PARTON OAM: EXPERIMENTAL LEADS

QCD EVOLUTION 2017
MAY 22-26, 2017
JEFFERSON LAB

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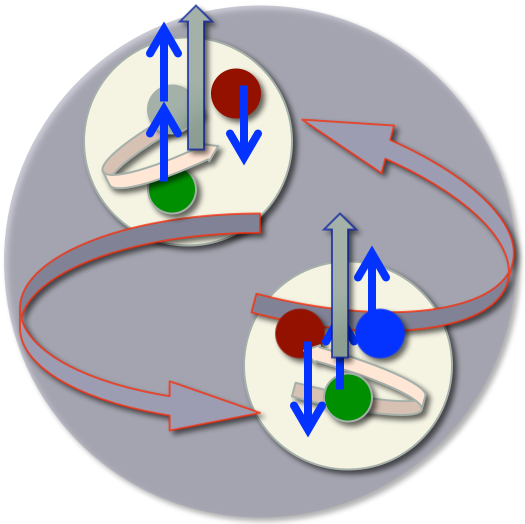
University of Virginia



Based on

PHYSICAL REVIEW D **94,** 034041 (2016)

Parton transverse momentum and orbital angular momentum distributions

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The quark orbital angular momentum component of proton spin, L_q, can be defined in QCD as the integral of a Wigner phase space distribution weighting the cross product of the quark's transverse position and momentum. It can also be independently defined from the operator product expansion for the off-forward Compton amplitude in terms of a twist-three generalized parton distribution. We provide an explicit link between the two definitions, connecting them through their dependence on partonic intrinsic transverse momentum. Connecting the definitions provides the key for correlating direct experimental determinations of L_q and evaluations through lattice QCD calculations. The direct observation of quark orbital angular momentum does not require transverse spin polarization but can occur using longitudinally polarized targets.

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…and A. Rajan et al., to be posted

Outline

1. Introduction
2. Definitions
3. Lorentz Invariant Relations ➔OAM is given by a twist three distribution
4. Equations of Motion Relations
5. A probe of QCD at the amplitude level
6. Process dependence of OAM distributions/universality
7. Conclusions

1. INTRODUCTION

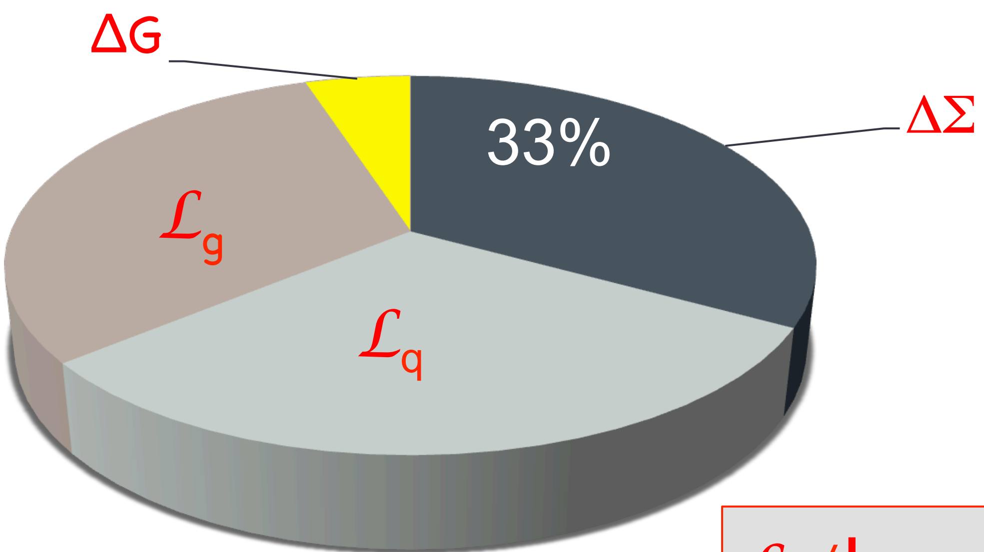
The spin crisis in a cartoon

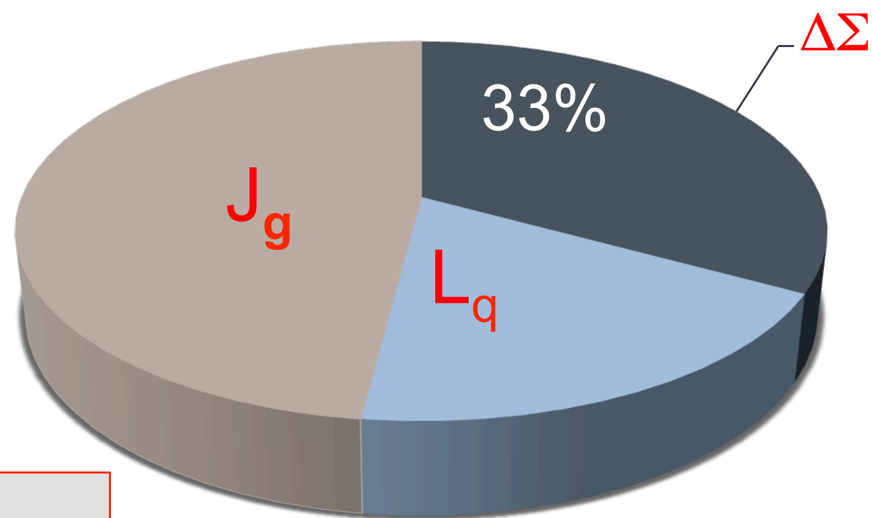
Jaffe Manohar

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

Ji

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$$

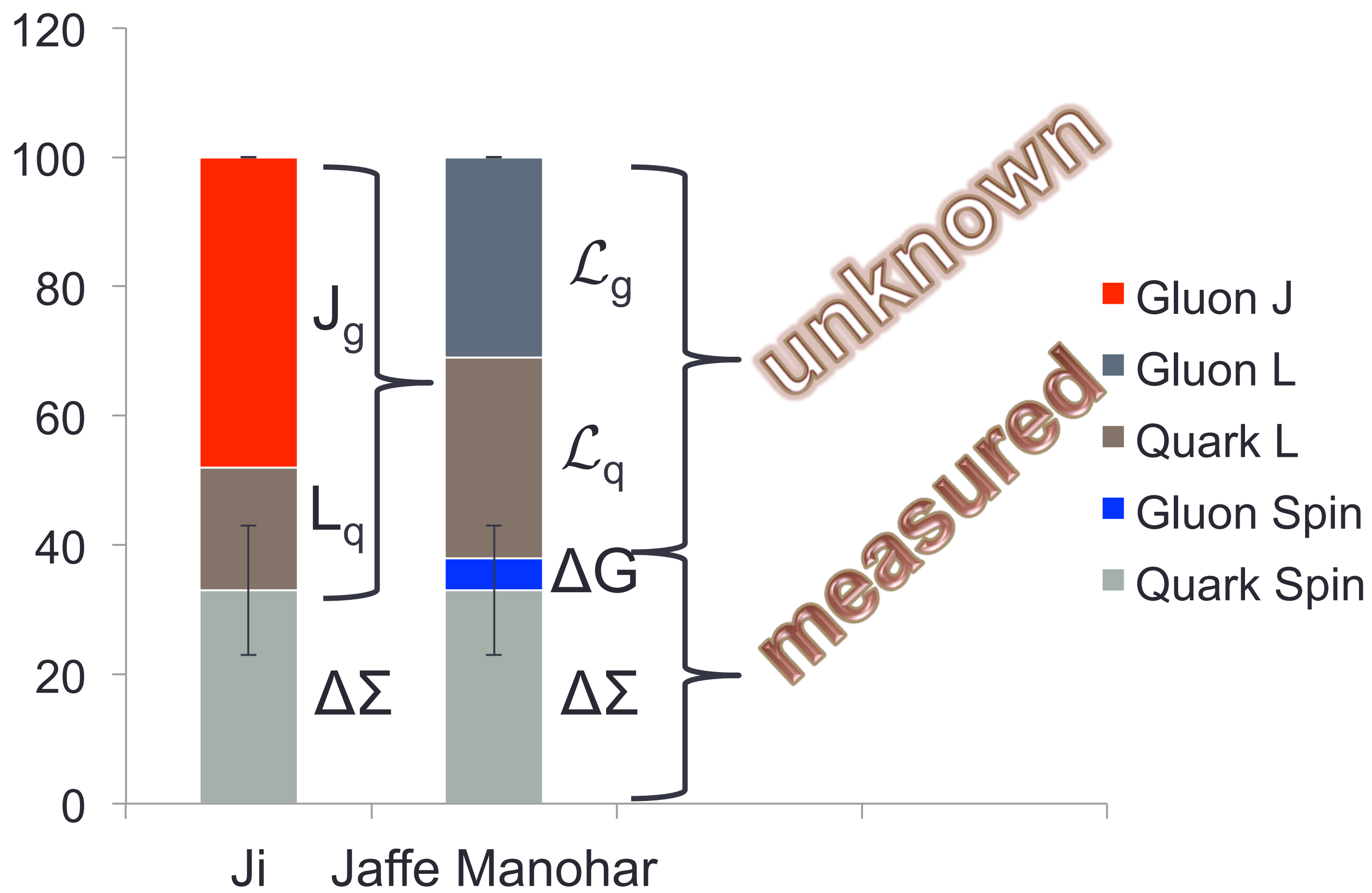




$\mathcal{L}_q \neq L_q$

$J_g \neq \mathcal{L}_g + \Delta G$

Angular Momentum Budget



The Starting Point

Jaffe and Manohar's field theoretical description of the quark and gluon orbital angular momentum through its relation to the QCD Energy Momentum Tensor,

$T^{\mu\nu} \rightarrow \boxed{M^{\mu\nu\lambda} = x^{\nu}T^{\mu\lambda} - x^{\lambda}T^{\mu\nu}}$ **Angular Momentum density**

Energy density (mass)

Shear stress

Momentum density

$$\begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}$$

Pressure

In QCD

$$\boxed{T^{\mu\nu} = \frac{1}{4} i q \bar{\psi}\left(\gamma^{\mu}\overleftrightarrow{D}^{\nu} + \gamma^{\nu}\overleftrightarrow{D}^{\mu}\right)\psi + Tr\left\{F^{\mu\alpha}F^{\nu}_{\alpha} - \frac{1}{2} g^{\mu\nu} F^{2}\right\}}$$

Jaffe Manohar: *

$$\boxed{M^{+12} = \psi^{\dagger}\sigma^{12}\psi + \psi^{\dagger}\left[\vec{x}\times(-i\partial)\right]^{3}\psi + Tr(\varepsilon^{+-ij}F^{+j}A^{j}) + 2i\,TrF^{+j}(\vec{x}\times\partial)A^{j}}$$

$\Delta\Sigma$ $\quad$ $\mathcal{L}_q$ $\quad$ ΔG $\quad$ $\mathcal{L}_g$

Ji:

$$\boxed{M^{+12} = \psi^{\dagger}\,\sigma^{12}\,\psi + \psi^{\dagger}\left[\vec{x}\times\left(-i\vec{D}\right)\right]^{3}\psi + \left[\vec{x}\times\left(\vec{E}\times\vec{B}\right)\right]^{3}}$$

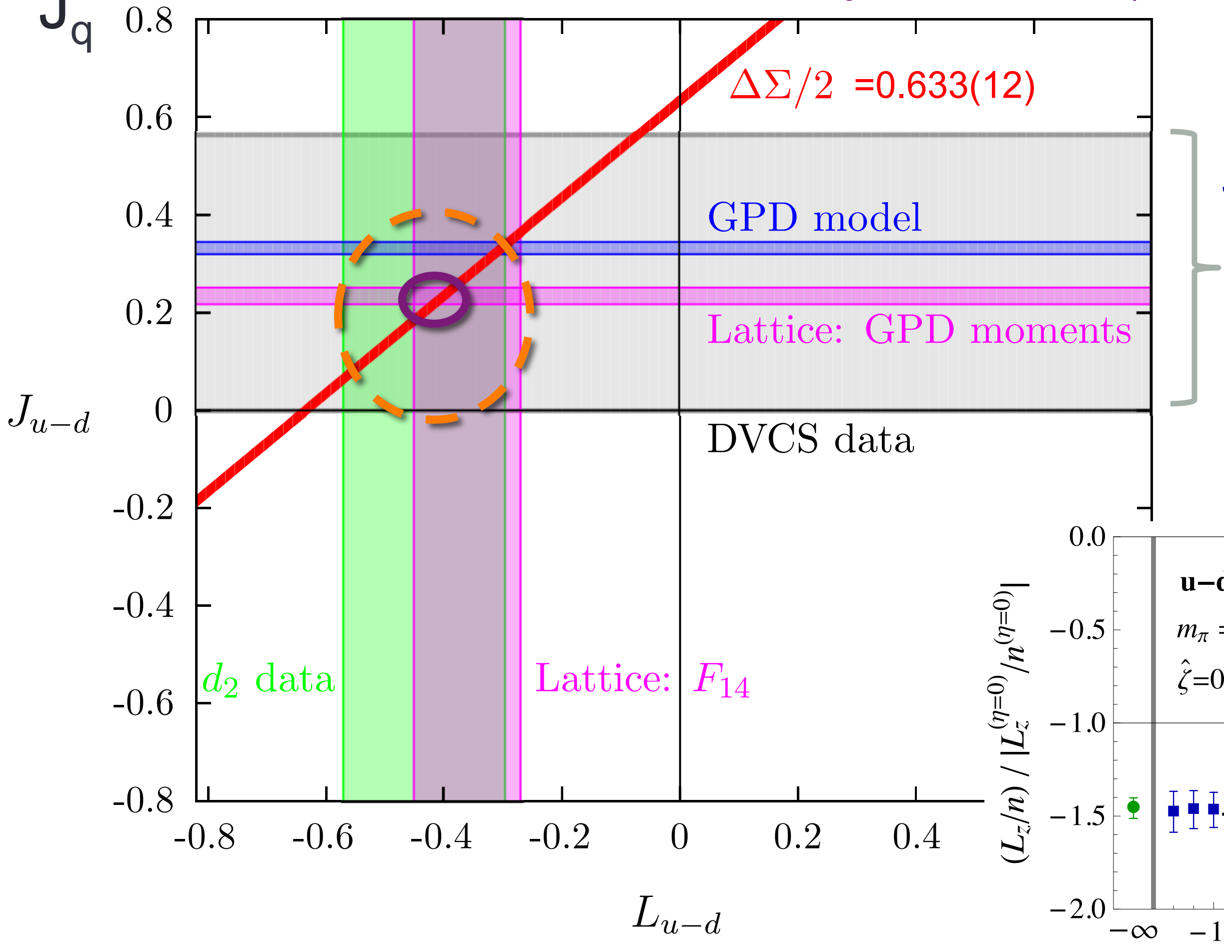
$$\underbrace{\Delta\Sigma \qquad L_q}_{J_q} \qquad J_g$$

*Chen, Goldman et al., are consistent with this definition (see K.F.Liu et al.)

Focus on quark sector : $J_q = L_q + \frac{1}{2}\Delta\Sigma_q$

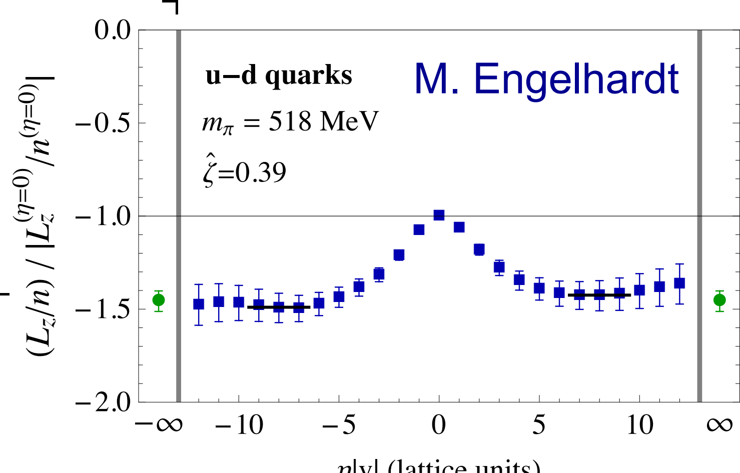
Validation of Sum Rule through three independent measurements

A. Rajan et al, PRD (2016) arXiv:1601.06117



JLAB, Mazous et al. PRL (2007)

arXiv:1701.01536



The first step towards an observable effect…

(Ji's) OAM is given by a twist 3 GPD $L_q(x)$

$$\underbrace{\frac{1}{M}\int d^2k_T\, k_T^2\, F_{14}(x,0,k_T^2,0,0)}_{k_T \text{ moment of a GTMD}} = -\int_x^1 dy\, \left[\widetilde{E}_{2T}(y,0,0) + \underbrace{H(y,0,0) + E(y,0,0)}_{\text{twist 3 GPD}}\right]$$

k_T moment of a GTMD
(Lorce, Pasquini, A. Mukherjee's talk)

twist 3 GPD
(Meissner Metz, Schlegel)

Spin Orbit interaction is given by a twist 3 GPD $(L_q \cdot S_q)(x)$

$$\frac{1}{M}\int d^2k_T\, k_T^2\, G_{11}(x,0,k_T^2,0,0) = \int_x^1 dy\, \left[2\tilde{H}'_{2T}(y,0,0) + E'_{2T}(y,0,0) + \tilde{H}(y,0,0)\right]$$

Obtained by studying the dynamics of GTMD's k_T dependence and twist 3 GPDs

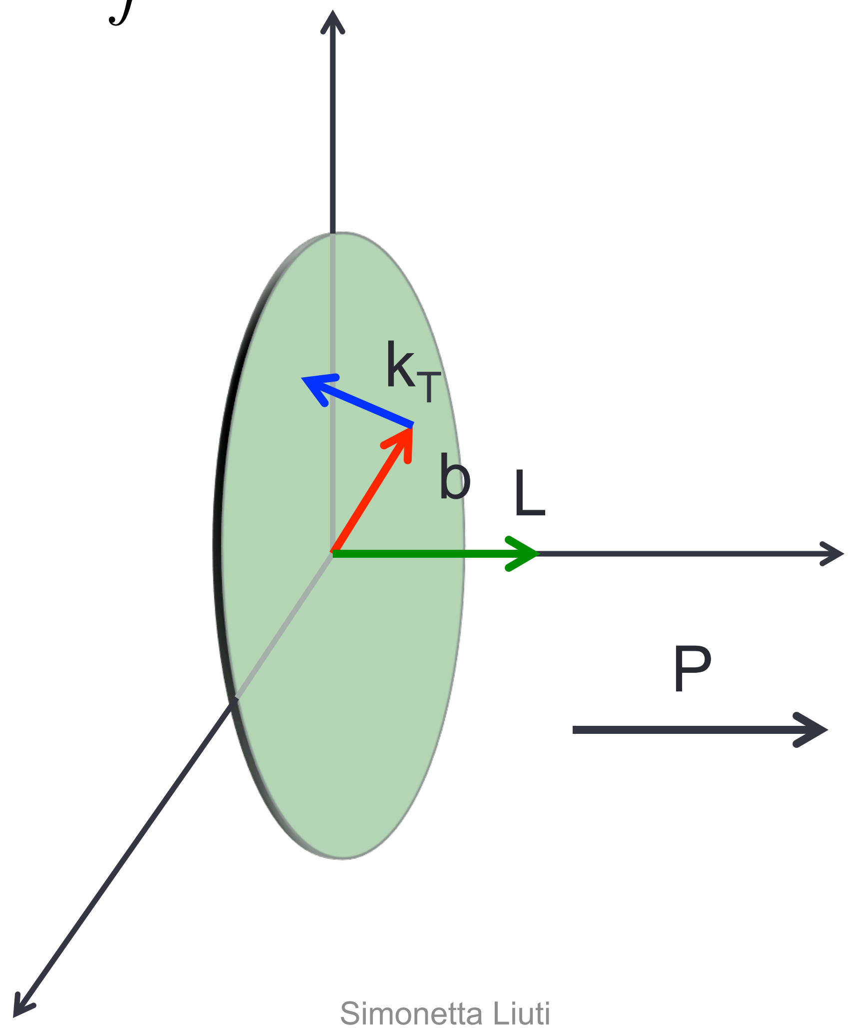
A. Rajan, A. Courtoy, M. Engelhardt, S.L., PRD (2016), arXiv:1601.06117

2. DEFINITIONS

Partonic OAM: Wigner Distributions

$$L_q^{\mathcal{U}} = \int dx \int d^2\mathbf{k}_T \int d^2\mathbf{b}\, (\mathbf{b} \times \mathbf{k}_T)_z\, \mathcal{W}^{\mathcal{U}}(x, \mathbf{k}_T, \mathbf{b})$$

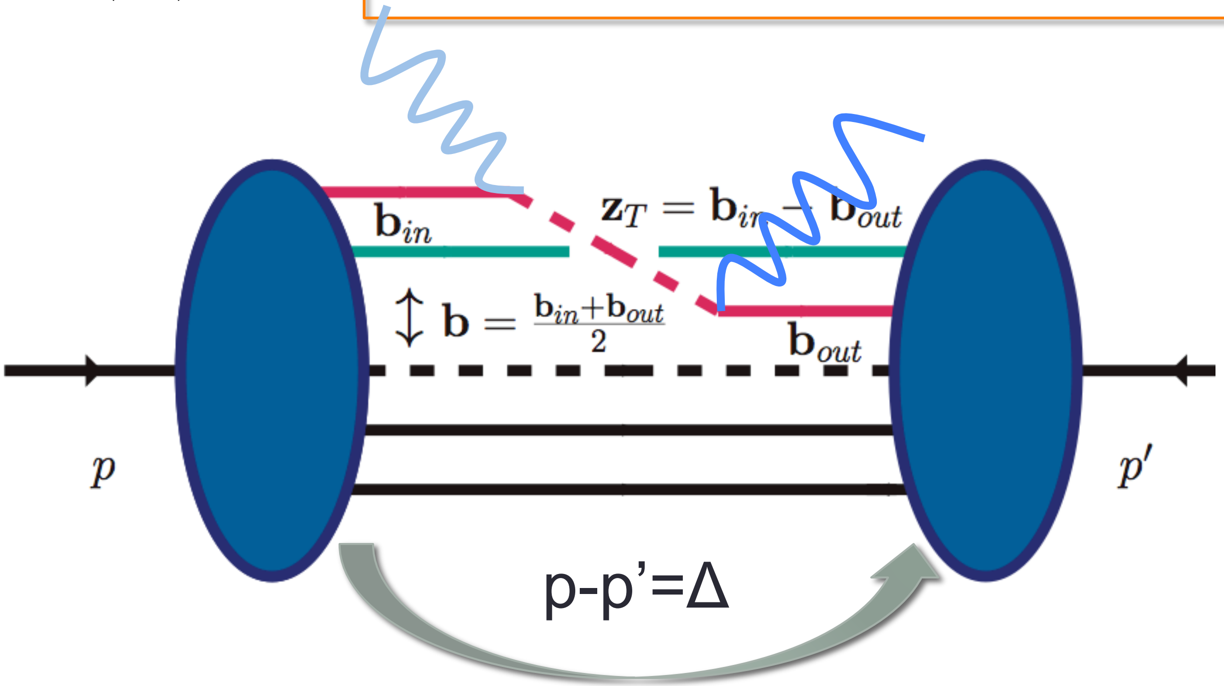
Hatta
Lorce, Pasquini,
Xiong, Yuan
Mukherjee



Wigner Distribution

$$\mathcal{W}^{\mathcal{U}} = \frac{1}{2}\int \frac{d^2\mathbf{\Delta}_T}{(2\pi)^2} e^{i\Delta_T\cdot b} \boxed{\int dz^- d^2\mathbf{z}_T\, e^{ikz} \langle P-\Delta, \Lambda' \mid \bar{q}(0)\gamma^+\mathcal{U}(0,z)q(z) \mid P, \Lambda\rangle \,|_{z^+=0}}$$

GTMD



- Δ_T Fourier conjugate: **b** = transverse position of the quark inside the proton
- k_T Fourier conjugate: $\mathbf{z}_T$= transverse distance traveled by the struck quark between the initial and final scattering

Which GTMD?

The quark-quark correlator for a spin ½ hadron has been parametrized up to twist four in terms of GTMDs, TMDs and GPDs, in a complete way in:

Generalized parton correlation functions for a spin-1/2 hadron

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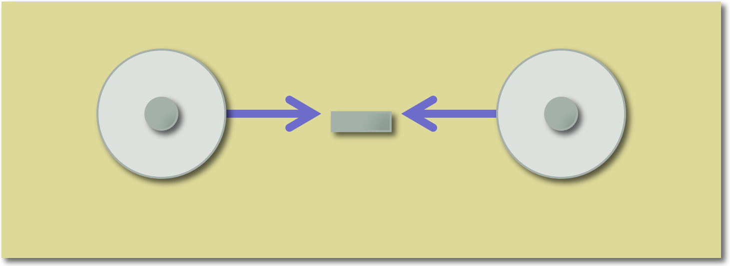
JHEP08(2009)

F_{14}

$$
\begin{aligned}
W^{\gamma^+}_{\Lambda\Lambda'} &= \frac{1}{2P^+}\overline{U}(p',\Lambda')\left[\gamma^+ F_{11} + \frac{i\sigma^{i+}\Delta^i_T}{2M}(2F_{13}-F_{11}) + \frac{i\sigma^{i+}\bar{k}^i_T}{2M}(2F_{12}) + \frac{i\sigma^{ij}\bar{k}^i_T\Delta^j_T}{M^2}F_{14}\right]U(p,\Lambda) \\
&= \delta_{\Lambda,\Lambda'}F_{11} + \delta_{\Lambda,-\Lambda'}\frac{-\Lambda\Delta_1 - i\Delta_2}{2M}(2F_{13}-F_{11}) + \delta_{\Lambda,-\Lambda'}\frac{-\Lambda\bar{k}_1 - i\bar{k}_2}{2M}(2F_{12}) + \delta_{\Lambda,\Lambda'}i\Lambda\frac{\bar{k}_1\Delta_2 - \bar{k}_2\Delta_1}{M^2}F_{14}
\end{aligned}
$$

helicity non-flip



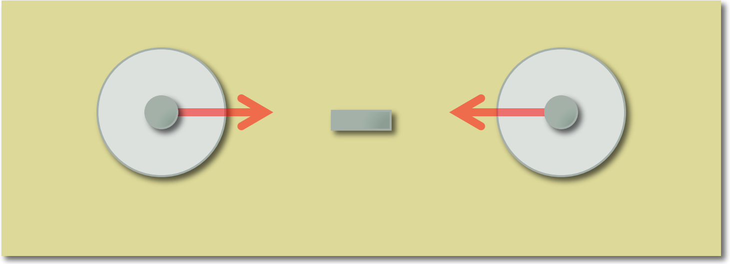
UL correlation: unpolarized quark density in a longitudinally polarized proton

G_{11} $(L \cdot S) \to \frac{1}{2}\int d^2\mathbf{b}\,(\mathbf{b}\times\mathbf{k}_T)_z\,\langle\bar{q}(0)\gamma^+\gamma_5 q(z)\rangle$

$$W^{\gamma^+\gamma_5}_{\Lambda\Lambda'} = \frac{1}{2M}\overline{U}(p',\Lambda')\left[-\frac{i\epsilon_T^{ij}k_T^i\Delta_T^j}{M^2}G_{11} + \frac{i\sigma^{i+}\gamma^5 k_T^i}{P^+}G_{12} + \frac{i\sigma^{i+}\gamma^5\Delta_T^i}{P^+}G_{13} + i\sigma^{+-}\gamma^5 G_{14}\right]U(p,\Lambda)$$

$$= \left[-\frac{i(k_1\Delta_2 - k_2\Delta_1)}{M^2}G_{11} + \Lambda G_{14}\right]\delta_{\Lambda\Lambda'} + \left[\frac{\Delta_1 + i\Lambda\Delta_2}{M}\left(G_{13} + \frac{i\Lambda(k_1\Delta_2 - k_2\Delta_1)}{2M^2}G_{11}\right) + \frac{k_1 + i\Lambda k_2}{M}G_{12}\right]\delta_{-\Lambda,\Lambda'}$$

$\boxed{\text{helicity non-flip}}$



UL correlation: longitudinally polarized quark density in an unpolarized proton

Integral relations

$$L_q = -\int_0^1 dx \int d^2k_T \, \frac{k_T^2}{M^2} \, F_{14} = -\int_0^1 dx \, F_{14}^{(1)}$$

$$L_q \cdot S_q = -\int_0^1 dx \int d^2k_T \, \frac{k_T^2}{M^2} \, G_{11} = -\int_0^1 dx \, G_{11}^{(1)}$$

Lorce, Pasquini, Xiong, Yuan
Hatta, Yoshida
Ji, Xiong, Yuan

2. LIR

Lorentz Invariance Relations (LIR)

(D. Pitonyak, M. Schlegel's talks)

- LIR in the off-forward sector: relations between twist-3 GPDs (➔PDFs) and k_T moments of GTMDs (➔TMDs)
- Based on the most general Lorentz invariant decomposition of the fully unintegrated quark-quark correlator
- LIRs are a consequence of there being a smaller number of independent unintegrated terms in the decomposition than the number of GTMDs

$$\Phi^{\mathcal{U}} = \int \frac{d^4 z}{(2\pi)^4} e^{i(k\cdot z)} \langle P', \Lambda' | \overline{\psi}(0) \gamma^+ \mathcal{U}(0,\infty | n) \psi(z) | P, \Lambda \rangle$$

➔ parametrized in terms of invariant functions A_1, A_2, …

(Meissner, Metz and Schlegel (2009), Mulders, Tangerman, Pijlman, Bacchetta….)

$$\tilde{\Phi}^{\mathcal{U}} = \int dk^- \Phi^{\mathcal{U}}$$

➔ parametrized in terms of invariant functions F_{11}, F_{12}, …, F_{21}, F_{22}…

Specifically, one finds the following relations (A. Rajan's talk)

tw 2

$$\begin{cases} \dfrac{k_T \cdot \Delta_T}{\Delta_T^2} F_{12} + F_{13} = 2P^+ \displaystyle\int dk^- \left(\frac{k_T \cdot \Delta_T}{\Delta_T^2} A_5 + A_6 - \frac{xP^2 - k \cdot P}{M^2} (A_8 + x A_9) \right) \\ F_{14} = 2P^+ \displaystyle\int dk^- \left(A_8 + x A_9 \right) \end{cases}$$

tw 3

$$\frac{k_T \cdot \Delta_T}{\Delta_T^2} F_{27} + F_{28} = 2P^+ \int dk^- \left(\frac{k_T \cdot \Delta_T}{\Delta_T^2} A_5 + A_6 + \frac{1}{M^2} \left(\frac{(k_T \cdot \Delta_T)^2}{\Delta_T^2} - k_T^2 \right) A_9 \right)$$

The OAM distribution function

$$\overbrace{F^{(1)}_{14} = -\int_x^1 dy\,(\tilde{E}_{2T} + H + E)}^{L_q(x)} \;\Rightarrow\; \overset{L_q}{\boxed{-L_q = \int_0^1 dx F^{(1)}_{14} = \int_0^1 dx\, x G_2}}$$

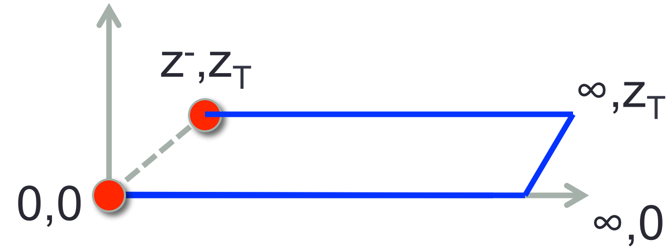
- F_{14} and $\tilde{E}_{2T}$ give us similar information on the distribution in x of OAM! new result
- In addition: we confirm and corroborate the global/integrated OAM result deducible from Ji et al

Different notation!

$$G_2 \rightarrow \tilde{E}_{2T} + H + E$$

Polyakov et al. Meissner, Metz and Schlegel, JHEP(2009)

Generalized LIR for a staple link



$$\frac{d}{dx}\int d^2k_T\,\frac{k_T^2}{M^2}\,F_{14} = \tilde{E}_{2T} + H + E + \mathcal{A}$$

LIR violating term

3. EQUATIONS OF MOTION

Equations of Motion (EoM) relation

Now insert the EoM in the correlator for a longitudinally polarized proton

$$\int \frac{dz^- d^2 z_T}{(2\pi)^3} e^{ixP^+ z^- - ik_T\cdot z_T} \langle p', \Lambda' \mid \overline{\psi}(-z/2)(\Gamma\, \mathcal{U}\, i\overrightarrow{\not{D}} + i\overleftarrow{\not{D}}\, \Gamma\, \mathcal{U})\psi(z/2) \mid p, \Lambda\rangle_{z^+=0} = 0$$

We find

$$\tilde{E}_{2T} = -\int_x^1 \frac{dy}{y}(H+E) + \left[\frac{\tilde{H}}{x} - \int_x^1 \frac{dy}{y^2}\tilde{H}\right] + \left[\frac{1}{x}\mathcal{M}_{F_{14}} - \int_x^1 \frac{dy}{y^2}\mathcal{M}_{F_{14}}\right]$$

$\int_0^1 dx \ldots$ relates to

L = J - S + 0

Consistent with OPE based relation

Polyakov et al.(2000), Hatta(2012)

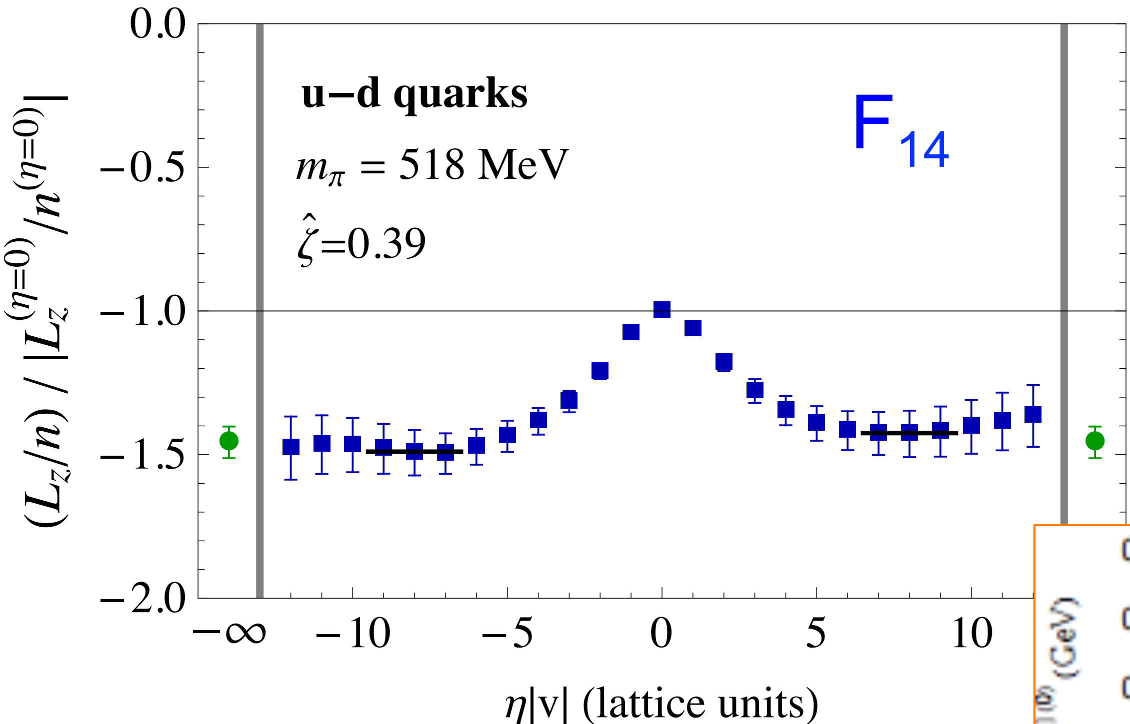
$$\int_0^1 dx\, xG_2 = -\frac{1}{2}\int_0^1 dx\, x(H+E) + \frac{1}{2}\int_0^1 dx\, \tilde{H}$$

$$\boxed{J_q = L_q + \frac{1}{2}\Delta\Sigma_q}$$

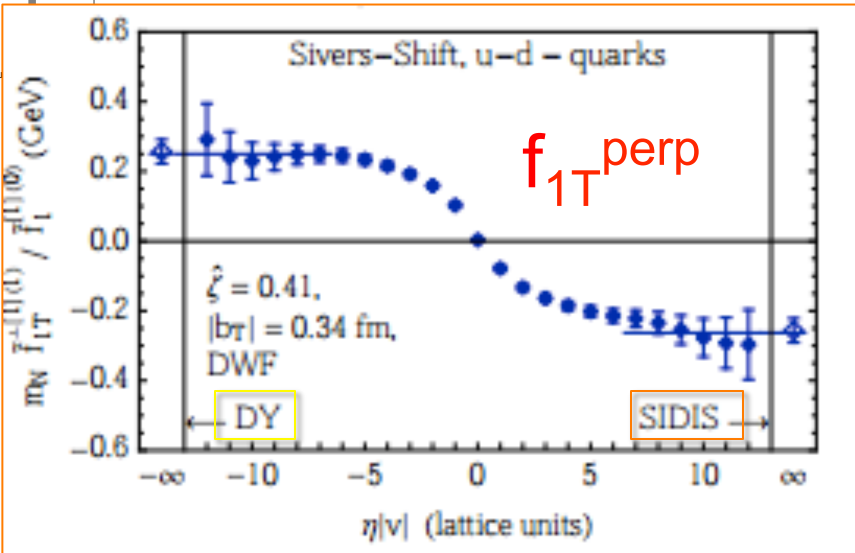
A generalized Wandzura Wilczeck relation obtained using OPE for twist 2 and twist 3 operators for the off-forward matrix elements

4. A PROBE OF QCD AT THE AMPLITUDE LEVEL

large effect from lattice (M. Engelhardt, arXiv:1701.01536)



PRD, arXiv:1111.4249



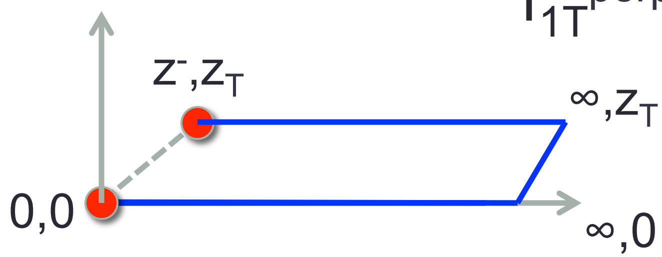
insight into non-perturbative aspects of QCD associated with dynamical chiral symmetry breaking

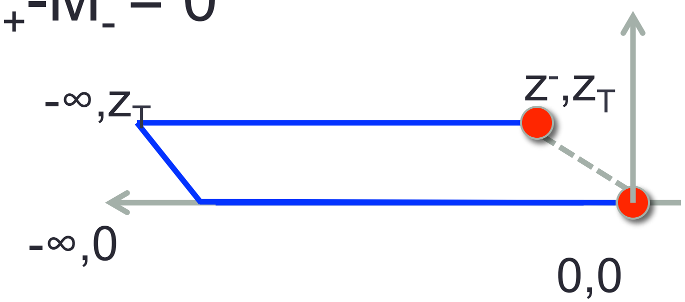
PT transformation

Forward case: Sivers function (J. Collins, 2002)

PT: $\underbrace{\langle P, S \mid \bar{\psi}(0)\gamma^{+}\psi(z) \mid P, S\rangle}_{M_+} = \underbrace{\langle P, -S \mid \bar{\psi}(0)\gamma^{+}\psi(z) \mid P, -S\rangle}_{M_-}$

$f_{1T}^{perp} = M_+ - M_- = 0$





PT:

$$\langle P, S \mid \bar{\psi}(0)\gamma^{+}U(v,z)\psi(z) \mid P, S\rangle = \langle P, -S \mid \bar{\psi}(0)\gamma^{+}U(-v,z)\psi(z) \mid P, -S\rangle$$

$M_+^{v} - M_-^{-v} = 0$ ⇒ $f_{1T}^{perp,SIDIS} = M_+^{v} - M_-^{v} = -f_{1T}^{perp,DY} = M_+^{-v} - M_-^{-v}$

Off forward case: F_{14}

PT:

$$\underbrace{\langle P-\Delta, S \mid \bar{\psi}(0)\gamma^{+}U(v,z)\psi(z) \mid P, S\rangle}_{L_{+}^{v,\Delta}} = \underbrace{\langle P, -S \mid \bar{\psi}(0)\gamma^{+}U(-v,z)\psi(z) \mid P-\Delta, -S\rangle}_{L_{-}^{-v,-\Delta}}$$

$L_{+}^{v,\Delta}-L_{-}^{-v,-\Delta} = 0$

$(k_T x \Delta_T)\ F_{14}^{\text{"SIDIS"}} = L_{+}^{v,\Delta}-L_{-}^{v,\Delta} = (k_T x \Delta_T)\ F_{14}^{\text{"DY"}} = L_{+}^{-v,\Delta}-L_{\_\_}^{v,\Delta}$

Genuine/intrinsic twist three term in Equation of Motion relation

$$\mathcal{M}^i_{\Lambda\Lambda'} = \frac{1}{4}\int \frac{dz^- d^2 z_T}{(2\pi)^3} e^{ixP^+z^- - ik_T\cdot z_T}$$

$$\langle p', \Lambda' \mid \overline{\psi}(-z/2) \boxed{\left[(\overrightarrow{\partial\!\!\!/} - ig A\!\!\!/)\mathcal{U}\Gamma\Big|_{-z/2} + \Gamma\mathcal{U}(\overleftarrow{\partial\!\!\!/} + ig A\!\!\!/)\Big|_{z/2} \right]} \psi(z/2) \mid p, \Lambda\rangle_{z^+=0}$$

$$-x\tilde{E}_{2T} = \tilde{H} - \int d^2 k_T \frac{k_T^2}{M^2} F_{14}^{\text{staple}} + \mathcal{M}_{F_{14}}^{\text{staple}}$$

$$-x\tilde{E}_{2T} = \tilde{H} - \int d^2 k_T \frac{k_T^2}{M^2} F_{14}^{\text{straight}} + \mathcal{M}_{F_{14}}^{\text{straight}}$$

$$\mathcal{A} = \frac{d}{dx}\left(\mathcal{M}^{\text{staple}} - \mathcal{M}^{\text{straight}}\right)$$

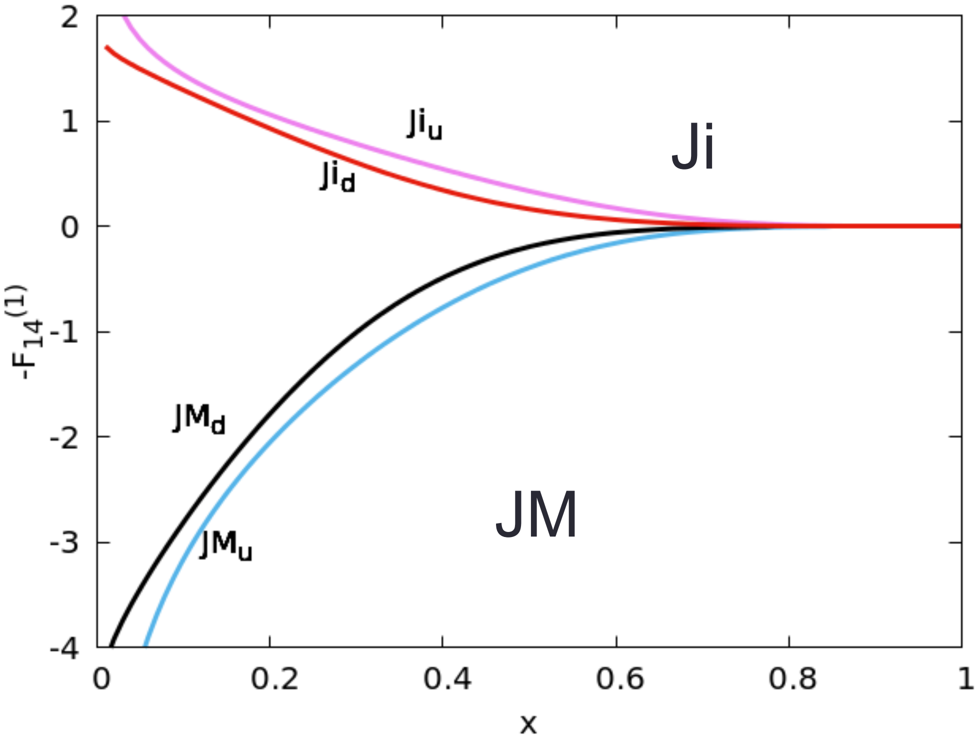
LIR violating term

Generalized Qiu Sterman term

$$\int d^2k_T\,\frac{k_T^2}{M^2}\,F_{14}^{JM} - \int d^2k_T\,\frac{k_T^2}{M^2}\,F_{14}^{Ji} = T_F(x,x,\Delta)$$

$F_{14}^{(1)}$

with B. Kriesten and A. Rajan, using diquark model

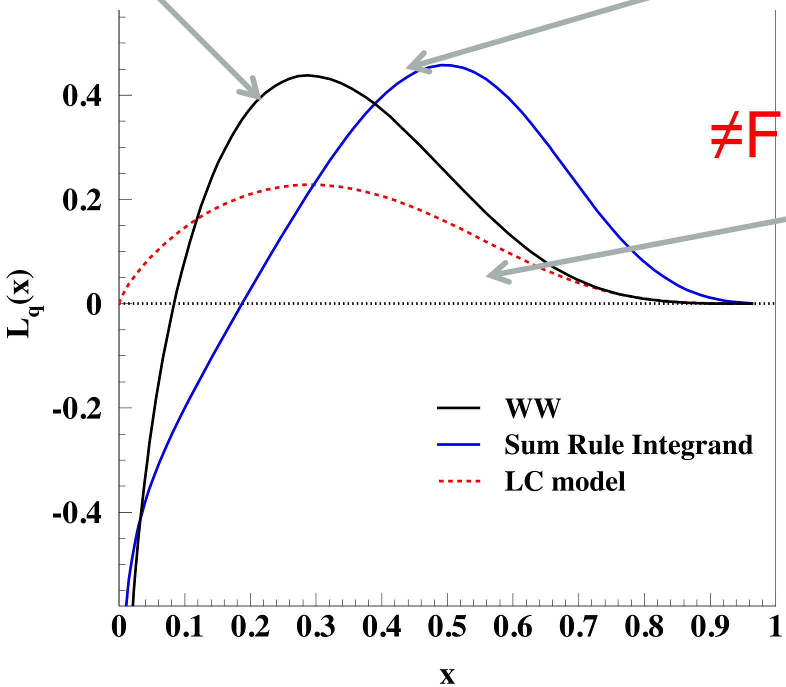


Ratio L_{Ji}/L_{JM}=0.72

Lattice Ratio L_{Ji}/L_{JM}=0.62±0.16(stat) (extrapolated at $\zeta=\infty$)

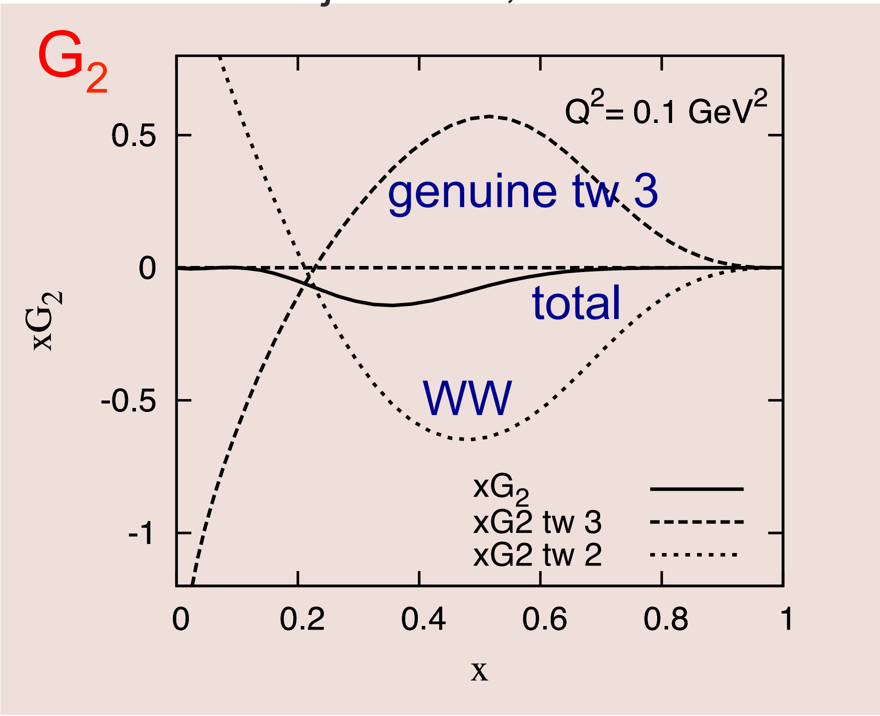
$$L_q(x,0,0) = x\int_x^1 \frac{dy}{y}(H_q(y,0,0)+E_q(y,0,0)) - x\int_x^1 \frac{dy}{y^2}\widetilde{H}_q(y,0,0),$$

Low x!



$\neq F_{14}$!

Abha Rajan et al., arXiv:1601.06117



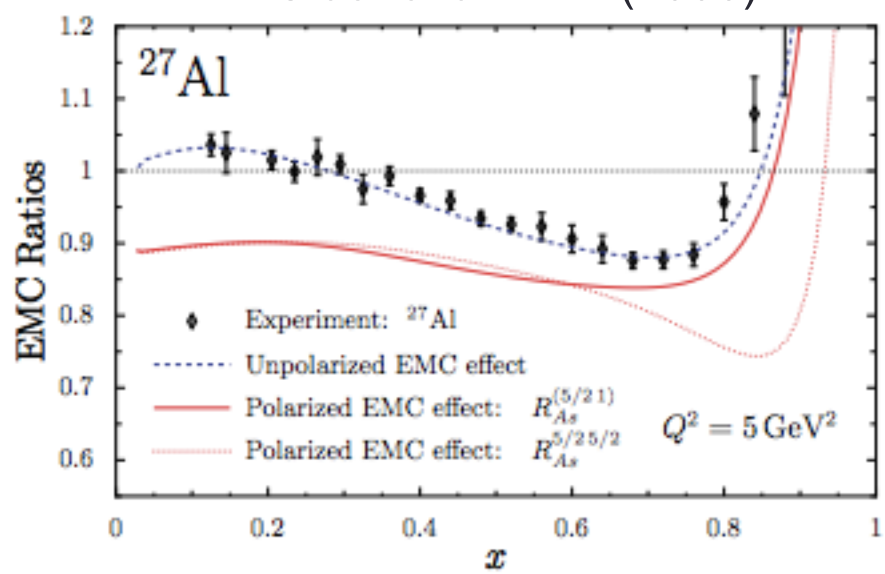
Finally, nuclei

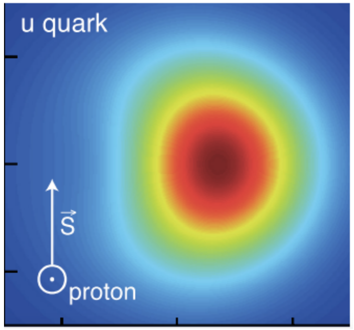
First Exclusive Measurement of Deeply Virtual Compton Scattering off ^{4}He: Toward the 3D Tomography of Nuclei

M. Hattawy,[1,2] N.A. Baltzell,[1,3] R. Dupré,[1,2,*] K. Hafidi,[1] S. Stepanyan,[3]
S. Bultmann,[4] R. De Vita,[5] A. El Alaoui,[1,6] L. El Fassi,[7] H. Egiyan,[3] F.X. Girod,[3]
M. Guidal,[2] D. Jenkins,[8] S. Liuti,[9] Y. Perrin,[10] B. Torayev,[4] and E. Voutier[10,2]
(The CLAS Collaboration)

The spin-orbit term can shed light on the polarized EMC effect
(work in progress with I. Cloet)

I. Cloet et al PLB (2006)





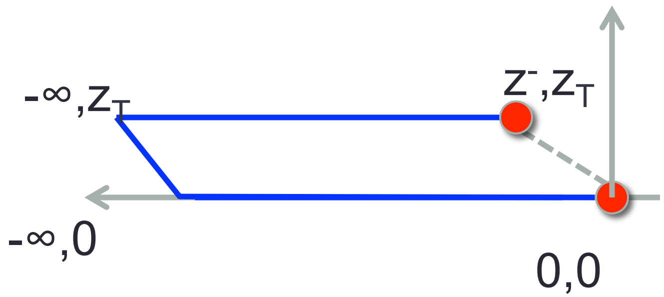
- <u>Spin and 3D structure of Deuteron</u>

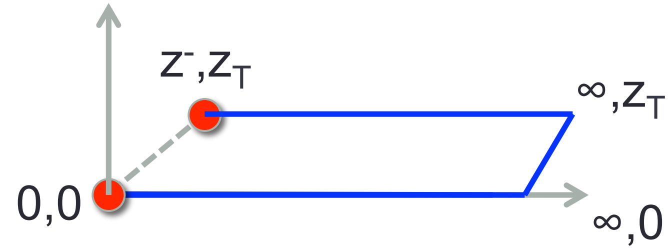
$$\frac{1}{2}\int_{-1}^{1} dx x\, [H_q(x,0,0) + E_q(x,0,0)] = J_q \longrightarrow \frac{1}{2}\int_{-1}^{1} dx x\, H_2^q(x,0,0) = J_q$$

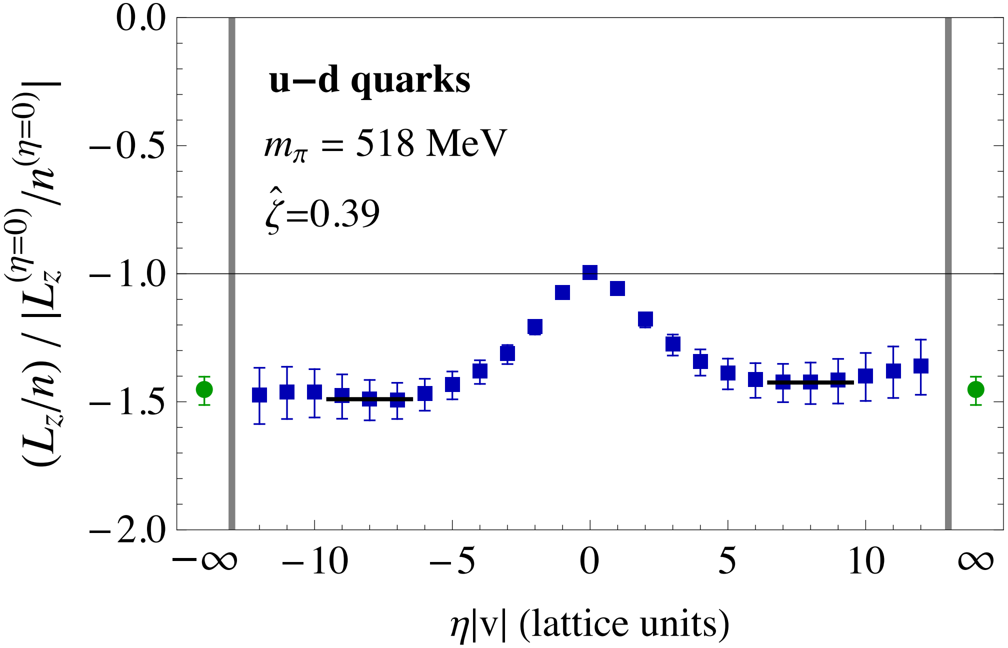
Nucleon (Ji, 1997)

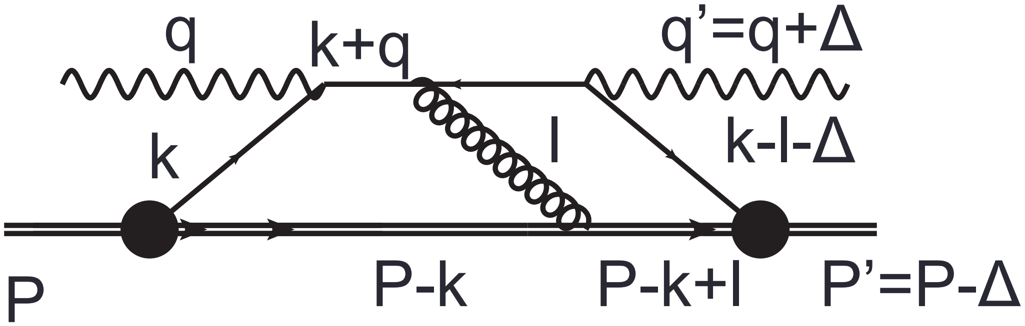
Deuteron (S.K. Taneja et al, 2012)

5. PROCESS DEPENDENCE


-∞,zT
z-,zT
∞,0
-∞,0
0,0

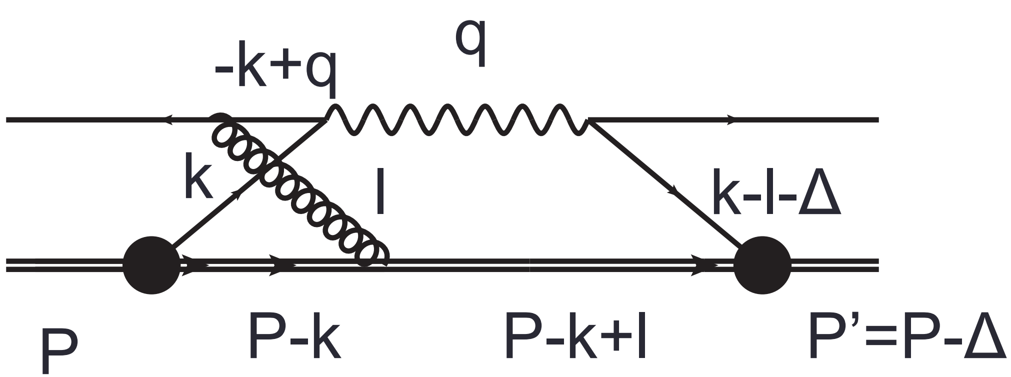

z-,zT
∞,zT
0,0
∞,0


u−d quarks
m_π = 518 MeV
$\hat{\zeta}$=0.39
$(L_z/n)\ /\ |L_z^{(\eta=0)}/n^{(\eta=0)}|$
0.0
−0.5
−1.0
−1.5
−2.0
−∞
−10
−5
0
5
10
∞
$\eta|v|$ (lattice units)



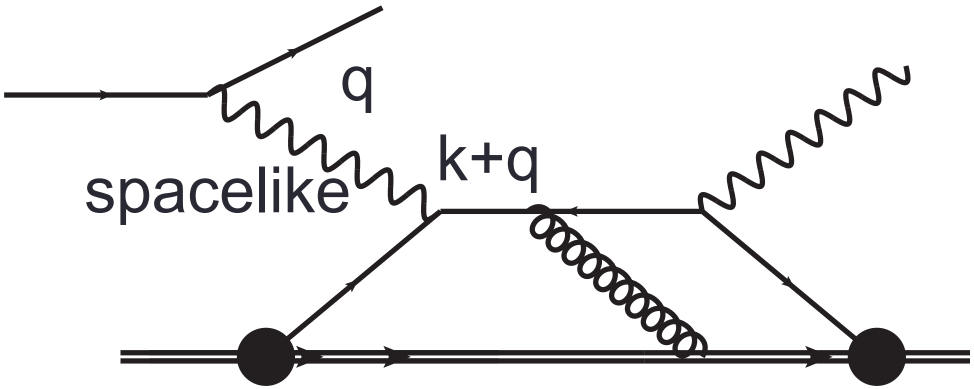
"SIDIS-like"

$$F_{1,4} = \int \frac{d^2 l}{(2\pi)^2} \frac{e_c^2 g_s^2 M^2 2P^+ (1-x)^2 \left(1 + \frac{l_T}{k_T}\cos\phi_l\right)}{2x(l_T^2 + m_g^2)((k-l)^2 - M_\Lambda^2)^2((k-\Delta)^2 - M_\Lambda^2)^2}$$

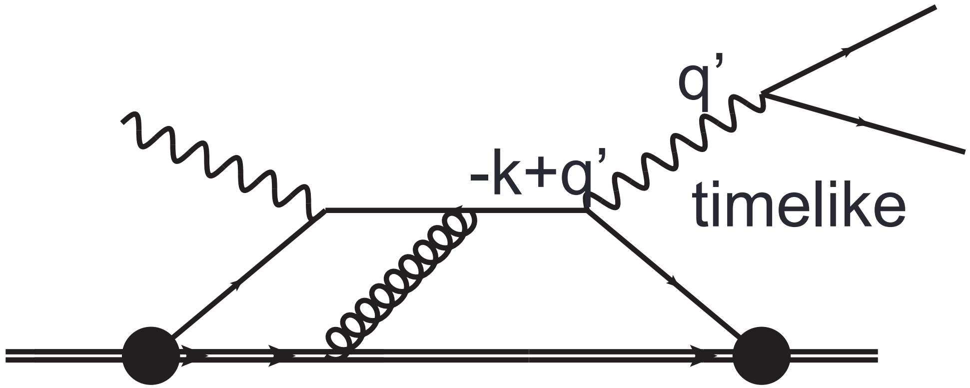


"DrellYan-like"

Two additional processes: DVCS and TCS twist three contributions



DVCS

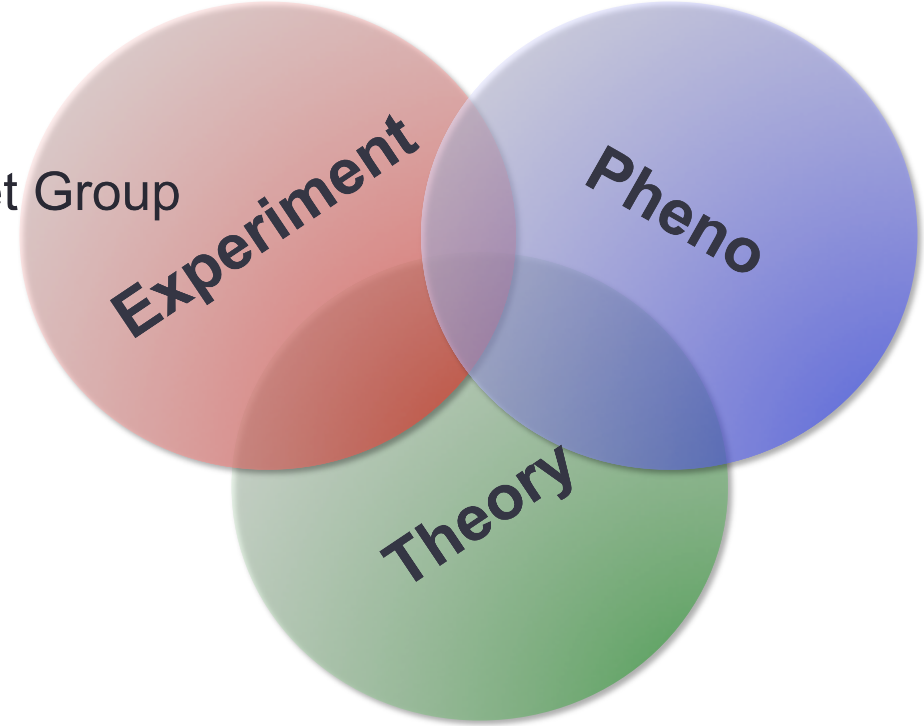


TCS

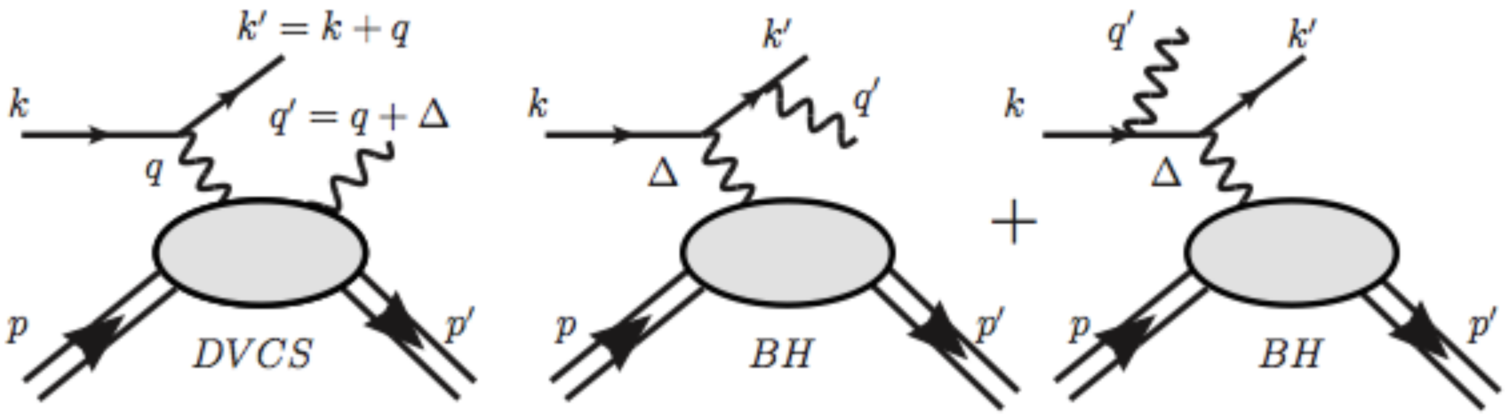
Extracting twist 3 GPDs from these processes will allow us to zoom into aspects of the "sign change"

How do we detect all this?

Dustin Keller & U.Va. Polarized Target Group



Deeply Virtual Exclusive Photoproduction



$$\frac{d^5\sigma}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} = \frac{\alpha^3}{16\pi^2(s-M^2)^2\sqrt{1+\gamma^2}}\left|T\right|^2 ,$$

$$T(k,p,k',q',p') = T_{DVCS}(k,p,k',q',p') + T_{BH}(k,p,k',q',p'),$$

From BKM formalism to “exact” Rosenbluth-like separation

Example 1

BH unpolarized cross section

$$\sigma_{BH} = \Gamma \left[A(y,t,\gamma,Q^2,\phi)\frac{F_1 + \tau F_2^2}{M^2} + B(y,t,\gamma,Q^2,\phi)\tau G_M^2(t) \right]$$

Example 2

DVCS unpolarized cross section

$$\sigma^{UU} = \frac{\Gamma}{Q^2(1-\epsilon)} \left[F_{UU,T} + \epsilon F_{UU,L} + \epsilon \cos 2\phi F_{UU}^{\cos 2\phi} + \sqrt{\epsilon(\epsilon+1)} \cos\phi F_{UU}^{\cos\phi} \right]$$

$$F_{UU,T} = 2(F^{11}_{++} + F^{11}_{+-} + F^{11}_{-+} + F^{11}_{--}),$$

Twist 2

$$F_{UU,L} = 2F^{00}_{++}$$

Twist 4

$$F_{UU}^{\cos\phi} = \mathrm{Re}\left[F^{01}_{++} + F^{01}_{--}\right]$$

Twist 3

$$F_{UU}^{\cos 2\phi} = \mathrm{Re}\left[F^{1-1}_{++} + F^{1-1}_{+-} + F^{1-1}_{-+} + F^{1-1}_{--}\right]$$

Photon helicity flip: transverse gluons

Helicity amplitudes

Virtual Photon helicities

$$F_{\Lambda\Lambda'}^{\Lambda_{\gamma^*}^{(1)}\Lambda_{\gamma^*}^{(2)}} = \sum_{\Lambda_{\gamma'}} \left(f_{\Lambda\Lambda'}^{\Lambda_{\gamma^*}^{(1)}\Lambda_{\gamma'}} \right)^* f_{\Lambda\Lambda'}^{\Lambda_{\gamma^*}^{(2)}\Lambda_{\gamma'}}$$

Initial and final proton helicities

Phase dependence

$$f \rightarrow e^{i\left[\Lambda_{\gamma*}-\Lambda_{\gamma'}-(\Lambda-\Lambda')\right]\phi}$$

The phase is determined by the virtual photon helicity which can be different for the amplitude and its conjugate

BASIC MODULE (based on helicity amplitudes)

$$\sum_{\Lambda'_\gamma,\,\Lambda'} \left(T^{h\Lambda'_\gamma}_{DVCS,\Lambda\Lambda'}\right)^* T^{h\Lambda'_\gamma}_{DVCS,\Lambda\Lambda'} =$$

$$\frac{1}{Q^2}\frac{1}{1-\epsilon}\left\{(F^{11}_{\Lambda+}+F^{11}_{\Lambda-}+F^{-1-1}_{\Lambda+}+F^{-1-1}_{\Lambda-})+\epsilon(F^{00}_{\Lambda+}+F^{00}_{\Lambda-})\right.$$

$$+2\sqrt{\epsilon(1+\epsilon)}\,\mathrm{Re}\,(-F^{01}_{\Lambda+}-F^{01}_{\Lambda-}+F^{0-1}_{\Lambda+}+F^{0-1}_{\Lambda-})+2\epsilon\,\mathrm{Re}\,(F^{1-1}_{\Lambda+}+F^{1-1}_{\Lambda-})$$

$$\boxed{\begin{aligned}&+(2h)\left[\sqrt{1-\epsilon^2}\,(F^{11}_{\Lambda+}+F^{11}_{\Lambda-}-F^{-1-1}_{\Lambda+}-F^{-1-1}_{\Lambda-})\right.\\ &\left.\left.-\;2\sqrt{\epsilon(1-\epsilon)}\,\mathrm{Re}\,(F^{01}_{\Lambda+}+F^{01}_{\Lambda-}+F^{0-1}_{\Lambda+}+F^{0-1}_{\Lambda-})\right]\right\}\end{aligned}}$$

polarized lepton

$$F^{11}_{++} = (1-\xi^2) \mid \mathcal{H} + \widetilde{\mathcal{H}} \mid^2 - \xi^2 \left[(\mathcal{H}^* + \widetilde{\mathcal{H}})^* (\mathcal{E} + \widetilde{\mathcal{E}}) + (\mathcal{H} + \widetilde{\mathcal{H}})(\mathcal{E}^* + \widetilde{\mathcal{E}}^*) \right]$$

$$F^{11}_{--} = (1-\xi^2) \mid \mathcal{H} - \widetilde{\mathcal{H}} \mid^2 - \xi^2 \left[(\mathcal{H}^* - \widetilde{\mathcal{H}})^* (\mathcal{E} - \widetilde{\mathcal{E}}) + (\mathcal{H} - \widetilde{\mathcal{H}})(\mathcal{E}^* - \widetilde{\mathcal{E}}^*) \right]$$

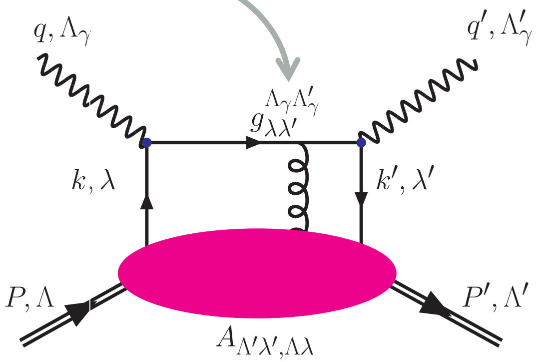
$$F^{11}_{+-} = \frac{t_0 - t}{4M^2} \mid \mathcal{E} + \xi \widetilde{\mathcal{E}} \mid^2$$

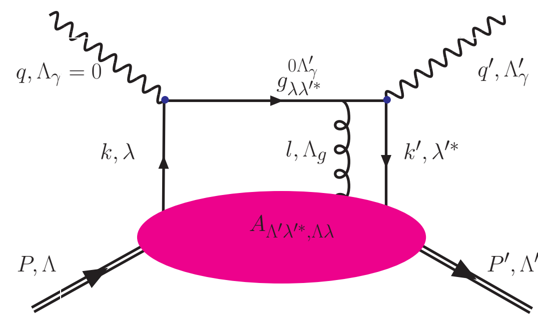
$$F^{11}_{-+} = \frac{t_0 - t}{4M^2} \mid \mathcal{E} - \xi \widetilde{\mathcal{E}} \mid^2$$

Twist 3

$$f^{01}_{\Lambda\Lambda'} = g^{01}_{-*+} \otimes A_{\Lambda'+,\Lambda-^*} + g^{01}_{-+^*} \otimes A_{\Lambda'+^*,\Lambda-} + g^{01}_{+^*-} \otimes A_{\Lambda'-,\Lambda+^*} + g^{01}_{+-^*} \otimes A_{\Lambda'-^*,\Lambda+}$$

"Bad" component (exchanged gluon flips the quark chirality)





Connecting the DVCS formalism with the TMD/GPD/GTMD comprehensive parametrizations

Bacchetta et al JHEP02 (2007), Meissner Metz and Schlegel, JHEP08 (2009)

Example

$$A_{+-,++^*} = \frac{1}{2}\left(\tilde{E}_{2T} - \overline{E}_{2T} + \tilde{E}'_{2T} + \overline{E}'_{2T}\right)$$

$$A_{+-^*,++} = \frac{1}{2}\left(-\tilde{E}_{2T} + \overline{E}_{2T} + \tilde{E}'_{2T} + \overline{E}'_{2T}\right)$$

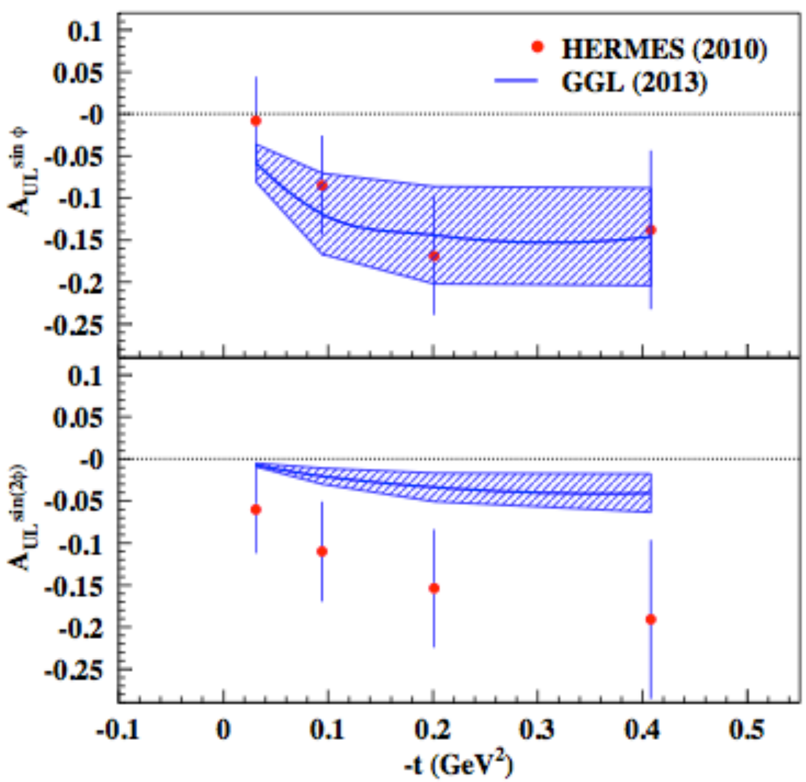
$$\vdots$$

Spin Orbit interaction

Orbital angular momentum

DVCS: bilinears of tw 2 and tw 3 CFFs

$$F^{01}_{++} = \mathcal{P}\left[\mathcal{H}^*(\tilde{\mathcal{E}}_{2T} - \bar{\mathcal{E}}_{2T} + ...), ...\right]$$

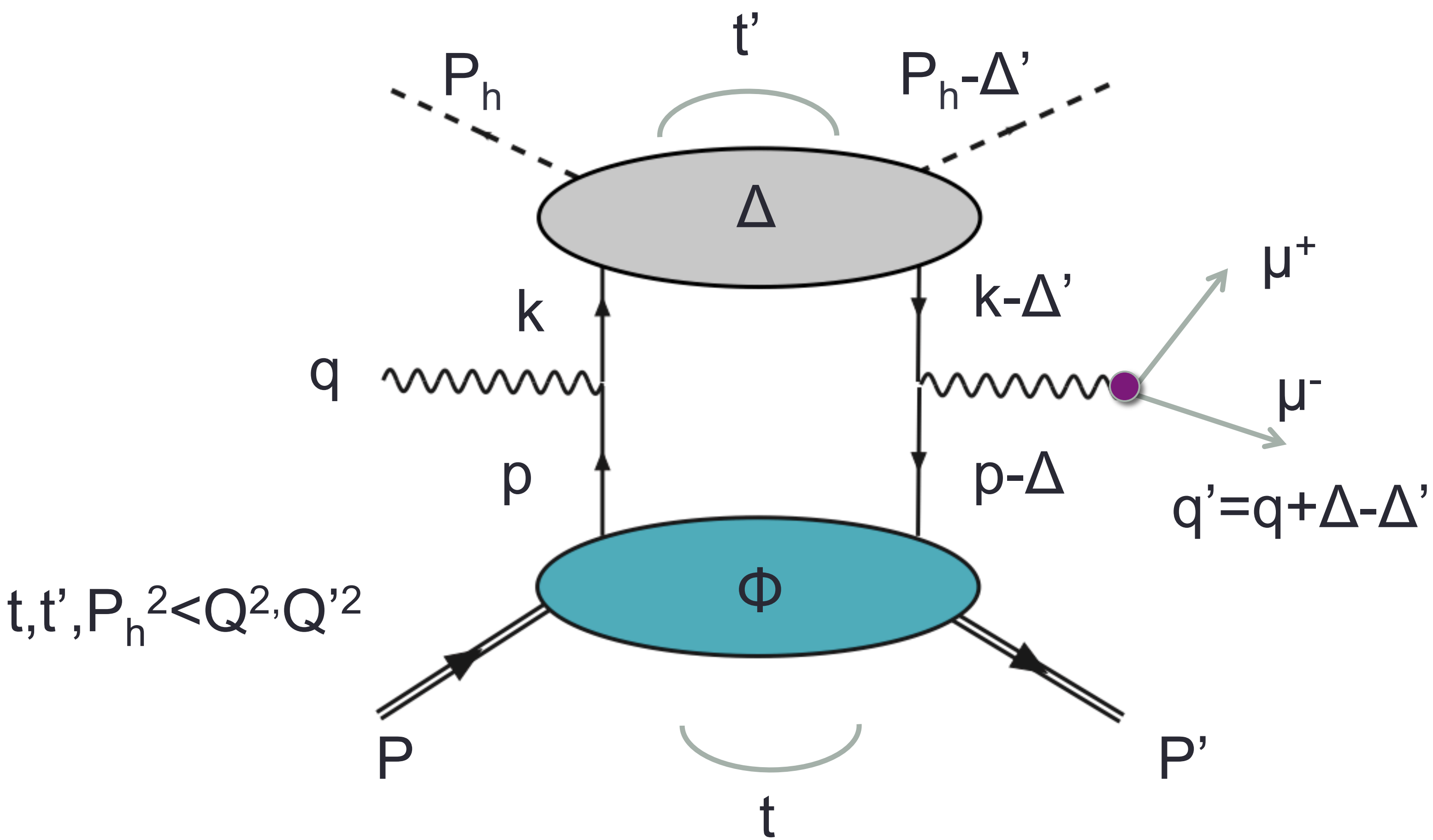


Extraction from experiment
using Wandzura Wilczek approximation

A.Courtoy, G.Goldstein, O.Gonzalez Hernandez, S.L. and A.Rajan, PLB 731(2014)

GTMDs from Double DVCS hadron production (off-forward SIDIS)

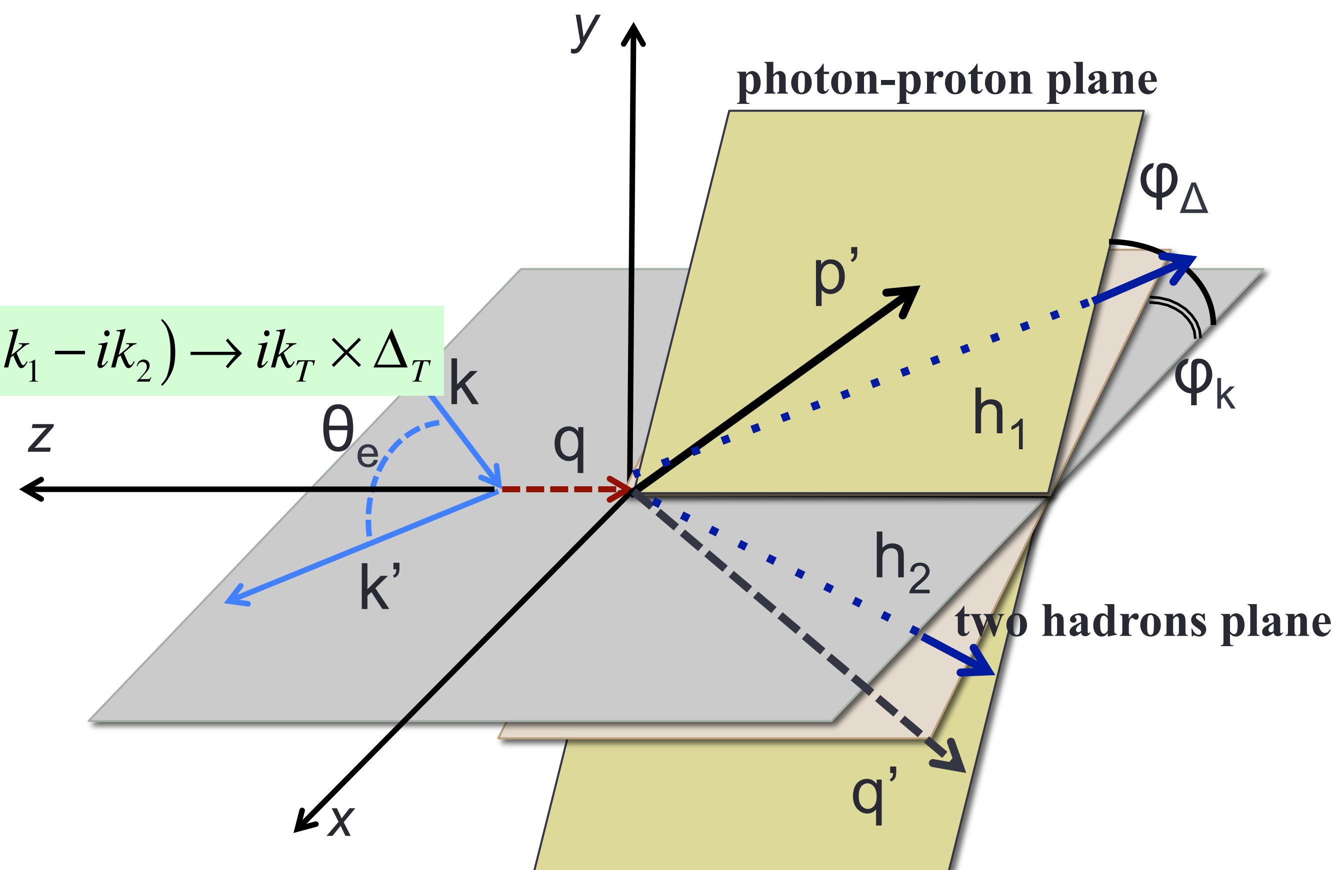
$$ep \to e'\pi^+\pi^-\mu^+\mu^-p'$$



Helicity amplitude formalism for DDVCS hadron production

- To measure F_{14} one has to be in a frame where the reaction cannot be viewed as a two-body quark-proton scattering
- In the CoM the amplitudes are imaginary ➔UL term goes to 0 unless one defines two hadronic planes

$$|\Delta| e^{i\varphi_\Delta} |k_T| e^{i\varphi_k} \left(\Delta_1 - i\Delta_2\right)\left(k_1 - ik_2\right) \to ik_T \times \Delta_T$$



Conclusions and Outlook

The connection we established through the new relations between (G)TMDs and Twist 3 GPDs, not only allows us to evaluate the angular momentum sum rule, it also opens many interesting avenues:

- It allows us to study in detail the role of quark-gluon correlations, in a framework where the role of k_T and off-shellness, k^2, is manifest.
- OAM was obtained so far by subtraction (also in lattice). We can now both calculate OAM on the lattice (GTMD) and validate this through measurements (twist 3 GPD)
- It provides an ideal setting to test renormalization issues, evolution etc…
- QCD studies at the amplitude level shed light on chiral symmetry breaking
- TWIST THREE GPDs ARE CRUCIAL TO STUDY QCD AT THE AMPLITUDE LEVEL

Back up

Helicity and Transverse Spin Structures of H+E

$$\left(A_{++,++}+A_{+-,+-}+A_{-+,-+}+A_{--,--}\right)+\left(A_{++,-+}+A_{+-,--}-A_{--,+-}-A_{-+,++}\right)$$

Helicity flips

=

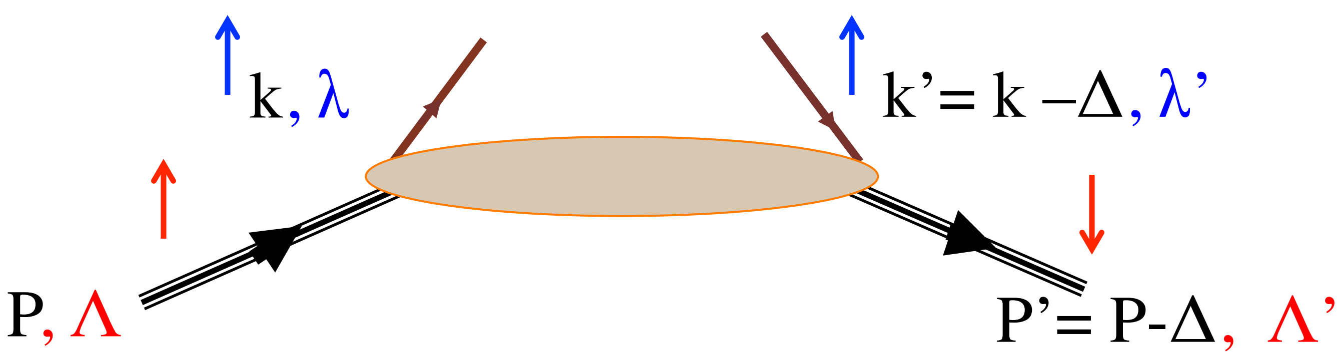
$$\left(A^X_{++,++}+A^X_{+-,+-}+A^X_{-+,-+}+A^X_{--,--}\right)+\left(A^X_{++,++}+A^X_{+-,+-}-A^X_{-+,-+}-A^X_{--,--}\right)$$

$$\approx H - i\Delta_2 E$$

Transv. spin conserved "non flip"

E measures J, not L, but a change of one unit of L (because of the helicity flip)

$$S_z = -1/2 \rightarrow 1/2 \Rightarrow \Delta L_z = 1$$ at fixed J



Brodsky and Drell '80s, Belitsky, Ji and Yuan, '90's