Connections between TMD and collinear twist-3 functions within the context of transverse single-spin asymmetries

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TMD Topical Collaboration

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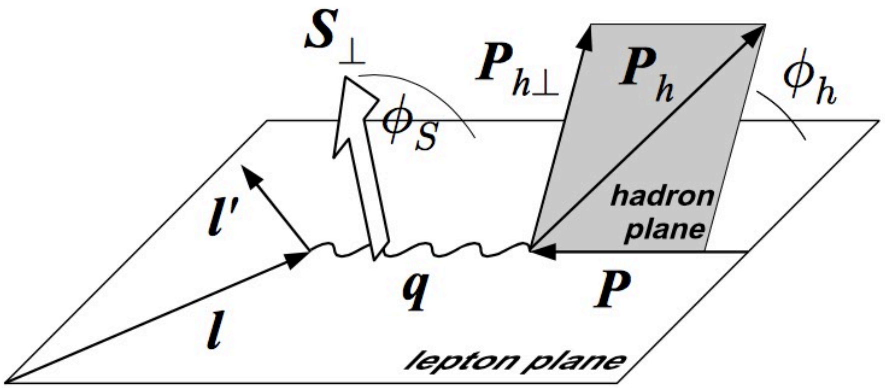
Outline

- Background
  - Transverse single-spin asymmetries
  - TMD and collinear twist-3 (CT3) functions
- TMD and CT3 observables
  - Sivers and Collins effects
  - A_N in $pp \rightarrow (\gamma \text{ or } \pi)\, X$
- Relations between TMD and CT3 functions
  - “Naïve” operator level
  - TMD evolution framework
- Towards a global analysis of TMD and CT3 observables
- Summary

Background

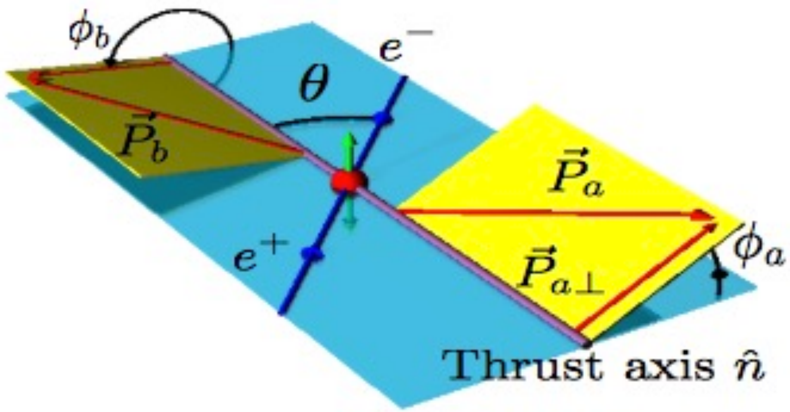
$e\,N \to e'\,h\,X$



Sivers ~ *sin(ϕ_h – ϕ_s)*, Collins ~ *sin(ϕ_h + ϕ_s)*, …

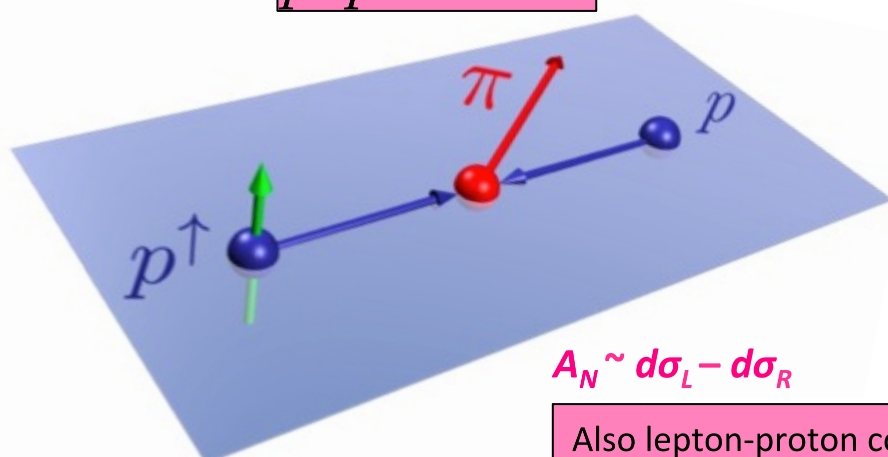
Also DY at COMPASS
(*πp -> e⁺e⁻ X)* and RHIC
(*pp -> {e⁺e⁻, γ, W/Z} X*)

$e^+e^- \to h_a h_b X$



Collins ~ *cos(ϕ_a + ϕ_b)*, …

$p^\uparrow p \to \pi\,X$



A_N ~ $d\sigma_L - d\sigma_R$

Also lepton-proton collisions at
HERMES (2013) and JLab Hall A (2013)

TMD PDFs (x, k_T)

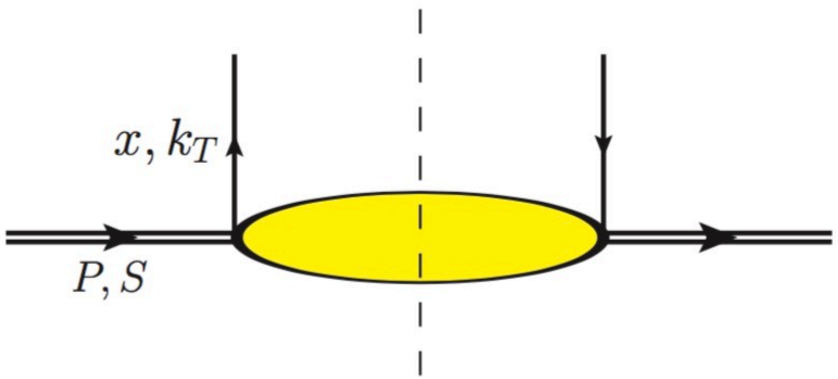
q pol. \ H pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_{1T} $h_{1T}^\perp$

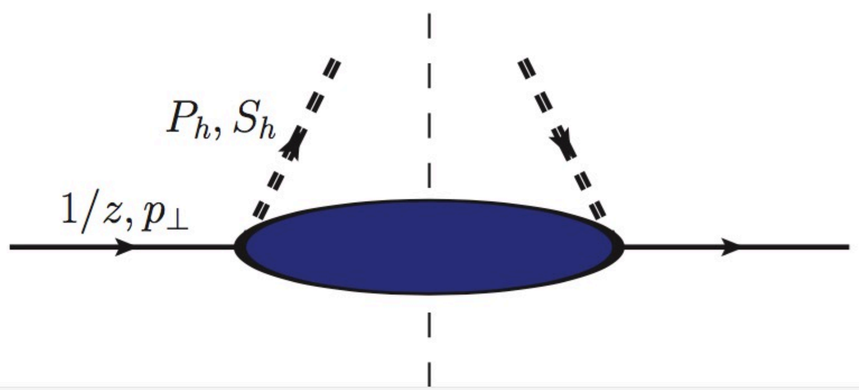
(Mulders, Tangerman (1996); Goeke, Metz, Schlegel (2005))

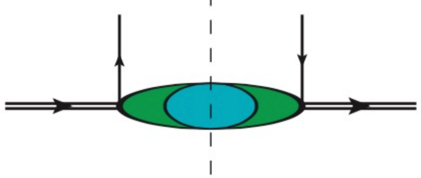
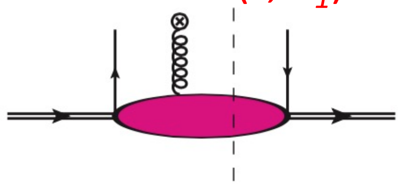
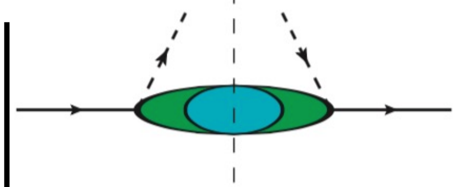
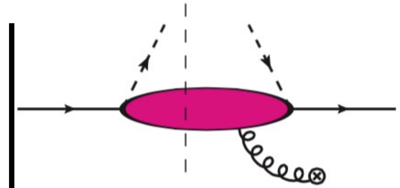
TMD FFs (z, $p_\perp$)

q pol. \ H pol.	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	H_{1T} $H_{1T}^\perp$

(Boer, Jakob, Mulders (1997))



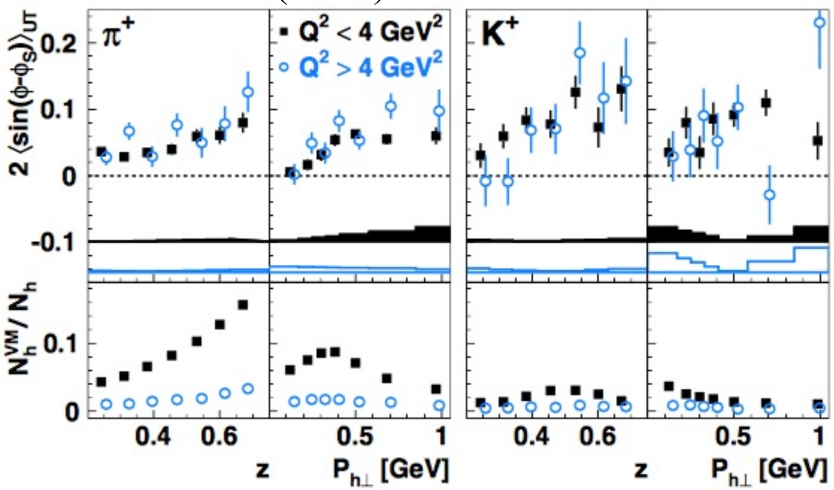


Hadron Pol.	CT3 PDF (x)		CT3 PDF (x, x_1)	CT3 FF (z)		CT3 FF (z, z_1)
						
	intrinsic	kinematical	dynamical	intrinsic	kinematical	dynamical
U	e	$h_1^{\perp(1)}$	H_{FU}	E, H	$H_1^{\perp(1)}$	$\hat{H}_{FU}^{\Re,\Im}$
L	h_L	$h_{1L}^{\perp(1)}$	H_{FL}	H_L, E_L	$H_{1L}^{\perp(1)}$	$\hat{H}_{FL}^{\Re,\Im}$
T	g_T	$f_{1T}^{\perp(1)}$, $g_{1T}^{\perp(1)}$	F_{FT}, G_{FT}	D_T, G_T	$D_{1T}^{\perp(1)}$, $G_{1T}^{\perp(1)}$	$\hat{D}_{FT}^{\Re,\Im}, \hat{G}_{FT}^{\Re,\Im}$

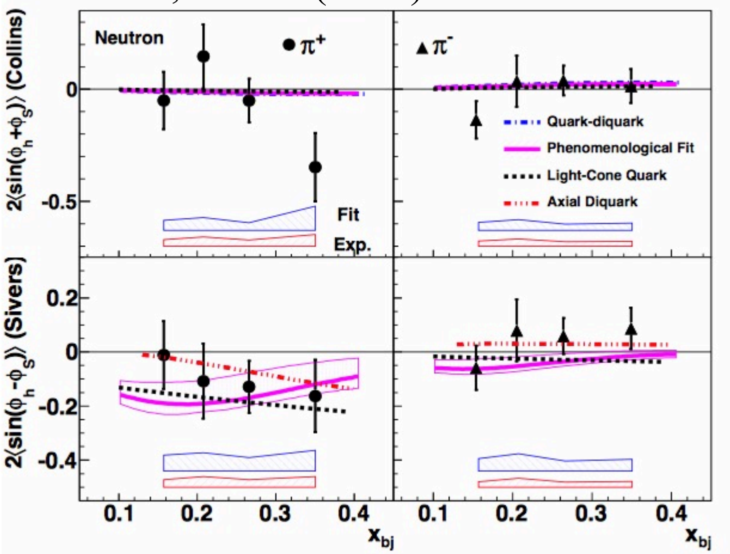
TMD and CT3 Observables

SIDIS Sivers effect (*sin(ϕ_h – ϕ_s)*)

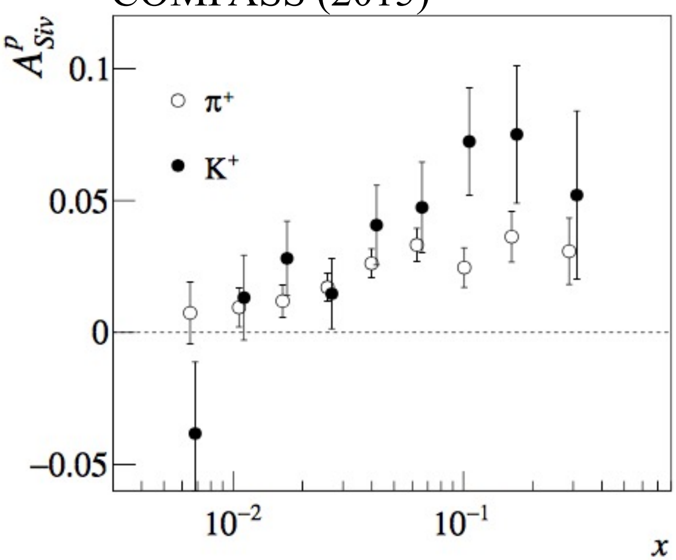
HERMES (2009)



JLab, Hall A (2011)

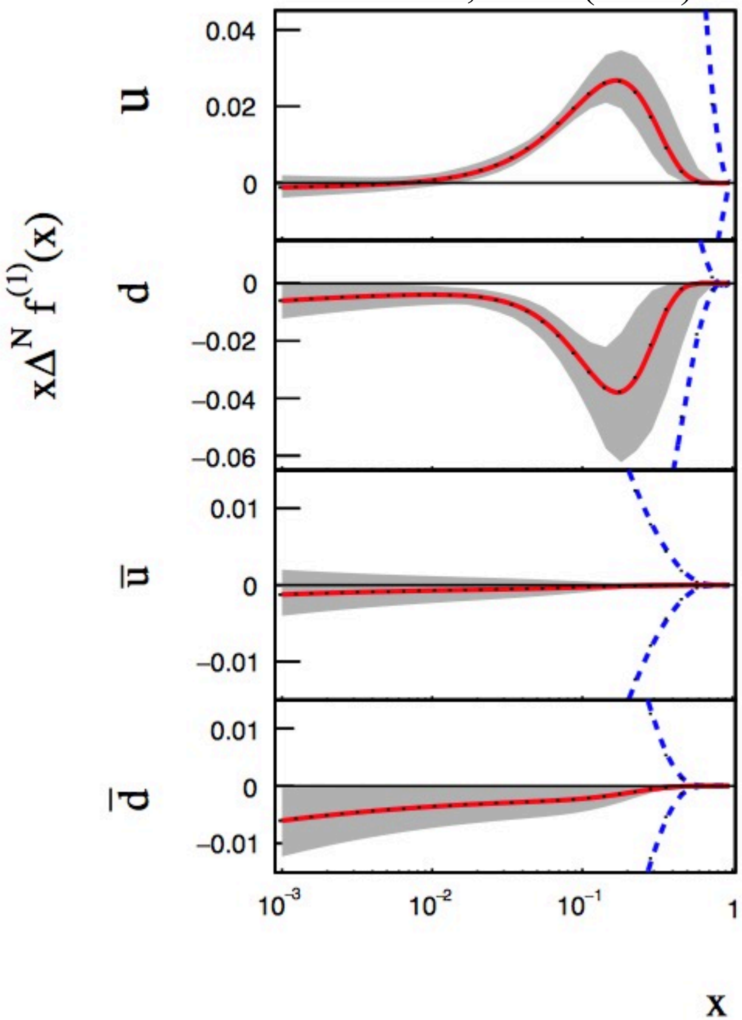


COMPASS (2015)

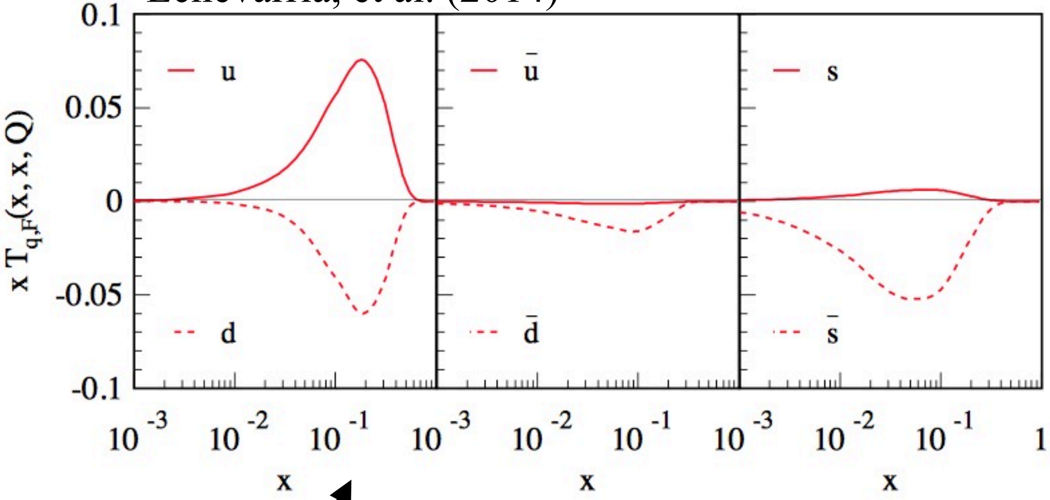


$$F_{UT}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{h}\cdot\vec{k}_T}{M}\,\boldsymbol{f_{1T}^{\perp}}\,D_1\right]$$

Anselmino, et al. (2017)



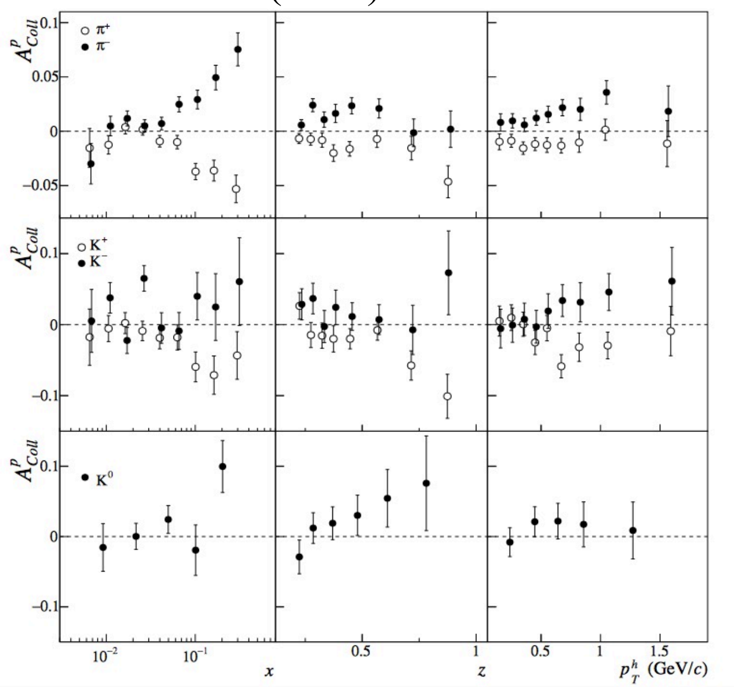
Echevarria, et al. (2014)



uses full TMD evolution

SIDIS Collins effect ($\sin(\phi_h + \phi_s)$)

COMPASS (2015)

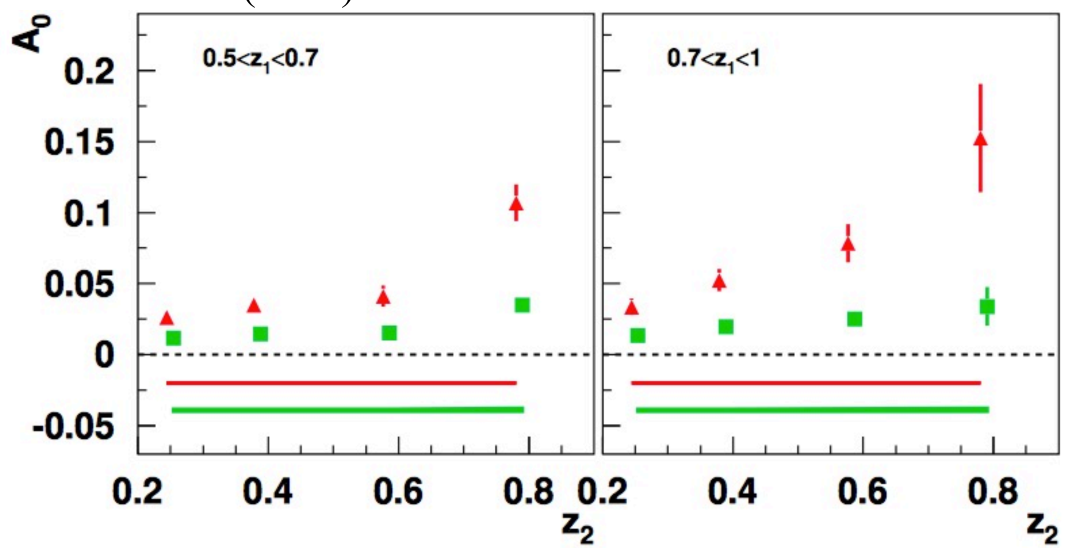


Also data from JLab Hall A (2011, 2014) and HERMES

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{h}\cdot\vec{p}_\perp}{M_h}\,\boldsymbol{h_1}\,\boldsymbol{H_1^\perp}\right]$$

e^+e^- Collins effect ($\cos(2\phi_0)$)

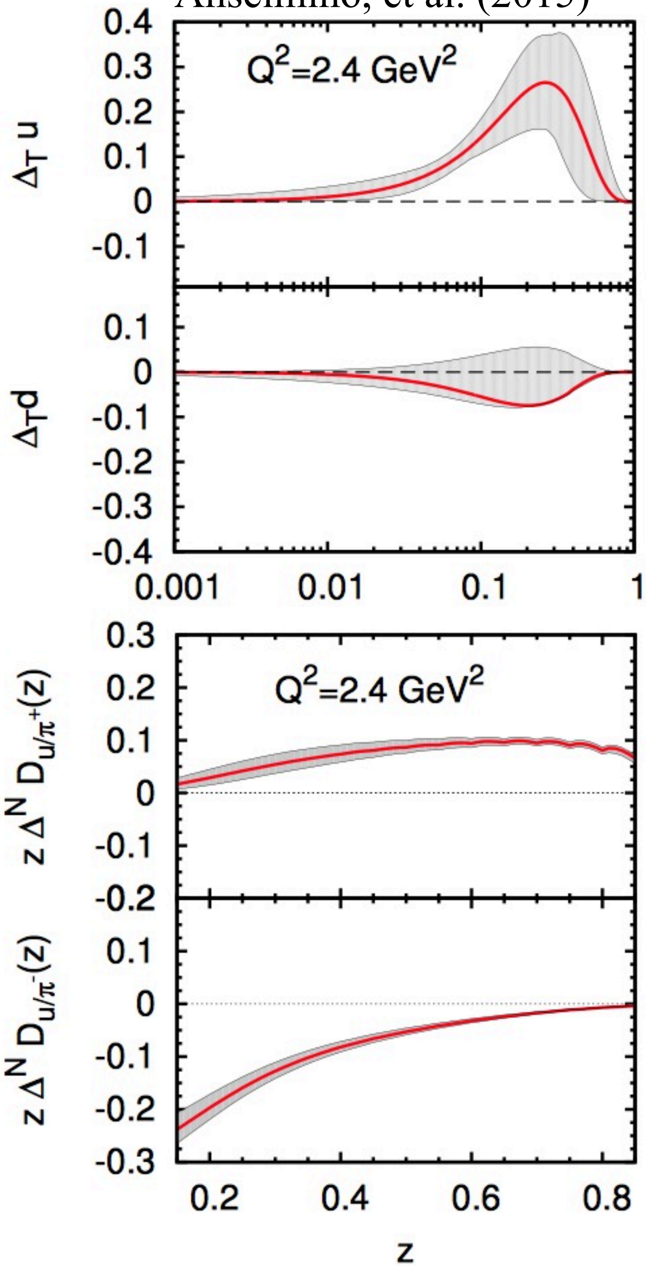
Belle (2008)



Also data from BaBar (2014) and BESIII (2016)

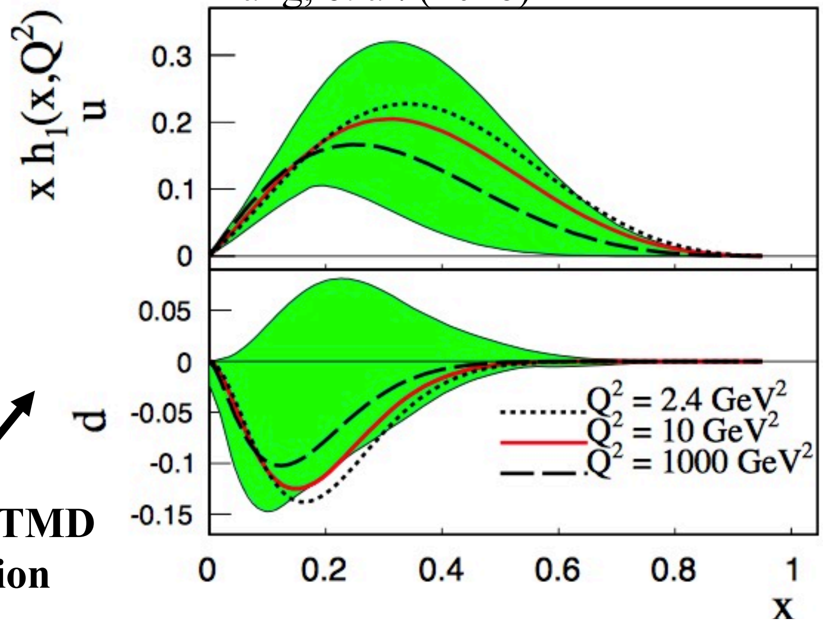
$$F_{UU}^{\cos(2\phi_0)} = \mathcal{C}\left[\frac{2\hat{h}\cdot\vec{p}_{a\perp}\,\hat{h}\cdot\vec{p}_{b\perp} - \vec{p}_{a\perp}\cdot\vec{p}_{b\perp}}{M_a M_b}\,\boldsymbol{H_1^\perp}\,\boldsymbol{\bar{H}_1^\perp}\right]$$

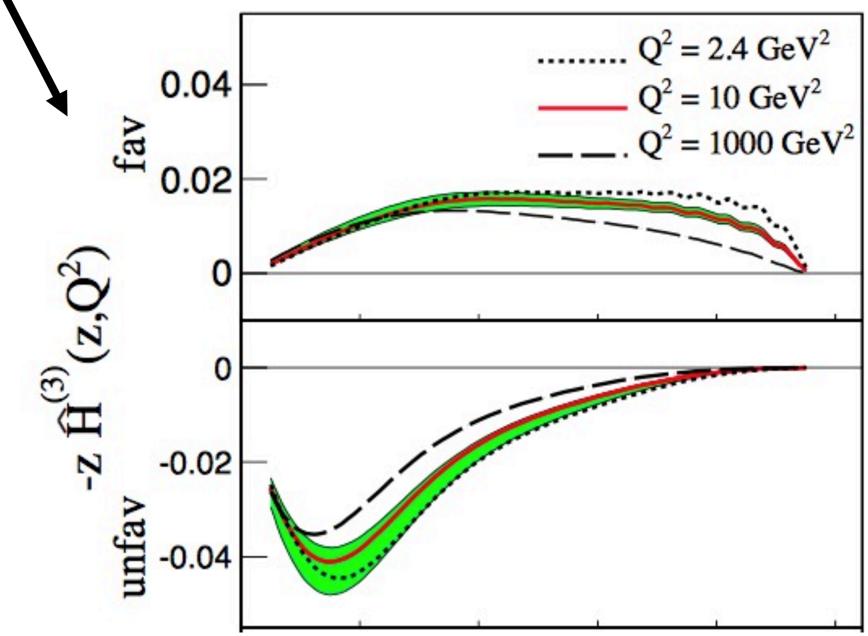
Anselmino, et al. (2015)

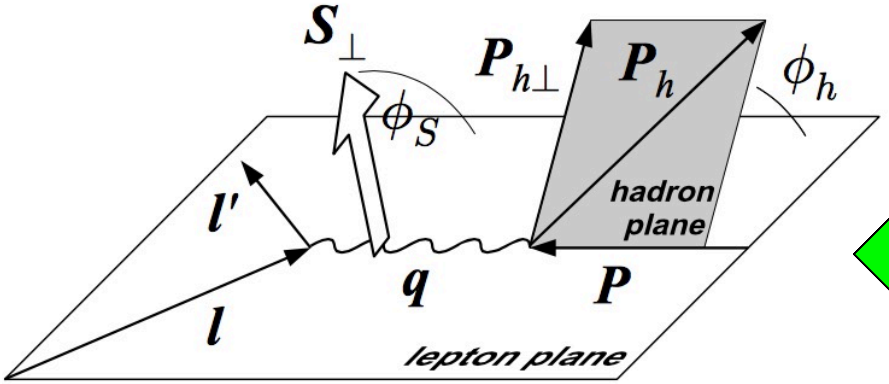


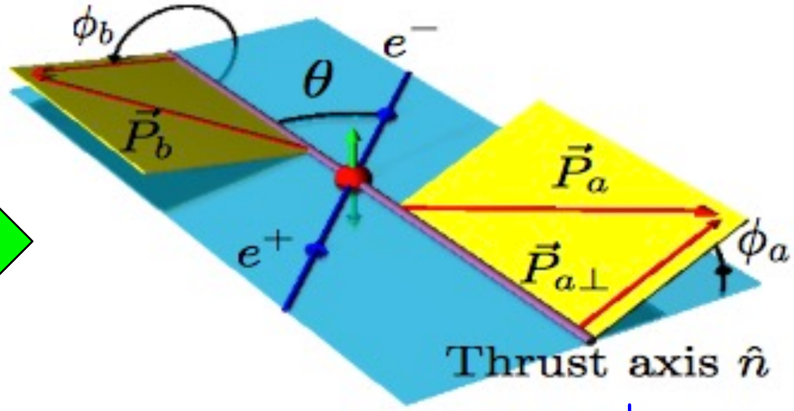
uses full TMD evolution

Kang, et al. (2016)





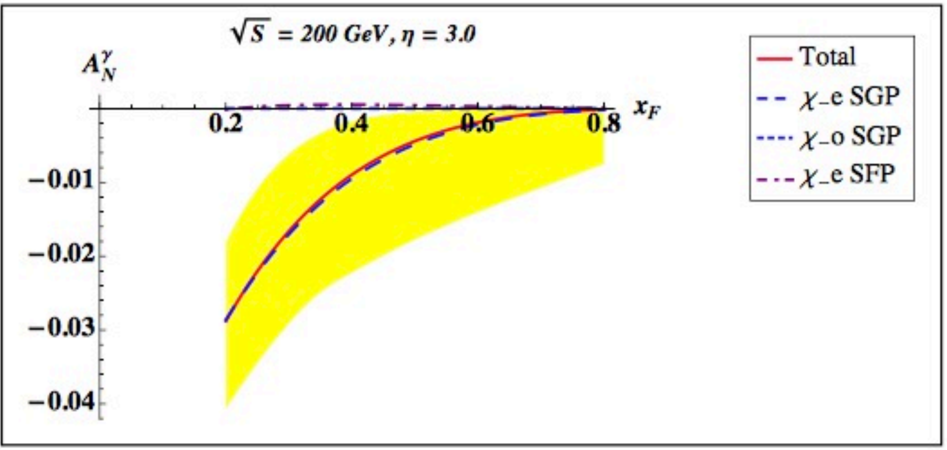


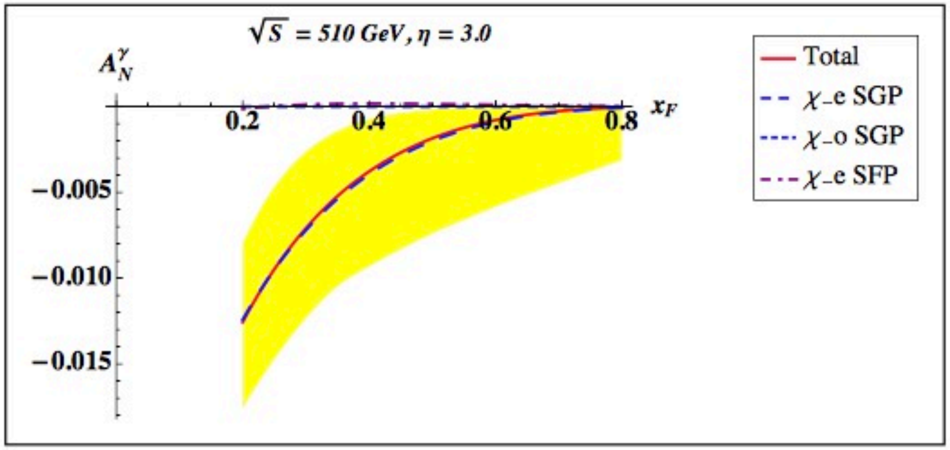


h_1, $f_{1T}^\perp$, $H_1^\perp$

$H_1^\perp$

A_N in $pp \to \gamma X$





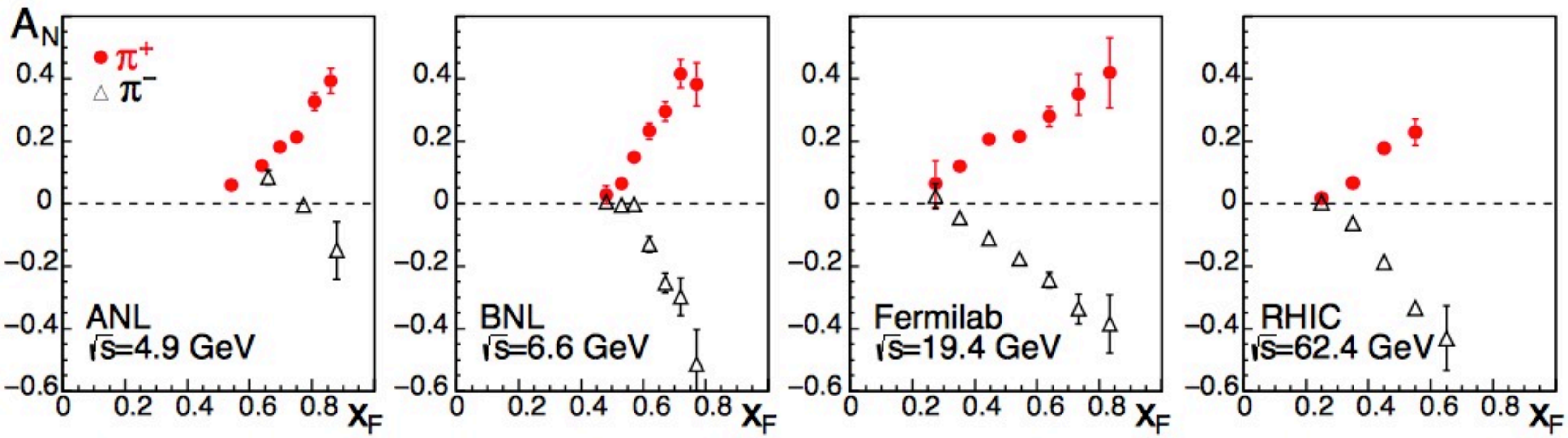
(Kanazawa, Koike, Metz, DP – PRD **91** (2015))
See also Gamberg, Kang, Prokudin (2013)

Qiu-Sterman term is the main cause of A_N in $pp \to \gamma X$

$$d\Delta\sigma^\gamma \sim H \otimes f_1 \otimes \boldsymbol{F_{FT}(x,x)}$$

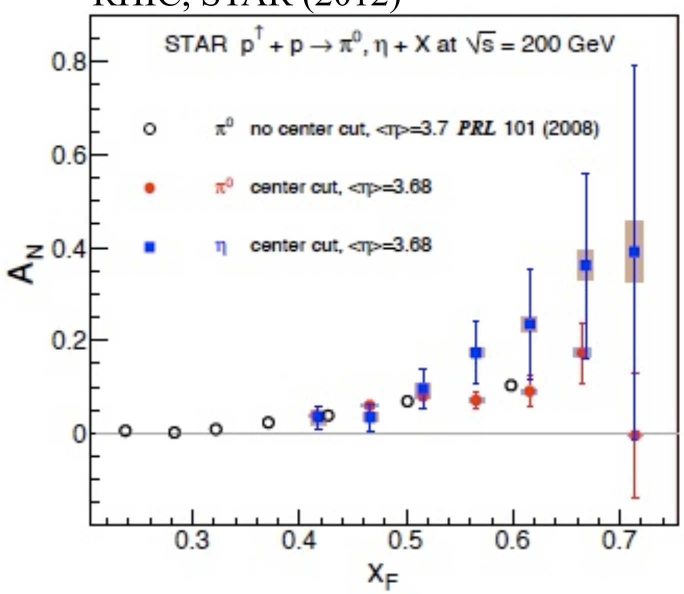
Qiu-Sterman function

A_N in *pp -> π X* – PUZZLE FOR 40+ YEARS!

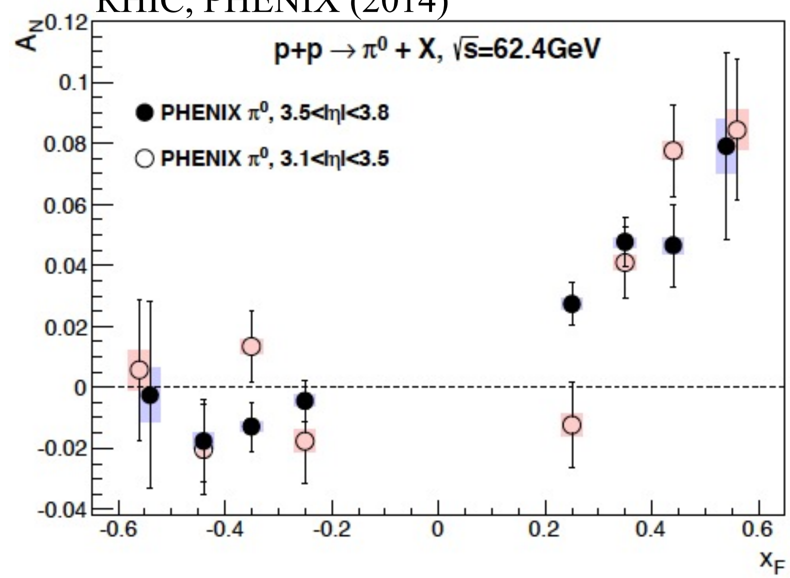


1976 →

RHIC, STAR (2012)



RHIC, PHENIX (2014)



$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{H}, \int \frac{\hat{\boldsymbol{H}}_{\boldsymbol{FU}}^{\Im}}{(1/z - 1/z_1)^2} \right)$$

(Metz and DP - PLB **723** (2013))

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{H}, \int \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

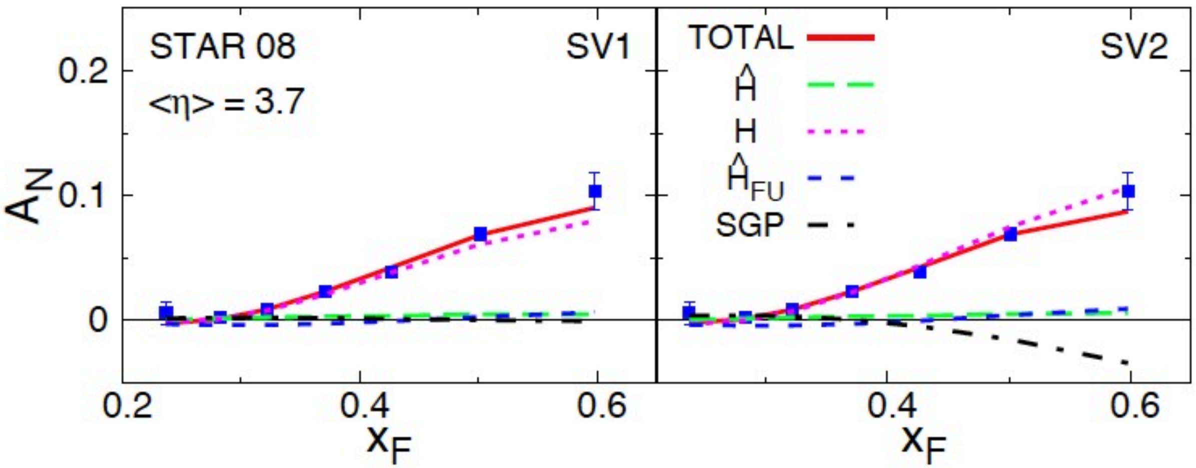
(Metz and DP - PLB **723** (2013))

$$H^q(z) = -2z\, H_1^{\perp(1),q}(z) + \underbrace{\boxed{2z \int_z^{\infty} \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{q,\Im}(z, z_1)}}_{\equiv \tilde{H}^q(z)}$$

QCD e.o.m. relation (EOMR)

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{H}, \int \frac{\hat{\boldsymbol{H}}_{\boldsymbol{FU}}^{\Im}}{(1/z - 1/z_1)^2} \right)$$

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \tilde{\boldsymbol{H}}, \int \frac{\hat{\boldsymbol{H}}_{\boldsymbol{FU}}^{\Im}}{(1/z - 1/z_1)^2} \right)$$



(Kanazawa, Koike, Metz, DP, PRD **89**(RC) (2014))

Fragmentation term is the main cause of A_N in *pp -> π X*

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \tilde{\boldsymbol{H}}, \int \frac{\hat{\boldsymbol{H}}_{\boldsymbol{FU}}^{\Im}}{(1/z - 1/z_1)^2} \right)$$

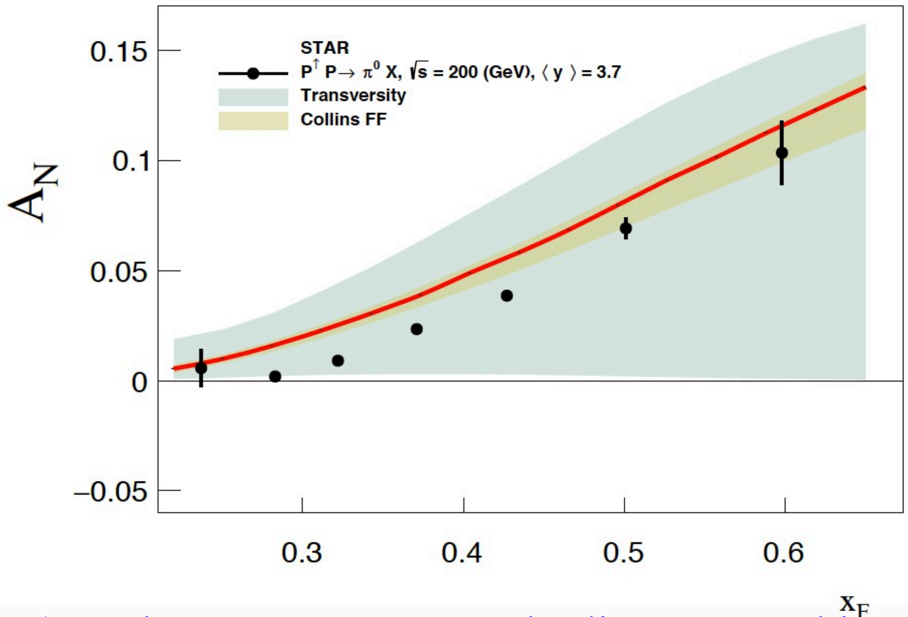
$$\frac{H^q(z)}{z} = -\left(1 - z\frac{d}{dz}\right) H_1^{\perp(1),q}(z) - \frac{2}{z}\int_z^{\infty} \frac{dz_1}{z_1^2} \frac{\hat{H}_{FU}^{q,\Im}(z,z_1)}{(1/z - 1/z_1)^2}$$

Lorentz invariance relation (LIR)

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

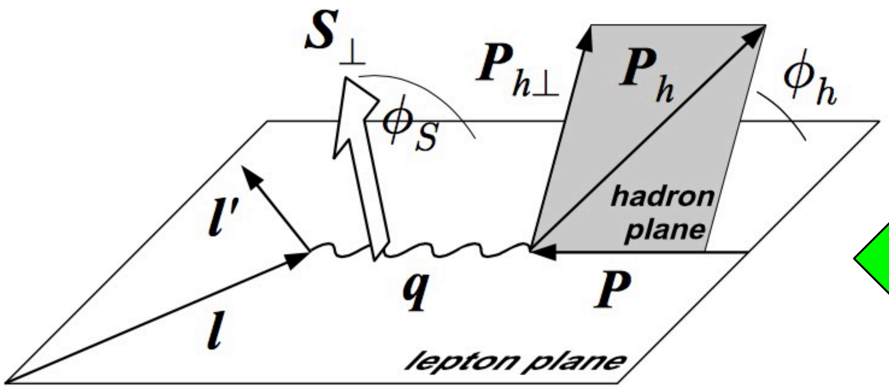
$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \left(H_1^{\perp(1)}, \tilde{H}, \int \frac{\hat{H}_{FU}^{\Im}}{(1/z - 1/z_1)^2} \right)$$

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \left(H_1^{\perp(1)}, \tilde{H} \right)$$

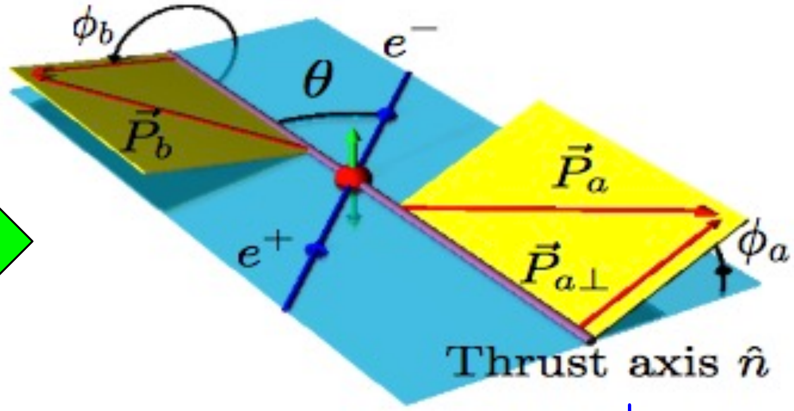


Fragmentation term is the main cause of A_N in *pp* -> *π X*

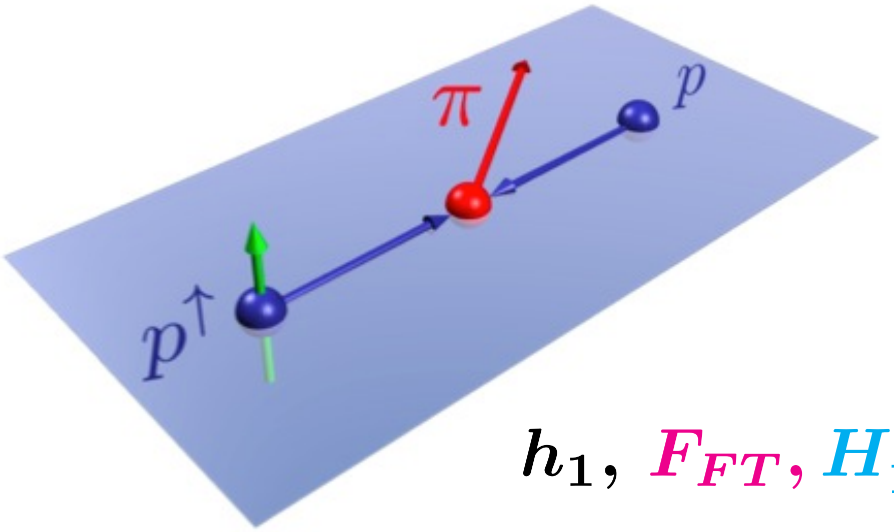
(Gamberg, Kang, DP, Prokudin, accepted in *Phys. Lett. B*)



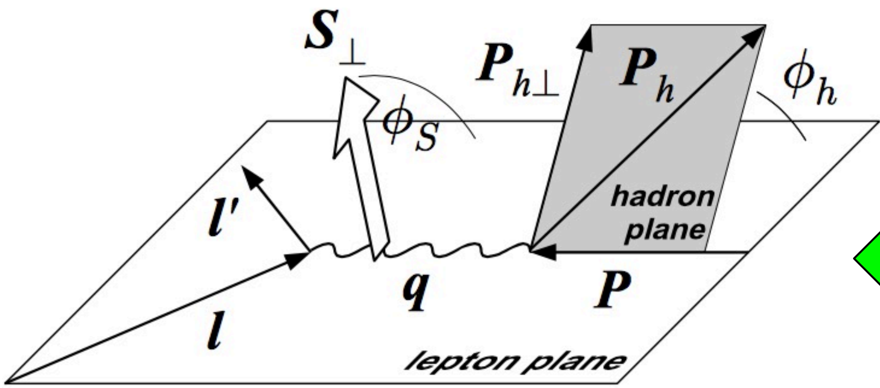
h_1, $f_{1T}^\perp$, $H_1^\perp$

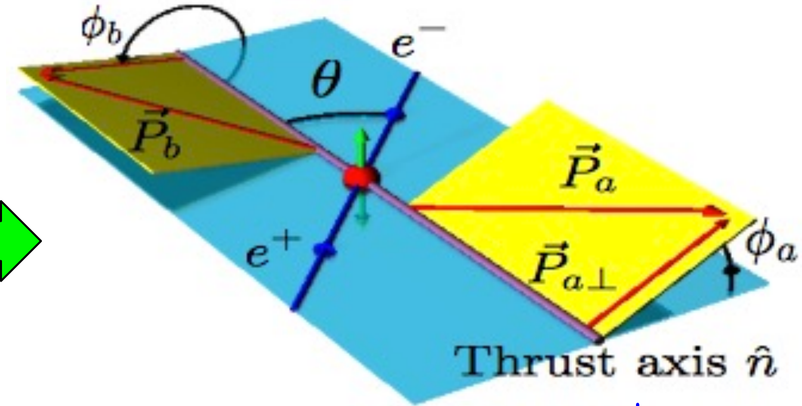


$H_1^\perp$



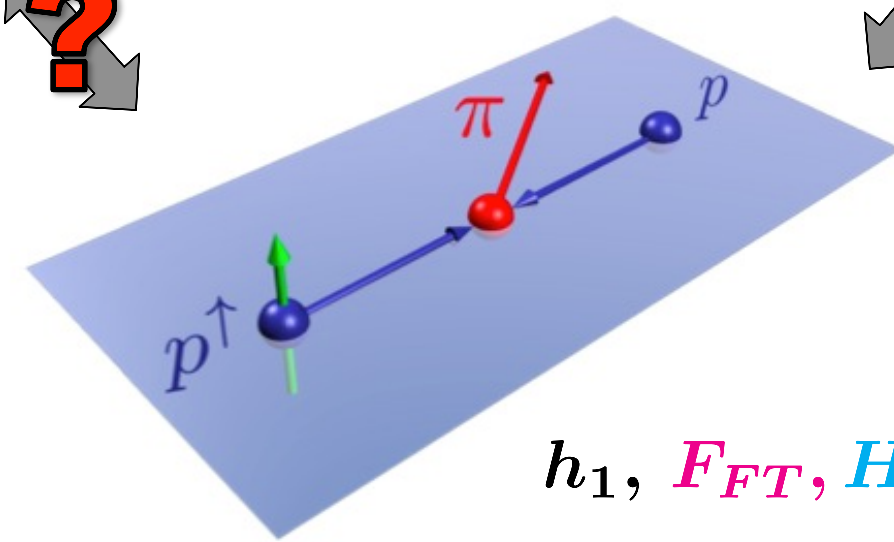
h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$





$h_1,\ f_{1T}^{\perp},\ H_1^{\perp}$

$H_1^{\perp}$



$h_1,\ F_{FT},\ H_1^{\perp(1)},\ \tilde{H}$

Relations between TMD and CT3 Functions

(and other relevant issues...)

Ongoing work with L. Gamberg, Z. Kang, A. Metz, A. Prokudin, T. Rogers, N. Sato, ...

“Naïve” OPERATOR-LEVEL

TMD (SIDIS) kinematical CT3 dynamical CT3

$$\int^{\mu} d^2\vec{k}_T \, \frac{\vec{k}_T^2}{2M^2} \, f_{1T}^{\perp}(x,\vec{k}_T^2) \;=\; f_{1T}^{\perp(1)}(x;\mu) \;=\; \pi F_{FT}(x,x;\mu)$$

Boer, Mulder, Pijlman (2003); Meissner (2009), ...

•
•
•

TMD kinematical CT3

$$\int^{\mu} d^2\vec{p}_\perp \, \frac{z^2\vec{p}_\perp^{\,2}}{2M_h^2} \, H_1^{\perp}(z,z^2\vec{p}_\perp^{\,2}) \;=\; H_1^{\perp(1)}(z;\mu)$$

Yuan and Zhou (2009)

•
•
•

TMD EVOLUTION – Original CSS

(Collins, Soper Sterman (1985); Ji, Ma; Yuan (2005); Collins (2011), ...)

TMD (SIDIS)

dynamical CT3

$$\mathcal{F}.\mathcal{T}.\left[k_T^\alpha\, \boldsymbol{f_{1T}^{\perp}(x,\vec{k}_T^2;Q)}\right] \quad \sim \quad i b^\alpha\, \boldsymbol{F_{FT}(x,x;u_{b_*})} \exp(-S_{pert}(Q,b_*(b_T)) - S_{NP}^{siv}(Q,b_T))$$

Kang, Xiao, Yuan (2011);
Aybat, Collins, Qiu, Rogers (2012); ...

TMD EVOLUTION – Original CSS

(Collins, Soper Sterman (1985); Ji, Ma; Yuan (2005); Collins (2011), ...)

TMD (SIDIS)

dynamical CT3

$$\mathcal{F}.\mathcal{T}.\left[k_T^\alpha\, \boldsymbol{f_{1T}^{\perp}(x,\vec{k}_T^2;Q)}\right] \sim ib^\alpha\, \boldsymbol{F_{FT}(x,x;u_{b_*})}\, \exp(-S_{pert}(Q,b_*(b_T)) - S_{NP}^{siv}(Q,b_T))$$

Kang, Xiao, Yuan (2011);
Aybat, Collins, Qiu, Rogers (2012); ...

same for unpol. and pol.

$$[-g_{siv}(x,b_T) - g_K(b_T)\ln(Q/Q_0)]$$

different for each TMD

universal

TMD EVOLUTION – Original CSS

(Collins, Soper Sterman (1985); Ji, Ma; Yuan (2005); Collins (2011), ...)

TMD (SIDIS)

dynamical CT3

$$\mathcal{F}.\mathcal{T}.\left[k_T^\alpha\, \boldsymbol{f_{1T}^{\perp}(x,\vec{k}_T^2;Q)}\right] \sim ib^\alpha\, \boldsymbol{F_{FT}(x,x;u_{b_*})} \exp(-S_{pert}(Q,b_*(b_T)) - S_{NP}^{siv}(Q,b_T))$$

Kang, Xiao, Yuan (2011);
Aybat, Collins, Qiu, Rogers (2012); ...

S_{pert}: same for unpol. and pol.

S_{NP}^{siv}: $[-g_{siv}(x,b_T) - g_K(b_T)\ln(Q/Q_0)]$

g_{siv}: different for each TMD

g_K: universal

$$b_*(b_T) \equiv \sqrt{\frac{b_T^2}{1+b_T^2/b_{\max}^2}} \qquad \mu_{b_*} = C_1/b_*$$

Note: $b_*(0) = 0$, $\mu_{b_* \to 0} \to \infty$

TMD EVOLUTION – Original CSS

(Collins, Soper Sterman (1985); Ji, Ma; Yuan (2005); Collins (2011), ...)

TMD (SIDIS) dynamical CT3

$$\mathcal{F}.\mathcal{T}.\left[k_T^\alpha\, \boldsymbol{f_{1T}^\perp(x,\vec{k}_T^2;Q)}\right] \quad \sim \quad ib^\alpha\, \boldsymbol{F_{FT}(x,x;u_{b_*})}\exp(-S_{pert}(Q,b_*(b_T)) - S_{NP}^{siv}(Q,b_T))$$

Kang, Xiao, Yuan (2011);
Aybat, Collins, Qiu, Rogers (2012); ...

TMD kinematical CT3

$$\mathcal{F}.\mathcal{T}.\left[\frac{p_\perp^\alpha}{z}\, \boldsymbol{H_1^\perp(z,z^2\vec{p}_\perp^{\,2};Q)}\right] \quad \sim \quad \frac{ib^\alpha}{z}\, \boldsymbol{H_1^{\perp(1)}(z;u_{b_*})}\exp(-S_{pert}(Q,b_*(b_T)) - S_{NP}^{col}(Q,b_T))$$

Echevarria, Idilbi, Scimemi (2014);
Kang, Prokudin, Sun, Yuan (2015); ...

TMD EVOLUTION – Original CSS

(Collins, Soper Sterman (1985); Ji, Ma; Yuan (2005); Collins (2011), ...)

TMD (SIDIS)

dynamical CT3

$$\mathcal{F}.\mathcal{T}.\left[k_T^\alpha\, \boldsymbol{f_{1T}^\perp(x,\vec{k}_T^2;Q)}\right] \sim ib^\alpha \boxed{\boldsymbol{F_{FT}(x,x;u_{b_*})}}\exp(-S_{pert}(Q,b_*(b_T)) - S_{NP}^{siv}(Q,b_T))$$

Kang, Xiao, Yuan (2011);
Aybat, Collins, Qiu, Rogers (2012); ...

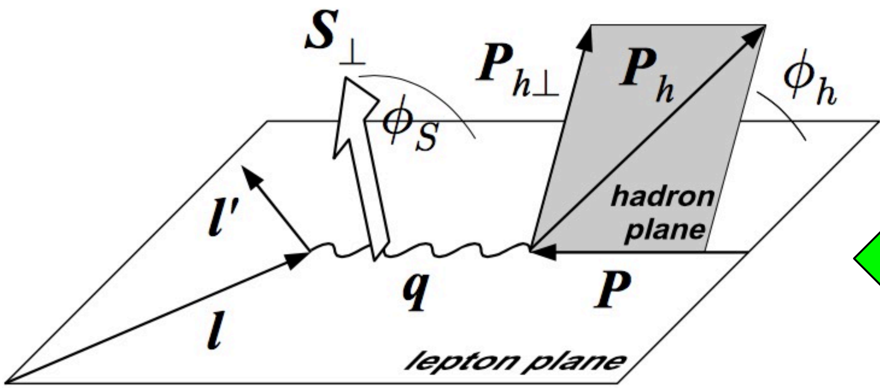
TMD

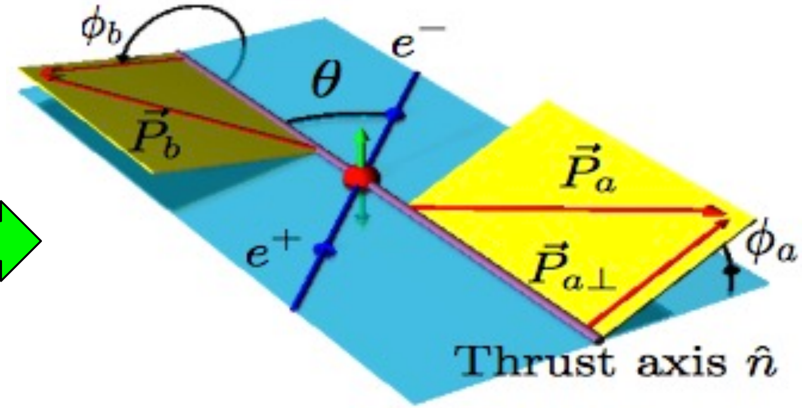
kinematical CT3

$$\mathcal{F}.\mathcal{T}.\left[\frac{p_\perp^\alpha}{z}\, \boldsymbol{H_1^\perp(z,z^2\vec{p}_\perp^{\,2};Q)}\right] \sim \frac{ib^\alpha}{z} \boxed{\boldsymbol{H_1^{\perp(1)}(z;u_{b_*})}}\exp(-S_{pert}(Q,b_*(b_T)) - S_{NP}^{col}(Q,b_T))$$

Echevarria, Idilbi, Scimemi (2014);
Kang, Prokudin, Sun, Yuan (2015); ...

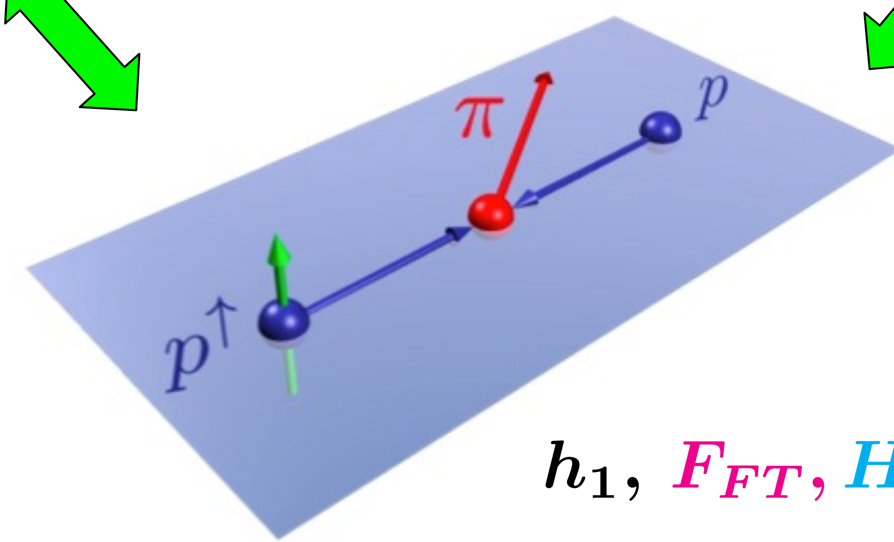
The **CT3 functions** are what get extracted in analyses of
TMD processes that include full TMD (CSS) evolution!
(Echevarria, Idilbi, Kang, Vitev (2014); Kang, Prokudin, Sun, Yuan (2016))



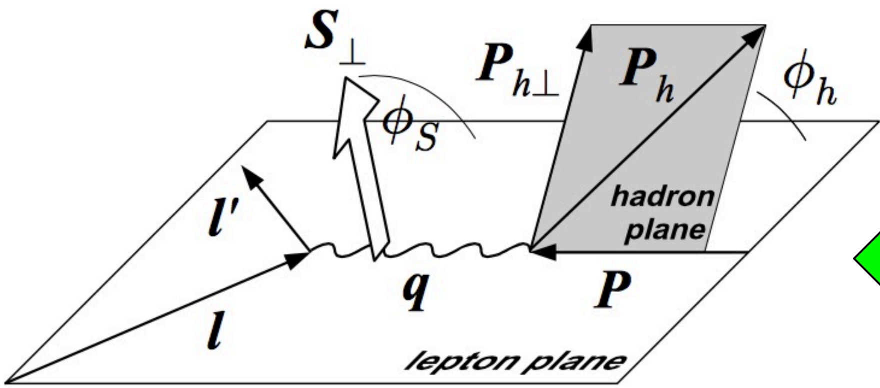


$h_1,\ F_{FT},\ H_1^{\perp(1)}$

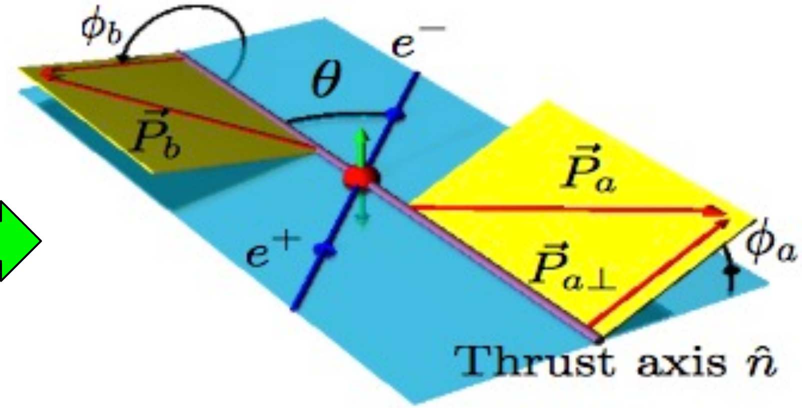
$H_1^{\perp(1)}$



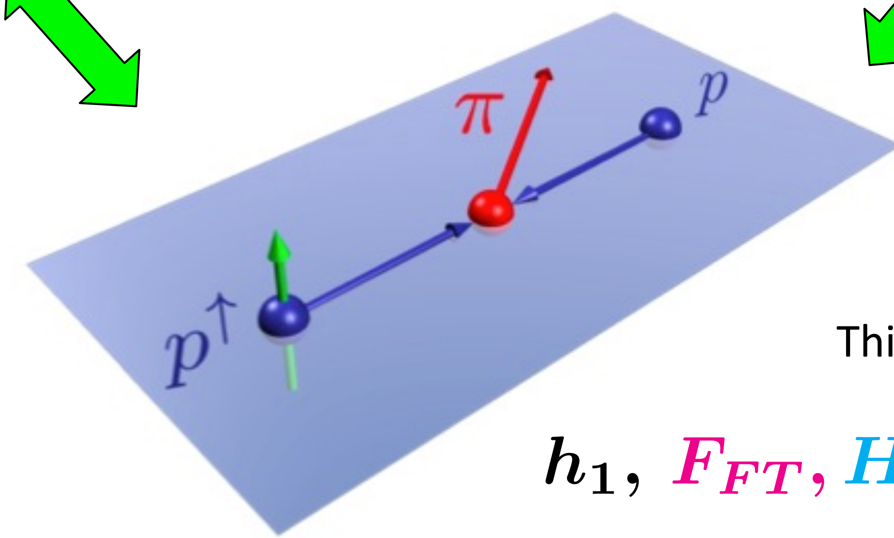
$h_1,\ F_{FT},\ H_1^{\perp(1)},\ \tilde{H}$



h_1, F_{FT}, $H_1^{\perp(1)}$



$H_1^{\perp(1)}$



This is NOT a "new" function!

h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$

$A_{UT}^{\sin\phi_S}$ in SIDIS integrated over P_T (Mulders, Tangerman (1996); Bacchetta, et al. (2007))

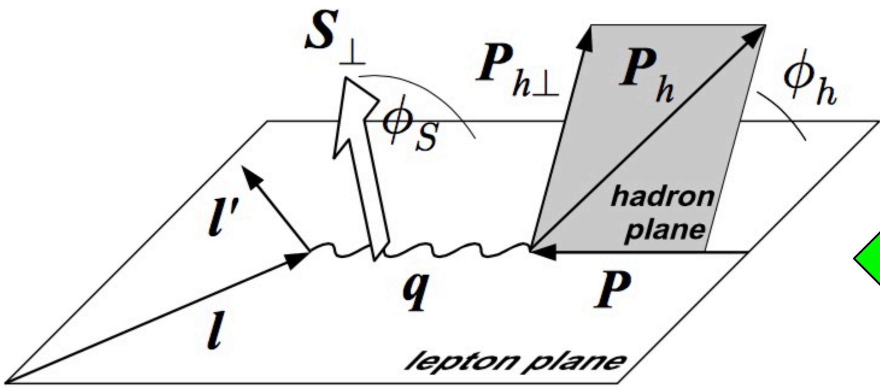
$$F_{UT}^{\sin\phi_S} \propto \sum_a e_a^2 \frac{2M_h}{Q} h_1^a(x) \frac{\tilde{H}^a(z)}{z}$$

$A_{UT}^{\sin\phi_S}$ in $e^+e^- \to h_1 h_2 X$ integrated over q_T (Boer, Jakob, Mulders (1997))

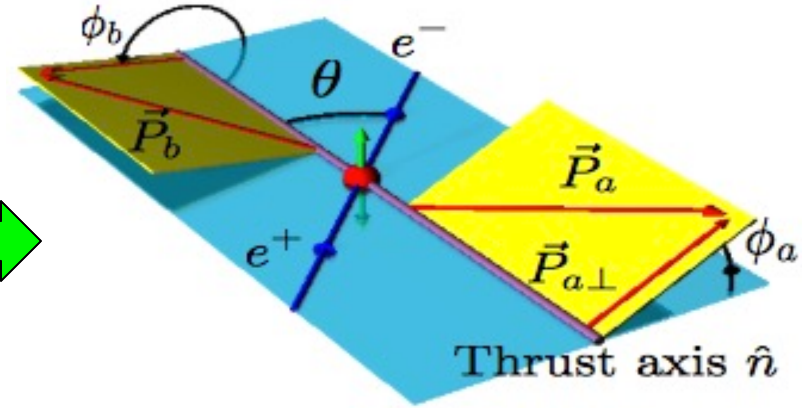
$$F_{UT}^{\sin\phi_S} \propto \sum_{a,\bar{a}} e_a^2 \left(\frac{2M_2}{Q} D_1^a(z_1) \frac{D_T^{\bar{a}}(z_2)}{z_2} + \frac{2M_1}{Q} \frac{\tilde{H}(z_1)}{z_1} H_1^{\bar{a}}(z_2) \right)$$

And also the TMD version of these (and other) observables (but with many more terms)

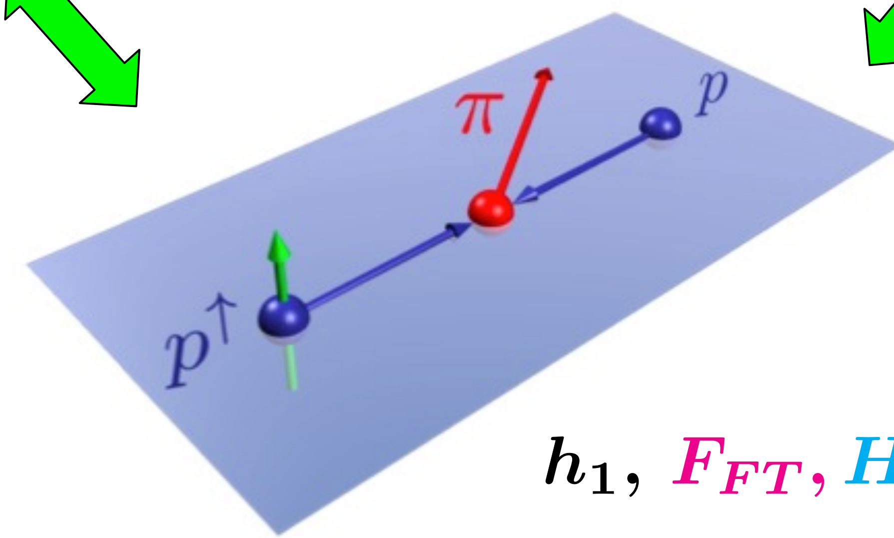
-Note: data from COMPASS, HERMES, and Belle show nonzero effects for the unintegrated version of the above asymmetries



$h_1,\ F_{FT},\ H_1^{\perp(1)},\ \tilde{H}$



$H_1^{\perp(1)},\ \tilde{H}$



$h_1,\ F_{FT},\ H_1^{\perp(1)},\ \tilde{H}$

"Naïve" OPERATOR-LEVEL

$$\int^{\mu} d^2\vec{k}_T \, \frac{\vec{k}_T^2}{2M^2} \quad \boldsymbol{f_{1T}^{\perp}(x, \vec{k}_T^2)} \quad = \quad \boldsymbol{f_{1T}^{\perp(1)}(x;\mu)} \quad = \quad \pi \boldsymbol{F_{FT}(x,x;\mu)}$$

$$\int^{\mu} d^2\vec{p}_\perp \, \frac{z^2\vec{p}_\perp^{\,2}}{2M_h^2} \quad \boldsymbol{H_1^{\perp}(z, z^2\vec{p}_\perp^{\,2})} \quad = \quad \boldsymbol{H_1^{\perp(1)}(z;\mu)}$$

TMD EVOLUTION – Original CSS

(Collins, Soper Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011), ...)

$$\mathcal{F}.\mathcal{T}.\left[k_T^\alpha \, \boldsymbol{f_{1T}^{\perp}(x, \vec{k}_T^2; Q)}\right] \quad \sim \quad i b^\alpha \, \boldsymbol{F_{FT}(x, x; u_{b_*})} \exp(-S_{pert}(Q, b_*) - S_{NP}^{siv}(Q, b))$$

$$\mathcal{F}.\mathcal{T}.\left[\frac{p_\perp^\alpha}{z} \, \boldsymbol{H_1^{\perp}(z, z^2\vec{p}_\perp^{\,2}; Q)}\right] \quad \sim \quad \frac{i b^\alpha}{z} \, \boldsymbol{H_1^{\perp(1)}(z; u_{b_*})} \exp(-S_{pert}(Q, b_*) - S_{NP}^{col}(Q, b))$$

"Naïve" OPERATOR-LEVEL

$$\int^{\mu} d^2\vec{k}_T \, \frac{\vec{k}_T^2}{2M^2} \quad f_{1T}^{\perp}(x,\vec{k}_T^2) \quad = \quad f_{1T}^{\perp(1)}(x;\mu) \quad = \quad \pi F_{FT}(x,x;\mu)$$

$$\int^{\mu} d^2\vec{p}_\perp \, \frac{z^2\vec{p}_\perp^{\,2}}{2M_h^2} \quad H_1^{\perp}(z,z^2\vec{p}_\perp^{\,2}) \quad = \quad H_1^{\perp(1)}(z;\mu)$$

TMD EVOLUTION – Original CSS

(Collins, Soper Sterman (1985), Ji, Ma, Yuan (2005), Collins (2011), ...)

$$\mathcal{F}.\mathcal{T}.\left[k_T^{\alpha}\, f_{1T}^{\perp}(x,\vec{k}_T^2;Q)\right] \quad \sim \quad i b^{\alpha}\, F_{FT}(x,x;u_{b_*})\,\exp(-S_{pert}(Q,b_*) - S_{NP}^{siv}(Q,b))$$

$$\mathcal{F}.\mathcal{T}.\left[\frac{p_\perp^{\alpha}}{z}\, H_1^{\perp}(z,z^2\vec{p}_\perp^{\,2};Q)\right] \quad \sim \quad \frac{i b^{\alpha}}{z}\, H_1^{\perp(1)}(z;u_{b_*})\,\exp(-S_{pert}(Q,b_*) - S_{NP}^{col}(Q,b))$$

Does the TMD q_T-differential cross section for TSSAs reduce to the collinear (twist-3) result after a (weighted) integration over q_T ?

TMD EVOLUTION – Original CSS

(Collins, Soper Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011),...)

$$\Gamma(q_T, Q) \equiv \frac{d\sigma}{d^2 q_T dQ \cdots} = W(q_T, Q) + Y(q_T, Q) + O((m/Q)^c)\,\Gamma(q_T, Q)$$

$q_T << Q$ region

$q_T \sim Q$ region

TMD EVOLUTION – Original CSS

(Collins, Soper Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011),...)

$$\Gamma(q_T, Q) \equiv \frac{d\sigma}{d^2 q_T dQ \cdots} = W(q_T, Q) + Y(q_T, Q) + O((m/Q)^c)\,\Gamma(q_T, Q)$$

$q_T << Q$ region

$q_T \sim Q$ region

$$W(q_T, Q) = \int \frac{d^2 b_T}{(2\pi)^2}\, e^{i q_T \cdot b_T}\, \tilde{W}(b_T, Q)$$

TMD EVOLUTION – Original CSS

(Collins, Soper Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011),...)

$$\Gamma(q_T, Q) \equiv \frac{d\sigma}{d^2 q_T dQ \cdots} = W(q_T, Q) + Y(q_T, Q) + O((m/Q)^c)\,\Gamma(q_T, Q)$$

q_T << Q region $\qquad$ q_T ~ Q region

$$W(q_T, Q) = \int \frac{d^2 b_T}{(2\pi)^2}\, e^{i q_T \cdot b_T}\, \tilde{W}(b_T, Q)$$

$$\tilde{W}(b_T, Q) = \tilde{W}^{\rm unp}(b_T, Q) - \frac{i}{2}\, \epsilon_{T\alpha\beta}\, b_T^{\alpha} S_T^{\beta}\, \tilde{W}^{\rm siv}(b_T, Q)$$

TMD EVOLUTION – Original CSS

(Collins, Soper Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011),...)

$$\Gamma(q_T, Q) \equiv \frac{d\sigma}{d^2 q_T dQ \cdots} = W(q_T, Q) + Y(q_T, Q) + O((m/Q)^c)\,\Gamma(q_T, Q)$$

$q_T << Q$ region

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NOT associated with the scale evolution

TMD EVOLUTION – Improved CSS

(Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

TMD (q_T << Q)

$$\Gamma^{\rm unp}(q_T, Q) \sim H \left[f_1(x, \vec{k}_T^2; Q) \otimes D_1(z, z^2\vec{p}_\perp^2; Q)\right]$$

. . .

LO Collinear

$$\int d^2q_T\, \Gamma(q_T, Q) \sim H\, [f_1(x;Q)D_1(z;Q)]$$

TMD EVOLUTION – Improved CSS

(Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

$$\int d^2q_T\, W(q_T, Q) = \tilde{W}^{\rm unp}(b_T \to 0, Q) = b_T^a \times (\text{log corrections}) \;\; \mathbf{= 0!}$$

The q_T-integrated cross section does NOT reduce to the LO collinear result!

TMD EVOLUTION – Improved CSS

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$$\int d^2q_T\, W(q_T, Q) = \tilde{W}^{\rm unp}(b_T \to 0, Q) = b_T^a \times (\text{log corrections}) \ \mathbf{= 0!}$$

The q_T-integrated cross section does NOT reduce to the LO collinear result!

Source of this issue: S_{pert} has large logs because, in the original CSS b_*-prescription, $b_*(0) = 0$

$$S_{pert} = \ln\frac{Q^2}{\mu_{b_*}^2}\tilde{K}(b_*(b_T);\mu_{b_*}) + \int_{\mu_{b_*}}^{\mu_Q}\frac{d\mu'}{\mu'}\left[2\gamma(\alpha_s(\mu');1) - \ln\frac{Q^2}{\mu'^2}\gamma_K(\alpha_s(\mu'))\right]$$

TMD EVOLUTION – Improved CSS

(Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

$$\int d^2 q_T\, W(q_T, Q) = \tilde{W}^{\rm unp}(b_T \to 0, Q) = b_T^a \times (\text{log corrections}) \;\; \mathbf{= 0!}$$

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$$S_{pert} = \ln \frac{Q^2}{\mu_{b_*}^2} \tilde{K}(b_*(b_T); \mu_{b_*}) + \int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'} \left[2\gamma(\alpha_s(\mu'); 1) - \ln \frac{Q^2}{\mu'^2} \gamma_K(\alpha_s(\mu')) \right]$$

Resolution: Place a lower cut-off on b. Also, explicitly cut off W at large q_T.

$$W^{\rm unp}(q_T, Q) \to W^{\rm unp}_{\rm New}(q_T, Q; \eta, C_5) \equiv \Xi\left(\frac{q_T}{Q}, \eta\right) \int \frac{d^2 b_T}{(2\pi)^2}\, e^{i q_T \cdot b_T}\, \tilde{W}^{\rm unp}(b_c(b_T), Q)$$

$$\text{where } \; b_c(b_T) = \sqrt{b_T^2 + b_0^2/(C_5 Q)^2} \;\longrightarrow\; \mu_{b_*} \text{ is cut off at } \mu_c \approx \frac{C_1 C_5 Q}{b_0}$$

Note: This also leads to a new Y-term.

With these modifications, one now finds,

$$\int d^2q_T\,\Gamma(q_T,Q) = H_{LO,jj}\,f_{1\,j/A}(x;\mu_c)\,D_{1\,B/j}(z;\mu_c) + O(\alpha_s(Q)) + O((m/Q)^p)$$

With these modifications, one now finds,

$$\int d^2q_T\,\Gamma(q_T,Q) = H_{LO,jj}\,f_{1\,j/A}(x;\mu_c)\,D_{1\,B/j}(z;\mu_c) + O(\alpha_s(Q)) + O((m/Q)^p)$$

Do the same issues arise for TSSAs and can the improved CSS formalism resolve them?

TMD (q_T << Q)

$$\Gamma^{\rm siv}(q_T,Q) \sim H\,\left[f_{1T}^{\perp}(x,\vec{k}_T^2;Q)\otimes D_1(z,z^2\vec{p}_\perp^{\,2};Q)\right]$$

. . .

LO Collinear (Twist-3)

$$\int d^2q_T\,\epsilon_{T\mu\nu}\,q_T^\mu S_T^\nu\,\Gamma(q_T,Q) \sim H\,\left[F_{FT}(x,x;Q)D_1(z;Q)\right]$$

Boer, Mulders (1997);

NLO $\left\{\begin{array}{r}\text{Kang, Vitev, Xing (2013);}\\ \text{Yoshida (2016)}\end{array}\right.$

TMD ($q_T << Q$)

$$\Gamma^{\rm siv}(q_T,Q) \sim H\left[f_{1T}^{\perp}(x,\vec{k}_T^2;Q)\otimes D_1(z,z^2\vec{p}_\perp^{\,2};Q)\right]$$

. . .

LO Collinear (Twist-3)

$$\int d^2q_T\,\epsilon_{T\mu\nu}\,q_T^\mu S_T^\nu\,\Gamma(q_T,Q)\sim H\,\left[F_{FT}(x,x;Q)D_1(z;Q)\right]$$

Original CSS...

$$\int d^2q_T\,\epsilon_{T\mu\nu}\,q_T^\mu S_T^\nu\;W(q_T,Q)\sim\frac{\partial}{\partial b_{T\mu}}\left[b_T^\alpha\,\tilde{W}^{\rm siv}(b_T,Q)\right]\Bigg|_{b_T=0}$$

$$=\frac{\partial}{\partial b_{T\mu}}\left[b_T^\alpha\cdot \underbrace{b_T^a\times(\log\text{ corrections})}_{S_{pert}\text{ is same for unpol. and pol.}}\right] \qquad \mathbf{= 0}$$

Improved CSS...

$$W^{\rm siv}(q_T,Q) \to W^{\rm siv}_{\rm New}(q_T,Q;\eta,C_5) \equiv \Xi\left(\frac{q_T}{Q},\eta\right)\int \frac{d^2b_T}{(2\pi)^2}\, e^{iq_T\cdot b_T}\, \tilde{W}^{\rm siv}(b_c(b_T),Q)$$

Improved CSS...

$$W^{\rm siv}(q_T,Q) \to W^{\rm siv}_{\rm New}(q_T,Q;\eta,C_5) \equiv \Xi\left(\frac{q_T}{Q},\eta\right) \int \frac{d^2 b_T}{(2\pi)^2}\, e^{iq_T\cdot b_T}\, \tilde{W}^{\rm siv}(b_c(b_T),Q)$$

$$\int d^2 q_T\, \epsilon_{T\mu\nu}\, q_T^\mu S_T^\nu\, \Gamma(q_T,Q) = \frac{1}{2}\epsilon_{T\mu\nu}\epsilon_{T\alpha\beta}\, S_T^\nu S_T^\beta\, \frac{\partial}{\partial b_{T\mu}}\left[b_T^\alpha\, \tilde{W}^{\rm siv}(b_c(b_T),Q)\right]\Bigg|_{b_T=0} + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

Improved CSS...

$$W^{\rm siv}(q_T,Q) \to W^{\rm siv}_{\rm New}(q_T,Q;\eta,C_5) \equiv \Xi\left(\frac{q_T}{Q},\eta\right) \int \frac{d^2b_T}{(2\pi)^2}\, e^{iq_T\cdot b_T}\, \tilde{W}^{\rm siv}(b_c(b_T),Q)$$

$$\int d^2q_T\, \epsilon_{T\mu\nu}\, q_T^\mu S_T^\nu\; \Gamma(q_T,Q) = \frac{1}{2}\epsilon_{T\mu\nu}\epsilon_{T\alpha\beta}\, S_T^\nu S_T^\beta\, \frac{\partial}{\partial b_{T\mu}}\left[b_T^\alpha \tilde{W}^{\rm siv}(b_c(b_T),Q)\right]\Big|_{b_T=0} + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

NOT replaced by $b_c(b_T)$

Improved CSS...

$$W^{\rm siv}(q_T,Q) \to W^{\rm siv}_{\rm New}(q_T,Q;\eta,C_5) \equiv \Xi\left(\frac{q_T}{Q},\eta\right)\int \frac{d^2b_T}{(2\pi)^2}\, e^{iq_T\cdot b_T}\,\tilde{W}^{\rm siv}(b_c(b_T),Q)$$

$$\int d^2q_T\,\epsilon_{T\mu\nu}\,q_T^\mu S_T^\nu\,\Gamma(q_T,Q) = \frac{1}{2}\epsilon_{T\mu\nu}\epsilon_{T\alpha\beta}\,S_T^\nu S_T^\beta\,\frac{\partial}{\partial b_{T\mu}}\left[b_T^\alpha\,\tilde{W}^{\rm siv}(b_c(b_T),Q)\right]\Big|_{b_T=0} + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

NOT replaced by $b_c(b_T)$

$$= \frac{1}{2}\tilde{W}^{\rm siv}(b_{\rm min},Q) + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

$O(1/Q)$

Improved CSS...

$$W^{\rm siv}(q_T,Q) \to W^{\rm siv}_{\rm New}(q_T,Q;\eta,C_5) \equiv \Xi\left(\frac{q_T}{Q},\eta\right)\int \frac{d^2b_T}{(2\pi)^2}\, e^{iq_T\cdot b_T}\, \tilde{W}^{\rm siv}(b_c(b_T),Q)$$

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NOT replaced by $b_c(b_T)$

$$= \frac{1}{2}\tilde{W}^{\rm siv}(b_{\rm min},Q) + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

O(1/Q)

O(Q)

$$= H^{\rm siv}_{LO,jj}(\mu_Q,Q)\left[\pi\, M F_{FT,\,j/A}(x,x;\mu_c)\right] D_{1\,B/j}(z;\mu_c) + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

Improved CSS...

$$W^{\rm siv}(q_T,Q) \to W^{\rm siv}_{\rm New}(q_T,Q;\eta,C_5) \equiv \Xi\left(\frac{q_T}{Q},\eta\right)\int \frac{d^2b_T}{(2\pi)^2}\, e^{iq_T\cdot b_T}\, \tilde{W}^{\rm siv}(b_c(b_T),Q)$$

$$\int d^2q_T\, \epsilon_{T\mu\nu}\, q_T^\mu S_T^\nu\; \Gamma(q_T,Q) = \frac{1}{2}\epsilon_{T\mu\nu}\epsilon_{T\alpha\beta}\, S_T^\nu S_T^\beta\, \frac{\partial}{\partial b_{T\mu}}\left[b_T^\alpha\,\tilde{W}^{\rm siv}(b_c(b_T),Q)\right]\Big|_{b_T=0} \qquad + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

NOT replaced by $b_c(b_T)$

$$= \frac{1}{2}\tilde{W}^{\rm siv}(b_{\rm min},Q) \qquad + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

O(1/Q)

O(Q)

$$= H^{\rm siv}_{LO,jj}(\mu_Q,Q)\,\left[\pi\, M F_{FT,\,j/A}(x,x;\mu_c)\right] D_{1\,B/j}(z;\mu_c) + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

Also can be shown with Bessel weighting

(Boer, Gamberg, Musch, Prokudin (2011))

Improved CSS...

$$W^{\rm siv}(q_T,Q) \to W^{\rm siv}_{\rm New}(q_T,Q;\eta,C_5) \equiv \Xi\left(\frac{q_T}{Q},\eta\right)\int\frac{d^2b_T}{(2\pi)^2}\,e^{iq_T\cdot b_T}\,\tilde{W}^{\rm siv}(b_c(b_T),Q)$$

$$\int d^2q_T\,\epsilon_{T\mu\nu}\,q_T^\mu S_T^\nu\,\Gamma(q_T,Q) = \frac{1}{2}\epsilon_{T\mu\nu}\epsilon_{T\alpha\beta}\,S_T^\nu S_T^\beta\,\frac{\partial}{\partial b_{T\mu}}\left[b_T^\alpha\tilde{W}^{\rm siv}(b_c(b_T),Q)\right]\Big|_{b_T=0} + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

NOT replaced by $b_c(b_T)$

$$= \frac{1}{2}\tilde{W}^{\rm siv}(b_{\rm min},Q) + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

O(1/Q)

O(Q)

$$= H^{\rm siv}_{LO,jj}(\mu_Q,Q)\left[\pi\,MF_{FT,\,j/A}(x,x;\mu_c)\right]D_{1\,B/j}(z;\mu_c) + \underbrace{O(\alpha_s(Q)) + O((m/Q)^{p'})}$$

from Y_{new} & (1−Ξ) terms, evolution,
NLO "C-factors", and OPE

Improved CSS...

$$W^{\rm siv}(q_T,Q) \to W^{\rm siv}_{\rm New}(q_T,Q;\eta,C_5) \equiv \Xi\left(\frac{q_T}{Q},\eta\right) \int \frac{d^2 b_T}{(2\pi)^2}\, e^{iq_T\cdot b_T}\, \tilde{W}^{\rm siv}(b_c(b_T),Q)$$

$$\int d^2 q_T\, \epsilon_{T\mu\nu}\, q_T^\mu S_T^\nu\, \Gamma(q_T,Q) = \frac{1}{2}\epsilon_{T\mu\nu}\epsilon_{T\alpha\beta}\, S_T^\nu S_T^\beta\, \frac{\partial}{\partial b_{T\mu}}\left[b_T^\alpha \tilde{W}^{\rm siv}(b_c(b_T),Q)\right]\Big|_{b_T=0} \quad + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

NOT replaced by $b_c(b_T)$

$$= \frac{1}{2}\tilde{W}^{\rm siv}(b_{\rm min},Q) \quad + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

O(1/Q)

O(Q)

$$= H^{\rm siv}_{LO,jj}(\mu_Q,Q)\left[\pi\, M F_{FT,\,j/A}(x,x;\mu_c)\right] D_{1\,B/j}(z;\mu_c) + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

The LO result is recovered, but what are the size of these corrections?
May not be the same as the unpolarized case because of the q_T weight...

Towards a Global Analysis of TMD and CT3 Observables

$$H^q(z) \;=\; -2z\, H_1^{\perp(1),q}(z) + 2z \int_z^\infty \frac{dz_1}{z_1^2}\,\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\,\hat{H}_{FU}^{q,\Im}(z,z_1)$$

QCD e.o.m. relation (EOMR)

$$\frac{H^q(z)}{z} = -\left(1 - z\frac{d}{dz}\right) H_1^{\perp(1),q}(z) - \frac{2}{z}\int_z^\infty \frac{dz_1}{z_1^2}\,\frac{\hat{H}_{FU}^{q,\Im}(z,z_1)}{(1/z - 1/z_1)^2}$$

Lorentz invariance relation (LIR)

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

$$H^q(z) = -2z\, H_1^{\perp(1),q}(z) + 2z \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{q,\Im}(z, z_1)$$

QCD e.o.m. relation (EOMR)

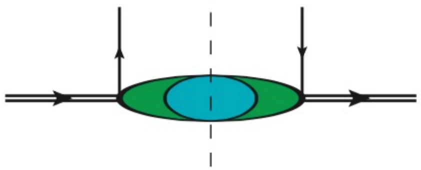
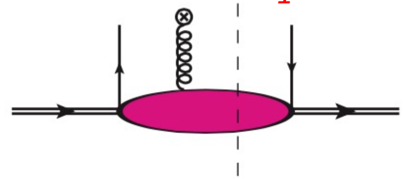
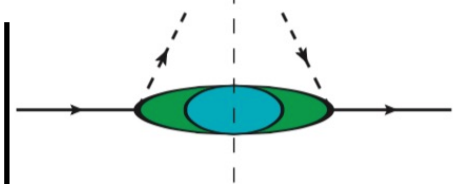
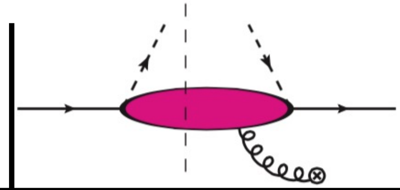
$$\frac{H^q(z)}{z} = -\left(1 - z\frac{d}{dz}\right) H_1^{\perp(1),q}(z) - \frac{2}{z} \int_z^\infty \frac{dz_1}{z_1^2} \frac{\hat{H}_{FU}^{q,\Im}(z, z_1)}{(1/z - 1/z_1)^2}$$

Lorentz invariance relation (LIR)

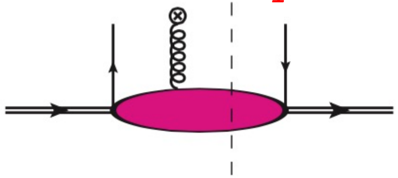
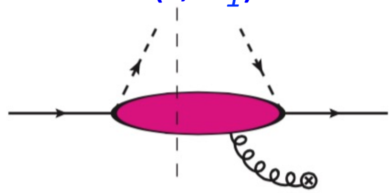
$$H(z) = \int_z^1 dz_1 \int_{z_1}^\infty \frac{dz_2}{z_2^2}\, 2 \left[\frac{\left(2(\frac{2}{z_1} - \frac{1}{z_2}) + \frac{1}{z_1}\left(\frac{1}{z_1} - \frac{1}{z_2}\right) \delta\left(\frac{1}{z_1} - \frac{1}{z}\right)\right)}{\left(\frac{1}{z_1} - \frac{1}{z_2}\right)^2} \hat{H}_{FU}^{\Im}(z_1, z_2) \right]$$

$$H_1^{\perp(1)}(z) = -\frac{2}{z} \int_z^1 dz_1 \int_{z_1}^\infty \frac{dz_2}{z_2^2} \frac{\left(\frac{2}{z_1} - \frac{1}{z_2}\right)}{\left(\frac{1}{z_1} - \frac{1}{z_2}\right)^2} \hat{H}_{FU}^{\Im}(z_1, z_2)$$

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

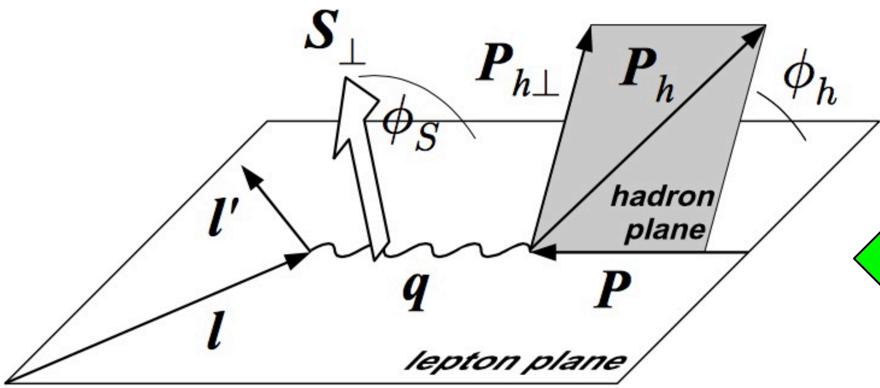
Hadron Pol.	PDF (x) 		PDF (x, x_1) 	FF (z) 		FF (z, z_1) 
	intrinsic	kinematical	dynamical	intrinsic	kinematical	dynamical
U	$\cancel{e}$	$\cancel{h_1^{\perp(1)}}$	H_{FU}	$\cancel{E}, \cancel{H}$	$\cancel{H_1^{\perp(1)}}$	$\hat{H}_{FU}^{\mathfrak{R},\mathfrak{I}}$
L	$\cancel{h_L}$	$\cancel{h_{1L}^{\perp(1)}}$	H_{FL}	$\cancel{E_L}, \cancel{H_L}$	$\cancel{H_{1L}^{\perp(1)}}$	$\hat{H}_{FL}^{\mathfrak{R},\mathfrak{I}}$
T	$\cancel{g_T}$	$\cancel{f_{1T}^{\perp(1)}}$, $\cancel{g_{1T}^{\perp(1)}}$	F_{FT}, G_{FT}	$\cancel{D_T}, \cancel{G_T}$	$\cancel{D_{1T}^{\perp(1)}}$, $\cancel{G_{1T}^{\perp(1)}}$	$\hat{D}_{FT}^{\mathfrak{R},\mathfrak{I}}, \hat{G}_{FT}^{\mathfrak{R},\mathfrak{I}}$

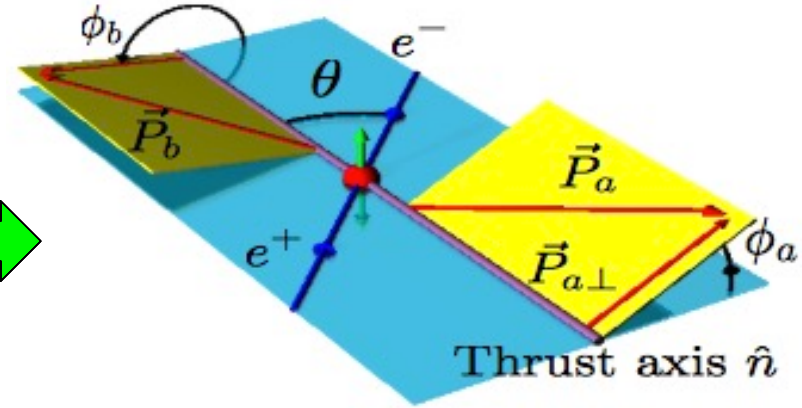
(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

Hadron Pol.	PDF (x, x_1) 	FF (z, z_1) 
	dynamical	dynamical
U	H_{FU}	$\hat{H}_{FU}^{\Re,\Im}$
L	H_{FL}	$\hat{H}_{FL}^{\Re,\Im}$
T	F_{FT}, G_{FT}	$\hat{D}_{FT}^{\Re,\Im}, \hat{G}_{FT}^{\Re,\Im}$

ALL *transverse spin observables* are driven by *multi-parton correlations*

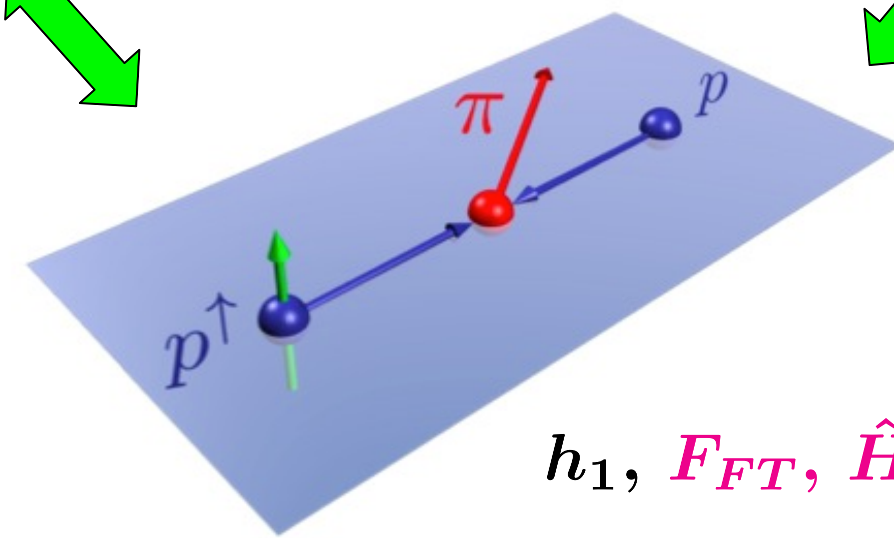
(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))



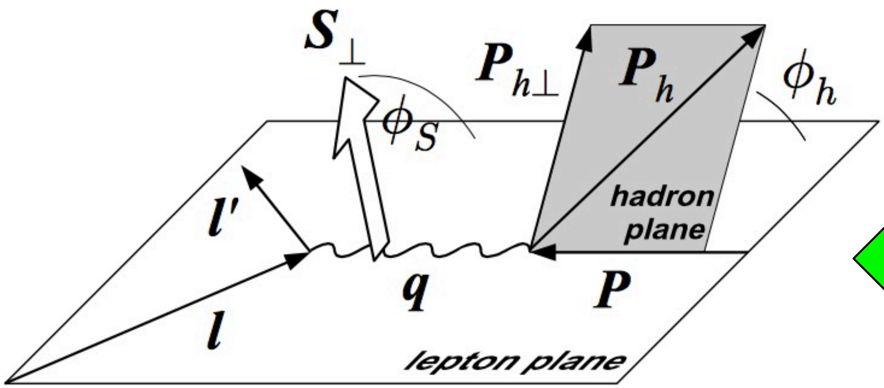


$h_1,\ F_{FT},\ \hat{H}_{FU}^{\Im}$

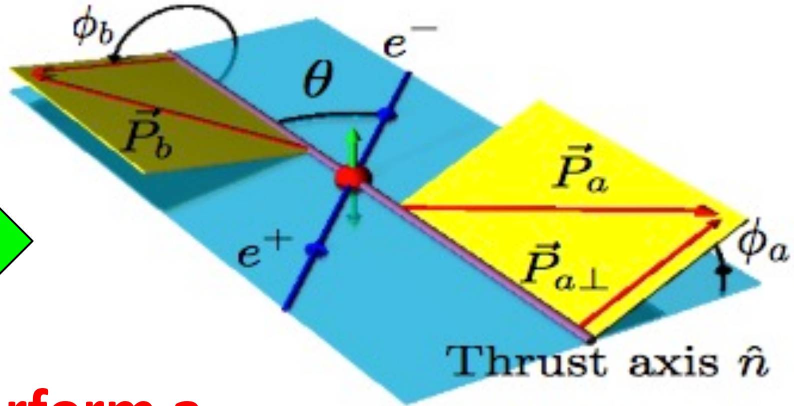
$\hat{H}_{FU}^{\Im}$



$h_1,\ F_{FT},\ \hat{H}_{FU}^{\Im}$

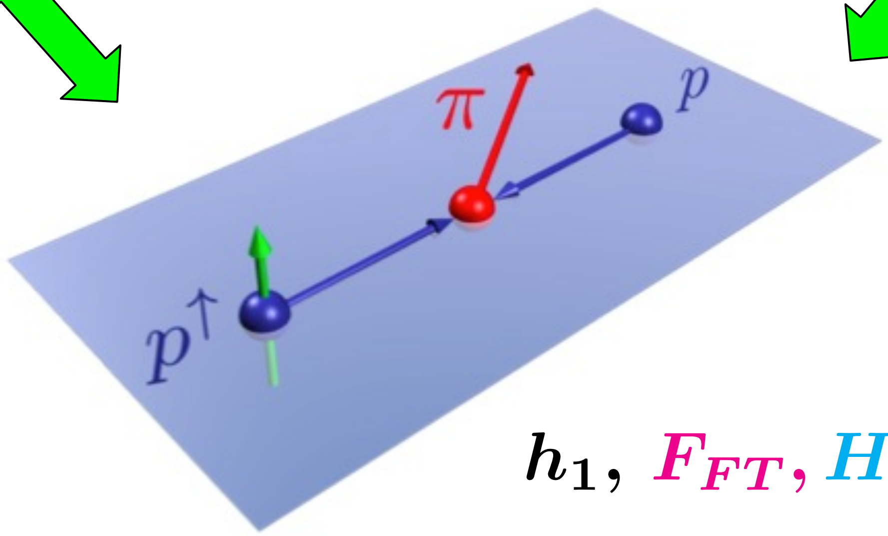


$h_1, \; F_{FT}, H_1^{\perp(1)}, \; \tilde{H}$



$H_1^{\perp(1)}, \; \tilde{H}$

Need to perform a "global" analysis!



$h_1, \; F_{FT}, H_1^{\perp(1)}, \tilde{H}$

Summary

- TSSAs have been studied in both TMD processes (SIDIS, e^+e^-, DY) and collinear processes (A_N in proton-proton & lepton-proton collisions)
- The current (improved CSS) TMD evolution formalism allows one to rigorously connect these two frameworks, although more work is needed
- (LIRs + EOMRs + TMD evolution) = ***ALL*** transverse spin observables are driven by 3-parton (dynamical) functions
- Global analysis of TMD *AND* collinear twist-3 observables is possible