Transverse Momentum Dependent Fragmenting Jet Functions (TMDFJF) with application to Quarkonium production

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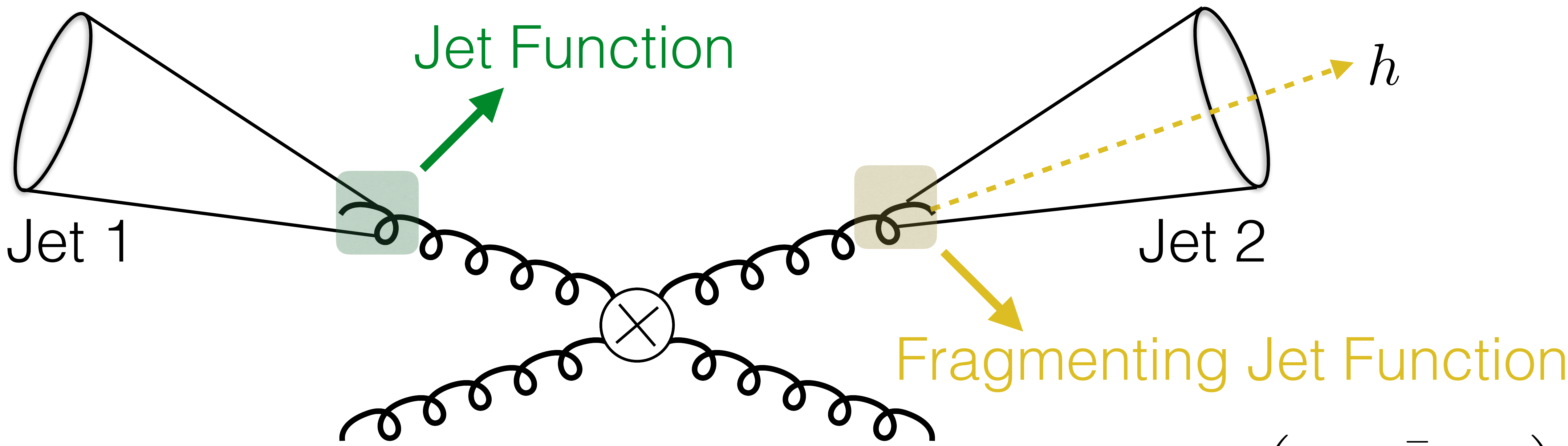
Based on: arXiv:1610.06508
In collaboration with Reggie Bain and Thomas Mehen

QCD Evolution, Newport News, VA, May 2017

Outline

- Introduction to Fragmenting Jet Functions (FJFs)
- The NRQCD factorization and Quarkonium production
- Transverse Momentum Dependent (TMD) FJFs
- Applications to Quarkonium production
- Summary

Hadron production in jets



$$J_i(\tau, \mu) \to \mathcal{G}_{i/h}\left(z = \frac{p_h^-}{k^-}, s, \mu\right)$$

Talk by I. Vitev

<u>Factorization in Soft-Collinear Effective Theory (SCET):</u>

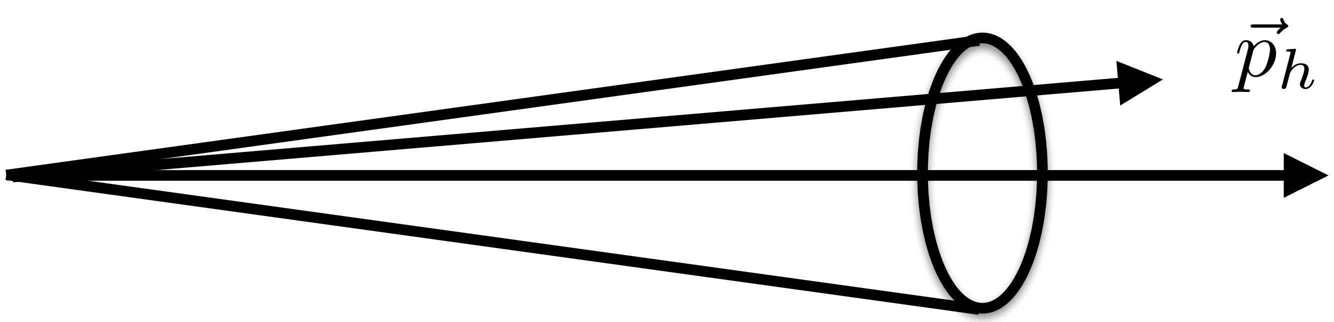
$$d\sigma = H_N(\mu) \times S_N^{n_1, n_2, \cdots, n_N}(\mu) \times \left(\prod_i^{N-1} J_{n_i}^{(i)}(\omega_i, \mu)\right) \times \mathcal{G}_{n_N}^{N\to h}(z, \omega_N, \mu)$$

Fragmenting Jet Functions (FJF)

$$\mathcal{G}_{i/h}(z,s,\mu) = [\mathcal{J}_{i/j}(s,\mu) \bullet D_{j/h}(\mu)](z) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{s}\right)$$

Short distance matching coefficients | Fragmentation Function

Unmeas. Jet	Meas. Jet		Unmeas. FJF	Meas. FJF
ωr	$\omega r,\, s$	$\longrightarrow$	$\omega r,\; z$	$\omega r,\; z,\; s$



Jet cone size: R

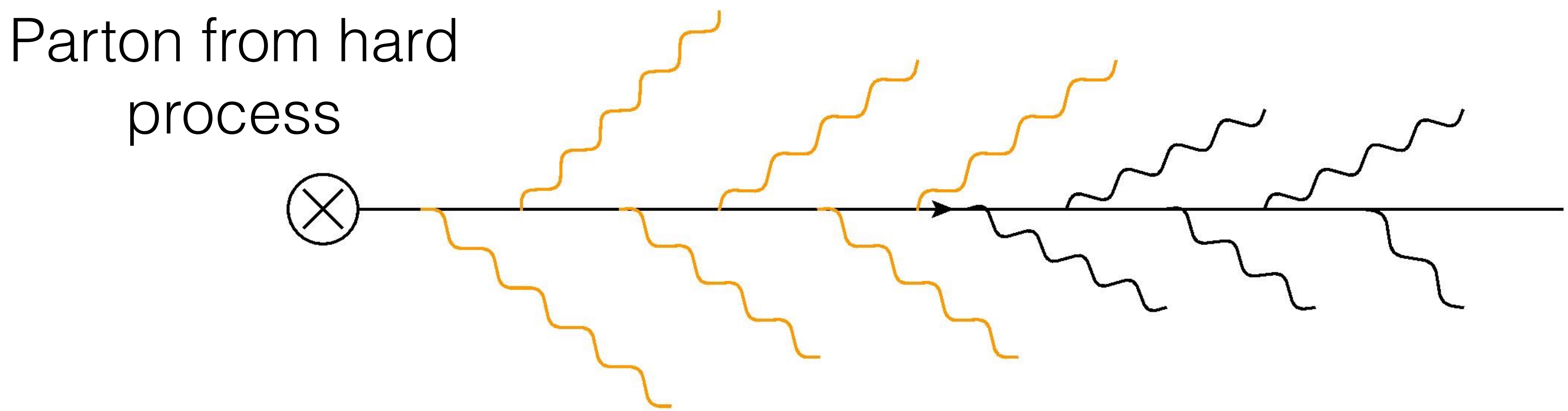
$$\vec{k} = \frac{\omega}{2}\hat{n}$$

$$s = \omega k^+ \qquad r = \tan(R/2)$$

Procura and Stewart [arXiv:0911.4980]

Procura and Waalewijn [arXiv:1111.6605], Jain, Procura and Waalewijn [arXiv:1101.4953]

FJF Evolution



Change in virtuality (s): Jet Evolution

$$\frac{d}{d\mu}\mathcal{G}(s,\mu) = \int ds' \; \gamma_J(s-s',\mu)\mathcal{G}(s',\mu)$$

Change in energy ratio (z): DGLAP Evolution

$$\frac{d}{d\mu}F_{i/h}(z,\mu) = \sum_j \int_z^1 \frac{dx}{x} \; P_{ij}(x)F_{j/h}\left(\frac{z}{x},\mu\right)$$

Application to Quarkonium production

Talk by J.P. Lansberg

NRQCD factorization Expansion in: $\alpha_s, \quad v$

$$d\sigma(a+b\to\mathcal{Q}+X)=\sum_n d\sigma(a+b\to Q\overline{Q}(n)+X)\langle\mathcal{O}_n^{\mathcal{Q}}\rangle$$

$$\mathcal{O}_n^{\mathcal{Q}}=\mathcal{O}_2^{n\,\dagger}\left(\sum_X|X+\mathcal{Q}\rangle\langle X+\mathcal{Q}|\right)\mathcal{O}_2^n \qquad \mathcal{O}_2^n=\psi^\dagger\mathcal{K}^n\chi$$

Leading Power (LP) Factorization Expansion in: $\alpha_s, \quad v, \quad \dfrac{m_{\mathcal{Q}}}{p_\perp}$

$$d\sigma(a+b\to\mathcal{Q}+X)=\sum_i d\sigma(a+b\to i+X)\otimes D_{i/\mathcal{Q}}(z)$$

$$D_{i/\mathcal{Q}}(z)=\sum_n d_n(z)\langle\mathcal{O}_n^{\mathcal{Q}}\rangle$$

Bodwin, Braaten, Lepage [hep-ph/9407339] Braaten, Chen [hep-ph/9604237] Braaten, Fleming [hep-ph/9411365]

Application to Quarkonium production

$$D_{i/\mathcal{Q}}(z) = \sum_n d_n(z) \langle \mathcal{O}_n^{\mathcal{Q}} \rangle$$

Extracted from data. Estimated size in the relative velocity scaling

Calculable in PT

For charmonium states: $\alpha_S(2m_c) \sim v^2 \sim 0.25$

Leading contributions from gluon fragmentation

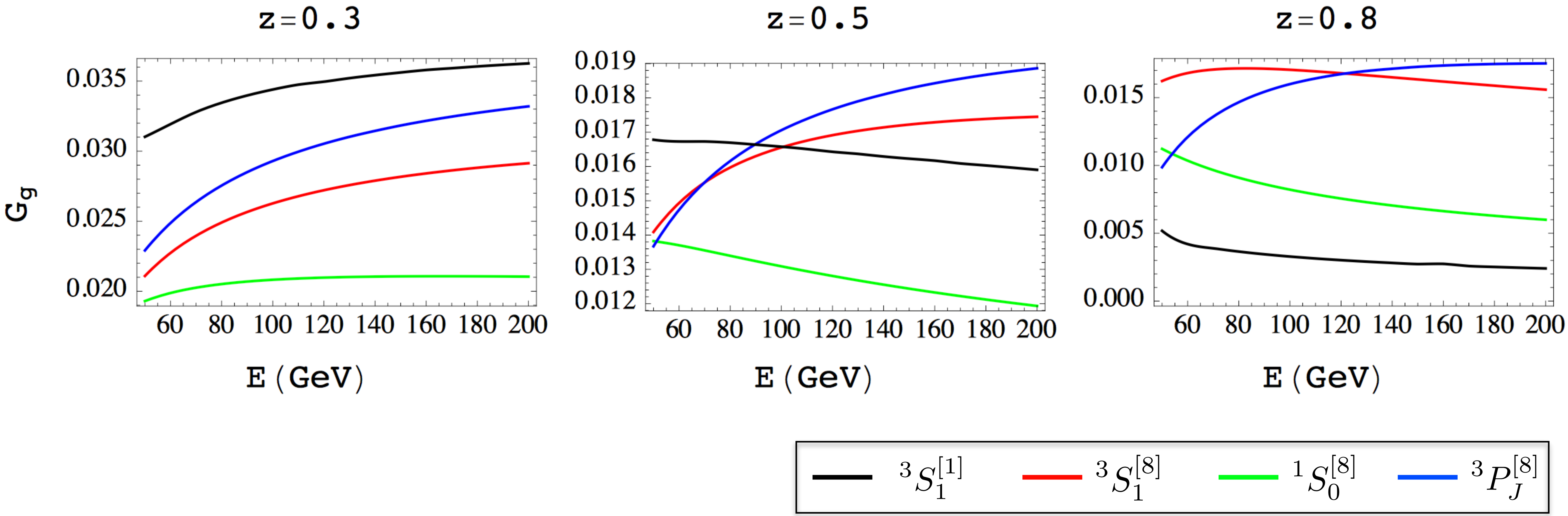
J/ψ:

$\langle \mathcal{O}(^3S_1^{(1)}) \rangle \sim v^3,$	$d(^3S_1^{(1)}) \sim \alpha_s^3,$	$\longrightarrow$	$^3S_1^{(1)} :\sim \alpha_s^3 v^3$
$\langle \mathcal{O}(^3S_1^{(8)}) \rangle \sim v^7,$	$d(^3S_1^{(8)}) \sim \alpha_s,$	$\longrightarrow$	$^3S_1^{(8)} :\sim \alpha_s v^7$
$\langle \mathcal{O}(^1S_0^{(8)}) \rangle \sim v^7,$	$d(^1S_0^{(8)}) \sim \alpha_s^2,$	$\longrightarrow$	$^1S_0^{(8)} :\sim \alpha_s^2 v^7$
$\langle \mathcal{O}(^3P_J^{(8)}) \rangle \sim v^7,$	$d(^3P_J^{(8)}) \sim \alpha_s^2,$	$\longrightarrow$	$^3P_J^{(8)} :\sim \alpha_s^2 v^7$

Can FJFs help discriminate between these production mechanisms?

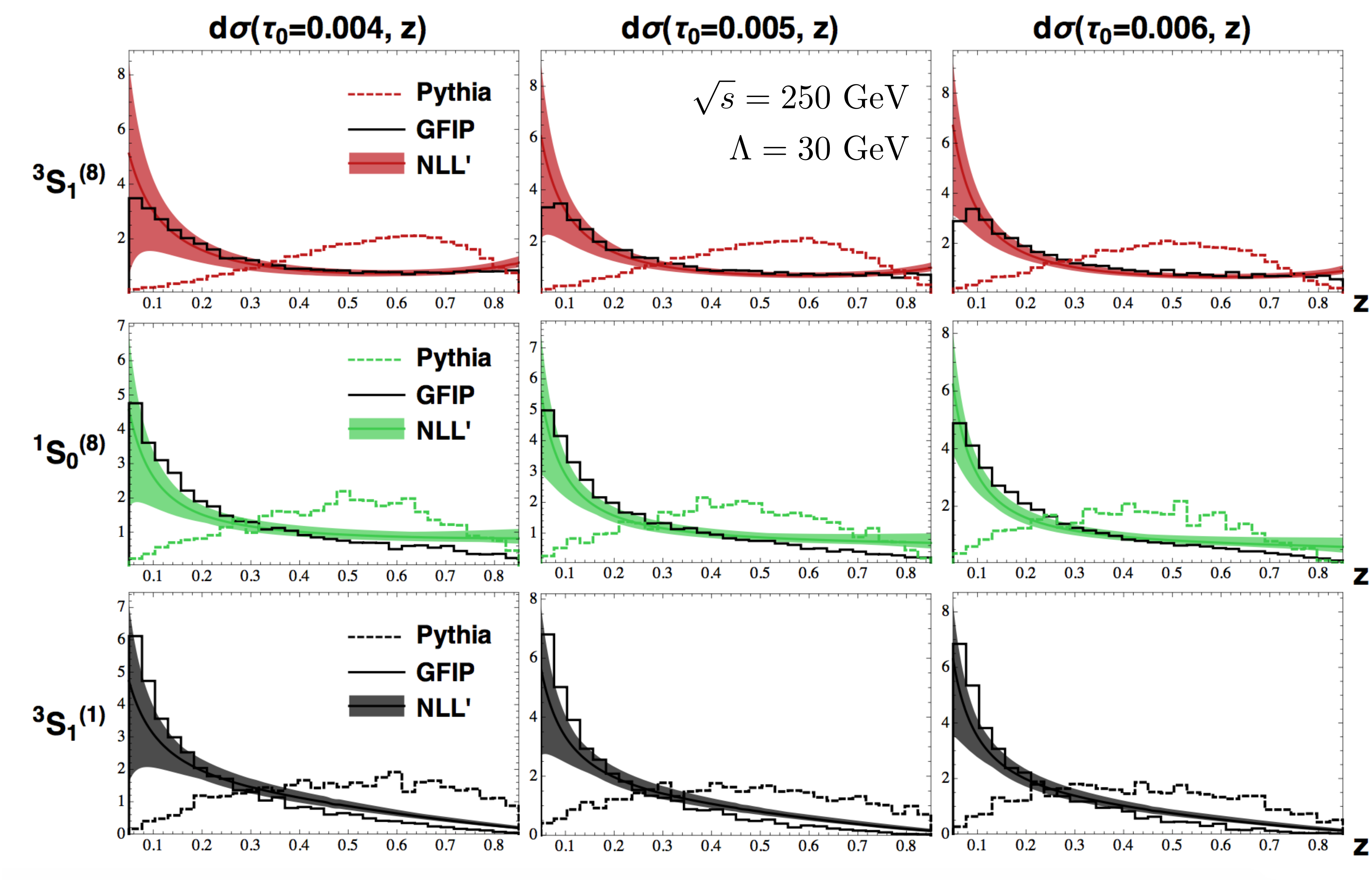
Fragmenting Jet Functions

Cone Jets: $R = 0.4$ Gluon fragmentation: $\mu = 2E\tan(R/2)$



Matthew Baumgart, Adam K. Leibovich, Thomas Mehen, Ira Z. Rothstein [arXiv:1406.2295]

Application to Quarkonium production



Bain, Dai, Hornig, Leibovich, YM, and Mehen [arXiv: arXiv:1603.06981]

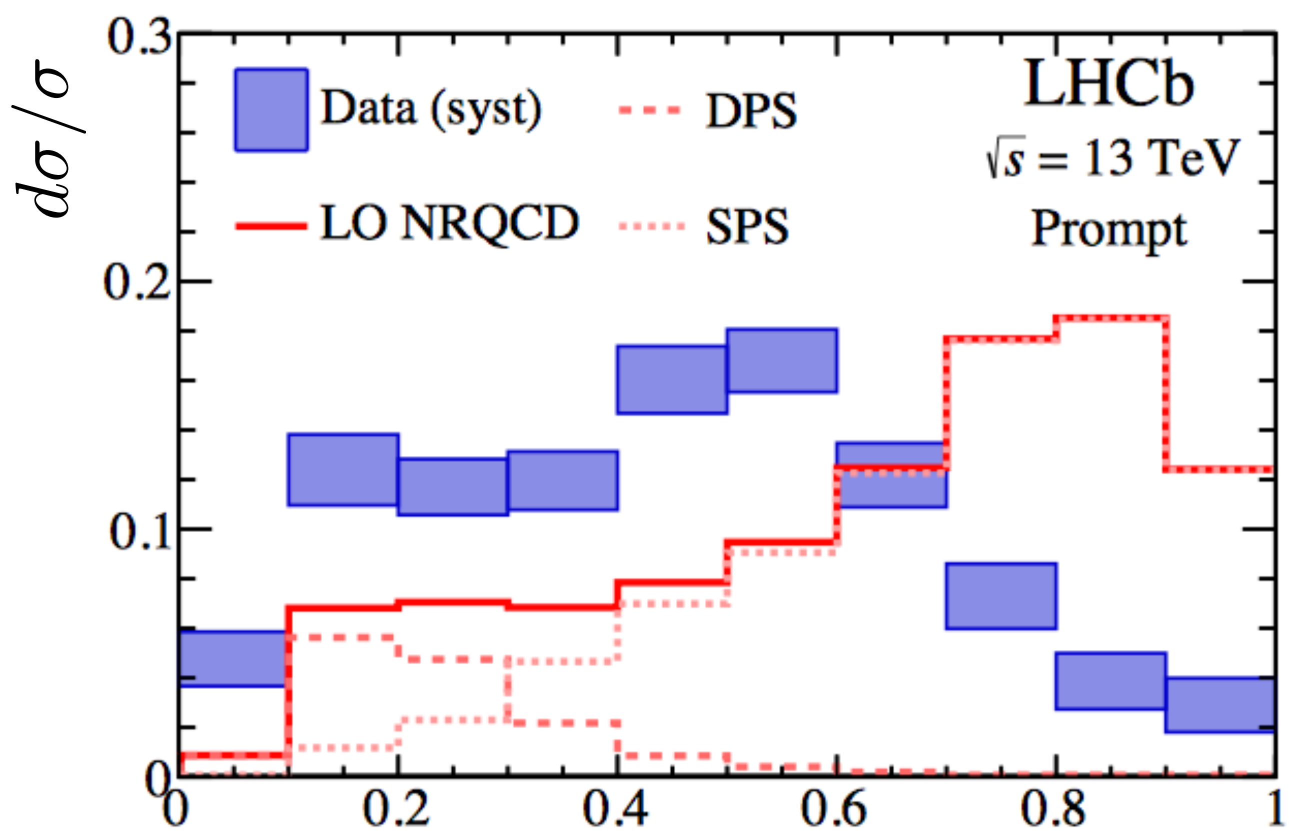
Quarkonium production in jets at LHC

Inclusive jet production (LHCb)

$$\frac{d\sigma}{dz}(pp \to \text{jet}(J/\psi) + X) \sim \sum_{i=c,\bar{c},g} \int dE \frac{d\hat{\sigma}^{(i)\text{LO}}}{dE}(E) \times \mathcal{G}_{i\to J/\psi}(z, \mu = 2E\tan(R/2))$$

Discretize and evaluate using monte-carlo integration (MadGraph)

LHCb Collaboration [arXiv:1701.05116]



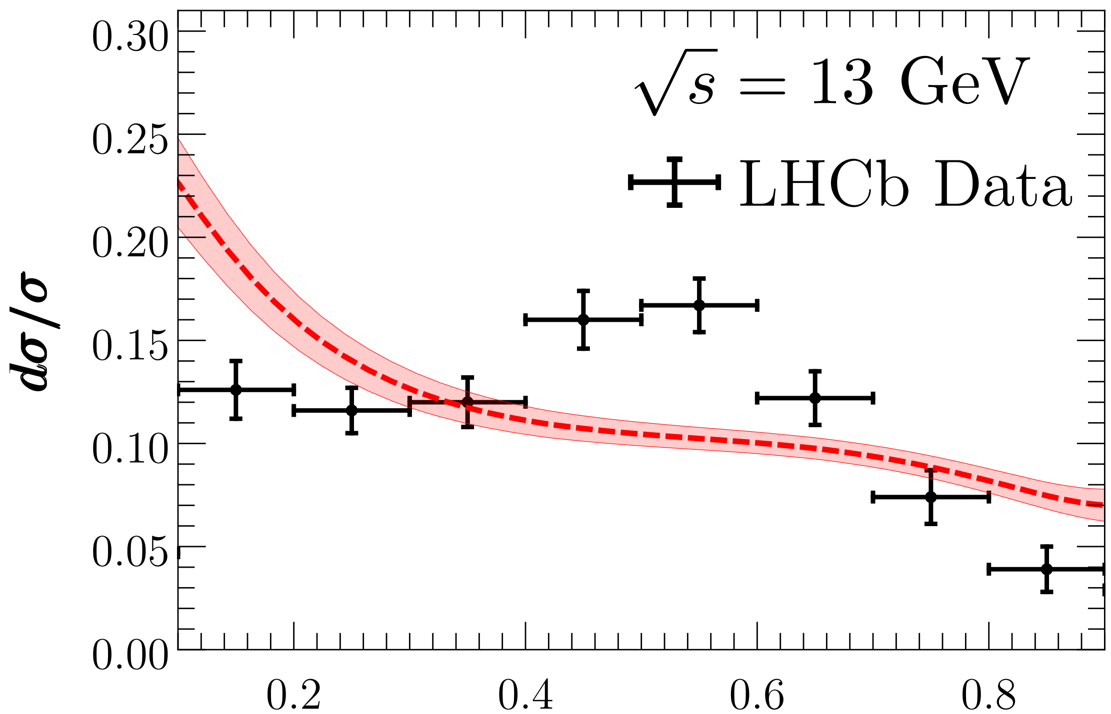
$$z(J/\psi) = \frac{E_{J/\psi}}{E_J}$$

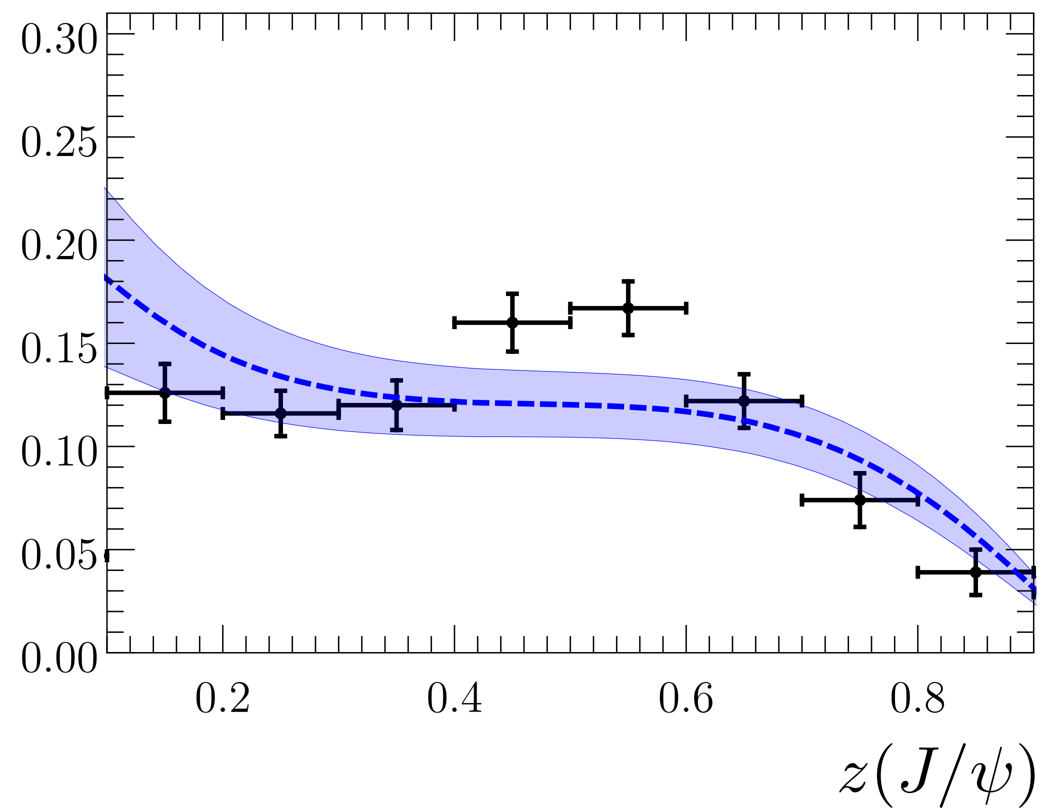
Experimental cuts:

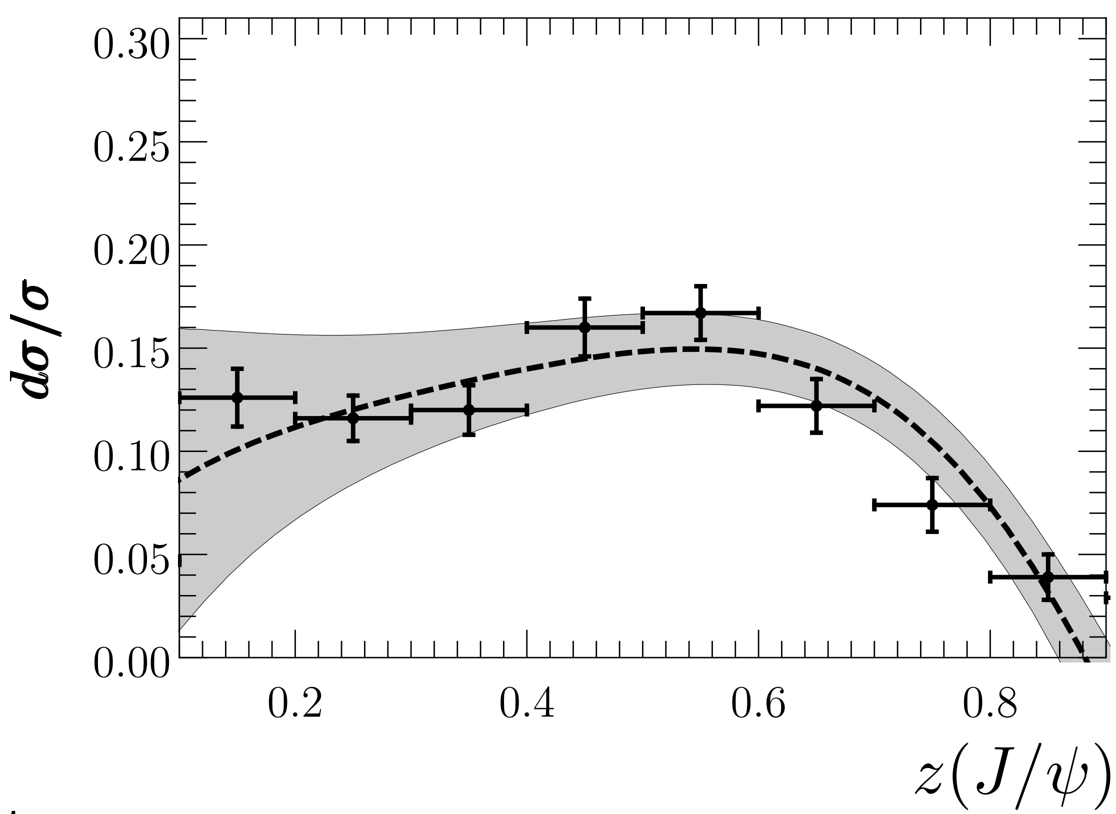
$$p^{\perp}_{\text{jet}} > 20 \text{ GeV}$$
$$E_\mu > 5 \text{ GeV}$$
$$2.5 < \eta_{J\psi} < 4.5$$
$$2.0 < \eta_{jet} < 4.0$$
$$2.5 < \eta_\mu < 4.5$$

Quarkonium production in jets at LHC

Bain, Dai, Leibovich,YM, and Mehen [arXiv:1702.05525]







NLO global fits to world's data

. Butenschoen and Kniehl, [arXiv:1201.1872].

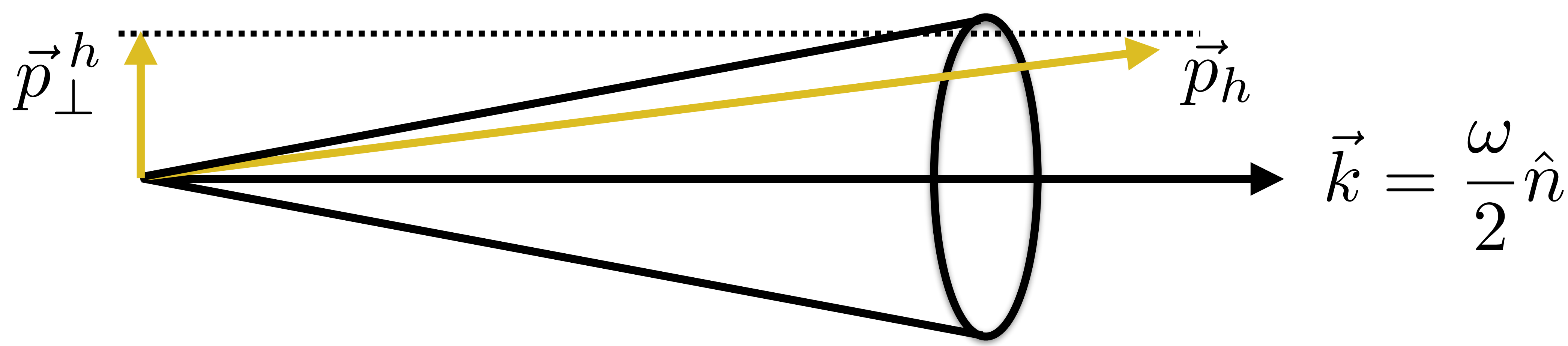
LP+NLO high pT fits (no singlet)

Bodwin, Chung, Kim, Lee: [arXiv:1403.3612]

NLO simultaneous fit of polarization and pT at high pT

.Chao, Ma, Shao, Wang, and Zhang, [arXiv:1201.2675].

TMD Fragmenting Jet Function



$J^{(i)}_\omega(\mu) \to \mathcal{G}_{i/h}(\mathbf{p}_h, z, \mu)$

Cone and kT-type jet algorithms

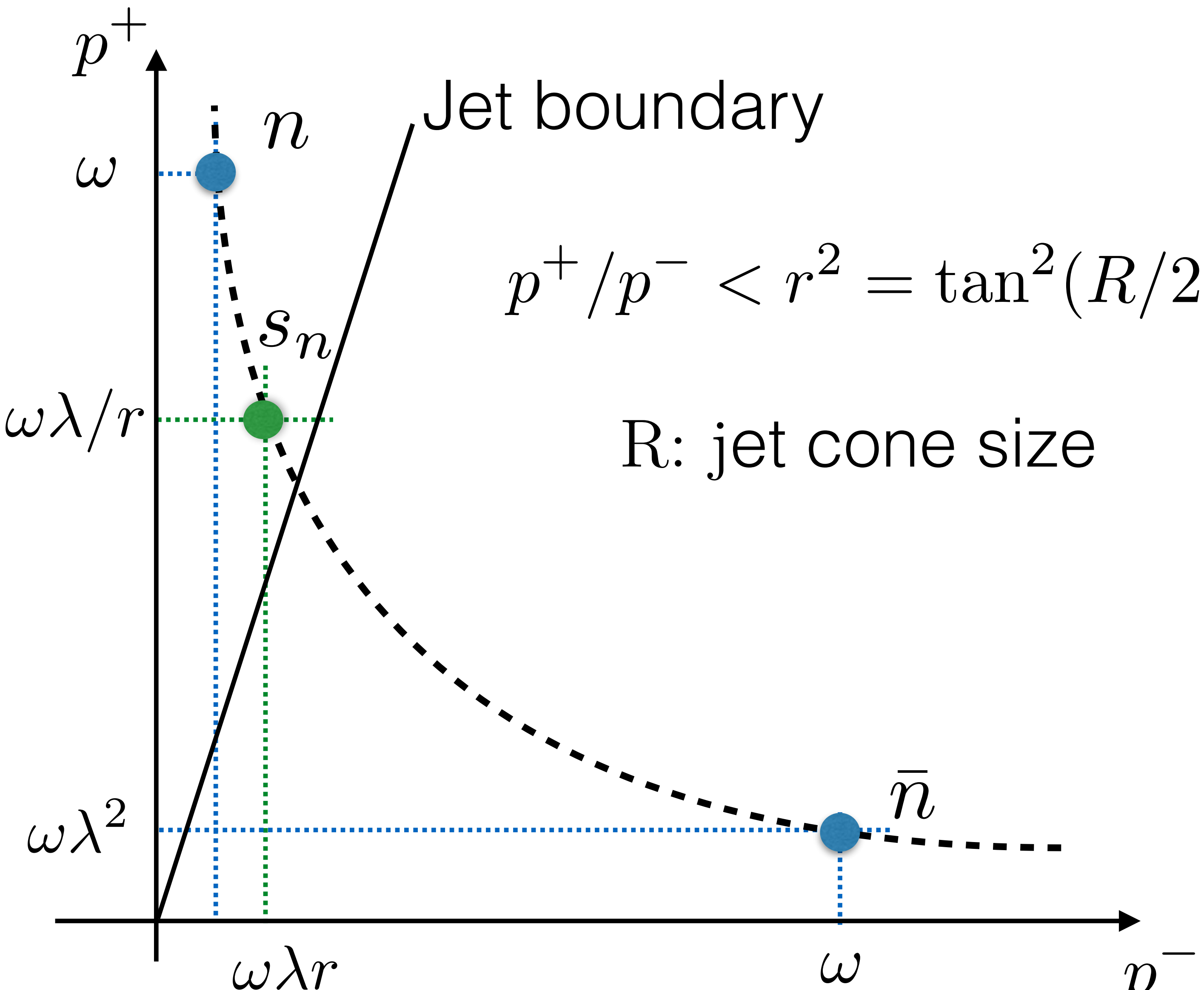
$$\mathcal{G}_{q/h}(\mathbf{p}_\perp, z, \mu) = \frac{1}{z}\sum_X \frac{1}{2N_c}\delta(p^-_{Xh;r})\delta^{(2)}(\mathbf{p}_\perp + \mathbf{p}_\perp^X)\,\mathrm{Tr}\left[\frac{\bar{\not{n}}}{2}\langle 0|\delta_{\omega,\overline{\mathcal{P}}}\chi_n^{(0)}(0)|Xh\rangle\langle Xh|\bar{\chi}_n^{(0)}(0)|0\rangle\right]$$

D. Neill, I. Scimemi, and W. Waalewijn [arXiv: 1612.04817] Transverse momentum measured with respect to a recoil-free axis.

Semi-Inclusive: Talk by F. Ringer

Transverse momentum measured with respect to the jet axis

Transverse Momentum Dependent FJF



$$p^+/p^- < r^2 = \tan^2(R/2)$$

R: jet cone size

$$\mathcal{G}_{i/h}(\vec{p}_\perp^{\,h}, z, \mu)$$

$$p_c \sim \omega(\lambda^2, 1, \lambda)$$

$$p_{cs} \sim p_h^\perp(r, 1/r, 1)$$

$$\lambda = p_h^\perp/\omega$$

Factorization

$$\mathcal{G}_{q/h}(\mathbf{p}_\perp, z, \mu) = H_+(\mu) \times \left[\mathcal{D}_{q/h} \otimes_\perp S_C\right](\mathbf{p}_\perp, z, \mu)$$

Matching + Normalization

$$H_+(\mu) = (2\pi)^2 N_c\, C_+^\dagger(\mu) C_+(\mu)$$

Collinear splittings within Jet + Fragmentation

$$\mathcal{D}_{i/h}(\mathbf{p}_\perp, z, \mu, \nu) = \int_z^1 \frac{dx}{x} \mathcal{J}_{i/j}(\mathbf{p}_\perp, x, \mu, \nu) D_{j/h}\left(\frac{z}{x}, \mu\right) \; + \; \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{|\mathbf{p}_\perp|^2}\right)$$

Collinear-Soft radiation + Jet boundary sensitivity

$$S_C(\mathbf{p}_\perp^S) \equiv \frac{1}{N_c} \sum_{X_{cs}} \mathrm{Tr}\left[\langle 0 | V_n^\dagger(0) U_n(0) \delta^{(2)}(\mathcal{P}_\perp + \mathbf{p}_\perp^S) | X_{cs} \rangle \langle X_{cs} | U_n^\dagger(0) V_n(0) | 0 \rangle\right]$$

Rapidity divergences/regulator

J.-Y. Chiu, A. Jain, D. Neill and I. Z. Rothstein [arXiv:1202.0814]

Rapidity Divergences:

See also: Talk by V. Vaidya

$$\mathcal{D}_{i/j}(\mathbf{p}_\perp, z, \mu, \nu_C)$$

$$W_n = \sum_{\text{perms}} \exp\left(-\frac{g\, w^2}{\bar{n}\cdot\mathcal{P}} \frac{|\bar{n}\cdot\mathcal{P}_g|^{-\eta}}{\nu^{-\eta}} \bar{n}\cdot A_n\right)$$

$$S_C^i(\mathbf{p}_\perp, \mu, \nu_S)$$

$$V_n = \sum_{\text{perms}} \exp\left(-\frac{g\, w}{\bar{n}\cdot\mathcal{P}} \frac{|\bar{n}\cdot\mathcal{P}_g|^{-\eta/2}}{\nu^{-\eta/2}} \bar{n}\cdot A_{n,cs}\right)$$

Rapidity divergences/regulator

Collinear splittings within Jet

$$\mathcal{D}_{i/j}(\mathbf{p}_\perp, z, \mu, \nu) = \delta_{ij}\delta(1-z)\delta^{(2)}(\mathbf{p}_\perp) + \frac{\alpha_s w^2 T_{ij}}{\pi}\Big\{\Big[-\frac{2}{\eta}\Big(\mathcal{L}_0(\mathbf{p}_\perp^2,\mu^2) - \frac{1}{2\epsilon}\delta^{(2)}(\mathbf{p}_\perp)\Big) + \frac{1}{2\epsilon}\Big(\ln\Big(\frac{\nu^2}{\omega^2}\Big) + \bar{\gamma}_i\Big)\Big]\delta_{ij}\delta(1-z) - \frac{1}{2\epsilon}P_{ij}(z) + \Big(\delta_{ij}\delta(1-z)\ln\Big(\frac{\omega^2}{\nu^2}\Big) + \bar{P}_{ij}(z)\Big)\mathcal{L}_0(\mathbf{p}_\perp^2,\mu^2) + c_{ij}(z)\delta^{(2)}(\mathbf{p}_\perp)\Big\}$$

$\nu_C = \omega$

Collinear-Soft radiation + Jet boundary sensitivity

$$S_C^{i,B}(\mathbf{p}_\perp,\mu,\nu) = \delta^{(2)}(\mathbf{p}_\perp) + \frac{\alpha_s w^2 C_i}{\pi}\Big\{\frac{2}{\eta}\Big(-\frac{1}{2\epsilon}\delta^{(2)}(\mathbf{p}_\perp) + \mathcal{L}_0(\mathbf{p}_\perp^2,\mu^2)\Big) + \delta^{(2)}(\mathbf{p}_\perp)\Big(\frac{1}{2\epsilon^2} + \frac{1}{2\epsilon}\ln\Big(\frac{\mu^2}{r^2\nu^2}\Big)\Big) - \mathcal{L}_0(\mathbf{p}_\perp^2,\mu^2)\ln\Big(\frac{\mu^2}{r^2\nu^2}\Big) + \mathcal{L}_1(\mathbf{p}_\perp^2,\mu^2) - \frac{\pi^2}{24}\delta^{(2)}(\mathbf{p}_\perp)\Big\}$$

$\nu_S = \mu/r$

Rapidity divergences/regulator

Rapidity divergences and scale cancel at the fixed order result:

$$\ln\left(\frac{\omega}{\nu}\right) - \ln\left(\frac{\mu}{\nu r}\right)$$

$$\mathcal{D}_{i/j} \otimes S^i_C(\mathbf{p}_\perp)|_{\nu_S=\nu_C=\nu} = \delta_{i,j}\delta(\mathbf{p}_\perp) + \frac{\alpha_s C_i}{\pi}\left\{2\ln\left(\frac{r\omega}{\mu}\right)\mathcal{L}_0(\mathbf{p}_\perp^2, \mu^2) + ...\right\}$$

Rapidity Renormalization Group Evolution

RGE:

$$\frac{d}{d\ln\nu}\tilde{F}(b,\mu,\nu)=\tilde{\gamma}_{\nu}^{F}(b,\mu,\nu)\tilde{F}(b,\mu,\nu)$$

Anomalous Dimension:

$$\tilde{\gamma}_{\nu}^{F}(b,\mu,\nu)=-\frac{\Gamma_{\nu}^{F}[\alpha_s]}{(2\pi)^2}\ln\left(\frac{\mu}{\mu_C(b)}\right)+\frac{\gamma_{\nu}^{F}[\alpha_s]}{(2\pi)^2}$$

Evolution:

$$\tilde{F}(b,\mu,\nu)=\tilde{F}(b,\mu,\nu_0)\mathcal{V}_F(b,\mu,\nu,\nu_0)$$

$$\mathcal{V}_F(b,\mu,\nu,\nu_0)=\exp\left[G_F(\mu,\nu,\nu_0)\right]\left(\frac{\mu}{\mu_C}\right)^{\eta_F(\mu,\nu,\nu_0)}$$

$$\eta_F(\mu,\nu,\nu_0)=-\frac{\Gamma_{\nu}^{F}[\alpha_s]}{(2\pi)^2}\ln\left(\frac{\nu}{\nu_0}\right)$$

$$G_F(\mu,\nu,\nu_0)=\frac{\gamma_{\nu}^{F}[\alpha_s]}{(2\pi)^2}\ln\left(\frac{\nu}{\nu_0}\right)$$

Anomalous dimensions

Renormalization Group (RG)

$$\gamma_\mu^{S_C}(\nu) = \frac{\alpha_s C_i}{\pi} \ln\left(\frac{\mu^2}{r^2\nu^2}\right)$$

$$\gamma_\mu^{\mathcal{D}}(\nu) = \frac{\alpha_s C_i}{\pi}\left(\ln\left(\frac{\nu^2}{\omega^2}\right) + \bar{\gamma}_i\right)$$

$$\gamma_\mu^{\mathcal{D}}(\nu) + \gamma_\mu^{S_C}(\nu) = \gamma_\mu^J = \frac{\alpha_s C_i}{\pi}\left(\ln\left(\frac{\mu^2}{r^2\omega^2}\right) + \bar{\gamma}_i\right)$$

TMDFJF evolves as Jet

Rapidity Renormalization Group (RRG)

$$\gamma_\nu^{S_C}(p_\perp, \mu) = +(8\pi)\alpha_s C_i \, \mathcal{L}_0(\mathbf{p}_\perp, \mu^2)$$

$$\gamma_\nu^{\mathcal{D}}(p_\perp, \mu) = -(8\pi)\alpha_s C_i \, \mathcal{L}_0(\mathbf{p}_\perp, \mu^2)$$

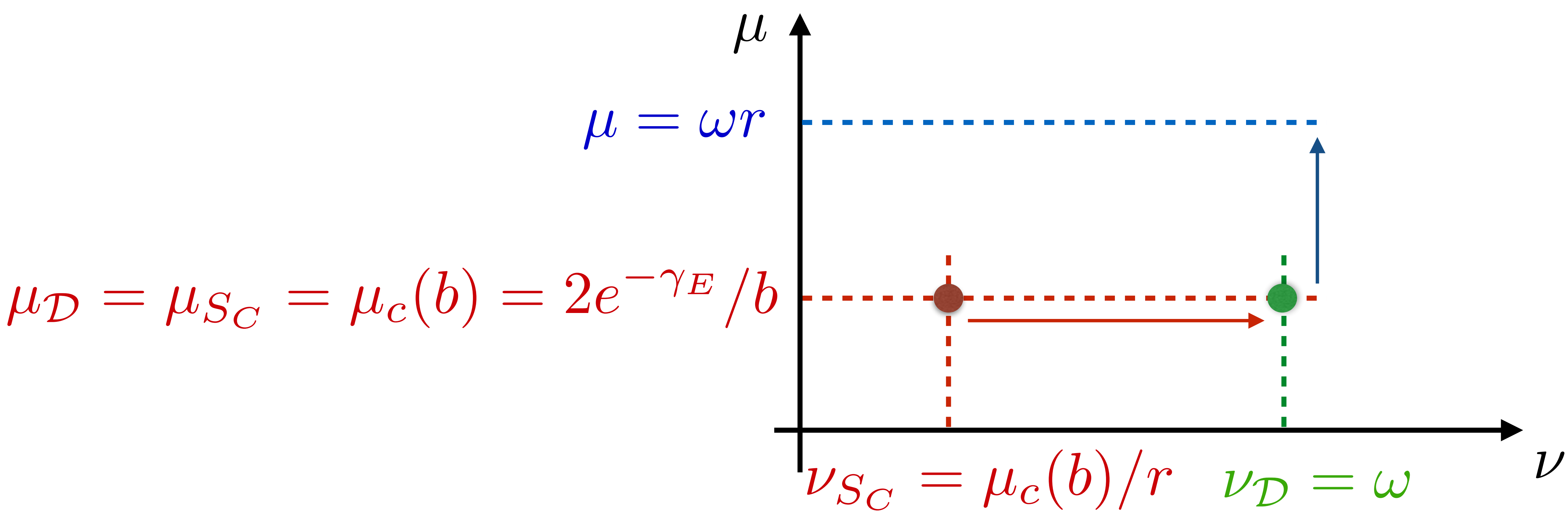
$$\gamma_\nu^{\mathcal{D}}(\mathbf{p}_\perp, \mu) + \gamma_\nu^{S}(\mathbf{p}_\perp, \mu) = 0$$

From cancelation of rapidity divergences

Function (F)	Γ_ν^F	γ_ν^F	Γ_F^0	γ_F^0
$\mathcal{D}_{i/h}$	$-(8\pi)\alpha_s C_i + \mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^2)$	0	$4C_i(\ln(\nu^2/\omega^2) + \bar{\gamma}_i)$
S_C^i	$(8\pi)\alpha_s C_i + \mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^2)$	$4C_i$	0

Renormalization Group Evolution

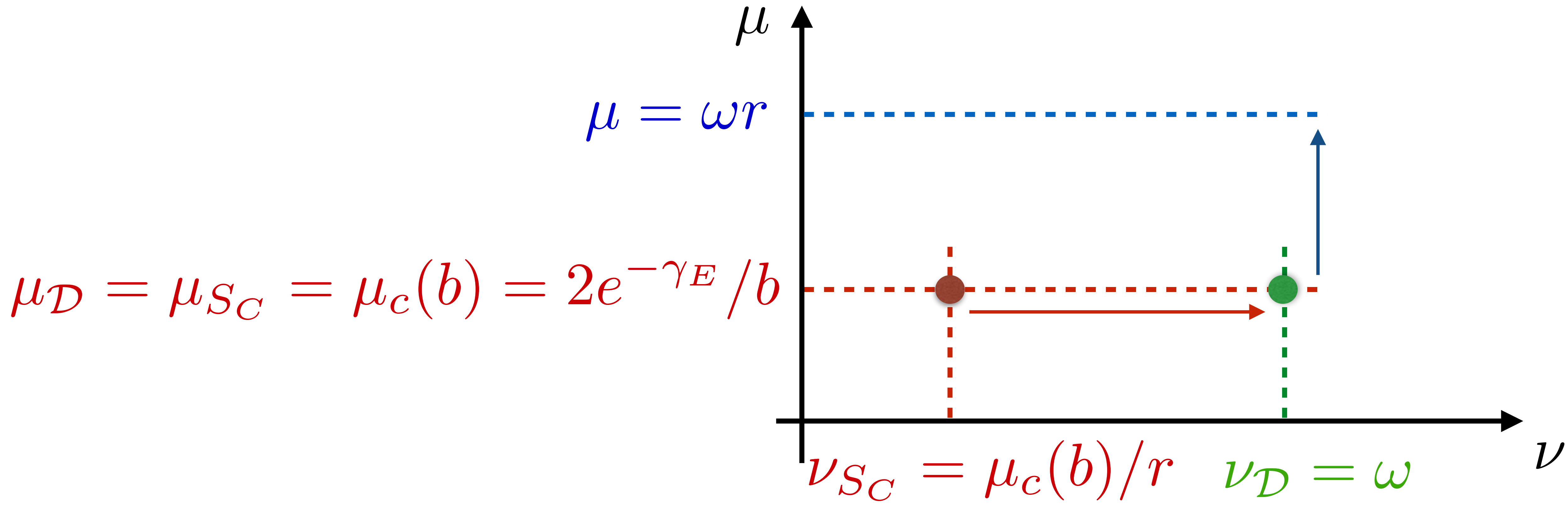
- Use of RRG evolution for improved PT at NLL: $\ln\left(\frac{r\omega}{\mu}\right)\Big|_{\mu\sim p_\perp}$
- Two dimensional Evolution: (μ,ν)
- Canonical scales in Fourier space:



Evolution in Fourier space

$$\mathcal{G}_{i/h}(p_\perp, z) = (2\pi)^2 p_\perp \int_0^\infty db\, b\, J_0(p_\perp b) \mathcal{U}_{S_C}(\mu, \mu_{S_C}, m_{S_C}) \mathcal{U}_{\mathcal{D}}(\mu, \mu_{\mathcal{D}}, 1)$$

$$\mathcal{V}(\mu_{S_C}, \nu_{\mathcal{D}}, \nu_{S_C}) \mathcal{FT}[\mathcal{D}_{i/h}(z, \nu_{\mathcal{D}}) \otimes_\perp S_C^i(\nu_S)]$$



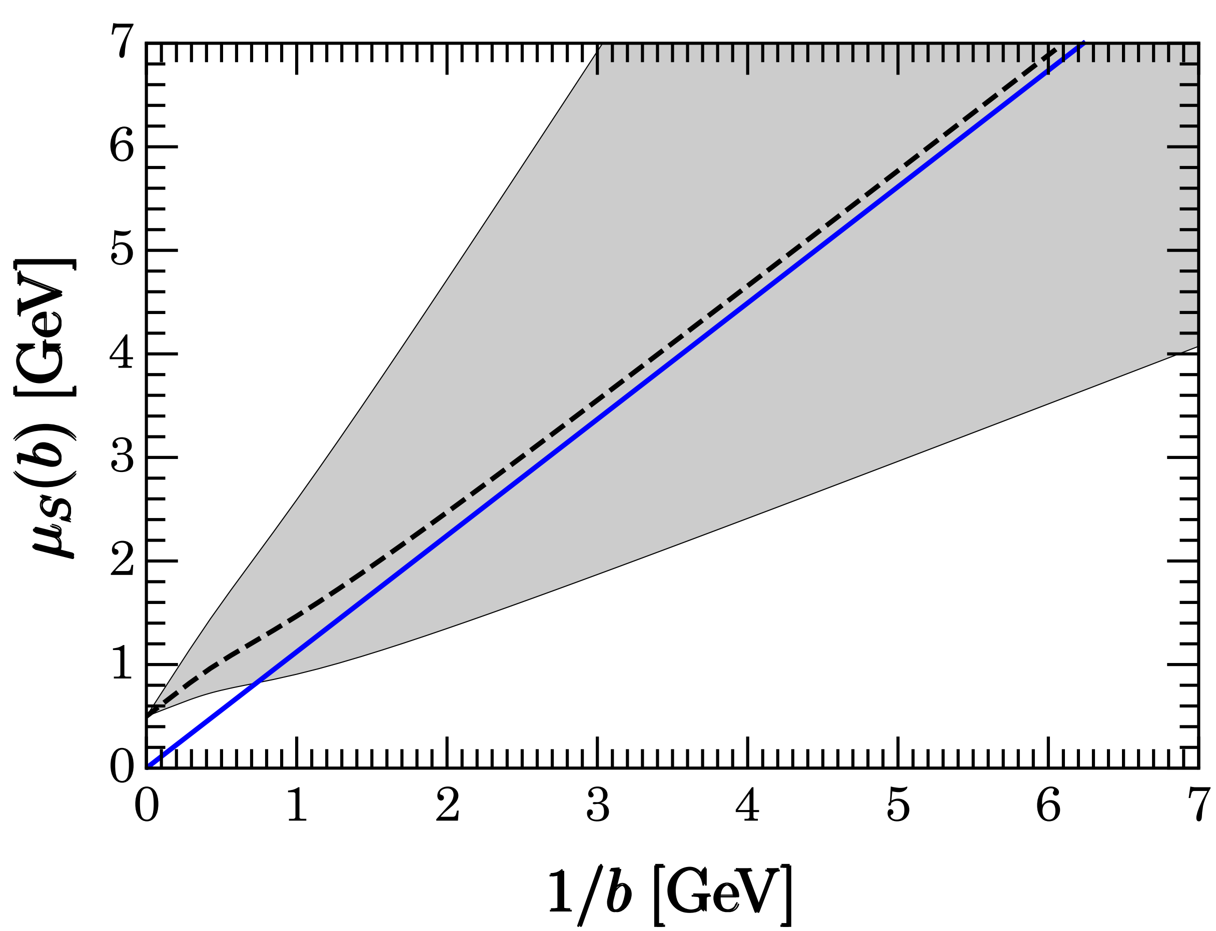
Evolution in Fourier space

$$\mathcal{G}_{i/h}(p_\perp, z) = (2\pi)^2 p_\perp \int_0^\infty db\, b\, J_0(p_\perp b) \mathcal{U}_{S_C}(\mu, \mu_{S_C}, m_{S_C}) \mathcal{U}_{\mathcal{D}}(\mu, \mu_{\mathcal{D}}, 1)$$

$$\mathcal{V}(\mu_{S_C}, \nu_{\mathcal{D}}, \nu_{S_C}) \mathcal{FT}[\mathcal{D}_{i/h}(z, \nu_{\mathcal{D}}) \otimes_\perp S_C^i(\nu_S)]$$

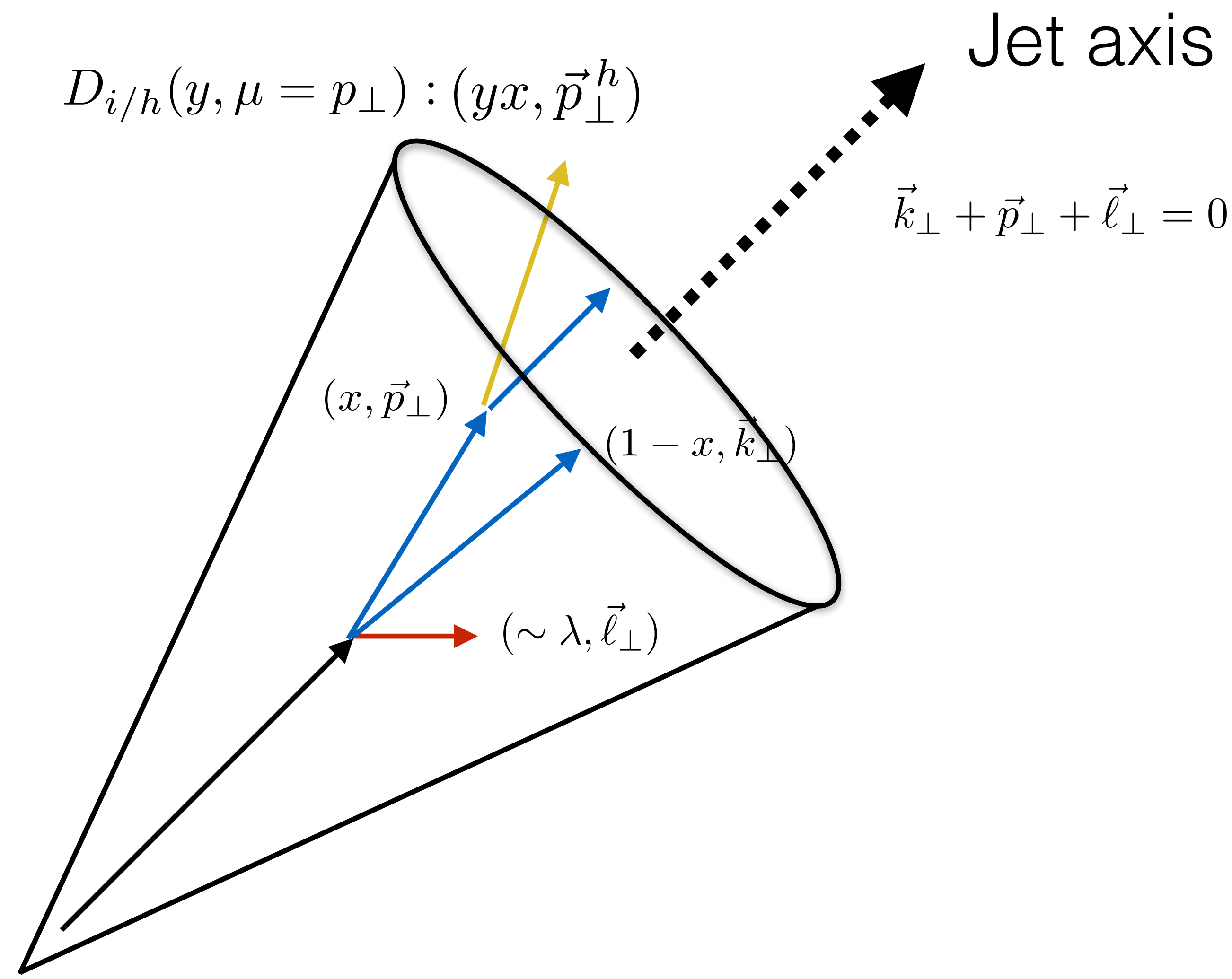
$$\mu_C^{\rm pf}(b) = \frac{2\exp(-\gamma_E)}{b} + \frac{1}{4}(1 - \tanh(0.6 - b))$$

The final result is independent of the profile form in the large b limit.



Fragmentation and matching onto FF

$$\mathcal{D}_{i/h}(\mathbf{p}_\perp, z) = \sum_j \int_z^1 \frac{dx}{x} \mathcal{J}_{i/j}(\mathbf{p}_\perp, x) D_{j/h}(z/x)$$

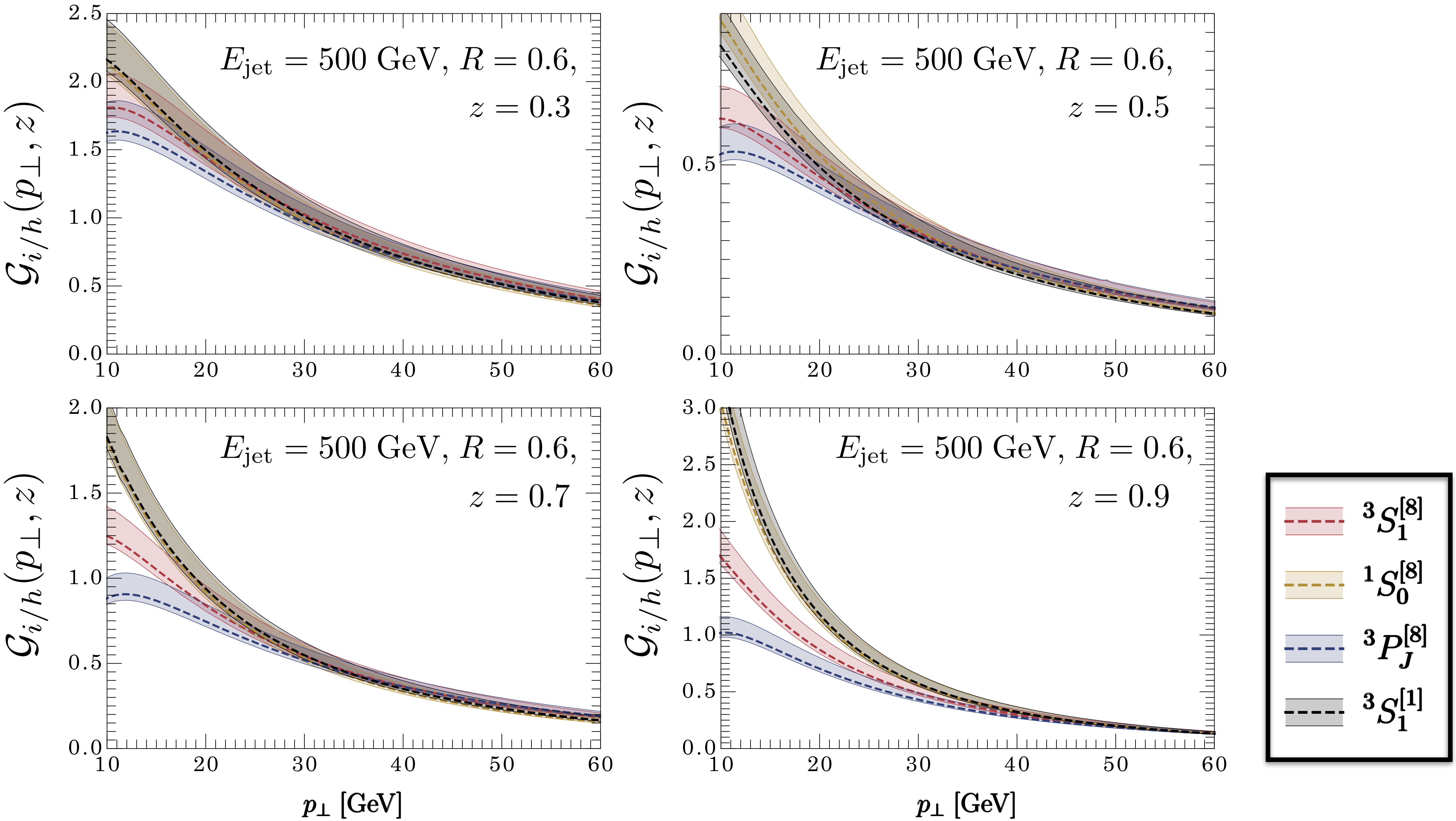


$$\mathcal{J}^{(0)}_{i/j}(\mathbf{p}_\perp, z) = \delta_{ij}\delta(1-x)\delta^{(2)}(\mathbf{p}_\perp)$$

$$\mathcal{J}^{(1)}_{i/j}(\mathbf{p}_\perp, z) = \mathcal{D}^{(1)}_{i/j}(\mathbf{p}_\perp, z) - D^{(1)}_{i/j}(z)\delta^{(2)}(\mathbf{p}_\perp)$$

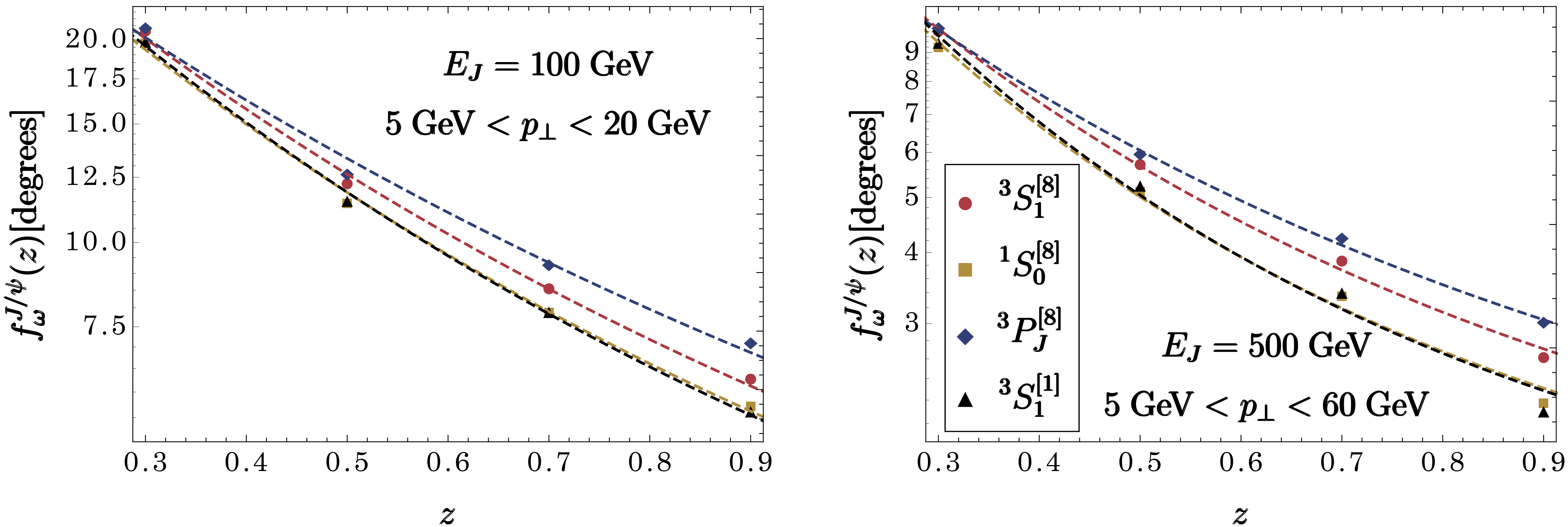
$$= \frac{\alpha_s T_{ij}}{\pi}\left\{\left(\delta_{ij}\delta(1-z)\ln\left(\frac{\omega}{\nu}\right) + \bar{P}_{ij}(z)\right)\mathcal{L}_0(\mathbf{p}_\perp^2, \mu^2) + c_{ij}(z)\delta^{(2)}(\mathbf{p}_\perp)\right\}$$

Application to Quarkonium production



Application to Quarkonium production

$$\langle\theta\rangle(z) \sim \frac{2\int dp_\perp\, p_\perp D_{g/h}(p_\perp, z, \mu)}{z\omega \int dp_\perp\, D_{g/h}(p_\perp, z, \mu)} \equiv f^h_\omega(z)$$



Summary

- Definition of Transverse Momentum Dependent FJF
- Applications to quarkonium production

  $J/\psi \longrightarrow$ Discriminating power for LO contributions

- NGLs considerations
- $z \to 1$ effects

Backup Slides

Angularities

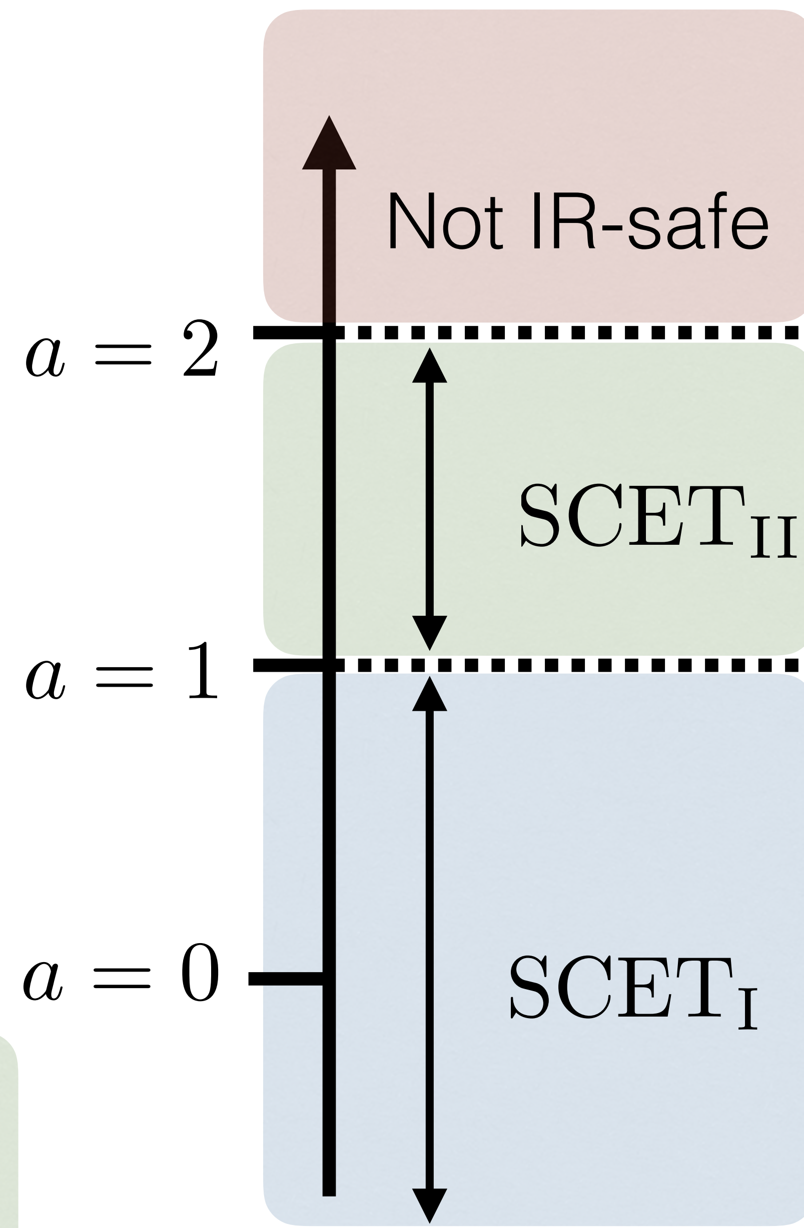
$$\tau_a = \frac{1}{\omega} \sum_{i \in \text{Jet}} (p_i^+)^{1-a/2} (p_i^-)^{a/2}$$

$$\omega = \sum_{i \in \text{Jet}} p_i^- \simeq 2E_{\text{Jet}}$$

$\tau_2 = 1$

$\tau_1 = B = \frac{1}{\omega} \sum_{i \in \text{Jet}} |\vec{p}_i^{\,\perp}|$ (Jet Broadening)

$\tau_0 = \tau = s/\omega^2 = m_J^2/\omega^2$ (Jet Mass)

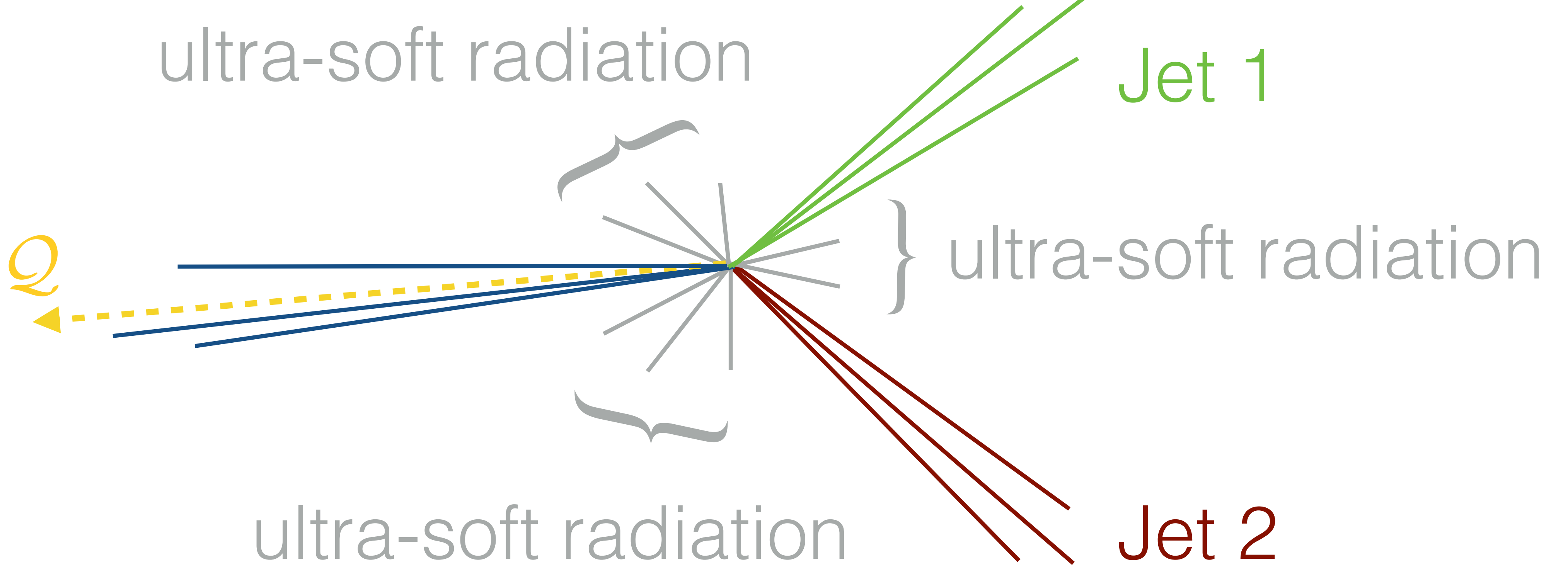


Application to Quarkonium production

$$e^+ + e^- \to 3\,\text{jets}(g \to J/\psi)$$

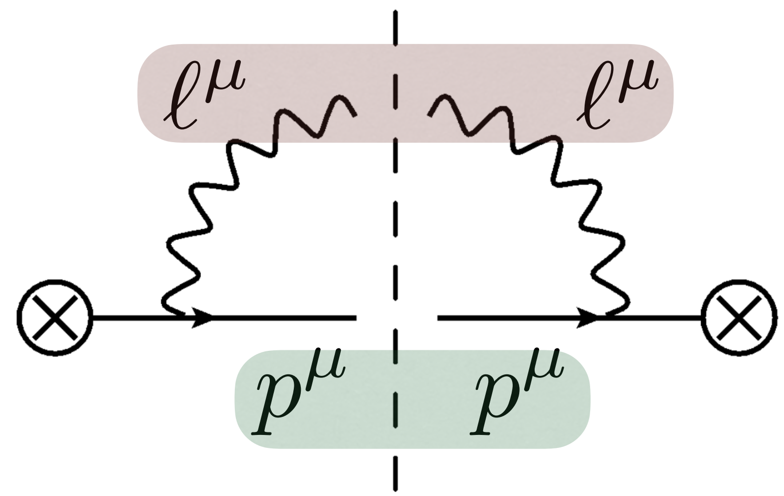
$$\frac{d\sigma^h}{dz d\tau_a} = \sum_n d_n(z, \tau_a) \langle \mathcal{O}_n^h \rangle$$

$$d_n(z,\tau_a) = \sigma_0 \times H_2(\mu) \times S^{\text{un}}(\mu) \times J_1(\omega_1,\mu) \times J_2(\omega_2,\mu) \left(S^{\text{ms}}(\mu) \otimes \frac{\mathcal{J}_{i/g}(z)}{2(2\pi)^3} \bullet d_n(z) \right)$$



FJFs with angularities

Bain, Dai, Hornig, Leibovich,YM, and Mehen [arXiv:1603.06981]



$$\mathcal{G}_i^h(\tau_a, z, \mu) = \int \frac{dk^+ dp_h^+}{2\pi} \int d^4y \, e^{-ik^+y^-/2}$$

$$\times \sum_X \frac{1}{4N_C} \mathrm{tr}\Big[\frac{\bar{n}\!\!\!/}{2} \langle 0|\chi_{n,\omega}(y)\delta(\tau_a - \hat{\tau}_a)|Xh\rangle\langle Xh|\bar{\chi}_{n,\omega}(0)|0\rangle\Big]$$

NLO

$$\delta(\tau_a - [(\ell^+)^{1-a/2}(\ell^-)^{a/2} + (p^+)^{1-a/2}(p^-)^{a/2}]/\omega)$$

Tests against MC

<u>What jet observable?</u>

Possible to calculate analytically.

Described by MC packages.

↓

Angularities

<u>Which hadron?</u>

Fragmentation Function.

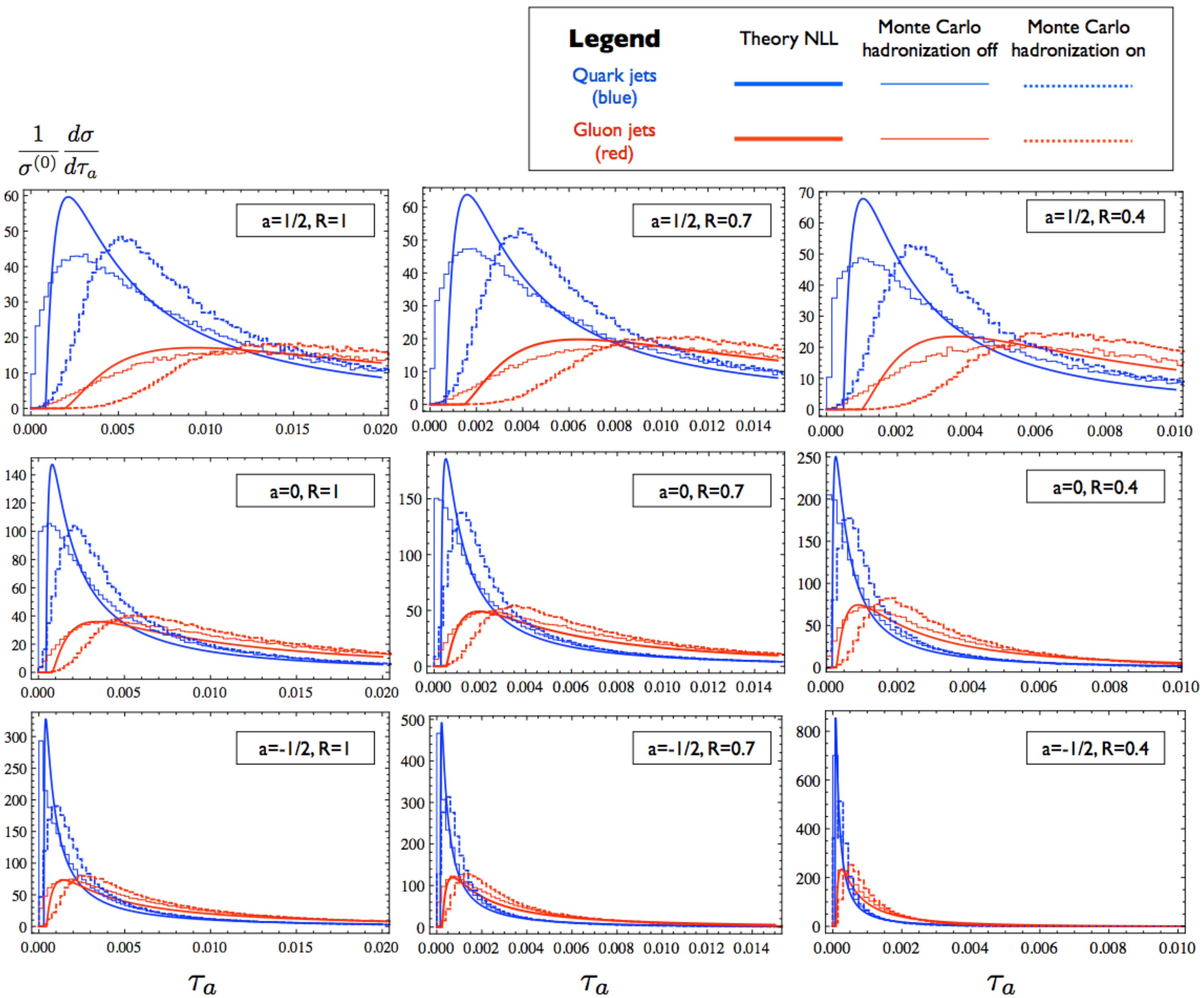
Described by MC packages.

↓

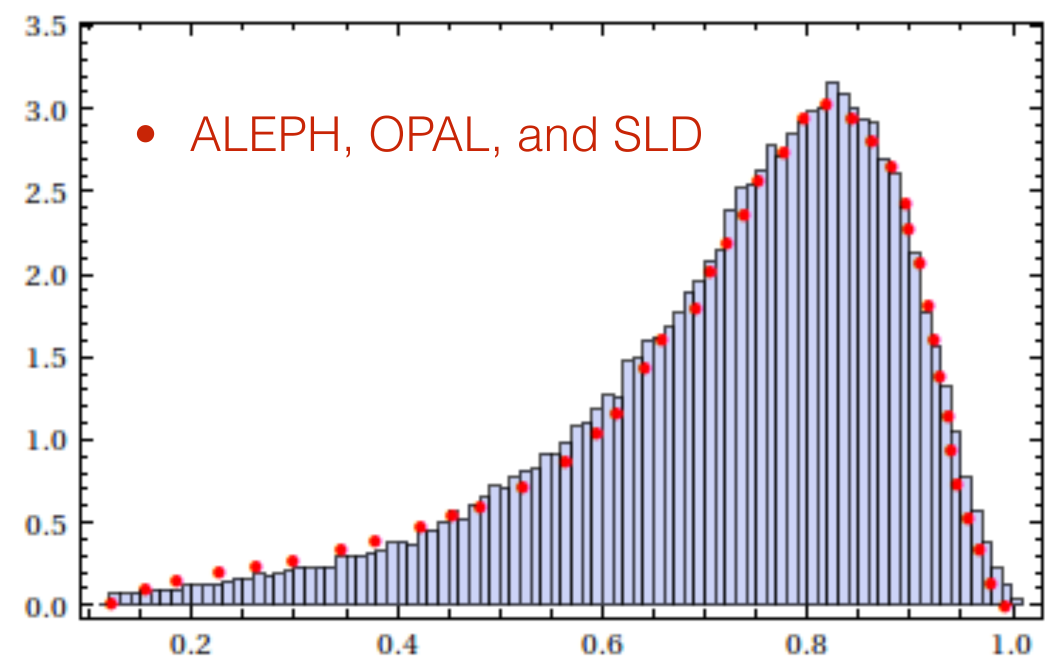
B-mesons

Tests against MC

Angularities cross section (PYTHIA+MG vs Theory [arXiv:1001.0014])



B messons (inclusive PYTHIA+MG vs LEP)



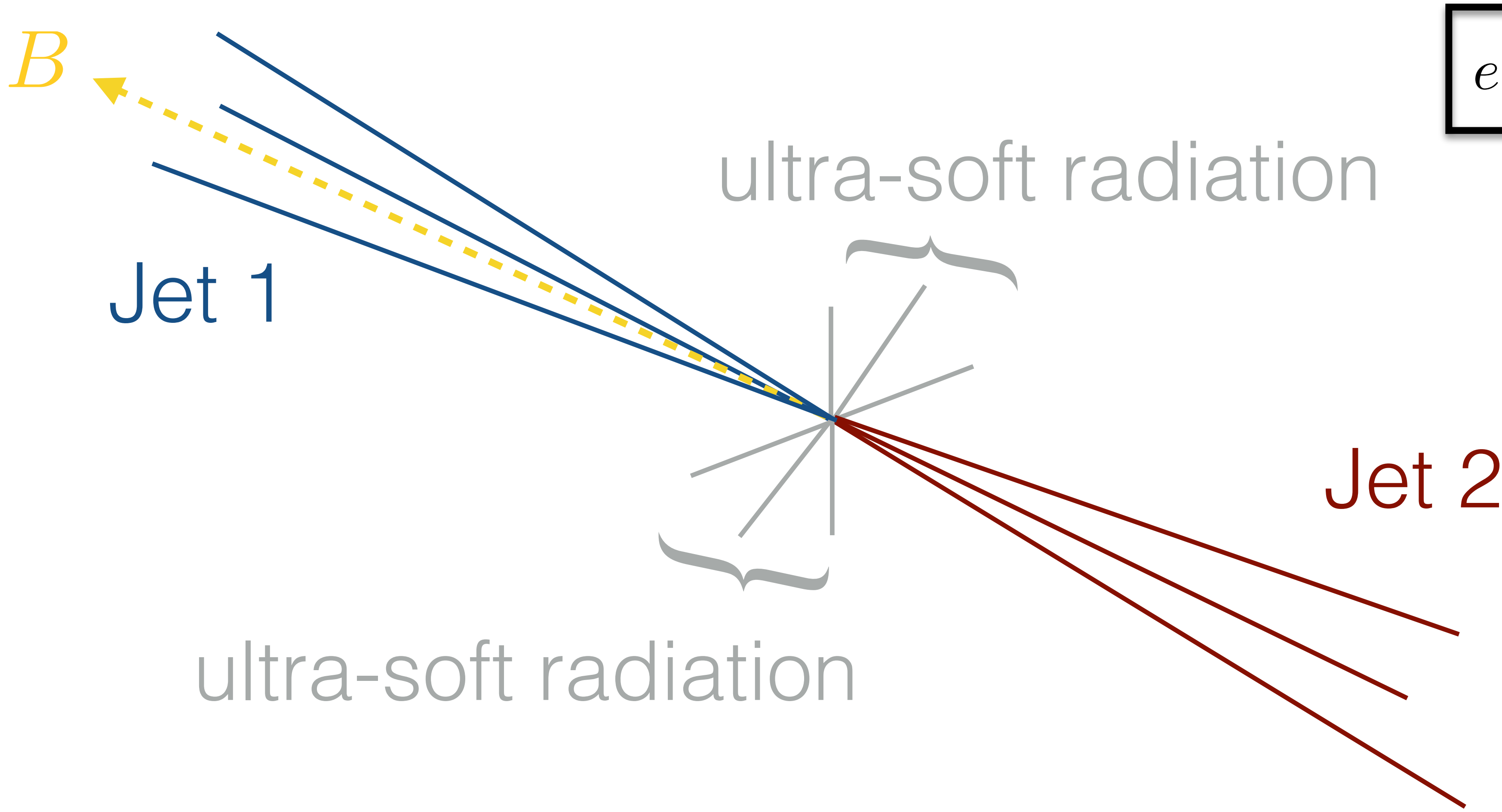
ALEPH: arXiv:hep-ex/0106051

OPAL: arXiv:hep-ex/0210031

SLD: arXiv:hep-ex/9912058

B-meson production in Jets

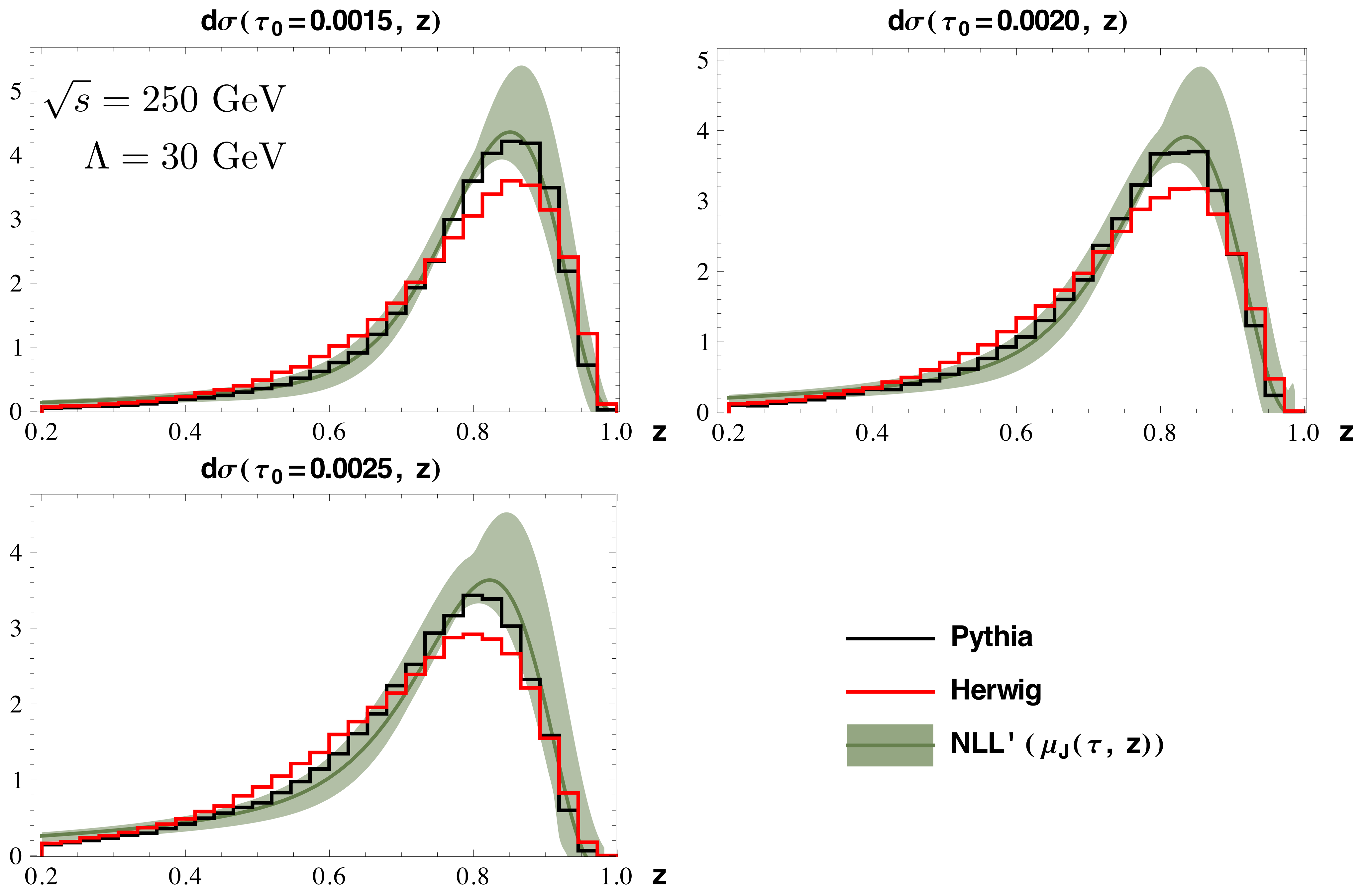
$$e^+ + e^- \to 2\ \mathrm{jets}(+B^0/B^+)$$



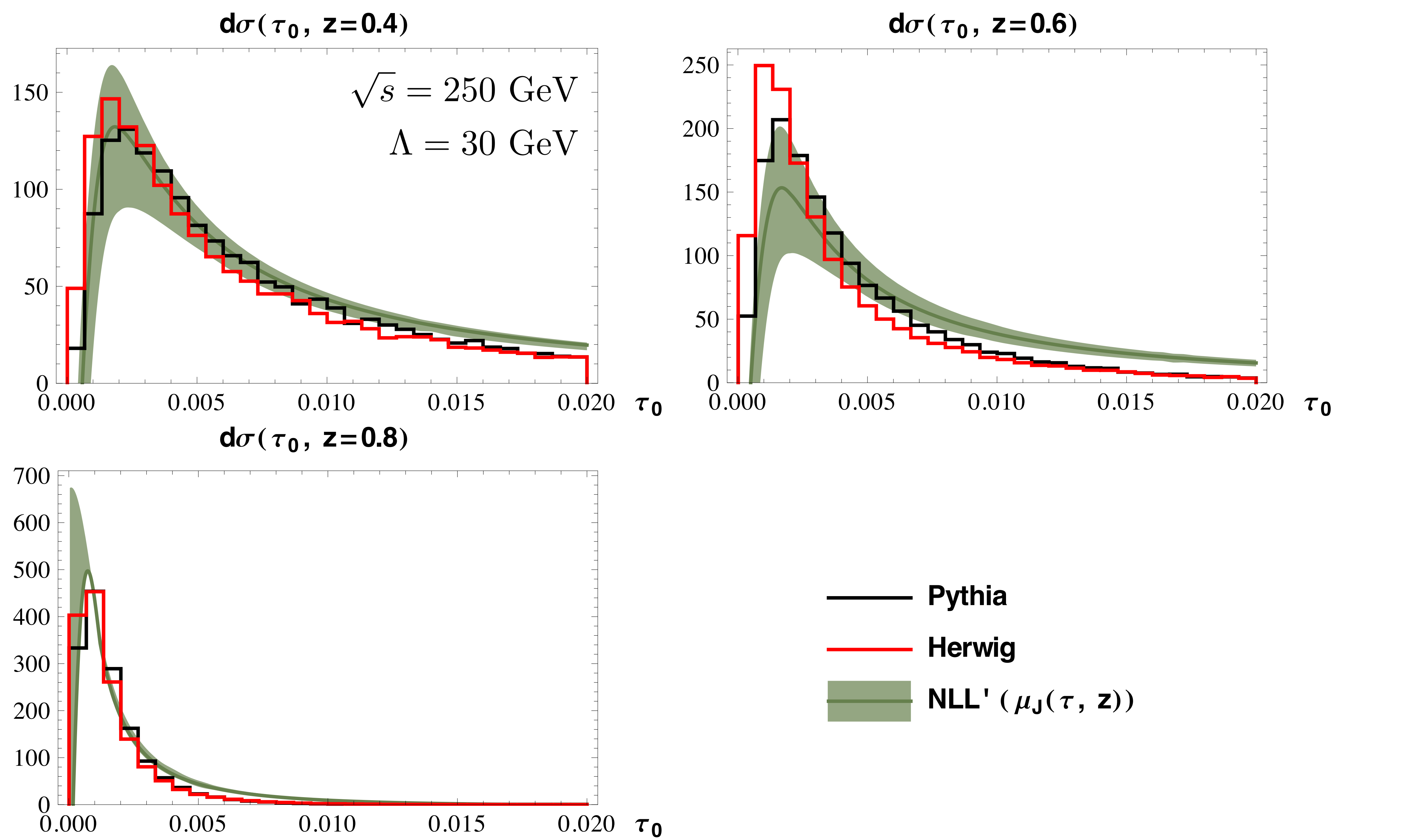
$$\frac{1}{\sigma_0}\frac{d\sigma^{(b)}}{d\tau_a dz} = H_2(\mu) \times S^{\mathrm{unmeas}}(\mu) \times J_{\bar{n}}^{(\bar{b})}(\mu) \times \sum_j \left[\left(S^{\mathrm{meas}}(\tau_a, \mu) \otimes \frac{\mathcal{J}_{bj}^{(b)}(\tau_a, z, \mu)}{2(2\pi)^3} \right) \bullet D_{j \to B}(z) \right]$$

Kniehl et al. : [arXiv: 0705.4392]

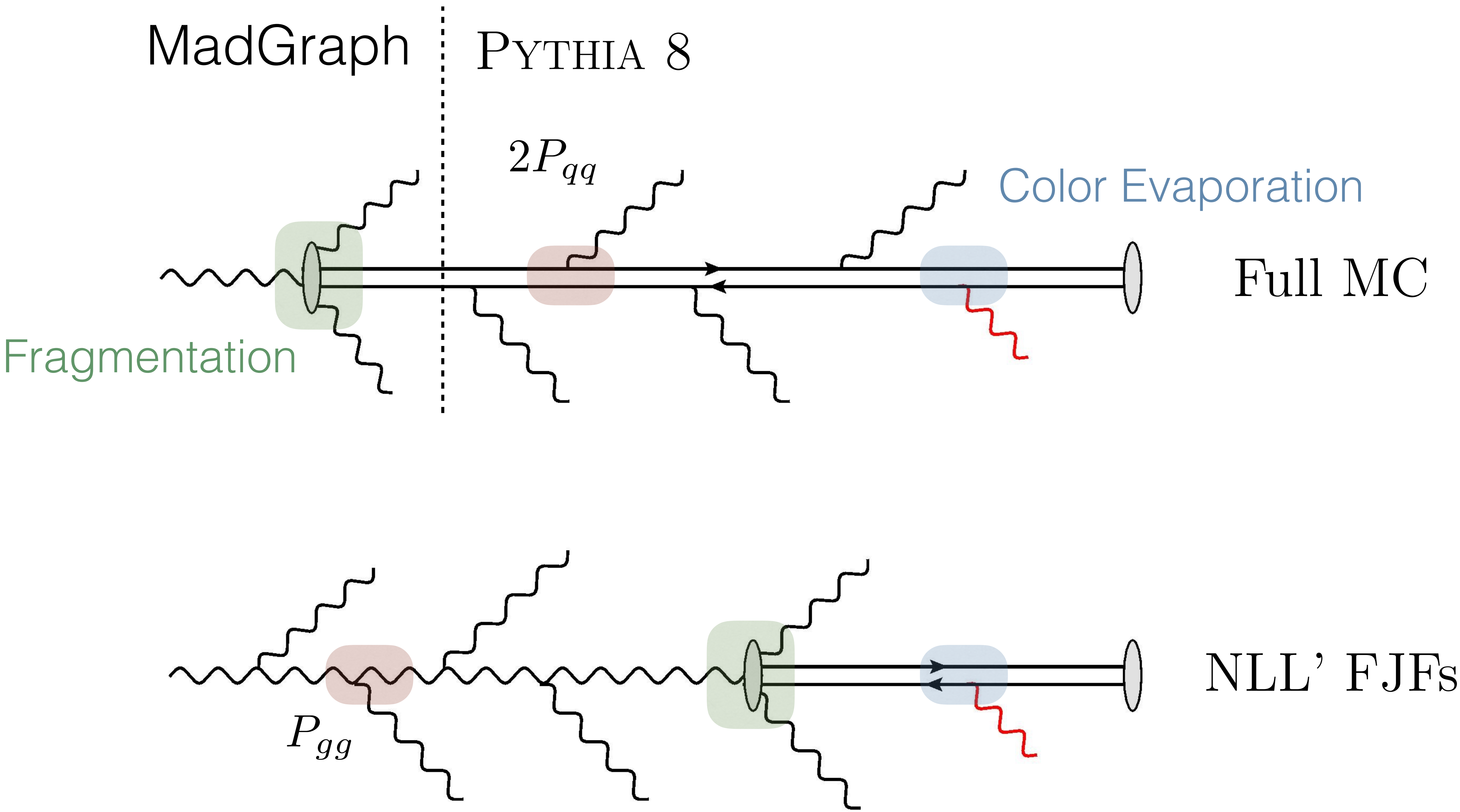
Energy ratio distributions (B-meson)



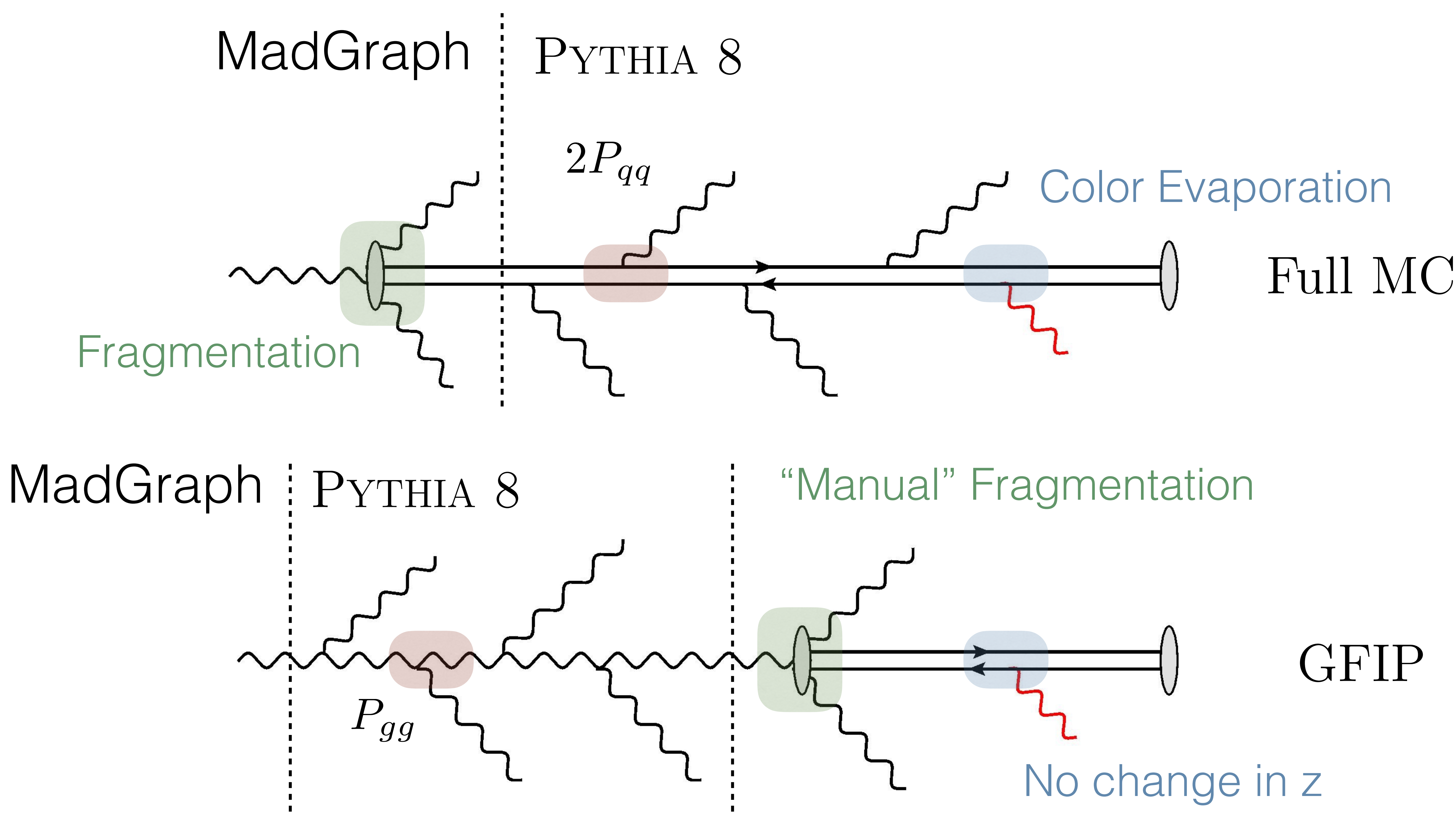
Angularity distributions (B-meson)



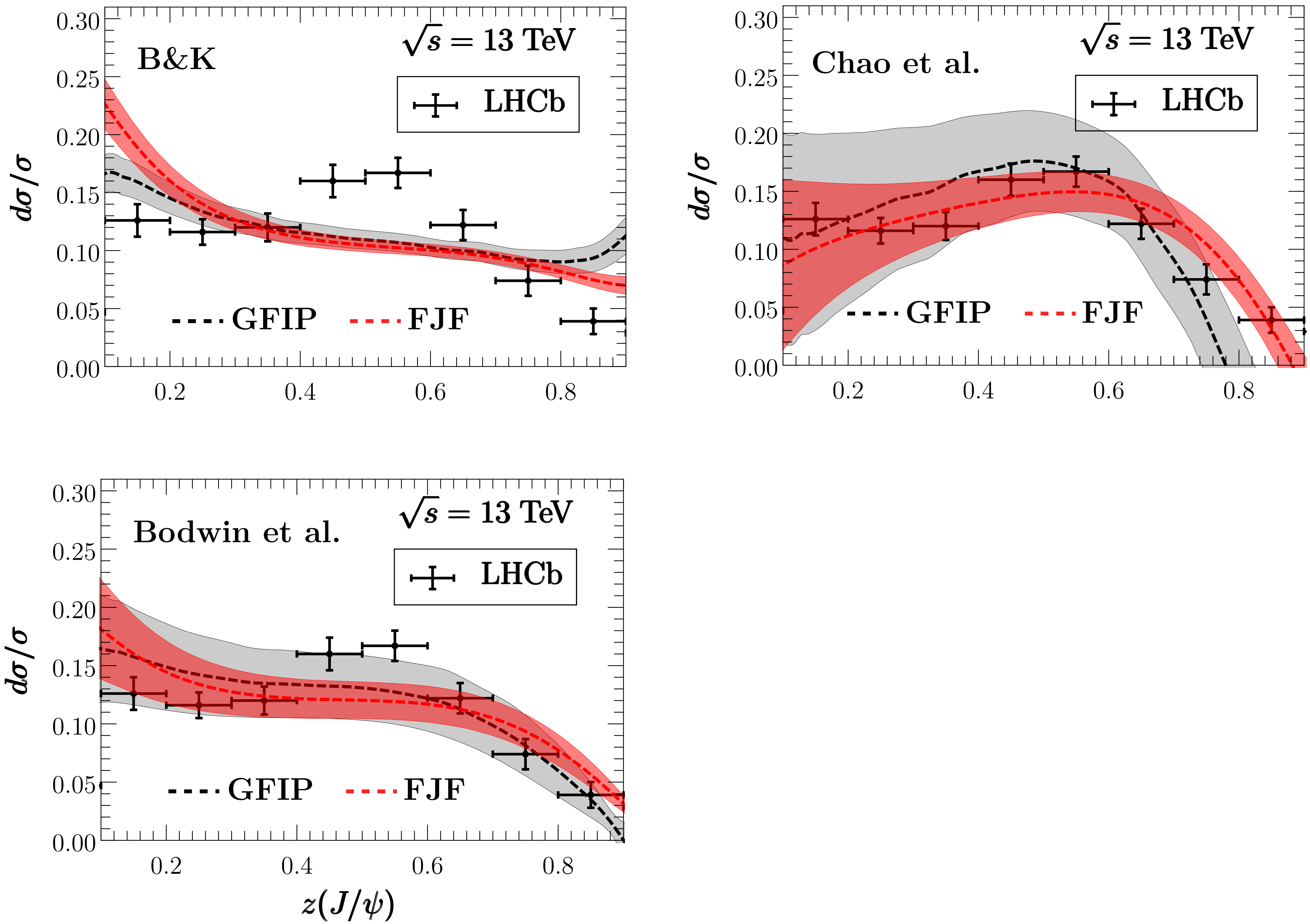
Pythia vs FJFs



Pythia vs GFIP



Quarkonium production in jets at LHC



Factorization

$$D_{q/h}(\mathbf{p}_\perp, z, \mu) = H_+(\mu) \times \Big[\mathcal{D}_{q/h} \otimes_\perp S_C\Big](\mathbf{p}_\perp, z, \mu)$$

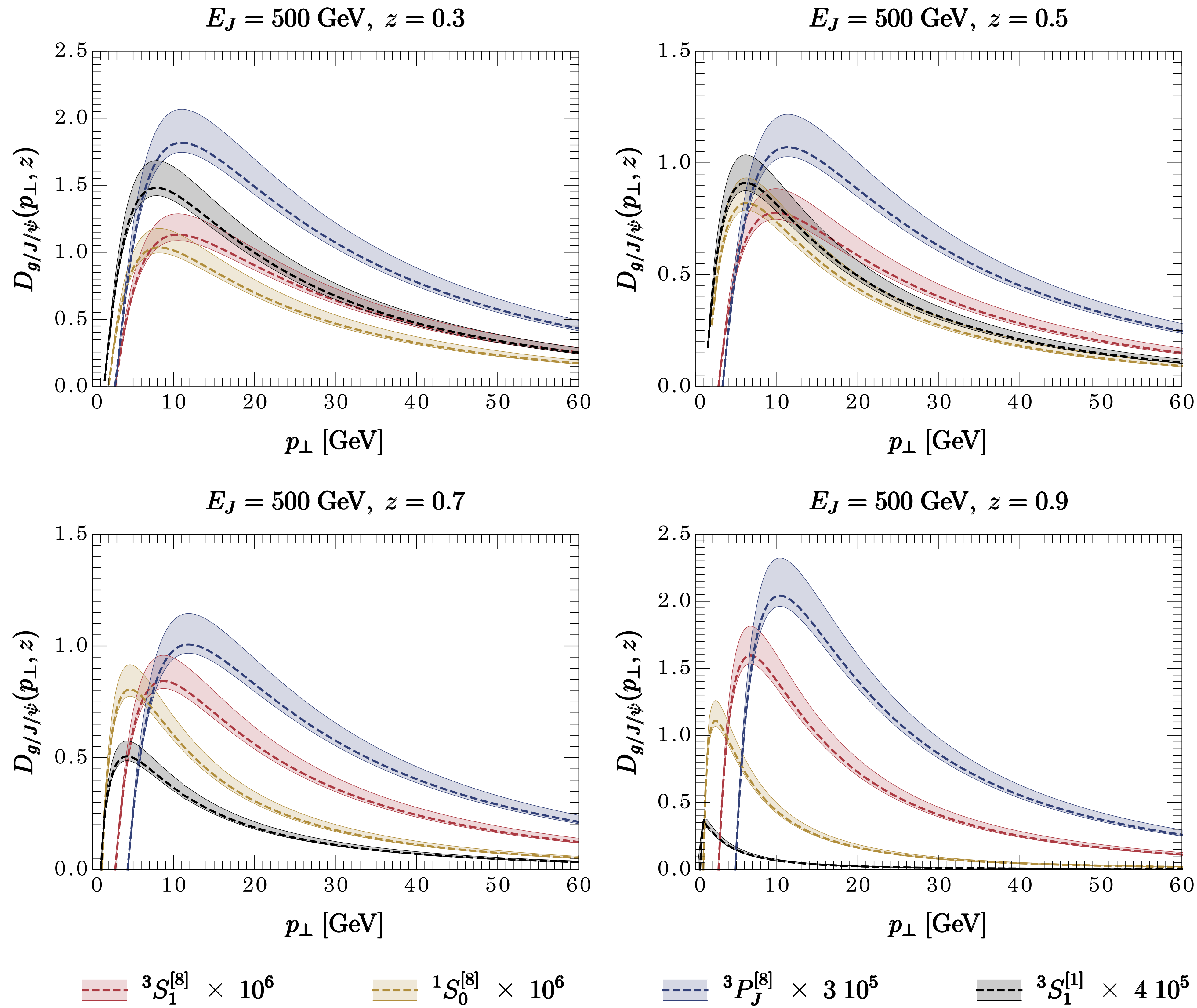
$$H_+(\mu) = (2\pi)^2 N_c\, C_+^\dagger(\mu) C_+(\mu)$$

$$\mathcal{D}_{q/h}(\mathbf{p}_\perp^{\mathcal{D}}, z) \equiv \frac{1}{z} \sum_{X_n} \frac{1}{2N_c} \delta(p^-_{Xh;r}) \delta^{(2)}(p^\perp_{Xh;r})\, \mathrm{Tr}\Big[\frac{\bar{\not{n}}}{2} \langle 0|\delta_{\omega,\overline{\mathcal{P}}} \chi_n(0) \delta^{(2)}(\mathcal{P}_\perp^{X_n} + \mathbf{p}_\perp^{\mathcal{D}})|X_n h\rangle \times \langle X_n h|\bar{\chi}_n(0)|0\rangle\Big]$$

$$\mathcal{D}_{i/h}(\mathbf{p}_\perp, z, \mu, \nu) = \int_z^1 \frac{dx}{x}\, \mathcal{J}_{i/j}(\mathbf{p}_\perp, x, \mu, \nu) D_{j/h}\left(\frac{z}{x}, \mu\right) \; + \; \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{|\mathbf{p}_\perp|^2}\right)$$

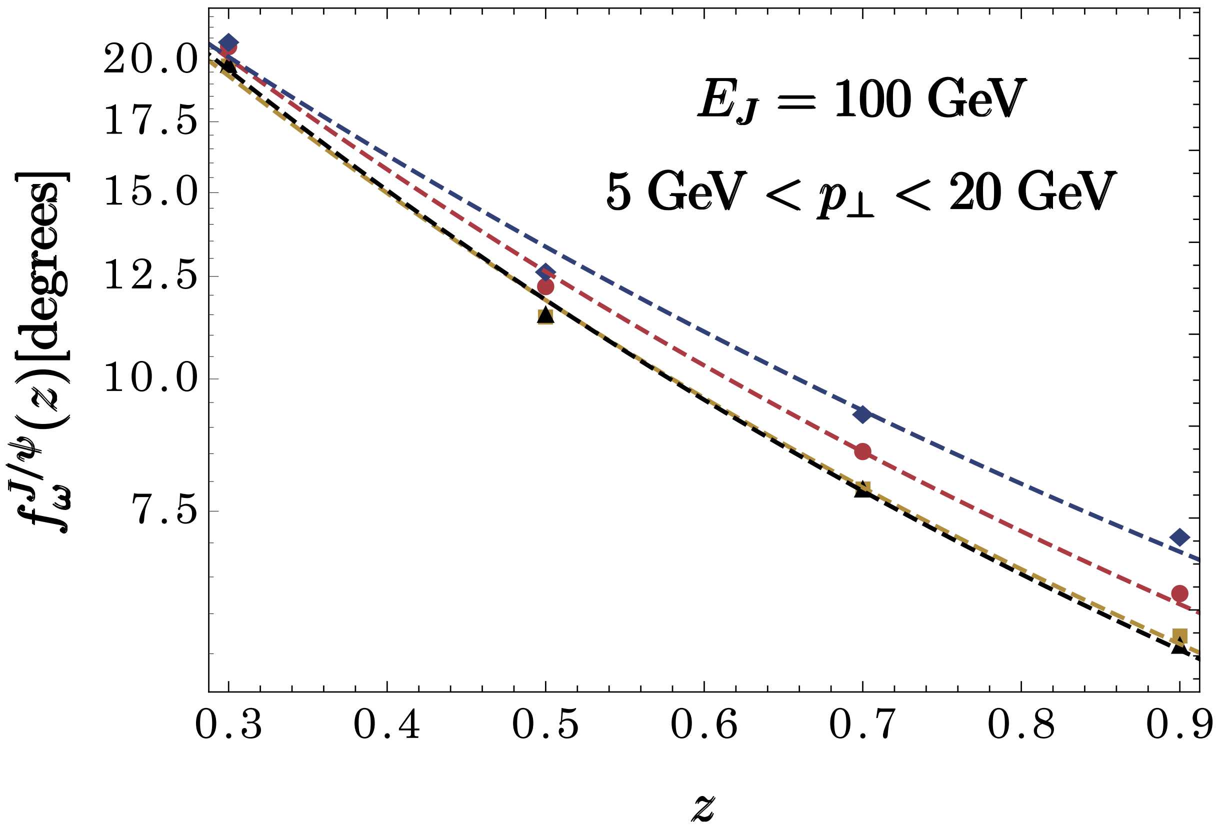
$$S_C(\mathbf{p}_\perp^S) \equiv \frac{1}{N_c} \sum_{X_{cs}} \mathrm{Tr}\Big[\langle 0|V_n^\dagger(0) U_n(0) \delta^{(2)}(\mathcal{P}_\perp + \mathbf{p}_\perp^S)|X_{cs}\rangle\langle X_{cs}|U_n^\dagger(0) V_n(0)|0\rangle\Big]$$

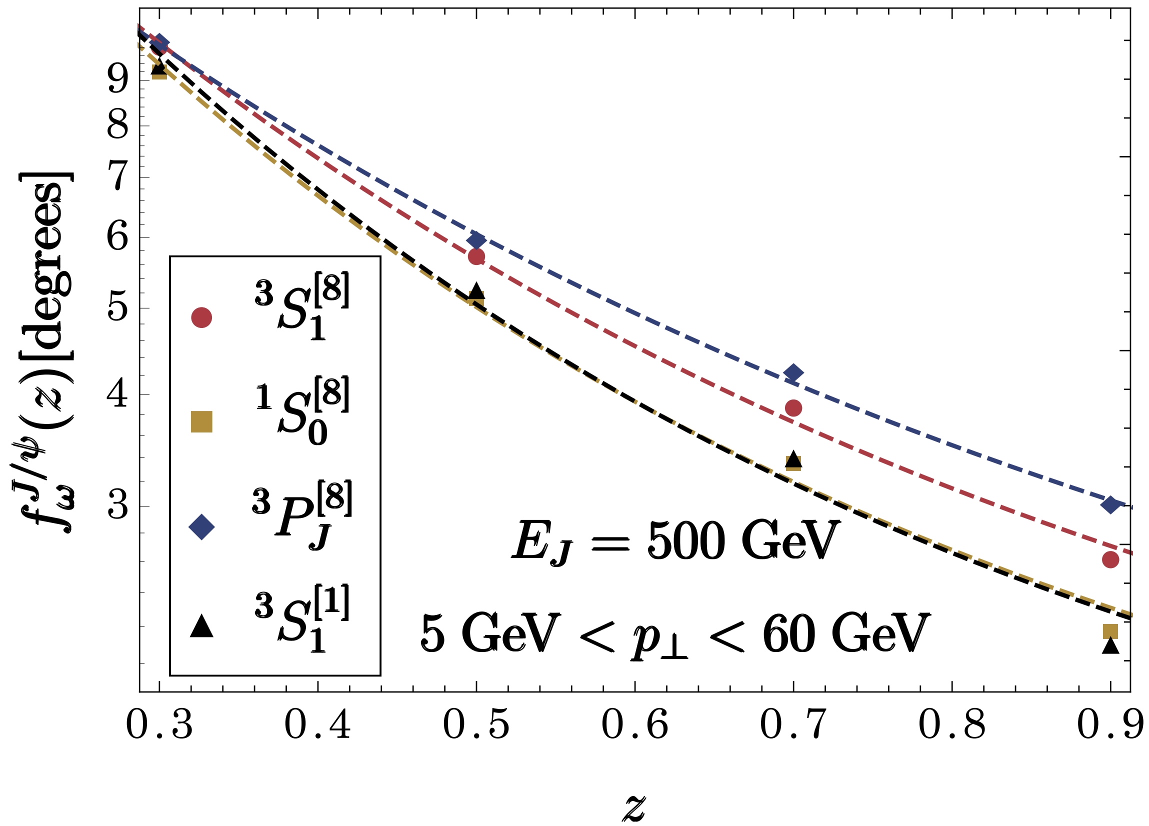
Application to Quarkonium production



Application to Quarkonium production

$$\langle\theta\rangle(z) \sim \frac{2\int dp_\perp\, p_\perp D_{g/h}(p_\perp, z, \mu)}{z\omega \int dp_\perp\, D_{g/h}(p_\perp, z, \mu)} \equiv f^h_\omega(z)$$





$E_J = 100$ GeV		
$^{2S+1}L_J^{[1,8]}$	C_0	C_1
$^3S_1^{[1]}$	3.92	0.92
$^3S_1^{[8]}$	3.86	0.84
$^1S_0^{[8]}$	3.88	0.90
$^3P_J^{[8]}$	3.75	0.74

$E_J = 500$ GeV		
$^{2S+1}L_J^{[1,8]}$	C_0	C_1
$^3S_1^{[1]}$	3.75	1.68
$^3S_1^{[8]}$	3.48	1.39
$^1S_0^{[8]}$	3.66	1.64
$^3P_J^{[8]}$	3.28	1.20

$$\ln(f(x)) = g(x; C_0, C_1) \text{ s.t. } g(x=0) = C_0$$

$$g_2(x) = C_0 \exp(-C_1 x)$$