



*Ignazio Scimemi (UCM)*

## Analysis of DY and vector boson production in TMD formalism

arXiv:1706....

Most recent results in collaboration with  
**Alexey Vladimirov**



Universität Regensburg



# Resummation, Evolution, Factorization 2017



## ▸ Main Menu

- Presentation
- Scientific Program
- Registration
- Important Dates
- Participants
- Accommodation
- Venue and

## ▸ Presentation

**Dates: 13/11/2017-17/11/2017**

**REF 2017** is the 6th workshop in the series of workshops on **Resummation, Evolution, Factorization**.  
Previous discussion meetings and workshops were

7-10 November 2016 Antwerp (Belgium)

2-5 November 2015 DESY Hamburg (Germany)

1-3 June 2015 Amsterdam (The Netherlands)

8-11 December 2014 Antwerp (Belgium)



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# Outline & Issues

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- ❖ Resume of TMD factorization and unpolarized TMDs
- ❖ **Limits and goals of DY fitting**
- ❖ **Scale prescriptions, convergence, models, theoretical errors,..**
- ❖ The impact of LHC
- ❖ arXiv:1403.7093



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# ....TMD factorization ....

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.. for DY and heavy boson production we have (Collins 2011, Echevarria, Idilbi, Scimemi (EIS) 2012 )

$$\frac{d\sigma}{dQ^2 dq_T dy} = \sum_q \sigma_q^\gamma H(Q^2, \mu^2) \int \frac{d^2\mathbf{b}}{4\pi} e^{-i\mathbf{q}_T \cdot \mathbf{b}} \Phi_{q/A}(x_A, \mathbf{b}, \zeta_A, \mu) \Phi_{q/B}(x_B, \mathbf{b}, \zeta_B, \mu)$$

$$\sqrt{\zeta_A \zeta_B} = Q^2$$

...and similar formulas are valid for SIDIS (EIC) and hadron production in ee colliders

The pathological behavior is associated to a particular kind of divergences: rapidity divergences

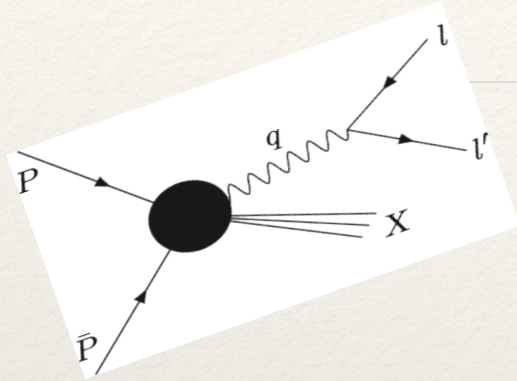
The renormalization of the rapidity divergences is responsible for the new resummation scale

We have new nonperturbative effects which cannot be included in PDFs.

**THE CASE OF UNPOLARIZED TMDs IS THE MOST STUDIED: FOR THE REST OF TMDs THE NNLO ERA IS JUST STARTED!**



## TMD's factorization and Operator Product Expansion: general outlook



Factorized hadronic tensor

$$q^2 = Q^2 \gg q_T^2$$

Q=M=di-lepton invariant mass

Factorization

$$q_T^2 \sim \Lambda_{QCD}^2$$



$$\tilde{M} = H(Q^2/\mu^2) \tilde{F}_n(x_n, b; Q^2, \mu^2) \tilde{F}_{\bar{n}}(x_{\bar{n}}, b; Q^2, \mu^2)$$

OPE

$$q_T^2 \gg \Lambda_{QCD}^2$$



$$\tilde{F}_n = \tilde{C}_{n \leftarrow j}(x_n, b, Q^2, \mu^2) \otimes f_{j \leftarrow h}(x_n, \mu^2) + \mathcal{O}(x_n b^2/B^2)$$

Very  
important

The factorization theorem predicts that each coefficient  
can be extracted on its own.

The evolution of TMD is universal (process independent)

Renormalons: power corrections are x-dependent

ALL THESE MATCHINGS ON COLLINEAR FUNCTIONS ARE JUST THE ASYMPTOTIC EXPANSION OF A  
MORE COMPLEX STRUCTURE: HOW CAN WE EXPLORE IT?



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# Status of unpolarized TMDs in perturbation theory

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- ❖ Evolution to N3LO Y. Li, H.X. Zhu, arXiv:1604.01404 A. Vladimirov, arXiv:1610.05791
- ❖ Soft function at NNLO M.G. Echevarría, I.S., A. Vladimirov, arXiv:1511.05590.
- ❖ NNLO coefficients for TMDPDFs M.G. Echevarría, I.S., A. Vladimirov, arXiv:1604.07869  
T. Lübbert, J. Oredsson, M. Stahlhofen, arXiv:1602.01829, T. Gehrmann, T. Lübbert, Li Lin Yang arXiv:1403.6451
- ❖ NNLO coefficients for TMD Fragmentation Functions M.G. Echevarría, I.S., A. Vladimirov, arXiv:1509.06392, arXiv:1604.07869

IT IS POSSIBLE TO MAKE A COMPLETE ANALYSIS OF UNPOLARIZED TMD IN DRELL-YAN AND SIDIS  
USING **NNLO** RESULTS

The study of polarized TMDs at the same precision is just started:

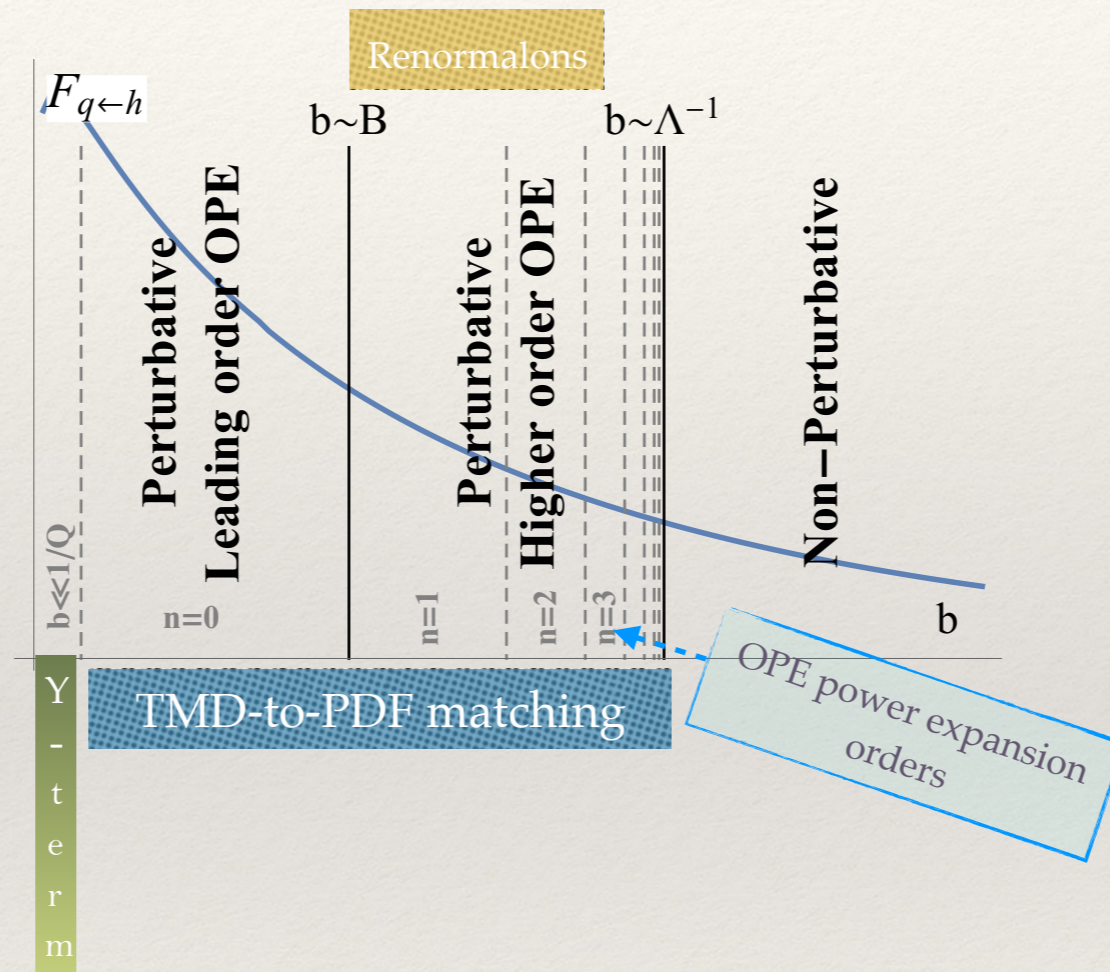
D. Gutierrez-Reyes, I.S., A. Vladimirov, arXiv:1702.06558

See also talk of T. Rogers



# Regions in $b$ -space

The factorization theorem works in  $b$ -space.  
 The perturbative expansion does not work on the whole space...



**EACH REGION NEEDS A PARTICULAR TREATMENT**

**NOT ALL REGIONS ARE EQUALLY IMPORTANT FOR EACH EXPERIMENT**



# TMD evolution

$$\mu^2 \frac{d}{d\mu^2} F_{f \leftarrow h}(x, \mathbf{b}; \mu, \zeta) = \frac{1}{2} \gamma_F^f(\mu, \zeta) F_{f \leftarrow h}(x, \mathbf{b}; \mu, \zeta),$$

$$\zeta \frac{d}{d\zeta} F_{f \leftarrow h}(x, \mathbf{b}; \mu, \zeta) = -\mathcal{D}^f(\mu, \mathbf{b}) F_{f \leftarrow h}(x, \mathbf{b}; \mu, \zeta),$$

$$\zeta \frac{d}{d\zeta} \frac{\gamma_F^f(\mu, \zeta)}{2} = \mu^2 \frac{d}{d\mu^2} (-\mathcal{D}^f(\mu, \mathbf{b})) = -\frac{\Gamma^f(\mu)}{2}$$

We have a double evolution in factorization and rapidity scales

$$F_{f \leftarrow h}(x, \mathbf{b}; \mu_f, \zeta_f) = R^f[\mathbf{b}; (\mu_f, \zeta_f) \leftarrow (\mu_i, \zeta_i)] F_{f \leftarrow h}(x, \mathbf{b}; \mu_i, \zeta_i)$$
$$R^f[\mathbf{b}; (\mu_f, \zeta_f) \leftarrow (\mu_i, \zeta_i)] = \exp \left[ \int_P \left( \gamma_F^f(\mu, \zeta) \frac{d\mu}{\mu} - \mathcal{D}^f(\mu, \mathbf{b}) \frac{d\zeta}{\zeta} \right) \right]$$

The evolution is **Path**  
independent only when all  
perturbative terms are included:  
Is there a best choice for initial  
and final scales?



# TMD evolution

The perturbative expression for the evolution kernel work only *up to a certain scale...*

$$R^f[\mathbf{b}; (\mu_f, \zeta_f) \leftarrow (\mu_i, \zeta_i); \mu_0] = \exp \left[ \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F^f(\mu, \zeta_f) - \int_{\mu_0}^{\mu_i} \frac{d\mu}{\mu} \Gamma^f(\mu) \ln \left( \frac{\zeta_f}{\zeta_i} \right) \right] \left( \frac{\zeta_f}{\zeta_i} \right)^{-\mathcal{D}_{\text{perp}}^f(\mu_0, \mathbf{b}) - g_K \mathbf{b}^2}.$$

...and in principle we include some (RENORMALON CONSISTENT) corrections

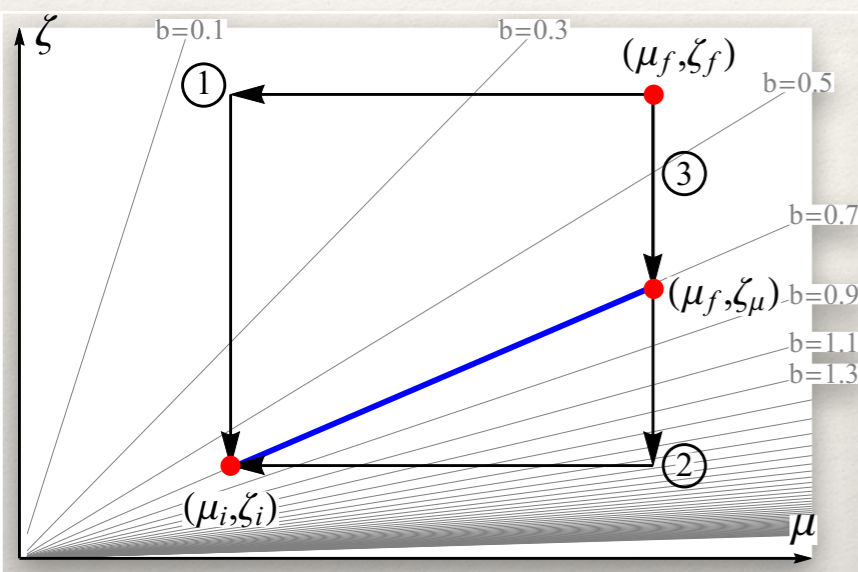
## What is the best prescription to choose scales?

*b\** prescription is not satisfactory:

- ❖ It is not consistent with renormalon calculations (I.S., A. Vladimirov 2016)
- ❖ It introduces undesired quadratic corrections (which alter model building)



# $\zeta$ -prescription



We choose  $\zeta = \zeta(\mu) \equiv \zeta_\mu$   
such that double logs are eliminated  
in PDF matching

$$C_{q \leftarrow q}(x, \mathbf{L}_\mu; \mu, \zeta) = \delta(\bar{x}) + a_s(\mu) C_F \left[ -2\mathbf{L}_\mu \left( \frac{2}{(1-x)_+} - 1 - x \right) + 2\bar{x} + \delta(\bar{x}) \left( -\mathbf{L}_\mu^2 + 2\mathbf{L}_\mu \mathbf{1}_\zeta - \frac{\pi^2}{6} \right) \right] \dots$$

In practice we implement..

$$\mu^2 \frac{dF(x, \mathbf{b}; \mu, \zeta_\mu)}{d\mu^2} = 0.$$

...and obtain iso-evolution curves..

$$\zeta_\mu = \frac{2\mu}{b} \exp \left( -\gamma_E + a_s \left[ \frac{11C_A - 4T_F N_f}{36} \mathbf{L}_\mu^2 + C_F \left( -\frac{3}{4} + \pi^2 - 12\zeta_3 \right) + C_A \left( \frac{649}{108} - \frac{17\pi^2}{12} + \frac{19}{2} \zeta_3 \right) + T_F N_f \left( -\frac{53}{27} + \frac{\pi^2}{3} \right) \right] + \mathcal{O}(a_s^2) \right).$$

See also the talk of V. Vaidya



# $\zeta$ -prescription

In this prescription the structure of coefficient is much simpler

$$C_{q \leftarrow q}(x, \mathbf{L}_\mu; \mu, \zeta_\mu) = \delta(\bar{x}) + a_s(\mu) C_F \left[ -2\mathbf{L}_\mu \left( \frac{2}{(1-x)_+} - 1 - x \right) + 2\bar{x} + \delta(\bar{x}) \left( -3\mathbf{L}_\mu - \frac{\pi^2}{6} \right) \right] + \dots$$

We do not introduce undesired power corrections

We have several proof of scale stability: TMD area, ...

$$\int_0^1 dx C_{q \leftarrow q}(x, \mathbf{L}_\mu; \mu, \zeta_\mu) = 1 + a_s(\mu) C_F \left( 1 - \frac{\pi^2}{6} \right) + \dots$$

Cancellation of logs

$$\mu^2 \frac{d}{d\mu^2} C_{f \leftarrow f'}(x, \mathbf{b}; \mu, \zeta_\mu) \otimes f_{f' \leftarrow h}(x, \mu) = 0$$

We are left with the freedom to choose

$$\mu = \mu_0 = \mu_b = \frac{C_0}{b} \sqrt{1 + \frac{b^2}{b_{max}^2}}$$



## Perturbative orders...

Name	$ C_V ^2$	$C_{f \leftarrow f'}$	$\Gamma$	$\gamma_V$	$\mathcal{D}$	PDF set	$a_s(\text{run})$	$\zeta_\mu$
NLL	$a_s^0$	$a_s^0$	$a_s^2$	$a_s^1$	$a_s^1$	nlo	nlo	NLL
NLO	$a_s^1$	$a_s^1$	$a_s^2$	$a_s^1$	$a_s^1$	nlo	nlo	NLO
NNLL	$a_s^1$	$a_s^1$	$a_s^3$	$a_s^2$	$a_s^2$	nnlo	nnlo	NNLL
NNLO	$a_s^2$	$a_s^2$	$a_s^3$	$a_s^2$	$a_s^2$	nnlo	nnlo	NNLO

**NEW!!**

## ...Theoretical uncertainties...

- Evolution factor
- Hard factorization scale
- TMD to PDF matching at small  $\underline{b}$

$$\mu_0 \rightarrow C_1 \mu_0$$

$$\mu_f \rightarrow C_2 \mu_f$$

$$\mu_i \rightarrow C_3 \mu_i$$



# DATA: Z-boson production....

	CDF run I	D0 run I
$\sqrt{s}$	1.8 TeV	1.8 TeV
process	$p + \bar{p} \rightarrow Z \rightarrow e^+e^-$	$p + \bar{p} \rightarrow Z \rightarrow e^+e^-$
$M_{ll}$ range	66-116 GeV	75-105 GeV
y	y-integrated	y-integrated
Observable	$\frac{d\sigma}{dq_T}$	$\frac{d\sigma}{dq_T}$
Exp. $\sigma_{\text{tot}}$ [pb]	$248 \pm 17$	$\sigma = 221 \pm 11$

	CDF run II	D0 run II
$\sqrt{s}$	1.96 TeV	1.96 GeV
process	$p + \bar{p} \rightarrow Z \rightarrow e^+e^-$	$p + \bar{p} \rightarrow Z \rightarrow e^+e^-$
$M_{ll}$ range	66-116 GeV	70-110 GeV
y	y-integrated	y-integrated
Observable	$\frac{d\sigma}{dq_T}$	$\frac{1}{\sigma} \frac{d\sigma}{dq_T}$
Exp. $\sigma_{\text{tot}}$ [pb]	$256 \pm 2.91$	$\sigma = 255$

	ATLAS	ATLAS
$\sqrt{s}$	7 TeV	8 TeV
process	$pp \rightarrow Z \rightarrow ee + \mu\mu$	$pp \rightarrow Z \rightarrow \mu\mu$
$M_{ll}$ range	66 - 116 GeV	66 - 116 GeV
lepton cuts	$p_T > 20$ GeV $ \eta  < 2.4$	$p_T > 20$ GeV $ \eta  < 2.4$
y	$-2.4 < y < 2.4$	$-2.4 < y < 2.4$
Observable	$\frac{1}{\sigma} \frac{d\sigma}{dq_T}$	$\frac{1}{\sigma} \frac{d\sigma}{dq_T}$

	CMS	CMS
$\sqrt{s}$	7 TeV	8 TeV
process	$pp \rightarrow Z \rightarrow ee + \mu\mu$	$pp \rightarrow Z \rightarrow \mu\mu$
$M_{ll}$ range	60-120 GeV	60-120 GeV
lepton cuts	$p_T > 20$ GeV $ \eta  < 2.1$	$p_T > 15$ GeV $ \eta  < 2.1$
y	$ y  < 2.1$	$ y  < 2.1$
Observable	$\frac{1}{\sigma} \frac{d\sigma}{dq_T}$	$\frac{1}{\sigma} \frac{d\sigma}{dq_T}$

	LHCb	LHCb	LHCb
$\sqrt{s}$	7 TeV	8 TeV	13 TeV
process	$pp \rightarrow Z \rightarrow \mu\mu$	$pp \rightarrow Z \rightarrow \mu\mu$	$pp \rightarrow Z \rightarrow \mu\mu$
$M_{ll}$ range	60-120 GeV	60-120 GeV	60-120 GeV
lepton cuts	$p_T > 20$ GeV $2 < \eta < 4.5$	$p_T > 20$ GeV $2 < \eta < 4.5$	$p_T > 20$ GeV $2 < \eta < 4.5$
y	$2 < y < 4.5$	$2 < y < 4.5$	$2 < y < 4.5$
Observable	$d\sigma(q_T)$	$d\sigma(q_T)$	$\frac{d\sigma}{dq_T}$
Norm. exp.	$\sigma = 76.0 \pm 3.1$ pb	$\sigma = 95.0 \pm 3.2$ pb	$\sigma = 198.0 \pm 13.3$ pb

**NEW!**



# DATA: and Drell-Yan...

**NEW!**

	E288 200	E288 300	E288 400
$\sqrt{s}$	19.4 GeV	23.8 GeV	27.4 GeV
process	p+Cu $\rightarrow \gamma \rightarrow \mu^+ \mu^-$	p+Cu $\rightarrow \gamma \rightarrow \mu^+ \mu^-$	p+Cu $\rightarrow \gamma \rightarrow \mu^+ \mu^-$
$Q$ range	4-9 GeV	4-9 GeV	5-14 GeV
$\Delta Q$ -bin	1 GeV	1 GeV	1 GeV
$y$	$y=0.4$	$y=0.21$	$y=0.03$
Observable	$E \frac{d^3\sigma}{d^3q}$	$E \frac{d^3\sigma}{d^3q}$	$E \frac{d^3\sigma}{d^3q}$

	ATLAS	ATLAS
$\sqrt{s}$	8 TeV	8 TeV
process	$pp \rightarrow Z/\gamma^* \rightarrow \mu\mu$	$pp \rightarrow Z/\gamma^* \rightarrow \mu\mu$
$M_{ll}$ range	46 - 66 GeV	116 - 150 GeV
lepton cuts	$p_T > 20$ GeV $ \eta  < 2.4$	$p_T > 20$ GeV $ \eta  < 2.4$
$y$	$-2.4 < y < 2.4$	$-2.4 < y < 2.4$
Observable	$\frac{1}{\sigma} \frac{d\sigma}{dq_T}$	$\frac{1}{\sigma} \frac{d\sigma}{dq_T}$

## Lepton cuts...

Lepton cuts have implemented numerically for LHC.

**However all experiments suffer from lepton cuts: they should always be reported!!**



# Normalization of the cross sections

Not all experiments provide a value for total cross sections:

- $N_{E288}=0.8$  fixed
  - For CDF, D0 we use DYNNLO
  - for LHC we normalize areas of partially integrated cross sections.  $N_{\text{th/exp}}$
- Generally good agreement, within errors

order	ATLAS Z-boson 7TeV	ATLAS Z-boson 8TeV	ATLAS 46-66 8TeV	ATLAS 116-150 8TeV	CMS 7TeV	CMS 8TeV	LHCb 7TeV	LHCb 8TeV	LHCb 13TeV
NLL	<i>0.80(1)</i>	0.79(1)	<b>0.86(1)</b>	<b>0.74(0)</b>	<i>0.83(1)</i>	<i>0.84(1)</i>	0.86(1)	0.87(1)	0.85(1)
NLO	<i>0.89(1)</i>	0.87(1)	<b>0.96(3)</b>	<b>0.81(1)</b>	<i>0.91(2)</i>	<i>0.93(2)</i>	0.95(2)	0.95(1)	<b>0.94(1)</b>
NNLL	<i>0.95(2)</i>	0.93(2)	<b>1.04(4)</b>	<b>0.86(1)</b>	<i>0.98(2)</i>	<i>1.00(2)</i>	<b>1.01(2)</b>	<b>1.01(2)</b>	<b>1.00(1)</b>
NNLO	<i>0.94(1)</i>	0.92(1)	<b>1.01(2)</b>	<b>0.85(1)</b>	<i>0.97(1)</i>	<i>0.99(1)</i>	<b>1.00(1)</b>	<b>1.01(1)</b>	<b>0.99(1)</b>



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# Models, data, stability

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Data are sensitive to models for non-perturbative part of TMDs.

We explore models with

- Minimal set of parameters
- renormalon consistency
- Independent on number of data points (Stability)
- We do not include Y-terms: we should select  $q_T/Q$  proper interval



To be checked on data!!





# Models fun

$$R^f[\mathbf{b}; (\mu_f, \zeta_f) \leftarrow (\mu_i, \zeta_i); \mu_0] = \exp \left[ \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F^f(\mu, \zeta_f) - \int_{\mu_0}^{\mu_i} \frac{d\mu}{\mu} \Gamma^f(\mu) \ln \left( \frac{\zeta_f}{\zeta_i} \right) \right] \left( \frac{\zeta_f}{\zeta_i} \right)^{-D_{\text{perp}}^f(\mu_0, \mathbf{b}) - g_K \mathbf{b}^2}$$

Renormalon for kernel

$$F_{q \leftarrow h}(x, \mathbf{b}; \mu, \zeta) = \int_x^1 \frac{dz}{z} \sum_f C_{q \leftarrow f}(z, \mathbf{b}; \mu, \zeta) f_{f \leftarrow h} \left( \frac{x}{z}, \mu \right) f_{NP}(z, \mathbf{b})$$

Non-perturbative corrections to TMD-PDF matching

$$f_{NP} = 1 \quad | \quad f_{NP} = e^{-\lambda_1 b^2} \quad | \quad f_{NP} = e^{-\lambda_1 b} \quad | \quad f_{NP} = 1, g_K \neq 0 \quad || \text{Basic (historical) ansatz}$$

Renormalon? It predicts  $zb^2$  corrections... I.S., A. Vladimirov 2016

**MODEL 1**

$$f_{NP}(\mathbf{b}) = e^{-\lambda_1 b} (1 + \lambda_2 b^2)$$

Ansatz in D'Alesio, Melis, I.S., M.G. Echevarria 2014

**MODEL 2**

$$f_{NP}(z, \mathbf{b}) = \exp \left( \frac{-\lambda_q z \mathbf{b}^2}{\sqrt{1 + z^2 \mathbf{b}^2 \frac{\lambda_q^2}{\lambda_1^2}}} \right) + \text{ren.}$$

**NEW** (renormalon consistent) ansatz!

$$\lambda_q = 10 \text{ GeV}^2, \text{ fixed}$$



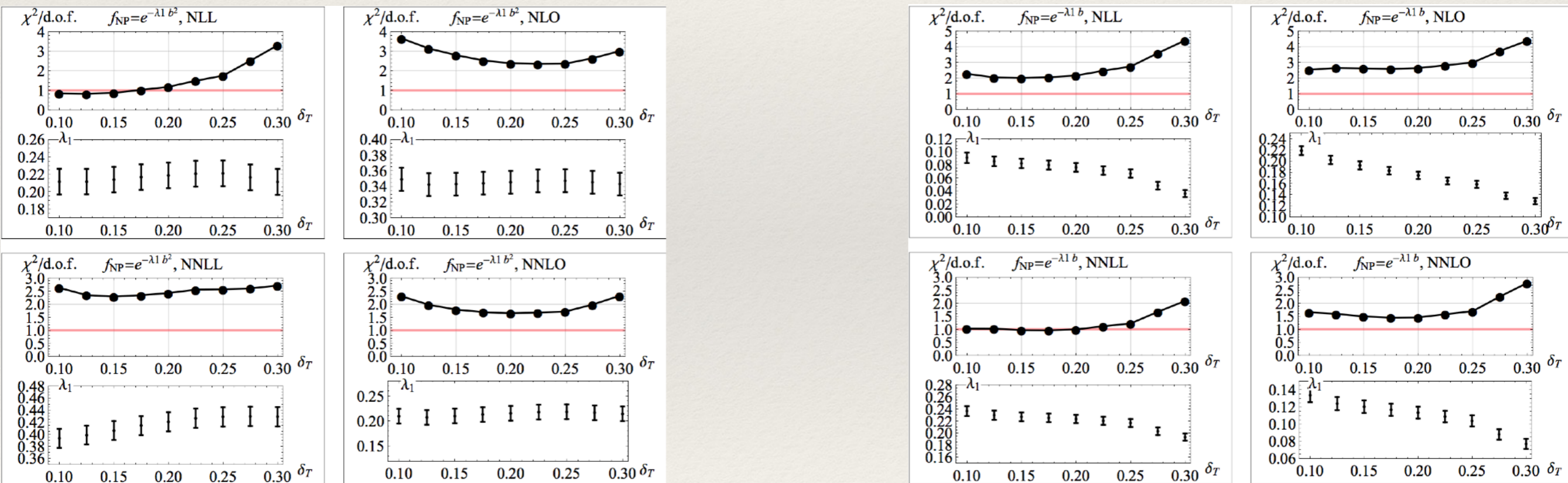


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# Stability

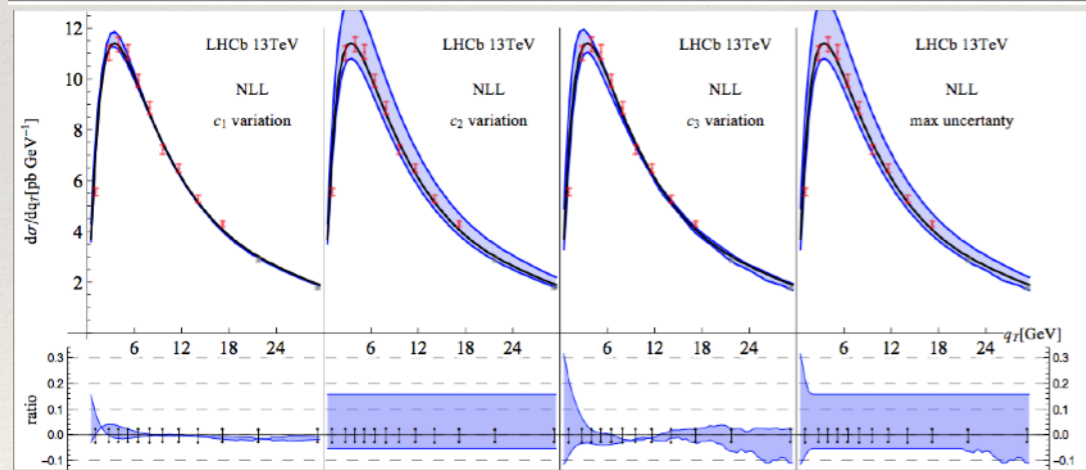
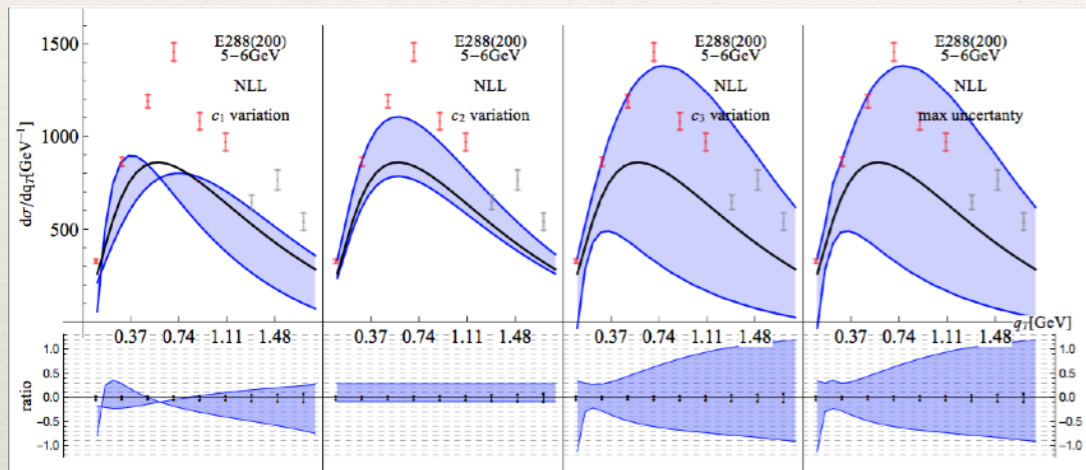
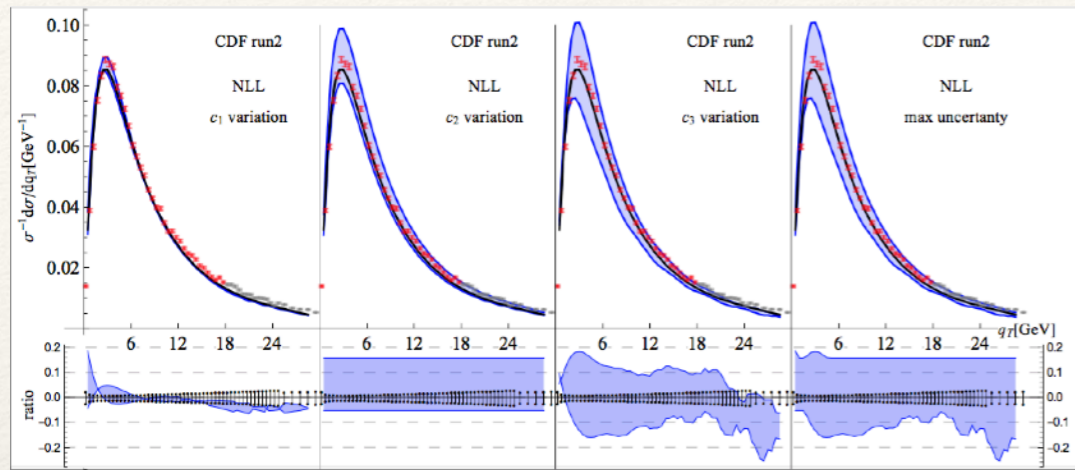
- Different models show different stability of  $\chi^2$ : *Gaussian disfavored/ Exponential favored*
- Stability increases with perturbative order: *NLL very unstable/NNLO very stable*
- We obtain a plateau only for  $\delta_T = q_T/Q \lesssim 0.2$



This addresses issues raised in the talks of C. Pisano, O. Gonzalez



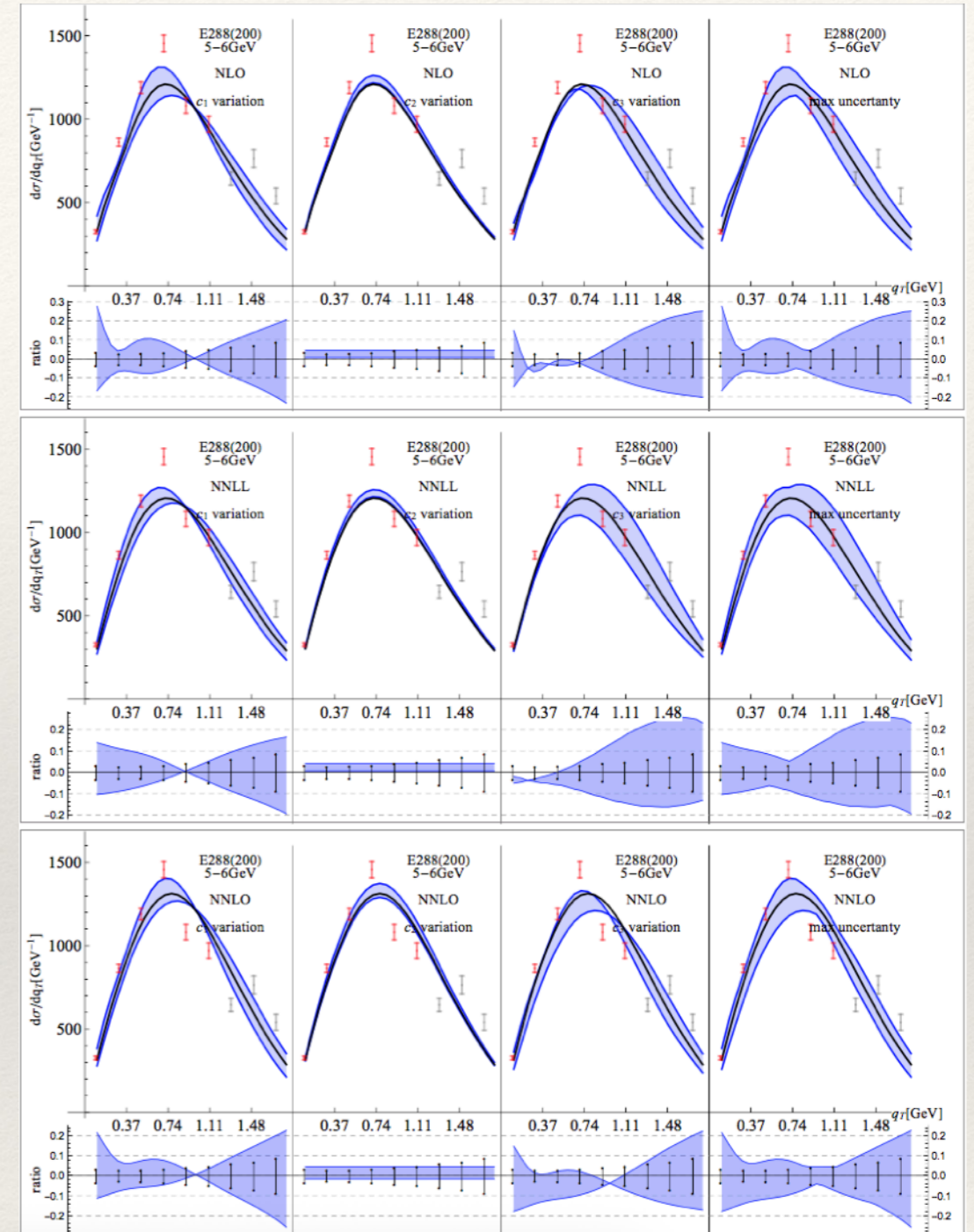
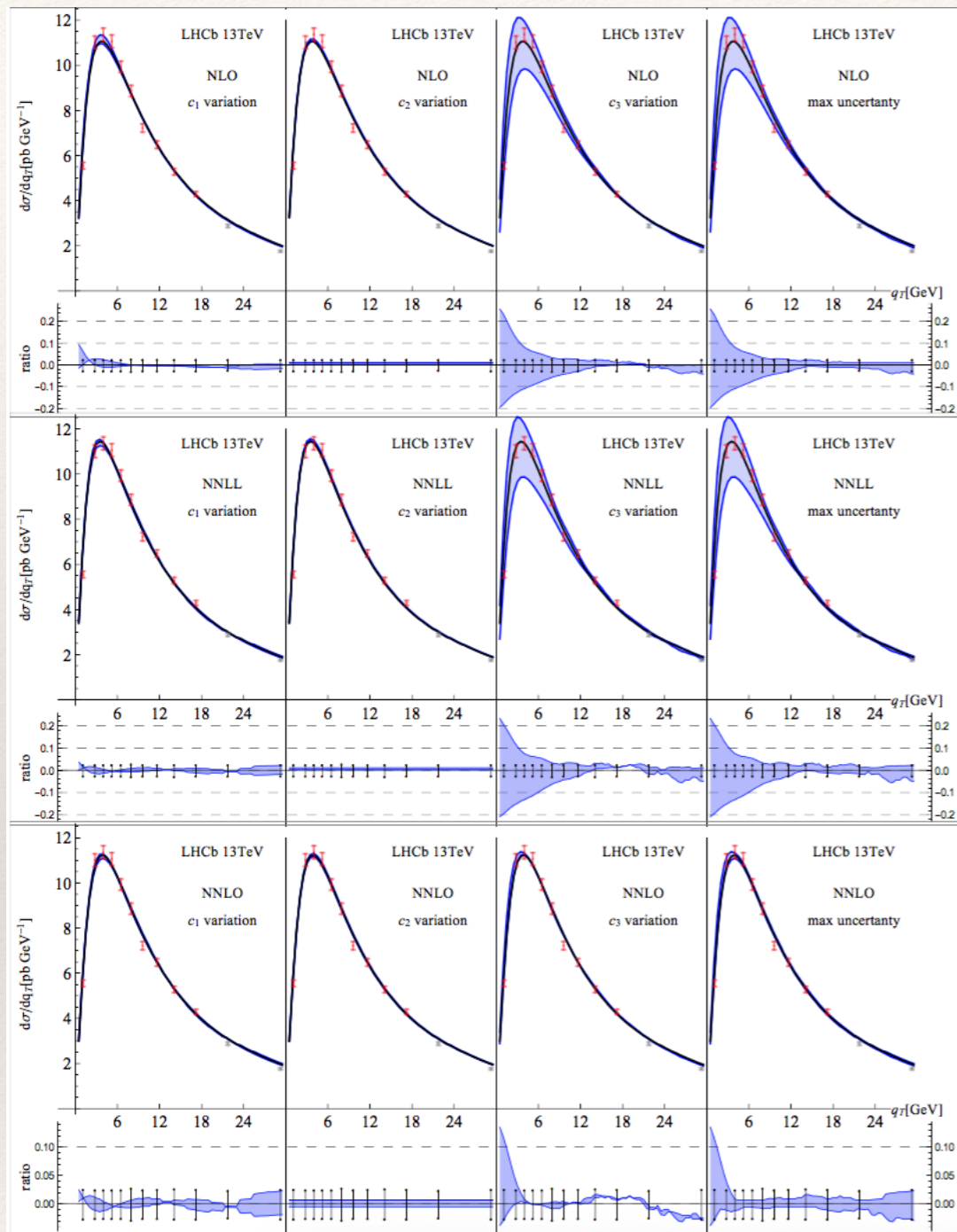
# arTeMiDe : theoretical errors NLL



NLL cannot be trusted for this kind of fits.



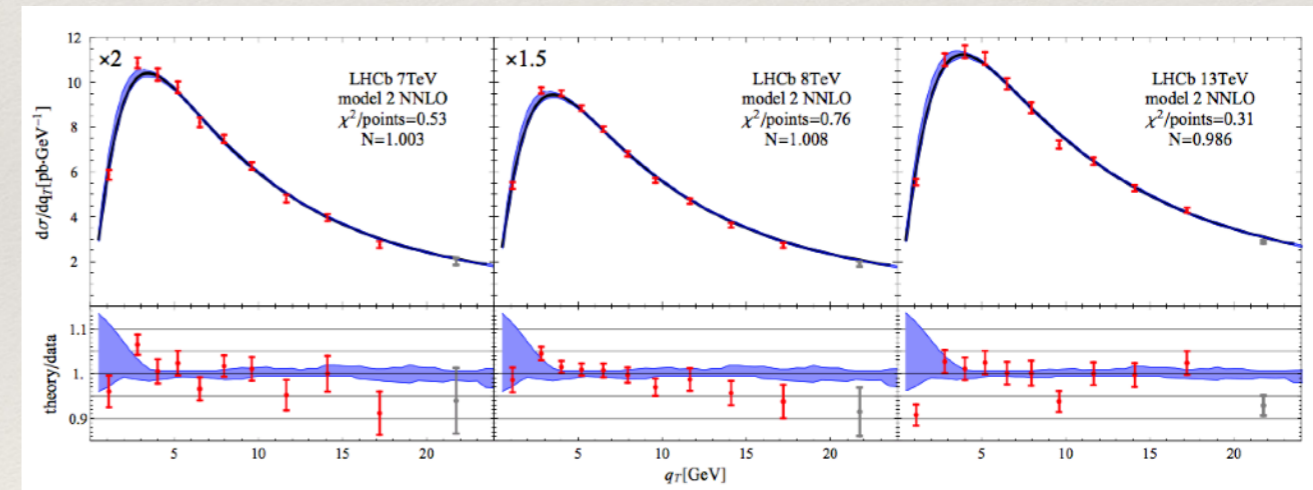
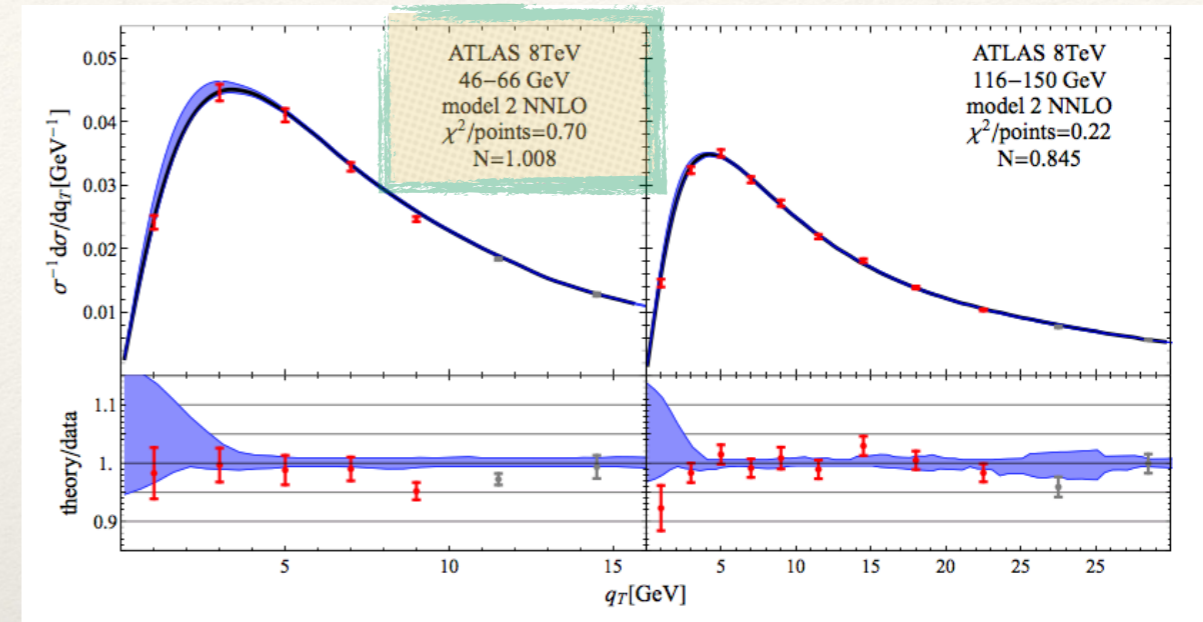
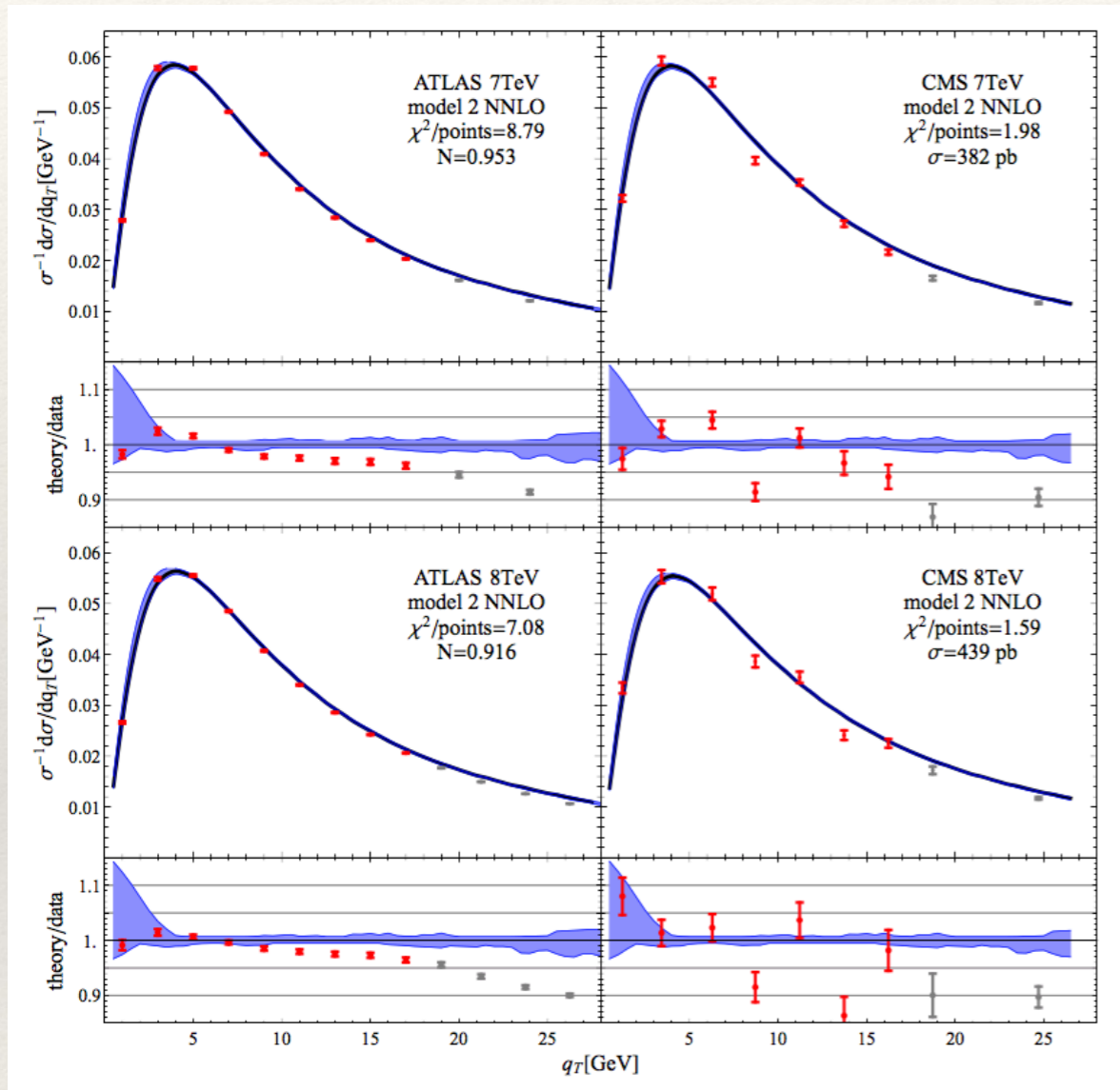
# arTeMiDe : theoretical errors NLO-NNLL-NNLO



- ❖ The origin of error is a bit different in high energy and low energy data
- ❖ A NNLO analysis is always necessary
- ❖ In E288 the lepton cuts are unknown...

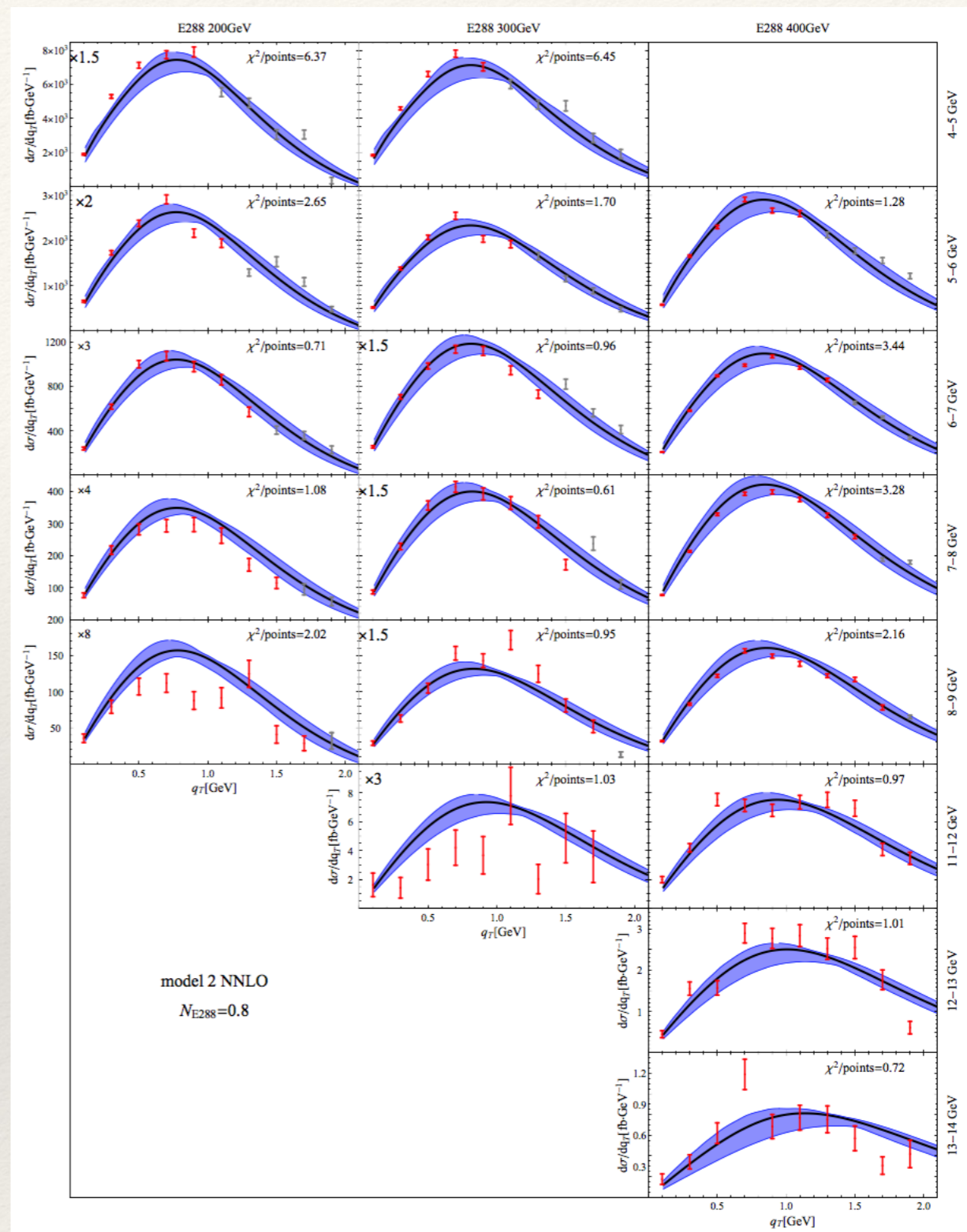


# results for LHC in Drell-Yan and Z-production at NNLO





# E288 at NNLO





# Fit results: high energy data

This fit includes Z-boson production (Tevatron, LHC) and the rest of LHC data

Order	$f_{NP} = 1$	$f_{NP} = e^{-\lambda_1 b^2}$	$f_{NP} = e^{-\lambda_1 b}$	$f_{NP} = 1, g_K \neq 0$
NLL	3.04	1.18 $\lambda_1 = 0.22(1)$	2.16 $\lambda_1 = 0.076(7)$	0.68 $g_K = 0.030(2)$
NLO	7.92	2.38 $\lambda_1 = 0.34(2)$	2.64 $\lambda_1 = 0.174(7)$	2.70 $g_K = 0.058(3)$
NNLL	9.16	2.43 $\lambda_1 = 0.42(2)$	0.99 $\lambda_1 = 0.223(7)$	3.49 $g_K = 0.067(3)$
NNLO	3.43	1.66 $\lambda_1 = 0.22(2)$	1.46 $\lambda_1 = 0.113(7)$	1.83 $g_K = 0.034(3)$

Non-perturbative part is needed

The only  $g_K$  correction is very unstable



# Fit results: all data

Order	$\frac{\chi^2}{d.o.f.}$	$\lambda_1$	$\lambda_2$
$f_{NP} = e^{-\lambda_1 b}$			
NLL	9.22	$0.007^{+0.001}_{-0.001}$	-
NLO	4.68	$0.153^{+0.001}_{-0.001}$	-
NNLL	5.07	$0.127^{+0.001}_{-0.001}$	-
NNLO	3.76	$0.142^{+0.002}_{-0.002}$	-
Model 1: $f_{NP} = e^{-\lambda_1 b}(1 + \lambda_2 b^2)$			
NLL	9.21	$0.0^{+0.002}$	$-0.66^{+0.06}_{-0.06} \times 10^{-2}$
NLO	2.62	$0.197^{+0.002}_{-0.002}$	$-3.15^{+0.05}_{-0.05} \times 10^{-2}$
NNLL	1.43	$0.172^{+0.006}_{-0.006}$	$-2.21^{+0.25}_{-0.24} \times 10^{-2}$
NNLO	1.84	$0.156^{+0.006}_{-0.006}$	$-3.79^{+0.27}_{-0.27} \times 10^{-2}$
Model 2: $f_{NP} = \exp\left(\frac{-z\lambda_q b^2}{\sqrt{1+z^2 b^2 \frac{\lambda_2}{\lambda_1}}}\right) + \text{ren.}$			
NLL	9.07	$0.0^{+0.0002}$	$0.31^{+0.03}_{-0.03} \times 10^{-2}$
NLO	2.64	$0.206^{+0.001}_{-0.001}$	$0.92^{+0.01}_{-0.01} \times 10^{-2}$
NNLL	1.46	$0.178^{+0.005}_{-0.005}$	$0.61^{+0.09}_{-0.09} \times 10^{-2}$
NNLO	1.79	$0.162^{+0.005}_{-0.005}$	$1.26^{+0.84}_{-0.84} \times 10^{-2}$

One more parameter is sufficient to have a reasonable fit

Renormalon effects



The value of this agrees with the fit of only high energy data (stability of fitted parameters with respect to data sets)



# Impact of the non-perturbative part of evolution kernel

Order	$\frac{\chi^2}{d.o.f.}$	$\lambda_1$	$\lambda_2$	$g_K$
$f_{NP} = e^{-\lambda_1 b}$				
NLL	8.90	$0.0^{+0.0001}$	-	$0.25_{-0.02}^{+0.03} \times 10^{-2}$
NLO	1.90	$0.171_{-0.006}^{+0.006}$	-	$2.25_{-0.14}^{+0.17} \times 10^{-2}$
NNLL	1.16	$0.171_{-0.005}^{+0.005}$	-	$1.17_{-0.12}^{+0.14} \times 10^{-2}$
NNLO	1.94	$0.174_{-0.006}^{+0.006}$	-	$1.50_{-0.13}^{+0.15} \times 10^{-2}$
Model 1: $f_{NP} = e^{-\lambda_1 b}(1 + \lambda_2 b^2)$				
NLL	2.44	$0.036_{-0.002}^{+0.002}$	$32.13_{-3.28}^{+3.30} \times 10^{-2}$	$4.71_{-0.24}^{+0.24} \times 10^{-2}$
NLO	1.89	$0.164_{-0.009}^{+0.009}$	$-1.06_{-0.79}^{+0.83} \times 10^{-2}$	$2.12_{-0.36}^{+0.35} \times 10^{-2}$
NNLL	1.08	$0.195_{-0.014}^{+0.013}$	$3.27_{-1.30}^{+1.39} \times 10^{-2}$	$1.98_{-0.35}^{+0.35} \times 10^{-2}$
NNLO	1.83	$0.158_{-0.009}^{+0.009}$	$-2.69_{-0.79}^{+0.79} \times 10^{-2}$	$0.61_{-0.58}^{+0.58} \times 10^{-2}$
Model 2: $f_{NP} = \exp\left(\frac{-z\lambda_q b^2}{\sqrt{1+z^2 b^2 \frac{\lambda_q^2}{\lambda_1^2}}}\right) + \text{ren.}$				
NLL	8.90	$0.00^{+0.0002}$	$-2.48_{-0.70}^{+0.70} \times 10^{-2}$	$0.95_{-0.16}^{+0.16} \times 10^{-2}$
NLO	1.91	$0.165_{-0.007}^{+0.008}$	$0.37_{-0.22}^{+0.23} \times 10^{-2}$	$2.07_{-0.35}^{+0.35} \times 10^{-2}$
NNLL	1.06	$0.185_{-0.009}^{+0.009}$	$-0.93_{-0.31}^{+0.30} \times 10^{-2}$	$2.02_{-0.33}^{+0.28} \times 10^{-2}$
NNLO	1.80	$0.178_{-0.69}^{+0.70}$	$0.55_{-0.27}^{+0.25} \times 10^{-2}$	$0.70_{-0.33}^{+0.33} \times 10^{-2}$

Non-perturbative corrections to the evolution kernel can be confused with quadratic corrections of different origin (check on lattice??)

$$\frac{\chi^2}{309 \text{ points}} = 1.86$$

$$g_K = 0.0015 \pm 0.002 \text{ GeV}^2$$

$$\lambda_2 = 0$$

$$\frac{\chi^2}{309 \text{ points}} = 1.77$$

$$g_K = 0$$

$$\lambda_2 = -0.038 \pm 0.003 \text{ GeV}^2$$

$$\frac{\chi^2}{309 \text{ points}} = 1.80$$

$$g_K = 0.006 \pm 0.0005 \text{ GeV}^2$$

$$\lambda_2 = -0.0269 \pm 0.008 \text{ GeV}^2$$

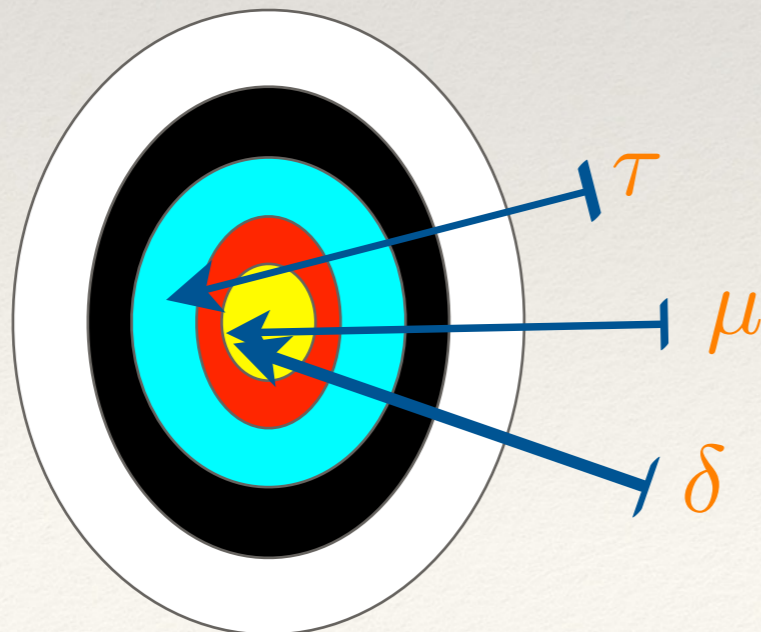


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# Conclusions

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- ❖ A NNLO ANALYSIS IS NECESSARY FOR FITTING DATA AND EXTRACTING TMD (MANY ISSUES SOLVED JUST INCREASING THE PERTURBATIVE ORDER).
- ❖ LHC PROVIDES VERY PRECISE DATA THAT SHOULD BE INCLUDED IN FITS (ESPECIALLY DATA OFF THE Z-BOSON PEAK). ATLAS AND CMS COULD DO BETTER AT 13 TEV!!
- ❖ WE HAVE DISCUSSED A NUMBER OF ISSUES WHICH ARE RELEVANT IN TMD ANALYSIS (DATA CHOICE, NORMALIZATIONS, PRESCRIPTIONS, SCALE CHOICES, STABILITY, THEORETICAL ERRORS,..ETC.)
- ❖ ALL THIS IS INCLUDED IN [arTeMiDe](#) (TO BE RELEASED SOON)





Back up



# Stability

- Different models show different stability of  $\chi^2$ : *Gaussian disfavored/ Exponential favored*
- Stability increases with perturbative order: *NLL very unstable/NNLO very stable*
- We obtain a plateau only for  $\delta_T = q_T/Q \lesssim 0.2$

