# Fragmentation functions and implications for semi-inclusive DIS 



## TMDs - a global approach




## TMDs - a global approach





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## Probing TMDs through fragmentation



in SIDIS*) couple PDFs to:
*) semi-inclusive DIS with unpolarized final state

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$\rightarrow \rightarrow$ FFs act as quark flavor-tagger and polarimeter
*) semi-inclusive DIS with unpolarized final state

## fragmentation in $e^{+} e^{-}$annihilation

- single-inclusive hadron production, $e^{+} e^{-} \rightarrow h X$
- $D_{1}$ fragmentation fctn.
- $D_{1 T}{ }^{\perp}$ spontaneous transv. pol.


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- product of $D_{1}$ or of Collins FFs
- flavor, transverse-momentum, and/or polarization tagging
- inclusive same-hemisphere hadron pairs, $e^{+} e^{-} \rightarrow h_{1} h_{2} X$
- dihadron fragmentation


Thrust (axis):

$$
T \stackrel{\max }{=} \frac{\sum_{h}\left|\mathbf{P}_{h}^{\mathrm{CMS}} \cdot \hat{\mathbf{n}}\right|}{\sum_{h}\left|\mathbf{P}_{h}^{\mathrm{CMS}}\right|}
$$



## $e^{+} e^{-}$annihilation at BaBar, Belle, and BESIII

- BaBar/Belle: asymmetric beam-energy e+e-collider near/at $\Upsilon(4 S)$ resonance (10.58 GeV)
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- integrated luminosities:


|  | $\Upsilon(4 S)$ <br> on resonance | $\Upsilon(4 S)$ <br> off resonance | other |
| :---: | :---: | :---: | :---: |
| BaBar | $424.2 \mathrm{fb}^{-1}$ | $43.9 \mathrm{fb}^{-1}$ |  |
| Belle | $(140+571) \mathrm{fb}^{-1}$ | $(15.6+73.8) \mathrm{fb}^{-1}$ |  |
| BESIII |  |  | $\left.\sim 62 \mathrm{pb}^{-1} @ 3.65 \mathrm{GeV} *\right)$ |

*) used for the Collins analysis presented here

## from hadron yields to cross sections

- hadron yields undergo series of corrections
- smearing unfolding [e.9., measured and true momentum might differ]
- particle (mis)identification [e.g., not every identified pion was a pion]
- non- $q \bar{q}$ processes [e.g., two-photon processes, $\Upsilon \rightarrow B B, \ldots]$
- " $4 \pi$ " correction [limited geometric acceptance and selection criteria]
- QED radiation [initial-state radiation (ISR)]
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- partially different approaches in different experiments/analyses


## single-hadron production

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- limits analysis of evolution and gluon fragmentation
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- now, results available from BaBar and Belle:

- BaBar Collaboration, Phys. Rev. D88 (2013) 032011: $\pi^{ \pm}, K^{ \pm}, ~ p+p$
- Belle Collaboration, Phys. Rev. Lett. 111 (2013) 062002: $\pi^{ \pm}$, K
- Belle Collaboration, Phys. Rev. $\operatorname{D92}$ (2015) 092007: $\pi^{ \pm}, K^{ \pm}, ~ p+p$


## single-hadron production

- very precise data for charged pions and kaons
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- included in recent DEHSS fits
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- also available: data for protons and anti-protons
- not (yet) included in DSS++
- similar z dependence as pions
- about $\sim 1 / 5$ of pion cross sections




## single-hadron production

- pion and(?) kaon data reasonably well described by Jetset
- protons difficult to reproduce, especially at large $z$
- MC overshoots data




## hadron-pair production

- single-hadron production has low discriminating power for parton flavor
- can use $2^{\text {nd }}$ hadron in opposite hemisphere to "tag" flavor (\& polarization)
- mainly sensitive to product of singlehadron FFs
- if hadrons in same hemisphere:

Thrust (axis):

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$$



- dihadron fragmentation a la de Florian \& Vanni [Phys. Lett. B 578 (2004) 139]
- dihadron fragmentation a la Collins, Heppelmann \& Ladinsky [Nucl. Phys. B 420 (1994) 565]; Boer, Jacobs \& Radici [Phys. Rev. D 67 (2003) 094003]



# hadron-pair production 

no hemisphere preference
[Phys. Rev. D92 (2015) 0920071


## hadron-pairs: weak-decay contributions

- not all hadrons originate from uds quarks but e.g., from $D$ decay
- here only $z_{1}=z_{2}$ diagonal bins
[Phys. Rev. D92 (2015) 092007]



## hadron-pairs: topology comparison

- any hemisphere vs. opposite- \& same-hemisphere pairs
- same-hemisphere pairs with kinematic limit at $z_{1}=z_{2}=0.5$

Phys. Rev. D92 (2015) 092007]


## hadron-pairs: comparison with PYTHIA

- generally good agreement at low z
- at large z only present Belle and PYTHIA default tunes satisfactory



## hadron-pairs: angular correlations

- angular correlations between nearly back-to-back hadrons used to tag transverse quark polarization -> Collins fragmentation functions
- RFO: one hadron as reference axis $\rightarrow \cos (2 \phi$ ) modulation
- RF12: thrust (or similar) axis $\quad-\cos \left(\phi_{1}+\phi_{2}\right)$ modulation

- RFO and RF12: different convolutions over transverse momenta
- debatable: MC used to "correct" thrust axis to $q \bar{q}$ axis


## hadron-pairs: angular correlations

- challenge: large modulations even without Collins effect (e.g., MC)



## hadron-pairs: angular correlations

- challenge: large modulations even without Collins effect (e.g., MC)
- construct double ratio of normalized-yield distributions $R_{12}$, e.g. unlike-/like-sign:

$$
\begin{equation*}
\frac{R_{12}^{U}}{R_{12}^{L}} \simeq \frac{1+\left\langle\frac{\sin ^{2} \theta_{\mathrm{th}}}{1+\cos ^{2} \theta_{\mathrm{th}}}\right\rangle G^{U} \cos \left(\phi_{1}+\phi_{2}\right)}{1+\left\langle\frac{\sin ^{2} \theta_{\mathrm{th}}}{1+\cos ^{2} \theta_{\mathrm{th}}}\right\rangle G^{L} \cos \left(\phi_{1}+\phi_{2}\right)} \tag{a}
\end{equation*}
$$



$$
\simeq 1+\left\langle\frac{\sin ^{2} \theta_{\mathrm{th}}}{1+\cos ^{2} \theta_{\mathrm{th}}}\right\rangle\left\{G^{U}-G^{L}\right\} \cos \left(\phi_{1}+\phi_{2}\right)
$$



- suppresses flavor-independent sources of modulations
- $G^{U / L}$ specific combinations of FFs
- remaining MC asym.'s: systematics
$\phi_{1}+\phi_{2}(\mathrm{rad})$


## Collins asymmetries (RFO)

- first measurement of Collins asymmetries by Belle [PRL 96 (2006) 232002, PRD 78 (2008) 032011, PRD 86 (2012) 039905(E)]
- significant asymmetries rising with z
- used for first transversity and Collins FF extractions



## Collins asymmetries (RFO)

[Phys. Rev. D90 (2014) 052003]


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- BaBar results [PRD 90 (2014) 052003] consistent with Belle
- BESIII [PRL 116 (2016) 042001] (a $\dagger$ smaller s) consistent with TMD evolution [Z.-B. Kang et al., PRD 93 (2016) 014009]



## Collins asymmetries - going further



- even larger effects seen for kaon pairs


## Collins asymmetries - going further



- even larger effects seen for kaon pairs

- p+ dependence for pions


## what to further expect from $e^{+} e^{-}$

- dihadron fragmentation function: $M_{h 1 h 2}$ dependence (Belle)
- helicity-dependent dihadron fragmentation $G_{1}{ }^{\perp}$ ("jet handedness") (Belle)
- kaon and pion-kaon pairs as well as $p_{+}$dependence of Collins asymmetries (Belle, BESIII)
- Collins asymmetries without double ratios (BaBar, BESIII)
- $\mathrm{k}_{\mathrm{T}}$-dependent $\mathrm{D}_{1}$ FFs (Belle)
- nearly back-to-back hadrons
- hadron-to-thrust
- transverse polarization of inclusively produced $\wedge^{0}$ hyperons (Belle)


## "pitfalls" in dihadron fragmentation



- dihadron FFs: alternative path to extract (collinear) transversity
- exploit orientation of hadron's relative momentum, correlate with target polarization
- complication: SIDIS cross section now differential in 9(!) variables
- integration over polar angle eliminates, in theory, a number of contributing FFs (partial waves)
- experimental constraints limit acceptance in polar angle, most prominently the minimum-momentum requirements


## simple case study

## basic assumptions:



- dihadron pair with equal-mass hadrons; here: pions
- $e^{+} e^{-}$annihilation, thus energy fractions $z$ translates directly to energy/momentum of particles/system as primary energy is "fixed" (-> simplifies Lorentz boost)
- without loss of generality, focus on B factory and use primary quark energy $E_{0}=5.79 \mathrm{GeV}$
- minimum energy of each pion in lab frame: $0.1 E_{0}$ (i.e., $z_{\text {min }}=0.1$ )


## application of Lorentz boost

- can easily apply Lorentz boost using the invariant mass of the dihadron $M$ and its energy $z E_{0}$ to arrive at condition on $\theta$, e.g., polar angle of pions in center-of-mass frame:

$$
\cos \theta \leq \frac{z-2 z_{\min }}{\sqrt{\left.\left[\left(z E_{0}\right)^{2}-M^{2}\right)\left(M^{2}-4 m_{\pi}^{2}\right)\right]}} E_{0} M
$$

- as both pions have to fulfill the constraint on the minimum energy:

$$
\cos (\pi-\theta)=-\cos \theta \leq \frac{z-2 z_{\min }}{\sqrt{\left.\left[\left(z E_{0}\right)^{2}-M^{2}\right)\left(M^{2}-4 m_{\pi}^{2}\right)\right]}} E_{0} M
$$

thus:

$$
|\cos \theta| \leq \frac{z-2 z_{\min }}{\sqrt{\left.\left[\left(z E_{0}\right)^{2}-M^{2}\right)\left(M^{2}-4 m_{\pi}^{2}\right)\right]}} E_{0} M
$$

- translates to a symmetric range around $\pi / 2$
(can be easily understood because at $\pi / 2$ the pions will have both the same energy in the lab and easily pass the $z_{\text {min }}$ requirement, while in the case of one pion going backward in the CMS, that pion will have less energy in the lab frame ... and maybe too little)


## impact of $z_{\text {min }}=0.1$ on accepted polar range

- (again without loss of generality) let's assume $M=0.5 \mathrm{GeV}$ :

- all theta below curve (and above its mirror curve relative to dashed line) are excluded
- clearly limited, especially at low z


## partial-wave expansion of dihadron FF

- partial-wave expansion worked out in Phys. Rev. D67 (2003) 094002
- for the particular case here, use Phys. Rev. D74 (2006) 114007, in particular Eq. (12), and (later on) Figure 5:

$$
\begin{align*}
D_{1}^{q}\left(z, \cos \theta, M_{h}^{2}\right) \approx & D_{1, o o}^{q}\left(z, M_{h}^{2}\right)+D_{1, o l}^{q}\left(z, M_{h}^{2}\right) \cos \theta \\
& +D_{1, l l}^{q}\left(z, M_{h}^{2}\right) \frac{1}{4}\left(3 \cos ^{2} \theta-1\right), \tag{12}
\end{align*}
$$

- it is the first contribution ( $\mathrm{D}_{1,00}$ ) that is used in "collinear extraction" of transversity (and subject of a current Belle analysis)
- it is also the only one surviving the integration over $\theta$
- the $D_{1,0}$ contribution vanishes upon integration over $\theta$ as long as the theta range is symmetric around $\pi / 2$ (as it is the case here)
- the $D_{1, \| l}$ term, however, will in general contribute in case of only partial integration over $\theta$ - the question is how much?


## $D_{1, I l}$ contribution to dihadron fragmentation

- $D_{1, \|}$ is unknown and can'† be calculated using first principles
- it can not be extracted from cross sections integrated over $\theta$
- upon (partial) integration there is no way to disentangle the two contributions
- in PRD74 (2006) 114007, a model for dihadron fragmentation was tuned to PYTHIA and used to estimate the various partial-wave contributions
- its Figure 5 gives an indication about the relative size of $D_{1, \|}$ vs. $D_{1,00}$ :




## effect of partial integration

- as both contributions - $D_{1, l l}$ and $D_{1,00}$ - will be affected by the partial integration, look at relative size of the $D_{1, \|}$ to $D_{1,00}$ modulations when subjected to integration:

$$
\frac{D_{1, \|}}{D_{1,00}} \frac{\int_{\cos \left(\pi-\theta_{0}\right)}^{\cos \theta_{0}} d \cos \theta \frac{1}{4}\left(3 \cos ^{2} \theta-1\right)}{\int_{\cos \left(\pi-\theta_{0}\right)}^{\cos \theta_{0}} d \cos \theta}=-\frac{1}{4}\left(1-\cos ^{2} \theta_{0}\right) \frac{D_{1, \|}}{D_{1,00}}
$$

- without limit in the polar-angular range $\left(\theta_{0}=0\right)$-> no contribution from $\mathrm{D}_{1, I I}$ (sanity check!)
- the relative size of the partial integrals reaches a maximum of $25 \%$ for $z=0.2$ (i.e., pions at 90 degrees in center-of-mass system)
- in order to estimate the $D_{1, \| l}$ contribution, one "just" needs the relative size of $D_{1, \| l}$ vs. $D_{1,00}$, e.g., Figure 5 of PRD74 (2006) 114007
- let's take for that size 0.5 (rough value for $M=0.5 \mathrm{GeV}$ )


## effect of partial integration

- ... $D_{1, I I} / D_{1,00} \sim 0.5$ results in an up to $O(10 \%)$ effect on the measured cross section:

- depending on the sign of $D_{1, \| 1}$, the partial integration thus leads to a systematic underestimation (positive $D_{1, I I}$ ) or overestimation (negative $D_{1, I I}$ ) of the "integrated" dihadron cross section
- leads to overestimate/underestimate of extracted transversity


## conclusions

- $e^{+} e^{-}$data has provided a rich precision data set for fragmentation studies
- input to $D_{1}$ FF phenomenology
- hadron-pair data could further constrain flavor dependence
- transverse-momentum dependence on the horizon
- Collins asymmetries available for pions and kaons, at different $s$ and by now also $\mathrm{P}_{T}$ dependent
- dihadron fragmentation for, e.g., collinear extraction of transversity


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- Collins asymmetries available for pions and kaons, at different s and by now also $\mathrm{P}_{T}$ dependent
- dihadron fragmentation for, e.g., collinear extraction of transversity
- however, precision $=/=$ accuracy (at least not always)
- e.g., partial-wave contributions can survive due to experimental constraints
- discussed for $e^{+} e^{-}$, but even more so for SIDIS or pp->h $h_{1} h_{2} X$
- important to keep in mind when aiming for precision measurements

