TMDs of a Spin-1 Target

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### Momentum Tomography

<table>
<thead>
<tr>
<th>leading twist</th>
<th>unpolared [U]</th>
<th>longitudinal [L]</th>
<th>transverse [T]</th>
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<tr>
<td>U</td>
<td>$f_1 = \uparrow$</td>
<td>$h_1^+ = \downarrow$</td>
<td>Boer-Mulders</td>
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<tr>
<td>L</td>
<td>$g_1 = \rightarrow \leftarrow$</td>
<td>$h_1^L = \rightarrow \leftarrow$</td>
<td>worm gear 1</td>
</tr>
<tr>
<td>T</td>
<td>$f_{1T} = \uparrow \rightarrow \downarrow$</td>
<td>$g_{1T} = \rightarrow \leftarrow$</td>
<td>transversity</td>
</tr>
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</table>

- A spin-1 target can have tensor polarization [associated with $\lambda = 0$]
- 3 additional $T$-even and 7 additional $T$-odd quark TMDs compared to nucleon
- Analogous situation for gluon TMDs [See talk of Mulders & Shanahan]
  - to fully expose role of gluons in nuclei need polarized nuclear targets [e.g. D, $^6$Li]
**TMDs of Spin-1 Targets**

- Spin 4-vector of a spin-1 particle moving in z-direction – with spin quantization axis \( S = (S_T, S_L) \) reads:
  - for given direction \( S \) the particle has the three possible spin projections \( \lambda = \pm 1, 0 \)
  - longitudinal polarization \( \implies S_T = 0, S_L = 1 \); transverse \( \implies |S_T| = 1, S_L = 0 \)

- Define quark TMDs of a spin-1 target with respect to the \( k_T \) dependent quark correlation function:

- At leading-twist:

\[
S^\mu(p) = \left( \frac{p_z}{m_h} S_L, S_T, \frac{p_0}{m_h} S_L \right)
\]

\[
\langle \gamma^+ \rangle_S^{(\lambda)}(x, k_T) = f(x, k_T^2) - \frac{3\lambda^2 - 2}{2} \left[ \left( S_L^2 - \frac{1}{3} \right) \theta_{LL}(x, k_T^2) \right.
\]

\[
+ \frac{(k_T \cdot S_T)^2 - \frac{1}{3} k_T^2}{m_h^2} \theta_{TT}(x, k_T^2) + S_L \frac{k_T \cdot S_T}{m_h} \theta_{LT}(x, k_T^2) \left. \right]
\]

\[
\langle \gamma^+ \gamma^5 \rangle_S^{(\lambda)}(x, k_T) = \ldots, \quad \langle \gamma^+ \gamma^i \gamma^5 \rangle_S^{(\lambda)}(x, k_T) = \ldots
\]
PDFs of Spin-1 Targets

Integrating over $k_T^2$ gives 4 leading-twist quark PDFs for a spin-1 target

$$f(x) = \int dk_T f(x, k_T^2), \quad \theta(x) = \int dk_T \left[ \theta_{LL}(x, k_T^2) - \frac{k_T^2}{2m_h^2} \theta_{TT}(x, k_T^2) \right], \ldots$$

For DIS on spin-1 target 4 additional structure functions $b_1 ... 4(x)$ appear; in Bjorken limit just one $b_1(x)$ [Hoodbhoy, Jaffe and Manohar, Nucl. Phys. B 312, 571 (1989)]

$$b_1(x) = \sum_q e_q^2 \left[ b_q(x) + b_{\bar{q}}(x) \right], \quad b_q = \frac{1}{2} \theta_q = \frac{1}{4} \left[ 2q_\lambda^{(\lambda=0)} - q_\lambda^{(\lambda=1)} - q_\lambda^{(\lambda=-1)} \right]$$

To measure $b_1(x)$ in DIS need tensor polarized target; HERMES has $^2H$ data, experiment planned at JLab

Seems impossible to explain HERMES data with only bound nucleons degrees of freedom

- need exotic QCD states: 6$q$ bags, etc
- JLab experiment is needed
The matrix $M$ is the antiquark–hadron forward scattering matrix

in hadron rest-frame $M$ is a $6 \times 6$ matrix in quark and hadron spin space

Positivity implies that eigenvalues of $M$ must be semi-positive for all $x$ & $k_T$

imposes 6 sufficient conditions on the 9 spin-1 quark TMDs (very complicated)

also sub-minors of $M$ must be semi-positive – imposes 63 necessarily conditions

For quark PDFs of a spin-1 target this gives 3 sufficient conditions:

\[
\begin{align*}
  f(x) & \geq 0, \\
  |g(x)| & \leq f(x) - \frac{1}{3} \theta(x) \\
  2 h(x)^2 & \leq \left( f(x) + \frac{2}{3} \theta(x) \right) \left( f(x) + g(x) - \frac{1}{3} \theta(x) \right) \\
\end{align*}
\]

spin-1 Soffer bound

Measuring TMDs of Spin-1 Targets

- Need longitudinal and tensor polarized spin-1 targets, e.g., deuteron and $^6\text{Li}$

- For SIDIS there are 41 structure functions; 18 for U+L which also appear for spin-1/2 and 23 associated with tensor polarization


- For proton + deuteron Drell-Yan there are 108 structure functions; 60 associated with tensor structure of deuteron


- Very challenging experimentally
  - need solid physics motivation and likely an EIC
The equations of motion of QCD \(\iff\) QCD’s Dyson–Schwinger equations
- an infinite tower of coupled integral equations
- must implement a symmetry preserving truncation

Most important DSE is QCD’s gap equation \(\implies\) dressed quark propagator

- ingredients – dressed gluon propagator & dressed quark-gluon vertex

\[
S(p) = \frac{Z(p^2)}{i\not{p} + M(p^2)}
\]

\(S(p)\) has correct perturbative limit

\(M(p^2)\) exhibits dynamical mass generation \(\iff\) DCSB

\(S(p)\) has complex conjugate poles
- no real mass shell \(\iff\) confinement

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QCD’s Dyson-Schwinger Equations

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TMDs for a Rho Meson

unpolarized

helicity

worm gear 2

worm gear 1

[Yu Ninomiya, ICC and Wolfgang Bentz, to appear]
Are spin-one TMDs interesting – do they contain new information?

Each of these six $T$-even spin-one TMDs that have a nucleon analogy

- each TMD is comparable in magnitude and shape
- however arguably contain few surprises; peak near $x \sim 1/2$, have power-law behavior $1/k_T^2$ for large transverse momentum

With only 2.2 MeV binding energy the deuteron helicity and transversity TMDs are likely much smaller . . . but maybe there are surprises c.f. $b_1(x)$
Tensor polarized TMDs have a number of surprising features

TMDs $\theta_{LL}(x, k_T^2) \& \theta_{LT}(x, k_T^2)$ identically vanishes at $x = 1/2$ for all $k_T^2$

- $x = 1/2$ corresponds to zero relative momentum between (the two) constituents, that is, $s$-wave contributions
- therefore $\theta_{LL} \& \theta_{LT}$ only receive contributions from $L \geq 1$ components of the wave function – *sensitive measure of orbital angular momentum*

Features hard to determine from a few moments – difficult for lattice QCD
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Spin-1 Fragmentation Functions: $q \rightarrow \rho + X$

Measuring the $\rho$ TMDs is clearly not possible for the foreseeable future

- for spin-1 need nuclear target

However, measuring the $q \rightarrow \rho$ TMD fragmentation functions is foreseeable

Fragmentation functions are particularly important

- potentially fragmentation functions can shed the most light on confinement and DCSB – because they describe how a fast moving (massless) quark becomes a tower of hadrons

Understanding the nature of confinement and its relation to DCSB is one of the most important challenges in hadron physics – origin of $\sim 98\%$ of mass in visible universe
Spin-0 TMDs – Pion
The Pion in QCD

Today the pion is understood as both a bound state of a dressed-quark and a dressed-antiquark in QFT and the Goldstone mode associated with DCSB in QCD.

This dichotomous nature has numerous ramifications, e.g.:

\[ \frac{m_\rho}{2} \sim \frac{M_N}{3} \sim 350 \text{ MeV} \quad \text{however} \quad \frac{m_\pi}{2} \simeq 0.2 \times 350 \text{ MeV} \]

- The pion is unusually light, the key is dynamical chiral symmetry breaking (DCSB).

In QFT a two-body bound state (e.g. a pion or rho) is described by the Bethe-Salpeter equation (BSE):

\[ \Gamma = \Gamma K \]

\[ K = \cdots + \cdots + \cdots \]

- The kernel must yield a solution that encapsulates the consequences of DCSB, e.g., in chiral limit \( m_\pi = 0 \) & \( m_\pi^2 \propto m_u + m_d \).

Pion BSE wave function has the general form:

\[ \chi_\pi(p, k) = S(k) \left[ E_\pi(p, k) + \not{p} F_\pi(p, k) + \not{k} k \cdot p G(p, k) + \sigma^{\mu\nu} k_\mu p_\nu H(p, k) \right] \gamma_5 S(k - p) \]
**Pion’s LFWFs**

- Leading LFWF is given by

\[ \psi_{\lambda\lambda'}(x, k_T) = \int dk^- \bar{u}_\lambda \gamma^+ \chi_{\text{BSE}}(p, k) \gamma^+ v_{\lambda'} \]

- LFWFs have many remarkable properties:
  - frame-independent; probability interpretation – as close as QFT gets to QM
  - boosts are kinematical – *not dynamical*

- Pion has two leading LFWFs: \( \psi_{\uparrow\downarrow}(x, k_T) \) & \( \psi_{\uparrow\uparrow}(x, k_T) \)
  - find broad concave functions in \( x \)
  - find same power-law behavior as predicted by perturbative QCD:
    \[ \psi_{\uparrow\downarrow} \sim 1/k_T^2 \] & \[ \psi_{\uparrow\uparrow} \sim 1/k_T^4 \]

- Parton distribution amplitudes (PDAs) are related to light-front wave functions

\[ \varphi(x) = \int d^2 k_T \psi_{\uparrow\downarrow}(x, k_T) \Leftrightarrow \varphi_{\pi}^{\text{asy}}(x) = 6x(1-x) \]
Pion’s Parton Distribution Amplitude

- pion’s PDA – $\varphi_\pi(x)$: *is a probability amplitude that describes the momentum distribution of a quark and antiquark in the bound-state’s valence Fock state*
  
- it’s a function of the light-cone momentum fraction $x = \frac{k^+}{p^+}$ and the scale $Q^2$

$$Q^2 F_\pi(Q^2) \rightarrow 16\pi f_\pi^2 \alpha_s(Q^2)$$

[Farrar and Jackson, PRL, 1979]
[Lepage and Brodsky, PRD, 1980]

PDAs enter numerous hard exclusive scattering processes
Pion PDA from the DSEs

Both DSE results – each using a different Bethe-Salpeter kernel – exhibit a pronounced broadening compared with the asymptotic pion PDA.

- scale of calculation is given by renormalization point $\xi = 2$ GeV

- A realization of DCSB on the light-front

- **ERBL evolution demonstrates that the pion’s PDA remains broad & concave for all accessible scales in current and conceivable experiments**

- Broading of PDA influences the $Q^2$ evolution of the pion’s EM form factor
Pion PDA from Lattice QCD

Currently, lattice QCD can determine only one non-trivial moment, e.g.:

\[ \int_0^1 dx \ (2x - 1)^2 \varphi_{\pi}(x) = 0.27 \pm 0.04 \]

[V. M. Braun et al., Phys. Rev. D 74, 074501 (2006)]

- scale is \( Q^2 = 4 \text{ GeV}^2 \)

- Standard practice to fit first coefficient of “asymptotic expansion” to moment

\[ \varphi_{\pi}(x, Q^2) = 6x(1-x) \left[ 1 + \sum_{n=2, 4, \ldots} a_n^{3/2}(Q^2) C_n^{3/2}(2x - 1) \right] \]

- however this expansion is guaranteed to converge rapidly only when \( Q^2 \to \infty \)
- method results in a double-humped pion PDA – not supported by BSE WFs

- Advocate using a generalized expansion

\[ \varphi_{\pi}(x, Q^2) = N_\alpha x^\alpha (1-x)^\alpha \left[ 1 + \sum_{n=2, 4, \ldots} a_n^{\alpha+1/2}(Q^2) C_n^{\alpha+1/2}(2x - 1) \right] \]

Find \( \varphi_{\pi} \simeq x^\alpha (1-x)^\alpha, \ \alpha = 0.35^{+0.32}_{-0.24} \); good agreement with DSE: \( \alpha \sim 0.52 \)
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Pion PDA from Lattice QCD – updated

Most recent lattice QCD moment:

\[ \int_0^1 dx \ (2x - 1)^2 \varphi_\pi(x) = 0.2361 (41) (39) (?) \]


DSE prediction:

\[ \int_0^1 dx \ (2x - 1)^2 \varphi_\pi(x) = 0.251 \]

Near complete agreement between DSE prediction and latest lattice QCD result

Conclude that the pion PDA is a broad concave function

double humped distributions are very likely for the pion
Direct, symmetry-preserving computation of pion form factor predicts maximum in $Q^2 F_\pi(Q^2)$ at $Q^2 \approx 6$ GeV$^2$

Magnitude of this product is determined by strength of DCSB at all accessible scales

The QCD prediction can be expressed as

$$Q^2 F_\pi(Q^2) \quad Q^2 \gg \Lambda_{\text{QCD}}^2 \quad 16 \pi f_\pi^2 \alpha_s(Q^2) w_\pi^2; \quad w_\pi = \frac{1}{3} \int_0^1 dx \frac{1}{x} \varphi_\pi(x)$$

Find consistency between the direct pion form factor calculation and the QCD hard-scattering formula – if DSE pion PDA is used

15% disagreement may be explained by higher order/higher-twist corrections

**Predict that QCD power law behavior – with QCD’s scaling law violations – sets in at** $Q^2 \sim 8$ GeV$^2$  

[Featured in 2015 NP Long Range Plan]
Pion Elastic Form Factor

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**Pion TMD from its LFWFs**

- DCSB results in broad pion LFWFs at hadronic scales
- this is reflected in DSE and lattice result for pion’s PDA

- Using pion’s LFWFs straightforward to make predictions for pion GPDs, TMDs, etc; For TMDs:

\[
f(x, k_T^2) \propto |\psi_{\uparrow\downarrow}(x, k_T^2)|^2 + k_T^2 |\psi_{\uparrow\uparrow}(x, k_T^2)|^2
\]

- Contrast our result with Pasquini & Schweitzer [PRD 90 014050 (2014)]

  - each result gives similar PDF but very different TMD
  
  - illustration of the potential for TMDs to differentiate between different frameworks & thereby expose quark-gluon dynamics in QCD
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Conclusion

Spin-1 targets present a rich quark and gluon structure that can help expose novel aspects of QCD

- find that TMDs associated with tensor polarization are sensitive to orbital angular momentum in target
- $\rho$ meson results a stepping stone to deuteron calculations

Find that because of DCSB pion’s LFWFs are broad and concave in $x$ – results have perturbative power-law behavior for large $k_T^2$

- find that PDFs can not distinguish between vastly different LFWFs
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Backup Slides
QCD Evolution & Asymptotic PDA

ERBL ($Q^2$) evolution for pion PDA [c.f. DGLAP equations for PDFs]

$$\mu \frac{d}{d\mu} \varphi(x, \mu) = \int_0^1 dy \, V(x, y) \varphi(y, \mu)$$

This evolution equation has a solution of the form

$$\varphi_\pi(x, Q^2) = 6 \, x \, (1 - x) \left[ 1 + \sum_{n=2, 4, \ldots} a_n^{3/2}(Q^2) C_n^{3/2}(2x - 1) \right]$$

$\alpha = 3/2$ because in $Q^2 \to \infty$ limit QCD is invariant under the collinear conformal group $SL(2; \mathbb{R})$

Gegenbauer-$\alpha = 3/2$ polynomials are irreducible representations $SL(2; \mathbb{R})$

The coefficients of the Gegenbauer polynomials, $a_n^{3/2}(Q^2)$, evolve logarithmically to zero as $Q^2 \to \infty$: $\varphi_\pi(x) \to \varphi_\pi^{\text{asy}}(x) = 6 \, x \, (1 - x)$

At what scales is this a good approximation to the pion PDA?

E.g., AdS/QCD find $\varphi_\pi(x) \sim x^{1/2} \, (1 - x)^{1/2}$ at $Q^2 = 1 \, \text{GeV}^2$; expansion in terms of $C_n^{3/2}(2x - 1)$ convergences slowly: $a_{32}^{3/2} / a_2^{3/2} \sim 10\%$
When is the Pion’s PDA Asymptotic

Under leading order $Q^2$ evolution the pion PDA remains broad to well above $Q^2 > 100 \text{ GeV}^2$, compared with $\varphi^{\text{asy}}_{\pi}(x) = 6x(1-x)$

Consequently, the asymptotic form of the pion PDA is a poor approximation at all energy scales that are either currently accessible or foreseeable in experiments on pion elastic and transition form factors

Importantly, $\varphi^{\text{asy}}_{\pi}(x)$ is only guaranteed be an accurate approximation to $\varphi_{\pi}(x)$ when pion valence quark PDF satisfies: $q^\pi_v(x) \sim \delta(x)$

This is far from valid at foreseeable energy scales
When is the Pion’s Valence PDF Asymptotic

LO QCD evolution of momentum fraction carried by valence quarks

\[
\langle x q_v(x) \rangle (Q^2) = \left( \frac{\alpha_s(Q^2)}{\alpha_s(Q^2_0)} \right)^\gamma_{qq}^{(0)2} / (2\beta_0) \langle x q_v(x) \rangle (Q^2_0)
\]

where \( \frac{\gamma_{qq}^{(0)2}}{2\beta_0} > 0 \)

- therefore, as \( Q^2 \to \infty \) we have \( \langle x q_v(x) \rangle \to 0 \) implies \( q_v(x) \propto \delta(x) \)
- At LHC energies valence quarks still carry 20% of pion momentum
  - the gluon distribution saturates at \( \langle x g(x) \rangle \sim 55\% \)

**Asymptotia is a long way away!**
To observe onset of perturbative power law behaviour – to differentiate from a monopole – optimistically need data at 8 GeV$^2$ but likely also at 10 GeV$^2$

- this is a very challenging task experimentally

- Scaling predictions are valid for both spacelike and timelike momenta

- timelike data show promise as the means of verifying modern predictions
Rigorously included transverse momentum of diquark correlations in TMDs

This has numerous consequences:

- scalar diquark correlations greatly increase $\langle k_T^2 \rangle$
- find deviation from Gaussian anzatz and that TMDs do not factorize in $x$ & $k_T^2$
- diquark correlations introduce a significant flavor dependence in the average $\langle k_T^2 \rangle$ [analogous to the quark-sector electromagnetic form factors]

Work is also underway for nucleon GPDs, WACS, etc
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