The transverse momentum distribution of hadrons within jets

Felix Ringer

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QCD evolution, 05/23/17

Outline

- Inclusive jets *Kang, FR, Vitev `16*
- In-jet fragmentation
    - Collinear FFs *Kang, FR, Vitev `16*
    - TMD FFs *Kang, Liu, FR, Xing `17*
- Conclusions

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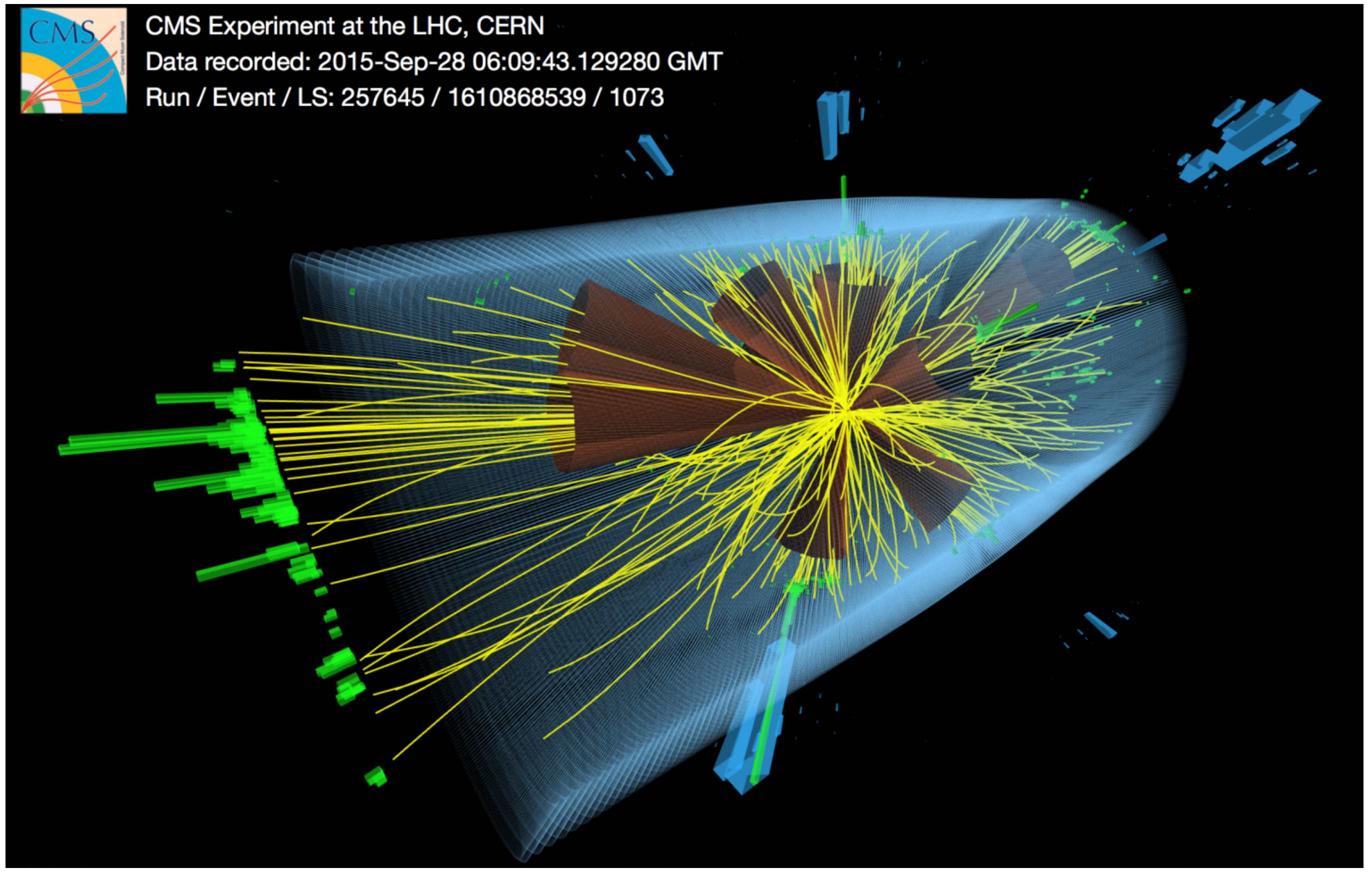
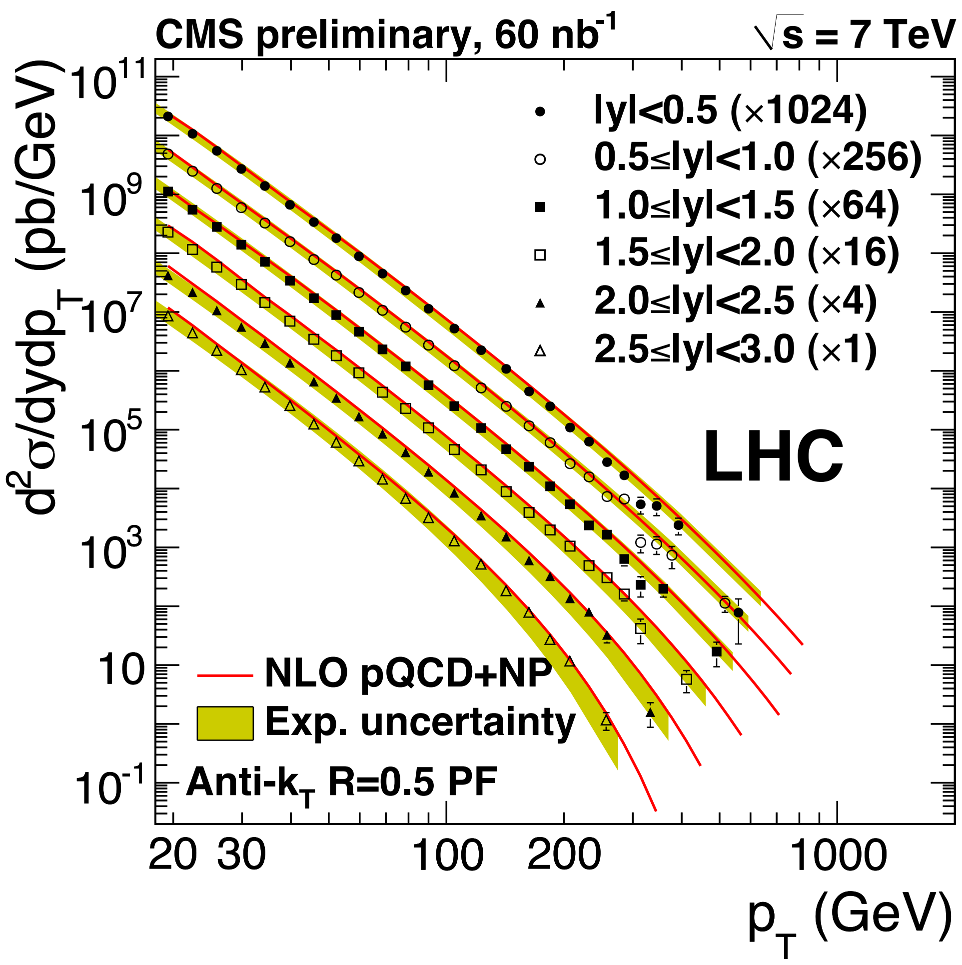
Inclusive Jet Production $pp \to \mathrm{jet}X$

- PDFs and α_s are constrained by collider jet data
- High p_T jets are a promising observable for the search of BSM physics at the LHC
- Baseline for jet quenching in heavy-ion collisions





Inclusive Jet Production $pp \to \text{jet}X$

Factorization

Kang, FR, Vitev `16

$R \sim 1$

$$\frac{d\sigma^{pp\to\text{jet}X}}{dp_T d\eta} = \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab}$$

$\longrightarrow$

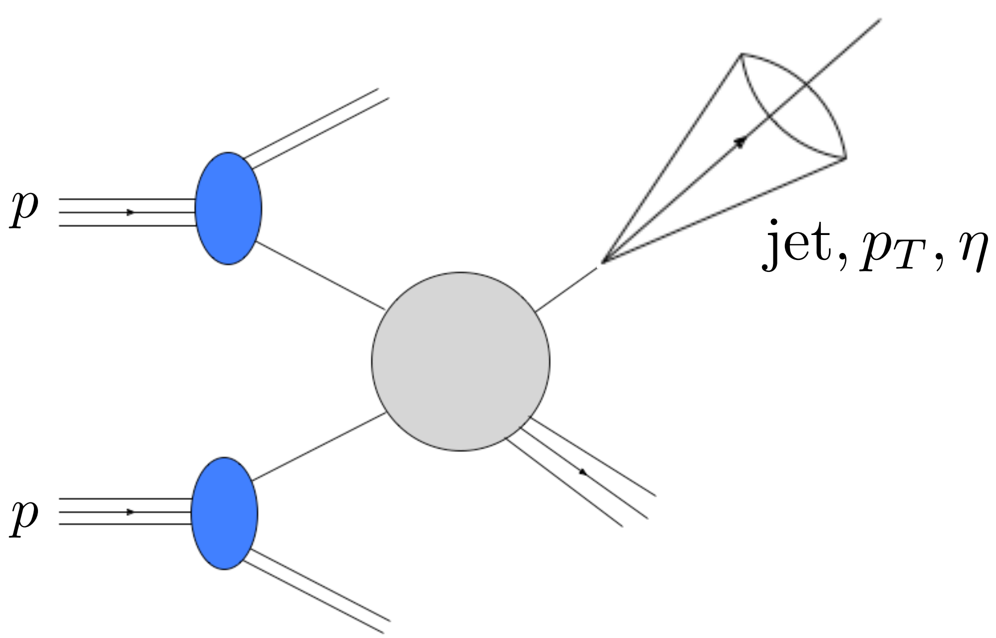
$R \ll 1$

$$\frac{d\sigma^{pp\to\text{jet}X}}{dp_T d\eta} = \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab}^c \otimes \boxed{J_c} + \mathcal{O}(R^2)$$

(e.g. <5% difference for R=0.7)

Ellis, Kunszt, Soper `90, Aversa, Chiappetta, Greco, Guillet `90,
Jäger, Stratmann, Vogelsang `04,
de Florian, Hinderer, Mukherjee, FR, Vogelsang `14
Dasgupta, Dreyer, Salam, Soyez `15, `16
Currie, Glover, Pires `16, …

see also:
Kaufmann, Mukherjee, Vogelsang `15
Dai, Kim, Leibovich `16



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Inclusive Jet Production $pp \to \mathrm{jet} X$

Kang, FR, Vitev `16

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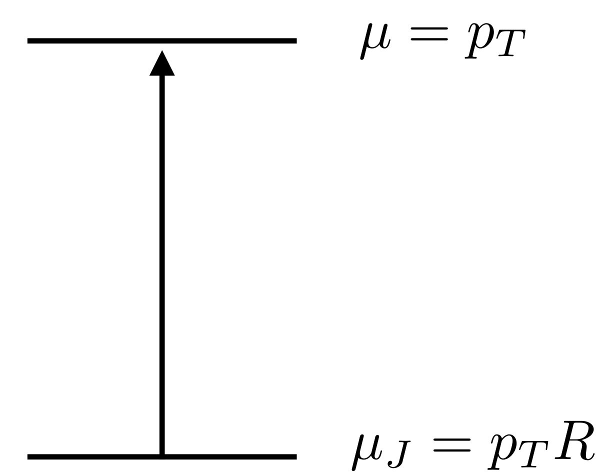
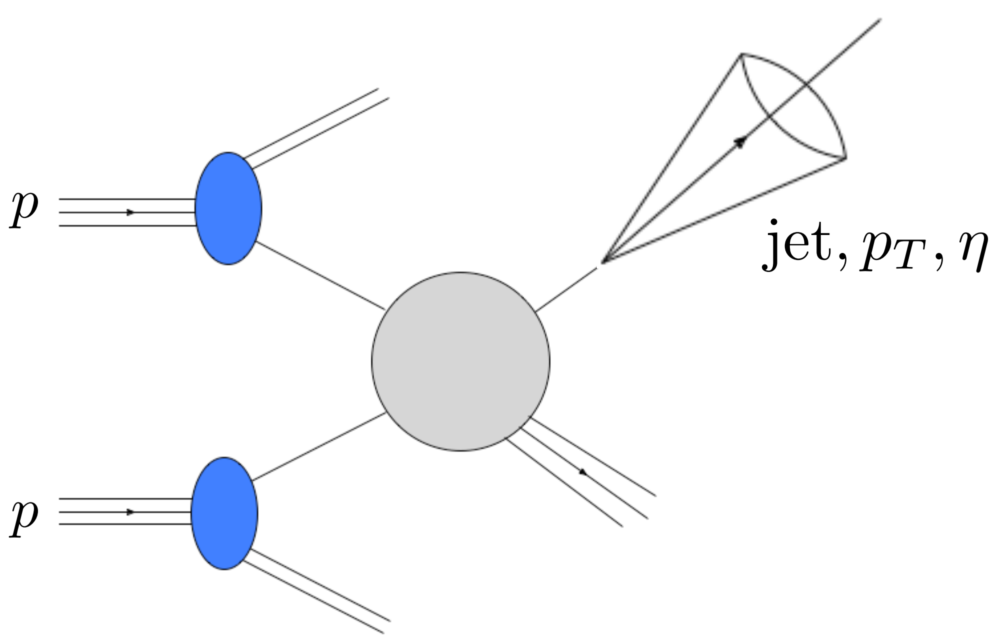
timelike DGLAP for <u>semi-inclusive jet function</u>

$$\boxed{\mu \frac{d}{d\mu} J_i = \sum_j P_{ji} \otimes J_j}$$





resummation of $\alpha_s^n \ln^n R$

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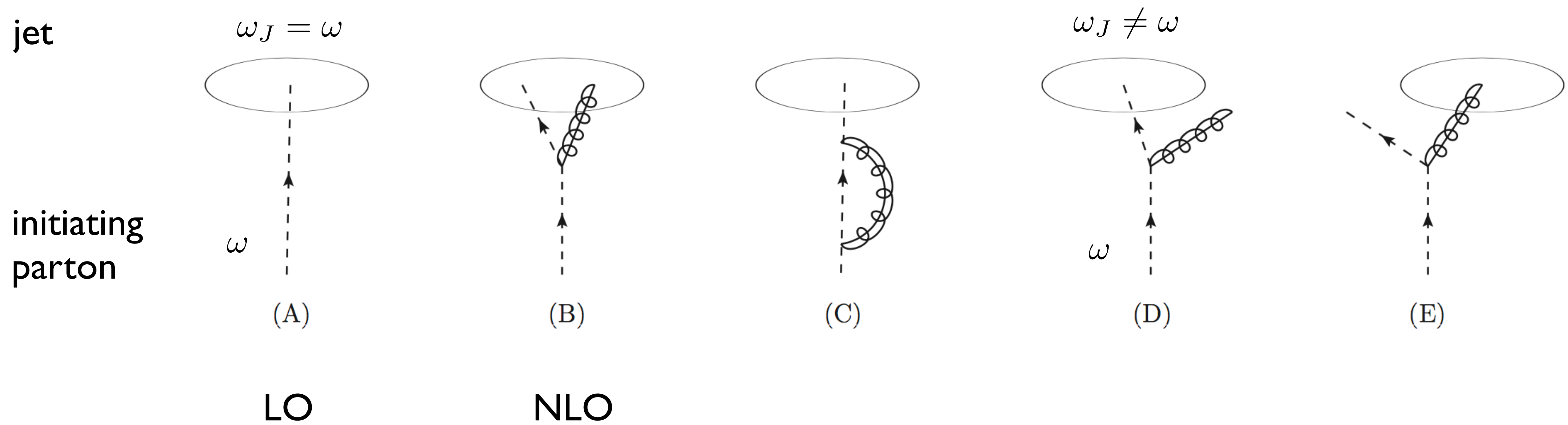
Semi-inclusive jet function in SCET up to NLO

- The siJFs describe how a parton is transformed into a jet with radius R and energy fraction z



where $z = \omega_J/\omega$ momentum sum rule: $\int_0^1 dz\, z J_i(z, \omega R, \mu) = 1$

Semi-inclusive jet function in SCET at NLO

Leading order $$J_q^{(0)}(z,\omega_J) = \delta(1-z)$$

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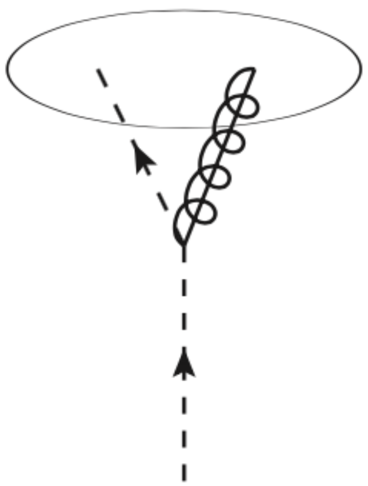
Semi-inclusive jet function in SCET at NLO

Leading order $J_q^{(0)}(z,\omega_J) = \delta(1-z)$

Next-to-leading order



(B)

$$J_q(z,\omega_J) = \delta(1-z)\frac{\alpha_s}{\pi}\left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^{\epsilon}\int_0^1 dx\hat{P}_{qq}(x,\epsilon)\int\frac{dq_\perp}{q_\perp^{1+2\epsilon}}\Theta_{\text{alg}}$$

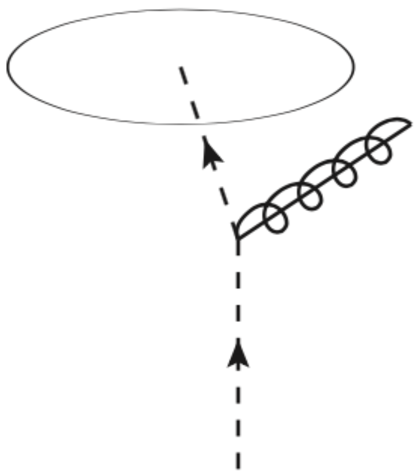
where: $\Theta_{\text{anti-k}_\text{T}} = \theta\left(x(1-x)\omega_J\tan\frac{R}{2} - q_\perp\right)$

$$\hat{P}_{qq}(x,\epsilon) = C_F\left[\frac{1+x^2}{1-x} - \epsilon(1-x)\right]$$

Semi-inclusive jet function in SCET at NLO

Leading order $$J_q^{(0)}(z,\omega_J) = \delta(1-z)$$

Next-to-leading order



(D)

$$J_{q\to q(g)}(z,\omega_J) = \frac{\alpha_s}{\pi}\left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^{\epsilon} \hat{P}_{qq}(z,\epsilon)\int \frac{dq_\perp}{q_\perp^{1+2\epsilon}}\Theta_{\mathrm{alg}}$$

where: $$\Theta_{\text{anti-k}_\mathrm{T}} = \theta\left(q_\perp - (1-z)\omega_J \tan\frac{R}{2}\right)$$

Semi-inclusive jet function in SCET at NLO

$\overline{\mathrm{MS}}$ scheme, anti-$\mathrm{k_T}$

$$J_q^{(1)}(z,\omega_J) = J_{q\to qg}(z,\omega_J) + J_{q\to q(g)}(z,\omega_J) + J_{q\to (q)g}(z,\omega_J)$$

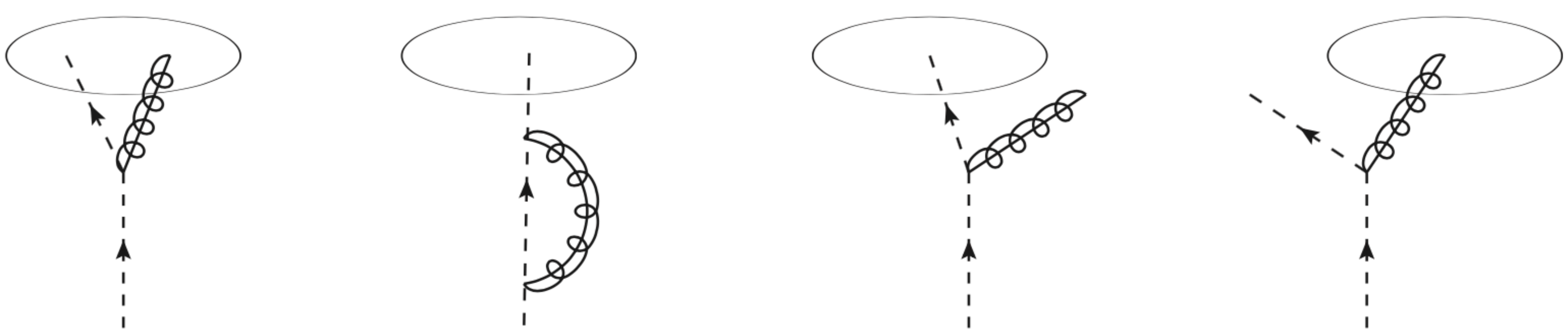
UV

$$= \frac{\alpha_s}{2\pi}\left[\frac{1}{\epsilon} + \ln\left(\frac{\mu^2}{\omega_J^2\tan^2(R/2)}\right)\right]\left[P_{qq}(z) + P_{gq}(z)\right]$$

$$- \frac{\alpha_s}{2\pi}\left\{C_F\left[2\left(1+z^2\right)\left(\frac{\ln(1-z)}{1-z}\right)_+ + (1-z)\right] - \delta(1-z)d_J^{q,\mathrm{alg}}\right.$$

$$\left. + P_{gq}(z)2\ln(1-z) + C_F z\right\},$$

$$d_J^{q,\mathrm{anti\text{-}k_T}} = C_F\left(\frac{13}{2} - \frac{2\pi^2}{3}\right)$$



(B) (C) (D) (E)

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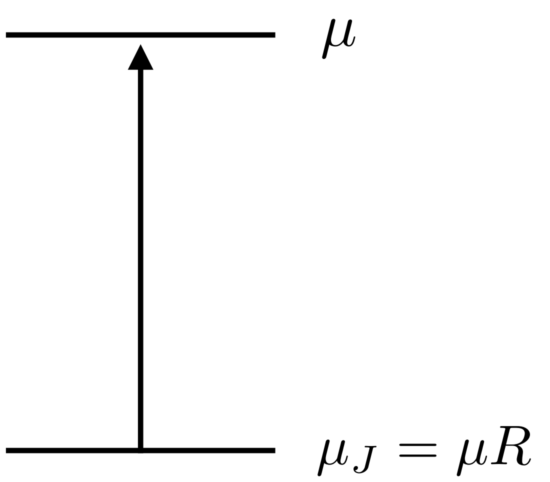
Renormalization and RG evolution

Bare - renormalized semi-inclusive jet function



$$J_{i,\mathrm{bare}}(z,\omega_J) = \sum_j \int_z^1 \frac{dz'}{z'} Z_{ij}\left(\frac{z}{z'},\mu\right) J_j(z',\omega_J,\mu)$$

RG equation:

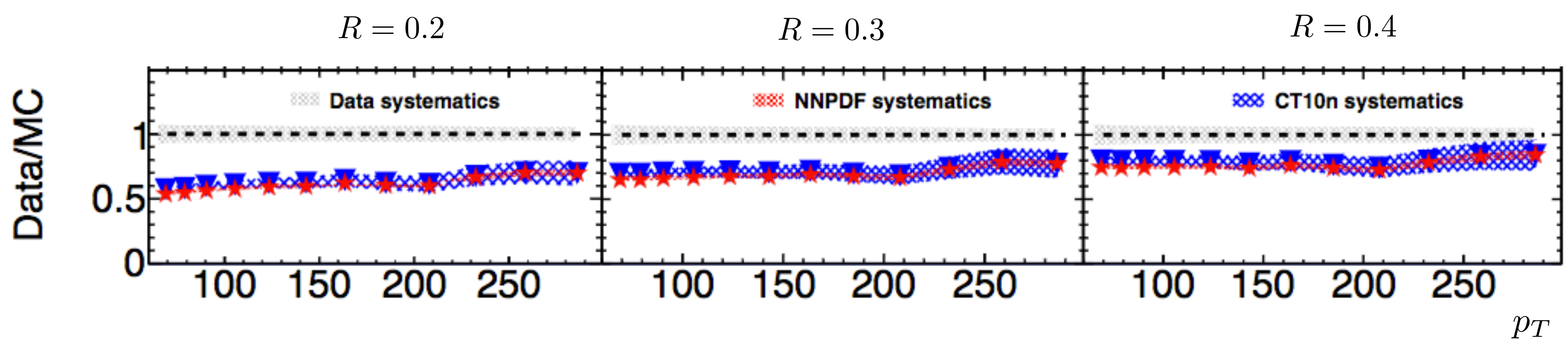
$$\boxed{\mu\frac{d}{d\mu} J_i(z,\omega_J,\mu) = \frac{\alpha_s(\mu)}{\pi} \sum_j \int_z^1 \frac{dz'}{z'} P_{ji}\left(\frac{z}{z'},\mu\right) J_j(z',\omega_J,\mu)}$$

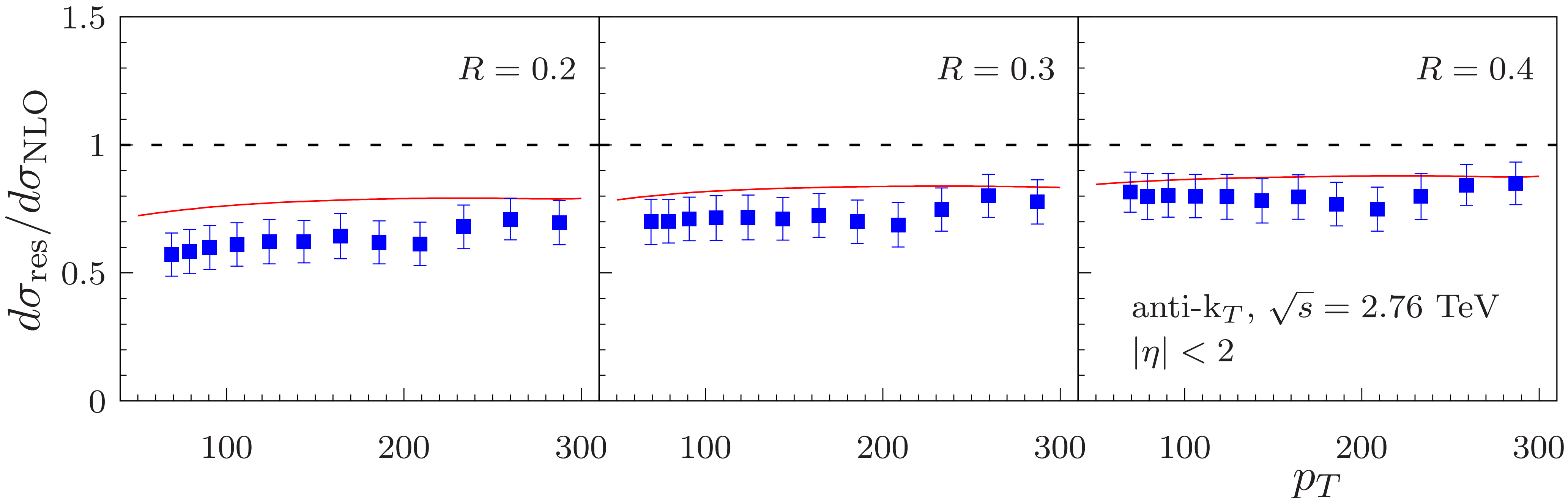
DGLAP evolution equation like for FFs. Resums single $\ln R$: $\mathrm{NLO{+}NLL}_R$

$\longrightarrow$ solve in Mellin moment space

see also
Dasgupta, Dreyer, Salam, Soyez `15, `16

Inclusive Jet Production in SCET $pp \to \text{jet}X$





CMS data arXiv:1609.05383

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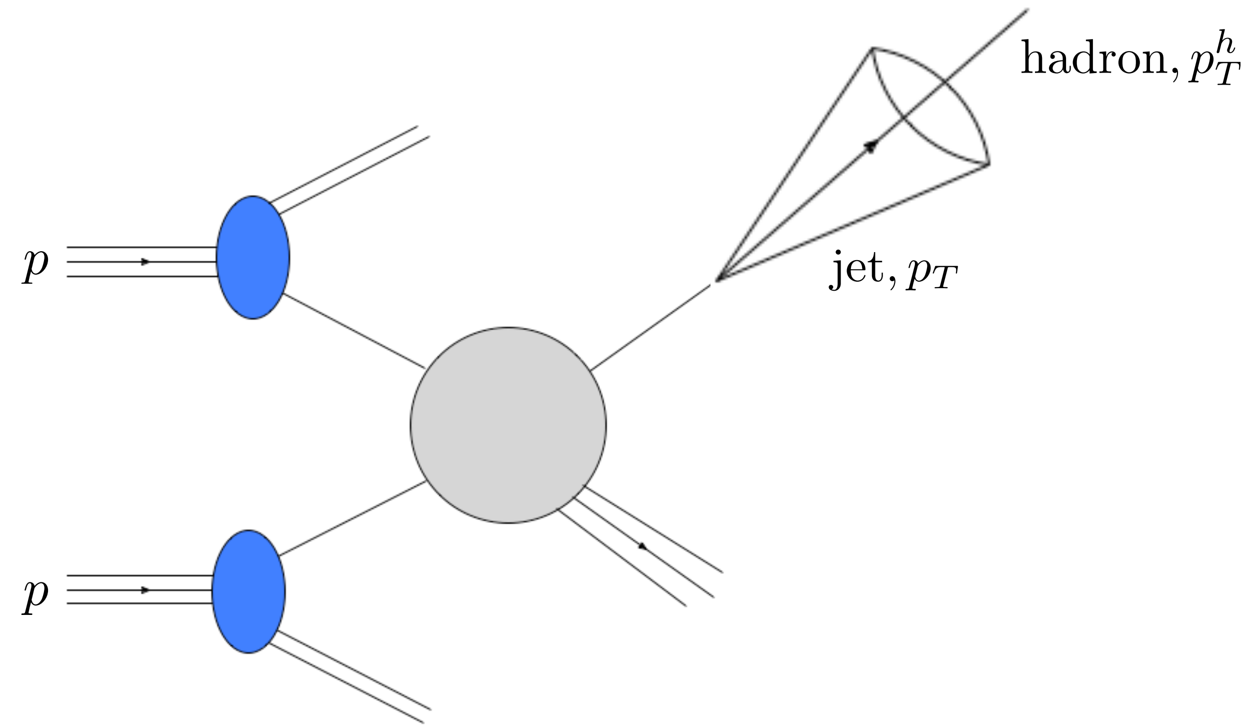
Jet fragmentation function

Definition: $F(z_h, p_T) = \dfrac{d\sigma^{pp\to(\text{jet}h)X}}{dp_T d\eta dz_h} \Big/ \dfrac{d\sigma^{pp\to\text{jet}X}}{dp_T d\eta}$

where $z_h = p_T^h / p_T$



The JFF describes the longitudinal momentum distribution of hadrons inside a reconstructed jet

Procura, Stewart `10; Liu `11; Jain, Procura, Waalewijn `11 and ’12; Procura, Waalewijn `12; Bauer, Mereghetti `14; Baumgart, Leibovich, Mehen, Rothstein `14, Chien, Kang, FR, Vitev, Xing `15, Bain, Dai, Hornig, Leibovich, Makris, Mehen `16, Bain, Makris, Mehen `16, Arleo, Fontannaz, Guillet, Nguyen `14, Kaufmann, Mukherjee, Vogelsang `15, Neill, Scimemi, Waalewijn `16 …

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Semi-inclusive fragmenting jet function

Factorized cross section:

$$\frac{d\sigma^{pp\to(\text{jet}\,h)X}}{dp_T d\eta dz_h} = \sum_{a,b,c} f_a(x_a,\mu)\otimes f_b(x_b,\mu)\otimes H_{ab}^c(x_a,x_b,\eta,p_T/z,\mu)\otimes \mathcal{G}_c^h(z,z_h,\omega_J,\mu)$$

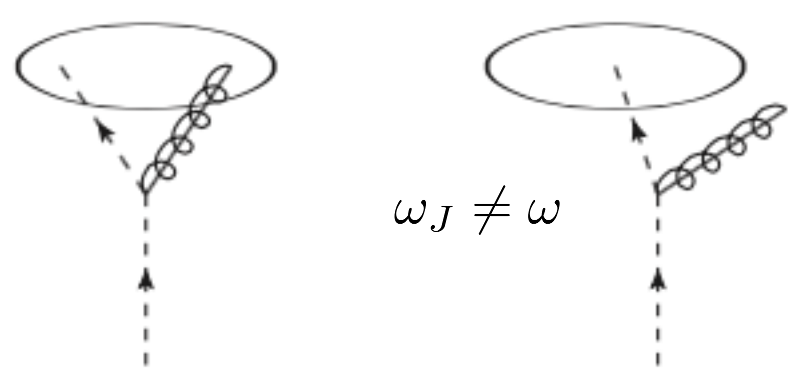
where
$$\mathcal{G}_i^h(z,z_h,\omega_J,\mu) = \sum_j \int_{z_h}^1 \frac{dz_h}{z_h'}\mathcal{J}_{ij}(z,z_h',\omega_J,\mu)D_j^h\left(\frac{z_h}{z_h'},\mu\right)$$

NLO: $z=\omega_J/\omega$ $\quad z_h=\omega_h/\omega_J$

- fragmenting parton
- jet
- initiating parton

$\omega_h \neq \omega_J$ $\quad \omega_h = \omega_J$

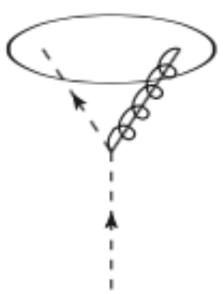
$\omega_J = \omega$ $\quad \omega_J \neq \omega$

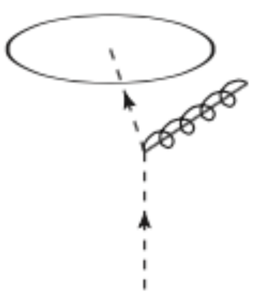


ω $\quad \omega$

Semi-inclusive fragmenting jet function

Quark-quark:



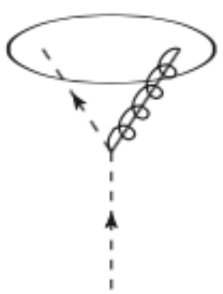


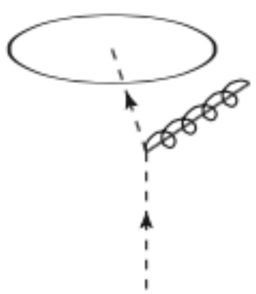
$$\mathcal{G}_{q,\mathrm{bare}}^{q}(z, z_h, \omega_J, \mu) = \delta(1-z)\delta(1-z_h) + \frac{\alpha_s}{2\pi}\left(-\frac{1}{\epsilon} - L\right) P_{qq}(z_h)\delta(1-z)$$
$$+ \frac{\alpha_s}{2\pi}\left(\frac{1}{\epsilon} + L\right) P_{qq}(z)\delta(1-z_h)$$
$$+ \delta(1-z)\frac{\alpha_s}{2\pi}\left[2C_F(1+z_h^2)\left(\frac{\ln(1-z_h)}{1-z_h}\right)_+ + C_F(1-z_h) + 2P_{qq}(z_h)\ln z_h\right]$$
$$- \delta(1-z_h)\frac{\alpha_s}{2\pi}\left[2C_F(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ + C_F(1-z)\right]$$

Semi-inclusive fragmenting jet function

Quark-quark:

IR





$$
\begin{aligned}
\mathcal{G}_{q,\mathrm{bare}}^{q}(z, z_h, \omega_J, \mu) = {} & \delta(1-z)\delta(1-z_h) + \frac{\alpha_s}{2\pi}\left(-\frac{1}{\epsilon} - L\right) P_{qq}(z_h)\delta(1-z) \\
& + \frac{\alpha_s}{2\pi}\left(\frac{1}{\epsilon} + L\right) P_{qq}(z)\delta(1-z_h) \\
& + \delta(1-z)\frac{\alpha_s}{2\pi}\left[2C_F(1+z_h^2)\left(\frac{\ln(1-z_h)}{1-z_h}\right)_+ + C_F(1-z_h) + 2P_{qq}(z_h)\ln z_h\right] \\
& - \delta(1-z_h)\frac{\alpha_s}{2\pi}\left[2C_F(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ + C_F(1-z)\right]
\end{aligned}
$$

Matching:

$$
\mathcal{G}_i^h(z, z_h, \omega_J, \mu) = \sum_j \int_{z_h}^1 \frac{dz_h}{z_h'} \mathcal{J}_{ij}(z, z_h', \omega_J, \mu) D_j^h\left(\frac{z_h}{z_h'}, \mu\right)
$$

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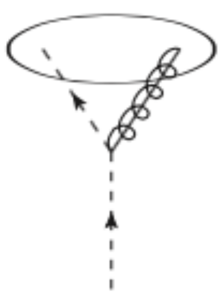
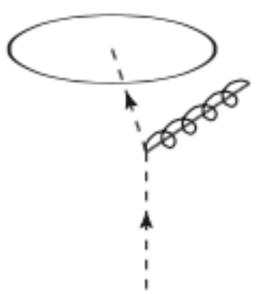
Semi-inclusive fragmenting jet function

Quark-quark:





$$
\begin{aligned}
\mathcal{G}_{q,\mathrm{bare}}^{q}(z, z_h, \omega_J, \mu) = {} & \delta(1-z)\delta(1-z_h) + \frac{\alpha_s}{2\pi}\left(\overset{\text{IR}}{-\frac{1}{\epsilon}} - L\right) P_{qq}(z_h)\delta(1-z) \\
& + \frac{\alpha_s}{2\pi}\left(\overset{\text{UV}}{\frac{1}{\epsilon}} + L\right) P_{qq}(z)\delta(1-z_h) \\
& + \delta(1-z)\frac{\alpha_s}{2\pi}\left[2C_F(1+z_h^2)\left(\frac{\ln(1-z_h)}{1-z_h}\right)_+ + C_F(1-z_h) + 2P_{qq}(z_h)\ln z_h\right] \\
& - \delta(1-z_h)\frac{\alpha_s}{2\pi}\left[2C_F(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ + C_F(1-z)\right]
\end{aligned}
$$

Matching:

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$$

$\ln R$ resummation:

$$
\mu\frac{d}{d\mu}\mathcal{G}_i^h(z, z_h, \omega_J, \mu) = \frac{\alpha_s(\mu)}{\pi}\sum_j \int_z^1 \frac{dz'}{z'} P_{ji}\left(\frac{z}{z'}\right) \mathcal{G}_j^h(z', z_h, \omega_J, \mu)
$$

Semi-inclusive fragmenting jet function

Quark-quark:

IR

UV

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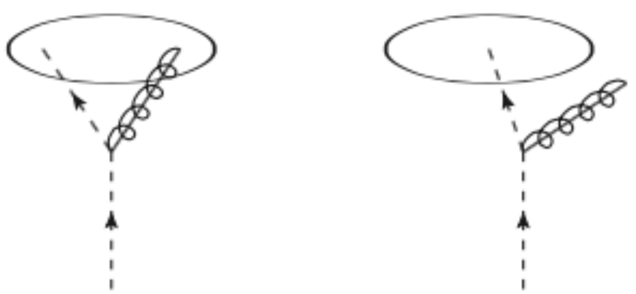
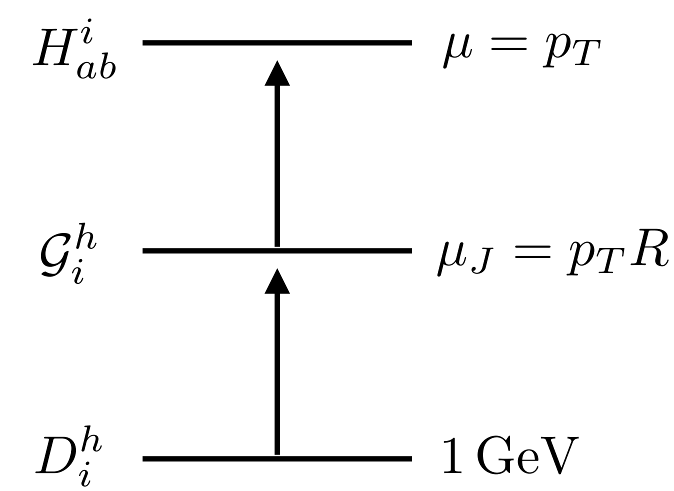


Matching:

$$\mathcal{G}_i^h(z, z_h, \omega_J, \mu) = \sum_j \int_{z_h}^1 \frac{dz_h'}{z_h'} \mathcal{J}_{ij}(z, z_h', \omega_J, \mu) D_j^h\left(\frac{z_h}{z_h'}, \mu\right)$$



$\ln R$ resummation:

$$\mu\frac{d}{d\mu}\mathcal{G}_i^h(z, z_h, \omega_J, \mu) = \frac{\alpha_s(\mu)}{\pi}\sum_j \int_z^1 \frac{dz'}{z'} P_{ji}\left(\frac{z}{z'}\right) \mathcal{G}_j^h(z', z_h, \omega_J, \mu)$$

… 2 DGLAPs now

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In-jet fragmentation at the LHC

- Light charged hadrons

  Arleo, Fontannaz, Guillet, Nguyen `14

  Kaufmann, Mukherjee, Vogelsang `15

  Kang, FR, Vitev `15

  Neill, Scimemi, Waalewijn `16

- Photons

  Kaufmann, Mukherjee, Vogelsang `16

- Heavy flavor mesons

  Chien, Kang, FR, Vitev, Xing `15

  Bain, Dai, Hornig, Leibovich, Makris, Mehen `16

see Daniele Anderle's talk

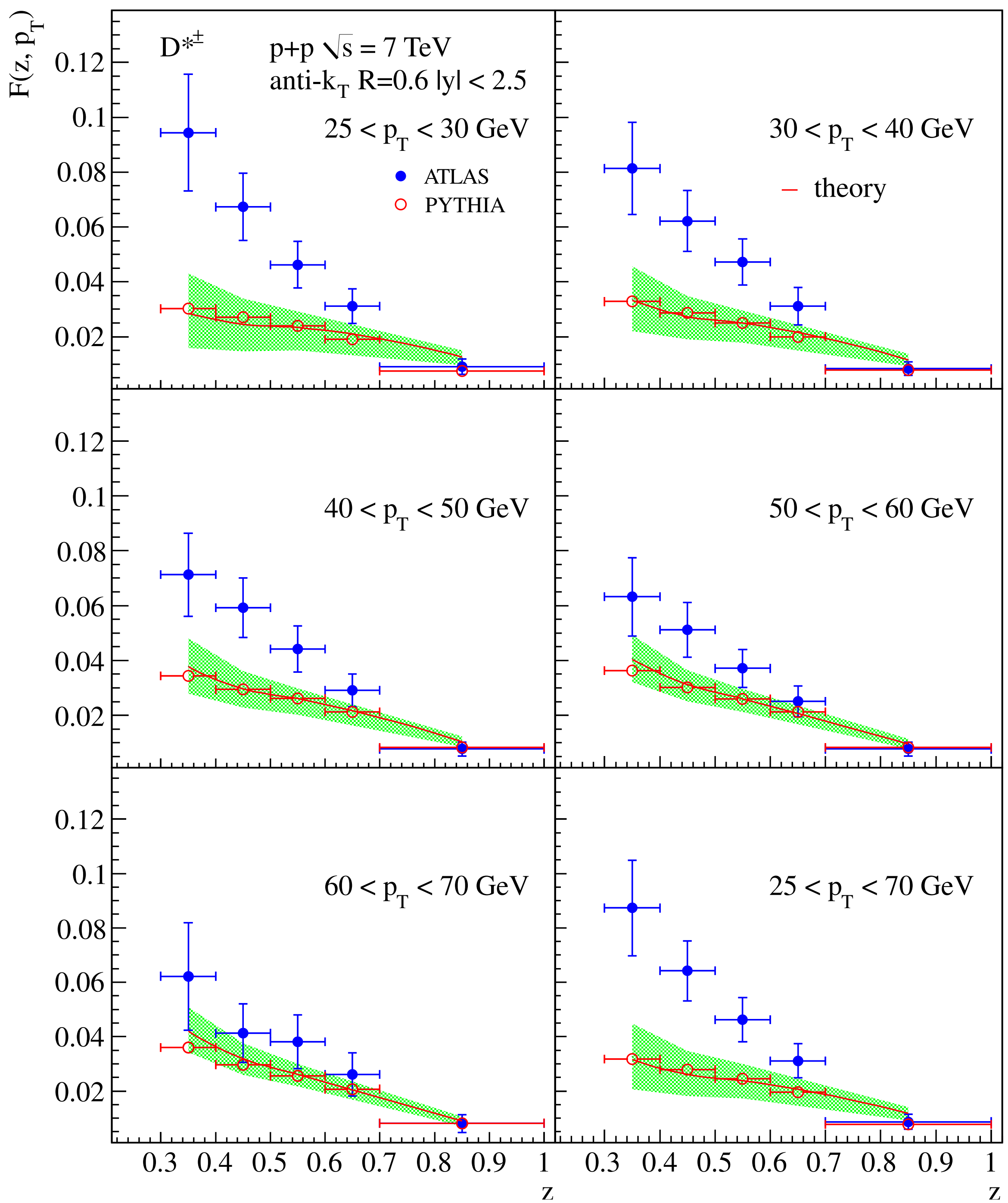
- Quarkonia

  Baumgart, Leibovich, Mehen, Rothstein `14

  Bain, Dai, Hornig, Leibovich, Makris, Mehen `16

  Kang, Qiu, FR, Xing, Zhang `17

  Bain, Dai, Leibovich, Makris, Mehen `17



D-mesons

Kneesch, Kniehl, Kramer, Schienbein `08

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- Conclusions

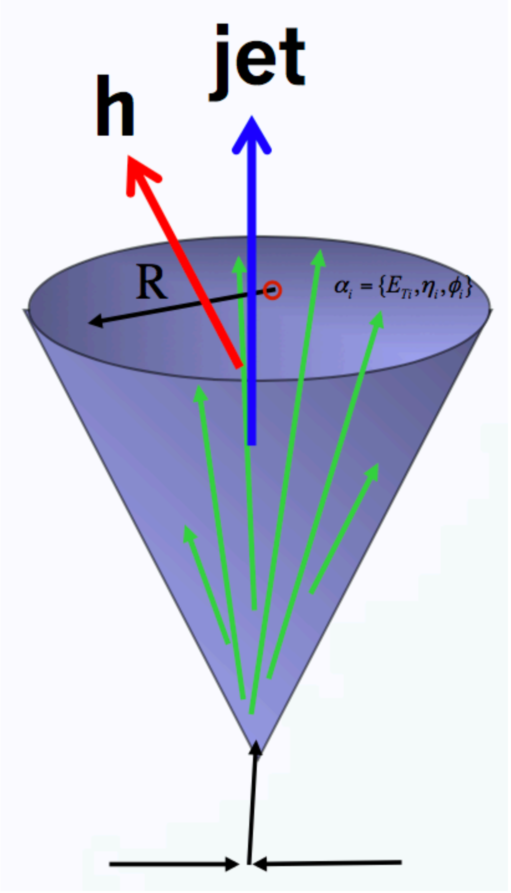
Jet fragmentation function $pp \to (\mathrm{jet}h)X$

- Measure in addition the relative transverse momentum of the hadron wrt. to the jet axis

momentum fraction z_h

transverse momentum $\boldsymbol{j}_\perp$

$$\frac{d\sigma^{pp\to(\mathrm{jet}\,h)X}}{dp_T d\eta dz_h d^2\boldsymbol{j}_\perp}$$



- See also:

Bain, Makris, Mehen `16 — TMD fragmenting jet functions with applications to quarkonium production

see Yiannis Makris's talk

Neill, Scimemi, Waalewijn `17 — Jet axes and universal TMD fragmentation

- *Kang, Liu, FR, Xing `17*
  Standard jet axis
  Inclusive jet sample
  Relation to usual TMD evolution and fits
  Light charged hadrons

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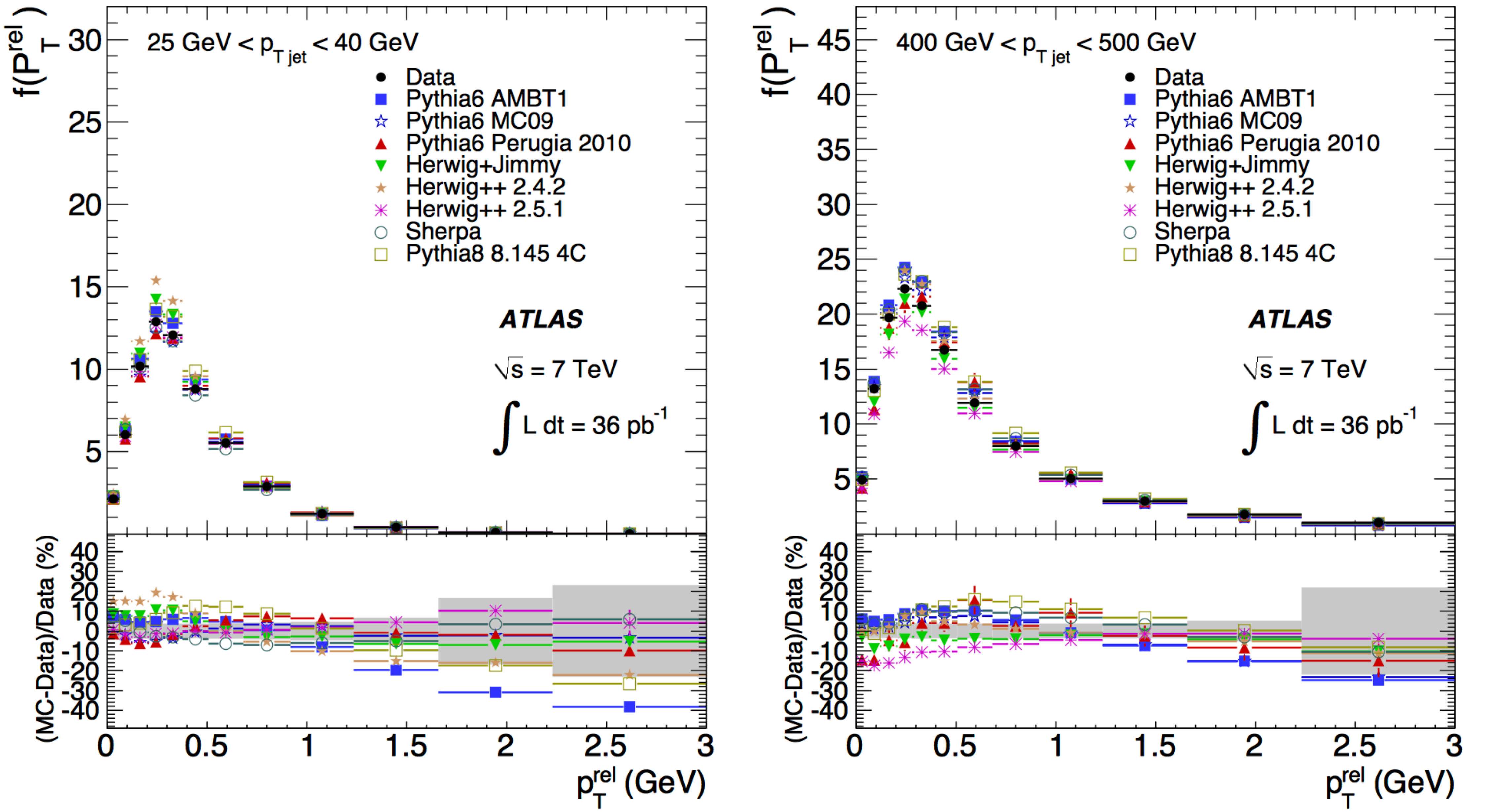


ATLAS Eur. Phys. J. C71 (2011) 1795
arXiv: 1109.5816

Jet fragmentation function $pp \to (\text{jet}h)X$

Factorized cross section $R \ll 1$

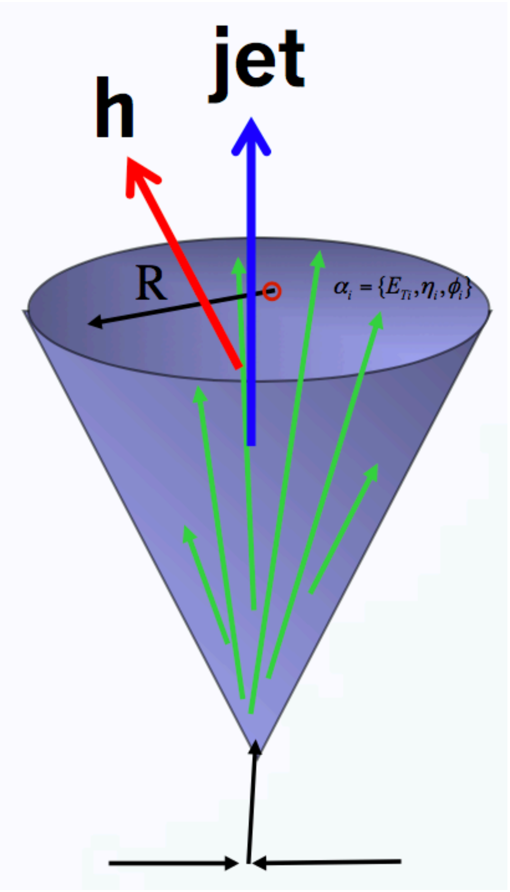
$$\frac{d\sigma^{pp\to(\text{jet }h)X}}{dp_T d\eta dz_h d^2\boldsymbol{j}_\perp} = \sum_{a,b,c} f_a(x_a,\mu) \otimes f_b(x_b,\mu) \otimes H_{ab}^c(x_a,x_b,\eta,p_T/z,\mu) \otimes \mathcal{G}_c^h(z,z_h,\boldsymbol{j}_\perp,\omega_J,\mu)$$

where for $|\boldsymbol{j}_\perp| \ll p_T R$

hard matching function

$$\mathcal{G}_c^h(z,z_h,\omega_J R,\boldsymbol{j}_\perp,\mu) = \mathcal{H}_{c\to i}(z,\omega_J R,\mu) \int d^2\boldsymbol{k}_\perp d^2\boldsymbol{\lambda}_\perp \delta^2\left(z_h\boldsymbol{\lambda}_\perp + \boldsymbol{k}_\perp - \boldsymbol{j}_\perp\right)$$
$$\times D_{h/i}(z_h,\boldsymbol{k}_\perp,\mu,\nu) S_i(\boldsymbol{\lambda}_\perp,\mu,\nu R)$$

TMD FF Soft function



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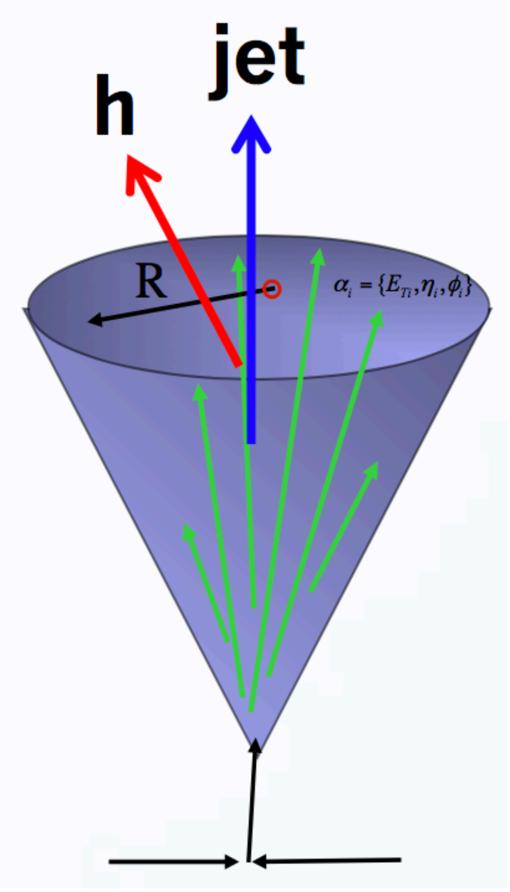
Jet fragmentation function $pp \to (\text{jet}h)X$



hard matching function

$$\mathcal{G}_c^h(z, z_h, \omega_J R, \boldsymbol{j}_\perp, \mu) = \mathcal{H}_{c\to i}(z, \omega_J R, \mu) \int d^2\boldsymbol{k}_\perp d^2\boldsymbol{\lambda}_\perp \delta^2\left(z_h \boldsymbol{\lambda}_\perp + \boldsymbol{k}_\perp - \boldsymbol{j}_\perp\right) \times D_{h/i}(z_h, \boldsymbol{k}_\perp, \mu, \nu) S_i(\boldsymbol{\lambda}_\perp, \mu, \nu R)$$

TMD FF

Soft function

Fourier transform to b-space:

$$\mathcal{G}_c^h(z, z_h, \omega_J R, \boldsymbol{j}_\perp, \mu) = \mathcal{H}_{c\to i}(z, \omega_J R, \mu) \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} e^{i\boldsymbol{j}_\perp\cdot\boldsymbol{b}/z_h} D_{h/i}(z_h, \boldsymbol{b}, \mu, \nu) S_i(\boldsymbol{b}, \mu, \nu R)$$

where

$$D_{h/i}(z_h, \boldsymbol{b}, \mu, \nu) = \frac{1}{z_h^2} \int d^2\boldsymbol{k}_\perp e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{b}/z_h} D_{h/i}(z_h, \boldsymbol{k}_\perp, \mu, \nu)$$

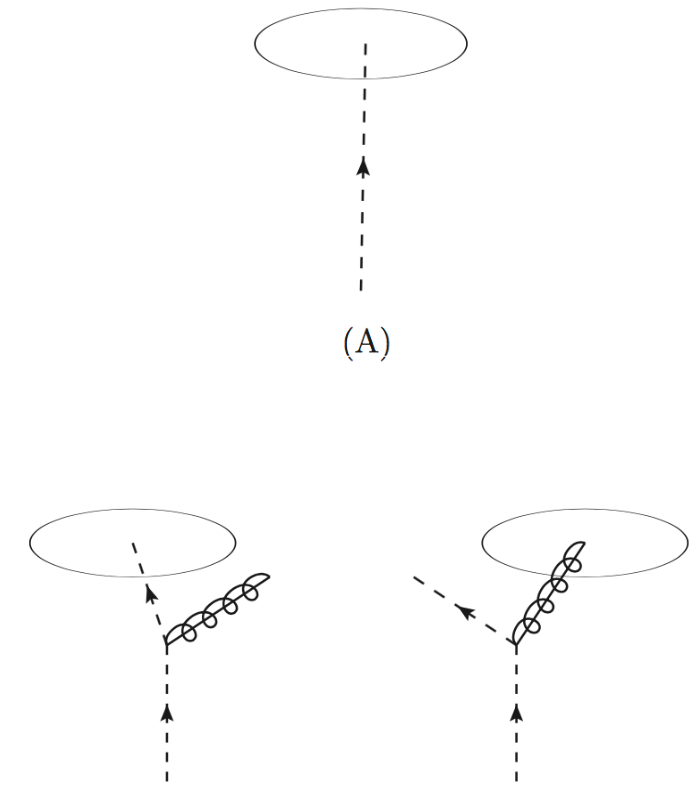
$$S_i(\boldsymbol{b}, \mu, \nu R) = \int d^2\boldsymbol{\lambda}_\perp e^{-i\boldsymbol{\lambda}_\perp\cdot\boldsymbol{b}} S_i(\boldsymbol{\lambda}_\perp, \mu, \nu R)$$

Jet fragmentation function $pp \to (\mathrm{jet}h)X$

- Hard matching functions

$$\mathcal{H}_{q\to q'}(z,\omega_J,\mu) = \delta_{qq'}\delta(1-z) + \delta_{qq'}\frac{\alpha_s}{2\pi}\Bigg[C_F\delta(1-z)\Big(-\frac{L^2}{2}-\frac{3}{2}L+\frac{\pi^2}{12}\Big) + P_{qq}(z)L - 2C_F(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ - C_F(1-z)\Bigg]$$

$$\mathcal{H}_{q\to g}(z,\omega_J,\mu) = \frac{\alpha_s}{2\pi}\Big[\Big(L-2\ln(1-z)\Big)P_{gq}(z) - C_F z\Big]$$



(A)

(D) (E)

- Evolution: modified DGLAP

$$\mu\frac{d}{d\mu}\mathcal{H}_{i\to j}(z,\omega_R,\mu) = \sum_k \int_z^1 \frac{dz'}{z'}\gamma_{ik}\left(\frac{z}{z'}\right)\mathcal{H}_{k\to j}(z',\omega_R,\mu)$$

where $\gamma_{ij}(z) = \delta_{ij}\delta(1-z)\Gamma_i + \frac{\alpha_s}{\pi}P_{ji}(z),$ $\quad\Gamma_q = \frac{\alpha_s}{\pi}C_F\left(-L-\frac{3}{2}\right)$

4 coupled equations with double logarithms. Characteristic scale $\mu_J = p_T R$

see also: Kang, FR, Waalewijn `17

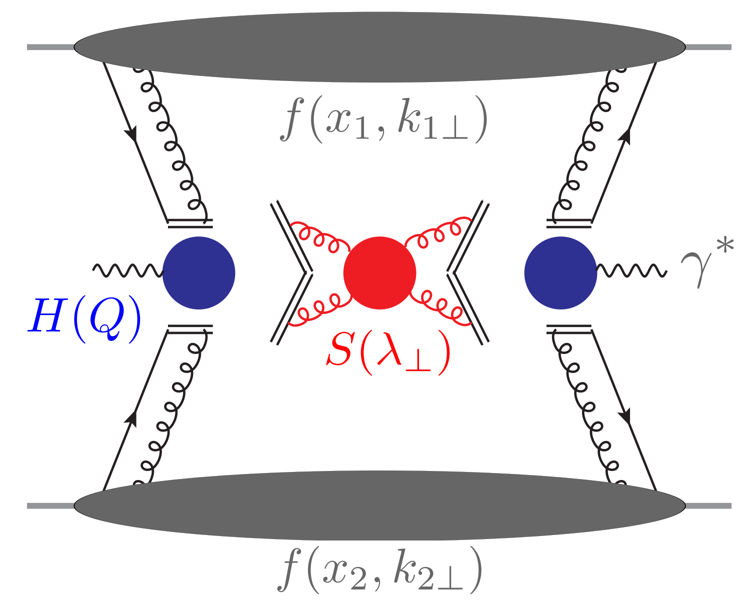
Drell-Yan $pp \to [\gamma^* \to \ell^+\ell^-]X$

Parton model interpretation

$$\frac{d\sigma}{dQ^2 dy d^2\boldsymbol{q}_\perp} \sim \int d^2\boldsymbol{k}_{1\perp}\, d^2\boldsymbol{k}_{2\perp}\, d^2\boldsymbol{\lambda}_\perp\, H(Q)\, f(x_1, \boldsymbol{k}_{1\perp})\, f(x_2, \boldsymbol{k}_{2\perp})\, S(\boldsymbol{\lambda}_\perp)\, \delta^2(\boldsymbol{k}_{1\perp} + \boldsymbol{k}_{2\perp} + \boldsymbol{\lambda}_\perp - \boldsymbol{q}_\perp)$$

$$= \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} e^{i\boldsymbol{q}_\perp \cdot \boldsymbol{b}}\, H(Q) f(x_1, \boldsymbol{b})\, f(x_2, \boldsymbol{b})\, S(\boldsymbol{b})$$

$$= \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} e^{i\boldsymbol{q}_\perp \cdot \boldsymbol{b}}\, H(Q) F(x_1, \boldsymbol{b})\, F(x_2, \boldsymbol{b})$$



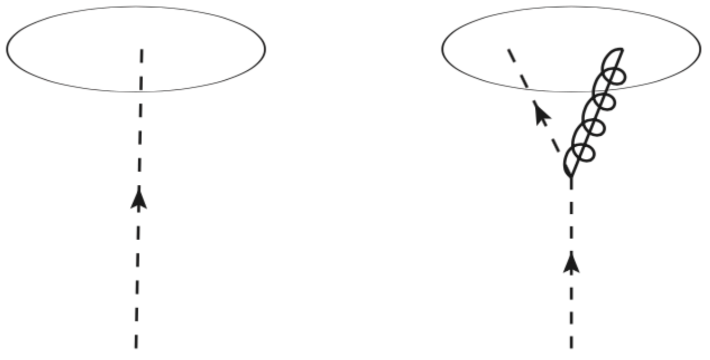
Rapidity divergences cancel in redefined TMD

$$F(x, \boldsymbol{b}) = f(x, \boldsymbol{b})\sqrt{S(\boldsymbol{b})}$$

Jet fragmentation function $pp \to (\mathrm{jet}h)X$

- Rapidity regulator η , scale ν *Chiu, Jain, Neill, Rothstein `12*
- (In-jet) quark TMD

$$D_q^q(z_h, \boldsymbol{k}_\perp, \mu, \nu) = \delta(1-z_h)\delta^2(\boldsymbol{k}_\perp) + \frac{\alpha_s}{2\pi^2} C_F \Gamma(1+\epsilon) e^{\gamma_E \epsilon} \frac{1}{\mu^2} \left(\frac{\mu^2}{\boldsymbol{k}_\perp^2}\right)^{1+\epsilon}$$
$$\times \left[\frac{2z_h}{(1-z_h)^{1+\eta}} \left(\frac{\nu}{\omega_J}\right)^\eta + (1-\epsilon)(1-z_h)\right]$$

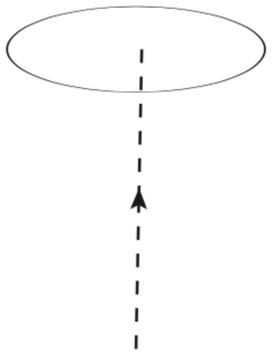


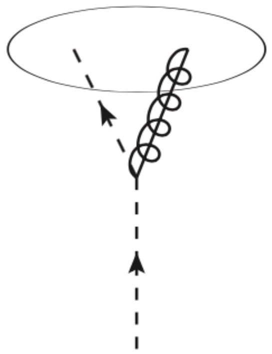
(A) (B)

Jet fragmentation function $pp \to (\mathrm{jet}h)X$

- Rapidity regulator η , scale ν *Chiu, Jain, Neill, Rothstein `12*
- (In-jet) quark TMD

$$D_q^q(z_h, \boldsymbol{k}_\perp, \mu, \nu) = \delta(1-z_h)\delta^2(\boldsymbol{k}_\perp) + \frac{\alpha_s}{2\pi^2} C_F \Gamma(1+\epsilon) e^{\gamma_E \epsilon} \frac{1}{\mu^2}\left(\frac{\mu^2}{\boldsymbol{k}_\perp^2}\right)^{1+\epsilon}$$
$$\times \left[\frac{2z_h}{(1-z_h)^{1+\eta}}\left(\frac{\nu}{\omega_J}\right)^{\eta} + (1-\epsilon)(1-z_h)\right]$$


(A)


(B)

b-space and expansion in $\eta,\ \epsilon$:

$$D_q^q(z_h, \boldsymbol{b}, \mu, \nu) = \frac{1}{z_h^2}\Bigg\{\delta(1-z_h)$$
$$+ \frac{\alpha_s}{2\pi} C_F \left[\frac{2}{\eta}\left(\frac{1}{\epsilon} + \ln\left(\frac{\mu^2}{\mu_b^2}\right)\right) + \frac{1}{\epsilon}\left(2\ln\left(\frac{\nu}{\omega_J}\right) + \frac{3}{2}\right)\right]\delta(1-z_h)$$
$$+ \frac{\alpha_s}{2\pi} C_F \left[-\frac{1}{\epsilon} - \ln\left(\frac{\mu^2}{z_h^2 \mu_b^2}\right)\right] P_{qq}(z_h)$$
$$+ \frac{\alpha_s}{2\pi} C_F \left[\ln\left(\frac{\mu^2}{\mu_b^2}\right)\left(2\ln\left(\frac{\nu}{\omega_J}\right) + \frac{3}{2}\right)\delta(1-z_h) + (1-z_h)\right]\Bigg\}$$

$$\mu_b = \frac{2e^{-\gamma_E}}{b}$$

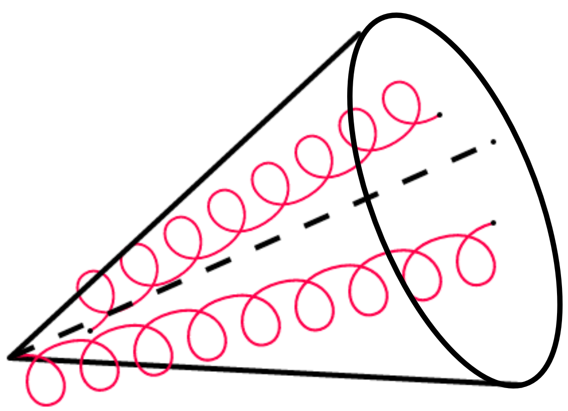
Jet fragmentation function $pp \to (\mathrm{jet}h)X$

- (In-jet) quark TMD

$$
\begin{aligned}
D_q^q(z_h, \boldsymbol{b}, \mu, \nu) = \frac{1}{z_h^2}\Bigg\{ & \delta(1-z_h) \\
& + \frac{\alpha_s}{2\pi} C_F \left[\frac{2}{\eta}\left(\frac{1}{\epsilon} + \ln\left(\frac{\mu^2}{\mu_b^2}\right)\right) + \frac{1}{\epsilon}\left(2\ln\left(\frac{\nu}{\omega_J}\right) + \frac{3}{2}\right)\right] \delta(1-z_h) \\
& + \frac{\alpha_s}{2\pi} C_F \left[-\frac{1}{\epsilon} - \ln\left(\frac{\mu^2}{z_h^2 \mu_b^2}\right)\right] P_{qq}(z_h) \\
& + \frac{\alpha_s}{2\pi} C_F \left[\ln\left(\frac{\mu^2}{\mu_b^2}\right)\left(2\ln\left(\frac{\nu}{\omega_J}\right) + \frac{3}{2}\right) \delta(1-z_h) + (1-z_h)\right]\Bigg\}
\end{aligned}
$$

- In-jet soft function

$$
\begin{aligned}
S_i(\boldsymbol{b}, \mu, \nu R) = 1 + \frac{\alpha_s}{2\pi} C_i \Bigg[& \frac{2}{\eta}\left(-\frac{1}{\epsilon} - \ln\left(\frac{\mu^2}{\mu_b^2}\right)\right) + \frac{1}{\epsilon^2} - \frac{1}{\epsilon}\ln\left(\frac{\nu^2 \tan^2(R/2)}{\mu^2}\right) \\
& - \ln\left(\frac{\mu^2}{\mu_b^2}\right) \ln\left(\frac{\nu^2 \tan^2(R/2)}{\mu_b^2}\right) + \frac{1}{2}\ln^2\left(\frac{\mu^2}{\mu_b^2}\right) - \frac{\pi^2}{12}\Bigg].
\end{aligned}
$$



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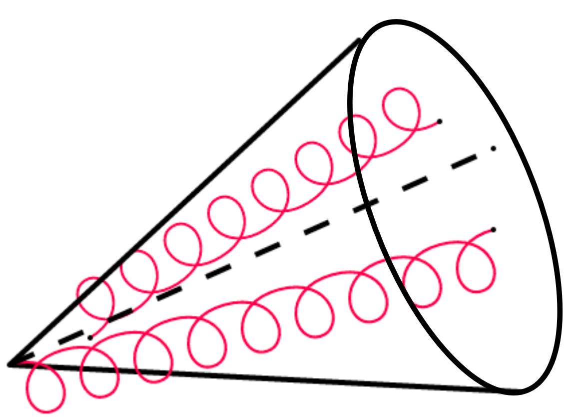
Jet fragmentation function $pp \to (\mathrm{jet}h)X$

- (In-jet) quark TMD

$$D_q^q(z_h, \boldsymbol{b}, \mu, \nu) = \frac{1}{z_h^2}\Bigg\{\delta(1-z_h) + \frac{\alpha_s}{2\pi}C_F\left[\frac{2}{\eta}\left(\frac{1}{\epsilon}+\ln\left(\frac{\mu^2}{\mu_b^2}\right)\right)+\frac{1}{\epsilon}\left(2\ln\left(\frac{\nu}{\omega_J}\right)+\frac{3}{2}\right)\right]\delta(1-z_h)$$
$$+\frac{\alpha_s}{2\pi}C_F\left[-\frac{1}{\epsilon}-\ln\left(\frac{\mu^2}{z_h^2\mu_b^2}\right)\right]P_{qq}(z_h)$$
$$+\frac{\alpha_s}{2\pi}C_F\left[\ln\left(\frac{\mu^2}{\mu_b^2}\right)\left(2\ln\left(\frac{\nu}{\omega_J}\right)+\frac{3}{2}\right)\delta(1-z_h)+(1-z_h)\right]\Bigg\}$$

- In-jet soft function

$$S_i(\boldsymbol{b}, \mu, \nu R) = 1 + \frac{\alpha_s}{2\pi}C_i\left[\frac{2}{\eta}\left(-\frac{1}{\epsilon}-\ln\left(\frac{\mu^2}{\mu_b^2}\right)\right)+\frac{1}{\epsilon^2}-\frac{1}{\epsilon}\ln\left(\frac{\nu^2\tan^2(R/2)}{\mu^2}\right)\right.$$
$$\left.-\ln\left(\frac{\mu^2}{\mu_b^2}\right)\ln\left(\frac{\nu^2\tan^2(R/2)}{\mu_b^2}\right)+\frac{1}{2}\ln^2\left(\frac{\mu^2}{\mu_b^2}\right)-\frac{\pi^2}{12}\right].$$



Jet fragmentation function $pp \to (\text{jet}h)X$

- Renormalization

$$D_{h/i}(z_h, \boldsymbol{b}, \mu, \nu) = Z_i^D(\boldsymbol{b}, \mu, \nu) D_{h/i}^{\text{bare}}(z_h, \boldsymbol{b}, \mu, \nu)$$

$$S_i(\boldsymbol{b}, \mu, \nu R) = Z_i^S(\boldsymbol{b}, \mu, \nu) S_i^{\text{bare}}(\boldsymbol{b}, \mu, \nu R)$$

- Evolution

$$\mu \frac{d}{d\mu} \ln S_i(\boldsymbol{b}, \mu, \nu R) = \gamma_{\mu,i}^S(b, \mu, \nu R)$$

$$\nu \frac{d}{d\nu} \ln S_i(\boldsymbol{b}, \mu, \nu R) = \gamma_{\nu,i}^S(b, \mu)$$

$$\mu \frac{d}{d\mu} \ln D_{h/i}(z_h, \boldsymbol{b}, \mu, \nu) = \gamma_{\mu,i}^D(\omega_J, \mu, \nu)$$

$$\nu \frac{d}{d\nu} \ln D_{h/i}(z_h, \boldsymbol{b}, \mu, \nu) = \gamma_{\nu,i}^D(b, \mu)$$

anomalous dimensions:

$$\gamma_{\mu,i}^S(b, \mu, \nu R) = -\frac{\alpha_s}{\pi} C_i \ln\left(\frac{\nu^2 \tan^2(R/2)}{\mu_b^2}\right)$$

$$\gamma_{\mu,q}^D(\omega_J, \mu, \nu) = \frac{\alpha_s}{\pi} C_F \left(2\ln\left(\frac{\nu}{\omega_J}\right) + \frac{3}{2}\right)$$

$$\gamma_{q,\nu}^D(\boldsymbol{b}, \mu) = -\gamma_{q,\nu}^S(\boldsymbol{b}, \mu) = \frac{\alpha_s}{\pi} C_F \ln\left(\frac{\mu^2}{\mu_b^2}\right)$$

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Jet fragmentation function $pp \to (\mathrm{jet}h)X$

- Proper TMD definitions

in-jet $\mathcal{D}_{h/i}^{R}(z_h, \boldsymbol{b}; \mu) \equiv D_{h/i}(z_h, \boldsymbol{b}, \mu, \nu)\, S_i(\boldsymbol{b}, \mu, \nu R)$

standard $\hat{\mathcal{D}}_{h/i}(z_h, \boldsymbol{b}; \mu) \equiv D_{h/i}(z_h, \boldsymbol{b}, \mu, \nu)\sqrt{\hat{S}_i(\boldsymbol{b}, \mu, \nu)}$

- Solution of the RG and RRG equations

$$\mathcal{D}_{h/i}^{R}(z_h, \boldsymbol{b}; \mu) = \mathcal{D}_{h/i}^{R}(z_h, \boldsymbol{b}; \mu_b) \exp\left[-\int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'}\left(\Gamma_{\mathrm{cusp}}^{i} \ln\left(\frac{\mu_J^2}{\mu'^2}\right) + \gamma^i\right)\right]$$
$$\hat{\mathcal{D}}_{h/i}(z_h, \boldsymbol{b}; \mu) = \hat{\mathcal{D}}_{h/i}(z_h, \boldsymbol{b}; \mu_b) \exp\left[-\int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'}\left(\Gamma_{\mathrm{cusp}}^{i} \ln\left(\frac{\mu^2}{\mu'^2}\right) + \gamma^i\right)\right]$$

Collins, Soper, Sterman `85

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Jet fragmentation function $pp \to (\mathrm{jet}\,h)X$

- Proper TMD definitions

in-jet $\mathcal{D}^R_{h/i}(z_h, \boldsymbol{b}; \mu) \equiv D_{h/i}(z_h, \boldsymbol{b}, \mu, \nu)\, S_i(\boldsymbol{b}, \mu, \nu R)$

standard $\hat{\mathcal{D}}_{h/i}(z_h, \boldsymbol{b}; \mu) \equiv D_{h/i}(z_h, \boldsymbol{b}, \mu, \nu)\sqrt{\hat{S}_i(\boldsymbol{b}, \mu, \nu)}$

- Solution of the RG and RRG equations

$$\mathcal{D}^R_{h/i}(z_h, \boldsymbol{b}; \mu) = \mathcal{D}^R_{h/i}(z_h, \boldsymbol{b}; \mu_b) \exp\left[-\int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'}\left(\Gamma^i_{\text{cusp}} \ln\left(\frac{\mu_J^2}{\mu'^2}\right) + \gamma^i\right)\right]$$
$$\hat{\mathcal{D}}_{h/i}(z_h, \boldsymbol{b}; \mu) = \hat{\mathcal{D}}_{h/i}(z_h, \boldsymbol{b}; \mu_b) \exp\left[-\int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'}\left(\Gamma^i_{\text{cusp}} \ln\left(\frac{\mu^2}{\mu'^2}\right) + \gamma^i\right)\right]$$

Collins, Soper, Sterman `85

write as:

$$\mathcal{D}^R_{h/i}(z_h, \boldsymbol{b}; \mu) = \hat{\mathcal{D}}_{h/i}(z_h, \boldsymbol{b}; \mu_J) \exp\left[-\int_{\mu_J}^{\mu} \frac{d\mu'}{\mu'}\left(\Gamma^i_{\text{cusp}} \ln\left(\frac{\mu_J^2}{\mu'^2}\right) + \gamma^i\right)\right]$$

same as for SIDIS and e^+e^-

canceled by Γ_i of the RG evolution for $\mathcal{H}_{c\to i}$

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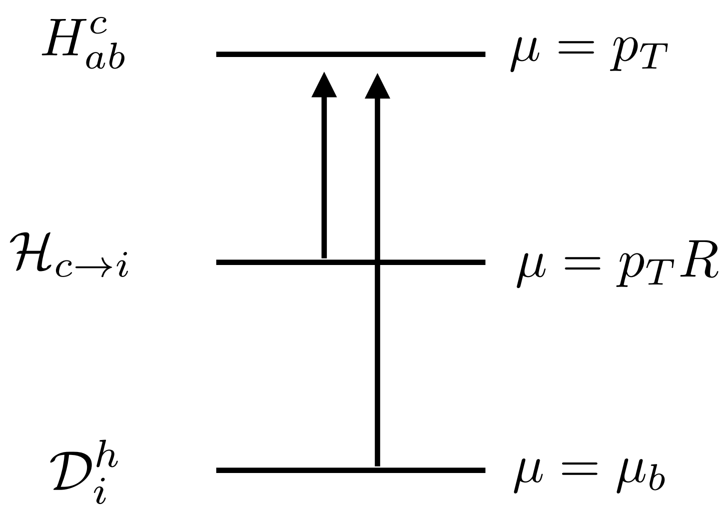
Evolution

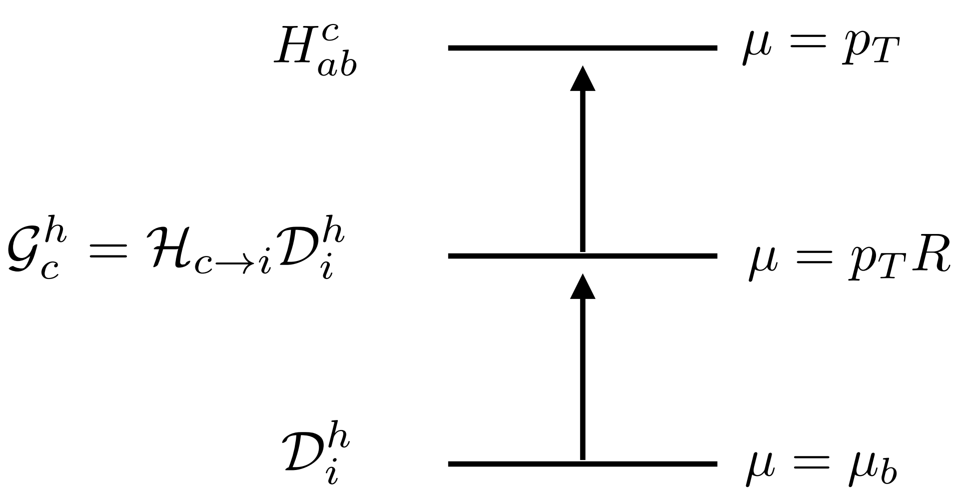
1.



using modified DGLAP for $\mathcal{H}_{c\to i}$

$$\mu \frac{d}{d\mu}\mathcal{H}_{c\to i} = \gamma_{ck} \otimes \mathcal{H}_{ki}$$

2.



using DGLAP for siTMDFJFs $\mathcal{G}_c^h$

$$\mu \frac{d}{d\mu}\mathcal{G}_c^h = P_{ic} \otimes \mathcal{G}_i^h$$

$$\gamma_{ii}^{\Gamma_i} + \gamma_{i,\mu}^S + \gamma_{i,\mu}^D = 0$$

Jet fragmentation function $pp \to (\mathrm{jet}h)X$

- The semi-inclusive TMD FJFs

$$\mathcal{G}_c^h(z, z_h, \omega_J R, \boldsymbol{j}_\perp, \mu) = \mathcal{H}_{c\to i}(z, \omega_J R, \mu) \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} e^{i\boldsymbol{j}_\perp \cdot \boldsymbol{b}/z_h} \mathcal{D}_{h/i}^R(z_h, \boldsymbol{b}; \mu)$$

where $$\hat{\mathcal{D}}_{h/i}(z_h, \boldsymbol{j}_\perp; \mu_J) = \frac{1}{z_h^2} \int \frac{b\, db}{2\pi} J_0(j_\perp b/z) C_{j\leftarrow i} \otimes D_{h/j}(z_h, \mu_{b_*}) e^{-S^i_{\mathrm{pert}}(b_*, \mu_J) - S^i_{\mathrm{NP}}(b, \mu_J)}$$

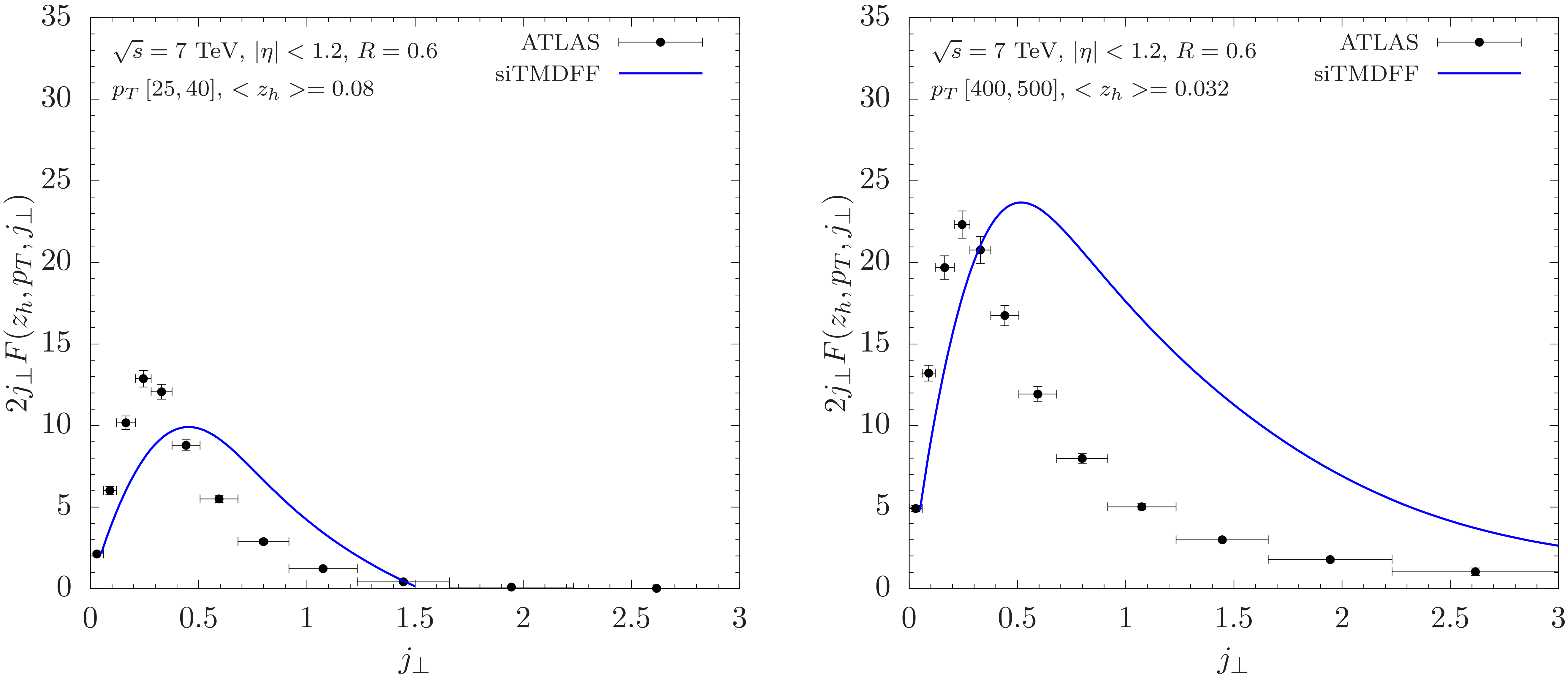
matching onto collinear FFs

b* prescription

with DGLAP evolution

$$\boxed{\mu \frac{d}{d\mu} \mathcal{G}_i^h(z, z_h, \omega_J R, \boldsymbol{j}_\perp, \mu) = \frac{\alpha_s}{2\pi} \sum_j \int_z^1 \frac{dz'}{z'} P_{ji}\left(\frac{z}{z'}\right) \mathcal{G}_j^h(z', z_h, \omega_J R, \boldsymbol{j}_\perp, \mu)}$$

Comparison to ATLAS data



b* prescription following *Sun, Kang, Prokudin, Yuan `16*

Outline

- Inclusive jets *Kang, FR, Vitev `16*
- In-jet fragmentation
  - Collinear FFs *Kang, FR, Vitev `16*
  - TMD FFs *Kang, Liu, FR, Xing `17*
- Conclusions

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Conclusions

- Inclusive jets and their substructure
- Identified hadrons within jets:
  light hardrons, open heavy flavor, quarkonia
- TMD FFs within jets

- Non-global logarithms, Y-term, non-perturbative Sudakov
- Spin asymmetries
- Extension for example to subjets