Rapidity factorization and TMDs

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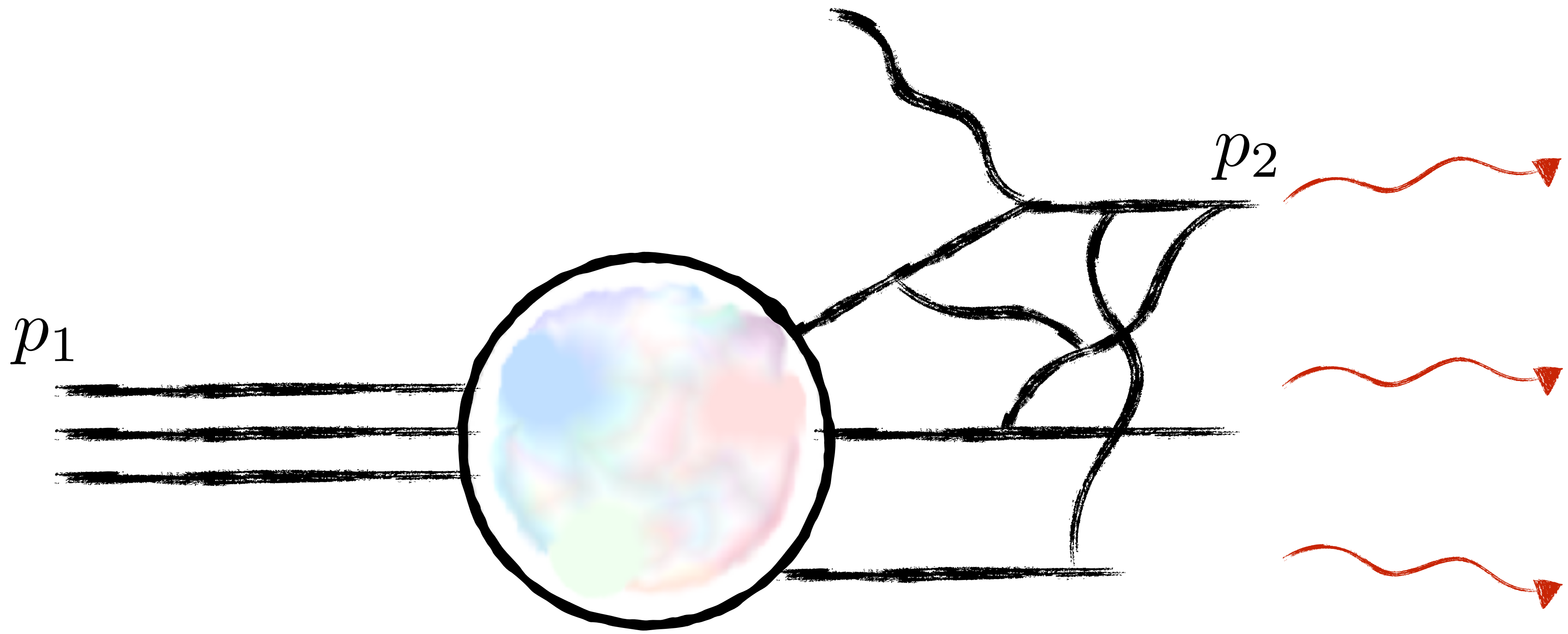
Deep inelastic scattering



An arbitrary momentum:

$$p = \alpha p_1 + \beta p_2 + p_\perp$$

Use this parameter to separate phases

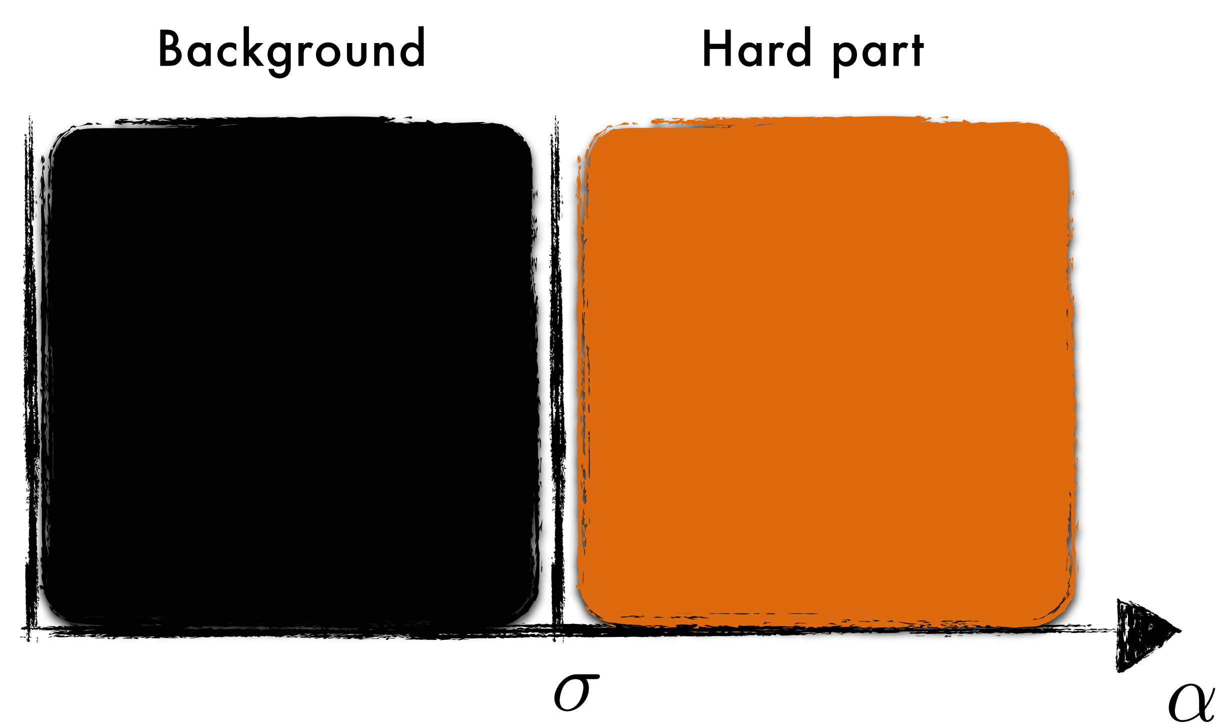
Rapidity factorization. DIS

I. Balitsky, hep-ph/0101042;
I. Balitsky and A. Tarasov, JHEP 10 (2015) 017; JHEP 06 (2016) 164

An arbitrary momentum:

$$p = \alpha p_1 + \beta p_2 + p_\perp$$

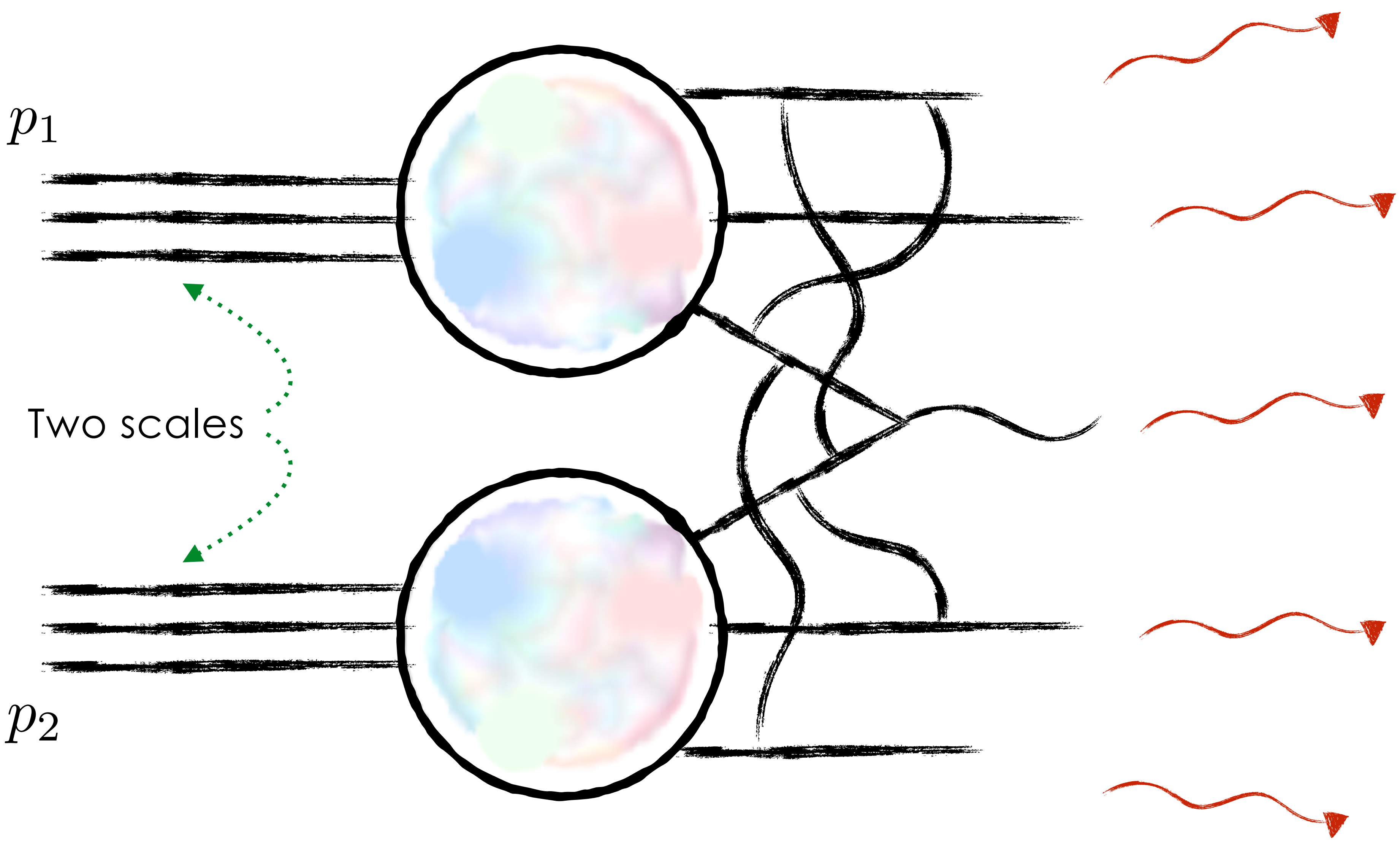
Use this parameter to separate phases



Factorization. Drell-Yan

$$p = \alpha p_1 + \beta p_2 + p_\perp$$

Can not use just one of these variables

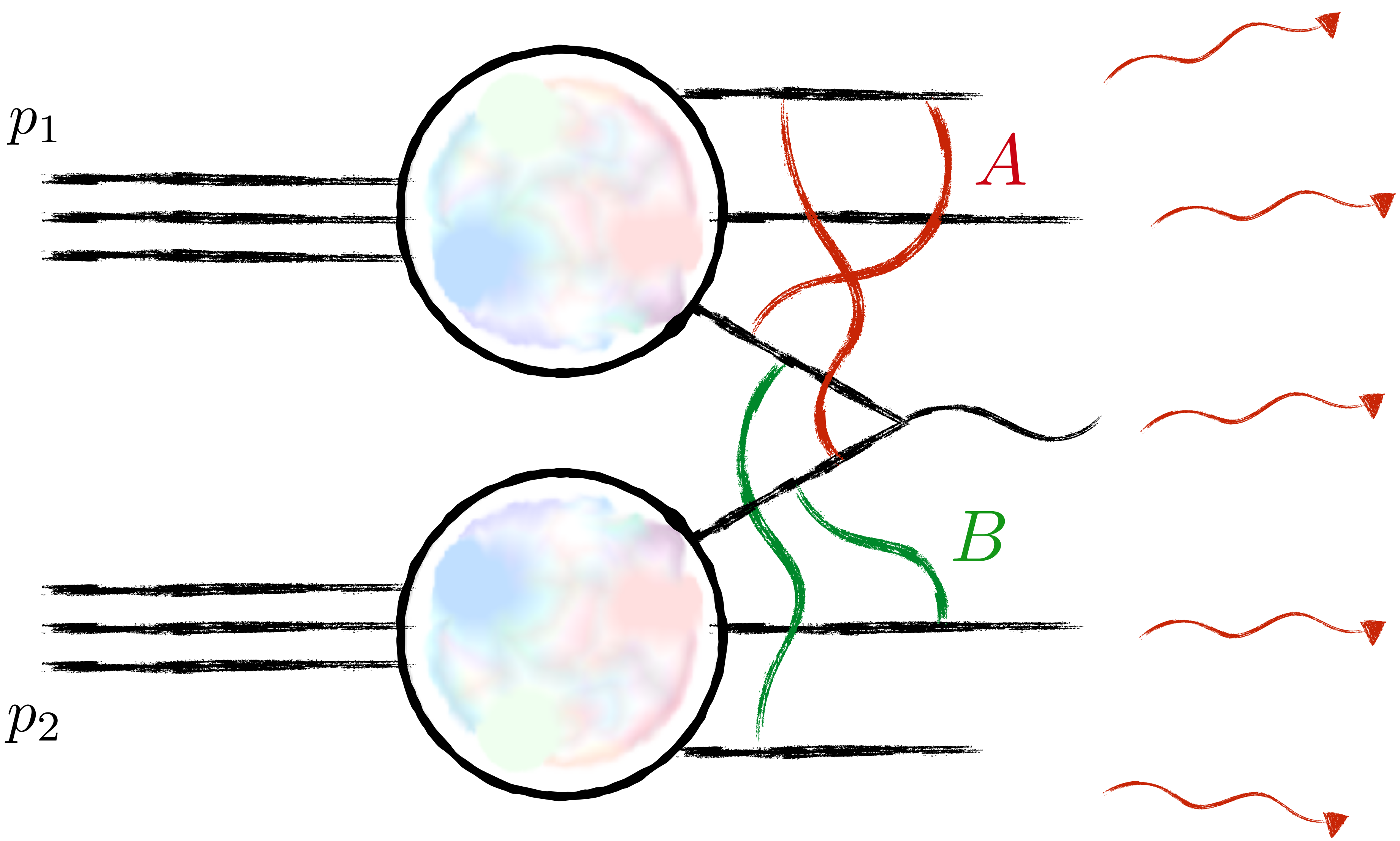


Factorization. Drell-Yan

Have to include two types of external background fields

$$p = \alpha p_1 + \beta p_2 + p_\perp$$

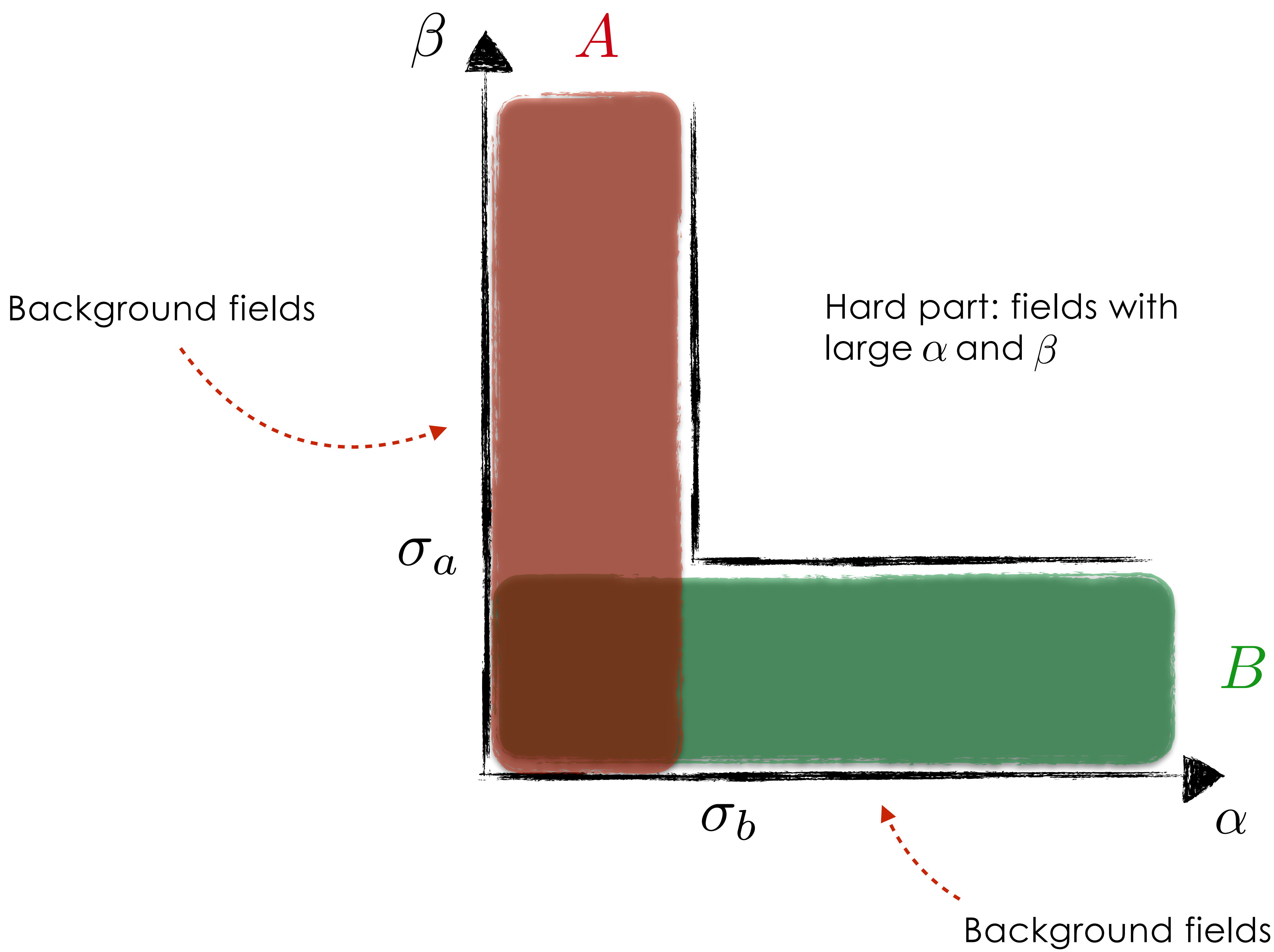
Can not use just one of these variables



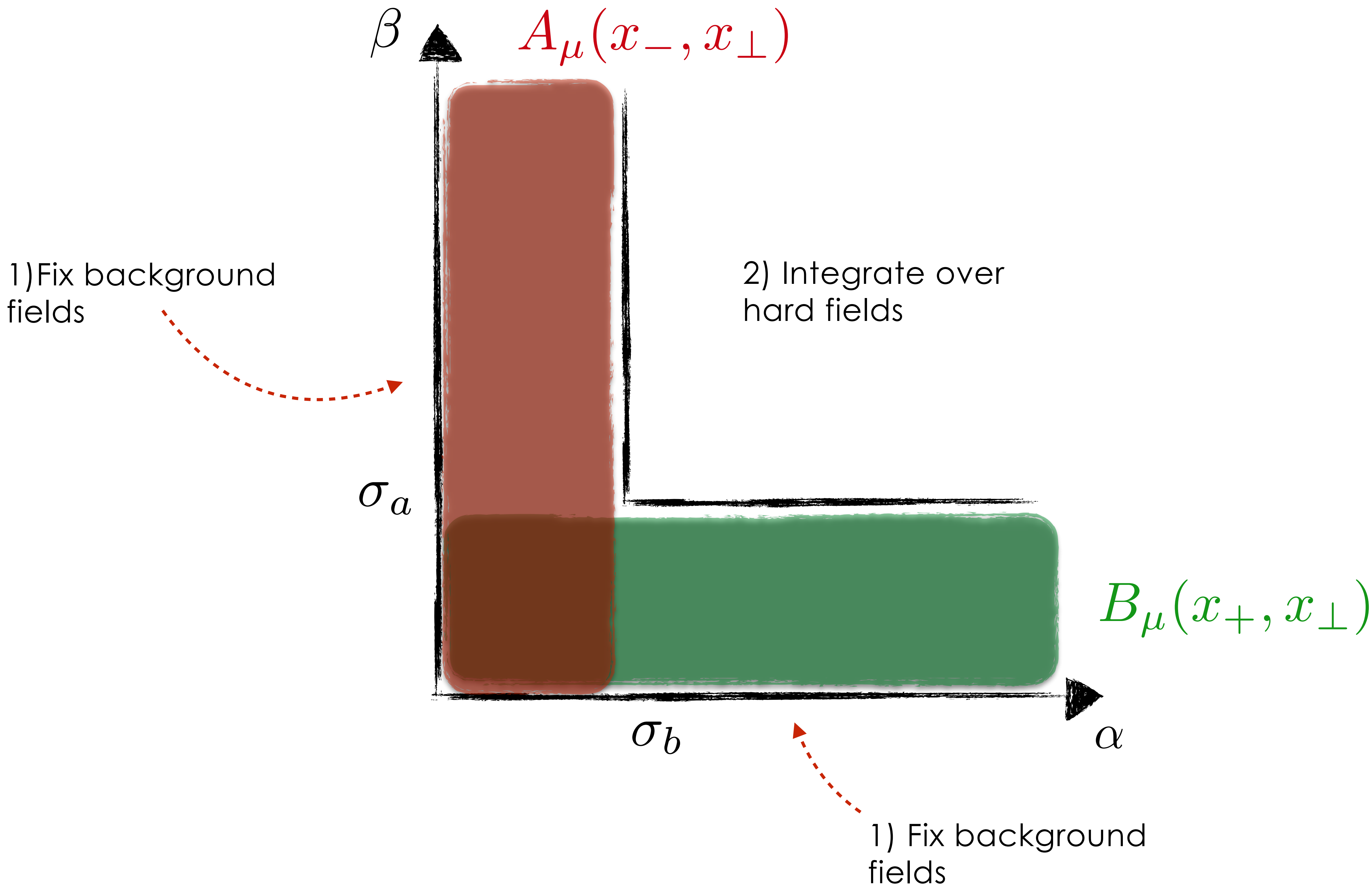
Two background fields

How do we define those fields?

Rapidity separation in hadron-hadron scattering

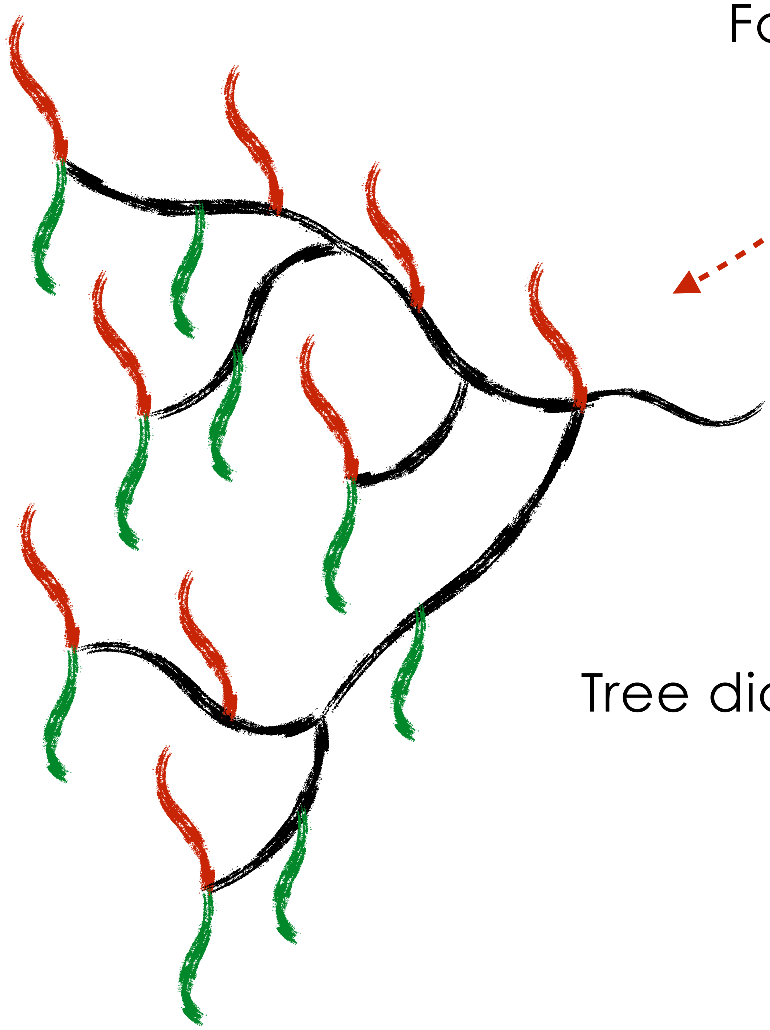


Rapidity separation in hadron-hadron scattering



Feynman diagrams in two background fields

Factorization?



Tree diagrams

moderate x

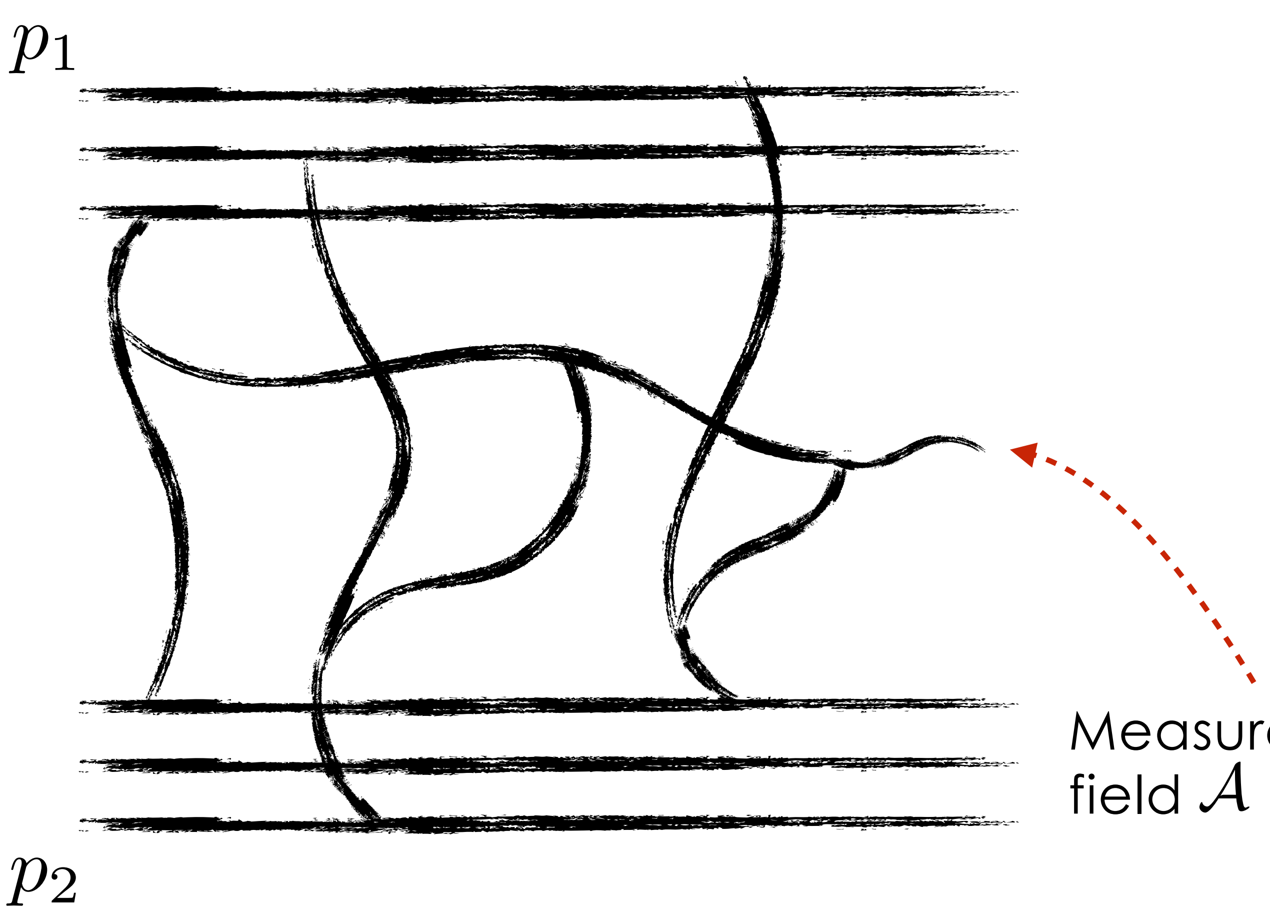
J.C. Collins, D.E. Soper and G. Sterman, Phys. Lett. B 109 (1982) 388;
J.C. Collins, D.E. Soper and G.F. Sterman, Nucl. Phys. B 250 (1985) 199;
G.T. Bodwin, Phys. Rev. D 31 (1985) 10;
X.-d. Ji, J.-p. Ma and F. Yuan, Phys. Rev. D 71 (2005) 034005;
M.G. Echevarria, A. Idilbi and I. Scimemi, JHEP 07 (2012) 002

small-x (shock-wave)

I. Balitsky, Phys. Rev. Lett. 81, 2024 (1998);
Yu. V. Kovchegov and A. H. Mueller, Nucl. Phys. B529, 451 (1998);
Yu. V. Kovchegov and K. Tuchin, Phys. Rev. D 65, 074026 (2002);
A. Dumitru and L. D. McLerran, Nucl. Phys. A700, 492 (2002);
J.P. Blaizot, F. Gelis, and R. Venugopalan, Nucl. Phys. A743, 13 (2004);
F. Gelis, T. Lappi, and R. Venugopalan, Phys. Rev. D 78, 054019 (2008)

1) Re-examine TMD factorization
2) Derive high-p_T corrections (Y-term)

YM equation of motion



Equation of motion:

$$\mathcal{D}^{\mu}_{\mathcal{A}}\mathcal{F}^{a}_{\mu\nu} + g\bar{\Psi}t^{a}\gamma_{\nu}\Psi = 0$$

Separation of the background field:

$$\mathcal{A}_{\mu}(x) = \bar{A}_{\mu}(x) + C_{\mu}(x)$$

Background field　　Quantum field

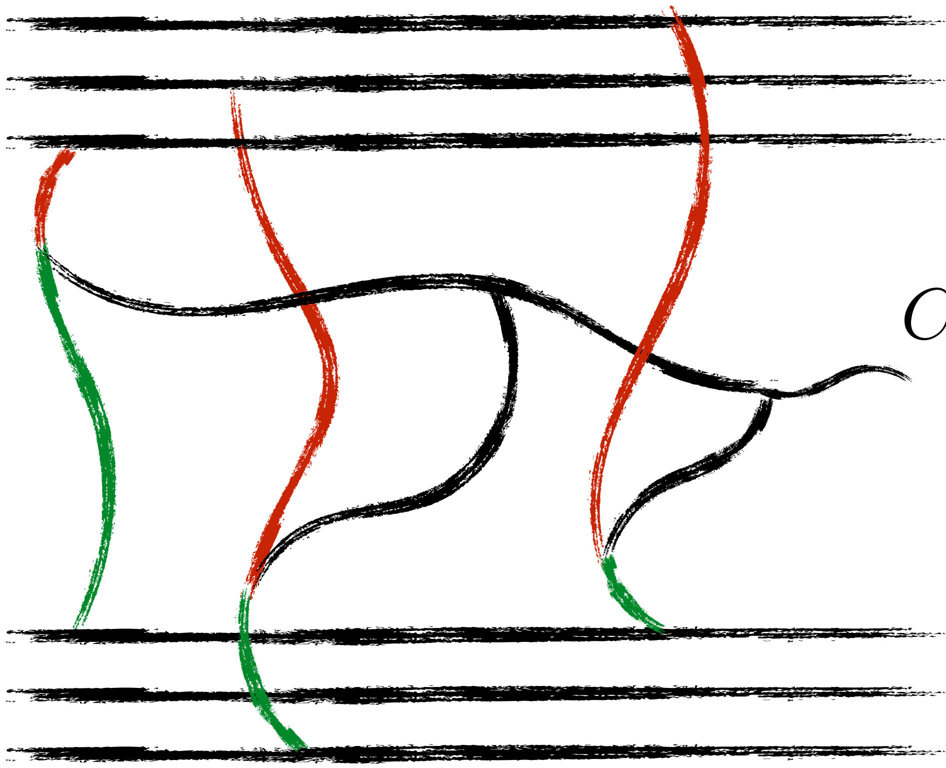
Initial condition:

$$\mathcal{A}_{\mu}(x) \stackrel{x_+\to-\infty}{=} A_{\mu}(x_-) \qquad \mathcal{A}_{\mu}(x) \stackrel{x_-\to-\infty}{=} B_{\mu}(x_+)$$

We construct solution of the YM equation of motion with the ***retarded*** propagators in the background field.

Note: this is equivalent to calculation of the "cross section" with Feynman propagator when background fields for two sides of the cut are the same: $\bar{\mathcal{A}}(x) = \bar{\mathcal{A}}'(x)$

YM equation of motion in background field



Shift of the field:

$$\mathcal{D}^\nu \mathcal{F}^a_{\mu\nu}(\bar{A}+C)=0$$

YM in the background field:

$$(g^{\mu\nu}\bar{\mathcal{P}}^2_{ab}+2ig\bar{\mathcal{F}}^{\mu\nu}_{ab})C^b_\nu=\bar{\mathcal{D}}_\nu\bar{\mathcal{F}}^{a\nu\mu}$$
$$+gf^{abc}(2C^{b\nu}\bar{\mathcal{D}}_\nu C^{c\mu}-C^{b\nu}\bar{\mathcal{D}}^\mu C^c_\nu)-g^2f^{abr}f^{cdr}C^b_\nu C^{c\mu}C^{d\nu}$$

Covariant derivative:

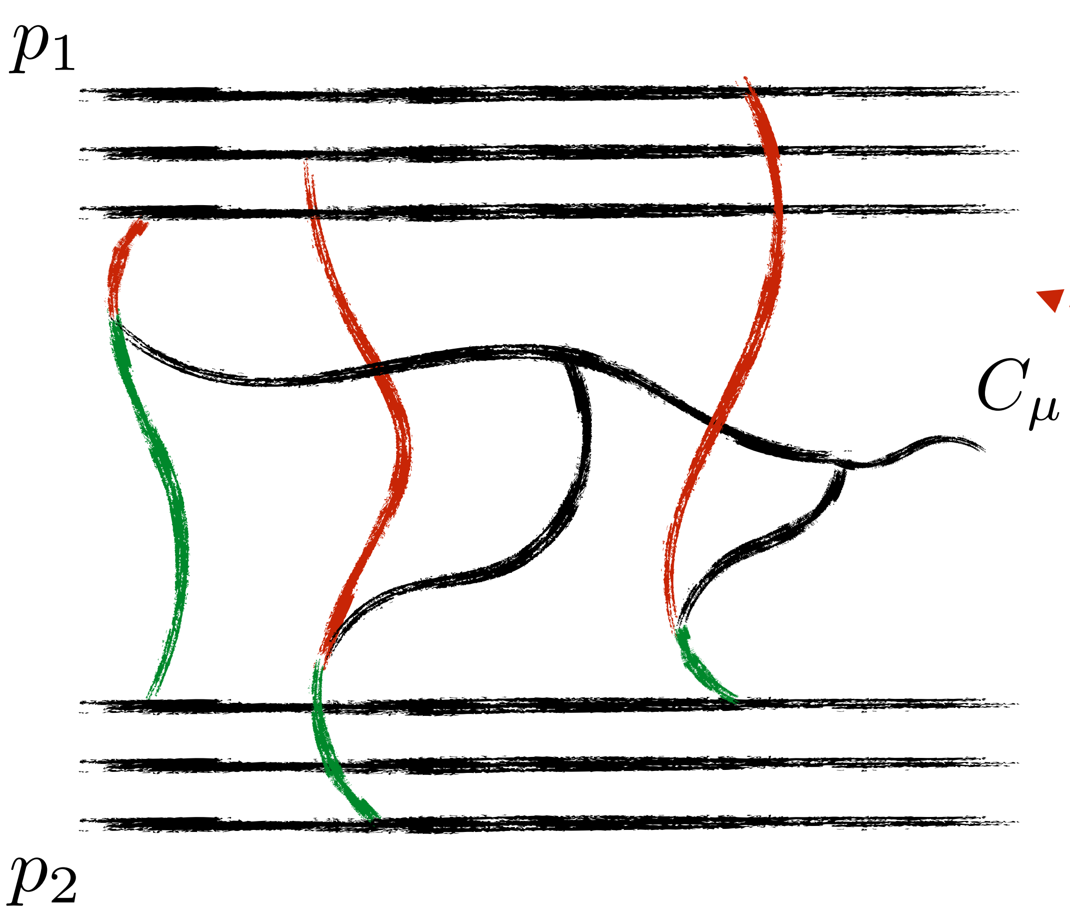
$$\bar{\mathcal{D}}_\mu=\partial_\mu-ig(A_\mu+B_\mu)$$

Initial condition:

$$C_\mu(x)\overset{x_+\to-\infty}{=}0 \qquad\qquad C_\mu(x)\overset{x_-\to-\infty}{=}0$$

1) Construct solution in two dimension (TMD factorization)

YM equation in two dimensions



Neglect transverse momentum
(two dimensional theory)

Solution in two dimensions is a pure gauge:

$$\mathcal{A}_+(x) = C_+(x) + \bar{A}_+(x) = \Omega(x) i\partial_+ \Omega^\dagger(x)$$

$$\mathcal{A}_-(x) = C_-(x) + \bar{A}_-(x) = \Omega(x) i\partial_- \Omega^\dagger(x)$$

One can reconstruct this matrix order by order:

$$\Omega = \frac{1}{2}[x_+, -\infty][x_-, -\infty] + \frac{1}{2}[x_-, -\infty][x_+, -\infty] + \ldots$$

1) Resummation without transverse momentum
2) Use this as initial condition in resummation with transverse momentum (corrections to TMD)

Important limits:

$$\Omega(x) \overset{x_+ \to -\infty}{=} [x_-, -\infty]$$

$$\Omega(x) \overset{x_- \to -\infty}{=} [x_+, -\infty]$$

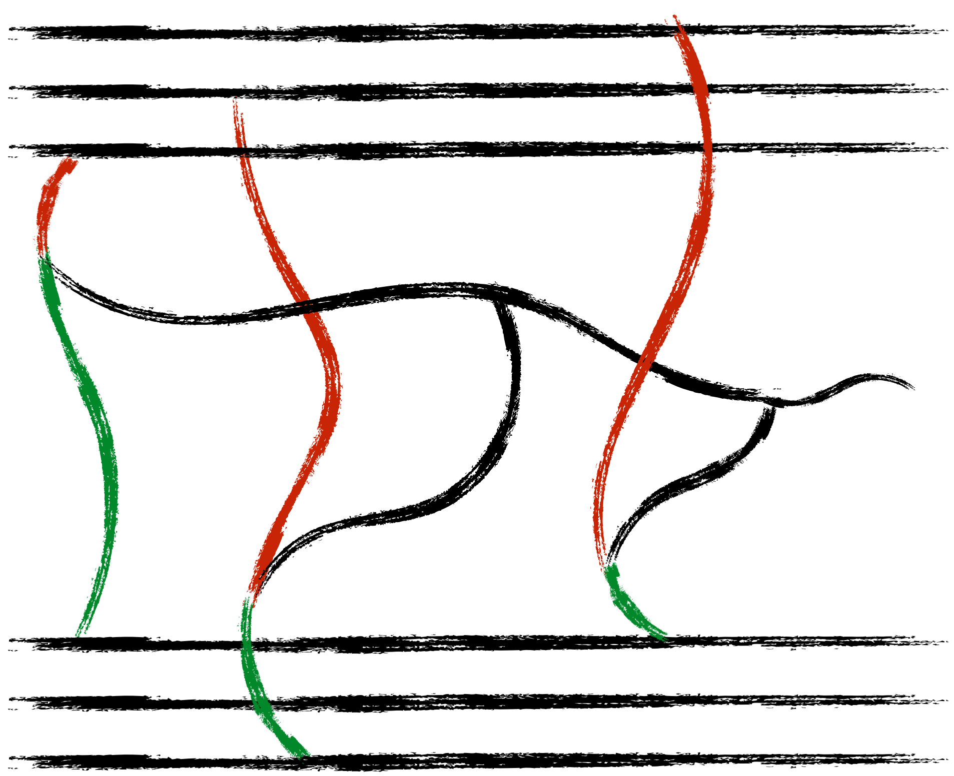
Wilson lines in TMDs

Gauge rotation of the background fields

New background field

$$U_\mu = \Omega^\dagger(x)\Big(\frac{i}{g}\partial_\mu + A_\mu(x)\Big)\Omega(x)$$

$$\Sigma_a(x_-, x_\perp) = \Omega^\dagger \xi_a(x_-, x_\perp)$$



New initial conditions:

$$\mathcal{A}_\mu(x) \overset{x_+ \to -\infty}{\to} U_\mu(x_-, x_\perp), \quad \psi(x) \overset{x_+ \to -\infty}{\to} \Sigma_a(x_-, x_\perp)$$

$$\mathcal{A}_\mu(x) \overset{x_- \to -\infty}{\to} V_\mu(x_+, x_\perp), \quad \psi(x) \overset{x_- \to -\infty}{\to} \Sigma_b(x_+, x_\perp)$$

New background field

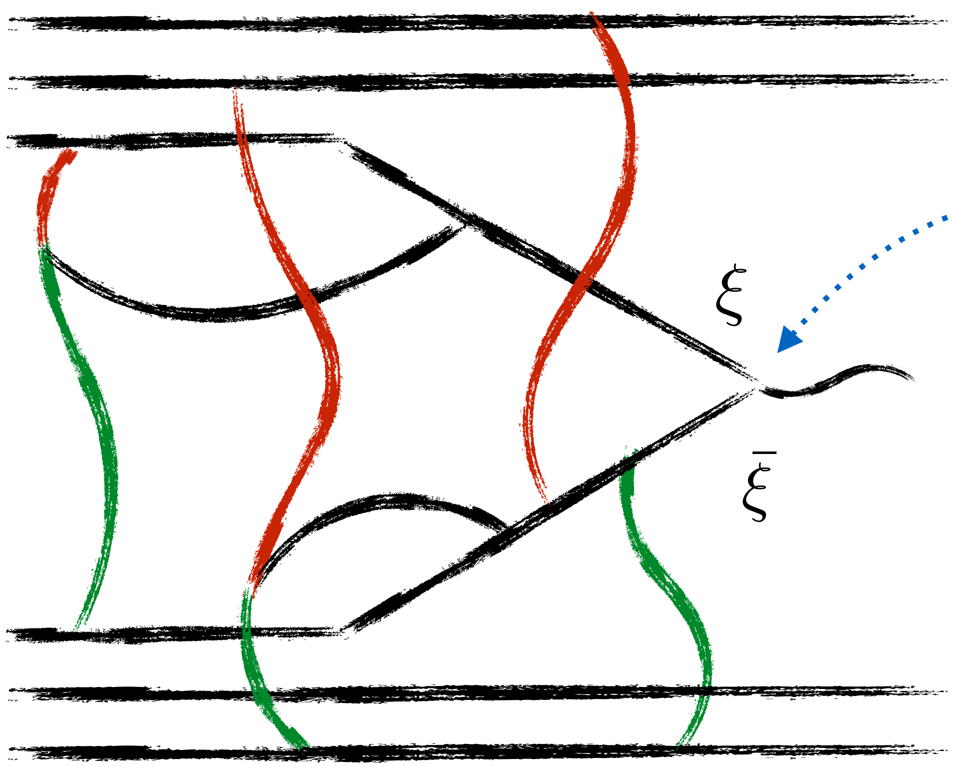
$$V_\mu = \Omega^\dagger(x)\Big(\frac{i}{g}\partial_\mu + A_\mu(x)\Big)\Omega(x)$$

$$\Sigma_b(x_-, x_\perp) = \Omega^\dagger \xi_b(x_+, x_\perp)$$

TMD factorization

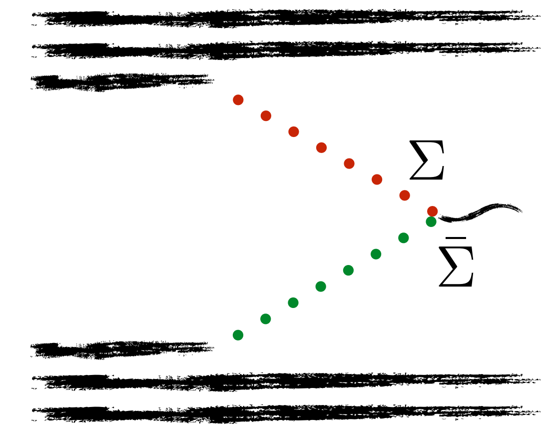
Resummation of tree diagrams at small $p_\perp$

$$\Sigma_a(x_-,x_\perp) = [-\infty, x_-]\xi_a(x_-,x_\perp)$$



$$\Sigma_b(x_+,x_\perp) = [-\infty, x_+]\xi_b(x_+,x_\perp)$$

Resummation of tree diagrams at small $p_\perp$



$$\bar{\Sigma}_b(x_+,x_\perp)\Sigma_a(x_-,x_\perp)$$
$$= [-\infty, x_+]\bar{\xi}_b(x_+,x_\perp)\xi_a(x_-,x_\perp)[x_-,-\infty]$$

1) Is it a leading order solution?
2) What are the corrections (Y-term)?

Parametrization of external fields. Lorentz boost

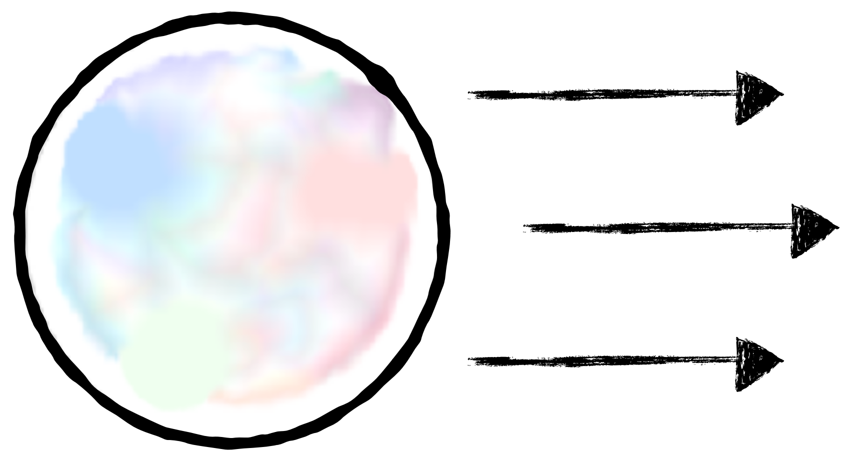
Hadron in the rest frame:

$$\mathcal{A}_+ \sim m, \quad \mathcal{A}_- \sim m, \quad \mathcal{A}_i \sim m$$

Look at the limit: $s \to \infty$

$$x_\bullet = \sqrt{\frac{s}{2}} x_-, \quad x_* = \sqrt{\frac{s}{2}} x_+$$

Boost the system



Expansion parameter:

$$m^2/s \sim p_\perp^2/s$$

$$\mathcal{A}_\bullet \sim s, \quad \mathcal{A}_* \sim m^2, \quad \mathcal{A}_i \sim m$$

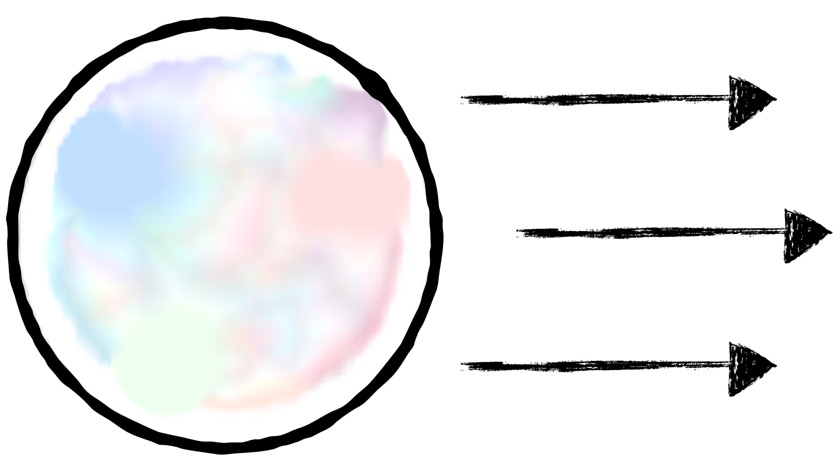
Use this parametrization to separate different contribution

Parametrization of fields after gauge rotation

$$U_\bullet(x_\bullet, x_\perp) \sim m^2, \quad U_*(x_\bullet, x_\perp) = 0, \quad U_i(x_\bullet, x_\perp) \sim m$$

$$U_{*i} \sim sm; \quad U_{*\bullet} \sim sm^2; \quad U_{ij} \sim m^2; \quad U_{\bullet i} \sim m^3$$

Consider YM equation in these background fields



$$V_\bullet(x_*, x_\perp) = 0; \quad V_*(x_*, x_\perp) \sim m^2; \quad V_i(x_*, x_\perp) \sim m$$

$$V_{\bullet i} \sim sm; \quad V_{*\bullet} \sim sm^2; \quad V_{ij} \sim m^2; \quad V_{*i} \sim m^3$$

YM equation in the background field

Equations of motion:

$$\mathcal{D}^{\mu}_{\mathcal{A}}\mathcal{F}^{a}_{\mu\nu} + g\bar{\Psi}t^a\gamma_\nu\Psi = 0; \quad i\not{\mathcal{D}}\Psi = 0$$

Find "quantum" solution $C, \ \Psi^{[c]}$ by expansion in parameter m^2/s

Separation of the background fields:

$$\mathcal{A}_\mu = C_\mu + U_\mu + V_\mu \equiv C_\mu + \mathcal{A}^{[0]}_\mu$$

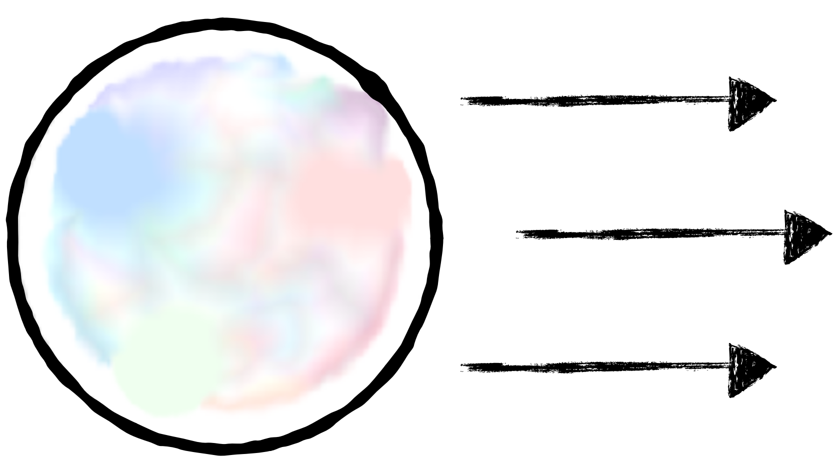
$$\Psi = \Psi^{[c]} + \Sigma_a + \Sigma_b \equiv \Psi^{[c]} + \Psi^{[0]}$$

Equation of motion for external field:

$$\mathcal{D}^{\mu}_{U}U^{a}_{\mu\nu} + g\bar{\Sigma}_a t^a\gamma_\nu\Sigma_a = 0$$

$$\mathcal{D}^{\mu}_{V}V^{a}_{\mu\nu} + g\bar{\Sigma}_b t^a\gamma_\nu\Sigma_b = 0$$

Equations of motion for external fields



YM equation in the background field

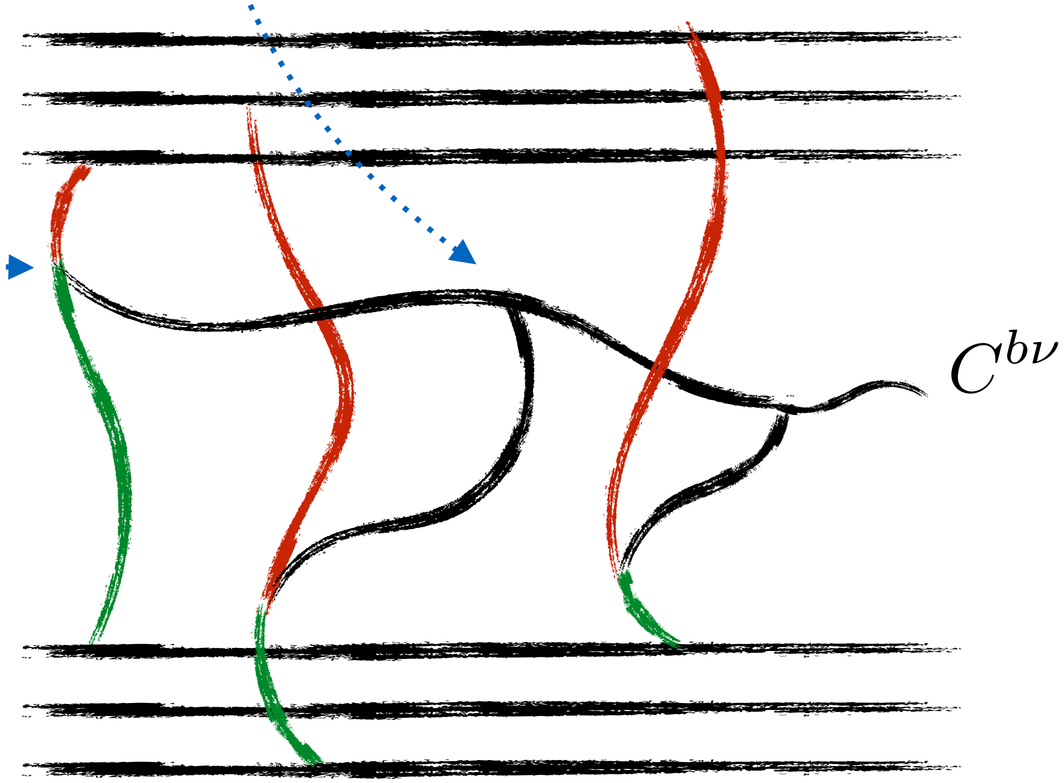
Construct perturbative solution of the equation of motion:

$$(g_{\mu\nu}\mathcal{P}^{[0]2}_{ab} + 2ig\mathcal{F}^{[0]ab}_{\mu\nu})C^{b\nu}$$
$$= L^a_\mu - 2igC^\nu_{ab}\mathcal{D}^{[0]}_\nu C^b_\mu + igC^\nu_{ab}(\mathcal{D}^{[0]bc}_\mu - igC^{bc}_\mu)C^c_\nu + g\bar{\Psi}^{[0]}t^a\gamma_\mu\Psi^{[c]} + g\bar{\Psi}^{[c]}t^a\gamma_\mu\Psi^{[0]}$$

Linear term

Three-gluon vertex

$$L^a_\nu \equiv \mathcal{D}^\mu_{\mathcal{A}}\mathcal{F}^a_{\mu\nu} + g\bar{\Psi}t^a\gamma_\nu\Psi\Big|_{\mathcal{A}^{[0]},\Psi^{[0]}}$$



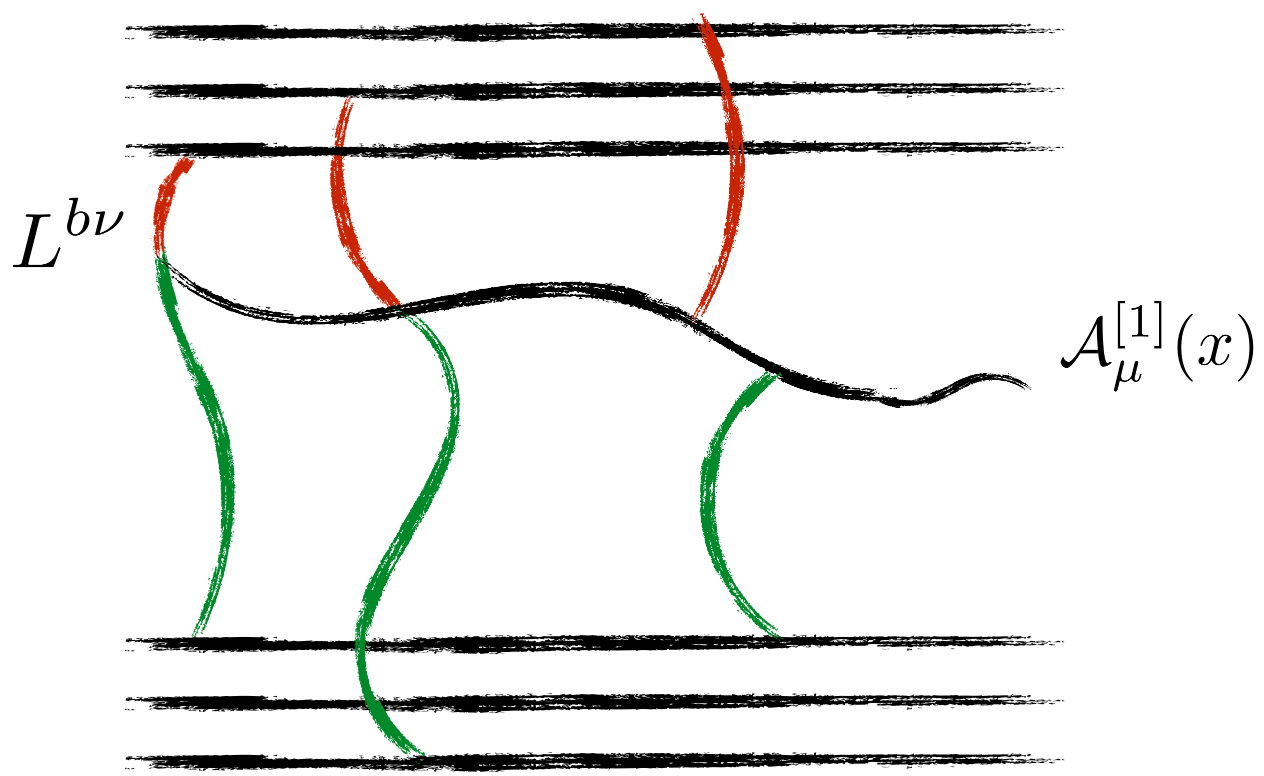
Perturbative expansion

Perturbative expansion:

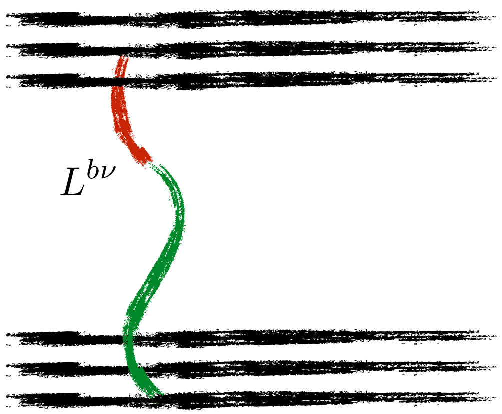
$$C_\mu = \mathcal{A}_\mu^{[1]} + \mathcal{A}_\mu^{[2]} + \ldots$$

$$\mathcal{A}_\mu^{[1]}(x) + \mathcal{A}_\mu^{[2]}(x) + \ldots$$
$$= \int d^4 z (x| \frac{1}{g^{\mu\nu}\mathcal{P}_{[0]}^2 + 2ig\mathcal{F}_{[0]}^{\mu\nu}} |z)^{ab} L^{b\nu}(z) + \ldots$$

Expand in parameter m^2/s



Linear term



Definition:

$$L^a_\nu \equiv \mathcal{D}^\mu_{\mathcal{A}} \mathcal{F}^a_{\mu\nu} + g\bar{\Psi} t^a \gamma_\nu \Psi \Big|_{\mathcal{A}^{[0]},\Psi^{[0]}}$$

Expansion:

$$L^a_\bullet = L^{(-1)a}_\bullet + L^{(0)a}_\bullet + L^{(1)a}_\bullet + \cdots \sim sm^2 + m^4 + \frac{m^6}{s} + \ldots$$

$$L^a_* = L^{(-1)a}_* + L^{(0)a}_* + L^{(1)a}_* + \cdots \sim sm^2 + m^4 + \frac{m^6}{s} + \ldots$$

$$L^a_i = L^{(0)a}_i + L^{(1)a}_i + \cdots \sim m^3 + \frac{m^5}{s} + \ldots$$

Use this to parameterize the perturbative expansion

Example:

$$L^{(-1)a}_\bullet = ig U^j_{ab} V^b_{\bullet j} \sim sm^2$$

$$L^{(0)a}_\bullet = igV^j_{ab} U^b_{\bullet j} + ig\mathcal{D}^{abj}_{U+V}(U^{bc}_\bullet V^c_j) + g\bar{\Sigma}_a(x_\bullet, x_\perp) t^a \gamma_\bullet \Sigma_b(x_*, x_\perp)$$
$$+ g\bar{\Sigma}_b(x_*, x_\perp) t^a \gamma_\bullet \Sigma_a(x_\bullet, x_\perp) - \frac{4ig}{s} U^{ab}_\bullet V^b_{*\bullet} \sim m^4$$

Parametrization of the solution

Equations of motion:

$$\mathcal{D}^{\mu}_{\mathcal{A}}\mathcal{F}^{a}_{\mu\nu} + g\bar{\Psi}t^{a}\gamma_{\nu}\Psi = 0$$

Perturbative expansion:

$$C_{\mu} = A^{[1]}_{\mu}(x) + A^{[2]}_{\mu}(x) \sim \frac{1}{s}L_{\mu} + \dots$$

Perturbative expansion:

$$A^{[1]}_{\bullet}(x) + A^{[2]}_{\bullet}(x) \sim c_1 m^2 + c_2\frac{m^4}{s} + c_3\frac{m^6}{s^2} + \dots = \frac{ig}{p_{\parallel}^2}U^{j}_{ab}V^{b}_{\bullet j} + \dots$$

$$A^{[1]}_{*}(x) + A^{[2]}_{*}(x) \sim c_1 m^2 + c_2\frac{m^4}{s} + c_3\frac{m^6}{s^2} + \dots = \frac{ig}{p_{\parallel}^2}V^{j}_{ab}U^{b}_{*j} + \dots$$

$$A^{[1]}_{i}(x) + A^{[2]}_{i}(x) \sim c_2\frac{m^3}{s} + c_3\frac{m^5}{s^2} + c_4\frac{m^7}{s^3} + \dots$$

Expansion in parameter:

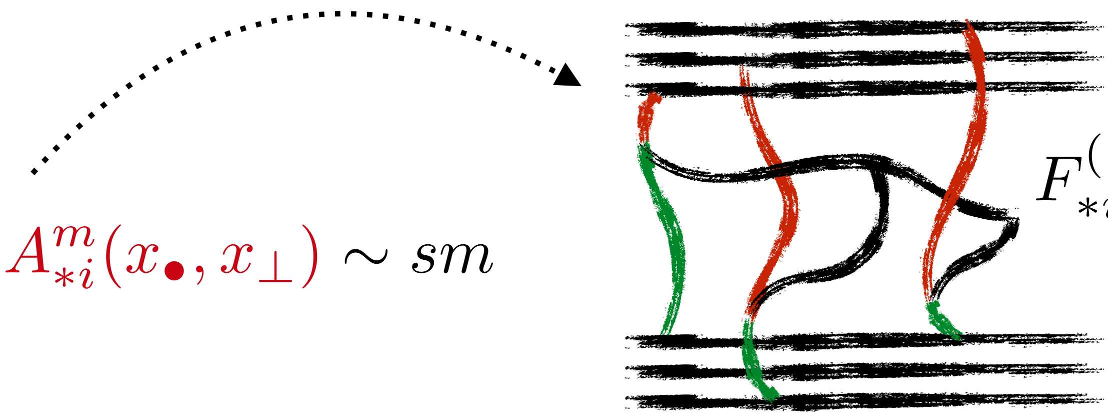
$$m^2/s \sim p_{\perp}^2/s$$

Parametrization of the strength tensor

Leading contributions:

$$F_{*i}^{a(-1)}(x_\bullet, x_\perp) = U_{*i}^a(x_\bullet, x_\perp) = [-\infty, x_\bullet]^{am} A_{*i}^m(x_\bullet, x_\perp) \sim sm$$

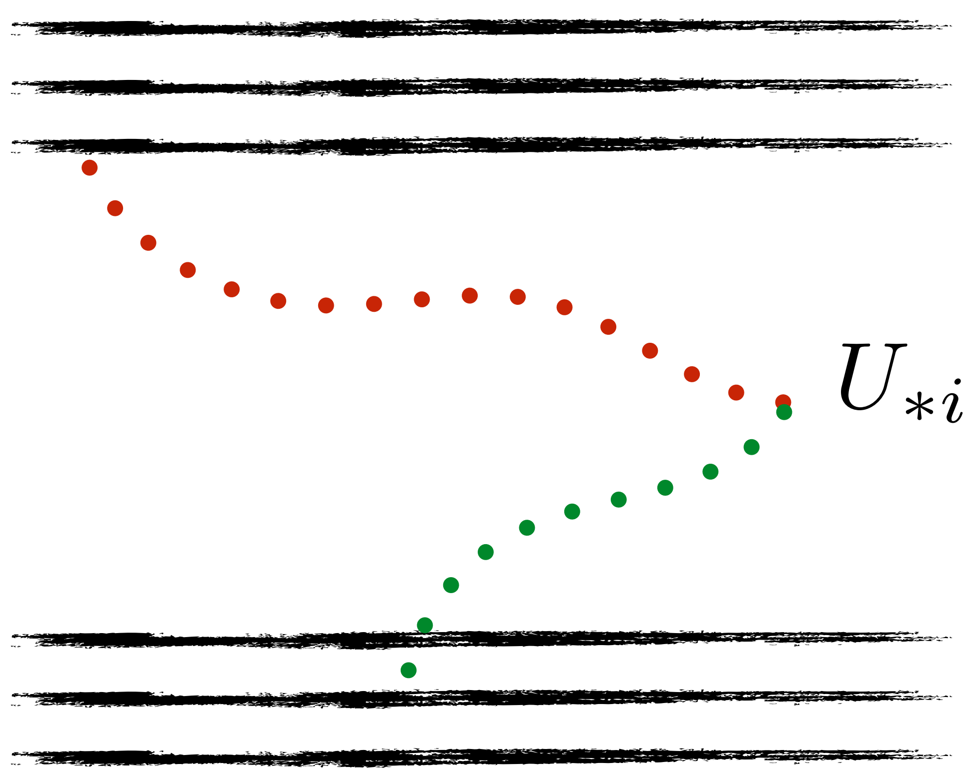
$$F_{\bullet i}^{a(-1)}(x_*, x_\perp) = V_{\bullet i}^a(x_*, x_\perp) = [-\infty, x_*]^{am} B_{\bullet i}^m(x_*, x_\perp) \sim sm$$



$F_{*i}^{(-1)} F_{\bullet i}^{(-1)}$

The product of these two strength tensor yields the TMD factorization formula

The same diagram in terms of U and V fields

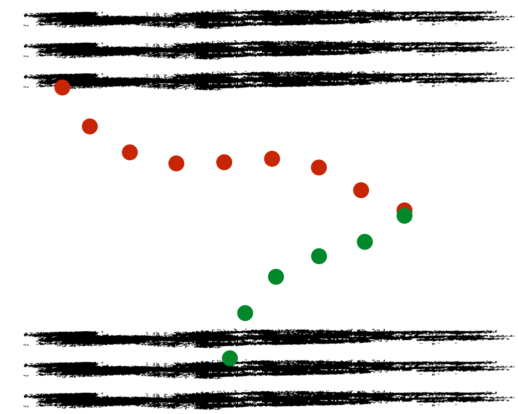


$U_{*i} V_{\bullet i}$

Can calculate corrections to this picture suppressed by $p_\perp^2/Q^2$

Parametrization of the strength tensor

Leading contributions:



$$F^{a(-1)}_{*i}(x_\bullet, x_\perp) = U^a_{*i}(x_\bullet, x_\perp) = [-\infty, x_\bullet]^{am} A^m_{*i}(x_\bullet, x_\perp) \sim sm$$

$$F^{a(-1)}_{\bullet i}(x_*, x_\perp) = V^a_{\bullet i}(x_*, x_\perp) = [-\infty, x_*]^{am} B^m_{\bullet i}(x_*, x_\perp) \sim sm$$

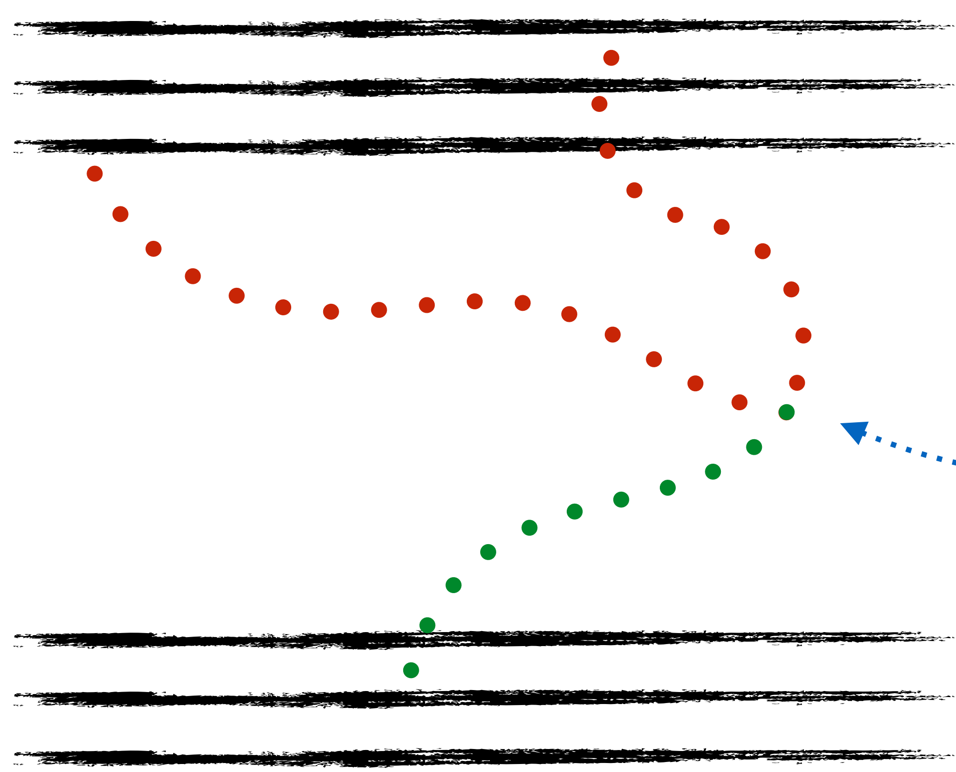
The product of these two strength tensor yields the TMD factorization formula

Subleading contribution:

$$F^{(-1)}_{*\bullet} = U_{*\bullet} + V_{*\bullet} - \frac{is}{2} U^{ab}_j V^{bj}$$

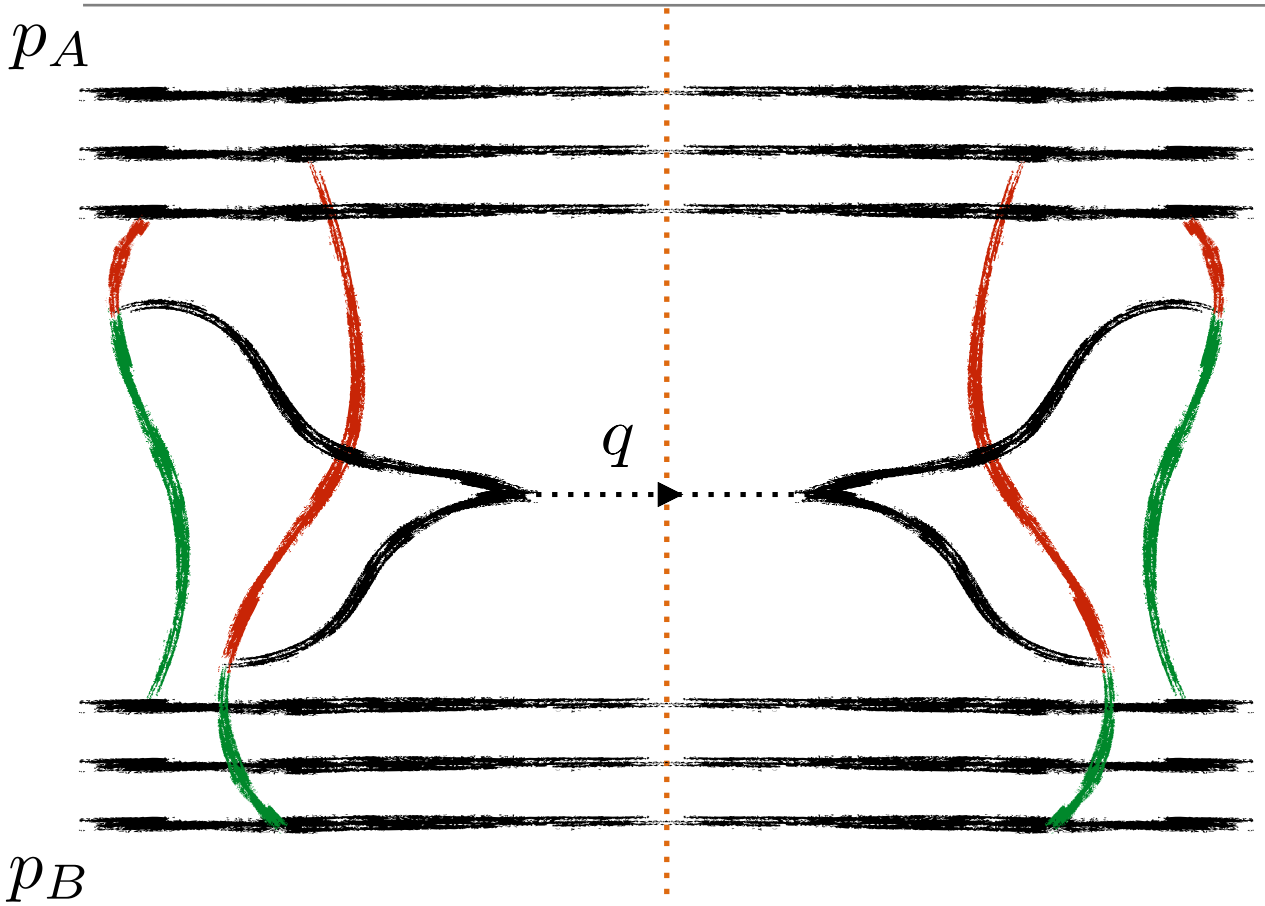
$$F^{(0)a}_{\bullet i} = U^a_{\bullet i} - igU^{ab}_\bullet V^b_i + \ldots$$

$$F^{(0)a}_{*i} = V^a_{*i} - igV^{ab}_* U^b_i + \ldots$$



High p_T corrections to TMD factorization

Higgs production



$$s \gg Q^2 \gg q_\perp^2$$

$$q^2 = Q^2 = M_H^2$$

One can construct expansion in $q_\perp^2/Q^2$

$$W(p_A, p_B, q) = \int d^4x e^{-iqx} \langle p_A, p_B | F^2(x) F^2(0) | p_A, p_B \rangle$$

Leading order

$$F^2(x) = \frac{8}{s} U^a_{*i}(x) V^{ai}_{\bullet}(x) + 2 f^{mac} f^{mbd} \Delta^{ij,kl} U^a_i(x) U^b_j(x) V^c_k(x) V^d_l(x) + \ldots$$

Subleading contribution

$$\Delta^{ij,kl} = g^{ij} g^{kl} - g^{ik} g^{jl} - g^{il} g^{jk}$$

Corrections to TMD factorization (Y-term)

$$W(p_A,p_B,q) = \frac{64/s^2}{N_c^2-1}\int d^2x_\perp e^{iq_\perp x_\perp}\frac{2}{s}\int dx_\bullet dx_* e^{-i\alpha_q x_\bullet - i\beta_q x_*}$$

$$\times\Big\{\langle p_A|U_*^{mi}(x_\bullet,x_\perp)U_*^{mj}(0)|p_A\rangle\langle p_b|V_{\bullet i}^n(x_*,x_\perp)V_{\bullet j}^n(0)|p_B\rangle$$

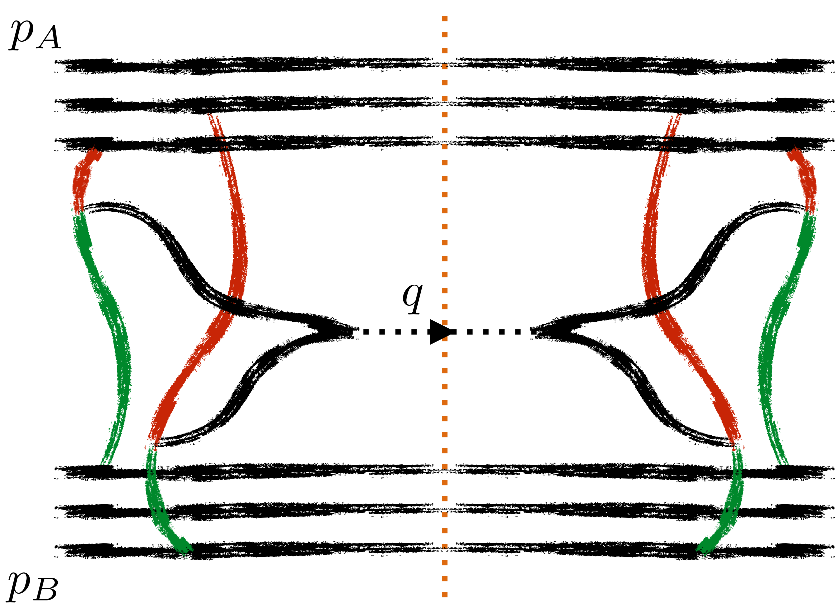
$$-\frac{N_c^2}{N_c^2-4}\frac{\Delta^{ij,kl}}{Q^2}\int_{-\infty}^{x_\bullet} d\frac{2}{s}x'_\bullet d^{abc}\langle p_A|U_{*i}^a(x_\bullet,x_\perp)U_{*j}^b(x'_\bullet,x_\perp)U_{*r}^c(0)|p_A\rangle$$

$$\times\int_{-\infty}^{x_*} d\frac{2}{s}x'_* d^{mns}\langle p_B|V_{\bullet k}^m(x_*,x_\perp)V_{\bullet l}^n(x'_*,x_\perp)V_\bullet^{sr}(0)|p_B\rangle + x\leftrightarrow 0$$

Leading order TMD distributions

High-$q_\perp$ corrections to TMD factorization in the region $s \gg Q^2 \gg q_\perp^2$

Ian Balitsky, A.T. (2017)



Conclusions

1) Solve YM equations of motion in the background field

2) Develop a parametrization scheme for external fields

3) In the leading order recover TMD factorization

4) In the next-to-leading order obtain high-$q_\perp$ corrections to the TMD factorization (Y-term)