Studying the *structure* of composite states

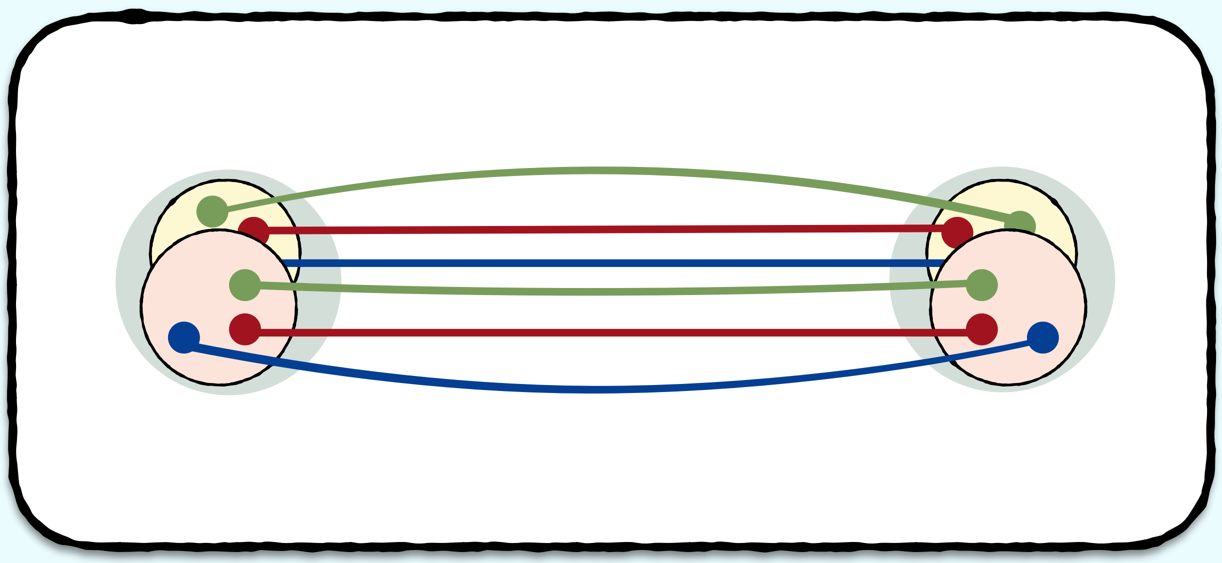
Raúl Briceño

hadspec

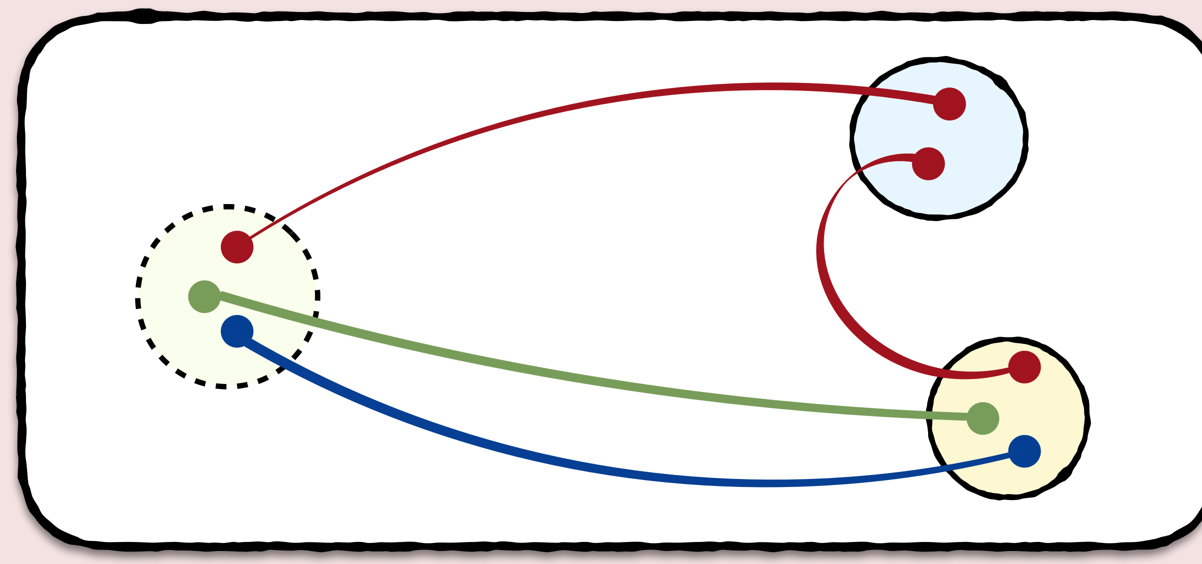
Jefferson Lab

Composite states

- The vast majority of QCD states are either:
  - unstable under QCD
  - *accidentally* stable



- Bound states
  - nuclei [e.g., deuteron]
  - hypernuclei
  - …



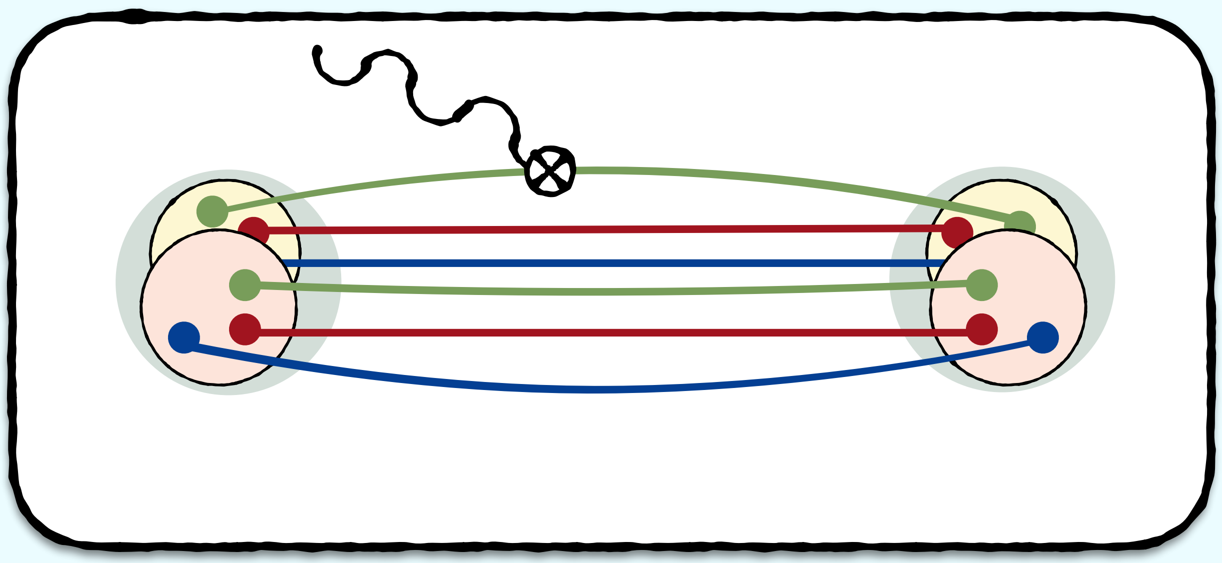
- Resonances
  - ρ [$\pi\pi$]
  - the Roper [Nπ, N$\pi\pi$,…]
  - …

QCD perspective:

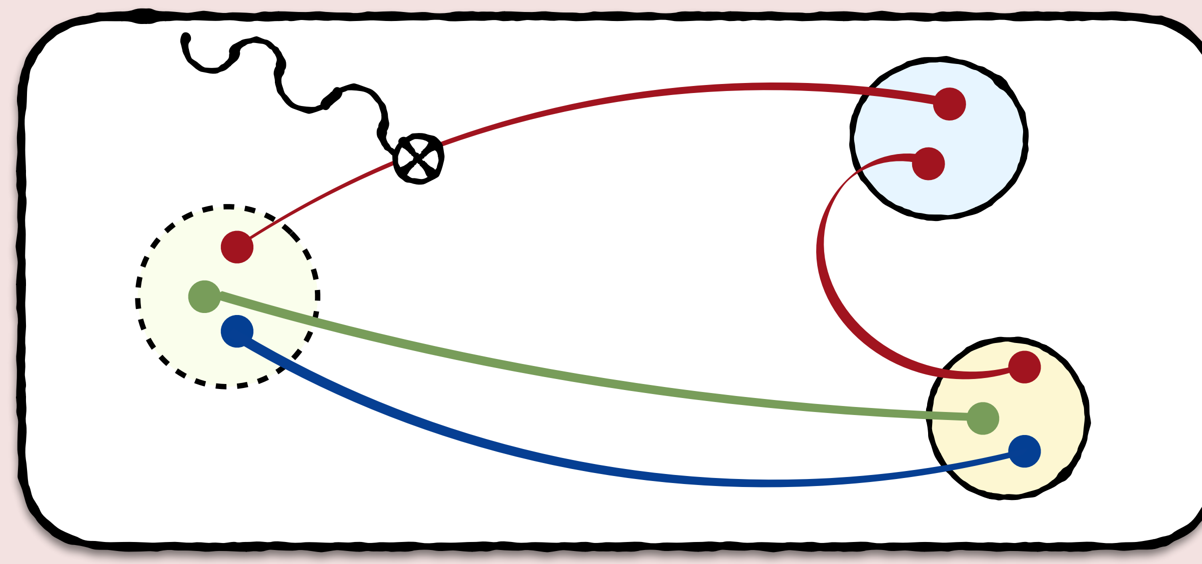
- not so different
- depending on the QCD parameters, a state can *"evolve"*

Structure of comp. states

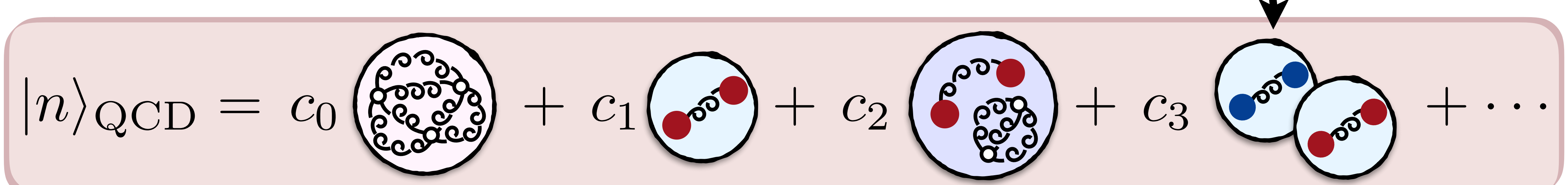
- Their structure could be accessed using external probes



- Molecules
- clustery
- EMC effect
- halo nuclei



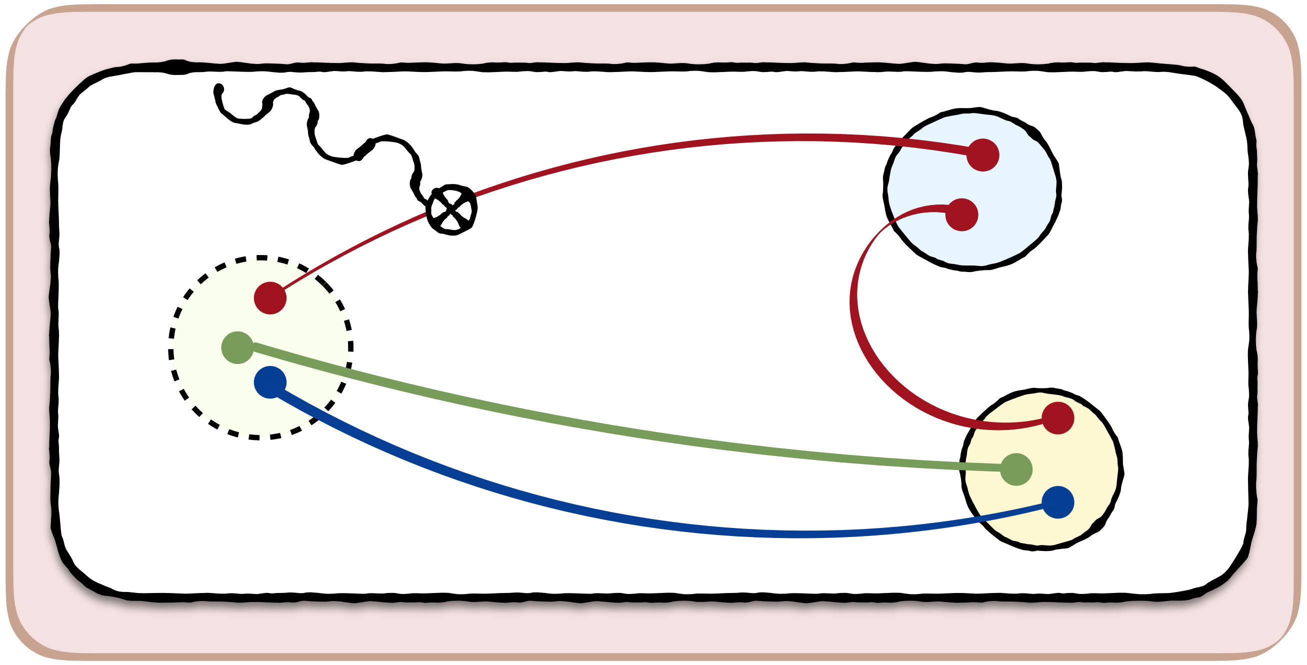
- Molecules
- exotic compositions
- glueballs
- hybrids }



Structure of comp. states

Let us focus on the more challenging task: resonance.

The conclusions drawn will be applicable for bound states as well.

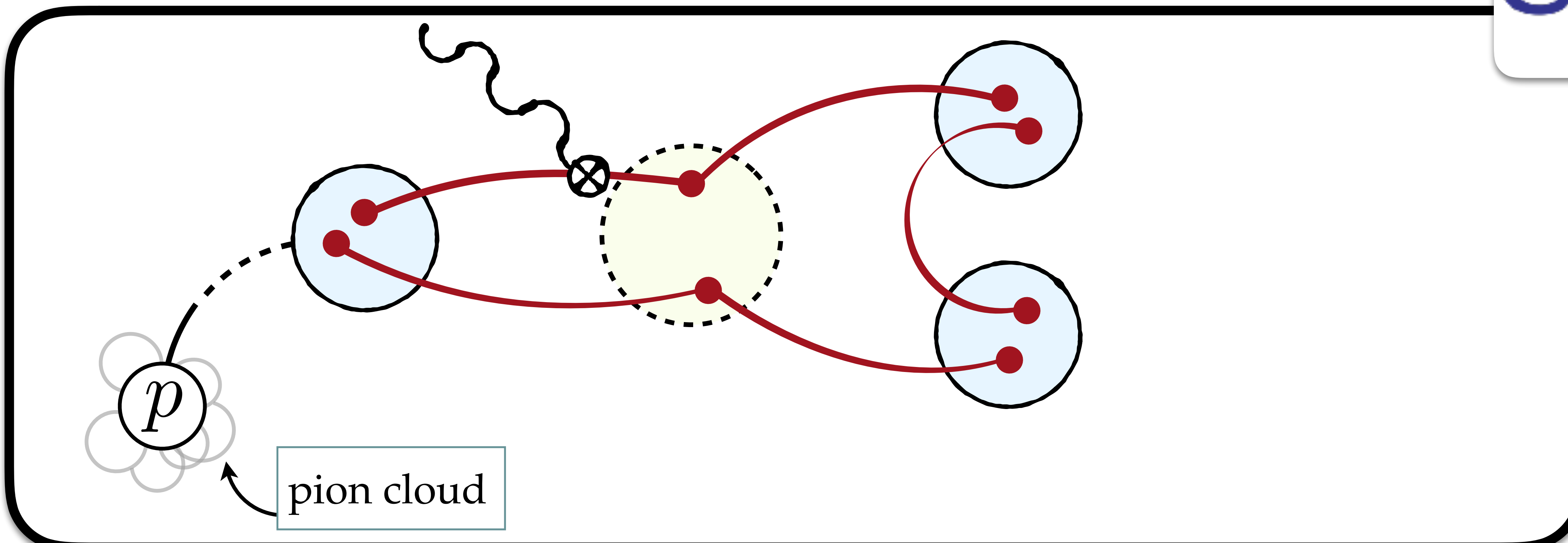


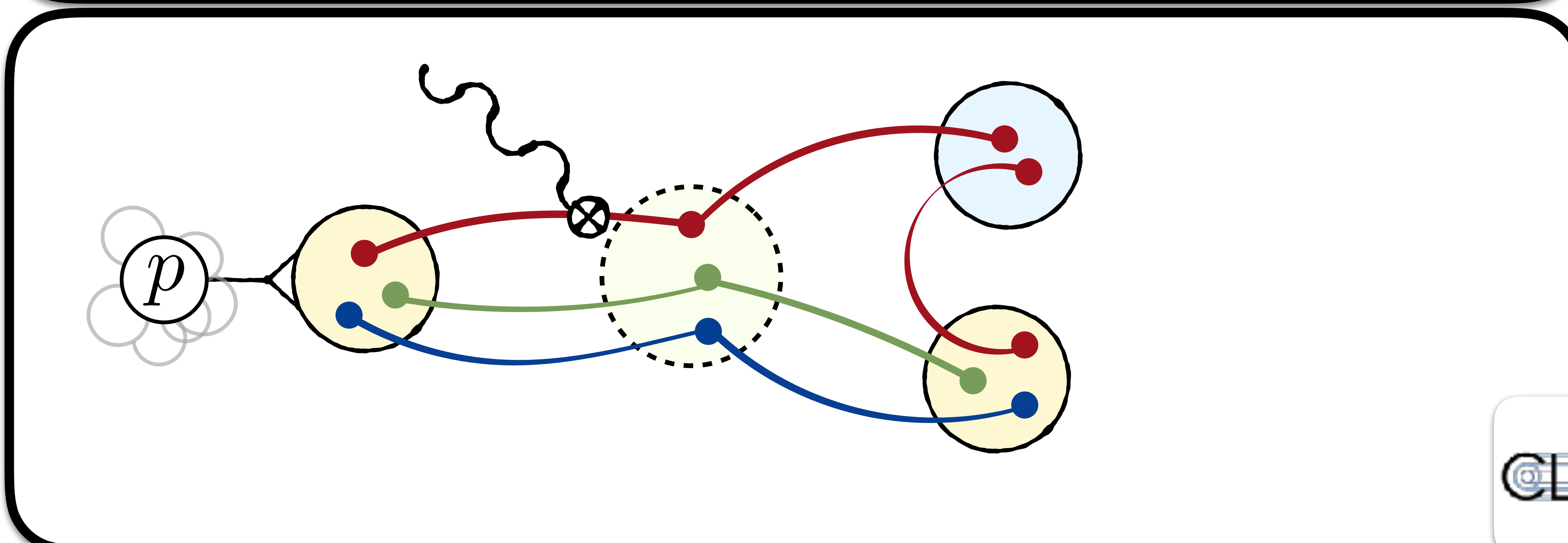
Challenges :

- Non-perturbative - lattice QCD
- Finite Euclidean spacetime vs. infinite Minkowski spacetimes
  - Ji (2012)
  - Hansen, Meyer & Robaina (2017)
- Compositeness in a finite spacetime **[RB & Hansen (2016)]**
  - form factors

Resonances in experiments

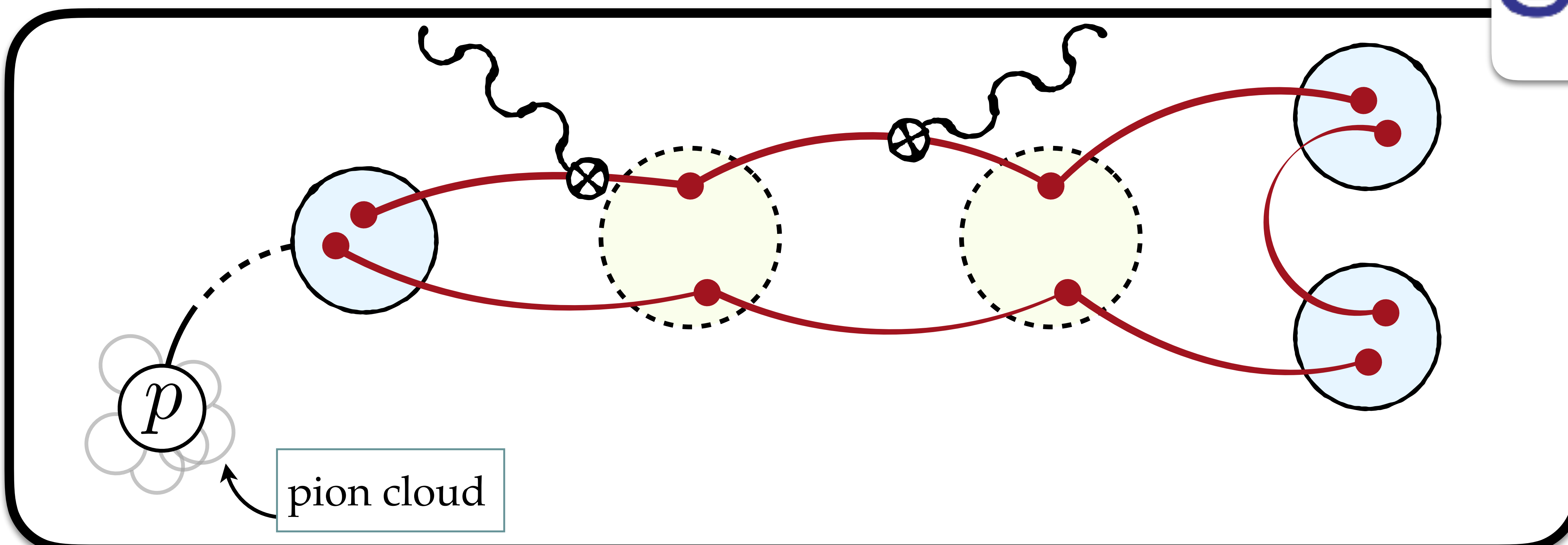
- Probing resonances experimentally is *"hard"*

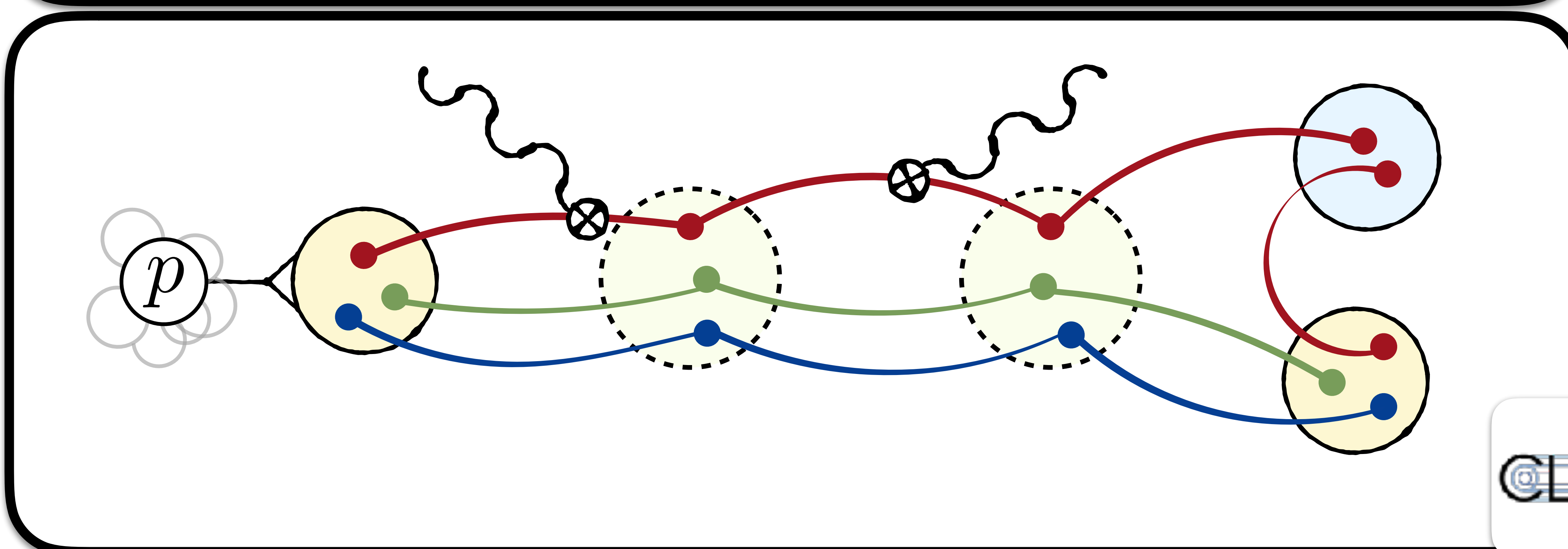




Resonances in experiments

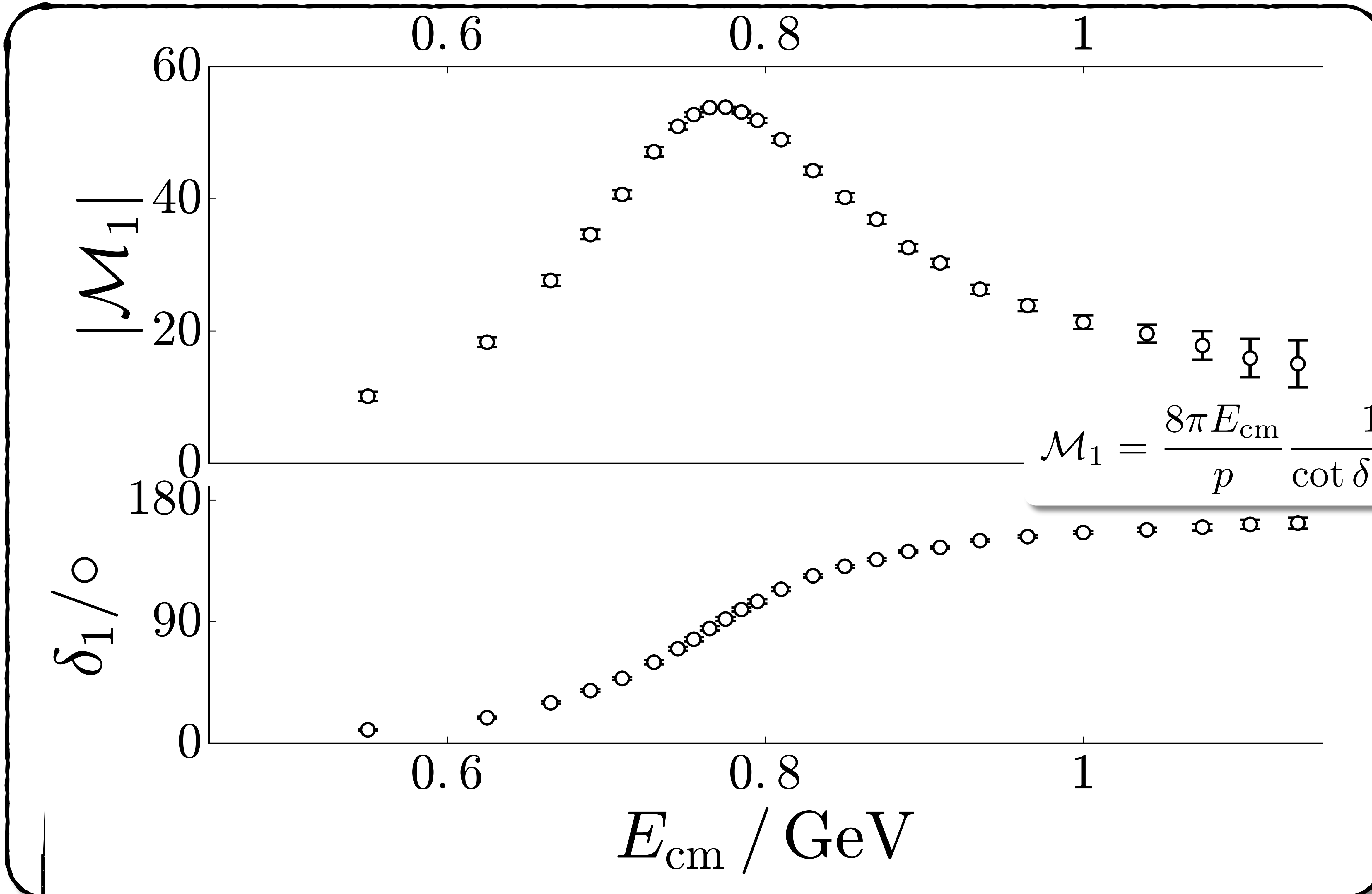
- Probing resonances experimentally is *“hard”*





A pseudo-quantitative definition

(bump in cross sections/amplitude - e.g., $\pi\pi$ scattering in ϱ-channel)

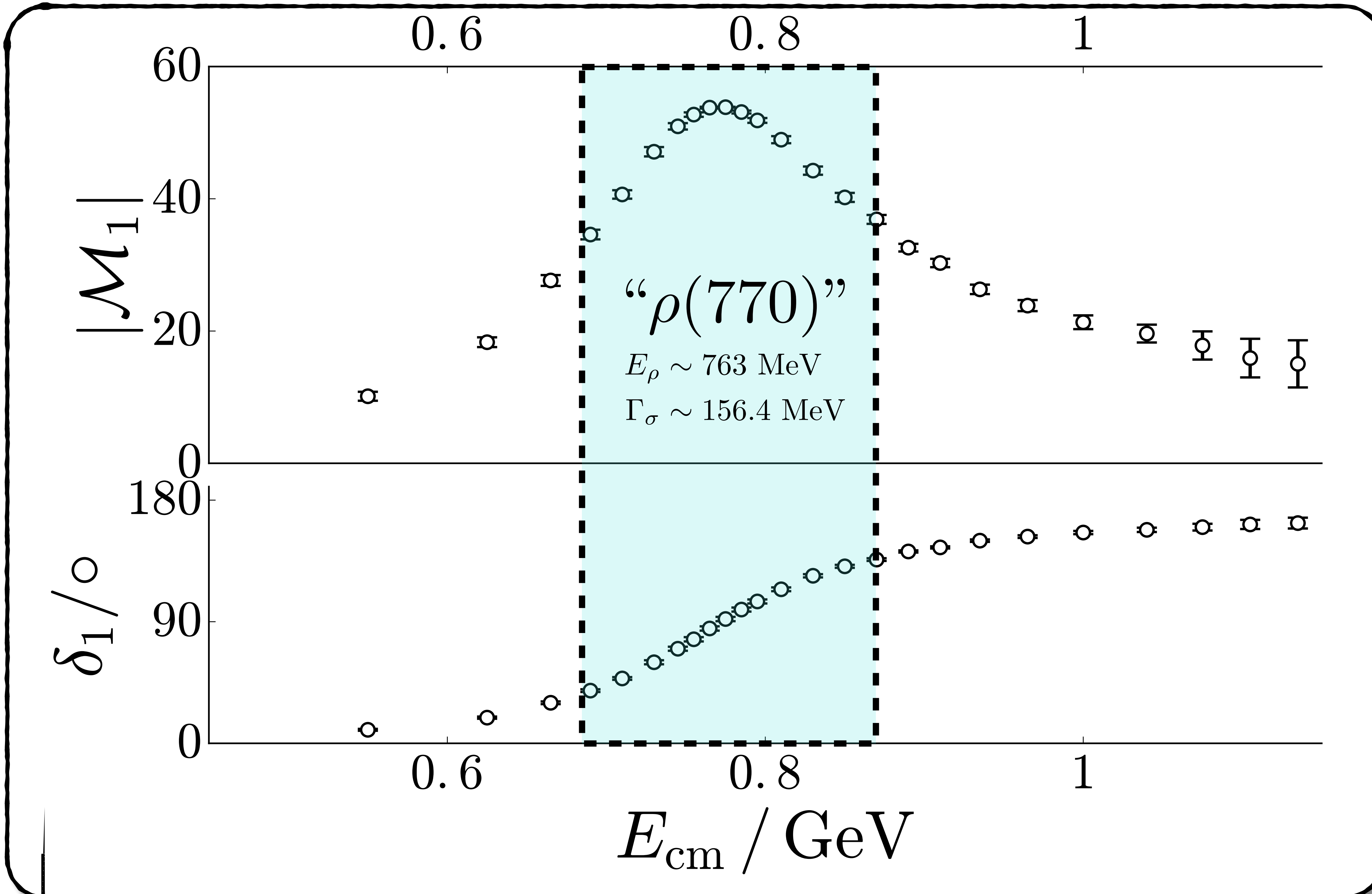


$$\mathcal{M}_1 = \frac{8\pi E_{\rm cm}}{p}\frac{1}{\cot\delta_1 - i}$$

Protopopescu et al. (1972)

A pseudo-quantitative definition

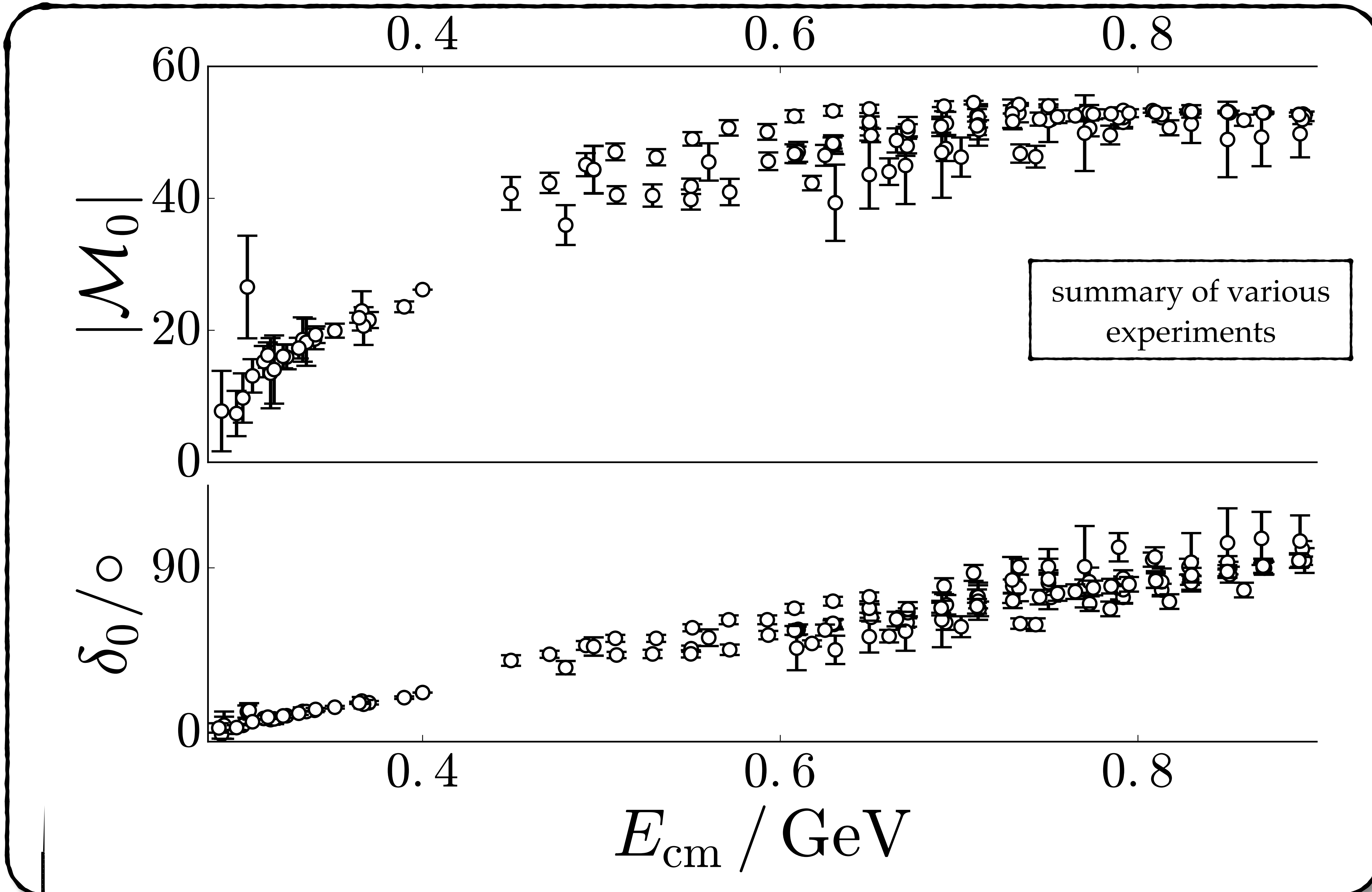
(bump in cross sections/amplitude - e.g., ππ scattering in ϱ-channel)



Protopopescu et al. (1972)

A counter example

(Isoscalar, scalar $\pi\pi$ scattering)



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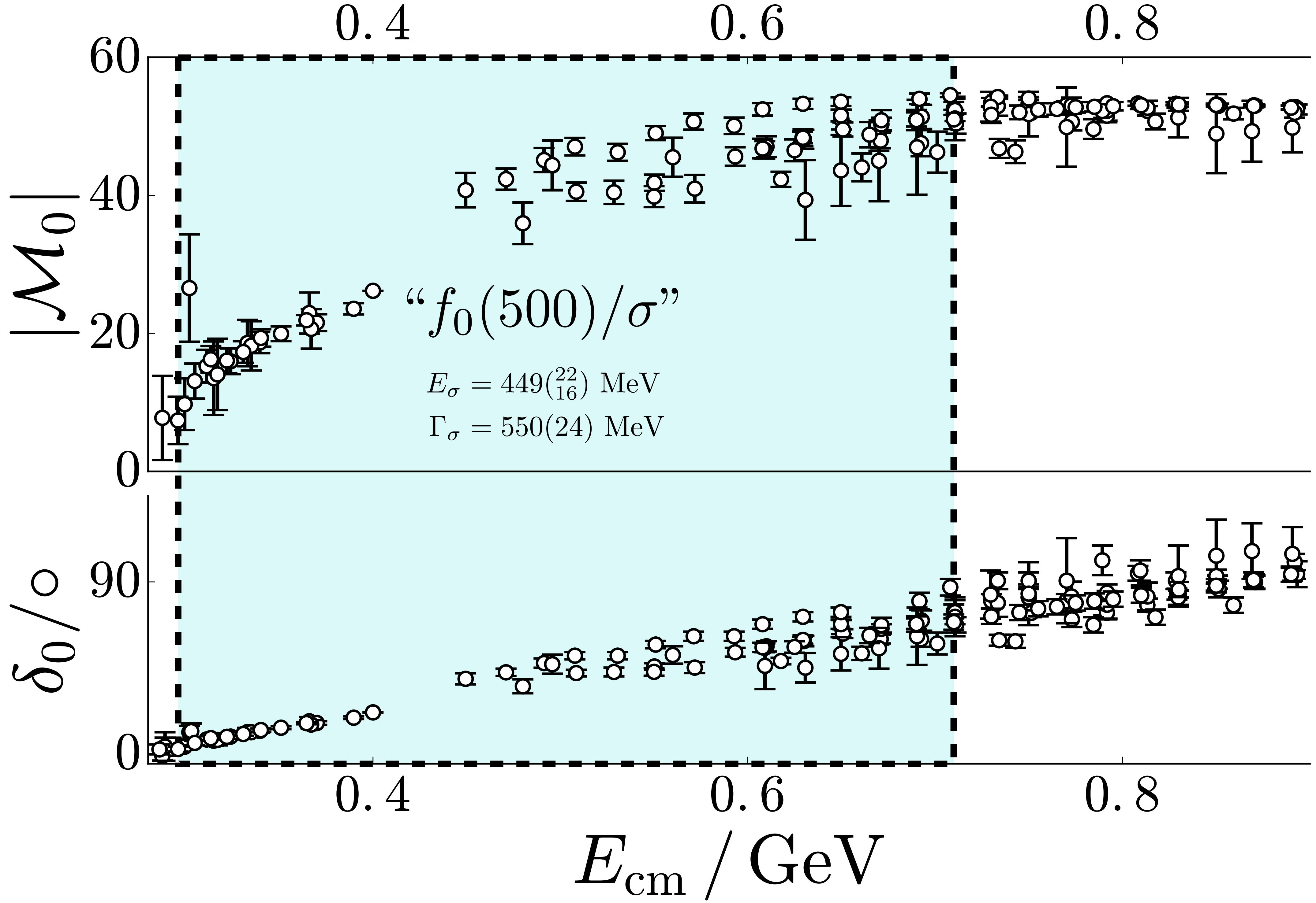
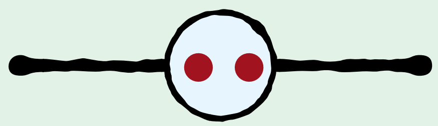


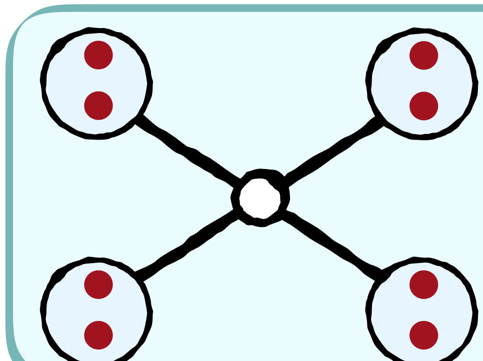
Quantitative definition

single-particle propagator:



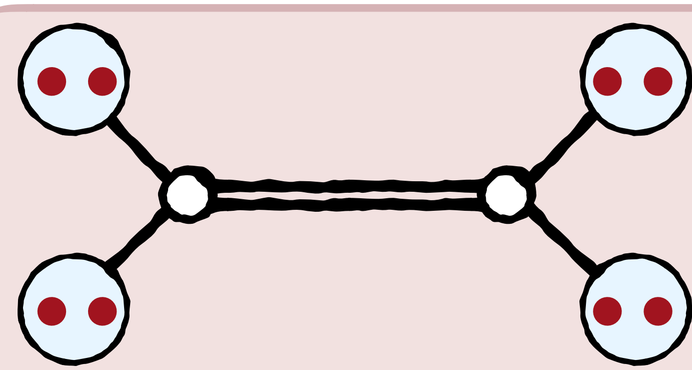
$$\sim \frac{iZ}{p^2 - m^2} = \frac{iZ}{s - m^2}$$

scattering amplitude:



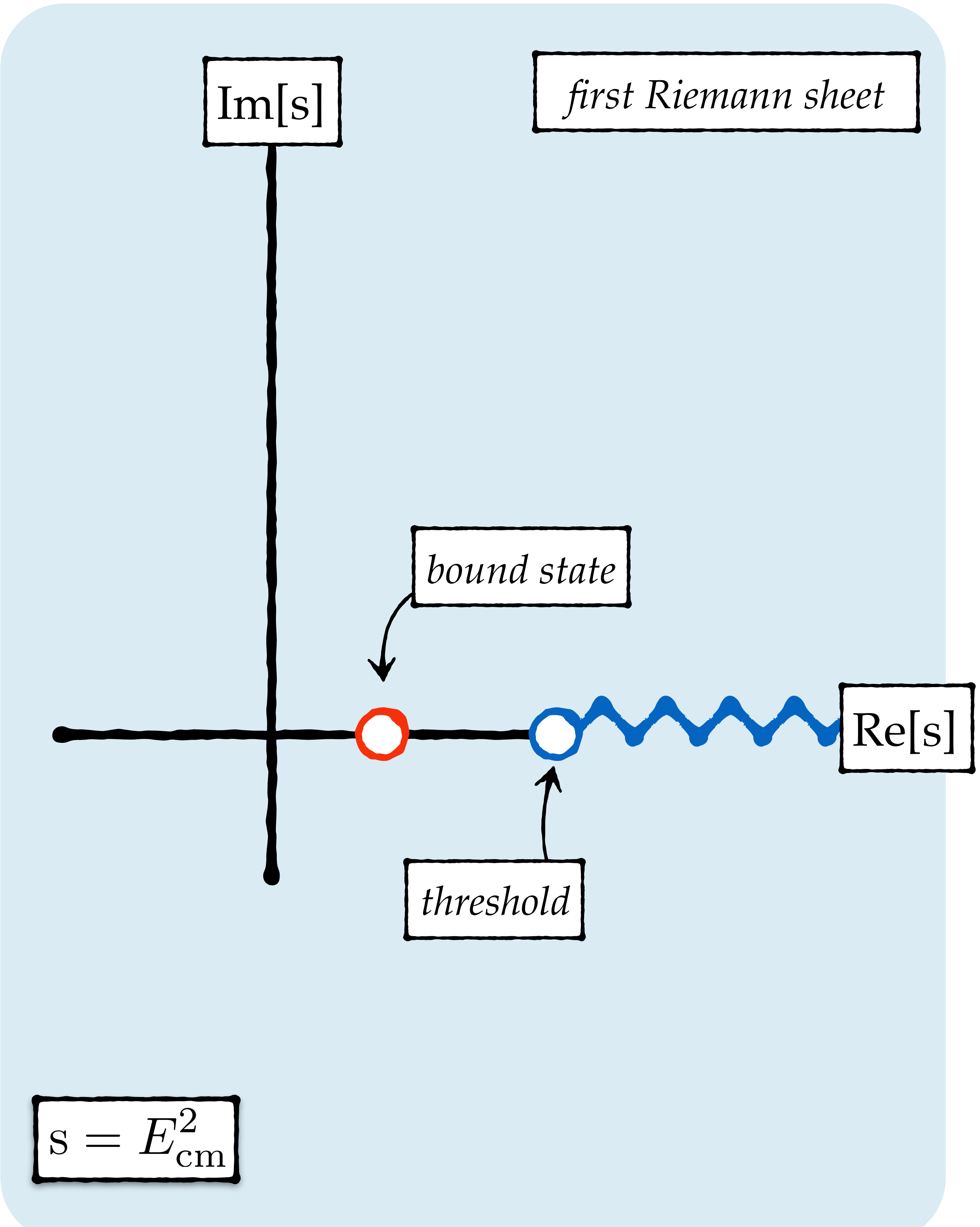
$$= i\mathcal{M}$$

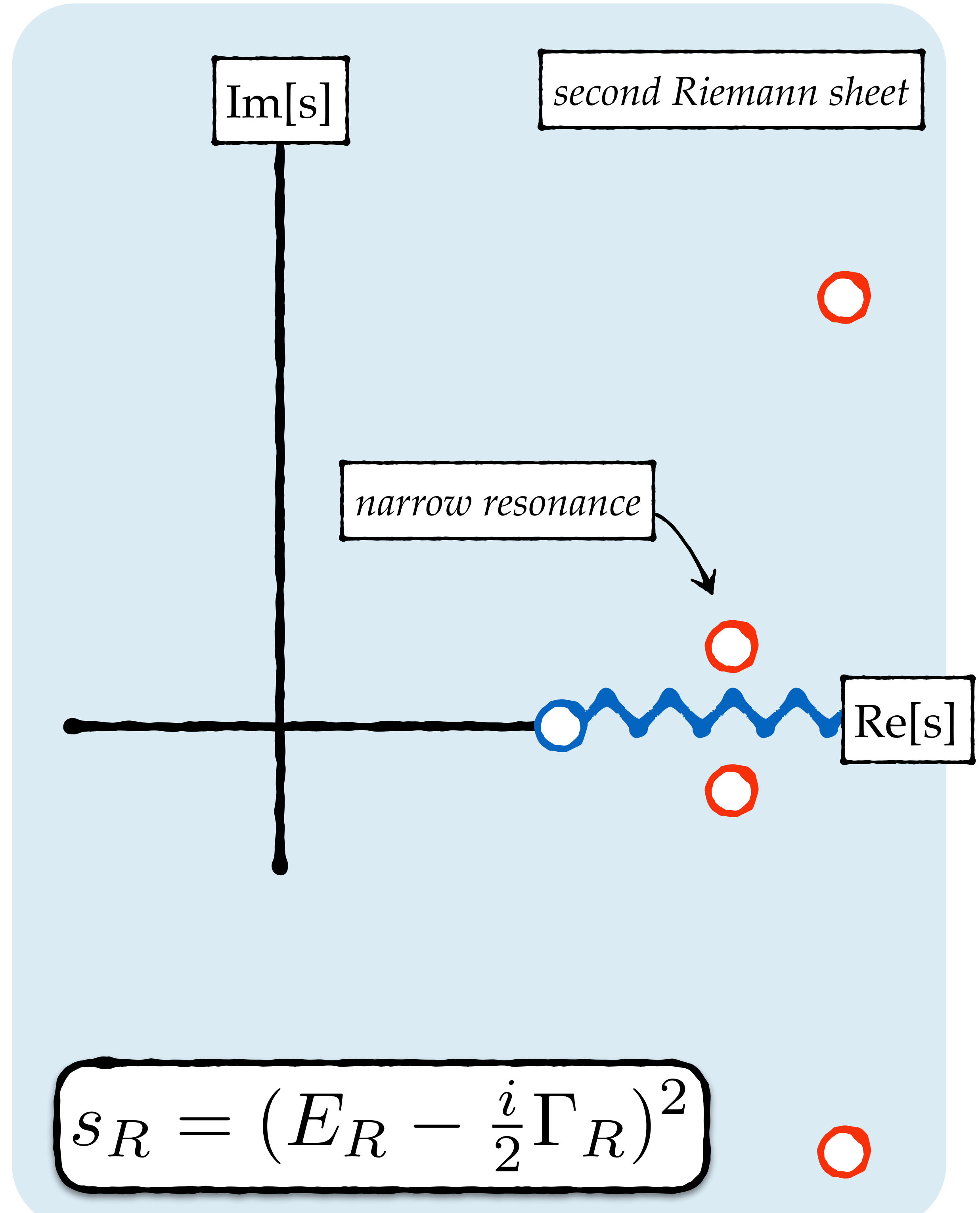
…near bound state or resonance:



$$\sim \frac{i(ig)^2}{s - s_0}, \quad s_0 = (m_R - \tfrac{i}{2}\Gamma_R)^2$$

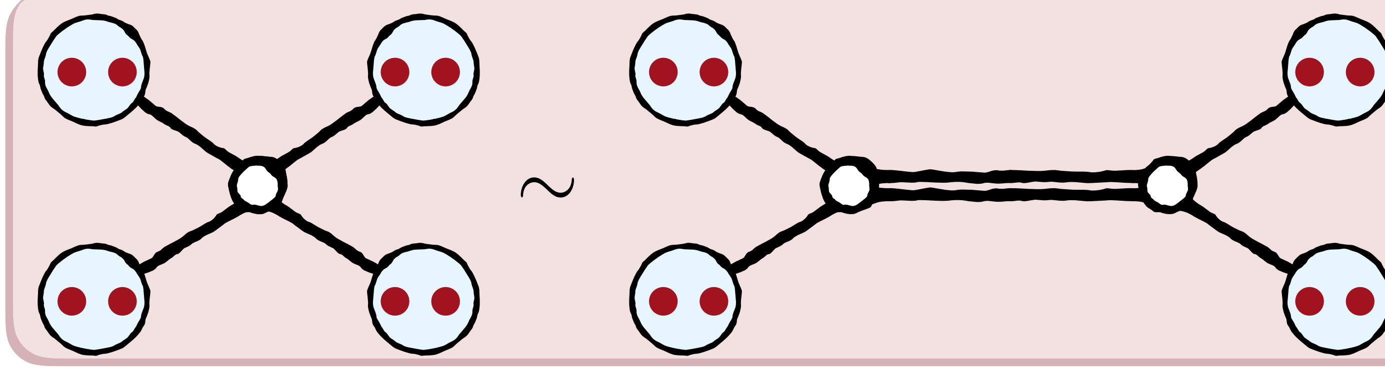
Composite states as poles





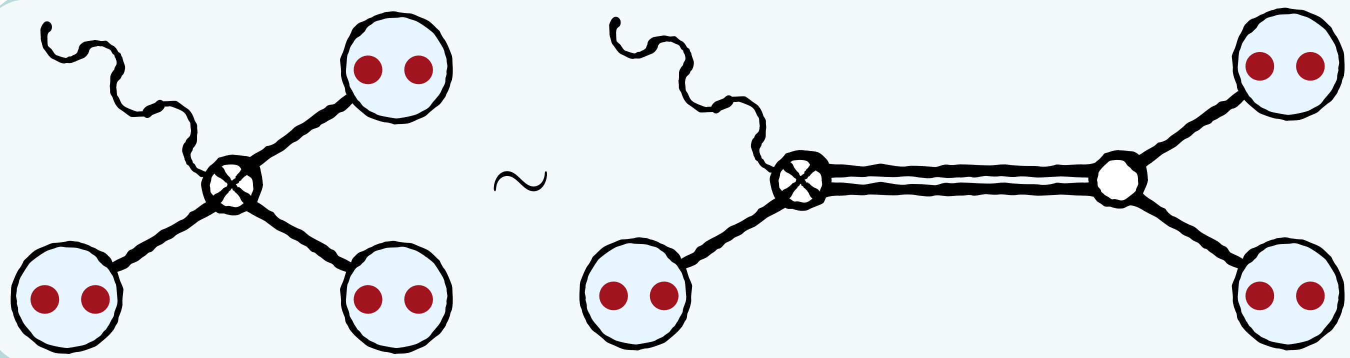
Goals

- *Obtain masses and width*



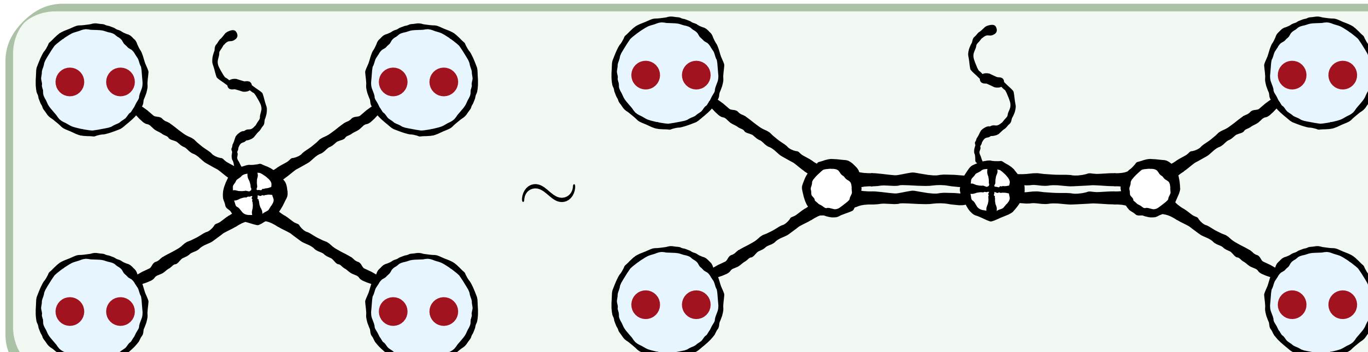
$$\sim \frac{i(ig)^2}{s - s_0}$$

- *Obtain transition form factors*



$$\sim -\frac{iF_{\text{tr.}}\, g}{s - s_0}$$

- *Obtain elastic form factors*

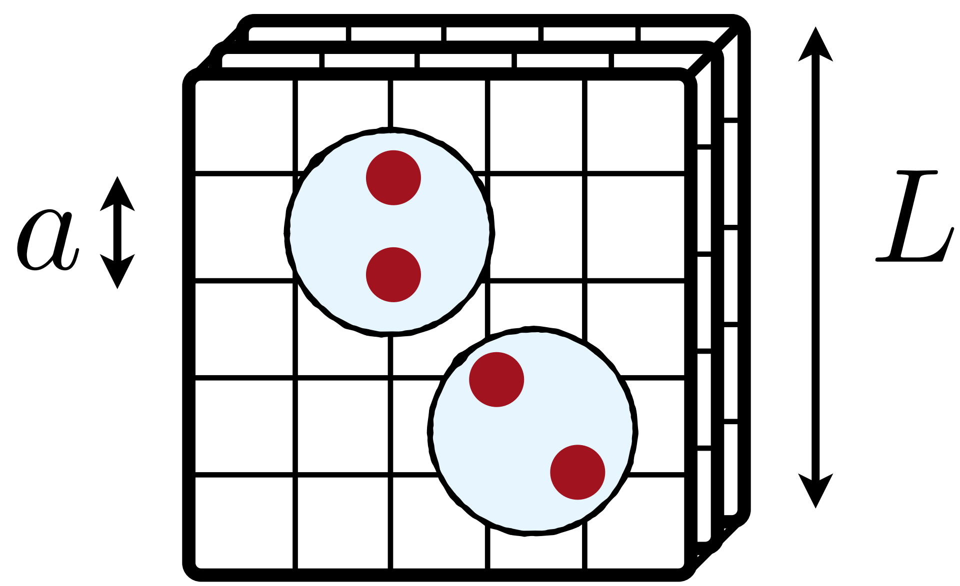


$$\sim \frac{g^2 iF_{\text{el.}}(Q^2)}{(s_i - s_0)(s_f - s_0)}$$

Lattice QCD

- Wick rotation [Euclidean spacetime]: $t_M \to -it_E$
- Monte Carlo sampling
- lattice spacing: $a \sim 0.03 - 0.15\ \text{fm}$
- finite volume

$$D_\mu = \Big(\qquad\Big)\updownarrow (L/a)^3 \times (T/a)$$



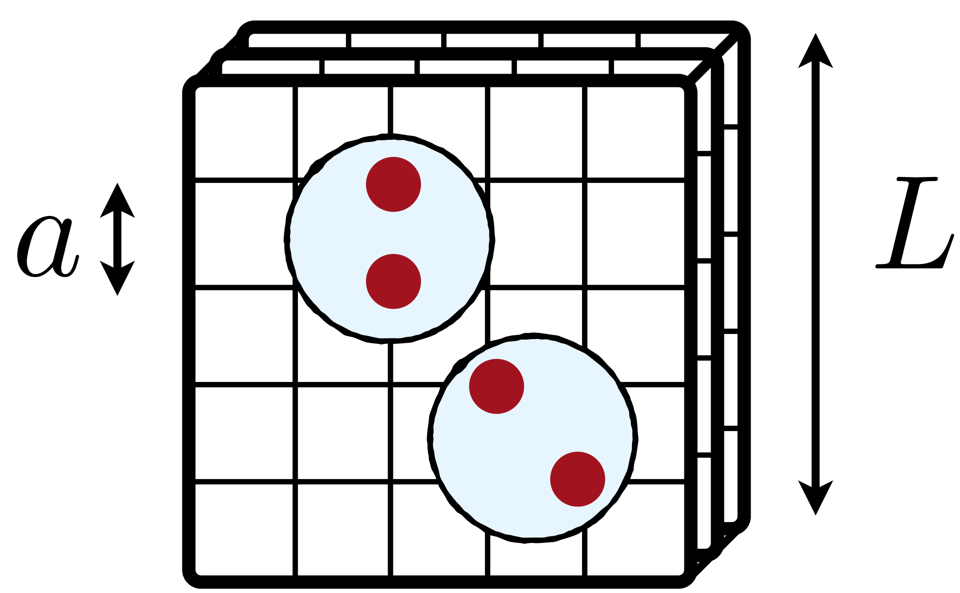
Never free!

No asymptotic states!

No scattering!

Lattice QCD

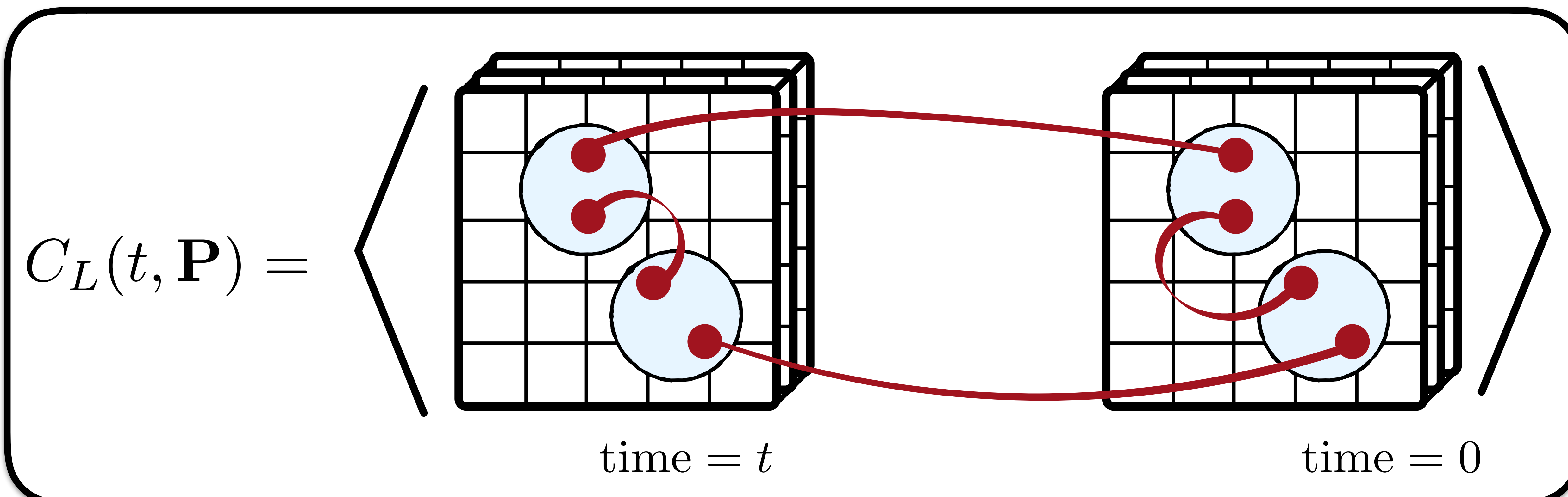
- Wick rotation [Euclidean spacetime]: $t_M \to -it_E$
- Monte Carlo sampling
- lattice spacing: $a \sim 0.03 - 0.15$ fm
- finite volume
- quark masses: $m_q \to m_q^{\text{phys.}}$



Advantage over experiment!

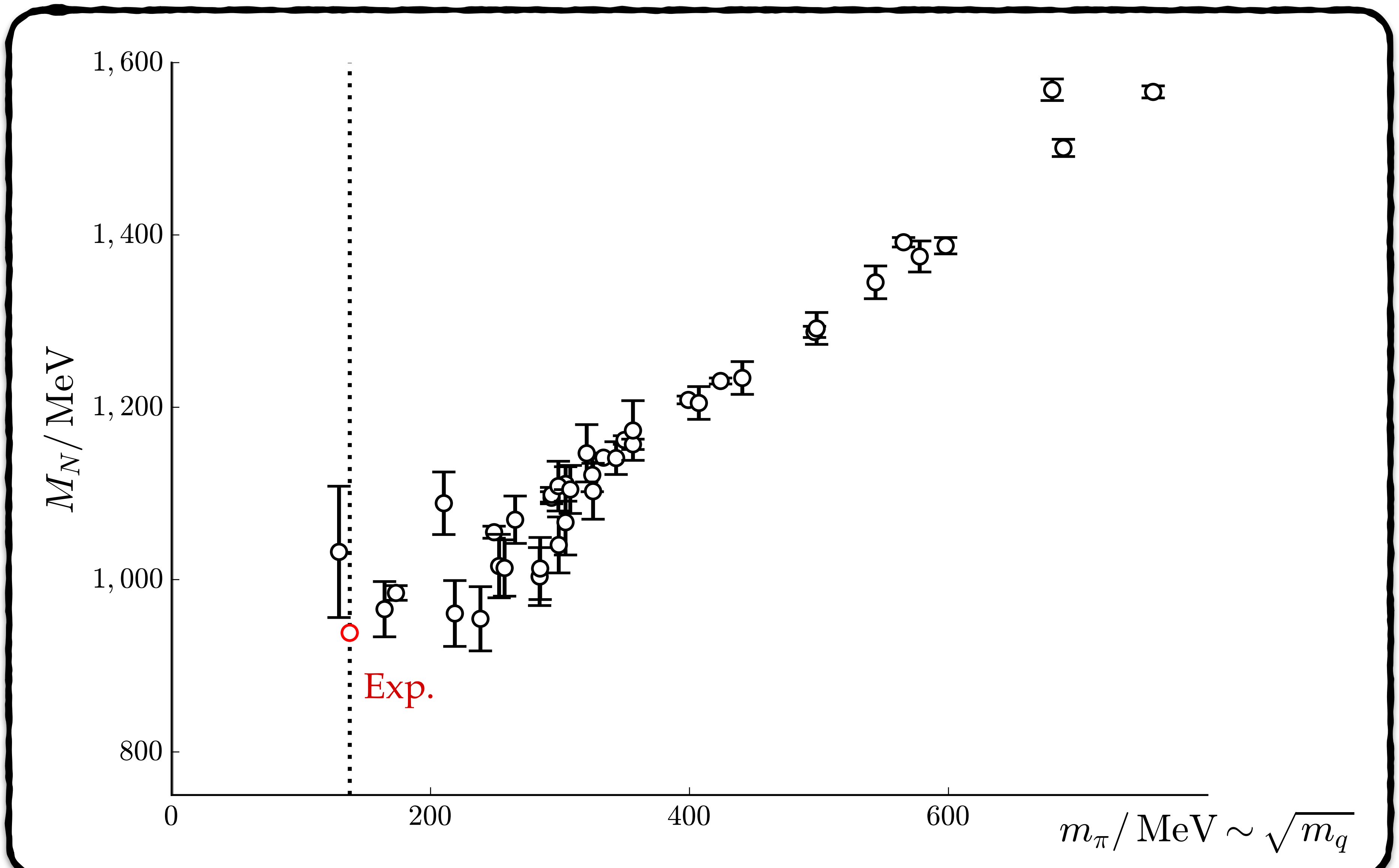
Lattice QCD

- Wick rotation [Euclidean spacetime]: $t_M \to -it_E$
- Monte Carlo sampling
- lattice spacing: $a \sim 0.03 - 0.15$ fm
- finite volume
- quark masses: $m_q \to m_q^{\text{phys.}}$
- Correlation functions: spectrum, matrix elements



Status of LQCD

- *Simple* properties of QCD stable states [non-composite states]
  - physical or lighter quark masses [down to m_π~120 MeV]
  - non-degenerate light-quark masses: N_f=1+1+1+1
  - dynamical QED



Status of LQCD

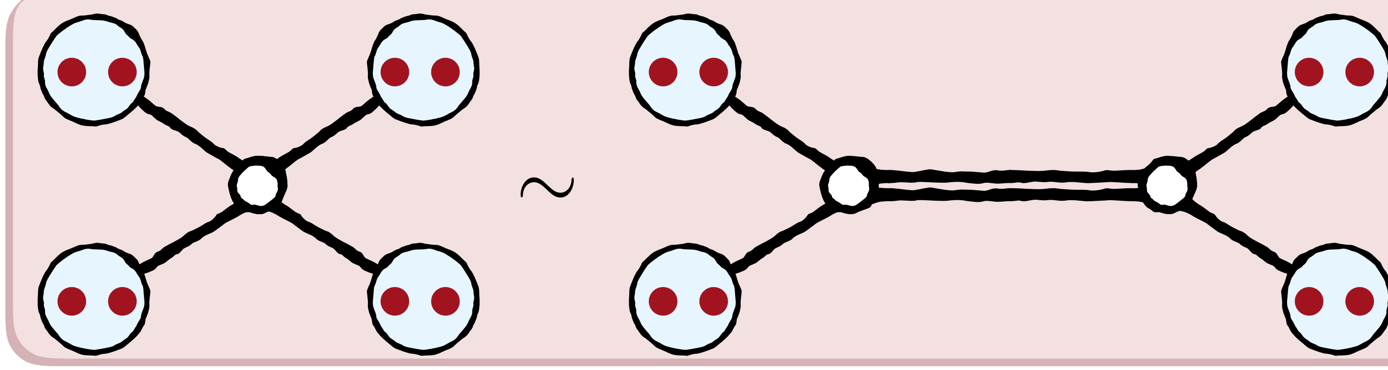
- *Simple* properties of QCD stable states [non-composite states]
  - physical or lighter quark masses [down to m_π~120 MeV]
  - non-degenerate light-quark masses: N_f=1+1+1+1
  - dynamical QED
- Frontier of lattice: multi-particle physics
  - scattering / reactions
  - composite states
    - bound states
    - hadronic resonances

Formal development:
- under way
- more needed

Benchmark calculations:
- exploratory
- proof of principle
- unphysical quark masses [m_π=236, 391 MeV]
- …

Goals

- *Obtain masses and width*



$$\sim \frac{i(ig)^2}{s - s_0}$$

- *Obtain transition form factors*

$$\sim -\frac{iF_{\mathrm{tr.}}\, g}{s - s_0}$$

- *Obtain elastic form factors*

$$\sim \frac{g^2 iF_{\mathrm{el.}}(Q^2)}{(s_i - s_0)(s_f - s_0)}$$

Two-point functions and spectrum

Two-point functions:

$$C_{ab}^{2pt.}(t,\mathbf{P}) \equiv \langle 0|\mathcal{O}_b(t,\mathbf{P})\mathcal{O}_a^\dagger(0,\mathbf{P})|0\rangle = \sum_n Z_{b,n}Z_{a,n}^\dagger e^{-E_n t}$$

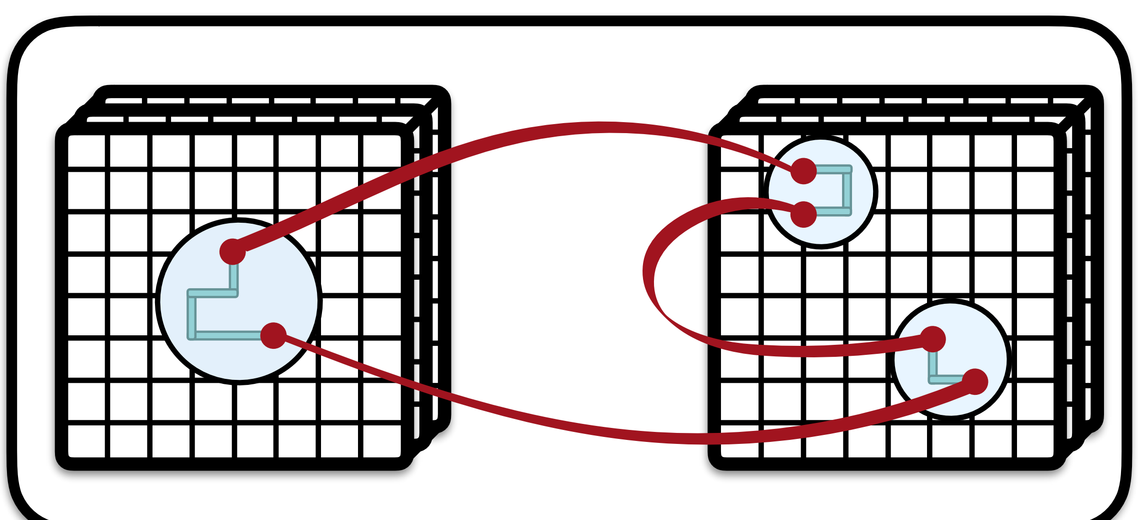
Is the finite-volume spectrum meaningful?

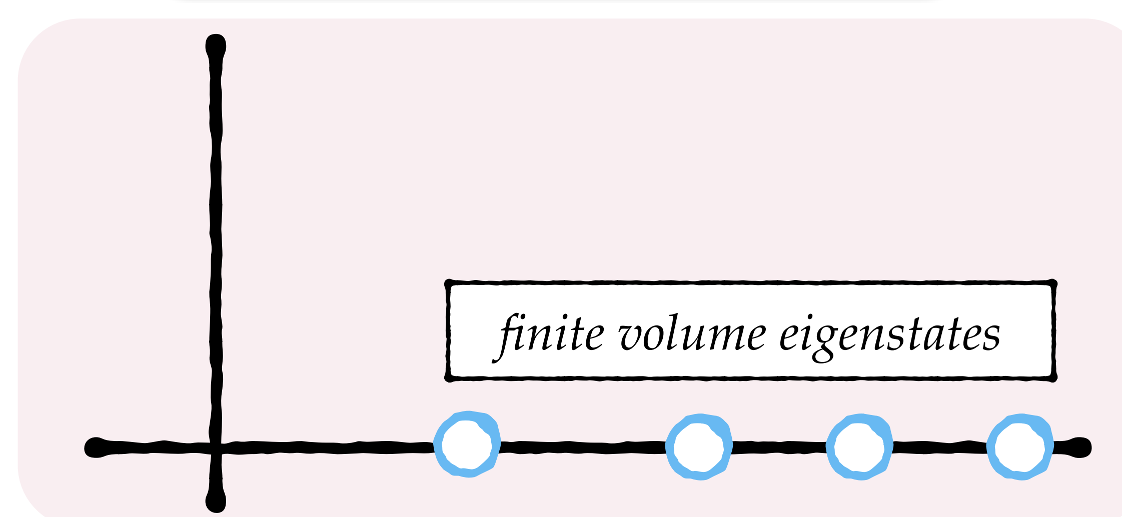
$$\hat{H}_{QCD,L} = \int_V d^3x\, \hat{\mathcal{H}}_{QCD} \neq \hat{H}_{QCD,\infty}$$

For QCD stable states: $E_L = E_\infty + \mathcal{O}(e^{-m_\pi L}) \approx E_\infty$

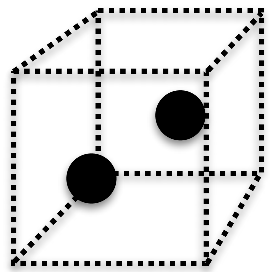
otherwise and in general: $E_L \not\approx E_\infty$

their relation must be obtained analytically





Two-particle spectrum

FV spectrum  → scattering amplitudes → resonance

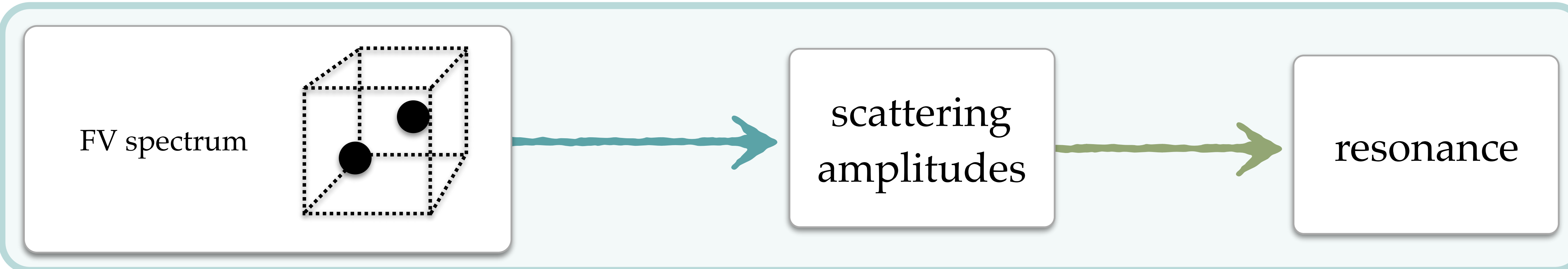
$$\det[F^{-1}(E_L, L) + \mathcal{M}(E_L)] = 0$$

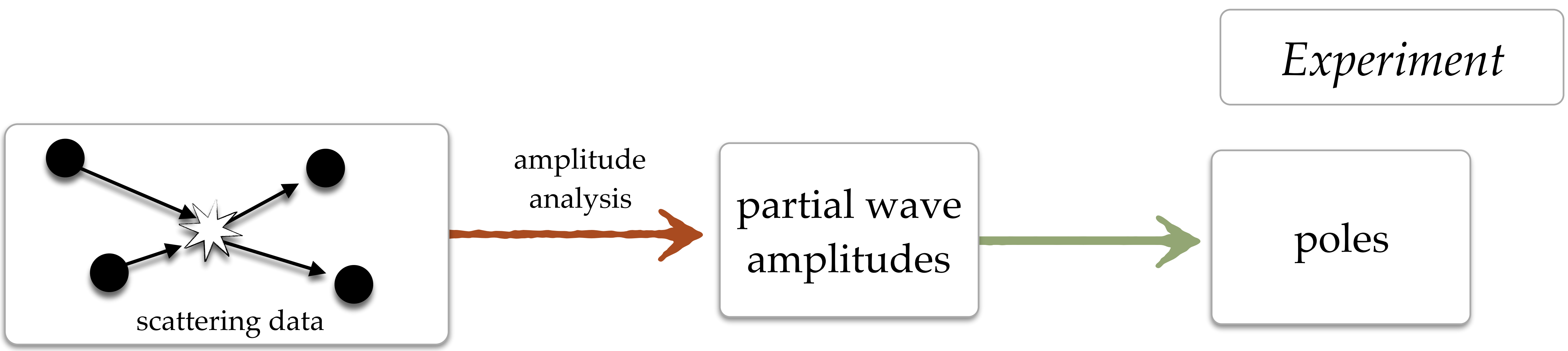
E_L = finite volume spec.
L = finite volume
F = known function
$\mathcal{M}$ = scattering amp.

not an extrapolation!

- Lüscher (1986, 1991) [elastic scalar bosons]
- Rummukainen & Gottlieb (1995) [moving elastic scalar bosons]
- Kim, Sachrajda, & Sharpe/Christ, Kim & Yamazaki (2005) [QFT derivation]
- Feng, Li, & Liu (2004) [inelastic scalar bosons]
- Hansen & Sharpe / RB & Davoudi (2012) [moving inelastic scalar bosons]
- RB (2014) [general 2-body result]

Two-particle spectrum

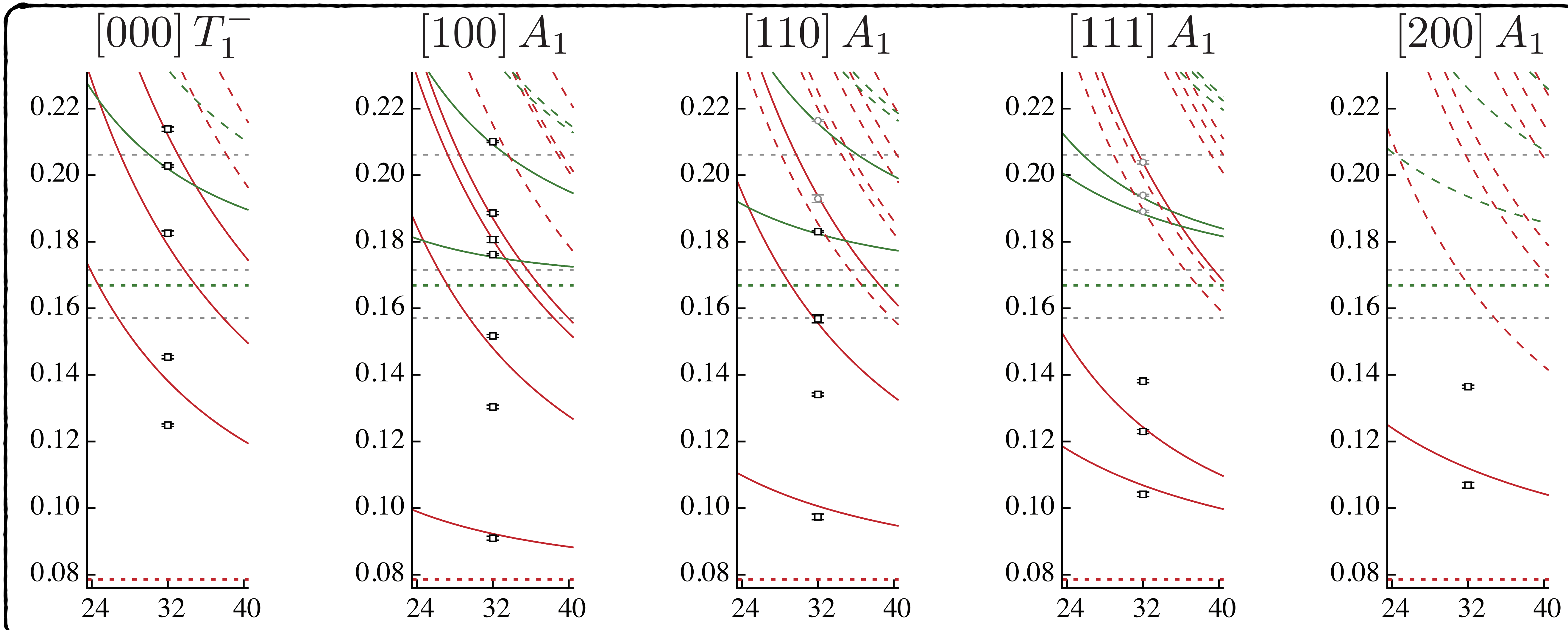




$\pi\pi$ Spectrum

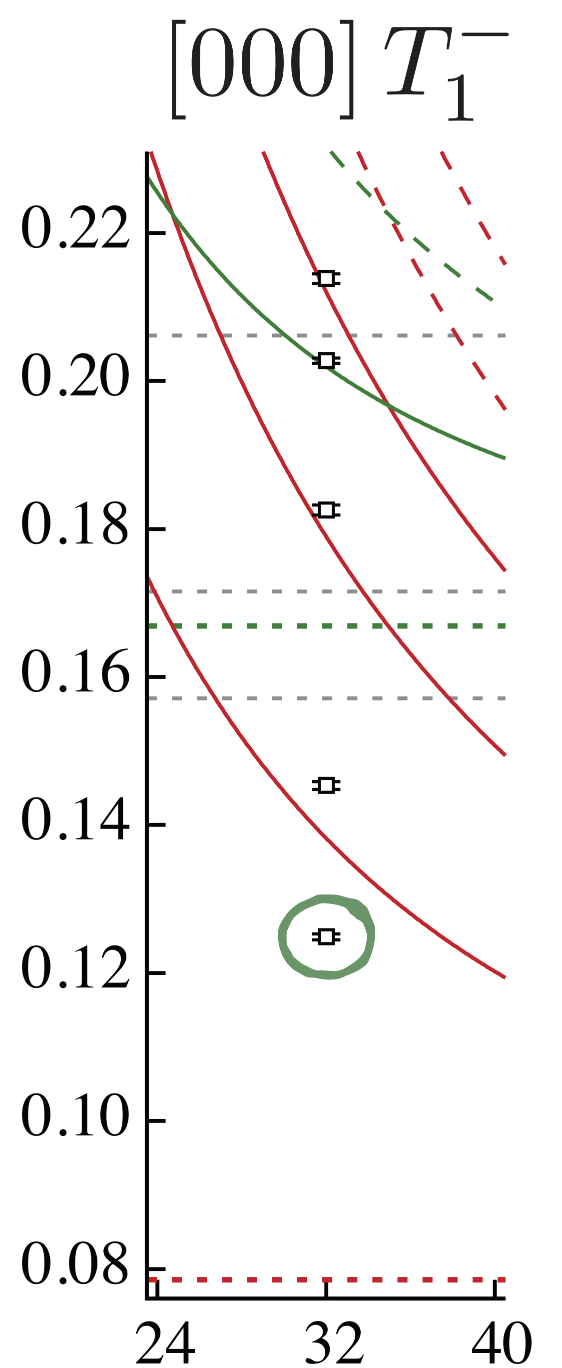
(I=1 channel)

Half of the spectrum:

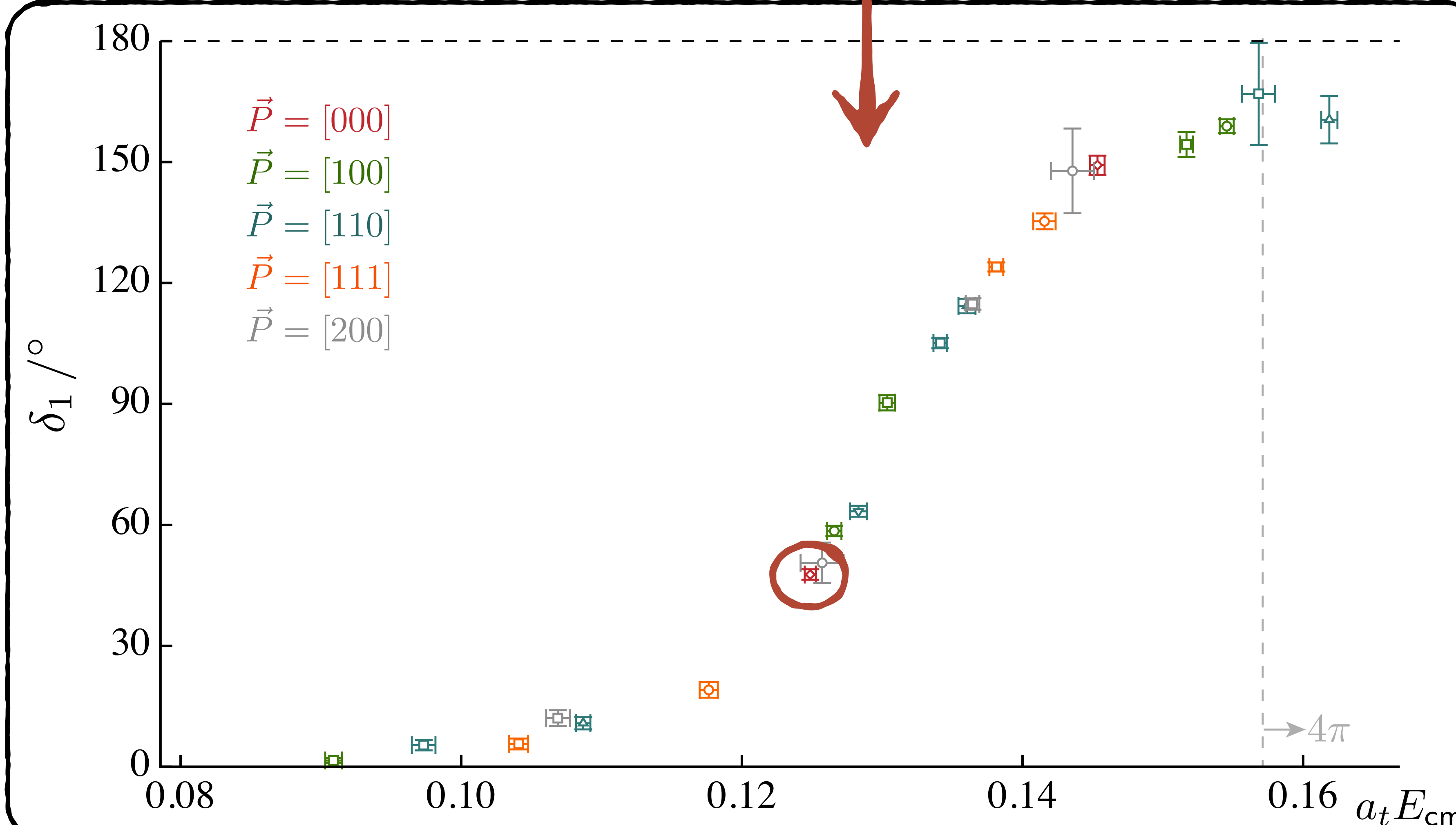


Wilson, RB, Dudek, Edwards & Thomas (2015)

Isovector $\pi\pi$ scattering



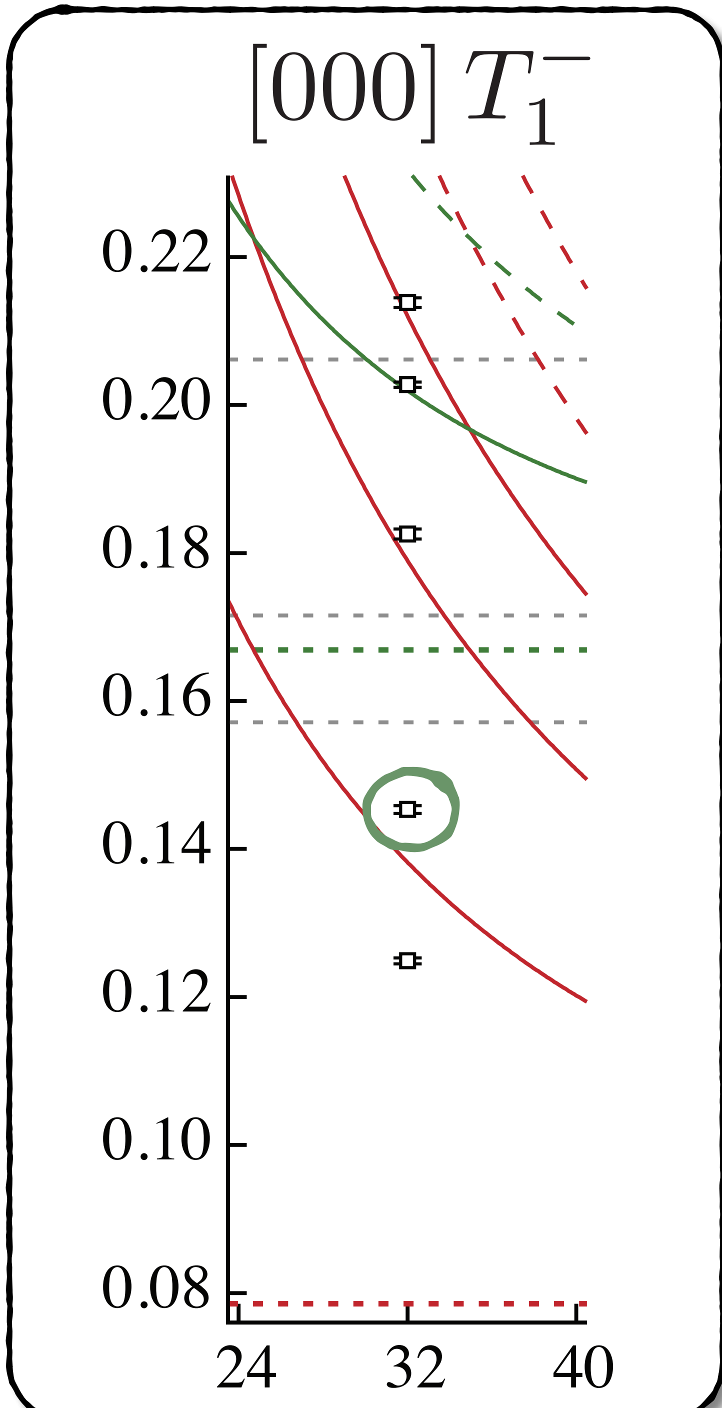
$$\det[F^{-1}(E_L, L) + \mathcal{M}(E_L)] = 0$$



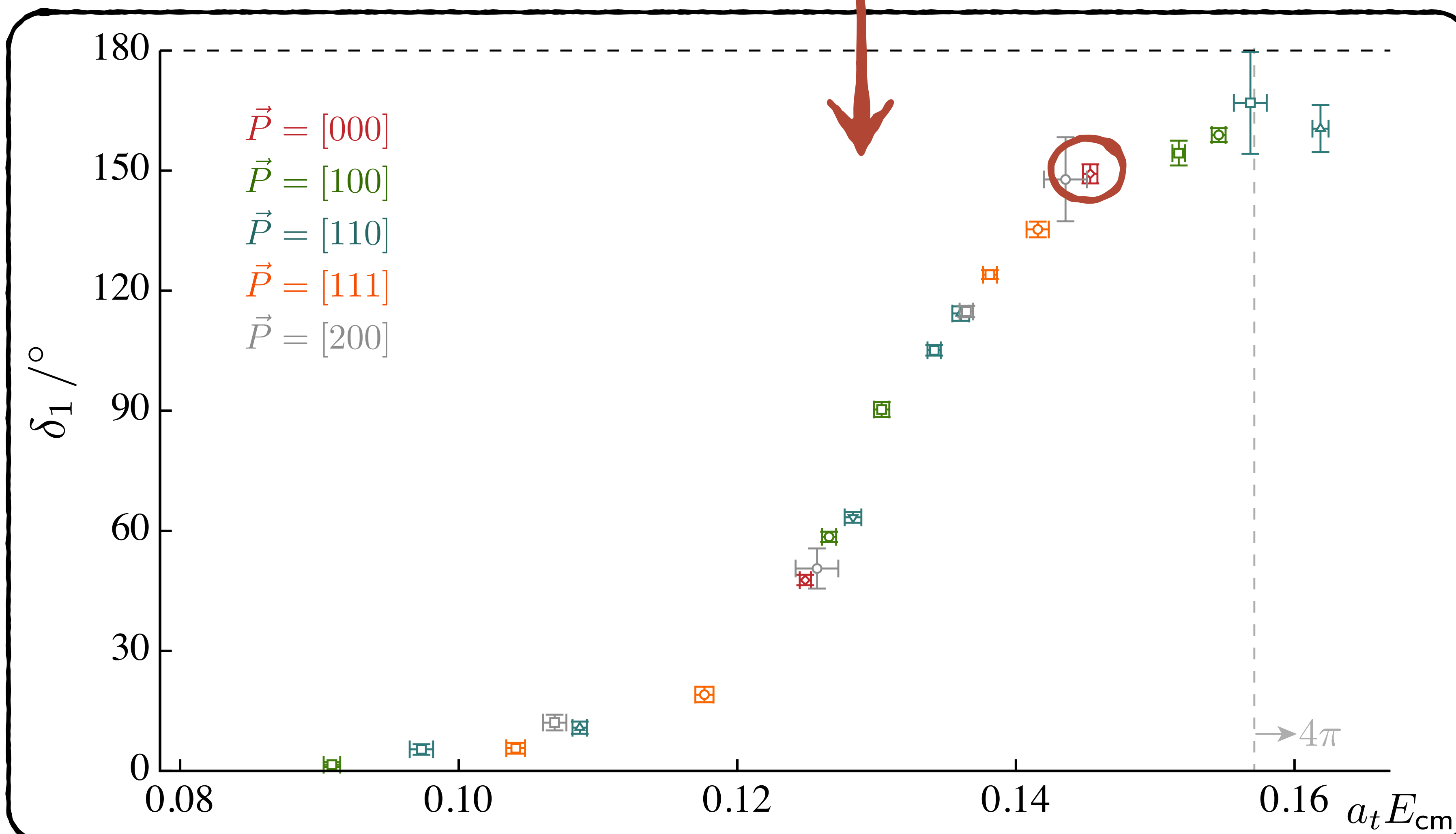
$$\mathcal{M} \propto \frac{1}{\cot \delta_1 - i}$$

Wilson, RB, Dudek, Edwards & Thomas (2015)

Isovector $\pi\pi$ scattering



$$\det[F^{-1}(E_L, L) + \mathcal{M}(E_L)] = 0$$



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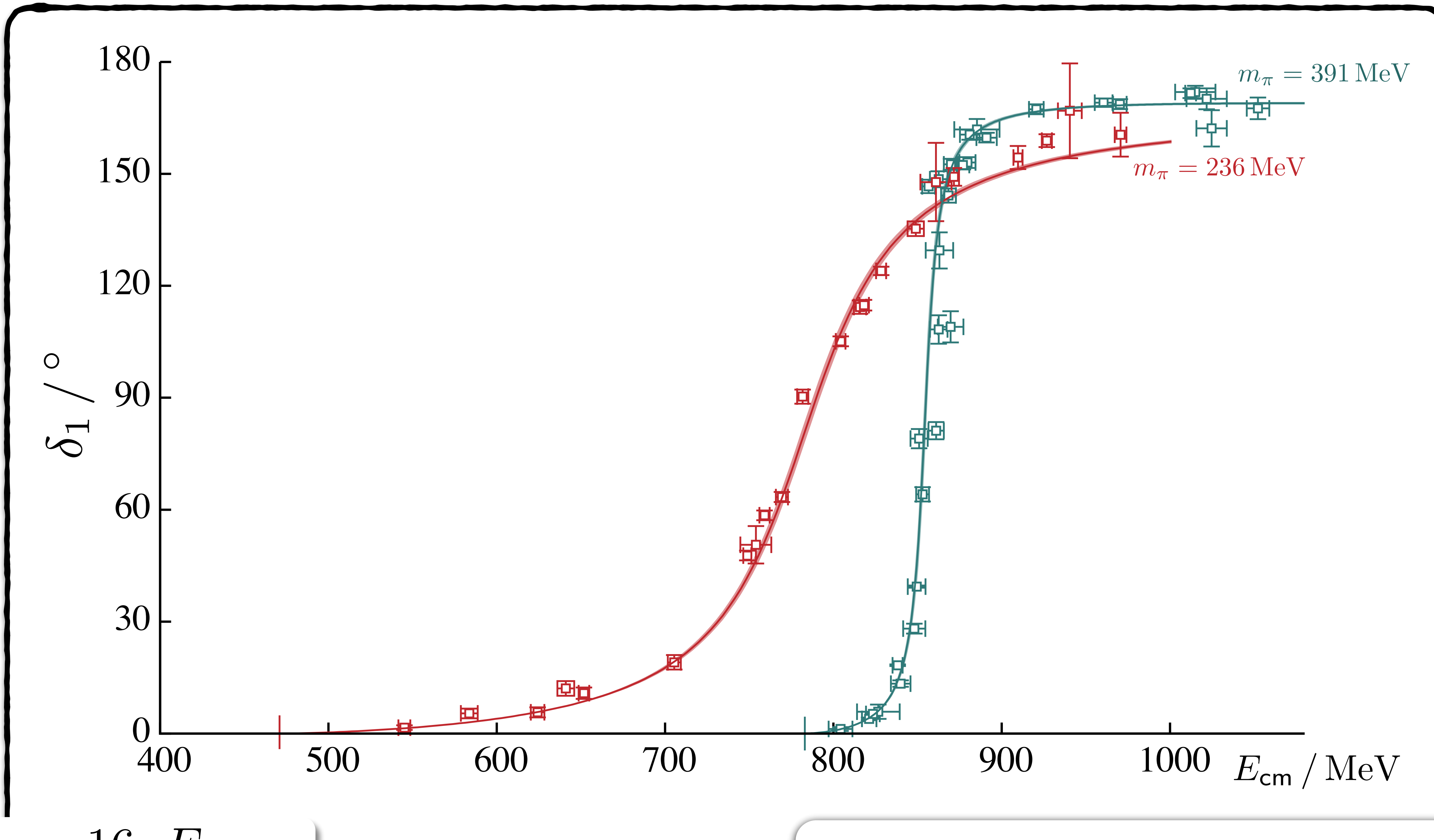
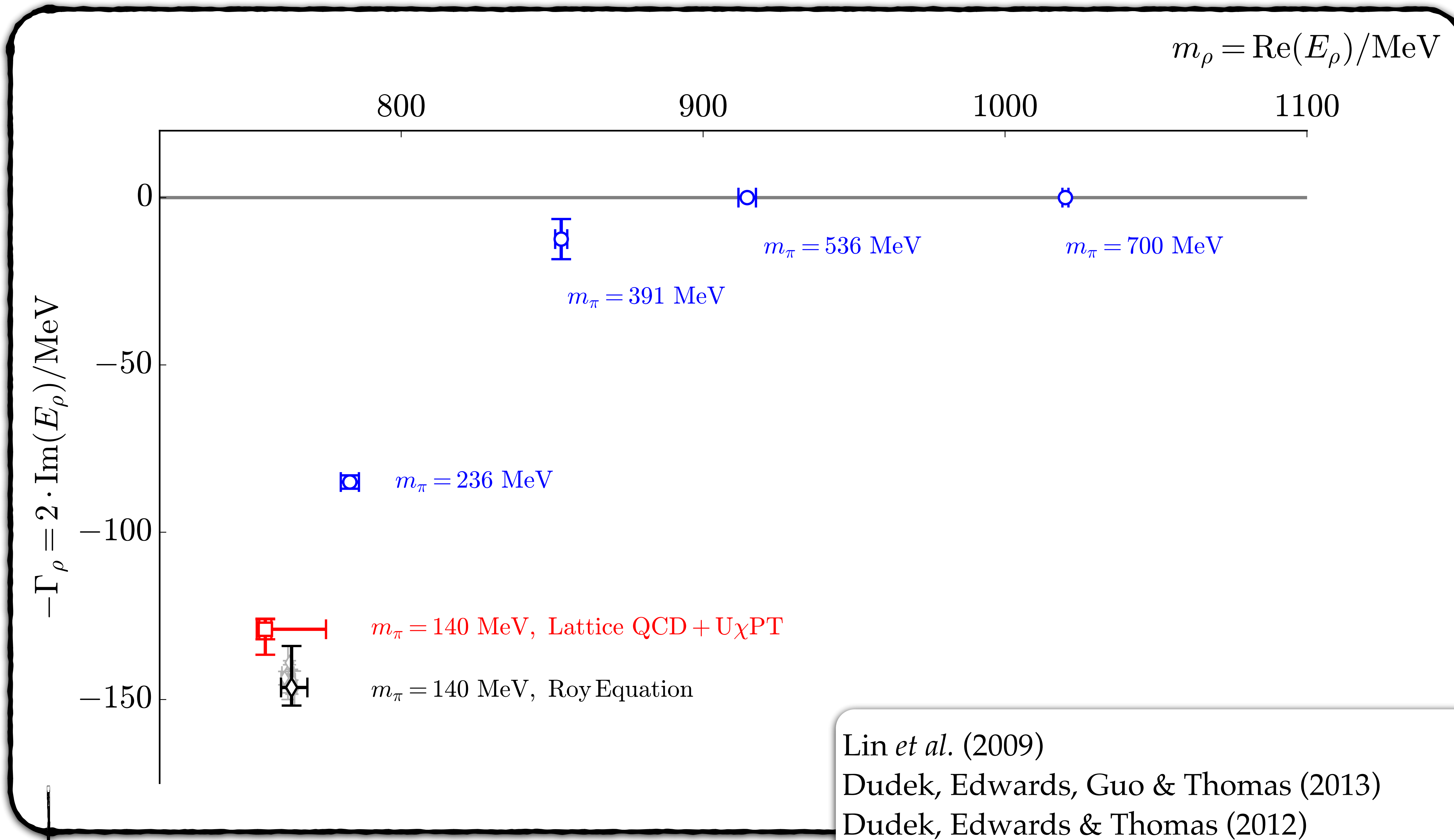


$$\mathcal{M}_1 = \frac{16\pi E_{\mathrm{cm}}}{p\cot\delta_1 - ip}$$

Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)

The ρ vs m_π



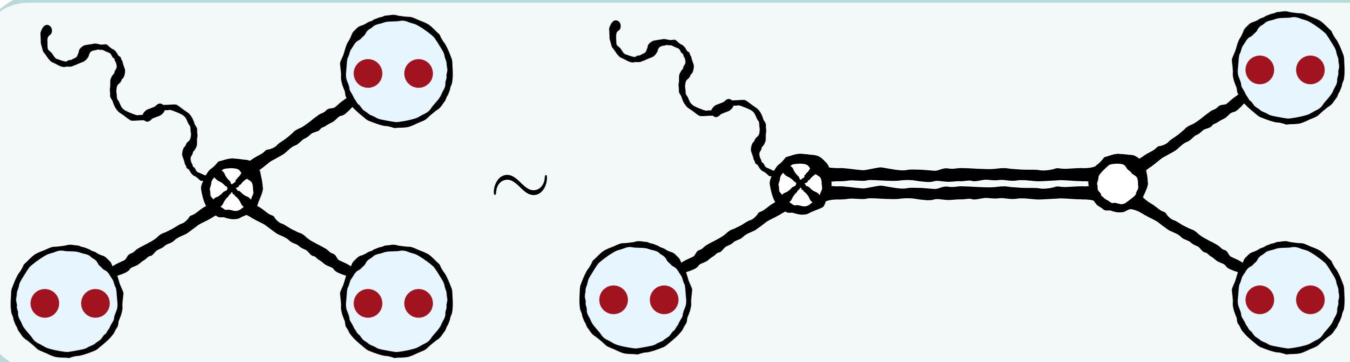
Lin *et al.* (2009)
Dudek, Edwards, Guo & Thomas (2013)
Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)
Bolton, RB & Wilson (2015)

Goals

- *Obtain masses and width*

$$\sim \frac{i(ig)^2}{s-s_0}$$

- *Obtain transition form factors*



$$\sim -\frac{iF_{\mathrm{tr.}}\, g}{s-s_0}$$

- *Obtain elastic form factors*

$$\sim \frac{g^2 iF_{\mathrm{el.}}(Q^2)}{(s_i-s_0)(s_f-s_0)}$$

Three-point functions and matrix elements

Three-point functions:

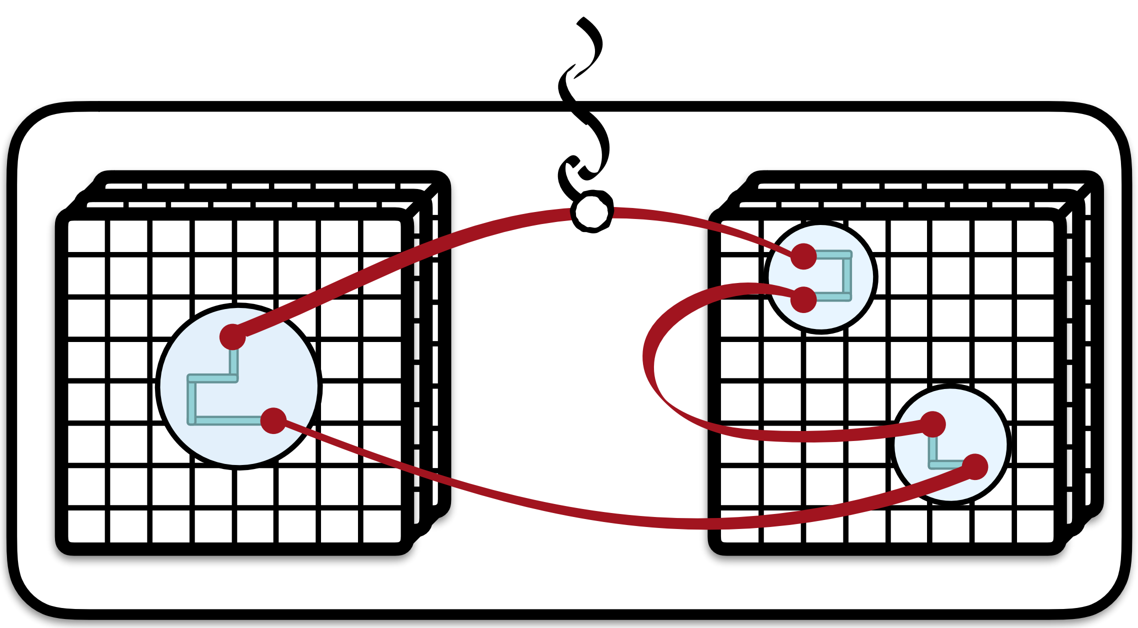
$$C^{3pt.}_{i\to f\mathcal{J}} = \langle 0|\mathcal{O}_f(\delta t)\mathcal{J}(t)\mathcal{O}_i^\dagger(0)|0\rangle_L = \sum_{n,n'} Z_{n,f} Z^*_{n',i} e^{-(\delta t - t)E_n} e^{-tE_{n'}} \langle n|\mathcal{J}|n'\rangle_L$$

Are finite-volume matrix elements meaningful?

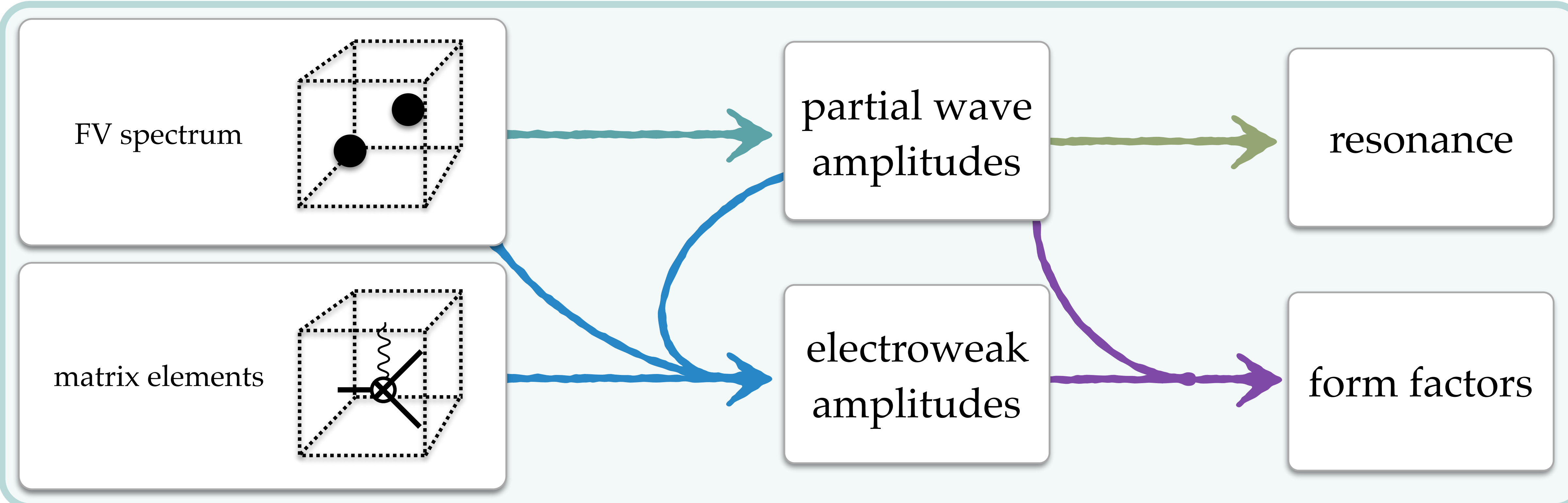
For QCD stable states: $\langle 1|\mathcal{J}|1\rangle_L = \langle 1|\mathcal{J}|1\rangle_\infty + \mathcal{O}(e^{-m_\pi L}) \approx \langle 1|\mathcal{J}|1\rangle_\infty$

otherwise and in general: $\langle n|\mathcal{J}|n'\rangle_L \not\approx \langle n|\mathcal{J}|n'\rangle_\infty$

just like with the spectrum, their relation must be obtained analytically



1-to-2 formalism

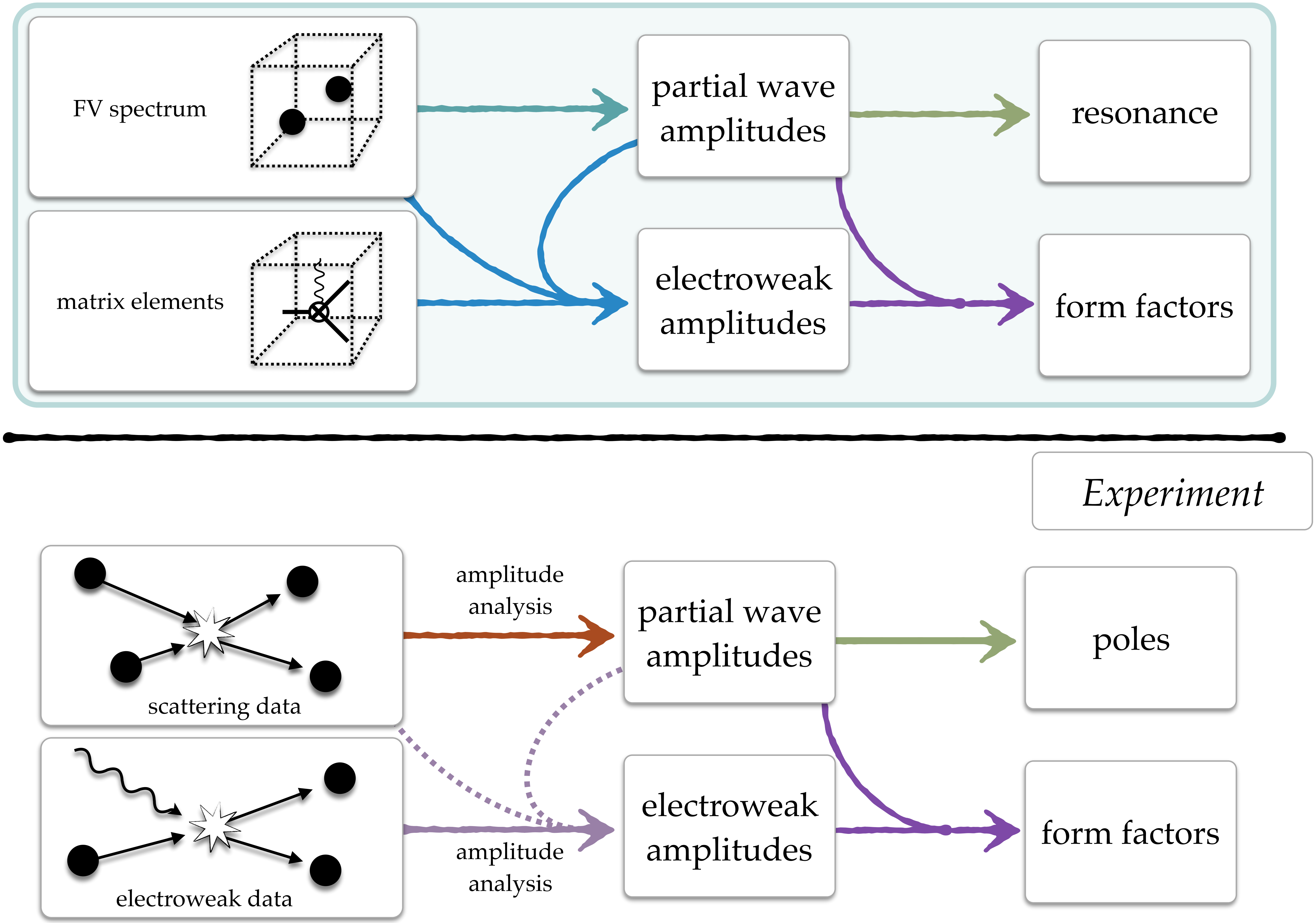


$$\left|\langle \mathbf{2}|\mathcal{J}|\mathbf{1}\rangle_L\right| = \sqrt{\mathcal{A}\,\mathcal{R}\,\mathcal{A}}$$

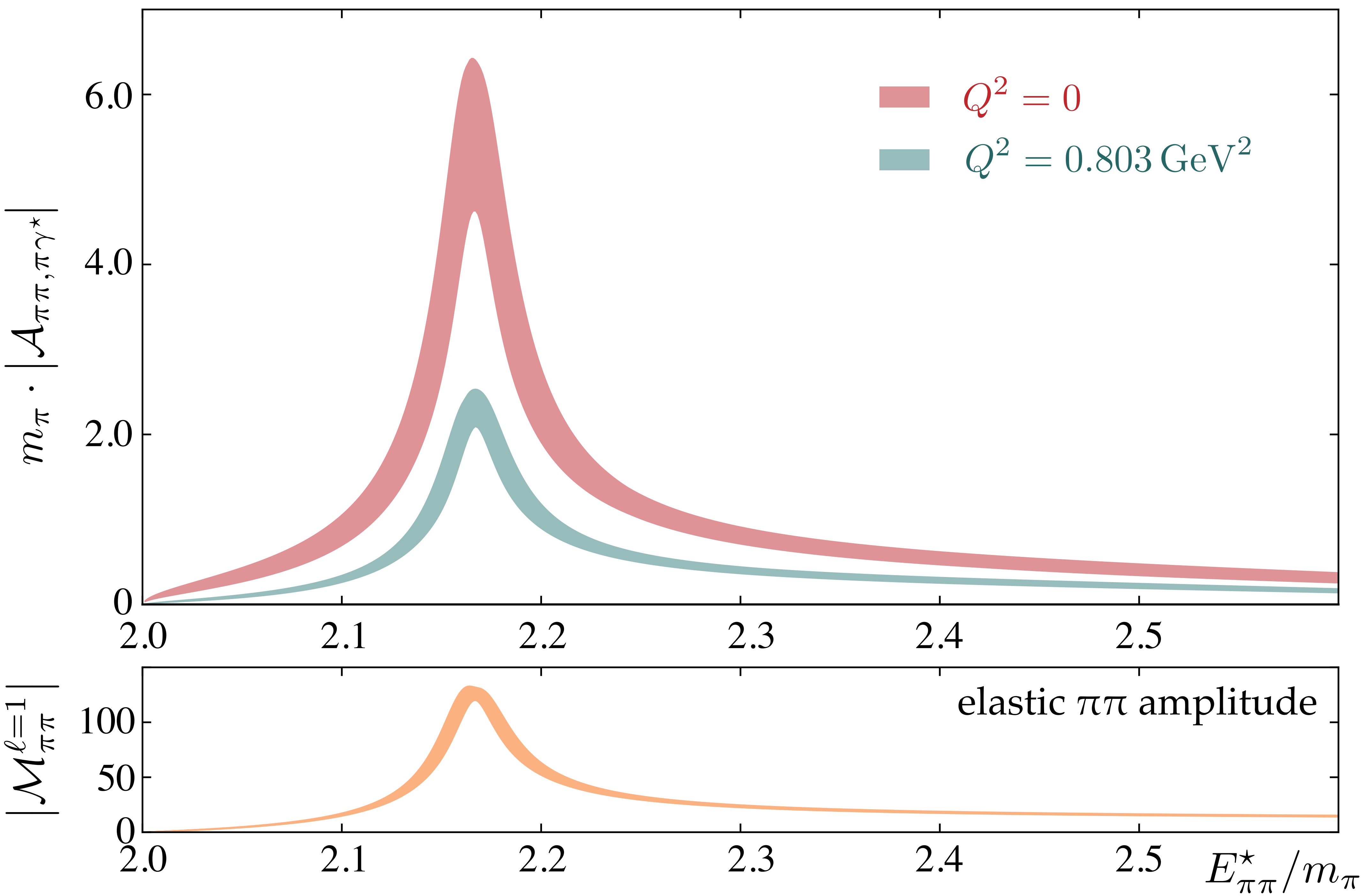
$\langle \mathbf{2}|\mathcal{J}|\mathbf{1}\rangle_L$ = FV matrix element
$\mathcal{R}$ = known function
$\mathcal{A}$ = electroweak amp.

- Lellouch & Lüscher (2000) [K-to-$\pi\pi$ at rest]
- Kim, Sachrajda, & Sharpe / Christ, Kim & Yamazaki (2005) [moving K-to-$\pi\pi$]
- …
- Hansen & Sharpe (2012) [D-to-$\pi\pi$ / KK]
- RB, Hansen Walker-Lou / RB & Hansen (2014-2015) [general 1-to-2 result]

1-to-2 formalism



$\pi\gamma^*$-to-$\pi\pi$ amplitude

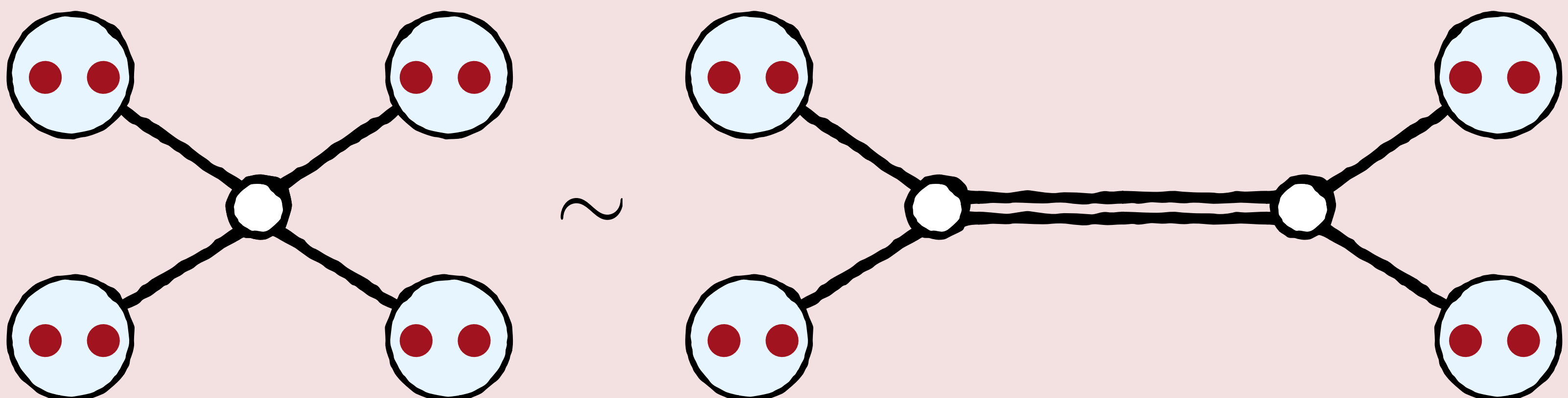


m_π=391 MeV

RB, Dudek, Edwards, Thomas, Shultz, Wilson - PRL (2015)

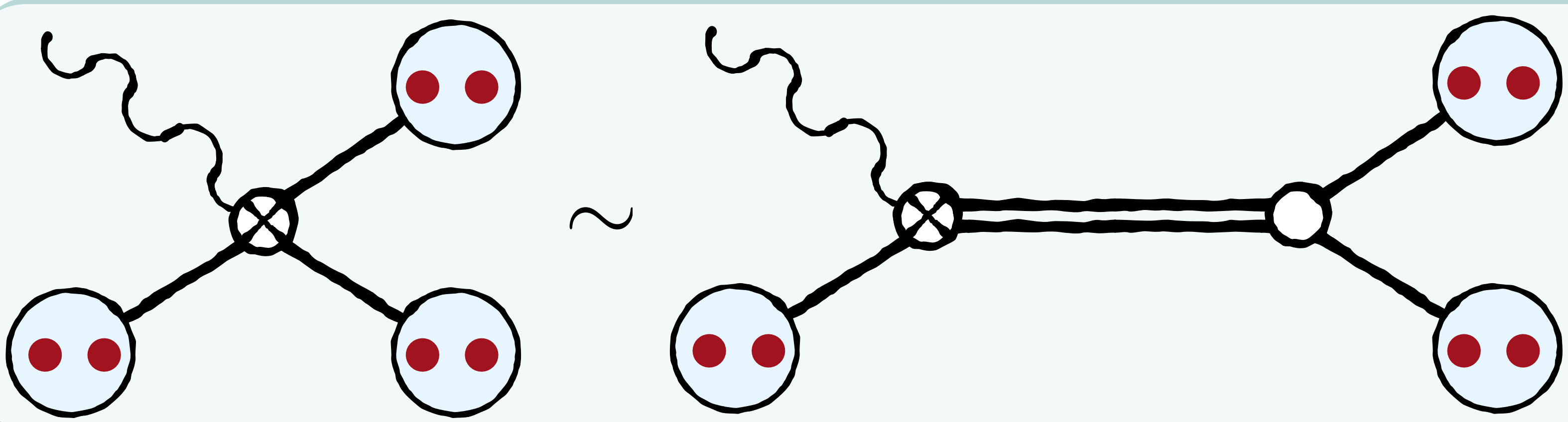
Explanation

- *$\pi\pi$-to-$\pi\pi$ amplitude:*



$$\sim \frac{i(ig_{\rho,\pi\pi})^2}{s - s_\rho}$$

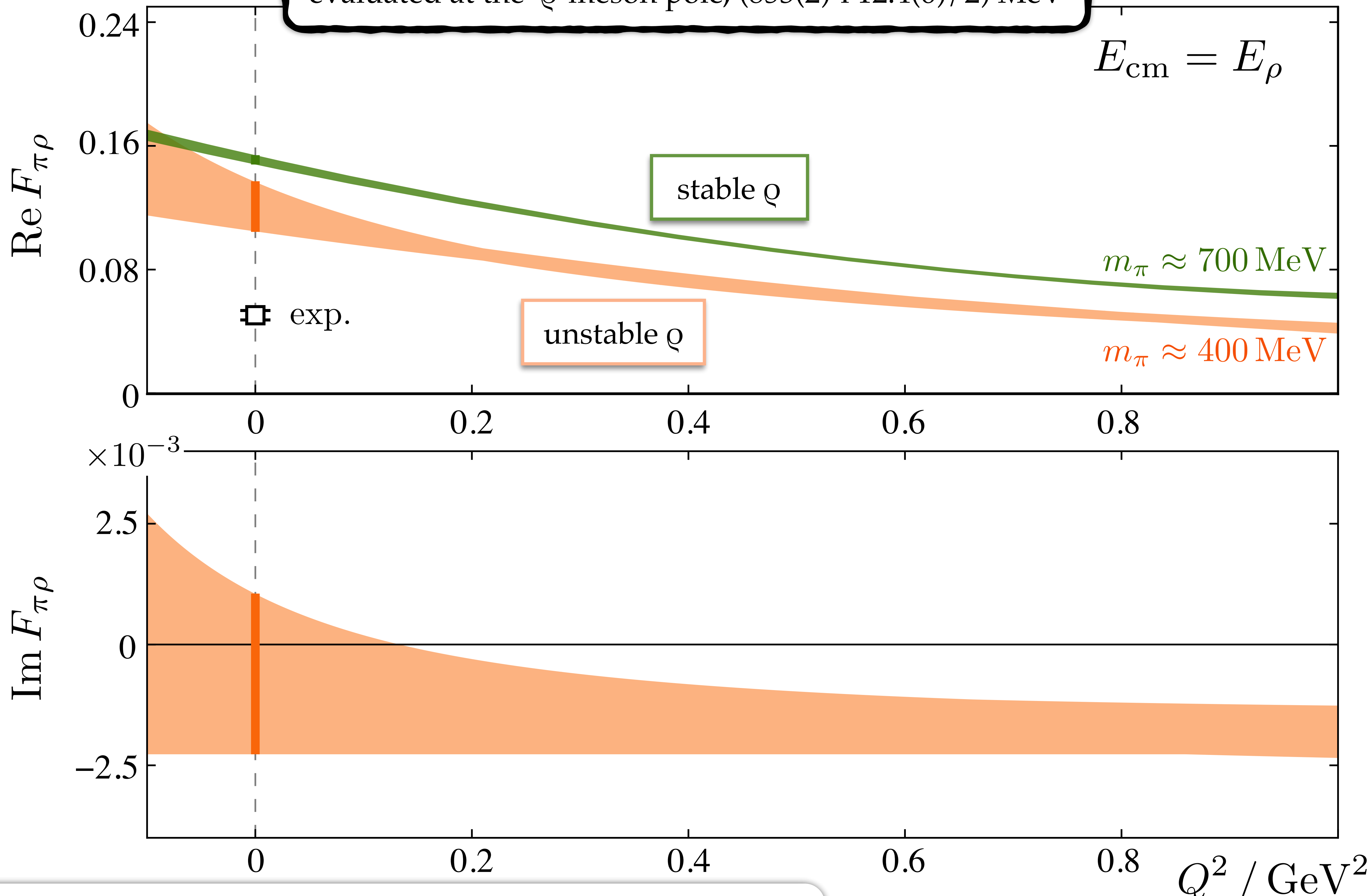
- *$\pi\gamma^*$-to-$\pi\pi$ amplitude:*



$$\sim -\frac{iF_{\pi\rho}\; g_{\rho,\pi\pi}}{s - s_\rho}$$

π-to-ϱ form factor

evaluated at the ϱ-meson pole, (853(2)-i 12.4(6)/2) MeV



Shultz, Dudek, & Edwards (2014)
RB, Dudek, Edwards, Shultz, Thomas & Wilson - PRL (2015)

Goals

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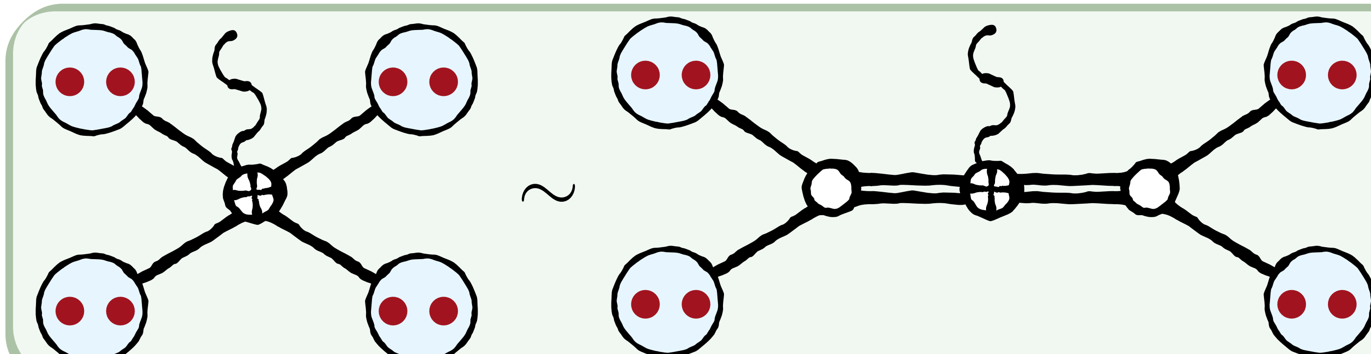
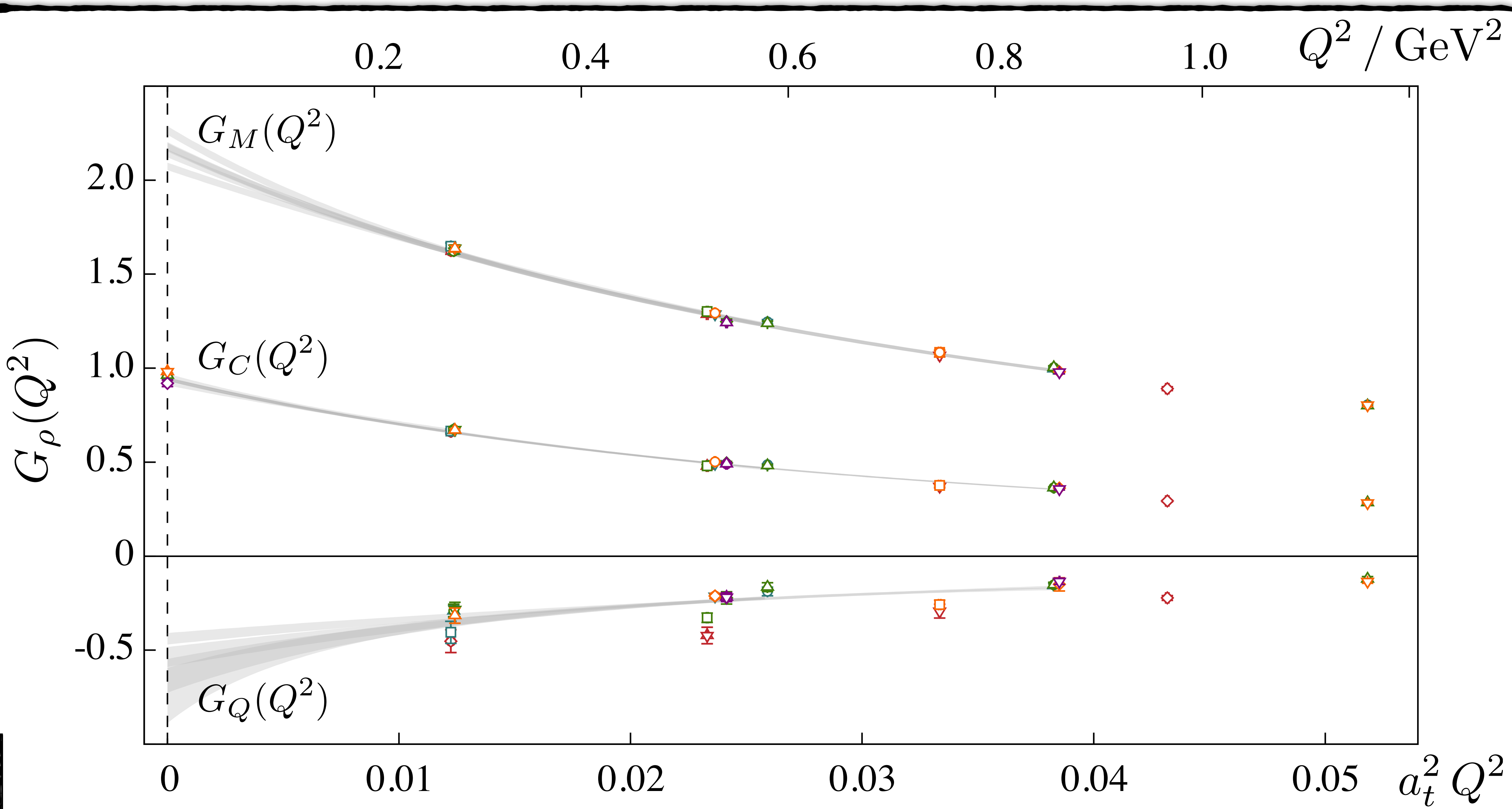
- *Obtain elastic form factors*



$$\sim \frac{g^2 iF_{\text{el.}}(Q^2)}{(s_i - s_0)(s_f - s_0)}$$

Elastic form factors

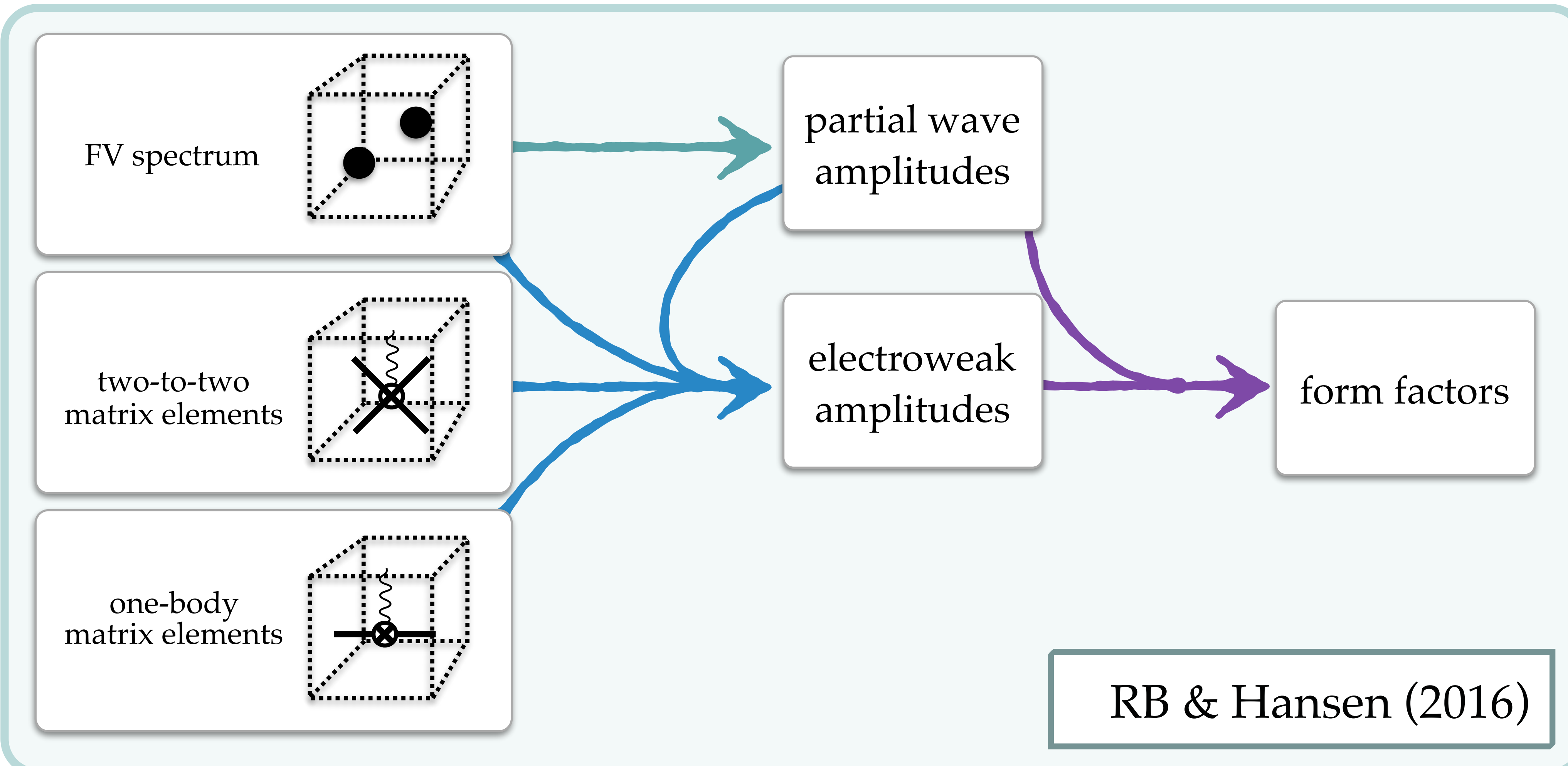
@ m_π= 700 MeV (*everything is stable!*)



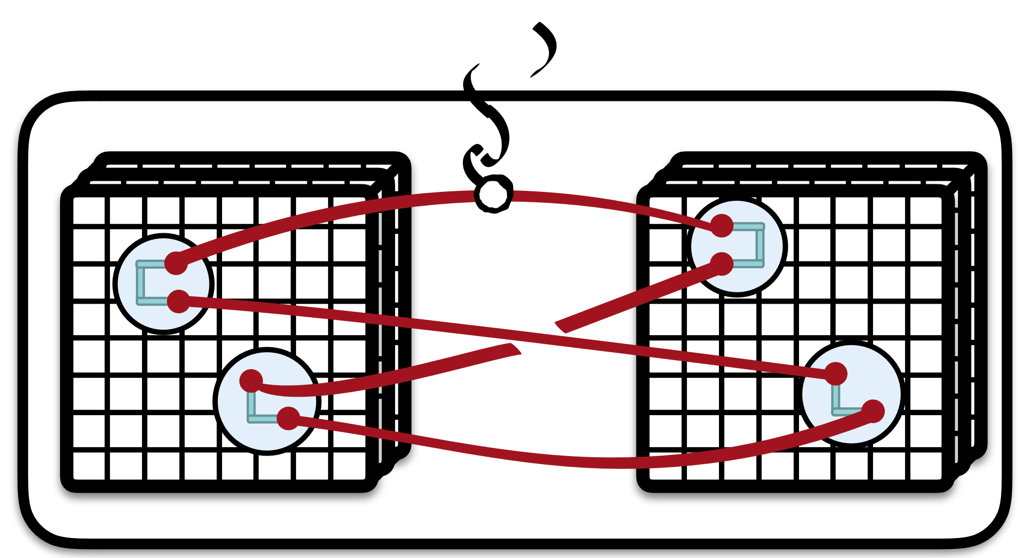
Shultz, Dudek, Edwards - PRD (2015)

2-to-2 formalism

- Formalism in place:



- necessary for:
  - scattering states
  - bound states
  - resonances
- untested!



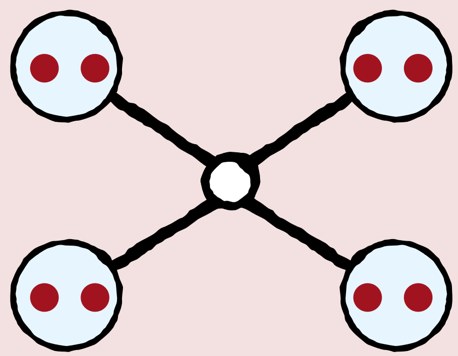
Some details…

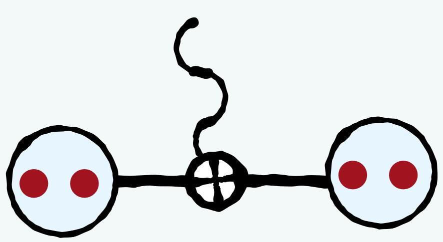
$$\left|\langle 2|\mathcal{J}|2\rangle\right|^2_L = \frac{1}{L^6}\,\mathrm{Tr}\left[\mathcal{R}\,\mathcal{W}_{L,\mathrm{df}}\,\mathcal{R}\,\mathcal{W}_{L,\mathrm{df}}\right]$$

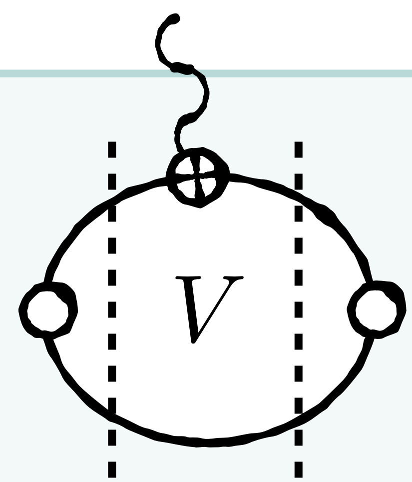
$\mathcal{R} =$ known function

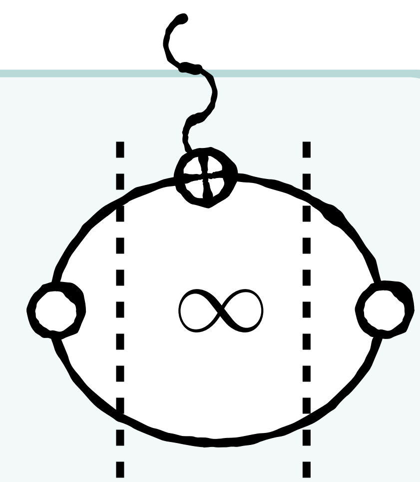
$$\mathcal{W}_{L,\mathrm{df}} = \mathcal{W}_{\mathrm{df}} + \mathcal{M}\;G(L,w)\;\mathcal{M}$$

"known"

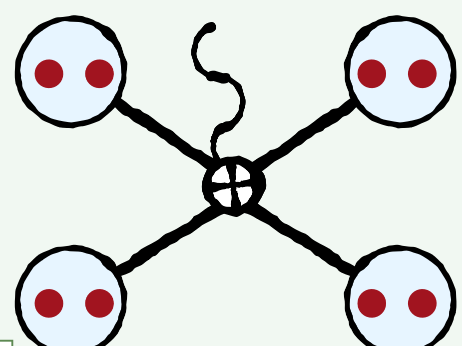
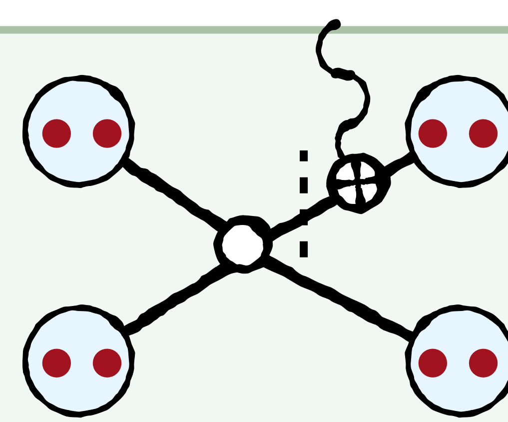
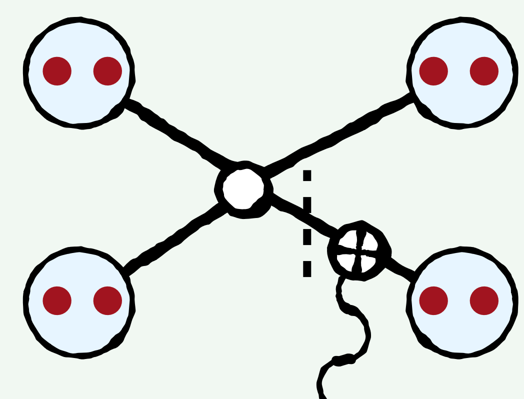
$i\mathcal{M} =$ 

$iw =$ 

$iG(L,w) =$ 

$-$ 

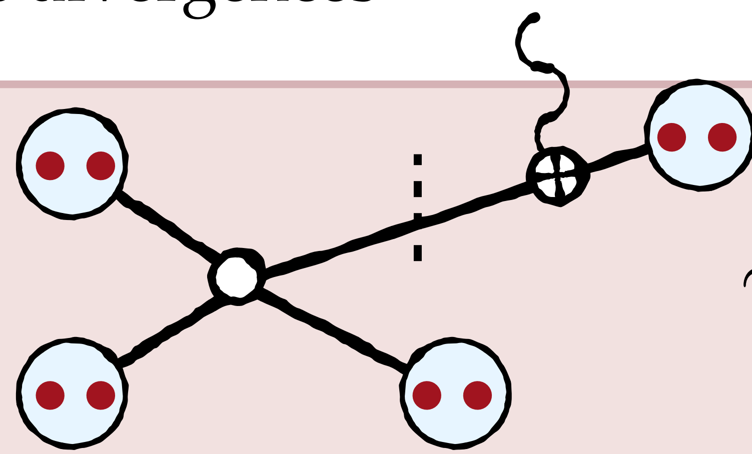
new

$i\mathcal{W}_{\mathrm{df}} =$  $-$  $-$  $- \cdots$

df = divergence free

Some more details…

- Kinematic divergences

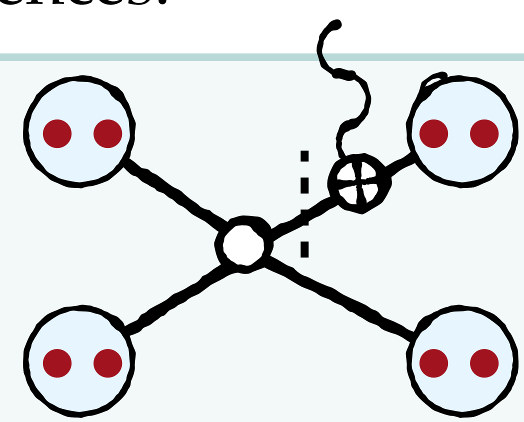


$$\sim i\mathcal{M}\,\frac{i}{2E_1'(E-E_1-E_1')}\,iw$$

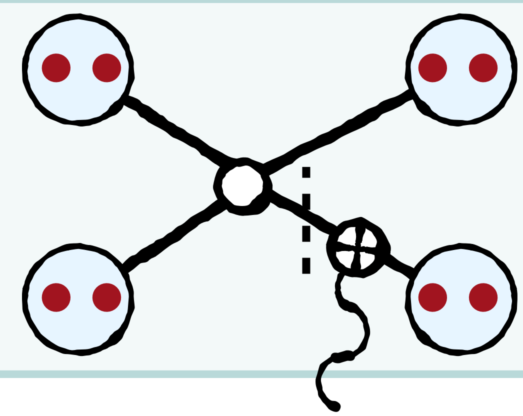
only depend on on-shell quantities

- One can add back divergences:

$$i\mathcal{W} = i\mathcal{W}_{\text{df}} +$$



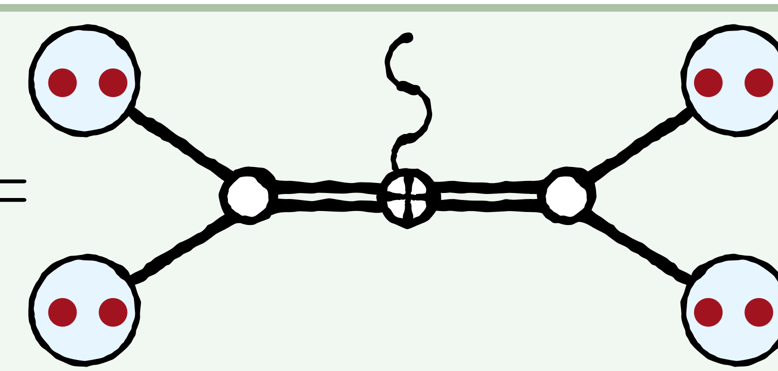
$$+$$



$$+\cdots$$

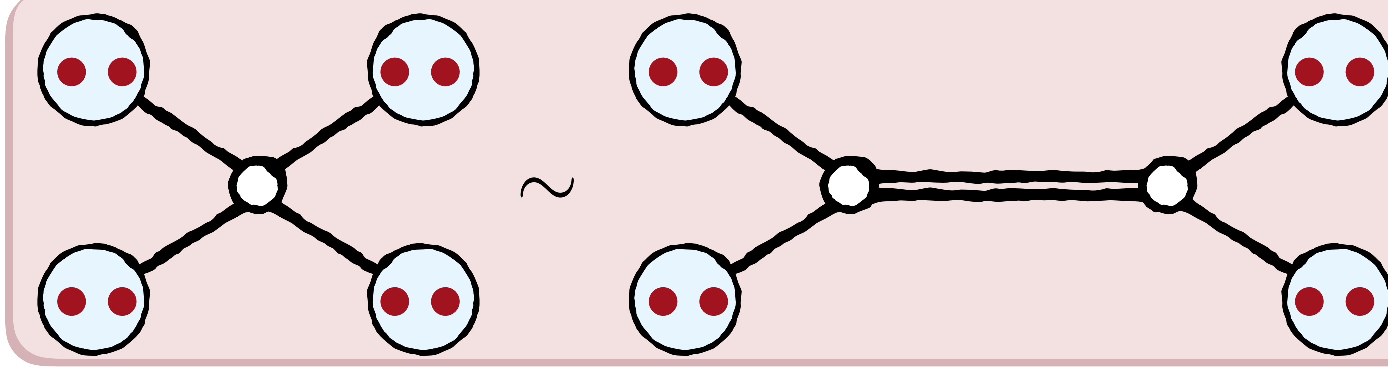
- Form factors of composite states:

$$\lim_{E_i,E_f\to E_R} i\mathcal{W} = \lim_{E_i,E_f\to E_R} i\mathcal{W}_{\text{df}} =$$



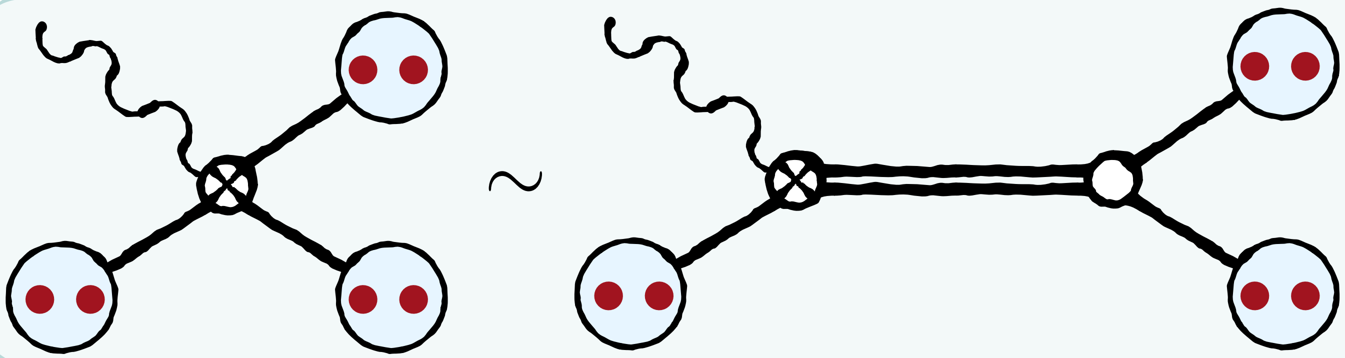
Almost there!

- *Obtain masses and width*



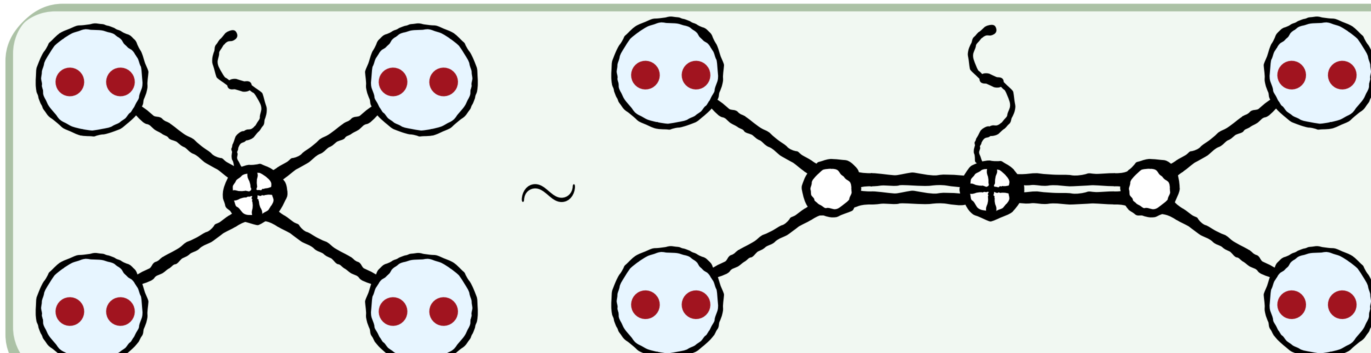
$$\sim \frac{i(ig)^2}{s - s_0}$$

- *Obtain transition form factors*



$$\sim -\frac{iF_{\text{tr.}}\, g}{s - s_0}$$

- *Obtain elastic form factors*



$$\sim \frac{g^2 iF_{\text{el.}}(Q^2)}{(s_i - s_0)(s_f - s_0)}$$

Collaborators & references

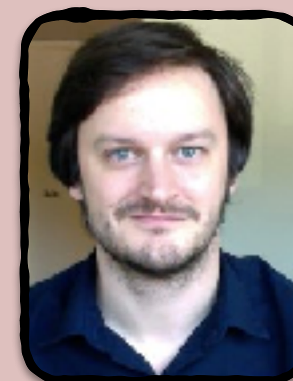

Baroni


Bolton


Hansen

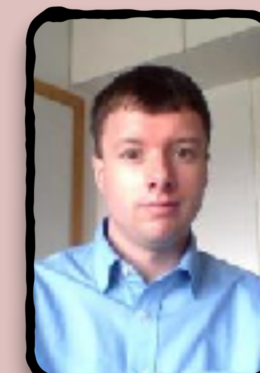

Walker-Loud


Chakraborty


Dudek


Edwards


Shultz


Thomas


Wilson

Formalism

PRD95 074510 (2017)
PRD94 013008 (2016)
PRD92 074509 (2015)
PRD91 034501 (2015)
PRD89 074507 (2014)

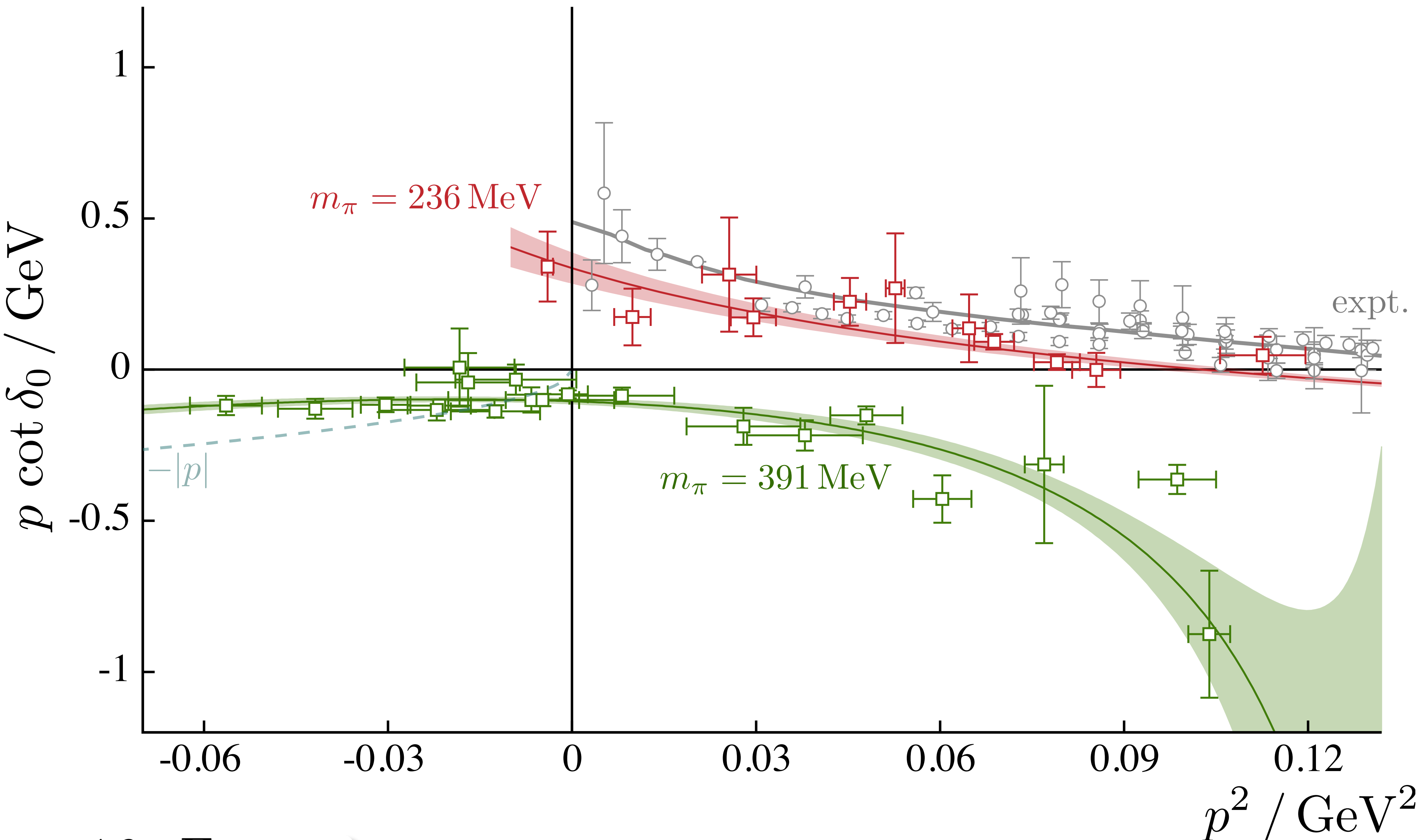
Numerical

PRD93 114508 (2016)
PLB 50-56 757 (2016)
PRL115 242001 (2015)
PRD92 094502 (2015)
PRD91 114501 (2015)

hadspec

backup slides

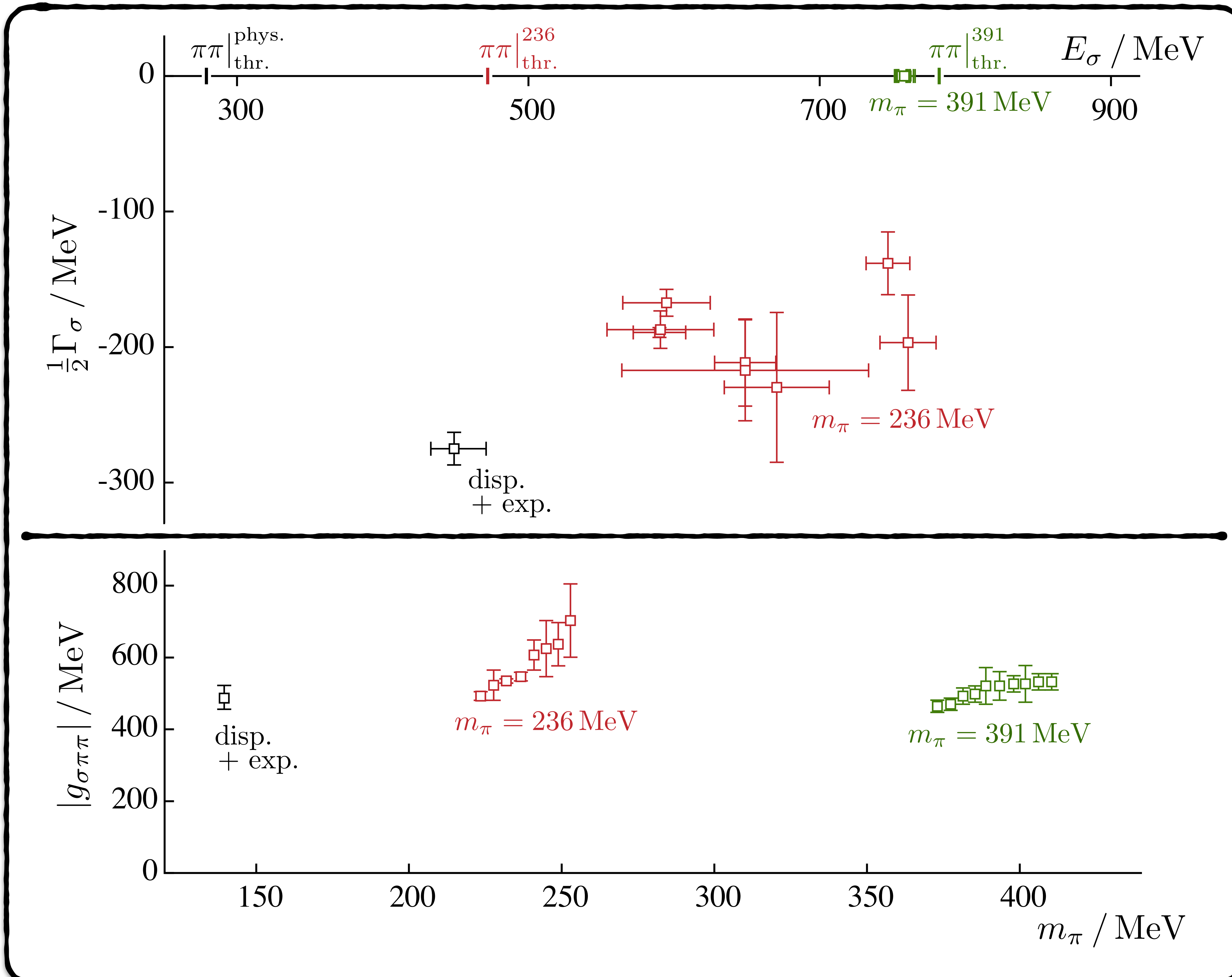
Isoscalar $\pi\pi$ scattering



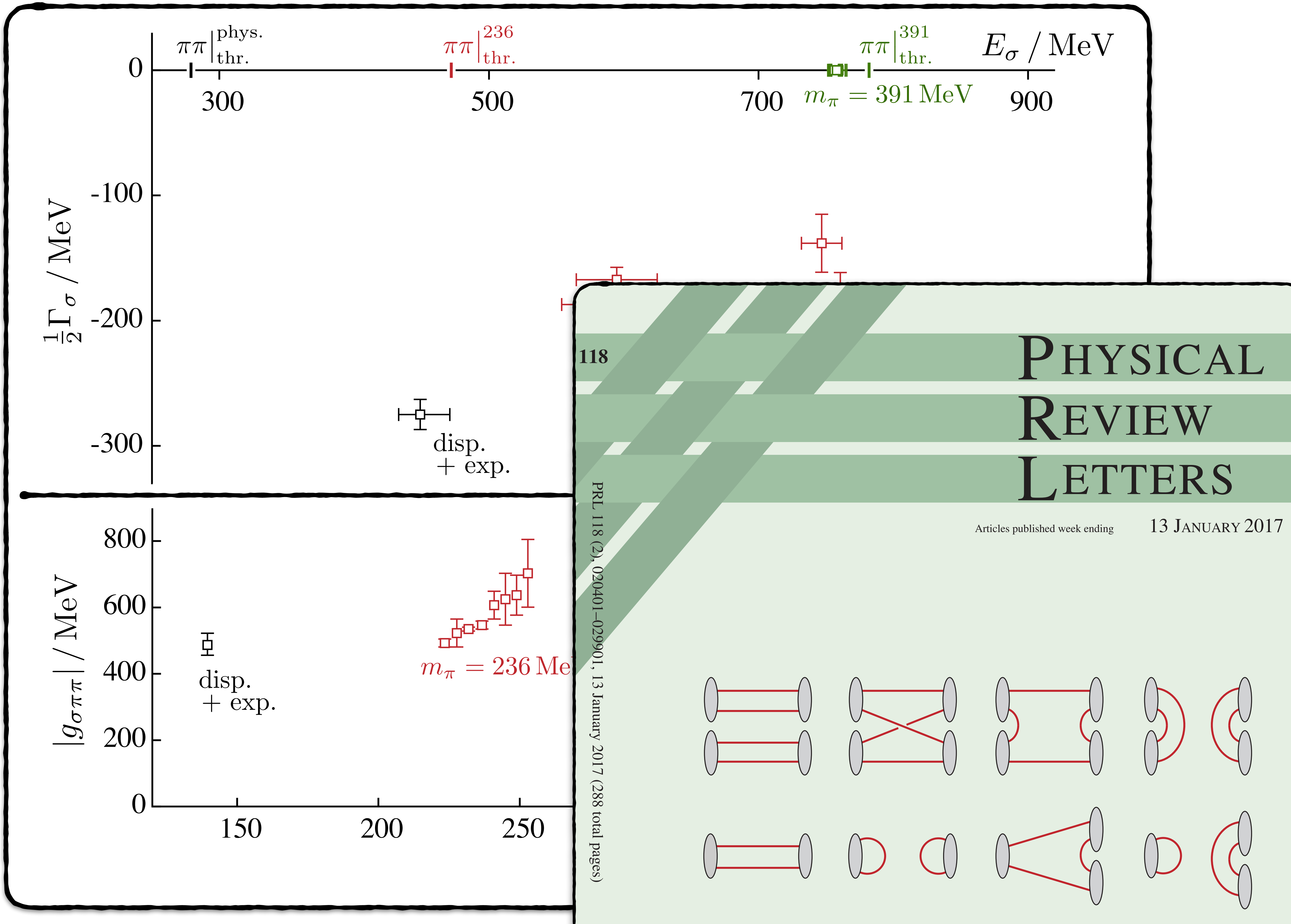
$$\mathcal{M}_0 = \frac{16\pi E_{\mathrm{cm}}}{p \cot \delta_0 - ip}$$

RB, Dudek, Edwards, Wilson - PRL (2017)

The σ / $f_0(500)$ vs m_π



The σ / f₀(500) vs m_π

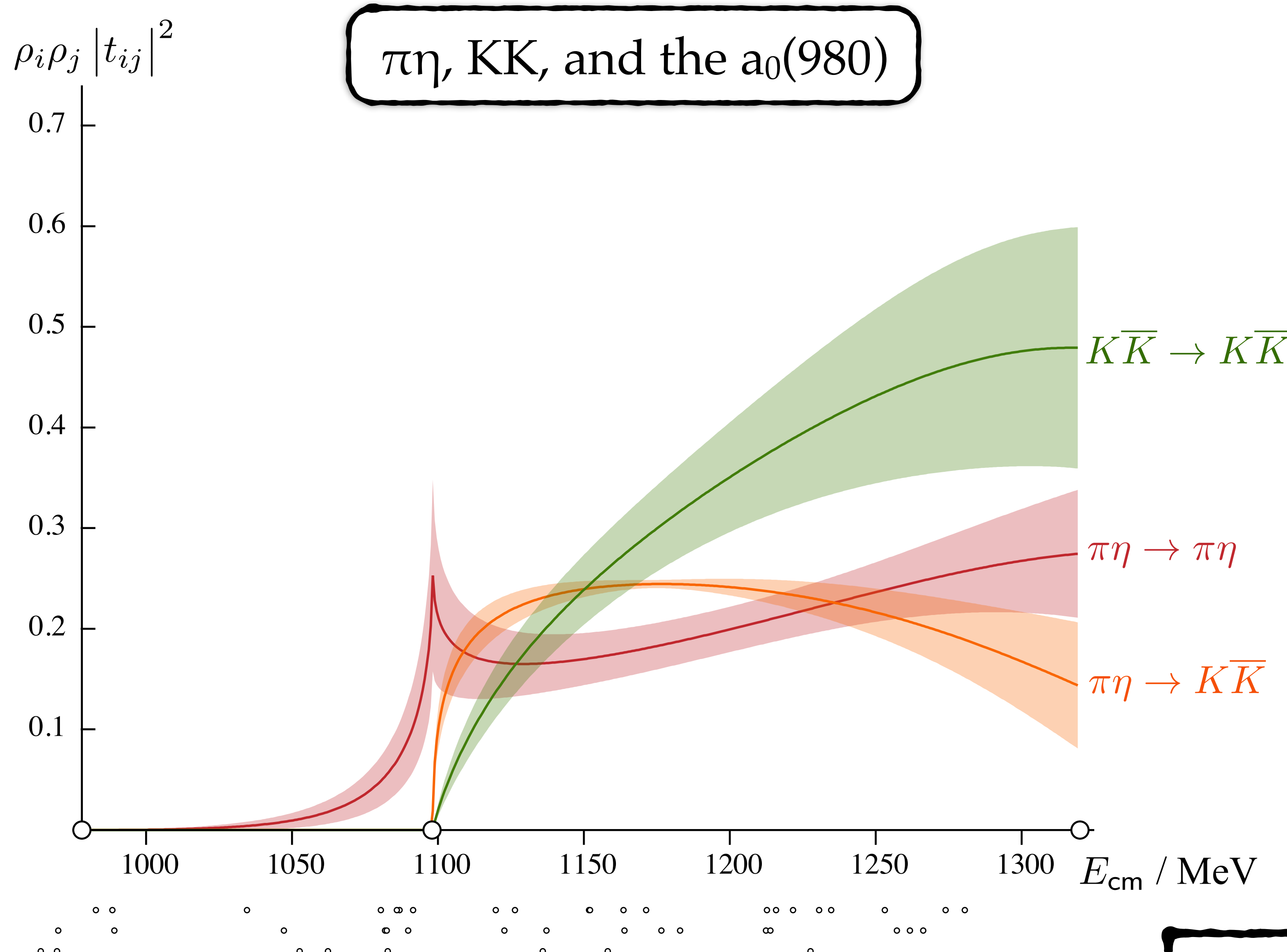


Coupled-channels systems

Four systems consider so far, all by hadspec

$K\pi$, $K\eta$:	Dudek, Edwards, Thomas, Wilson - PRL (2015)
	Wilson, Dudek, Edwards, Thomas - PRD (2015)
$\pi\pi$, KK:	Wilson, RB, Dudek, Edwards - PRD (2015)
$\pi\eta$, KK:	Dudek, Edwards, Wilson - PRD (2016)
$D\pi$, $D\eta$, D_sK:	Moir, Peardon, Ryan, Thomas, Wilson - JHEP (2016)

$\pi\eta$, KK, and the $a_0(980)$



m_π=391 MeV

Going higher in energy

- *Beyond two particles:*

$$\det\left[1+\begin{pmatrix} F_2 & 0 \\ 0 & F_3 \end{pmatrix}\begin{pmatrix} \mathcal{K}_2 & \mathcal{K}_{23} \\ \mathcal{K}_{32} & \mathcal{K}_{\mathrm{df},3} \end{pmatrix}\right]=0$$

RB, Hansen & Sharpe (2017)

"could be used to extract 3N forces, the Roper and much more"