The spin-dependent quark beam function at two loops

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In collaboration with R. Boughezal, F. Petriello and U. Schubert
arXiv: 1704.05457

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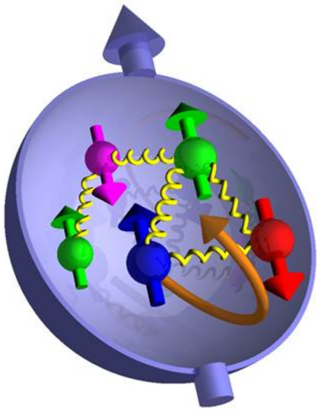
Spin configuration of proton

- Proton helicity sum rule

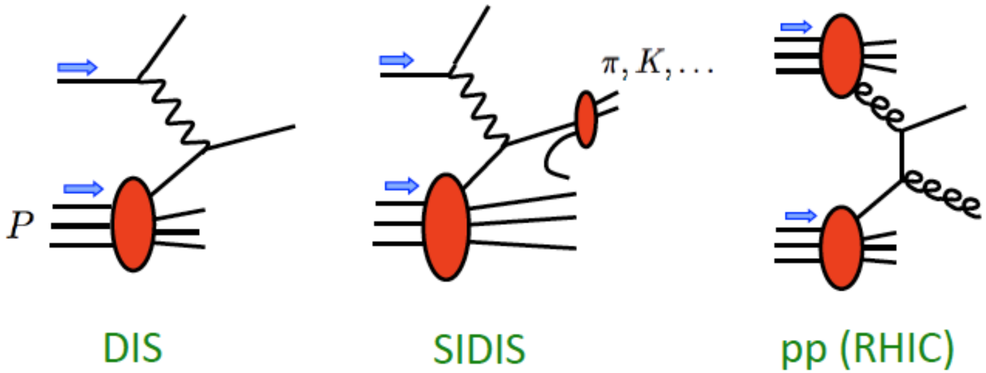
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

quark spin $\Delta \sum = \int_0^1 dx \Delta f_q(x)$

gluon spin $\Delta G = \int_0^1 dx \Delta f_g(x)$



- Probes are used so far

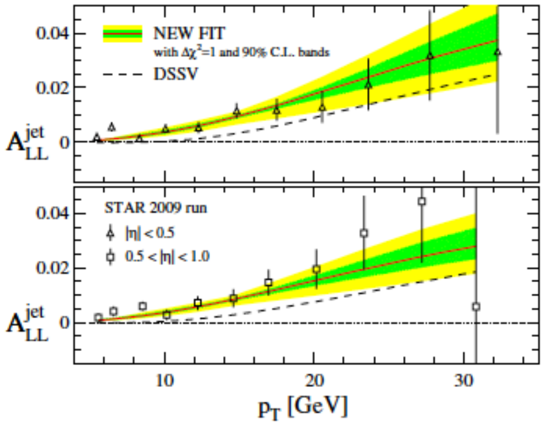


- QCD factorization for inclusive hadron production in pp

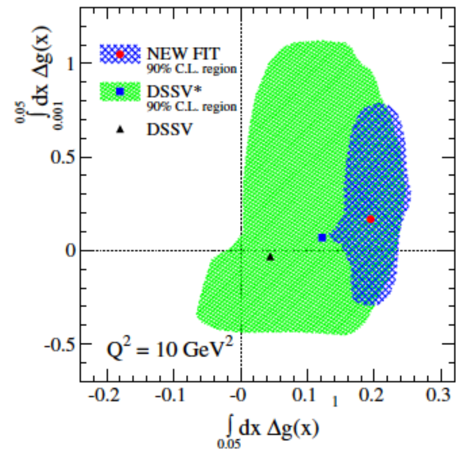
$$d\Delta\sigma = \sum_{a,b,c} \Delta f_a \otimes \Delta f_b \otimes d\Delta\hat{\sigma}_{ab}^c \otimes D_c^\pi$$

Global extraction of helicity PDFs

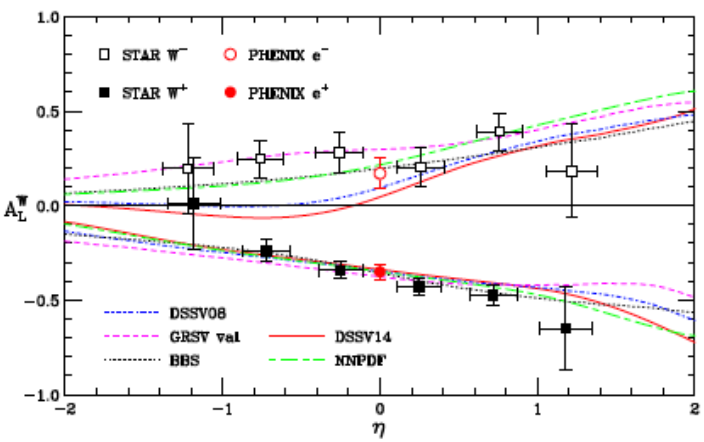
- Global fitting (GRV, DSSV, NNPDF …)



DSSV 2014

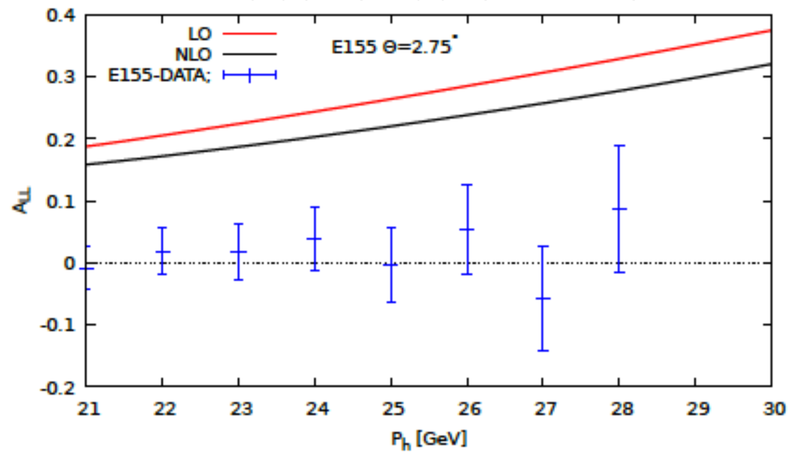


- NLO predictions



Ringer and Vogelsang, PRD 2015

Inclusive hadron in DIS



Hinderer, Schlegel, and Vogelsang, 1703.10872

- How to proceed to NNLO?

N-jettiness

- N-jettiness is a global event shape variable designed to veto final state jets

Stewart, Tackmann, Waalewijn 0910. 0467

$$\tau_N = \sum_k \min_i \left\{ \frac{2p_i \cdot q_k}{Q_i} \right\}$$

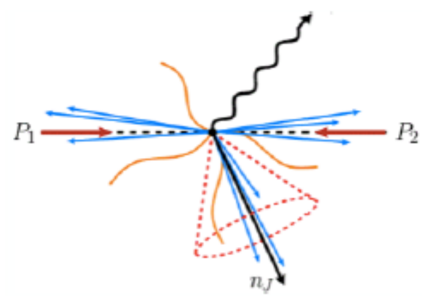
p_i Momenta of initial state beams and final state jets

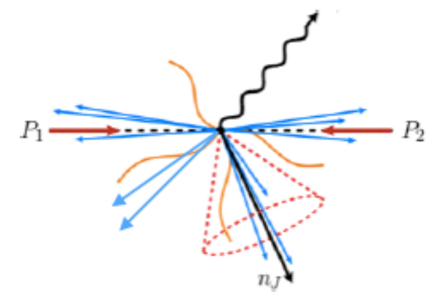
q_k Momenta of all final state partons

Q_i Measure of the jet hardness

- Use N-jettiness to separate N jet event and more-than-N-jet event

1 jet ← small — τ_1 — large → at least 2 jets





- $\tau_1 = 0$ forces an 1-jet final state, q_k must be soft or collinear to one of p_i
- τ_1 controls all the IR behaviors for 1-jet, which is universal for any physical IR safe measurement on 1 jets

N-jettiness subtraction

- Introduce τ_N^{cut} to partition the phase space, identify IR behavior

$$\sigma_{NNLO} = \int d\Phi_N |M_N|^2 + \int d\Phi_{N+1} |M_{N+1}|^2 \theta_N^< + \int d\Phi_{N+2} |M_{N+2}|^2 \theta_N^<$$

$$+ \int d\Phi_{N+1} |M_{N+1}|^2 \theta_N^> + \int d\Phi_{N+2} |M_{N+2}|^2 \theta_N^>$$

$$\equiv \sigma_{NNLO}(\tau_N < \tau_N^{cut}) + \sigma_{NNLO}(\tau_N > \tau_N^{cut})$$

$$\theta_N^< = \theta(\tau_N^{cut} - \tau_N) \qquad \theta_N^> = \theta(\tau_N - \tau_N^{cut})$$

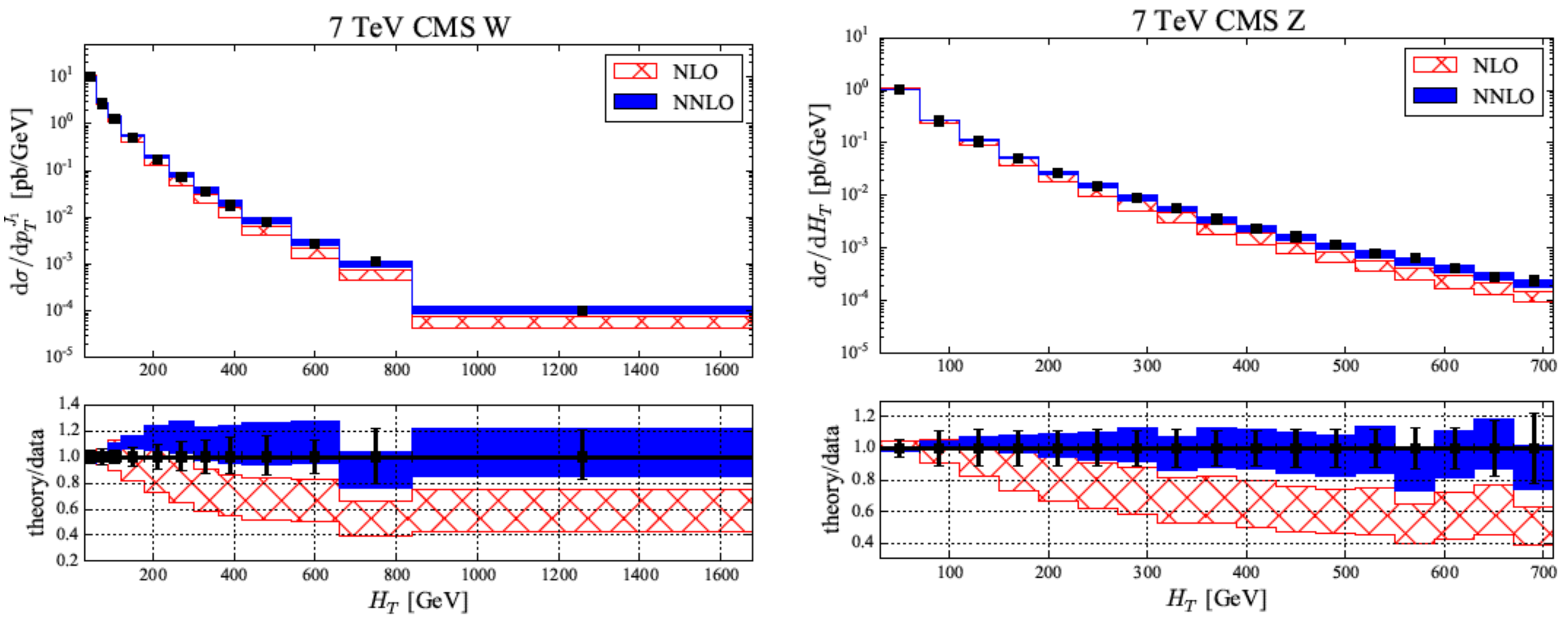
τ_N^{cut} has to be small, so we can safely neglect power corrections

Boughezal, Focke, Liu, Petriello (2015); Gaunt, Stahlhofen, Tackmann, Walsh (2015)

- Achievements of NNLO N-jettiness subtraction for LHC physics

$$\sigma_{NNLO} = \sigma_{NNLO}(\tau_N < \tau_N^{cut}) + \sigma_{NNLO}(\tau_N > \tau_N^{cut})$$

SCET — Fixed order NLO, MCFM

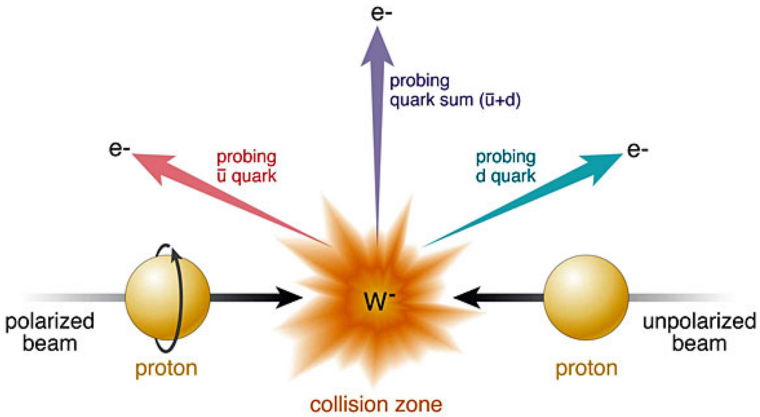


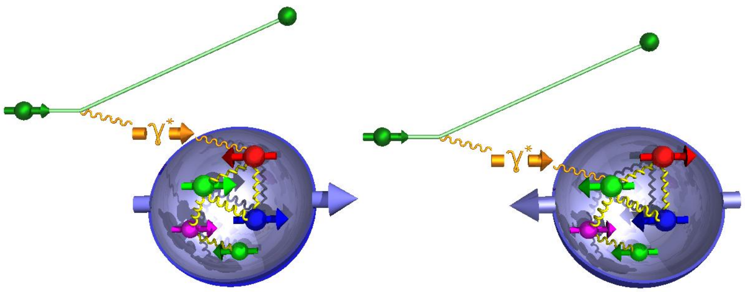
Boughezal, Liu, Petriello, 2016

Impact: reduce the Higgs cross section uncertainty from PDF by 30%

Boughezal, Guffanti, Petriello, Ubiali, 2017

N-jettiness in polarized collisions





$$\frac{d\sigma_{LU}}{d\tau_N} = \Delta H \otimes \Delta B_a \otimes B_b \otimes S \otimes \left[\prod_n^N J_n\right] + \cdots$$

$$\frac{d\sigma_{LL}}{d\tau_N} = \Delta H \otimes \Delta B \otimes S \otimes \left[\prod_n^N J_n\right] + \cdots$$

- Hard function: two loop virtual corrections, done for DIS and DY!

$$\Delta H = H^+ - H^-$$

- Soft function: remains the same as unpolarized (R. Boughezal, X. Liu, F. Petriello, 15)
- Jet function: remains the same as unpolarized (Becher, Neubert 06, Becher, Bell 11)
- Beam function: not available, the only missing ingredient

$$\Delta B = B^+ - B^-$$

Polarized quark beam function

- Operator definition

$$\Delta B_q(t,x,\mu) = \langle p_n(P^-),+|\theta(\omega)\bar{\chi}_n(0)\delta(t-\omega\hat{p}^+)\frac{\bar{n}\cdot\gamma\gamma_5}{2}[\delta(\omega-\overline{\mathcal{P}}_n)\chi_n(0)]|p_n(P^-),+\rangle$$

Composite quark operator

$$\chi_n(y) = W_n^\dagger(y)\xi_n(y)$$

Wilson line

$$W_n(y) = \left[\sum_{\text{perms}} \exp\left(-\frac{g}{\overline{\mathcal{P}}_n}\bar{n}\cdot A_n(y)\right)\right]$$

- Renormalization and RGE (double log resummation)

$$\Delta B_i^{bare}(t,z) = \int dt' Z_i(t-t',\mu)\Delta B_i(t',z,\mu)$$

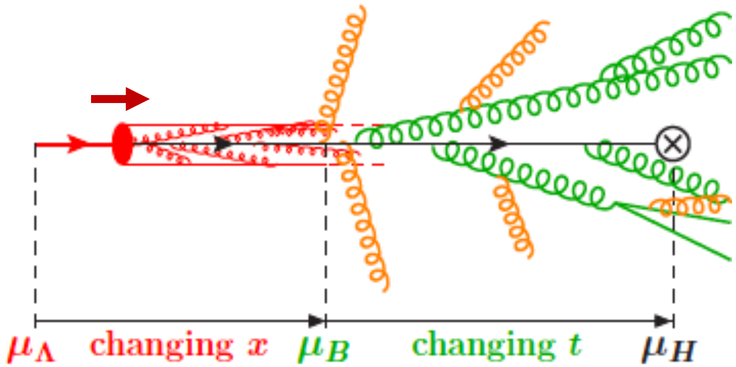
$$\mu\frac{d}{d\mu}\Delta B_i(t',z,\mu) = \int dt' \gamma_B^i(t-t',\mu)\Delta B_i(t',z,\mu)$$

- anomalous dimension

$$\gamma_B^i(t,\mu) = -\int dt'(Z_i)^{-1}(t-t',\mu)\mu\frac{d}{d\mu}Z_i(t',\mu)$$

$$\int dt'(Z_i)^{-1}(t-t',\mu)Z_i(t',\mu) = \delta(t)$$

Initial state radiation



- Single log resummation: DGLAP for polarized PDFs

$$\frac{d}{d\ln\mu^2}\Delta f_j = \Delta P_{jk} \otimes \Delta f_k$$

- Beam function matches to PDFs $\quad t >> \Lambda^2_{QCD}$

$$\Delta B_i(t,x,\mu) = \sum_j \int_x^1 \frac{d\xi}{\xi} \Delta I_{ij}\left(t, \frac{x}{\xi}\right) \Delta f_j(\xi,\mu)$$

t is the virtuality of the parton that enters the hard interaction

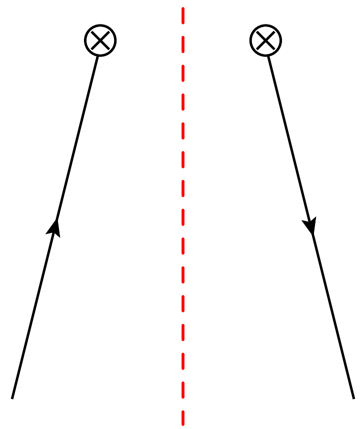
- Matching coefficient

ΔI_{ij} describes initial state radiation, can be computed perturbatively

- Calculate partonic beam function

$$\Delta B_{ij}(t,z,\mu) = \sum_k \int_z^1 \frac{dz'}{z'} \Delta\mathcal{I}_{ik}(t,z',\mu)\Delta f_{kj}\left(\frac{z}{z'}\right)$$

- Leading order

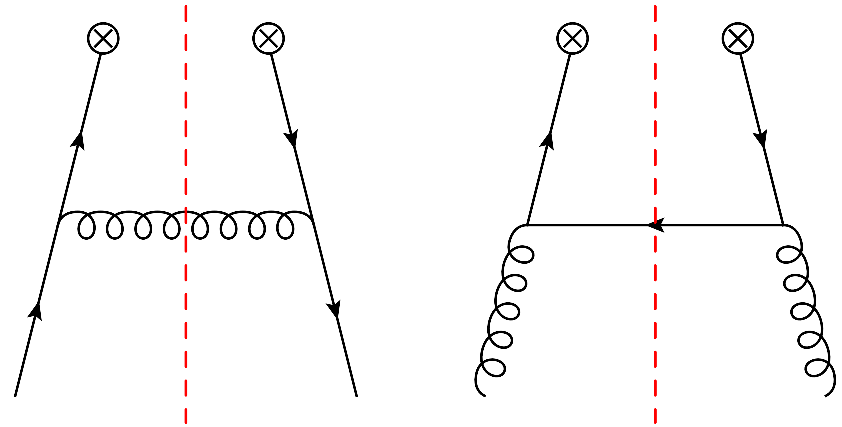


$$\begin{aligned}\Delta B_{qq}^{(0)}(t,z,\mu) =& \langle q_n(p),+|\theta(\omega)\bar{\chi}_n(0)\delta(t-\omega\hat{p}^+)\frac{\bar{n}\cdot\gamma\gamma_5}{2}[\delta(\omega-\overline{\mathcal{P}}_n)\chi_n(0)]|q_n(p),+\rangle \\ =& \delta(t)\delta(1-\omega/p^-)\end{aligned}$$

$$\mathcal{I}_{qq}^{(0)}(t,z,\mu) = \mathcal{I}_{\bar{q}\bar{q}}^{(0)}(t,z,\mu) = \delta(t)\delta(1-z)$$

$$\mathcal{I}_{qg}^{(0)}(t,z,\mu) = \mathcal{I}_{gq}^{(0)}(t,z,\mu) = 0$$

Next-to-leading order



$$\left(\frac{\alpha_s}{4\pi}\right)\Delta B_{qq}^{bare(1)}(t,z) = \frac{g^2}{N_c}\left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^{\epsilon}\int d\mathrm{PS}^{(1)}\mathrm{Tr}\left[\frac{\bar{n}\cdot\gamma\gamma_5}{2}\ell\cdot\gamma\gamma^{\rho}\mathcal{P}_R p\cdot\gamma\gamma^{\sigma}\ell\cdot\gamma\right]d_{\rho\sigma}(k)\frac{1}{\ell^2}\frac{1}{\ell^2}\mathrm{Tr}[\mathrm{T^aT^a}]$$

- γ_5 in d-dimension – HVBM scheme

$$\gamma_5 \equiv \frac{i}{4!}\epsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_\nu\gamma_\sigma\gamma_\rho \quad \Longrightarrow \quad \{\gamma_5,\tilde{\gamma}_\mu\} = 0, \quad [\gamma_5,\hat{\gamma}_\mu] = 0$$

Maintain the four-dimension definition

anticommute in 4-dimension
commute in d-4 dimension

- Final state phase

$$\int d\mathrm{PS}^{(1)} = \int \frac{d^dk}{(2\pi)^{d-1}}d^d\ell\;\delta(k^2)\delta(\omega-\ell^-)\delta(t-\omega k^+)\delta^d(p-k-\ell)$$

$$= \frac{1}{(4\pi)^{2-\epsilon}}\frac{1}{\Gamma(-\epsilon)}\frac{1}{\omega}\int_0^{t\frac{1-z}{z}} d\hat{k}_\perp^2(\hat{k}_\perp^2)^{-1-\epsilon}$$

d-4 dimension momentum

- Bare quark beam function at NLO

UV divergence

$$\Delta B_{qq}^{bare(1)}(t,z) = \boxed{\frac{4}{\epsilon^2}C_F\delta(t)\delta(1-z) - \frac{4}{\epsilon}C_F\frac{1}{\mu^2}\mathcal{L}_0\left(\frac{t}{\mu^2}\right)\delta(1-z) + \frac{3}{\epsilon}C_F\delta(t)\delta(1-z)} - \frac{2}{\epsilon}C_F\delta(t)\Delta P_{qq}^{(0)}(z)$$
$$+ 4C_F\frac{1}{\mu^2}\mathcal{L}_1\left(\frac{t}{\mu^2}\right)\delta(1-z) + 2C_F\frac{1}{\mu^2}\mathcal{L}_0\left(\frac{t}{\mu^2}\right)\mathcal{L}_0(1-z)(1+z^2)$$
$$+ 2C_F\delta(t)\left[\mathcal{L}_1(1-z)(1+z^2) - \frac{1+z^2}{1-z}\ln z - 3(1-z) - \frac{\pi^2}{6}\delta(1-z)\right]$$

- renormalized quark beam function at NLO

IR divergence

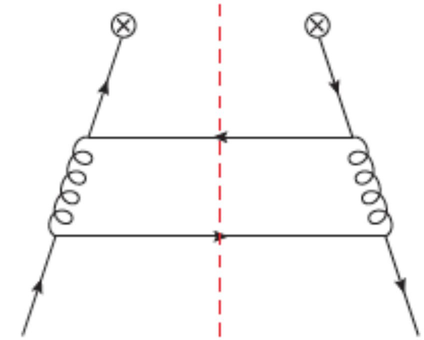
$$\Delta B_{qq}^{(1)}(t,z,\mu^2) = \boxed{-\frac{2}{\epsilon}\delta(t)\Delta P_{qq}^{(0)}(z)} + 4C_F\frac{1}{\mu^2}\mathcal{L}_1\left(\frac{t}{\mu^2}\right)\delta(1-z) + 2C_F\frac{1}{\mu^2}\mathcal{L}_0\left(\frac{t}{\mu^2}\right)\mathcal{L}_0(1-z)(1+z^2)$$
$$+ 2C_F\delta(t)\left[\mathcal{L}_1(1-z)(1+z^2) - \frac{1+z^2}{1-z}\ln z - 3(1-z) - \frac{\pi^2}{6}\delta(1-z)\right]$$

- Matching coefficient at NLO

$$\Delta\tilde{I}_{qq}^{(1)}(z) = 2C_F\left[\mathcal{L}_1(1-z)(1+z^2) - \frac{1+z^2}{1-z}\ln z - 3(1-z) - \frac{\pi^2}{6}\delta(1-z)\right]$$

Finite!

NNLO



- q’ -> q as an example

$$B_{qq'}^{\mathrm{bare},(2)}(t,z) = \sum_i c_i' \mathcal{I}_i$$

$$\mathcal{I}_i = \mathcal{I}_i(i_1,i_2,i_3,i_4,i_5,i_6,i_7) = C \int \frac{d^d k_1}{(2\pi)^{(d-3)}} \frac{d^d k_2}{(2\pi)^{(d-3)}} \frac{1}{D_1^{i_1} D_2^{i_2} D_3^{i_3} D_4^{i_4} D_5^{i_5} D_6^{i_6} D_7^{i_7}}$$

- IBP reduction

$$\Delta B_{qq'}^{bare(2)}(t,z) = C_F T_R \left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^{2\epsilon} \sum_{i=1}^{4} C_i(t,z,\epsilon) I_i^{RR}$$

- master integrals

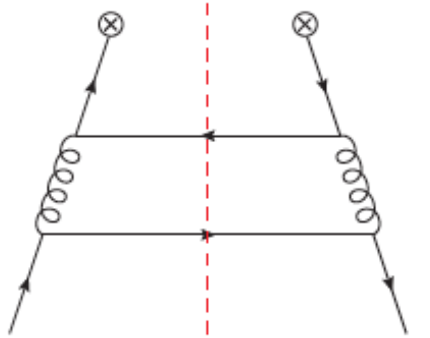
$$I_1^{RR} = \int d\mathrm{PS}^{(2)} \times 1 = 4\frac{(16\pi)^{-3+2\epsilon}}{\omega\Gamma\left(\frac{3}{2}-\epsilon\right)^2} t^{1-2\epsilon}\left(\frac{1-z}{z}\right)^{1-2\epsilon},$$

$$I_2^{RR} = \int d\mathrm{PS}^{(2)} \times \frac{1}{\bar{n}\cdot(p-k_1)} = 4\frac{(16\pi)^{-3+2\epsilon}}{\omega^2\Gamma\left(\frac{3}{2}-\epsilon\right)^2} t^{1-2\epsilon}(1-z)^{1-2\epsilon} z^{2\epsilon}\, {}_2F_1(1,1-\epsilon;2-2\epsilon;1-z),$$

$$I_3^{RR} = \int d\mathrm{PS}^{(2)} \times \frac{1}{(p-k_1-k_2)^2} = -4\frac{(16\pi)^{-3+2\epsilon}}{\omega\Gamma\left(\frac{3}{2}-\epsilon\right)^2} t^{-2\epsilon}(1-z)^{1-2\epsilon} z^{2\epsilon}\, {}_2F_1(1,1-\epsilon;2-2\epsilon;1-z),$$

$$I_4^{RR} = \int d\mathrm{PS}^{(2)} \times \frac{1}{(p-k_1-k_2)^2}\frac{1}{\bar{n}\cdot(p-k_1)} = -4\frac{(16\pi)^{-3+2\epsilon}}{\omega^2\Gamma\left(\frac{3}{2}-\epsilon\right)^2} t^{-2\epsilon}(1-z)^{1-2\epsilon} z^{1+2\epsilon}\, {}_2F_1(1,1-\epsilon;2-2\epsilon;1-z$$

NNLO



- q' -> q as an example

$$B_{qq'}^{\mathrm{bare},(2)}(t,z) = \sum_i c_i' \mathcal{I}_i$$

$$\mathcal{I}_i = \mathcal{I}_i(i_1,i_2,i_3,i_4,i_5,i_6,i_7) = C\int \frac{d^d k_1}{(2\pi)^{(d-3)}} \frac{d^d k_2}{(2\pi)^{(d-3)}} \frac{1}{D_1^{i_1} D_2^{i_2} D_3^{i_3} D_4^{i_4} D_5^{i_5} D_6^{i_6} D_7^{i_7}}$$

- IBP reduction

$$\Delta B_{qq'}^{bare(2)}(t,z) = C_F T_R \left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^{2\epsilon} \sum_{i=1}^{4} C_i(t,z,\epsilon) I_i^{RR}$$

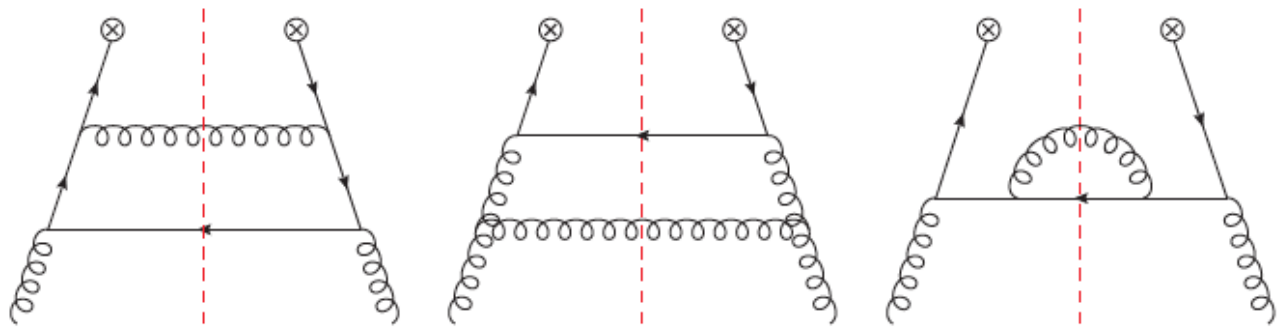
- No UV divergence
- IR divergence

$$-\frac{2}{\epsilon^2}\delta(t)\Delta P_{qg}^{(0)}(z)\otimes\Delta P_{gq'}^{(0)}(z) + \frac{2}{\epsilon}\delta(t)\Delta P_{qq'}^{(1)}(z) + \frac{2}{\epsilon}\Delta\mathcal{I}_{qg}^{(1)}(t,z,\mu)\otimes\Delta P_{gq'}^{(0)}(z)$$

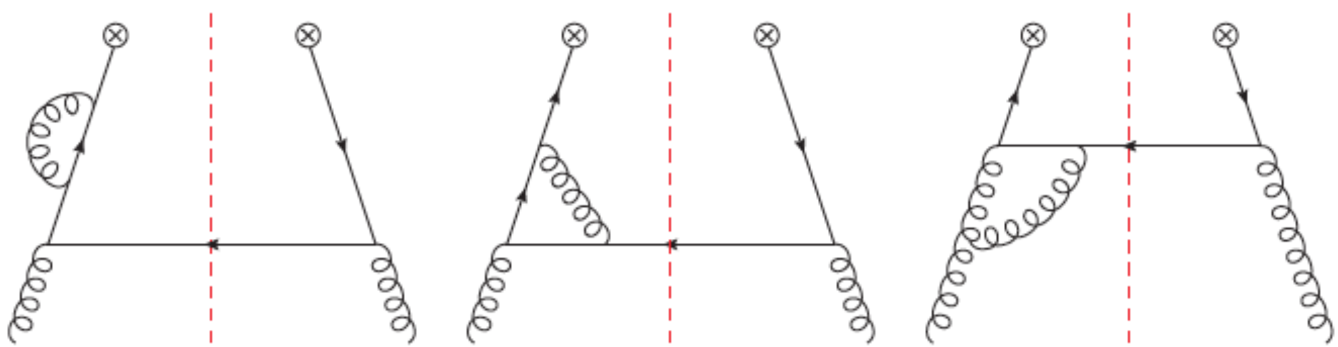
- Finite matching coefficient

gluon -> quark

- Real-real



- Real-virtual



- virtual-virtual: scaleless and vanish

- UV divergence 1: alphas renormalization

$$Z_\alpha = 1 - \frac{\alpha_s}{4\pi}\frac{\beta_0}{\epsilon} + \mathcal{O}(\alpha_s^2)$$

$$\Delta B_{qg}^{bare(2)}(t,z) = \Delta B_{qg}^{bare(2)}(\alpha_s^{(0)},t,z) - \frac{\beta_0}{\epsilon}\Delta B_{qg}^{bare(1)}(\alpha_s^{(0)},t,z)$$

- UV divergence 2: beam function renormalization

$$\Delta B_{qg}^{(2)}(t,z,\mu) = \Delta B_{qg}^{bare(2)}(t,z) - \int dt' Z_q^{(1)}(t-t',\mu)\Delta B_{qg}^{(1)}(t',z,\mu)$$

- IR divergence cancels with PDFs

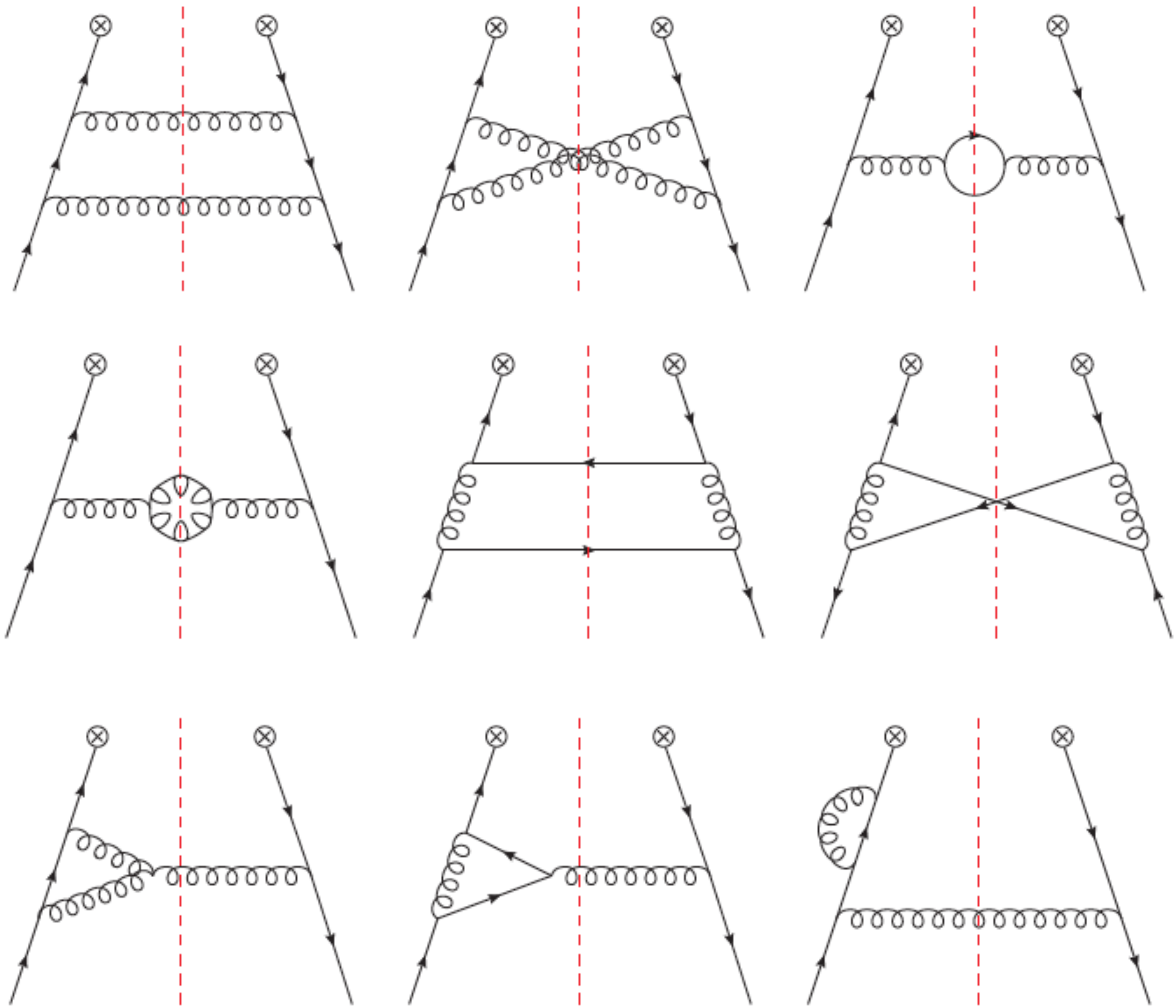
$$\Delta \mathcal{I}_{qg}^{(2)}(t,z,\mu) = \Delta B_{qg}^{(2)}(t,z,\mu) - 4\delta(t)\Delta f_{qg}^{(2)}(z) - 2\Delta \mathcal{I}_{qq}^{(1)}(t,z,\mu)\otimes \Delta f_{qg}^{(1)}(z) - 2\Delta \mathcal{I}_{qg}^{(1)}(t,z',\mu)\otimes \Delta f_{gg}^{(1)}(z)$$

$$\Delta f_{ij}^{(1)}(z) = -\frac{1}{\epsilon}\Delta P_{ij}^{(0)}(z),$$

$$\Delta f_{ij}^{(2)}(z) = \frac{1}{2\epsilon^2}\sum_k \Delta P_{ik}^{(0)}(z)\otimes \Delta P_{kj}^{(0)}(z) + \frac{\beta_0}{4\epsilon^2}\Delta P_{ij}^{(0)}(z) - \frac{1}{2\epsilon}\Delta P_{ij}^{(1)}(z)$$

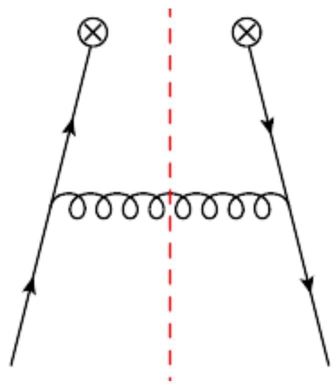
- Finite matching coefficient

quark -> quark



One more renormalization: scheme transformation

- helicity conservation requires $\Delta I_{qq}^{(1)} = I_{qq}^{(1)}$



However, in our naïve calculation by using HV scheme

$$\Delta \tilde{I}_{qq}^{(1)} \neq I_{qq}^{(1)}$$

- Drawback of HV scheme: Violation of helicity conservation

Due to the broken of anti-commutativity of gamma 5 and Dirac matrix in d-dimension $\{\gamma_5, \tilde{\gamma}_\mu\} = 0, \quad [\gamma_5, \hat{\gamma}_\mu] = 0$

- Fixed by extra renormalization constant Z^5

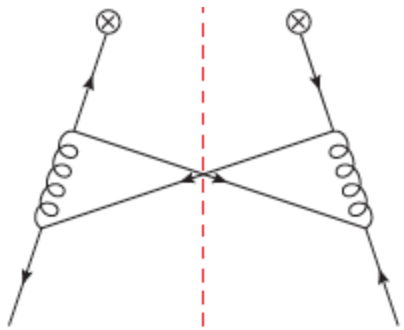
$$\Delta B = \left(\Delta \tilde{I} \otimes \bar{Z}^5\right) \otimes \left(Z^5 \otimes \Delta \tilde{f}\right) = \Delta I \otimes \Delta f$$

$$\Delta I^{(1)} = \Delta \tilde{I}^{(1)} \otimes \bar{Z}^5 = \Delta \tilde{I}^{(1)} - z_{qq}^{(1)} = I^{(1)}$$

$$z_{qq}^{(1)} = -8C_F(1-z)$$

NNLO scheme transformation

- helicity conservation requires $\Delta I_{q\bar{q}}^{(V,2)} = -I_{q\bar{q}}^{(V,2)}$



Again, not true in naïve calculation

- Scheme transformation - splitting functions

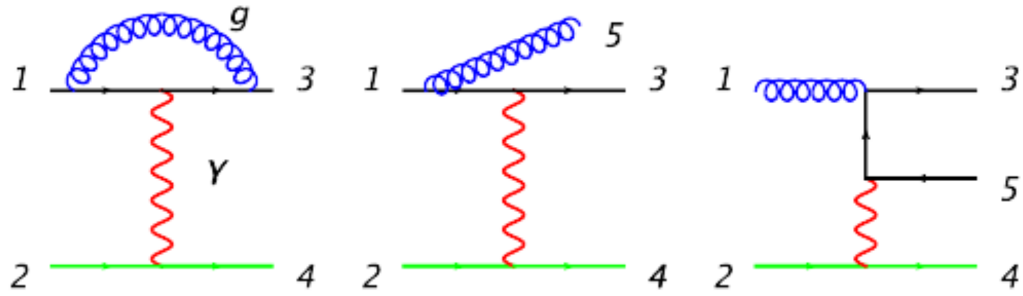
$$\Delta P_{qq}^{(1)} = \Delta \tilde{P}_{qq}^{(1)} - \frac{1}{4}\beta_0 z_{qq}^{(1)}$$
$$\Delta P_{qg}^{(1)} = \Delta \tilde{P}_{qg}^{(1)} + \frac{1}{2} z_{qq}^{(1)} \otimes \Delta \tilde{P}_{qg}^{(0)}$$

- Scheme transformation - matching coefficient

$$\Delta I_{qq}^{(2,V)} = \Delta \tilde{I}_{qq}^{(2,V)} - \Delta \tilde{I}_{qq}^{(1)} \otimes z_{qq}^{(1)} + z_{qq}^{(1)} \otimes z_{qq}^{(1)} - z_{qq}^{(2,V)}$$
$$\Delta I_{q\bar{q}}^{(2,V)} = \Delta \tilde{I}_{q\bar{q}}^{(2,V)} - z_{q\bar{q}}^{(2,V)}$$
$$\Delta I_{qq}^{(2,S)} = \Delta \tilde{I}_{qq}^{(2,S)} - z_{qq}^{(2,S)}$$

$$\Delta I_{q_i q_j}^{(2)} = \delta_{ij} \Delta I_{qq}^{(2,V)} + \Delta I_{qq}^{(2,S)}$$

DIS NLO

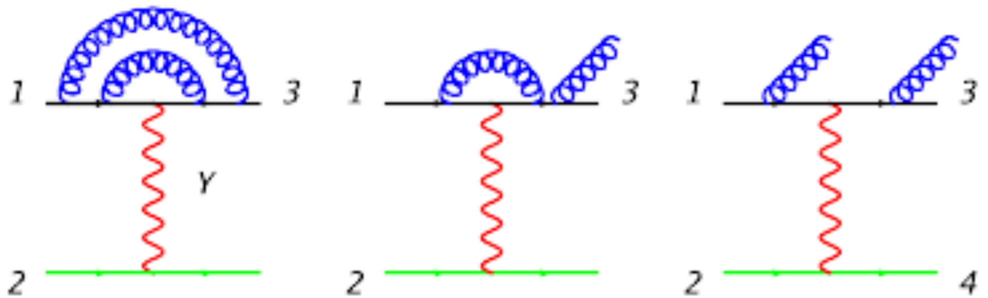


- Above cut $\theta_N^{>} = \theta(\tau_N - \tau_N^{cut})$
  - Only real radiation contributes, the radiated gluon/quark is resolved, this region of phase space contains the tree diagram to the 2 jet process.
  - Tree level calcualtion

- Below cut $\theta_N^{<} = \theta(\tau_N^{cut} - \tau_N)$
  - Virtual: τ_N is zero
  - Real: the radiated gluon/quark is unresolved. Purely IR divergent region
  - Calculate this part from SCET

$$\frac{d\sigma_{LL}}{d\tau_N} = \Delta H \otimes \Delta B \otimes S \otimes \left[\prod_n^N J_n\right] + \cdots$$

DIS NNLO



- Above cut $\theta_N^{>} = \theta(\tau_N - \tau_N^{cut})$
  - In RR: at lease one of the two additional radiations that appear is resolved, this region of phase space contains the NLO correction to the 2 jet process.
  - In RV: the radiation has to be hard, this is NLO virtual correction to 2 jet production.
  - Recycle this part from available NLO tool.
- Below cut $\theta_N^{<} = \theta(\tau_N^{cut} - \tau_N)$
  - VV: τ_N is zero
  - RV and RR: both additional radiations are unresolved. Purely IR divergent region
  - two loop, soft and collinear radiation
  - Calculate this part from SCET

Summary

- We calculated the matching coefficients between the polarized quark beam function and PDF at two-loop order.

- Our results are an important ingredient for high order calculations in using N-jettiness subtraction scheme, as well as for the resummation for observables such as global event shape.

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Thanks!