Renormalization Issues of quasi-PDFs

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In collaboration with:

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QCD Evolution 2017

May 23, 2017

IN THIS TALK

A. Introduction to quasi-PDFs

B. quasi-PDFs in Lattice QCD

C. Perturbative Renormalization

D. Linear Divergence Fit

E. Non-perturbative Renormalization & Linear Divergence

E. Discussion

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A. Introduction to quasi-PDFs

B. quasi-PDFs in Lattice QCD

C. Perturbative Renormalization

D. Linear Divergence Fit Backup Slides

E. Non-perturbative Renormalization & Linear Divergence

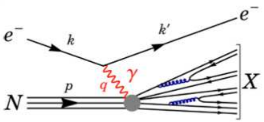
E. Discussion

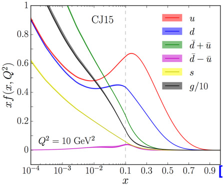
A

INTRODUCTION TO

quasi-PDFs

Probing Nucleon Structure





CJ15 PDFs

[A. Accardi et al., arXiv:1602.03154]

Parton Distribution Functions

- ★ **powerful tool to describe the structure of a nucleon**
- ★ **necessary for the analysis of Deep inelastic scattering (DIS) data**
- ★ **Parametrization of off-forward matrix of a bilocal quark operator (light-like)**

$$F_\Gamma(x,\xi,q^2) = \frac{1}{2}\int\frac{d\lambda}{2\pi}e^{ix\lambda}\langle p'|\bar{\psi}(-\lambda n/2)\,\mathcal{O}\,\underbrace{\mathcal{P}e^{ig\int_{-\lambda/2}^{\lambda/2} d\alpha n\cdot A(n\alpha)}}_{\text{gauge invariance}}\psi(\lambda n/2)|p\rangle$$

$q = p' - p$, $\bar{P} = (p' + p)/2$, n: **light-cone vector** ($\bar{P}.n = 1$), $\xi = -n \cdot \Delta/2$

PDFs on the Lattice

- **Unpolarized** (vector current)
- **Polarized** (axial current)
- **Transversity** (tensor current)

★ **first principle calculations of PDFs are necessary**

★ **On the lattice: long history of moments of PDFs**

$$\boxed{f^n = \int_{-1}^{1} dx\, x^n f(x)}$$

★ **rely on OPE to reconstruct the PDFs** (difficult task):

- **signal-to-noise is bad for higher moments**
- **n $>$ 3: operator mixing (unavoidable!)**
- **gluon moments: limited progress**
  (discon. diagram, signal quality, operator mixing)

PDFs on the Lattice

Novel direct approach: [X.Ji, arXiv:1305.1539]

- ★ **compute quasi-PDF on the lattice**
- ★ **contact with physical PDFs on two steps:**

PDFs on the Lattice

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★ **compute quasi-PDF on the lattice**

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1. **Renormalization of quasi-PDFs in Lattice Regularization**
2. **Matching procedure (LaMET)**

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Exploratory studies are maturing:

- ★ **Nucleon Matrix Elements**

  [H-W. Lin et al., arXiv:1402.1462], [C.Alexandrou et al., arXiv:1504.07455],

  [J.-W. Chen et al., arXiv:1603.06664], [C.Alexandrou et al., arXiv:1610.03689]

- ★ **Matching to physical PDFs**

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- ★ **Linear divergence / renormalization of quasi-PDFs**

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Talks by:
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- ★ Renormalization and mixing of quasi-PDFs: This Talk

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K. Orginos
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B

quasi-PDFs

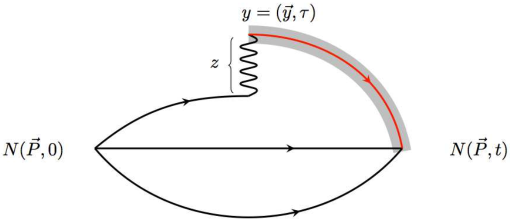
IN LATTICE QCD

Access of PDFs on a Euclidean Lattice

★ **quasi-PDF purely spatial for nucleons with finite momentum**

$$\boxed{\tilde{q}(x,\mu^2,P_3) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z} \langle N(P_3)|\bar{\Psi}(z)\,\gamma^z\,\mathcal{A}(z,0)\Psi(0)|N(P_3)\rangle_{\mu^2}}$$

- $\mathcal{A}(z,0)$: Wilson line from $0 \to z$
- z: distance in any spatial direction **(momentum boost in z direction)**



★ **At finite but feasibly large momenta on the lattice:**

a large momentum EFT can relate Euclidean $\tilde{q}$ to PDFs through a factorization theorem

★ **use of Perturbation Theory for the matching**

Bare Nucleon Matrix Elements

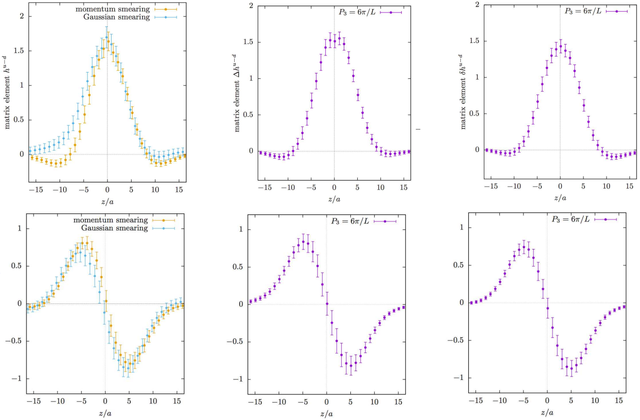
[C. Alexandrou et al. (ETMC), arXiv:1504.07455, arXiv:1610.03689]

Twisted Mass Fermions, m_π=375MeV, P_3=$6\pi/L$

Unpolarized **Polarized** **Transversity**

Real

Imaginary



★ **Momentum smearing allows to reach higher momenta**

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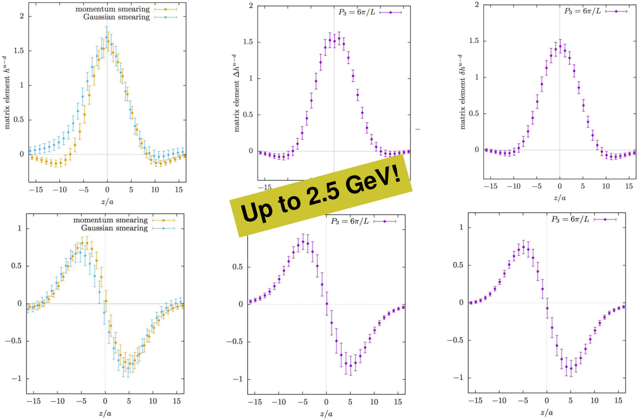

Up to 2.5 GeV!

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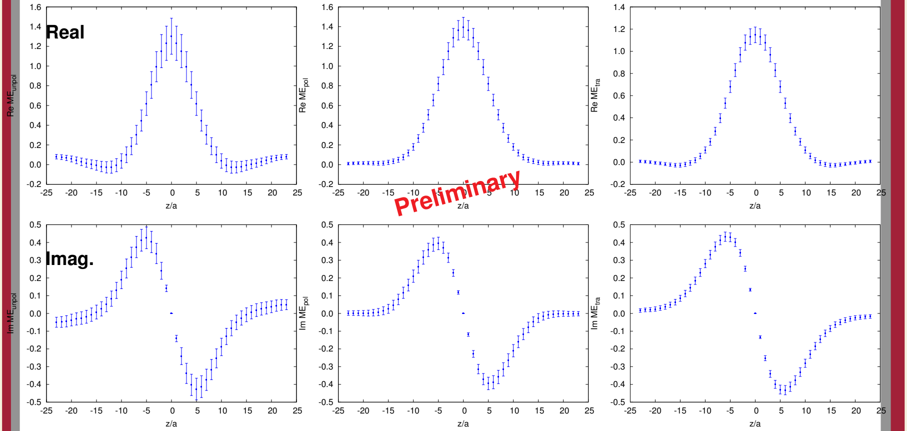
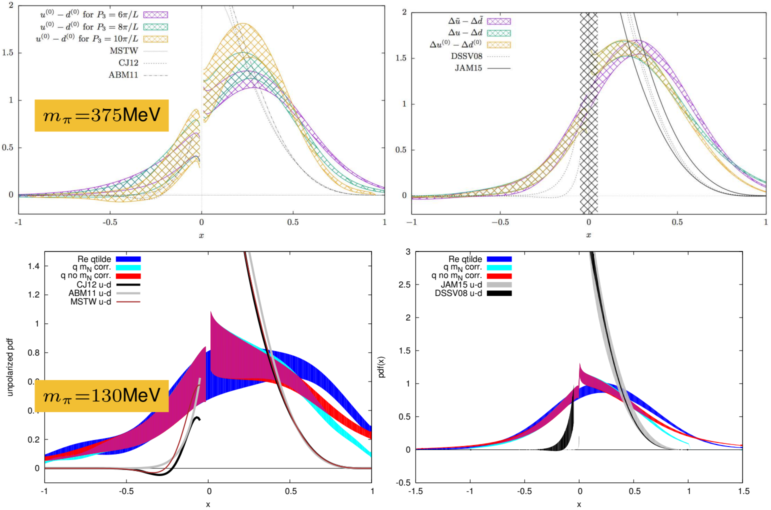


Twisted Mass Fermions & clover term, $m_\pi = 130$MeV

- ★ **Currently increasing momentum to $P_3 = 12\pi/L$**

Matching to Physical PDFs

Unpolarized **Polarized**



★ **Qualitative comparison (bare quasi-PDFs)**

C

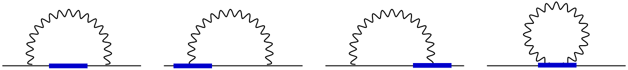
PERTURBATIVE RENORMALIZATION

Perturbative Calculation

- ★ **Operators**

  Scalar, Pseudoscalar, Vector, Axial, Tensor

- ★ **Feynman Diagrams**



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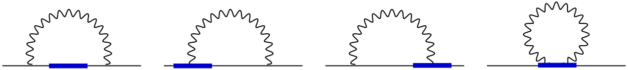

★ **Main components of calculation**

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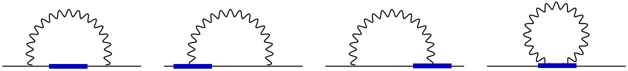

★ **Main components of calculation**

A Dimensional Regularization (DR)

- **Compute conversion factor between $\overline{MS}$ and RI'**

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Scalar, Pseudoscalar, Vector, Axial, Tensor

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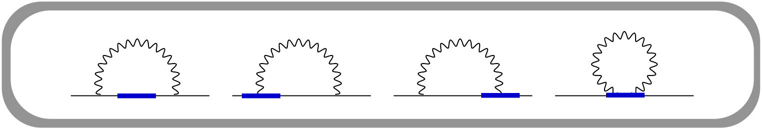


★ **Main components of calculation**

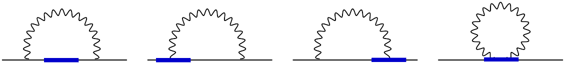
A Dimensional Regularization (DR)

- **Compute conversion factor between $\overline{MS}$ and RI'**

B Lattice Regularization (LR)

- **Extract proper Z-factors using Green's functions in both DR and LR**

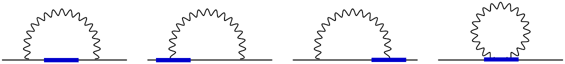
A. Dimensional Regularization



Features of DR Calculation

- ★ **No linear divergence**
- ★ **Z-factors in $\overline{\mathrm{MS}}$: real function**
- ★ **Conversion factor: a complex function**

A. Dimensional Regularization



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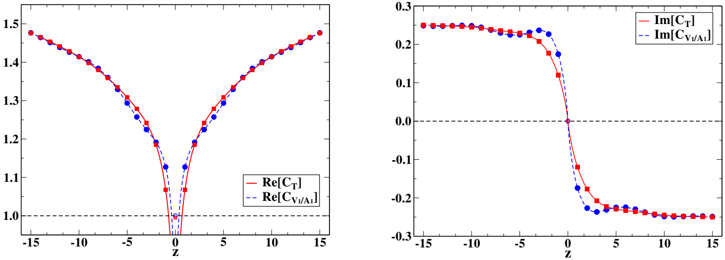
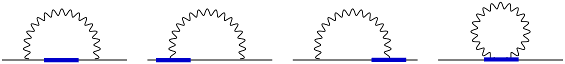


Parameters chosen based on the ETMC ensemble, Nucleon momentum: $P_3\!=\!4$

- ★ **Necessary ingredient for non-perturbative renormalization**

B. Lattice Regularization



- ★ **Linear divergence from tadpole diagram:** $\propto |z|/a$
- ★ **To all orders in pert. theory:** $e^{-c\,\frac{|z|}{a}}$ **[Dotsenko et al., NPB169 (1980) 527]**
- ★ **Green's functions complicated functions of external momentum**

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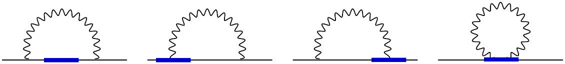

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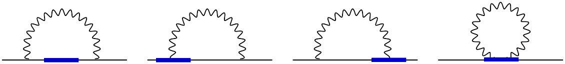
- ★ **Extraction of Z-factor**

$$\boxed{\langle\psi\,\mathcal{O}_\Gamma\,\bar{\psi}\rangle^{DR,\overline{\rm MS}}_{\rm amp} - \langle\psi\,\mathcal{O}_\Gamma\,\bar{\psi}\rangle^{LR}_{\rm amp} = \frac{g^2\,C_f}{16\,\pi^2}\,e^{i\,q_\mu\,z}\,\times\,\mathcal{F}}$$

$$\mathcal{F} = \Gamma\Big(c_1 + c_2\,\beta + c_3\,\frac{|z|}{a} + \log\left(a^2\bar{\mu}^2\right)(4-\beta)\Big) + (\Gamma\cdot\gamma_\mu + \gamma_\mu\cdot\Gamma)\,\Big(c_4 + c_5\,c_{\rm SW}\Big)$$

B. Lattice Regularization



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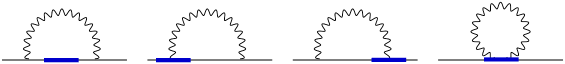
linear divergence

B. Lattice Regularization



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Results

$$Z_{\mathcal{O}}^{LR,\overline{\mathrm{MS}}} = 1 + \frac{g^2\, C_f}{16\,\pi^2}\, \left(e_1 + e_2\, \frac{|z|}{a} + e_3\, c_{\mathrm{SW}} + e_4\, c_{\mathrm{SW}}^2 - 3\log\left(a^2 \bar{\mu}^2\right) \right)$$

$$Z_{mix}^{LR,\overline{\mathrm{MS}}} = 0 + \frac{g^2\, C_f}{16\,\pi^2}\, \left(e_5 + e_6\, c_{\mathrm{SW}} \right)$$

Wherever mixing occurs

c_{SW}**: simulation parameter**

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Consequenses:

★ **Dirac structure along the Wilson line:**

Unpolarized PDF mix

Polarized & Transversity PDF do not mix

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⇒ **Perturbative results can guide simulations on quasi-PDFs**

⇒ **Mixing MUST be taken into account in quasi-PDF results**

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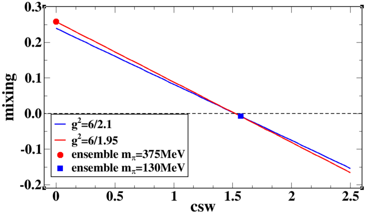
c_{SW}: **simulation parameter**

Results for selected actions

Action	e_1	e_2	e_3	e_4	e_5	e_6
Wilson	24.306	-19.955	-2.249	-1.397	14.450	-8.285
Iwasaki	12.558	-12.978	-1.601	-0.973	9.937	-6.528



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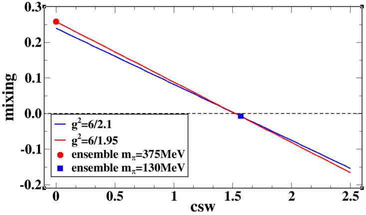

← guidance for simulations

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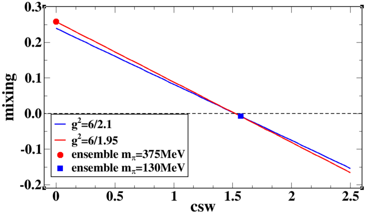

← guidance for simulations

★ **Mixing vanishes at** $c_{\rm SW} = -\, e_5/e_6$

★ **Numerical values with** $c_{\rm SW} \sim 1.5$:

mixing suppressed

E

NON-PERTURBATIVE RENORMALIZATION

Non-perturbative Renormalization

- ★ **Similar process as the renormalization of the local currents**

- ★ **Compute Z-factor on each value of z (length of Wilson Line)**

RI**-scheme:**

$$Z_{\mathcal{O}}^{LR,RI}\, Z_{\psi}^{-1}\, \frac{1}{12}\, \mathrm{Tr}\left[\mathcal{V}_{\mathcal{O}}^{LR}\, \mathcal{V}_{\mathcal{O}}^{tree}\right] = 1$$

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$$Z_{\mathcal{O}}^{LR,RI}\, Z_{\psi}^{-1}\, \tfrac{1}{12}\, \mathrm{Tr}\left[\mathcal{V}_{\mathcal{O}}^{LR}\, \mathcal{V}_{\mathcal{O}}^{tree}\right] = 1$$

- ★ $Z_{\mathcal{O}}^{LR,RI}$ **includes the linear divergence**

$$\boxed{Z_{\mathcal{O}}^{LR,RI} \equiv \mathcal{Z}_{\mathcal{O}}\, e^{-c\,\frac{|z|}{a}}}$$

(The vertex function $\mathcal{V}$ has the same divergence as the nucleon matrix element)

- ★ **Use 1-loop conversion factor to convert to the $\overline{\mathrm{MS}}$ at 2 GeV.**

Numerical Results

- ★ **Twisted Mass fermions, m_π=375MeV, $32^3 \times 64$, HYP smearing**
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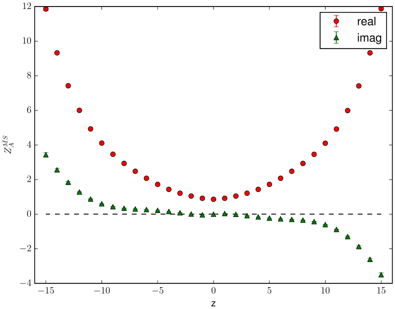

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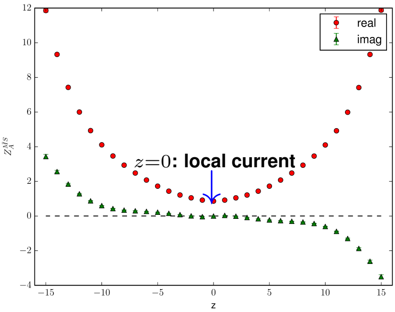

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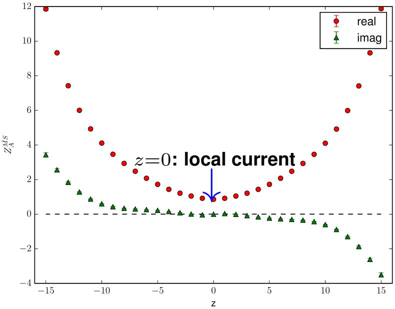
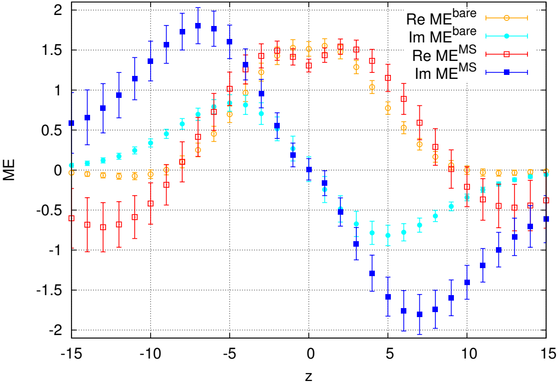


- ★ **Systematics need to be addresses:**
  - **truncation of Conversion factor (only 1-loop)**
  - **large lattice artifacts (non-diagonal momenta)**

Renormalized quasi-PDFs

Polarized case



Next step: Address systematics related to renormalization

Refining Renormalization

★ **Improvement Technique:**

- **Computation of 1-loop lattice artifacts to $\mathcal{O}(g^2\, a^\infty)$**
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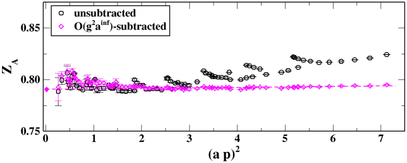

[M. Constantinou et al., arXiv:1509.00213]

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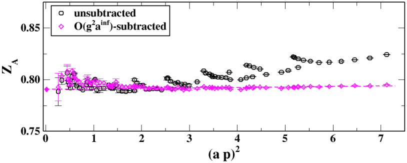

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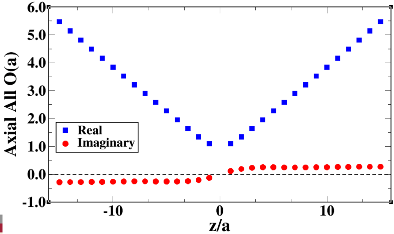
★ **Application to the quasi-PDFs:**



← **All orders in a for polarized case**

momentum $P_3\!=\!4\,\frac{2\pi}{32}$

must subtract $\mathcal{O}(a^0)$ to extract only lattice artifacts

F

DISCUSSION

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Progress in renormalization of quasi-PDFs

- ★ **Techniques to understand and remove linear divergence**
- ★ **Study of multiplicative renormalization**

  (perturbatively and non-perturbatively)
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THANK YOU!

BACKUP SLIDES

D

LINEAR DIVERGENCE FIT

Linear Divergence

Absence of mixing:

$$\mathcal{R} = \frac{q(P_3, z)}{q(P_3', z')} = e^{\left(-\frac{c}{a}+c_0\right)(|z|-|z'|)} \left(\frac{P_3}{P_3'}\right)^{-6\frac{g^2 C_f}{16\pi^2}}$$

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[R. Sommer, arXiv[1501.03060]]

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open symbols:

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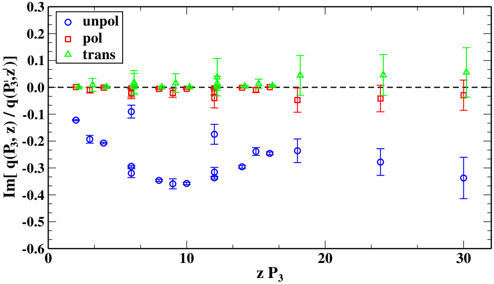

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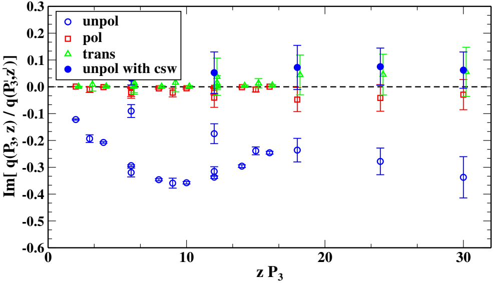

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filled symbols:

Twisted Mass
& c_{SW}

$m_\pi = 130$**MeV**

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- ★ **Presence of c_{sw} suppresses mixing**