OAM Peripheral Transverse Densities from Chiral Dynamics

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Outline

- Motivation
- Transverse Densities
  Definitions, properties
- Orbital angular momentum in impact parameter space
  Spectral Functions, ChPT calculation
- Light front formulation
  Wave functions and χGPDs
- Transverse densities of orbital angular momentum

Motivation

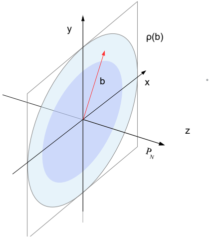
General

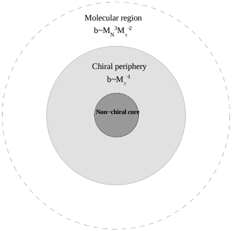
- Complement and extend current understanding of the structure of nucleons, at large distance scales
- Identify the dynamical picture defining structure,
- Identify contributions to the nucleon's intrinsic properties from relevant constituents
- Test the practical reach of fundamental theories

Focus on peripheral orbital angular momentum,

- Define and probe OAM distributions. Use transverse densities
- Provide a mechanical picture of OAM distributions. Light-front formulation of dynamics in the chiral periphery.

Transverse Densities





M.Strikman, C.Weiss PRC82(2010)

CG, C.Weiss JHEP 1401 (2014)

Connect Form Factors and GPDs to nucleon intrinsic spacial structure

$$F(-\Delta_T^2) = \int d^2 b e^{i\boldsymbol{\Delta}_T\cdot\boldsymbol{b}}\rho(b)$$

$$\rho_1(b) = \int dx \frac{d^2\Delta_T}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_T\cdot\boldsymbol{b}} H(x,-\Delta_T^2)$$

G.A. Miller,ARNPS 60 (2010)

M.Burkardt,PRD62(2000)

Boost invariants; structure of the nucleon as a relativistic multiparticle systems.

Nucleon transverse profile; Define dynamical regions in impact parameter space; e.g., non chiral, chiral $[M_\pi^{-1}]$, and molecular $[M_N^2 M_\pi^{-3}]$

Transverse density of orbital angular momentum

- From form factors of energy-momentum tensor

$$\langle N_2|T_{\mu\nu}(0)|N_1\rangle = \bar{u}_2\left[A(\Delta^2)\gamma_{(\mu}p_{\nu)} + B(\Delta^2)p_{(\mu}i\sigma_{\nu)\alpha}\frac{\Delta^\alpha}{2M_N} + C(\Delta^2)\left(\frac{\Delta_\mu\Delta_\nu - \Delta^2 g_{\mu\nu}}{M_N}\right) + \tilde{C}(\Delta^2)M_N g_{\mu\nu}\right]u_1,$$

and the condition,

$$\int d^2b\rho_I(b) = A(0) + B(0) = 1.$$

- Various alternatives,

M. V. Polyakov, PLB **555**, 57 (2003)

L. Adhikari and M. Burkardt, PRD **94**, no. 11, 114021 (2016)

$$\rho_J(b)^{\mathrm{GP}} = \frac{1}{3}\left[(\rho_A(b) + \rho_B(b)) - b\frac{\partial}{\partial b}(\rho_A(b) + \rho_B(b))\right],$$

or

$$\rho_J(b)^{\mathrm{IMF}} \equiv -\frac{1}{2}b\frac{d}{db}(\rho_A(b) + \rho_B(b))$$

- $\rho_J(b)^{\mathrm{GP}}$ not from a relativistic interpretation, and $\rho_J(b)^{\mathrm{IMF}}$ fails to match Jaffes partonic definition (quaks and gluons).
- We look at $\rho_J(b)^{\mathrm{IMF}}$ in the Chiral periphery!

EMT form factors

$$
\begin{aligned}
\langle N_2|T_{\mu\nu}(0)|N_1\rangle &= \bar{u}_2\Big[A(\Delta^2)\gamma_{(\mu}\, p_{\nu)} + B(\Delta^2)p_{(\mu}\, i\sigma_{\nu)\alpha}\frac{\Delta^\alpha}{2M_N} \\
&\quad + C(\Delta^2)\left(\frac{\Delta_\mu\Delta_\nu - \Delta^2 g_{\mu\nu}}{M_N}\right) + \tilde{C}(\Delta^2)M_N g_{\mu\nu}\Big]u_1,
\end{aligned}
$$

Working in the $\Delta^+ = 0$ reference frame, one can write the EMT form factors in terms of matrix elements of light front components of the EMT. Solving for the form factors A and B one has,

$$
\left.\begin{array}{c} A(-\boldsymbol{\Delta}_T^2) \\ \\ B(-\boldsymbol{\Delta}_T^2) \end{array}\right\} = \frac{1}{2(p^+)^2}\sum_{\sigma_1\sigma_2}\langle N_2|\,T^{++}(0)\,|N_1\rangle \left\{\begin{array}{c} \frac{1}{2}\delta(\sigma_1,\sigma_2) \\ \frac{2M_N}{\boldsymbol{\Delta}_T^2}\,(-i)(\boldsymbol{\Delta}_T\times\boldsymbol{e}_z)\cdot\boldsymbol{S}_T(\sigma_1,\sigma_2) \end{array}\right\}
$$

while for the combination $A + B$,

$$
A(-\boldsymbol{\Delta}_T^2) + B(-\boldsymbol{\Delta}_T^2) = \frac{4}{2p^+}\,\frac{i(\boldsymbol{\Delta}_T\times\boldsymbol{e}_z)}{\boldsymbol{\Delta}_T^2}\cdot\sum_{\sigma_1\sigma_2}\langle N_2|\,T^{+T}(0)\,|N_1\rangle S_z(\sigma_1,\sigma_2)
$$

Transverse density of orbital angular momentum

- Study longitudinal component

$$\langle L_z(\boldsymbol{b})\rangle = \frac{1}{2p^+}\langle \boldsymbol{b} \times T^{+T}(\boldsymbol{b})\rangle \cdot \boldsymbol{e}_z,$$

- Matrix elements

$$\langle L_z(\boldsymbol{b})\rangle = -\frac{\langle S_z\rangle}{2} b \frac{d}{db}\left(\rho_A(b) + \rho_B(b)\right).$$

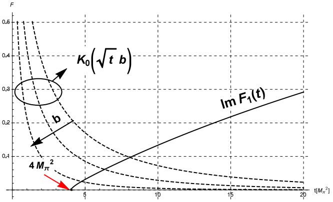
- One can then identify the quantity,

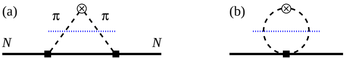
$$\rho_{l_z}(b) \equiv -\frac{1}{2} b \frac{d}{db}\left(\rho_A(b) + \rho_B(b)\right)$$

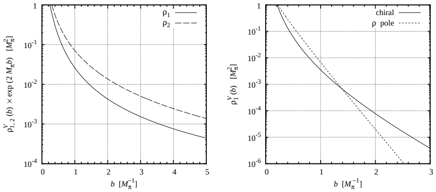
as a transverse density of longitudinal orbital angular momentum. It satisfies the condition,

$$\int d^2 b \rho_{l_z}(b) = A(0) + B(0).$$

TD from spectral functions in χPT







M.Strikman, C.Weiss PRC82(2010)

CG, C.Weiss JHEP 1401 (2014)

Filter high momentum contributions

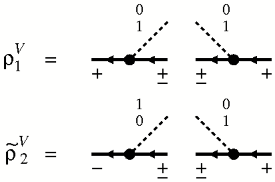
$$\rho(b) = \int_{4M_\pi^2}^{\infty} \frac{dt}{2\pi} K_0(\sqrt{t}b) \frac{\text{Im}F(t+i0)}{\pi}$$

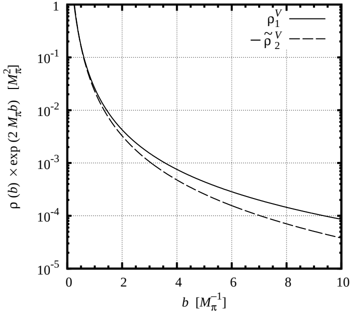
FF from leading order χPT

$$\mathcal{L}_{\pi N} = -\frac{g_A}{F_\pi}\bar{\psi}\gamma^\mu\gamma_5\boldsymbol{\tau}\psi\cdot\partial_\mu\boldsymbol{\pi} - \frac{1}{4F_\pi^2}\bar{\psi}\gamma^\mu\boldsymbol{\tau}\psi\cdot\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi}$$

- subtreshold singularity (on-shell intermediate nucleon)Molecular region
- convergent HB expansions in chiral region
- chiral component becomes dominant for b> 2fm

Charge and magnetization densities from LF dynamics





C.G , C. Weiss, JHEP **1507**, 170 (2015)

Light front current matrix as wavefunction overlap,

$$\frac{J(b)}{2p^+} = \frac{1}{2\pi} \int \frac{dy}{y\bar{y}} \Phi^\dagger \left(y, \frac{b}{\bar{y}} \right) \Phi \left(y, \frac{b}{\bar{y}} \right),$$

then in terms of radial functions

$$\left. \begin{array}{c} \rho_1^V(b) \\ \widetilde{\rho}_2^V(b) \end{array} \right\} = \frac{1}{2\pi} \int \frac{dy}{y\bar{y}^3} \left\{ \begin{array}{c} [U_0(y, b/\bar{y})]^2 \; + \; [U_1(y, b/\bar{y})]^2 \\ -2\, U_0(y, b/\bar{y}) \; U_1(y, b/\bar{y}) \end{array} \right\}.$$

Inequality and positive definiteness of light front current,

$$|\rho_1(b)| > \widetilde{\rho}_2(b) \Rightarrow J^+(b) > 0$$

weakly bound pions.

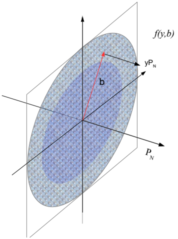
Near parametric equality,

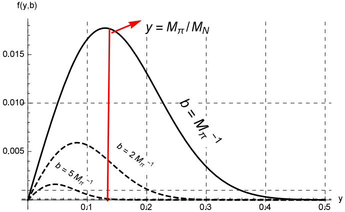
$\rho_1(b) \approx -\widetilde{\rho}_2(b) \Rightarrow$ relativistic pion-nucleon system

Explain through left right asymmetry in Transverse densities .

CG, C. Weiss, PRC **92**, no. 2, 025206 (2015)

Charge magnetization densities and χGPDs





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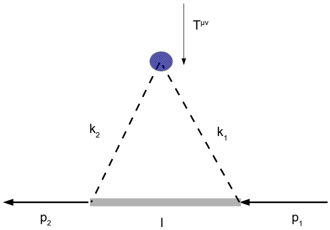
then in terms of radial functions

$$\left. \begin{array}{l} \rho_1^V(b) \\ \widetilde{\rho}_2^V(b) \end{array} \right\} = \frac{1}{2\pi} \int \frac{dy}{y\bar{y}^3} \left\{ \begin{array}{c} [U_0(y, b/\bar{y})]^2 + [U_1(y, b/\bar{y})]^2 \\ -2\, U_0(y, b/\bar{y})\, U_1(y, b/\bar{y}) \end{array} \right\}.$$

Integrands correspond to Fourier trans. of GPDs, $H(y,t)$ and $E(y,t)$, i.e., $f_1(y,b)$, $f_2(y,b)$ respectively,

$$\left. \begin{array}{l} f_1^V(y, b) \\ \widetilde{f}_2^V(y, b) \end{array} \right\} = \frac{1}{2\pi} \frac{1}{y\bar{y}^3} \left\{ \begin{array}{c} [U_0(y, b/\bar{y})]^2 + [U_1(y, b/\bar{y} \\ -2U_0(y, b/\bar{y})U_1(y, b/\bar{y} \end{array} \right.$$

Energy momentum tensor and OAM in χPT



- Use $\mathcal{L}_{eff}$ and

$$T_{\mu\nu} = \mathrm{Tr}\left(\partial_\mu \pi \partial_\nu \pi - \frac{1}{2} g_{\mu\nu} (\partial_\sigma \pi)^2\right)$$

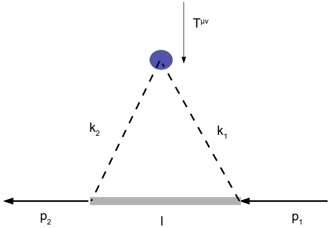
to obtain

$$\left\langle N_2 | T^{N\pi}_{\mu\nu}(0) | N_1 \right\rangle = -\frac{3}{4}\frac{g_A^2}{F_\pi^2}\int \frac{d^4 l}{(2\pi)^4}\left[-i\bar{u}_2 \not{k}_2 \gamma^5 (\not{l} + M_N)\not{k}_1 \gamma^5 u_1 D_N(l)\right.$$
$$\left.(k_{2\mu} k_{1\nu} + k_{1\mu} k_{2\nu} - g_{\mu\nu} k_2 k_1) D_\pi(k_2) D_\pi(k_1)\right].$$

From imaginary part of $T^{\pi N}$,

$$\frac{1}{\pi}\mathbf{Im}(A(t)+B(t)) = \frac{3}{4}\frac{g_A^2}{F_\pi^2} M_N^2 \frac{(t/2 - M_\pi^2)^3}{(4\pi)^2 \sqrt{P^2}^5 \sqrt{t}}\left(\frac{2}{3}x^3 + x - \left(x^2+1\right)\mathbf{arctan}(x)\right) \quad (1)$$

Energy momentum tensor and OAM in χPT



- Use $\mathcal{L}_{eff}$ and

$$T_{\mu\nu} = \mathrm{Tr}\left(\partial_\mu\pi\partial_\nu\pi - \frac{1}{2}g_{\mu\nu}(\partial_\sigma\pi)^2\right)$$

to obtain

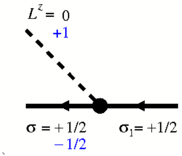
$$\left\langle N_2|T^{N\pi}_{\mu\nu}(0)|N_1\right\rangle = -\frac{3}{4}\frac{g_A^2}{F_\pi^2}\int\frac{d^4l}{(2\pi)^4}\left[-i\bar{u}_2\not{k}_2\gamma^5(\not{l}+M_N)\not{k}_1\gamma^5u_1D_N(l)\right.$$
$$\left.(k_{2\mu}k_{1\nu}+k_{1\mu}k_{2\nu}-g_{\mu\nu}k_2k_1)D_\pi(k_2)D_\pi(k_1)\right].$$

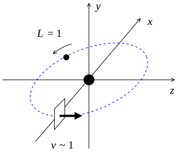
In LF variables ($v^\pm = v^0 \pm v^z$),

$$\left\langle N_2|\frac{T^{++}_{N\pi}(0)}{2p^+}|N_1\right\rangle = \frac{3}{4}p^+\int\frac{dy}{y\bar{y}}\frac{d^2\boldsymbol{k}_T}{(2\pi)^3}y\Psi^*\left(y,\boldsymbol{k}_T+\bar{y}\frac{\boldsymbol{\Delta}_T}{2};\right)\Psi\left(y,\boldsymbol{k}_T-\bar{y}\frac{\boldsymbol{\Delta}_T}{2}\right),$$

$$\left\langle N_2|\frac{T^{+T}_{N\pi}(0)}{2p^+}|N_1\right\rangle = \frac{3}{4}\int\frac{dy}{y\bar{y}}\frac{d^2\boldsymbol{k}_T}{(2\pi)^3}\boldsymbol{k}_T\Psi^*\left(y,\boldsymbol{k}_T+\bar{y}\frac{\boldsymbol{\Delta}_T}{2};\right)\Psi\left(y,\boldsymbol{k}_T-\bar{y}\frac{\boldsymbol{\Delta}_T}{2}\right)$$

Light front wave functions





Eigenfunctions of LF Hamiltonian.

Allow quantum mechanical description of peripheral dynamics.

Computable at leading order in chiral periphery

C.G , C. Weiss, JHEP **1507**, 170 (2015)

$$\Psi(y, \widetilde{\boldsymbol{k}}_T = \boldsymbol{k}_T + y p_{1T}) \equiv \frac{\Gamma(y, \widetilde{\boldsymbol{k}}_T)}{\underbrace{\Delta \mathcal{M}^2(y, \widetilde{\boldsymbol{k}}_T)}_{\text{Inv. Mass difference}}}$$

$$\begin{aligned} \Gamma(y, \widetilde{\boldsymbol{k}}_T) &\approx \frac{g_A M_N}{F_\pi} \bar{u}(y, \boldsymbol{k}_T) i \gamma_5 u(p_{1T}) \\ &= \frac{2 i g_A M_N^2}{F_\pi \sqrt{\bar{y}}} \left[y \boldsymbol{S}_z + \frac{\widetilde{\boldsymbol{k}}_T \cdot \boldsymbol{S}_T}{M_N} \right] \end{aligned}$$

while in transverse coordinate space,

$$\begin{aligned} \Phi(y, r_T) &= \int \frac{d^2 \widetilde{\boldsymbol{k}}_T}{(2\pi)^2} e^{\widetilde{\boldsymbol{k}}_T \cdot \boldsymbol{r}_T} \Psi(y, \widetilde{\boldsymbol{k}}_T) \\ &= -2i \left[U_0(y, r_T) S^z + i \frac{U_1(y, r_T) \boldsymbol{r}_T \cdot \boldsymbol{S}_T}{r_T} \right] \end{aligned}$$

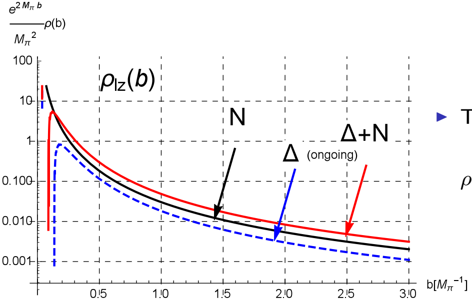
with radial functions,

$$\left. \begin{aligned} U_0(y, r_T) \\ U_1(y, r_T) \end{aligned} \right\} = \frac{g_A M_N \, y \sqrt{\bar{y}}}{2\pi F_\pi} \left\{ \begin{aligned} y M_N \;\; & K_0(M_T r_T) \\ M_T \;\; & K_1(M_T r_T) \end{aligned} \right. ,$$

and transverse mass

$$M_T^2 = \bar{y}^2 M_\pi + y^2 M_N^2$$

OAM in Chiral Periphery



- Transverse densities from spectral functions.

$$\rho_{lz)} = \int_{4M_\pi^2}^{\infty} \frac{dt}{2\pi} \frac{\sqrt{t}b}{2} K_1(\sqrt{t}b) \frac{\boldsymbol{Im}[A+B](t+i0)}{\pi}$$

In the LF-formulation

$$\rho_{l_z}(b) = \frac{3}{2}\frac{1}{2\pi}\int \frac{dy}{y\bar{y}^3}\bar{y}\mathrm{Tr}\left[S_z\Phi^\dagger(y,\boldsymbol{r}_T)\left(\boldsymbol{r}_T\times(-i)\frac{\partial}{\partial\boldsymbol{r}_T}\right)\Phi(y,\boldsymbol{r}_T)\right]|_{\boldsymbol{r}_T=\boldsymbol{b}/\bar{y}},$$

Validates a probabilistic quantum-mechanical interpretation of $\rho_{lz}(b)$ as a density of OAM in the nucleon's periphery!
This cannot be achieved with other definitions of $\rho_{lz}(b)$

Summary and Outlook

Transverse densities computed in Chiral EFT were used in a model independent approach to quantify the distribution of orbital angular momentum in the periphery of the nucleon.

By using LF variables, a propose density of angular momentum is written as a proper density in LF-quantum mechanics of a pion-nucleon fluctuation of a nucleon.

This framework opens the possibility of fully exploring the role that chiral dynamics plays constraining the nucleons internal structure.

- Transverse densities associated with form factors of the energy momentum tensor.Distributions of matter in impact parameter space.
- Expand on different intermediate baryons. (Ongoing) work on intermediate Δ probes large Nc limit properties of LCWF and allows the study of higher orbital modes