Renormalization Issues of quasi-PDFs

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QCD Evolution 2017
May 23, 2017
A. Introduction to quasi-PDFs
B. quasi-PDFs in Lattice QCD
C. Perturbative Renormalization
D. Linear Divergence Fit
E. Non-perturbative Renormalization & Linear Divergence
E. Discussion
IN THIS TALK

A. Introduction to quasi-PDFs
B. quasi-PDFs in Lattice QCD
C. Perturbative Renormalization
D. Linear Divergence Fit Backup Slides
E. Non-perturbative Renormalization & Linear Divergence
E. Discussion
A

INTRODUCTION TO

quasi-PDFs
Parton Distribution Functions

★ powerful tool to describe the structure of a nucleon
★ necessary for the analysis of Deep inelastic scattering (DIS) data
★ Parametrization of off-forward matrix of a bilocal quark operator
  (light-like)

\[
F_\Gamma (x, \xi, q^2) = \frac{1}{2} \int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle p' | \bar{\psi} (-\lambda n/2) \ O \ P e^{-\lambda/2} \psi (\lambda n/2) | p \rangle
\]

\[
q = p' - p, \ \bar{P} = (p' + p)/2, \ n: \text{light-cone vector} \ (\bar{P} \cdot n = 1), \ \xi = -n \cdot \Delta/2
\]
PDFs on the Lattice

- Unpolarized (vector current)
- Polarized (axial current)
- Transversity (tensor current)

★ first principle calculations of PDFs are necessary
★ On the lattice: long history of moments of PDFs

\[ f^n = \int_{-1}^{1} dx \, x^n f(x) \]

★ rely on OPE to reconstruct the PDFs (difficult task):
  - signal-to-noise is bad for higher moments
  - \( n > 3 \): operator mixing (unavoidable!)
  - gluon moments: limited progress
    (discon. diagram, signal quality, operator mixing)
PDFs on the Lattice


★ compute quasi-PDF on the lattice
★ contact with physical PDFs on two steps:
PDFs on the Lattice


- compute quasi-PDF on the lattice
- contact with physical PDFs on two steps:
  1. Renormalization of quasi-PDFs in Lattice Regularization
  2. Matching procedure (LaMET)
PDFs on the Lattice


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Exploratory studies are maturing:

- **Nucleon Matrix Elements**
  - [H-W. Lin et al., arXiv:1402.1462], [C.Alexandrou et al., arXiv:1504.07455],

- **Matching to physical PDFs**
  - [X. Xiong et al., arXiv:1310.7471], [Y.-Q. Ma et al., arXiv:1412.2688],

- **Linear divergence / renormalization of quasi-PDFs**
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★ Linear divergence / renormalization of quasi-PDFs

Talks by:
K. Orginos
Y. Zhao
Y.-B. Yang
C. Monahan
PDFs on the Lattice

Novel direct approach: \([X. Ji, \text{arXiv:1305.1539}]\)

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Exploratory studies are maturing:

- **Nucleon Matrix Elements**

- **Matching to physical PDFs**
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- **Linear divergence / renormalization of quasi-PDFs**

- **Renormalization and mixing of quasi-PDFs:** This Talk

Talks by:
- K. Orginos
- Y. Zhao
- Y.-B. Yang
- C. Monahan
B

quasi-PDFs

IN LATTICE QCD
Access of PDFs on a Euclidean Lattice

☆ quasi-PDF purely spatial for nucleons with finite momentum

\[ \tilde{q}(x, \mu^2, P_3) = \int \frac{dz}{4\pi} e^{-ixP_3z} \langle N(P_3)|\bar{\Psi}(z)\gamma^z A(z, 0)\Psi(0)|N(P_3)\rangle_{\mu^2} \]

- \(A(z, 0)\): Wilson line from 0 → z
- \(z\): distance in any spatial direction (momentum boost in \(z\) direction)

☆ At finite but feasibly large momenta on the lattice:

- a large momentum EFT can relate Euclidean \(\tilde{q}\) to PDFs through a factorization theorem
- use of Perturbation Theory for the matching
Bare Nucleon Matrix Elements


Twisted Mass Fermions, \( m_\pi = 375 \text{MeV}, \ P_3 = 6\pi/L \)

Unpolarized

Polarized

Transversity

★ Momentum smearing allows to reach higher momenta
Bare Nucleon Matrix Elements


Twisted Mass Fermions, $m_\pi = 375\text{MeV}$, $P_3 = 6\pi/L$

Unpolarized

Polarized

Transversity

Momentum smearing allows to reach higher momenta up to 2.5 GeV!
Bare Nucleon Matrix Elements

$P_3 = 6\pi/L$

**Unpolarized**

**Polarized**

**Transversity**

Twisted Mass Fermions & clover term, $m_\pi = 130$MeV

★ Currently increasing momentum to $P_3 = 12\pi/L$
Matching to Physical PDFs

Unpolarized

Polarized

$m_\pi = 375$ MeV

$m_\pi = 130$ MeV

★ Qualitative comparison (bare quasi-PDFs)
PERTURBATIVE RENORMALIZATION
Perturbative Calculation

★ Operators
 Scalar, Pseudoscalar, Vector, Axial, Tensor

★ Feynman Diagrams

[Diagram of Feynman diagrams with wavy lines and lines indicating particle paths]
Perturbative Calculation

★ Operators
Scalar, Pseudoscalar, Vector, Axial, Tensor

★ Feynman Diagrams

★ Main components of calculation
Perturbative Calculation

★ Operators
Scalar, Pseudoscalar, Vector, Axial, Tensor

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★ Main components of calculation

A Dimensional Regularization (DR)

- Compute conversion factor between $\overline{MS}$ and $RI'$
Perturbative Calculation

★ Operators
Scalar, Pseudoscalar, Vector, Axial, Tensor

★ Feynman Diagrams

★ Main components of calculation
A Dimensional Regularization (DR)
• Compute conversion factor between $\overline{MS}$ and $RI'$

B Lattice Regularization (LR)
• Extract proper Z-factors using Green’s functions in both DR and LR
A. Dimensional Regularization

Features of DR Calculation

★ No linear divergence
★ Z-factors in \( \overline{\text{MS}} \): real function
★ Conversion factor: a complex function
A. Dimensional Regularization

Features of DR Calculation

★ No linear divergence
★ Z-factors in $\overline{\text{MS}}$: real function
★ Conversion factor: a complex function

Parameters chosen based on the ETMC ensemble, Nucleon momentum: $P_3 = 4$

★ Necessary ingredient for non-perturbative renormalization
B. Lattice Regularization

★ **Linear divergence from tadpole diagram:** $\propto \frac{|z|}{a}$

★ **To all orders in pert. theory:** $e^{-c \frac{|z|}{a}}$  
[Dotsenko et al., NPB169 (1980) 527]

★ **Green’s functions complicated functions of external momentum**
B. Lattice Regularization

★ Linear divergence from tadpole diagram: \( \propto \frac{|z|}{a} \)

★ To all orders in pert. theory: \( e^{-c \frac{|z|}{a}} \)

[Dotsenko et al., NPB169 (1980) 527]

★ Green’s functions complicated functions of external momentum

★ Extraction of Z-factor

\[
\langle \psi \mathcal{O}_\Gamma \bar{\psi} \rangle_{\text{amp}}^{DR, \overline{\text{MS}}} - \langle \psi \mathcal{O}_\Gamma \bar{\psi} \rangle_{\text{amp}}^{LR} = \frac{g^2 C_f}{16 \pi^2} e^{i q_\mu z} \times \mathcal{F}
\]

\[
\mathcal{F} = \Gamma \left( c_1 + c_2 \beta + c_3 \frac{|z|}{a} + \log \left( a^2 \bar{\mu}^2 \right) (4 - \beta) \right) + (\Gamma \cdot \gamma_\mu + \gamma_\mu \cdot \Gamma) \left( c_4 + c_5 \, \text{cSW} \right)
\]
B. Lattice Regularization

★ **Linear divergence from tadpole diagram:** $\propto |z|/a$

★ **To all orders in pert. theory:** $e^{-c \frac{|z|}{a}}$ [Dotsenko et al., NPB169 (1980) 527]

★ **Green’s functions complicated functions of external momentum**

★ **Extraction of Z-factor**

\[
\langle \psi \mathcal{O} \Gamma \bar{\psi} \rangle_{DR, \text{MS}}^{\text{amp}} - \langle \psi \mathcal{O} \Gamma \bar{\psi} \rangle_{LR}^{\text{amp}} = \frac{g^2 C_f}{16 \pi^2} e^{i q \cdot z} \times \mathcal{F}
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\mathcal{F} = \Gamma \left( c_1 + c_2 \beta + c_3 \frac{|z|}{a} + \log (a^2 \bar{\mu}^2) (4-\beta) \right) + (\Gamma \cdot \gamma_\mu + \gamma_\mu \cdot \Gamma) \left( c_4 + c_5 \cdot c_{SW} \right)
\]

linear divergence
B. Lattice Regularization

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$$\langle \psi \ O_{\Gamma} \ \bar{\psi} \rangle_{\text{amp}}^{DR, \text{MS}} - \langle \psi \ O_{\Gamma} \ \bar{\psi} \rangle_{\text{amp}}^{LR} = \frac{g^2 C_f}{16 \pi^2} e^{i q_{\mu} z} \times \mathcal{F}$$

$$\mathcal{F} = \Gamma \left( c_1 + c_2 \beta + c_3 \frac{|z|}{a} + \log \left( a^2 \bar{\mu}^2 \right) (4-\beta) \right) + (\Gamma \cdot \gamma_{\mu} + \gamma_{\mu} \cdot \Gamma) \left( c_4 + c_5 c_{SW} \right)$$

linear divergence

mixing term
Results

\[ Z_{LR, \overline{MS}}^{O} = 1 + \frac{g^2 C_f}{16 \pi^2} \left( e_1 + e_2 \frac{|z|}{a} + e_3 c_{SW} + e_4 c_{SW}^2 - 3 \log \left( a^2 \bar{\mu}^2 \right) \right) \]

\[ Z_{LR, \overline{MS}}^{mix} = 0 + \frac{g^2 C_f}{16 \pi^2} \left( e_5 + e_6 c_{SW} \right) \]

Wherever mixing occurs

c_{SW}: simulation parameter
Results

\[ Z_{\mathcal{O}}^{LR, \overline{\text{MS}}} = 1 + \frac{g^2 C_f}{16 \pi^2} \left( e_1 + e_2 \frac{|z|}{a} + e_3 c_{SW} + e_4 c_{SW}^2 - 3 \log \left( a^2 \bar{\mu}^2 \right) \right) \]

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Wherever mixing occurs

\( c_{SW} \): simulation parameter

Consequences:

★ Dirac structure along the Wilson line:

Unpolarized PDF mix

Polarized & Transversity PDF do not mix
Results

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Z^{LR,\overline{\text{MS}}}_O = 1 + \frac{g^2 C_f}{16 \pi^2} \left( e_1 + e_2 \frac{|z|}{a} + e_3 c_{SW} + e_4 \frac{c_{SW}^2}{C_f} - 3 \log \left( a^2 \bar{\mu}^2 \right) \right) \\
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★ Dirac structure perpendicular to Wilson line:

Unpolarized PDF does not mix
Polarized & Transversity PDF mix
Results

\[ Z_{\text{LR,MS}} = 1 + \frac{g^2 C_f}{16 \pi^2} \left( e_1 + e_2 \frac{|z|}{a} + e_3 c_{\text{SW}} + e_4 c_{\text{SW}}^2 - 3 \log \left( a^2 \tilde{\mu}^2 \right) \right) \]

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Unpolarized PDF does not mix
Polarized & Transversity PDF mix

⇒ Perturbative results can guide simulations on quasi-PDFs
⇒ Mixing MUST be taken into account in quasi-PDF results
Results

\[ Z_{LR, \overline{MS}}^O = 1 + \frac{g^2 C_f}{16 \pi^2} \left( e_1 + e_2 \frac{|z|}{a} + e_3 c_{SW} + e_4 c_{SW}^2 - 3 \log \left( a^2 \mu^2 \right) \right) \]

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Results for selected actions

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← guidance for simulations
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\[ \leftarrow \text{guidance for simulations} \]

★ Mixing vanishes at \( c_{SW} = - \frac{e_5}{e_6} \)

★ Numerical values with \( c_{SW} \approx 1.5: \)

mixing suppressed
E

NON-PERTURBATIVE RENORMALIZATION
Non-perturbative Renormalization

★ Similar process as the renormalization of the local currents

★ Compute Z-factor on each value of $z$ (length of Wilson Line)

$RI$-scheme:

$$Z_{O}^{LR,RI} Z_{\psi}^{-1} \frac{1}{12} \text{Tr} \left[ \mathcal{V}_{O}^{LR} \mathcal{V}_{O}^{tree} \right] = 1$$
Non-perturbative Renormalization

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★ $Z_{LR,RI}^{O}$ includes the linear divergence

$Z_{LR,RI}^{O} \equiv Z_{O} e^{-c \frac{|z|}{a}}$

(The vertex function $\mathcal{V}$ has the same divergence as the nucleon matrix element)

★ Use 1-loop conversion factor to convert to the $\overline{\text{MS}}$ at 2 GeV.
Numerical Results

★ Twisted Mass fermions, \( m_\pi = 375 \text{MeV} \), \( 32^3 \times 64 \), HYP smearing
★ Renormalization scale: same as nucleon momentum ( \( 4 \frac{2\pi}{32} \) )
★ Conversion & Evolution to \( \overline{\text{MS}}(2\text{GeV}) \) \hspace{1cm} (Perturbatively)
Numerical Results

⭐ Twisted Mass fermions, $m_\pi = 375 \text{MeV}$, $32^3 \times 64$, HYP smearing

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★ Renormalization scale: same as nucleon momentum ($\frac{4 \cdot 2\pi}{32}$)
★ Conversion & Evolution to $\overline{\text{MS}}(2\text{GeV})$ (Perturbatively)

★ Systematics need to be addressed:
  • truncation of Conversion factor (only 1-loop)
  • large lattice artifacts (non-diagonal momenta)
Renormalized quasi-PDFs

Polarized case

Next step: Address systematics related to renormalization
Refining Renormalization

★ Improvement Technique:

- Computation of 1-loop lattice artifacts to $\mathcal{O}(g^2 a^\infty)$
- Subtraction of lattice artifacts from non-perturbative estimated
Refining Renormalization

★ Improvement Technique:

- Computation of 1-loop lattice artifacts to $O(g^2 a^\infty)$
- Subtraction of lattice artifacts from non-perturbative estimated

★ Method is successful for Z-factors, e.g. axial charge

![Graph showing Z_A versus (a p)^2](M. Constantinou et al., arXiv:1509.00213)
Refining Renormalization

★ Improvement Technique:

- Computation of 1-loop lattice artifacts to $\mathcal{O}(g^2 a^\infty)$
- Subtraction of lattice artifacts from non-perturbative estimated

★ Method is successful for Z-factors, e.g. axial charge

[M. Constantinou et al., arXiv:1509.00213]

★ Application to the quasi-PDFs:

All orders in $a$ for polarized case

momentum $P_3 = 4 \frac{2\pi}{32}$

must subtract $\mathcal{O}(a^0)$ to extract only lattice artifacts
F

DISCUSSION
Progress in renormalization of quasi-PDFs

★ Techniques to understand and remove linear divergence

★ Study of multiplicative renormalization
  (perturbatively and non-perturbatively)

★ Eliminate mixing where present
Progress in renormalization of quasi-PDFs

- Techniques to understand and remove linear divergence
- Study of multiplicative renormalization (perturbatively and non-perturbatively)
- Eliminate mixing where present

Many more things to be done

- In the process of renormalizing the nucleon matrix elements
- Subtraction of lattice artifacts using perturbative results
- Mixing elimination non-perturbatively
- Conversion factor to 2 loops
- Investigation of cases with gamma matrix perpendicular to the Wilson line (to avoid mixing)
DISCUSSION

Progress in renormalization of quasi-PDFs

★ Techniques to understand and remove linear divergence
★ Study of multiplicative renormalization
  (perturbatively and non-perturbatively)
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Many more things to be done

★ In the process of renormalizing the nucleon matrix elements
★ Subtraction of lattice artifacts using perturbative results
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★ Conversion factor to 2 loops
★ Investigation of cases with gamma matrix perpendicular to the Wilson line (to avoid mixing)

THANK YOU!
BACKUP SLIDES
D

LINEAR

DIVERGENCE FIT
Absence of mixing:

\[ \mathcal{R} = \frac{q(P_3, z)}{q(P'_3, z')} = e^{-\frac{c}{\alpha} + c_0} (|z| - |z'|) \left( \frac{P_3}{P'_3} \right)^{-6 \frac{g^2 C_f}{16 \pi^2}} \]

★ \( \mathcal{R} \): real ⇒ Imaginary part of simulation data should be zero

presence of \( c_0 \):
[R. Sommer, arXiv[1501.03060]]
Linear Divergence

Absence of mixing:

\[ \mathcal{R} = \frac{q(P_3, z)}{q(P'_3, z')} = e^{\left(\frac{-c}{\alpha} + c_0\right)(|z| - |z'|)} \left( \frac{P_3}{P'_3} \right)^{-6 \frac{g^2 C_f}{16 \pi^2}} \]

★ \( \mathcal{R} \): real ⇒ Imaginary part of simulation data should be zero

★ Test the ratio on your lattice quasi-PDF data!

presence of \( c_0 \):

[R. Sommer, arXiv[1501.03060]]
Linear Divergence

Absence of mixing:

\[ \mathcal{R} = \frac{q(P_3, z)}{q(P'_3, z')} = e \left( -\frac{c}{a} + c_0 \right) (|z| - |z'|) \left( \frac{P_3}{P'_3} \right) - 6 \frac{g^2 C_f}{16 \pi^2} \]

- ★ \( \mathcal{R} \): real ⇒ Imaginary part of simulation data should be zero
- ★ Test the ratio on your lattice quasi-PDF data!

![Graph](image)

- open symbols: Twisted Mass
- \( m_\pi = 375 \text{MeV} \)

- ★ Mixing must be treated for the unpolarized case
Absence of mixing:

\[ R = \frac{q(P_3, z)}{q(P_3', z')} = e \left( -\frac{c}{a} + c_0 \right) (|z| - |z'|) \left( P_3 \right)^{-6 \frac{g^2 c_f}{16 \pi^2}} \]

★ \( R \) : real \( \Rightarrow \) Imaginary part of simulation data should be zero

★ Test the ratio on your lattice quasi-PDF data!

- \( \text{Im} \left[ \frac{q(P_3, z)}{q(P_3', z')} \right] \)

open symbols: Twisted Mass  
\( m_\pi = 375 \text{MeV} \)

filled symbols: Twisted Mass & \( c_{SW} \)  
\( m_\pi = 130 \text{MeV} \)

★ Mixing must be treated for the unpolarized case

★ Presence of \( c_{SW} \) suppresses mixing

[R. Sommer, arXiv[1501.03060]]