (2003)]



- $S(p)$ has correct perturbative limit
- $M(p^2)$ exhibits dynamical mass generation $\Longleftrightarrow$ DCSB
- $S(p)$ has complex conjugate poles
  - no real mass shell $\Longleftrightarrow$ confinement

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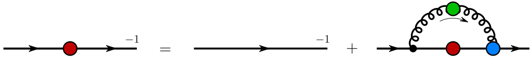

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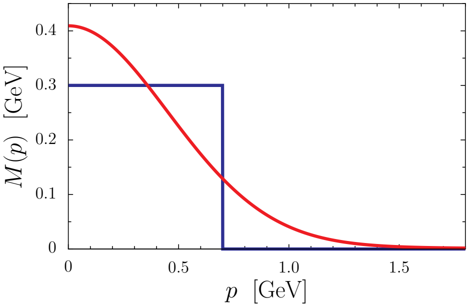
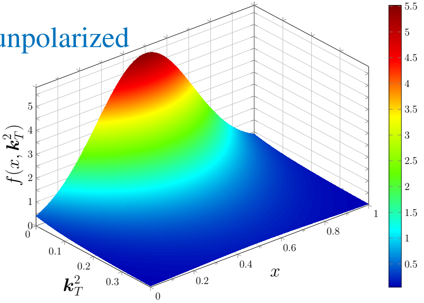


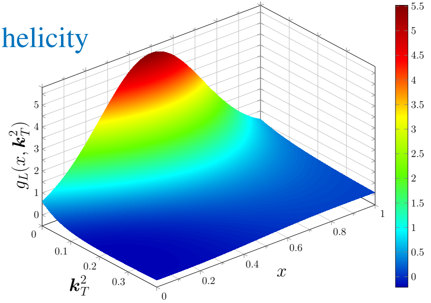
TMDs for a Rho Meson

[Yu Ninomiya, ICC and Wolfgang Bentz, *to appear*]

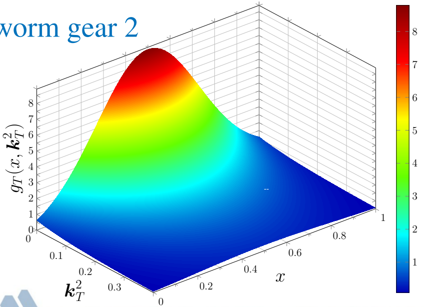
unpolarized



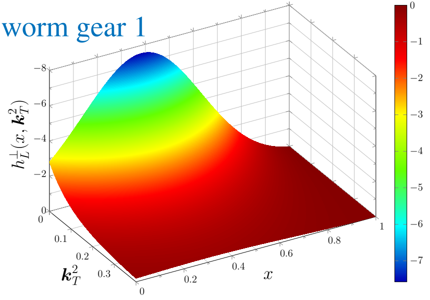
helicity



worm gear 2



worm gear 1



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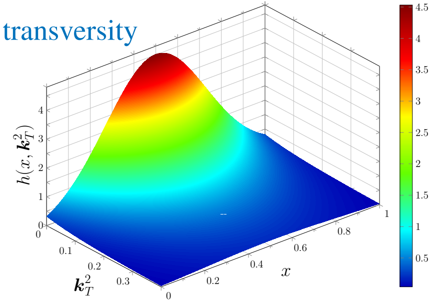
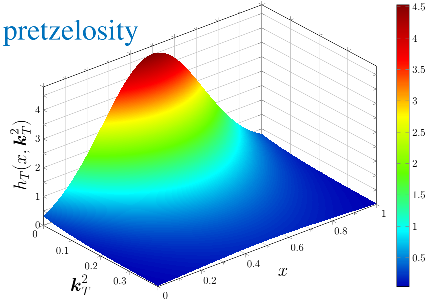
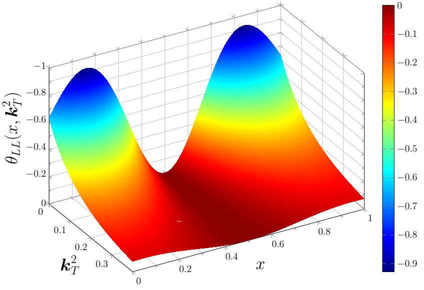
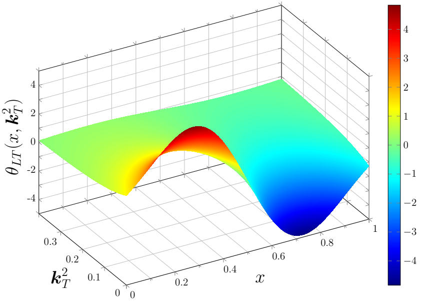




- Are spin-one TMDs interesting – do they contain new information?
- Each of these six T-even spin-one TMDs that have a nucleon analogy
  - each TMD is comparable in magnitude and shape
  - however arguably contain few surprises; peak near $x \sim 1/2$, have power-law behavior $1/\boldsymbol{k}_T^2$ for large transverse momentum
- With only 2.2 MeV binding energy the deuteron helicity and transversity TMDs are likely much smaller ... but maybe there are surprises c.f. $b_1(x)$

TMDs for a Rho Meson – Tensor Polarization





- Tensor polarized TMDs have a number of surprising features

$$\theta(x, \boldsymbol{k}_T^2) = \theta_{LL} - \frac{\boldsymbol{k}_T^2}{2\, m_h^2}\, \theta_{TT}$$

- TMDs $\theta_{LL}(x\, \boldsymbol{k}_T^2)$ & $\theta_{LT}(x\, \boldsymbol{k}_T^2)$ identically vanishes at $x = 1/2$ for all $\boldsymbol{k}_T^2$
  - $x = 1/2$ corresponds to zero relative momentum between (the two) constituents, that is, s-wave contributions
  - therefore θ_{LL} & θ_{LT} only receive contributions from $L \geqslant 1$ components of the wave function – *sensitive measure of orbital angular momentum*
- Features hard to determine from a few moments – difficult for lattice QCD

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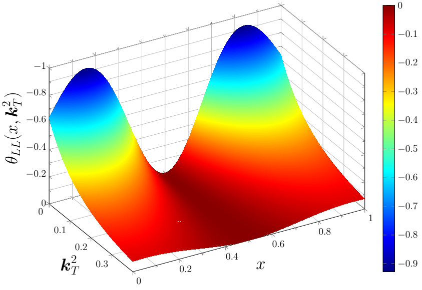

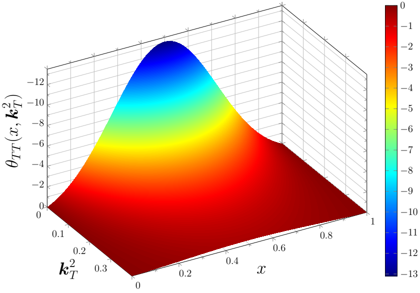

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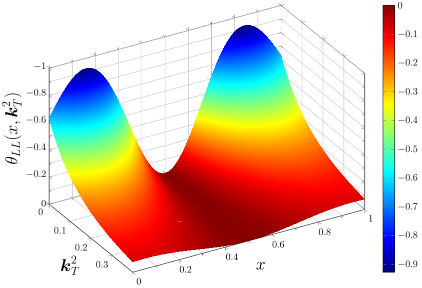

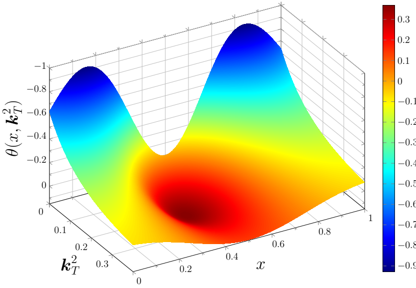

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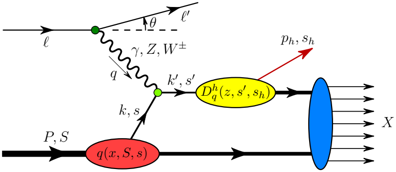
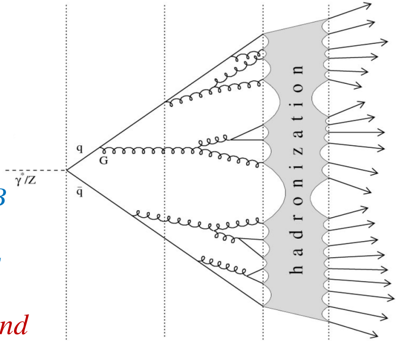
Spin-1 Fragmentation Functions: $q \to \rho + X$

- Measuring the ρ TMDs is clearly not possible for the forseeable future
  - for spin-1 need nuclear target



- However, measuring the $q \to \rho$ TMD fragmentation functions is forseeable

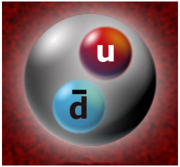
- Fragmentation functions are particularly important
  - *potentially fragmentation functions can shed the most light on confinement and DCSB – because they describe how a fast moving (massless) quark becomes a tower of hadrons*



- *Understanding the nature of confinement and its relation to DCSB is one of the most important challenges in hadron physics – origin of ~98% of mass in visible universe*

Spin-0 TMDs – Pion

The Pion in QCD

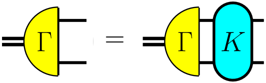


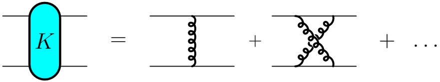
- Today the pion is understood as both a bound state of a *dressed-quark* and a *dressed-antiquark* in QFT and the Goldstone mode associated with DCSB in QCD
- This dichotomous nature has numerous ramifications, e.g.:

$$m_\rho/2 \sim M_N/3 \sim 350\,\text{MeV} \qquad \text{however} \qquad m_\pi/2 \simeq 0.2 \times 350\,\text{MeV}$$

  - pion is unusually light, the key is *dynamical chiral symmetry breaking* (DCSB)

- In QFT a two-body bound state (e.g. a pion or rho) is described by the Bethe-Salpeter equation (BSE):





  - the kernel must yield a solution that encapsulates the consequences of DCSB, e.g., in chiral limit $m_\pi = 0$ & $m_\pi^2 \propto m_u + m_d$

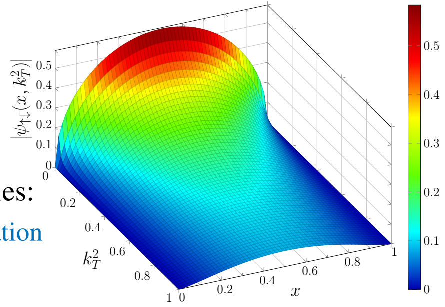
- Pion BSE wave function has the general form

$$\chi_\pi(p,k) = S(k)\Big[E_\pi(p,k) + \not{p}\,F_\pi(p,k) + \not{k}\,k\cdot p\,\mathcal{G}(p,k) + \sigma^{\mu\nu}k_\mu p_\nu\,\mathcal{H}(p,k)\Big]\gamma_5\,S(k-p)$$

Pion's LFWFs

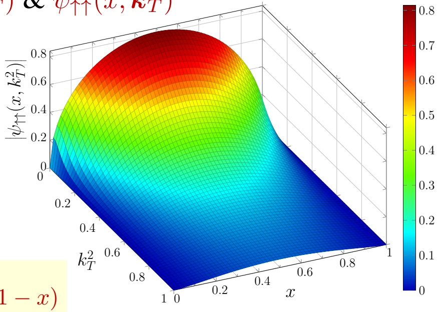
- Leading LFWF is given by

$$\psi_{\lambda\lambda'}(x, \boldsymbol{k}_T) = \int dk^- \; \bar{u}_\lambda \, \gamma^+ \, \chi_{\text{BSE}}(p,k) \, \gamma^+ \, v_{\lambda'}$$



- LFWFs have many remarkable properties:
  - frame-independent; probability interpretation – as close as QFT gets to QM
  - boosts are kinematical – *not dynamical*

- Pion has two leading LFWFs: $\psi_{\uparrow\downarrow}(x, \boldsymbol{k}_T)$ & $\psi_{\uparrow\uparrow}(x, \boldsymbol{k}_T)$
  - find broad concave functions in x
  - find same power-law behavior as predicted by perturbative QCD: $\psi_{\uparrow\downarrow} \sim 1/\boldsymbol{k}_T^2$ & $\psi_{\uparrow\uparrow} \sim 1/\boldsymbol{k}_T^4$



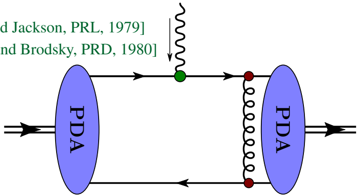
- Parton distribution amplitudes (PDAs) are related to light-front wave functions

$$\varphi(x) = \int d^2\boldsymbol{k}_T \; \psi_{\uparrow\downarrow}(x, \boldsymbol{k}_T) \;\; \Leftrightarrow \;\; \varphi_\pi^{\text{asy}}(x) = 6\, x\,(1-x)$$

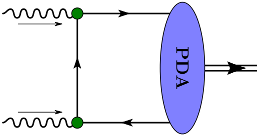
Pion's Parton Distribution Amplitude

- pion's PDA – $\varphi_\pi(x)$: *is a probability amplitude that describes the momentum distribution of a quark and antiquark in the bound-state's valence Fock state*
  - it's a function of the light-cone momentum fraction $x = \frac{k^+}{p^+}$ and the scale Q^2

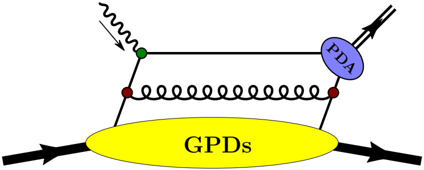
[Farrar and Jackson, PRL, 1979]
[Lepage and Brodsky, PRD, 1980]

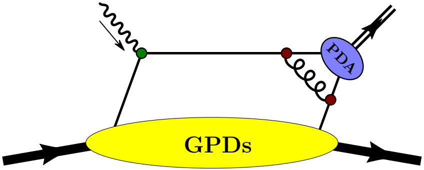


$$Q^2\, F_\pi(Q^2) \to 16\pi\, f_\pi^2\, \alpha_s(Q^2)$$



$$Q^2\, F_{\gamma^*\gamma\pi}(Q^2) \to 2\, f_\pi$$

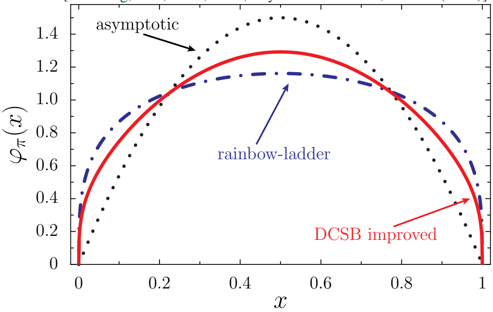




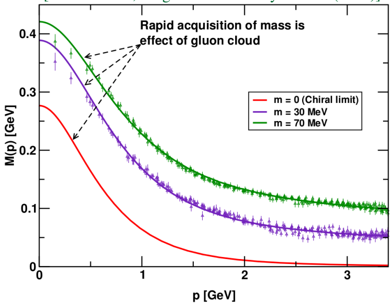
- PDAs enter numerous hard exclusive scattering processes

Pion PDA from the DSEs

[L. Chang, ICC, CDR, *et al.*, Phys. Rev. Lett. **110**, 132001 (2013)]



[C.D. Roberts, Prog. Part. Nucl. Phys. 61 50 (2008)]



- Both DSE results – each using a different Bethe-Salpeter kernel – exhibit a pronounced broadening compared with the asymptotic pion PDA
  - scale of calculation is given by renormalization point $\xi = 2\,\text{GeV}$
- A realization of DCSB on the light-front
- *ERBL evolution demonstrates that the pion's PDA remains broad & concave for all accessible scales in current and conceivable experiments*
- Broading of PDA influences the Q^2 evolution of the pion's EM form factor

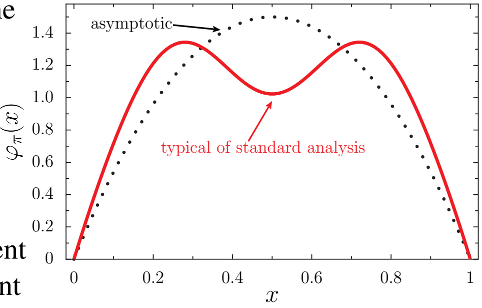
Pion PDA from Lattice QCD

- Currently, lattice QCD can determine only one non-trivial moment, e.g.:

$$\int_0^1 dx\,(2\,x-1)^2\varphi_\pi(x) = 0.27 \pm 0.04$$

[V. M. Braun *et al.*, Phys. Rev. D **74**, 074501 (2006)]

  - scale is $Q^2 = 4\,\mathrm{GeV}^2$



- Standard practice to fit first coefficient of "*asymptotic expansion*" to moment

$$\varphi_\pi(x,Q^2) = 6\,x\,(1-x)\left[1+\sum_{n=2,\,4,\ldots} a_n^{3/2}(Q^2)\,C_n^{3/2}(2x-1)\right]$$

  - however this expansion is guaranteed to converge rapidly only when $Q^2 \to \infty$
  - method results in a *double-humped* pion PDA – not supported by BSE WFs

- Advocate using a *generalized expansion*

$$\varphi_\pi(x,Q^2) = N_\alpha\,x^\alpha(1-x)^\alpha\left[1+\sum_{n=2,\,4,\ldots} a_n^{\alpha+1/2}(Q^2)\,C_n^{\alpha+1/2}(2x-1)\right]$$

- Find $\varphi_\pi \simeq x^\alpha(1-x)^\alpha,\ \alpha = 0.35^{+0.32}_{-0.24}$; good agreement with DSE: $\alpha \sim 0.52$

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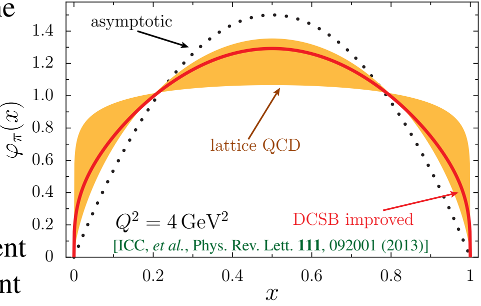

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Pion PDA from Lattice QCD – updated

- Most recent lattice QCD moment:

$$\int_0^1 dx\,(2\,x-1)^2\varphi_\pi(x) = 0.2361\,(41)\,(39)\,(?)$$

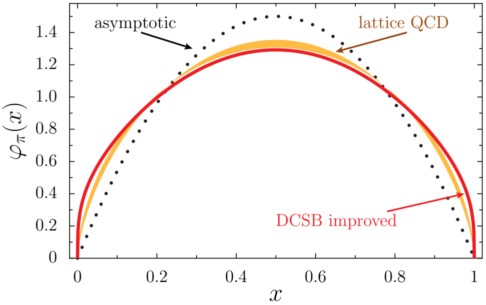
[V. M. Braun, *et al.*, Phys. Rev. D **92**, no. 1, 014504 (2015)]

- DSE prediction:

$$\int_0^1 dx\,(2\,x-1)^2\varphi_\pi(x) = 0.251$$

- *Near complete agreement between DSE prediction and latest lattice QCD result*

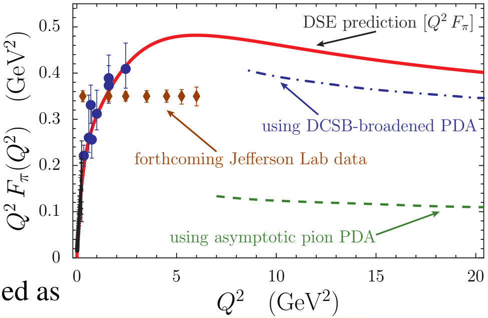
- Conclude that the pion PDA is a broad concave function
  - *double humped distributions are very likely for the pion*



Pion Elastic Form Factor

- Direct, symmetry-preserving computation of pion form factor predicts maximum in $Q^2\,F_\pi(Q^2)$ at $Q^2 \approx 6\,\text{GeV}^2$
  - magnitude of this product is determined by strength of DCSB at all accessible scales

[L. Chang, ICC, CDR, *et al.*, Phys. Rev. Lett. **111**, 141802 (2013)]



- The QCD prediction can be expressed as

$$Q^2 F_\pi(Q^2) \overset{Q^2 \gg \Lambda^2_{\rm QCD}}{\sim} 16\,\pi\, f_\pi^2\, \alpha_s(Q^2)\, w_\pi^2; \qquad w_\pi = \frac{1}{3}\int_0^1 dx\, \frac{1}{x}\, \varphi_\pi(x)$$

- Find consistency between the *direct pion form factor calculation* and the QCD hard-scattering formula – if DSE pion PDA is used
  - 15% disagreement may be explained by higher order/higher-twist corrections

- *Predict that QCD power law behavior – with QCD's scaling law violations – sets in at $Q^2 \sim 8\,\text{GeV}^2$* [*Featured in 2015 NP Long Range Plan*]

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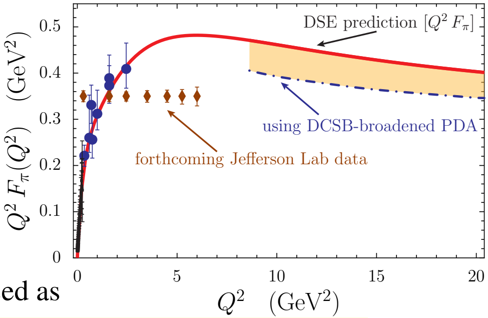

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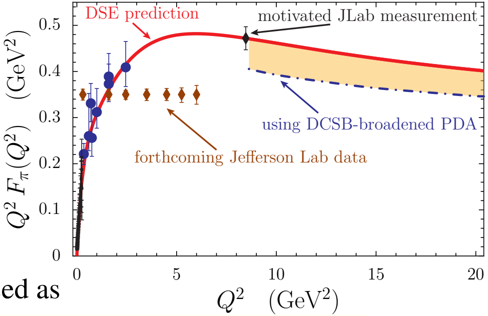

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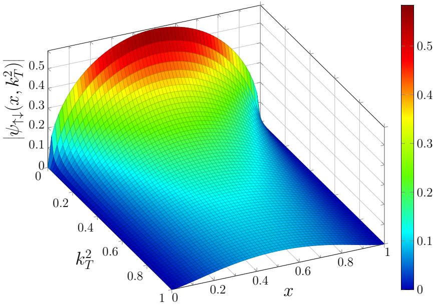
Pion TMD from its LFWFs

- DCSB results in broad pion LFWFs at hadronic scales
  - this is reflected in DSE and lattice result for pion's PDA

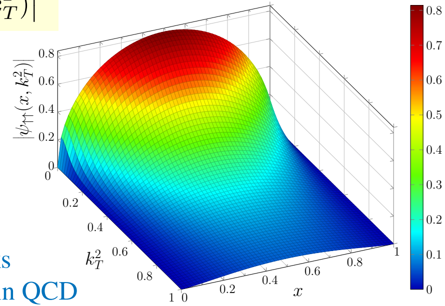
- Using pion's LFWFs straightforward to make predictions for pion GPDs, TMDs, etc; For TMDs:

$$f(x, \boldsymbol{k}_T^2) \propto \left|\psi_{\uparrow\downarrow}(x, \boldsymbol{k}_T^2)\right|^2 + \boldsymbol{k}_T^2 \left|\psi_{\uparrow\uparrow}(x, \boldsymbol{k}_T^2)\right|^2$$

- Contrast our result with Pasquini & Schweitzer [PRD **90** 014050 (2014)]
  - each result gives similar PDF but very different TMD
  - illustration of the potential for TMDs to differentiate between different frameworks & thereby expose quark-gluon dynamics in QCD



[Chao Shi and ICC, *to appear*]



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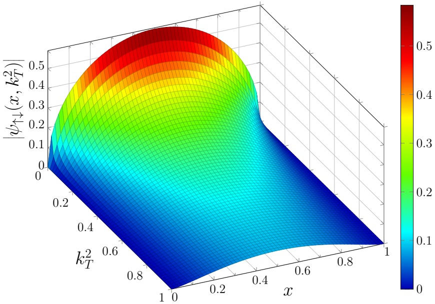

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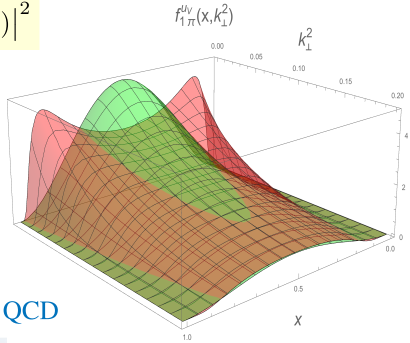

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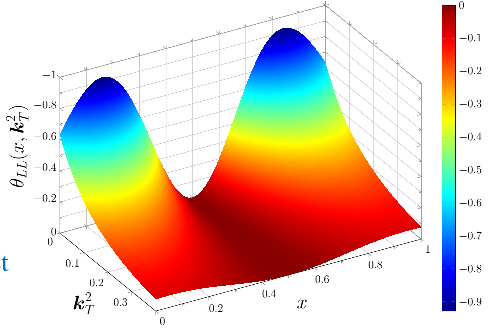
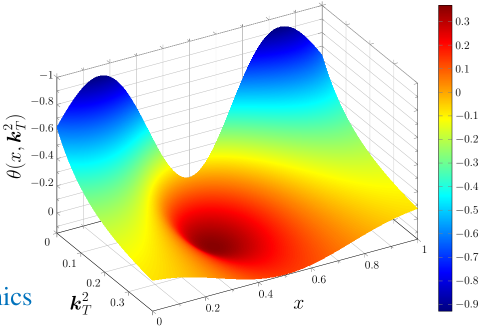
Conclusion

- Spin-1 targets present a rich quark and gluon structure that can help expose novel aspects of QCD
  - find that TMDs associated with tensor polarization are sensitive to orbital angular momentum in target
  - ρ meson results a stepping stone to deuteron calculations
- Find that because of DCSB pion's LFWFs are broad and concave in x – results have perturbative power-law behavior for large $\boldsymbol{k}_T^2$
  - find that PDFs can not distinguish between vastly different LFWFs
  - however TMDs are a powerful tool to expose underlying quark/gluon dynamics





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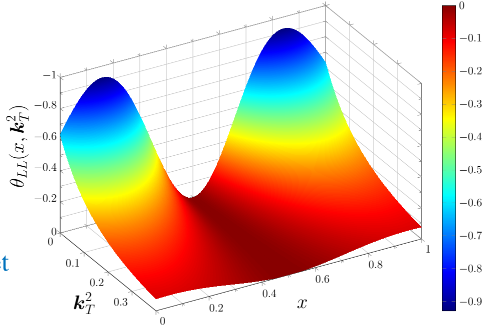
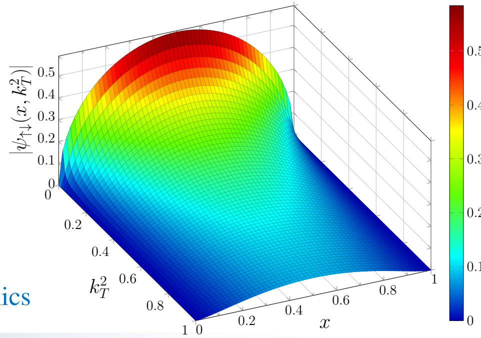




Backup Slides

QCD Evolution & Asymptotic PDA

- ERBL (Q^2) evolution for pion PDA [c.f. DGLAP equations for PDFs]

$$\mu \frac{d}{d\mu} \varphi(x,\mu) = \int_0^1 dy\, V(x,y)\, \varphi(y,\mu)$$

- This evolution equation has a solution of the form

$$\varphi_\pi(x,Q^2) = 6\,x\,(1-x)\left[1 + \sum_{n=2,\,4,\dots} a_n^{3/2}(Q^2)\, C_n^{3/2}(2x-1)\right]$$

  - $\alpha = 3/2$ because in $Q^2 \to \infty$ limit QCD is invariant under the collinear conformal group $SL(2;\mathbb{R})$
  - Gegenbauer-$\alpha = 3/2$ polynomials are irreducible representations $SL(2;\mathbb{R})$

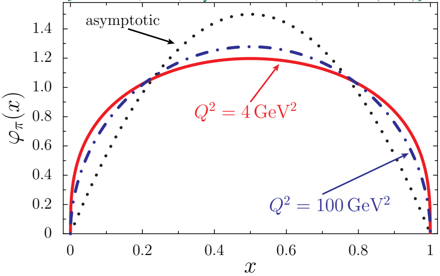
- The coefficients of the Gegenbauer polynomials, $a_n^{3/2}(Q^2)$, evolve logarithmically to zero as $Q^2 \to \infty$: $\varphi_\pi(x) \to \varphi_\pi^{\text{asy}}(x) = 6\,x\,(1-x)$

- At what scales is this a good approximation to the pion PDA?

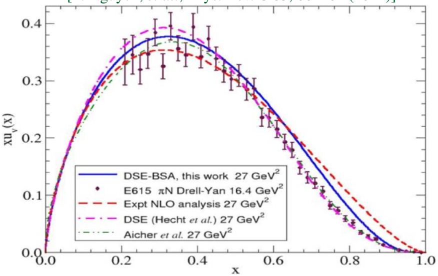
- E.g., AdS/QCD find $\varphi_\pi(x) \sim x^{1/2}\,(1-x)^{1/2}$ at $Q^2 = 1\,\text{GeV}^2$; expansion in terms of $C_n^{3/2}(2x-1)$ convergences slowly: $a_{32}^{3/2} / \, a_2^{3/2} \sim 10\,\%$

When is the Pion's PDA Asymptotic

[I. C. Cloët, *et al.*, Phys. Rev. Lett. **111**, 092001 (2013)]

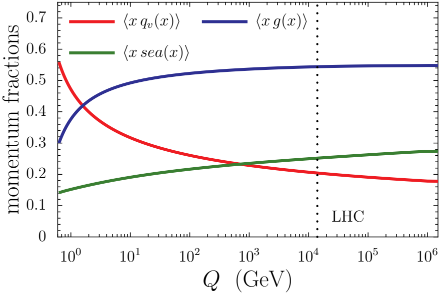


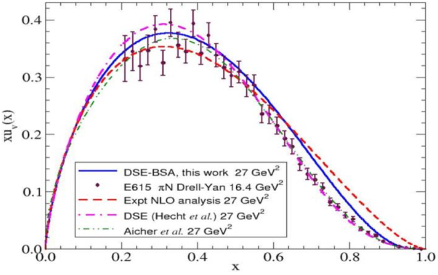
[T. Nguyen, *et al.*, Phys. Rev. C **83**, 062201 (2011)]



- Under leading order Q^2 evolution the pion PDA remains broad to well above $Q^2 > 100\,\text{GeV}^2$, compared with $\varphi_\pi^{\text{asy}}(x) = 6\,x\,(1-x)$
- *Consequently, the asymptotic form of the pion PDA is a poor approximation at all energy scales that are either currently accessible or foreseeable in experiments on pion elastic and transition form factors*
- Importantly, $\varphi_\pi^{\text{asy}}(x)$ is only guaranteed be an accurate approximation to $\varphi_\pi(x)$ when pion valence quark PDF satisfies: $q_v^\pi(x) \sim \delta(x)$
- This is far from valid at forseeable energy scales

When is the Pion's Valence PDF Asymptotic



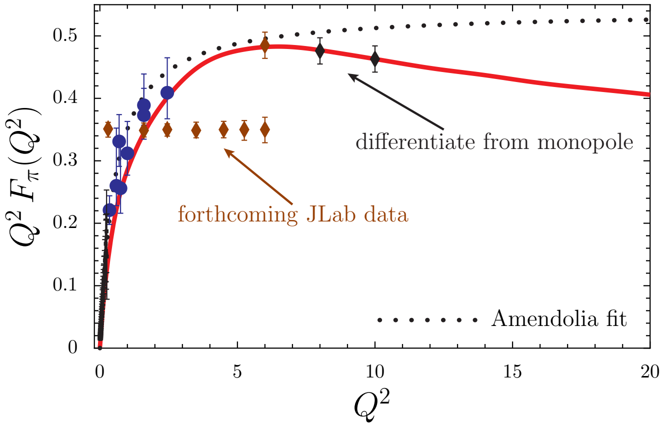


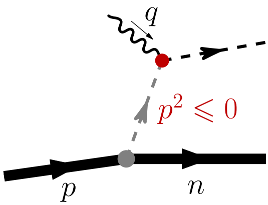
- LO QCD evolution of momentum fraction carried by valence quarks

$$\langle x\, q_v(x)\rangle (Q^2) = \left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{\gamma_{qq}^{(0)2}/(2\beta_0)} \langle x\, q_v(x)\rangle (Q_0^2) \quad \text{where} \quad \frac{\gamma_{qq}^{(0)2}}{2\beta_0} > 0$$

  - therefore, as $Q^2 \to \infty$ we have $\langle x\, q_v(x)\rangle \to 0$ implies $q_v(x) \propto \delta(x)$
- At LHC energies valence quarks still carry 20% of pion momentum
  - the gluon distribution saturates at $\langle x\, g(x)\rangle \sim 55\%$
- *Asymptotia is a long way away!*

Measuring onset of Perturbative scaling

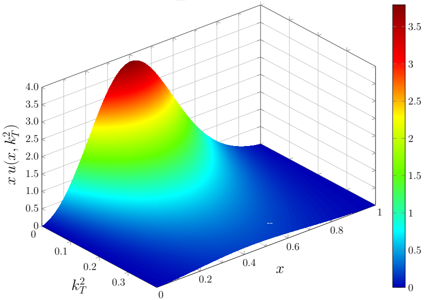


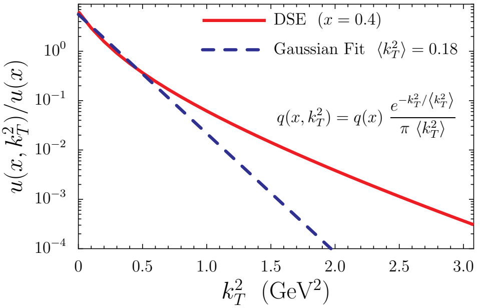


Sullivan Process

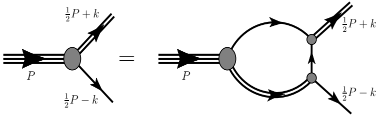
- To observe onset of perturbative power law behaviour – *to differentiate from a monopole* – optimistically need data at $8\,\mathrm{GeV}^2$ but likely also at $10\,\mathrm{GeV}^2$
  - this is a very challenging task experimentally
- Scaling predictions are valid for both spacelike and timelike momenta
  - timelike data show promise as the means of verifying modern predictions

TMDs, Diquarks & Flavor Dependence





- Rigorously included transverse momentum of diquark correlations in TMDs



- This has numerous consequences:
  - scalar diquark correlations greatly increase $\langle k_T^2\rangle$
  - find deviation from Gaussian anzatz and that TMDs do not factorize in x & k_T^2
  - diquark correlations introduce a significant flavor dependence in the average $\langle k_T^2\rangle$ [analogous to the quark-sector electromagnetic form factors]
- Work is also underway for nucleon GPDs, WACS, etc

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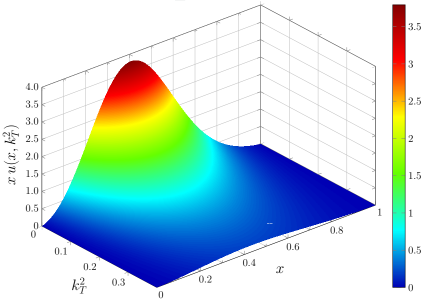

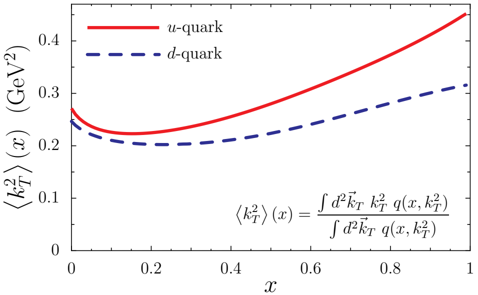

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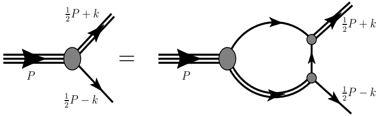

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