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# Wigner Distributions of Quarks and Gluons

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# Plan of the talk

- Wigner distributions for quarks
- Wigner distributions for different polarization of target and quark
- Calculation for a dressed quark at one loop
- Numerical results
- Wigner distribution for gluons
- Summary and Conclusions

# Wigner Distribution for Quarks

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Partonic picture of nucleons in terms of quarks and gluons : joint position and momentum space information. In classical physics phase space distributions

Quantum mechanics : because of uncertainty principle position and momentum cannot be determined simultaneously. One cannot have density interpretation of such phase space variables. They are positive definite only in the classical limit

For a one-dimensional quantum system with wave function  $\psi(x)$  the Wigner function is defined as

$$W(x, p) = \int dy e^{ip \cdot y} \psi^*(x - y/2) \psi(x + y/2)$$

Matrix element of the Wigner operator for a nucleon state can be interpreted as distribution of partons in 6 D (3 position and 3 momentum)

# Wigner Distributions

5 D Wigner distribution in infinite momentum frame : boost invariant description

Lorce, Pasquini, PRD 84, 014015 (2011)

Integrating over transverse momentum Wigner distributions reduce to generalized parton distributions (GPDs) in impact parameter space; integrating over transverse position, they become transverse momentum dependent pdfs (TMDs)

“Mother Distributions” : contain information coded in GPDs and TMDs and even more

Unintegrated off-forward quark correlators were introduced in the context of studying orbital angular momentum in

Schaefer, AM, Hagler, PLB 582, 55 (2004)



# Wigner Distributions

Related to GTMDs ; give information on orbital angular momentum of quarks as well as spin-orbit correlation

Meissner, Metz, Schlegel, JHEP 08 (2009) 056

Inclusion of soft factor in the definition and evolution of GTMDs discussed

Echevarria, Idilbi, Kanazawa, Lorce, Metz, Pasquini, Schlegel, PLB 759, 336 (2016)

Accessing quark GTMDs in exclusive double Drell-Yan Process

Bhattacharya, Metz, Zhou, 1702.04387 [hep-ph]

# Wigner distributions for quarks

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$$\rho^{[\Gamma]}(b_{\perp}, k_{\perp}, x, \sigma) = \int \frac{d^2 \Delta_{\perp}}{(2\pi)^2} e^{-i \Delta_{\perp} \cdot b_{\perp}} W^{[\Gamma]}(\Delta_{\perp}, k_{\perp}, x, \sigma);$$

$\Delta_{\perp}$  Momentum transfer in the transverse direction

$b_{\perp}$  Impact parameter conjugate to  $\Delta_{\perp}$

$$W^{[\Gamma]}(\Delta_{\perp}, k_{\perp}, x, \sigma) = \frac{1}{2} \int \frac{dz^{-} d^2 z_{\perp}}{(2\pi)^3} e^{i(xp^{+} z^{-} / 2 - k_{\perp} \cdot z_{\perp})}$$

$$\left\langle p^{+}, \frac{\Delta_{\perp}}{2}, \sigma \left| \bar{\psi}\left(-\frac{z}{2}\right) \Omega \Gamma \psi\left(\frac{z}{2}\right) \right| p^{+}, -\frac{\Delta_{\perp}}{2}, \sigma \right\rangle \Big|_{z^{+}=0}.$$

$k_{\perp}$  Average transverse momentum of quark conjugate to  $z_{\perp}$

$\Omega$  : gauge link,  $\Gamma$  : Dirac matrix

We use light-cone gauge and take the gauge link to be unity

# Dressed Quark Target

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Instead of a proton, we take the target to be a quark dressed with a gluon

State is expanded in Fock space in terms of multi-particle light-front wave functions

$$\begin{aligned} |p^+, p_\perp, \sigma\rangle = & \Phi^\sigma(p) b_\sigma^\dagger(p) |0\rangle + \sum_{\sigma_1 \sigma_2} \int [dp_1] \int [dp_2] \sqrt{16\pi^3 p^+} \delta^3(p - p_1 - p_2) \\ & \Phi_{\sigma_1 \sigma_2}^\sigma(p; p_1, p_2) b_{\sigma_1}^\dagger(p_1) a_{\sigma_2}^\dagger(p_2) |0\rangle; \end{aligned}$$

Two-component formalism, light-front gauge

$\Phi_{\sigma_1 \sigma_2}^\sigma(p; p_1, p_2)$  Two-particle LFWF;

Zhang, Harindranath, PRD (1993)

$\Phi^\sigma(p)$  Gives normalization of the state      Related to boost invariant LFWF

Composite spin  $\frac{1}{2}$  state with a gluonic degree of freedom, two-particle LFWF calculated analytically

# Wigner Distribution for Quarks

Unpolarized target and different quark polarizations

$$\rho_{UU}(b_{\perp}, k_{\perp}, x) = \frac{1}{2} \left[ \rho^{[\gamma^+]}(b_{\perp}, k_{\perp}, x, \hat{e}_z) + \rho^{[\gamma^+]}(b_{\perp}, k_{\perp}, x, -\hat{e}_z) \right]$$

$$\rho_{UL}(b_{\perp}, k_{\perp}, x) = \frac{1}{2} \left[ \rho^{[\gamma^+ \gamma^5]}(b_{\perp}, k_{\perp}, x, \hat{e}_z) + \rho^{[\gamma^+ \gamma^5]}(b_{\perp}, k_{\perp}, x, -\hat{e}_z) \right]$$

$$\rho_{UT}^j(b_{\perp}, k_{\perp}, x) = \frac{1}{2} \left[ \rho^{[i\sigma^{+j} \gamma^5]}(b_{\perp}, k_{\perp}, x, \hat{e}_z) + \rho^{[i\sigma^{+j} \gamma^5]}(b_{\perp}, k_{\perp}, x, -\hat{e}_z) \right]$$

$\rho_{UL}$  and  $\rho_{LU}$  are equal in this model .  $e_z$  : Polarization of the target state

$\rho_{UL}$  : no TMD or GPD limit. Represents quark spin-orbit correlation

$\rho_{UT}$  : related to Boer-Mulders function in TMD limit, to  $\tilde{H}_T$  in GPD limit

# Wigner Distribution for Quarks

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Longitudinally polarized target and different quark polarization

$$\rho_{LU}(b_{\perp}, k_{\perp}, x) = \frac{1}{2} \left[ \rho^{[\gamma^+]}(b_{\perp}, k_{\perp}, x, \hat{e}_z) - \rho^{[\gamma^+]}(b_{\perp}, k_{\perp}, x, -\hat{e}_z) \right]$$

$$\rho_{LL}(b_{\perp}, k_{\perp}, x) = \frac{1}{2} \left[ \rho^{[\gamma^+ \gamma^5]}(b_{\perp}, k_{\perp}, x, \hat{e}_z) - \rho^{[\gamma^+ \gamma^5]}(b_{\perp}, k_{\perp}, x, -\hat{e}_z) \right]$$

$$\rho_{LT}^j(b_{\perp}, k_{\perp}, x) = \frac{1}{2} \left[ \rho^{[i\sigma^{+j} \gamma^5]}(b_{\perp}, k_{\perp}, x, \hat{e}_z) - \rho^{[i\sigma^{+j} \gamma^5]}(b_{\perp}, k_{\perp}, x, -\hat{e}_z) \right]$$

TMD limit :  $\rho_{LT}$  related to the worm-gear function  $h_{1L}^{\perp}$

Related to the GPDs  $H_T$  and  $\tilde{H}_T$

$\rho_{LU}$  : related to orbital angular momentum of the quark

# Wigner Distribution for Quarks

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Transversely polarized target and different quark polarizations

$$\rho_{TU}^i(b_{\perp}, k_{\perp}, x) = \frac{1}{2} \left[ \rho^{[\gamma^+]}(b_{\perp}, k_{\perp}, x, \hat{e}_i) - \rho^{[\gamma^+]}(b_{\perp}, k_{\perp}, x, -\hat{e}_i) \right]$$

$$\rho_{TL}^i(b_{\perp}, k_{\perp}, x) = \frac{1}{2} \left[ \rho^{[\gamma^+ \gamma^5]}(b_{\perp}, k_{\perp}, x, \hat{e}_i) - \rho^{[\gamma^+ \gamma^5]}(b_{\perp}, k_{\perp}, x, -\hat{e}_i) \right]$$

$$\rho_{TT}(b_{\perp}, k_{\perp}, x) = \frac{1}{2} \delta_{ij} \left[ \rho^{[i\sigma^+ j \gamma^5]}(b_{\perp}, k_{\perp}, x, \hat{e}_i) - \rho^{[i\sigma^+ j \gamma^5]}(b_{\perp}, k_{\perp}, x, -\hat{e}_i) \right]$$

$$\rho_{TT}^{\perp}(b_{\perp}, k_{\perp}, x) = \frac{1}{2} \epsilon_{ij} \left[ \rho^{[i\sigma^+ j \gamma^5]}(b_{\perp}, k_{\perp}, x, \hat{e}_i) - \rho^{[i\sigma^+ j \gamma^5]}(b_{\perp}, k_{\perp}, x, -\hat{e}_i) \right]$$

Pretzelous Wigner distribution : quark and target transversely polarized in orthogonal directions : zero in our model

$\rho_{TL}$  : TMD limit is related to the other worm-gear function  $g_{1T}$  ;



# Analytical Results

Two-particle light-front wave function is given by

$$\Psi_{s_1 s_2}^{sa}(x, q^\perp) = \frac{1}{\left[ m^2 - \frac{m^2 + (q^\perp)^2}{x} - \frac{(q^\perp)^2}{1-x} \right]} \frac{g}{\sqrt{2}(2\pi)^3} T^a \chi_{s_1}^\dagger \frac{1}{\sqrt{1-x}} \\ \times \left[ -\frac{2q^\perp}{1-x} - \frac{(\sigma^\perp \cdot q^\perp) \sigma^\perp}{x} + \frac{im \sigma^\perp (1-x)}{x} \right] \chi_s(\epsilon_{s_2}^\perp)^*$$

Two-component formalism

Harindranath and Zhang, PRD, 1993

Used light-front gauge

$m$  : mass of the quark, parameter in our model

Boost invariant LFWF

Two-component spinor  $\chi$

Wigner distributions can be expressed as overlaps of LFWF

# Overlap Formula

$$W_{s s'}^{[\gamma^+]}(\Delta_{\perp}, k_{\perp}, x) = \sum_{\lambda'_1, \lambda_1, \lambda_2} \Psi_{\lambda'_1 \lambda_2}^{*s'}(x, q'^{\perp}) \chi_{\lambda'_1}^{\dagger} \chi_{\lambda_1} \Psi_{\lambda_1 \lambda_2}^s(x, q^{\perp})$$

$$W_{s s'}^{[\gamma^+ \gamma^5]}(\Delta_{\perp}, k_{\perp}, x) = \sum_{\lambda'_1, \lambda_1, \lambda_2} \Psi_{\lambda'_1 \lambda_2}^{*s'}(x, q'^{\perp}) \chi_{\lambda'_1}^{\dagger} \sigma_3 \chi_{\lambda_1} \Psi_{\lambda_1 \lambda_2}^s(x, q^{\perp})$$

$$W_{s s'}^{[i\sigma^{+j} \gamma^5]}(\Delta_{\perp}, k_{\perp}, x) = \sum_{\lambda'_1, \lambda_1, \lambda_2} \Psi_{\lambda'_1 \lambda_2}^{*s'}(x, q'^{\perp}) \chi_{\lambda'_1}^{\dagger} \sigma_j \chi_{\lambda_1} \Psi_{\lambda_1 \lambda_2}^s(x, q^{\perp})$$

$$q_{\perp} = k_{\perp} + \frac{\Delta_{\perp}}{2}(1 - x); \quad q'_{\perp} = k_{\perp} - \frac{\Delta_{\perp}}{2}(1 - x)$$

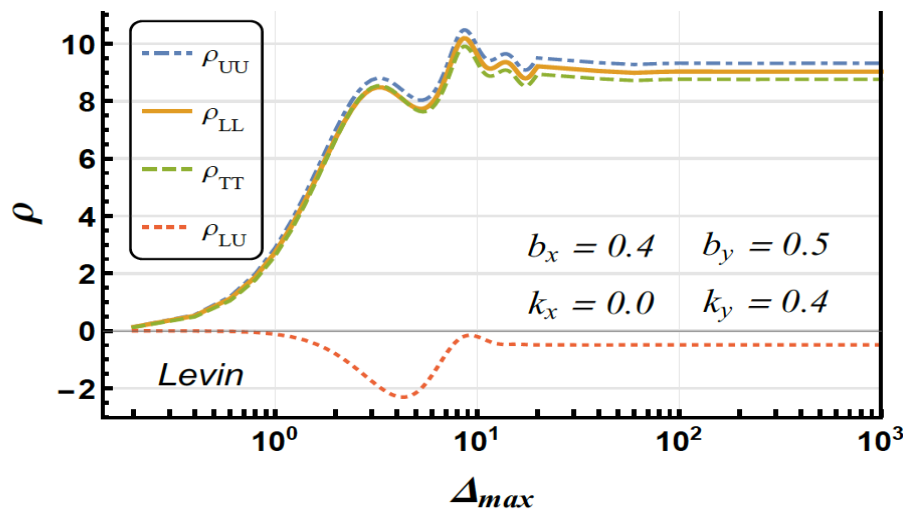
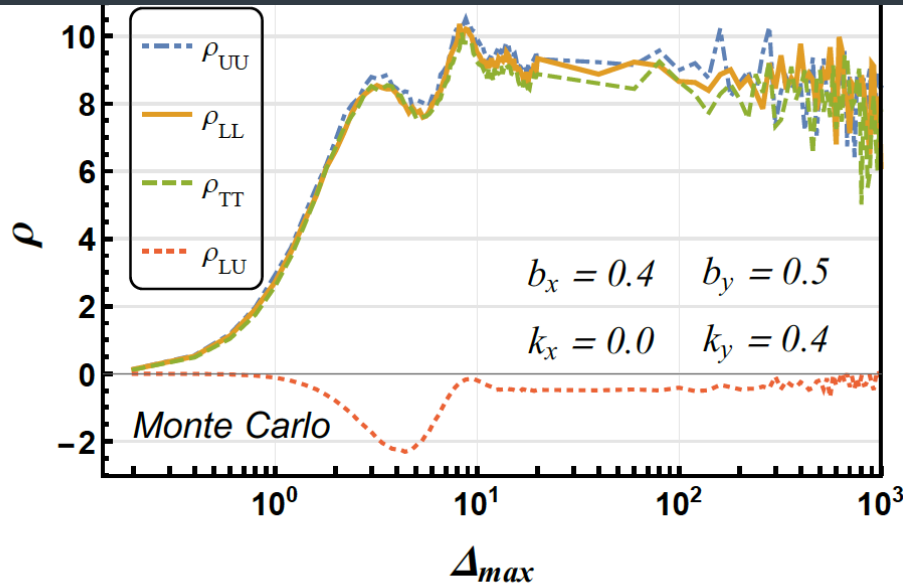
Wigner distributions are Fourier transform of these



# Analytic Results

$$\begin{aligned}
 \rho_{UU}(b_{\perp}, k_{\perp}, x) &= N \int \frac{d^2 \Delta_{\perp}}{2(2\pi)^2} \frac{\cos(\Delta_{\perp} \cdot b_{\perp})}{D(q_{\perp})D(q'_{\perp})} \\
 &\times \left[ \frac{\left(4k_{\perp}^2 - \Delta_{\perp}^2(1-x)^2\right)(1+x^2)}{x^2(1-x)^3} + \frac{4m^2(1-x)}{x^2} \right] \\
 \rho_{UL}(b_{\perp}, k_{\perp}, x) &= N \int \frac{d^2 \Delta_{\perp}}{2(2\pi)^2} \frac{\sin(\Delta_{\perp} \cdot b_{\perp})}{D(q_{\perp})D(q'_{\perp})} \left[ \frac{4\left(k_y \Delta_x - k_x \Delta_y\right)(1+x)}{x^2(1-x)} \right] \\
 \rho_{UT}^x(b_{\perp}, k_{\perp}, x) &= N \int \frac{d^2 \Delta_{\perp}}{2(2\pi)^2} \frac{\sin(\Delta_{\perp} \cdot b_{\perp})}{D(q_{\perp})D(q'_{\perp})} \left[ \frac{4m\Delta_x}{x^2} \right] \\
 D(q_{\perp}) &= \left[ m^2 - \frac{m^2 + \left(k_{\perp} + \frac{\Delta_{\perp}(1-x)}{2}\right)^2}{x} - \frac{\left(k_{\perp} + \frac{\Delta_{\perp}(1-x)}{2}\right)^2}{1-x} \right] \\
 D(q'_{\perp}) &= \left[ m^2 - \frac{m^2 + \left(k_{\perp} - \frac{\Delta_{\perp}(1-x)}{2}\right)^2}{x} - \frac{\left(k_{\perp} - \frac{\Delta_{\perp}(1-x)}{2}\right)^2}{1-x} \right]
 \end{aligned}$$

# Integration Technique



Earlier study : MC integration method. Low value of upper integration limit for convergence :  $\Delta_{max}$  dependence

AM, Nair, Ojha, PRD (2015)

Levin method : for oscillatory integrand. Better convergence. Results agree for smaller values of cutoff

$$\Delta_{max} = 20 \text{ GeV}, m=0.33 \text{ GeV}$$

Results are independent of cutoff

# Contribution from single particle sector

3 D plots of 'transverse' Wigner distributions in  $b$  and  $k$  space  
 $x$  integrated in  $b$  space from 0 to 1 and in  $k$  space from 0 to 0.9

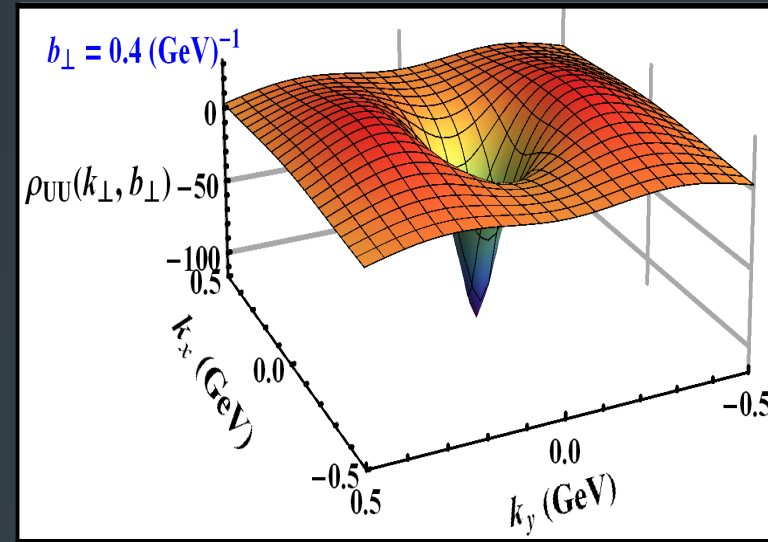
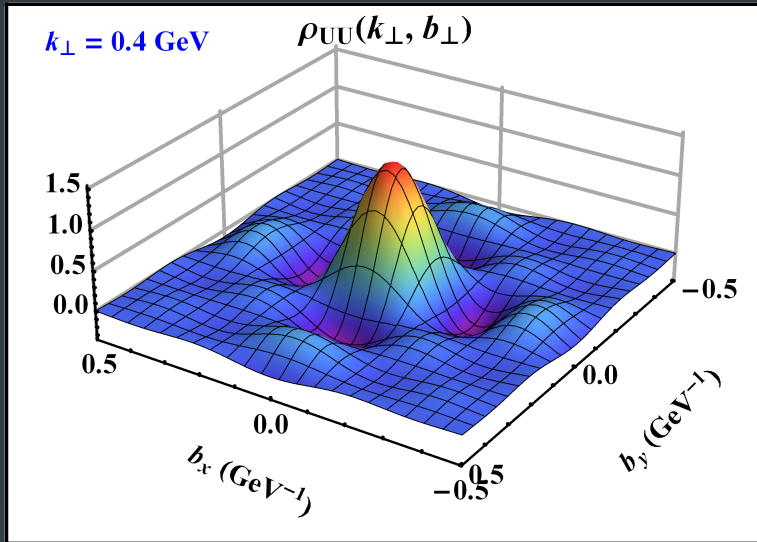
To get the correct result at  $x=1$ , contribution from single particle sector needs to be taken into account. This contributes to  $\rho_{UU}$ ,  $\rho_{LL}$ , and  $\rho_{TT}$

This is of the form  $N\delta(1-x)\delta^2(b_{\perp})\delta^2(k_{\perp})$

There is also a contribution due to the normalization of the state

Contribution from the normalization of the state combines with the contribution from the two particle sector to give the familiar plus distribution in the pdf for a dressed quark

# Numerical Results : $\rho_{UU}$



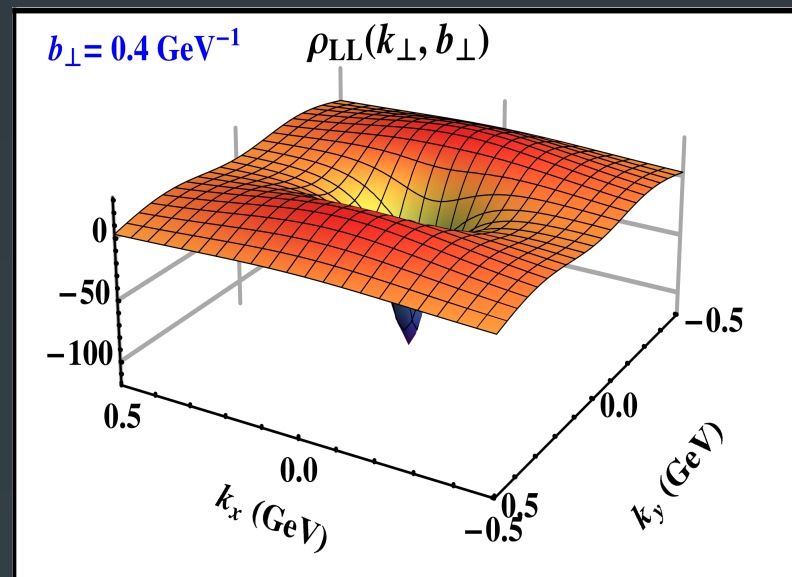
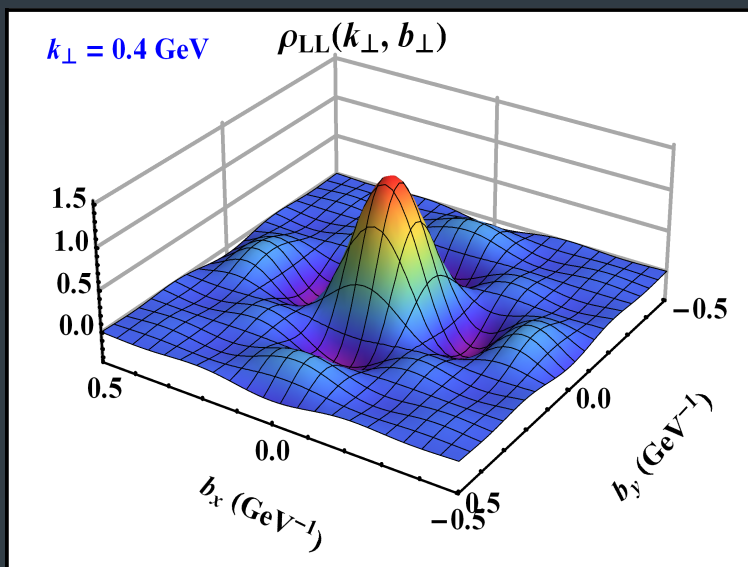
$b(k)$  space plot : for fixed value of  $k_{\perp}$  ( $b_{\perp}$ )

Single particle contribution does not affect this plot

Positive peak in  $b$  space similar to LFCQM (Lorce and Pasquini, PRD 93, 034040 (2016)); spectator model (Liu and Ma, PRD 91, 034019 (2015))

More, AM, Nair, PRD 95, 074039 (2017)

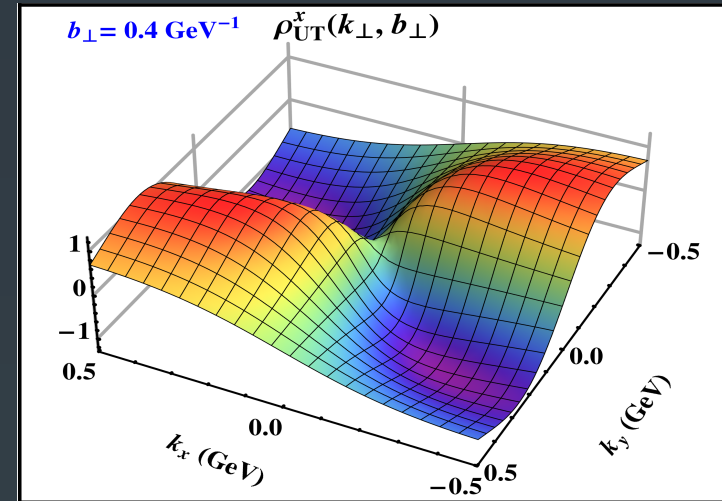
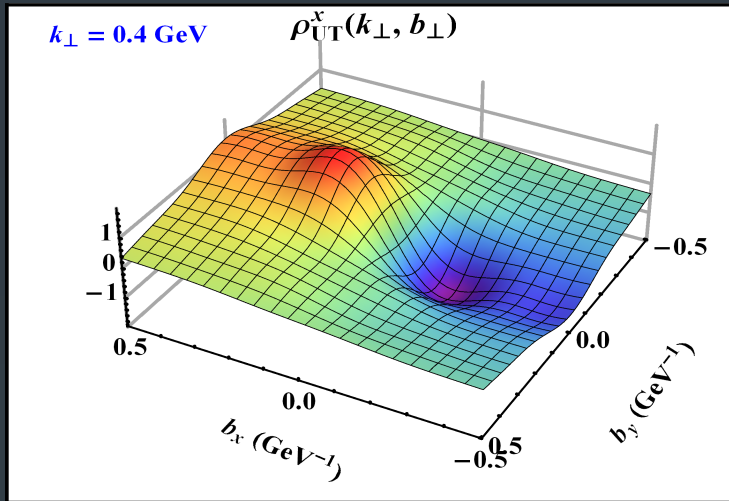
# Numerical Results : $\rho_{LL}$



Analytic expression similar to  $\rho_{UU}$ , difference in mass term

Positive peak in  $b$  space similar to other model calculations

# Numerical Results : $\rho_{UT}^x$



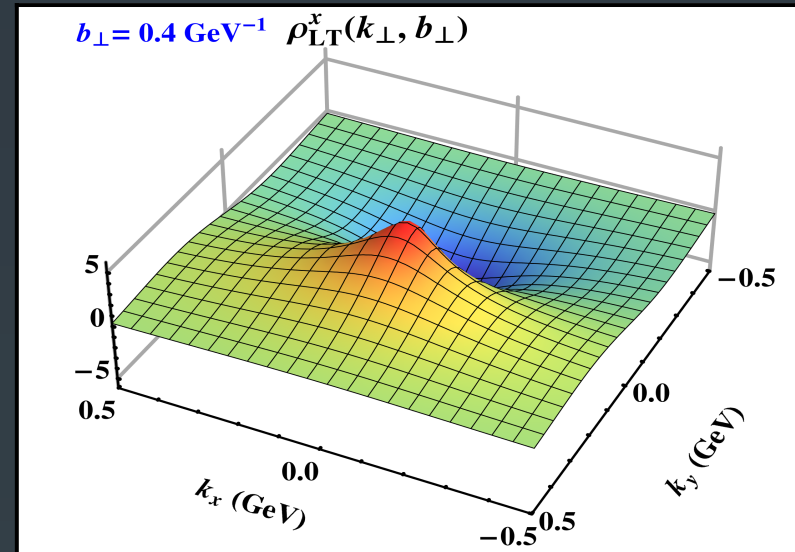
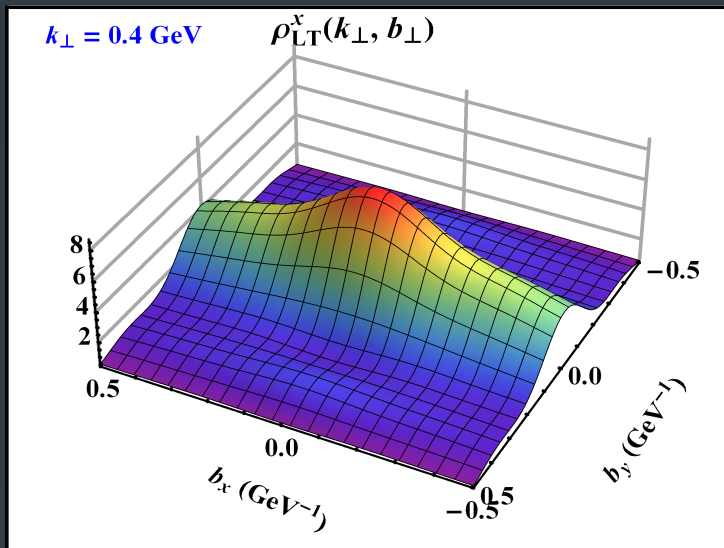
Transversely polarized quark in unpolarized target; quark polarization in x direction

Dipole behaviour in  $b$  space similar to spectator model, quadrupole behaviour in  $k$  space

Vanishes in TMD limit : as we have not considered the gauge link. Boer-Mulders function is zero in our model.

# Numerical Result : $\rho_{LT}^x$

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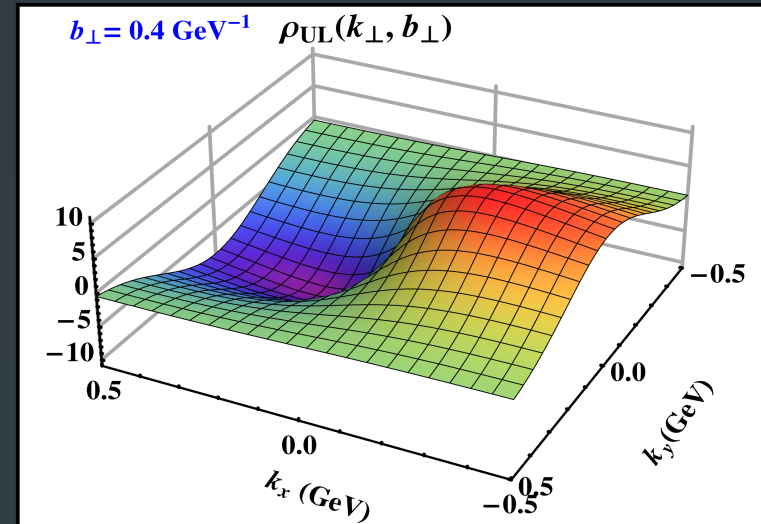
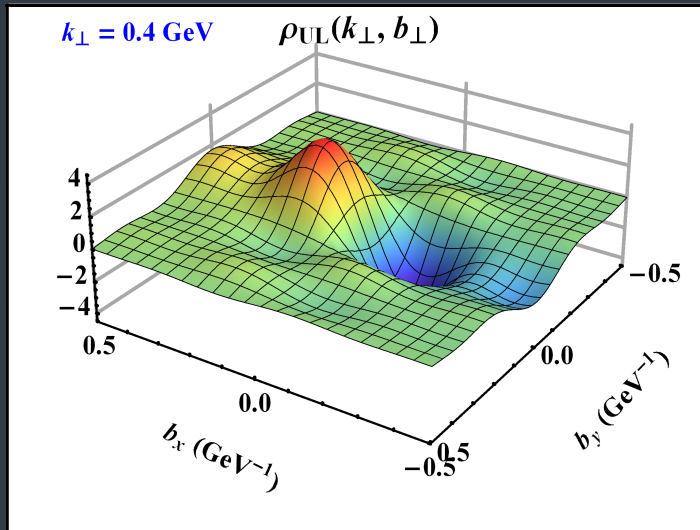
Transversely polarized quark in longitudinally polarized target, quark polarization in x direction : TMD limit related to worm gear function

Dipole behaviour in k space, similar to spectator model; different in b space

Similar to LFCQM (Lorce and Pasquini, PRD 93, 034040 (2016))

More, AM, Nair, PRD 95, 074039 (2017)

# Numerical Result : $\rho_{UL}$



Dipole structure of  $\rho_{UL}$  as observed in other models

Lorce and Pasquini, PRD 84, 014015 (2011); Liu and Ma, PRD 91, 034019 (2015)

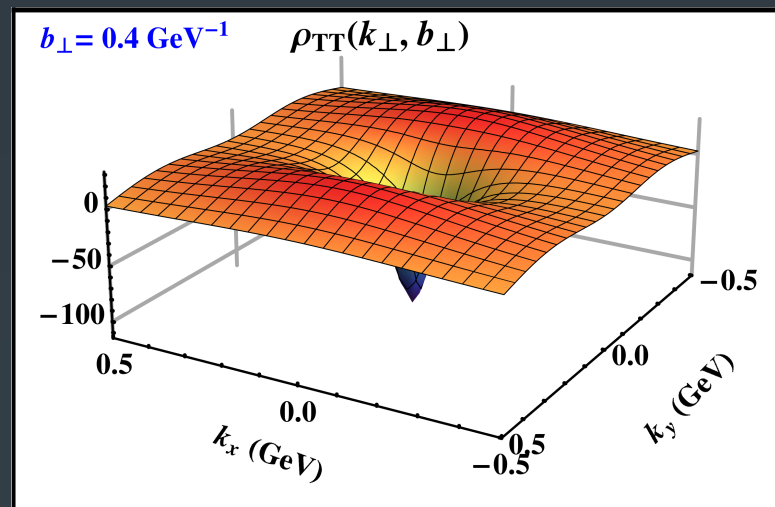
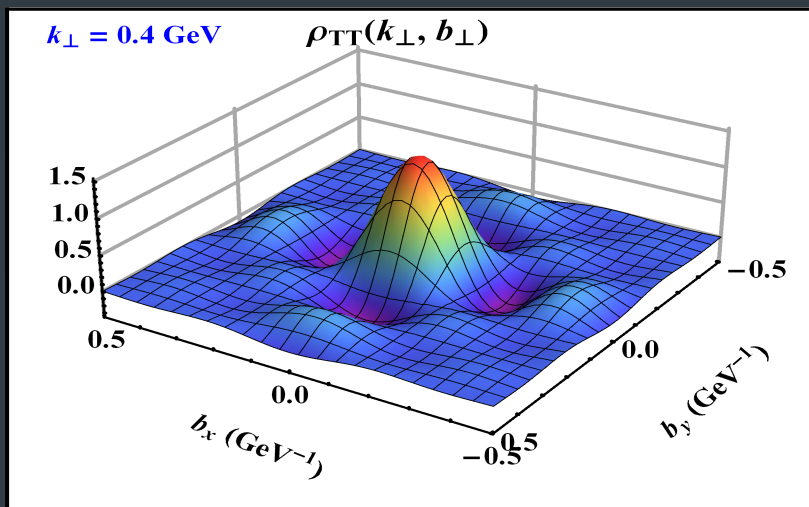
Represents spin-orbit correlation of the quark

Does not have TMD or GPD limit

More, AM, Nair, PRD 95, 074039 (2017)



# Numerical Result : $\rho_{TT}$

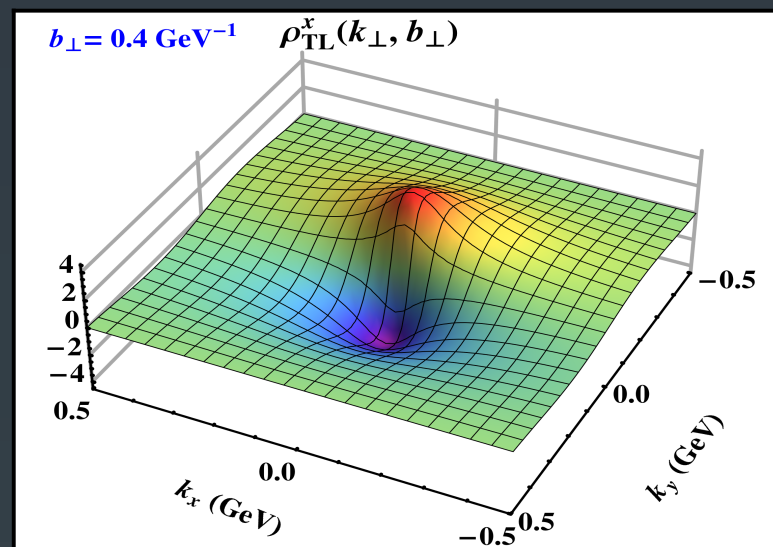
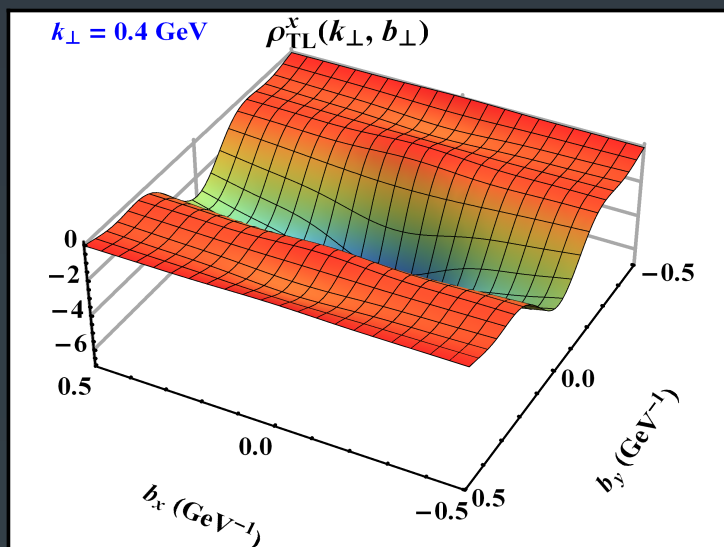


Behaviour of  $\rho_{TT}$  similar to  $\rho_{UU}$  and  $\rho_{LL}$ , analytic result different

Similar behaviour as in light-front spectator model

More, AM, Nair, PRD 95, 074039 (2017)

# Numerical Result : $\rho_{\text{TL}}^x$



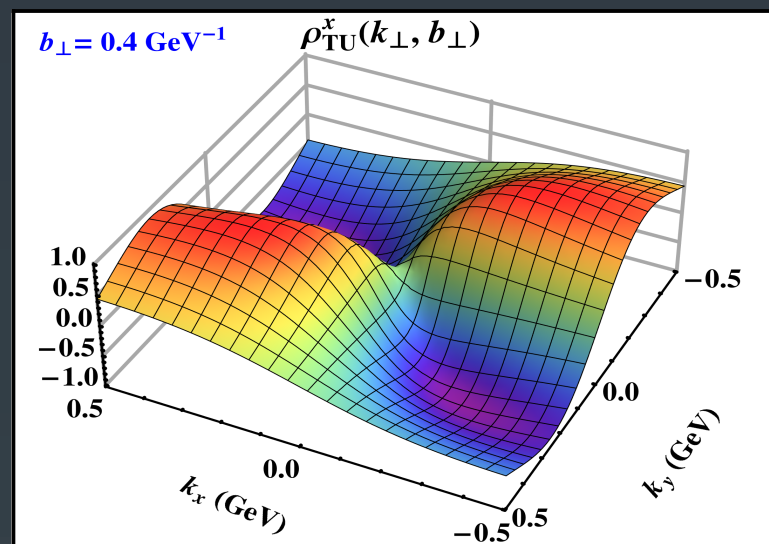
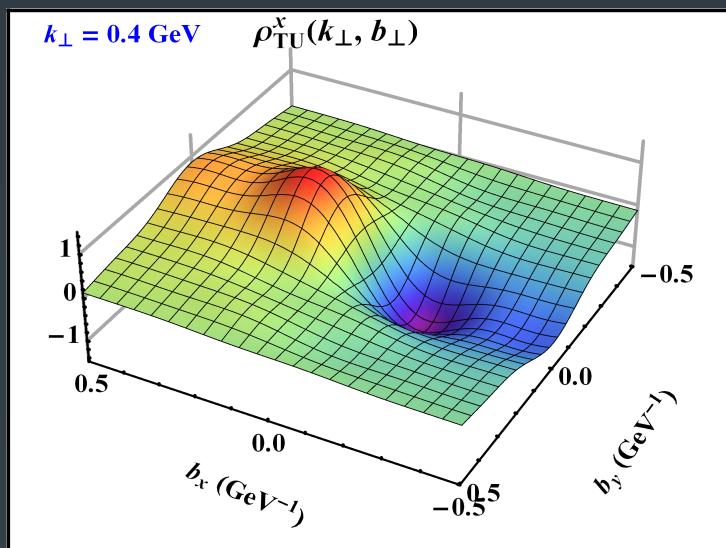
Longitudinally polarized quark in a transversely polarized target state

Behave similarly as  $\rho_{\text{LT}}$ , with sign difference, analytic expression slightly different

TMD limit related to worm gear function  $g_{1\text{T}}$  (transverse helicity)

More, AM, Nair, PRD 95, 074039 (2017)

# Numerical Result : $\rho_{\text{TU}}^x$



Unpolarized quark in transversely polarized target state

TMD limit related to Sivers function (T odd), and GPD limit to H and E with other functions (T even). In our model Sivers function is zero

Dipole behaviour in  $b$  space similar to spectator model, behaves differently in  $k$  space

# Gluon Wigner Distributions

Gluon GTMDs have been discussed in the context of diffractive vector meson production and Higgs production at Tevatron and LHC

Martin, Ryskin, Teubner, PRD 62, 014022 (2000);  
Khoze, Martin, Ryskin, EPJC 14, 525 (2000)

Wigner distribution for gluons

Ji, Xiong, Yuan, PRD 88, 014041 (2013); Lorce and Pasquini,  
JHEP 09, 138 (2013)

Gluon Wigner distributions at small  $x$  in hard diffractive dijet production in DIS ,  
and in DVCS

Hatta, Xiao, Yuan, PRL 116, 202301 (2016); 1703.02085 [hep-ph]

# Gluon Husimi Distributions

Gluon Wigner, Husimi and GTMDs at small  $x$

Hagiwara, Hatta, Ueda, PRD 94 , 094036 (2016)

Husimi distributions have a Gaussian regularization factor in the integrand that keeps them positive in the entire range of transverse space coordinate

Upon integration over  $b_{\perp}$  they do not reduce to TMDs

However, upon integration over both  $b_{\perp}$  and  $k_{\perp}$  they give the pdfs

# Gluon Wigner Distributions

$$\begin{aligned}
 xW_{\sigma,\sigma'}(x, k_{\perp}, b_{\perp}) &= \int \frac{d^2\Delta_{\perp}}{(2\pi)^2} e^{-i\Delta_{\perp} \cdot b_{\perp}} \int \frac{dz^- d^2z_{\perp}}{2(2\pi)^3 p^+} e^{ik \cdot z} \\
 &\times \left\langle p^+, -\frac{\Delta_{\perp}}{2}, \sigma' \left| \Gamma^{ij} F^{+i} \left( -\frac{z}{2} \right) F^{+j} \left( \frac{z}{2} \right) \right| p^+, \frac{\Delta_{\perp}}{2}, \sigma \right\rangle \Big|_{z^+=0}
 \end{aligned}$$

Where  $F^{+j}$  is the gluon field strength tensor and  $\Gamma^{ij}$  denote the polarization combinations

Ji, Xiong, Yuan, PRD 88, 014041 (2013); Lorce and Pasquini, JHEP 09, 138 (2013)

Gauge links are needed for color gauge invariance, two gauge links for each gluon correlator

Process dependence and color structure more involved than quark Wigner distributions

We choose light cone gauge and take the gauge link to be unity

# Gluon Correlators

Gluon-gluon correlator in terms of overlaps of two-particle LFWFs

$$\Gamma^{ij} = \delta^{ij}$$

$$W_{\sigma\sigma'}^1(x, k_{\perp}, \Delta_{\perp}) = - \sum_{\sigma_1, \lambda_1, \lambda_2} \left[ \Psi_{\sigma_1 \lambda_1}^{*\sigma'}(\hat{x}, \hat{q}'_{\perp}) \Psi_{\sigma_1 \lambda_2}^{\sigma}(\hat{x}, \hat{q}_{\perp}) \left( \epsilon_{\lambda_2}^1 \epsilon_{\lambda_1}^{*1} + \epsilon_{\lambda_2}^2 \epsilon_{\lambda_1}^{*2} \right) \right]$$

$$\Gamma^{ij} = \epsilon_{\perp}^{ij}$$

$$W_{\sigma\sigma'}^2(x, k_{\perp}, \Delta_{\perp}) = -i \sum_{\sigma_1, \lambda_1, \lambda_2} \left[ \Psi_{\sigma_1 \lambda_1}^{*\sigma'}(\hat{x}, \hat{q}'_{\perp}) \Psi_{\sigma_1 \lambda_2}^{\sigma}(\hat{x}, \hat{q}_{\perp}) \left( \epsilon_{\lambda_2}^1 \epsilon_{\lambda_1}^{*2} - \epsilon_{\lambda_2}^2 \epsilon_{\lambda_1}^{*1} \right) \right]$$

# Gluon Correlators

$\Gamma^{RR}$

$$W_{\sigma\sigma'}^3(x, k_{\perp}, \Delta_{\perp}) = - \sum_{\sigma_1, \lambda_1, \lambda_2} \left[ \Psi_{\sigma_1 \lambda_1}^{*\sigma'}(\hat{x}, \hat{q}'_{\perp}) \Psi_{\sigma_1 \lambda_2}^{\sigma}(\hat{x}, \hat{q}_{\perp}) \epsilon_{\lambda_2}^R \epsilon_{\lambda_1}^{*R} \right]$$

$\Gamma^{LL}$

$$W_{\sigma\sigma'}^4(x, k_{\perp}, \Delta_{\perp}) = - \sum_{\sigma_1, \lambda_1, \lambda_2} \left[ \Psi_{\sigma_1 \lambda_1}^{*\sigma'}(\hat{x}, \hat{q}'_{\perp}) \Psi_{\sigma_1 \lambda_2}^{\sigma}(\hat{x}, \hat{q}_{\perp}) \epsilon_{\lambda_2}^L \epsilon_{\lambda_1}^{*L} \right]$$

$$\hat{x} = (1 - x), \quad \hat{q}_{\perp} = -q_{\perp} \quad \epsilon_{\lambda}^{R(L)} = \epsilon_{\lambda}^1 \pm i\epsilon_{\lambda}^2$$

Wigner distributions are the Fourier transforms of these correlators



# Gluon Wigner Distributions

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Unpolarized target and different gluon polarization

$$W_{UU}(x, k_{\perp}, b_{\perp}) = \frac{1}{2} \left[ W^1(x, k_{\perp}, b_{\perp}, \hat{e}_z) + W^1(x, k_{\perp}, b_{\perp}, -\hat{e}_z) \right]$$

$$W_{UL}(x, k_{\perp}, b_{\perp}) = \frac{1}{2} \left[ W^2(x, k_{\perp}, b_{\perp}, \hat{e}_z) + W^2(x, k_{\perp}, b_{\perp}, -\hat{e}_z) \right]$$

$$W_{UT}^{(R)}(x, k_{\perp}, b_{\perp}) = \frac{1}{2} \left[ W^3(x, k_{\perp}, b_{\perp}, \hat{e}_z) + W^3(x, k_{\perp}, b_{\perp}, -\hat{e}_z) \right]$$

$$W_{UT}^{(L)}(x, k_{\perp}, b_{\perp}) = \frac{1}{2} \left[ W^4(x, k_{\perp}, b_{\perp}, \hat{e}_z) + W^4(x, k_{\perp}, b_{\perp}, -\hat{e}_z) \right]$$

More, AM, Nair, in preparation

# Numerical Results

We calculate the gluon Wigner distributions for a quark state dressed with a gluon

Use Levin's method of integration for better convergence, take

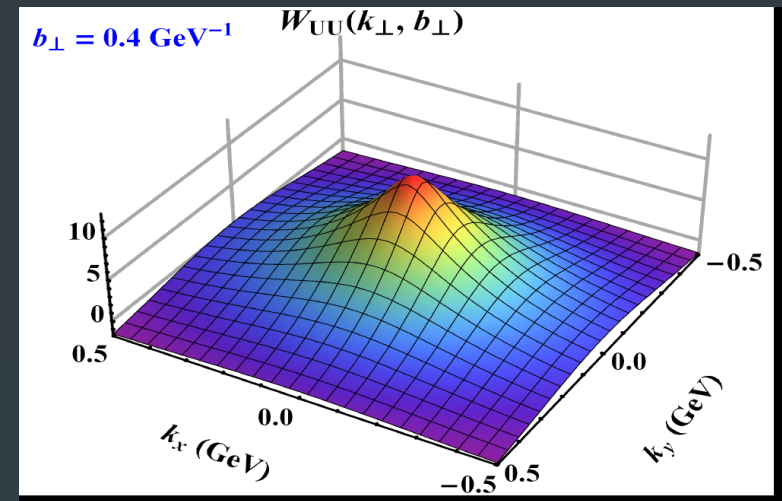
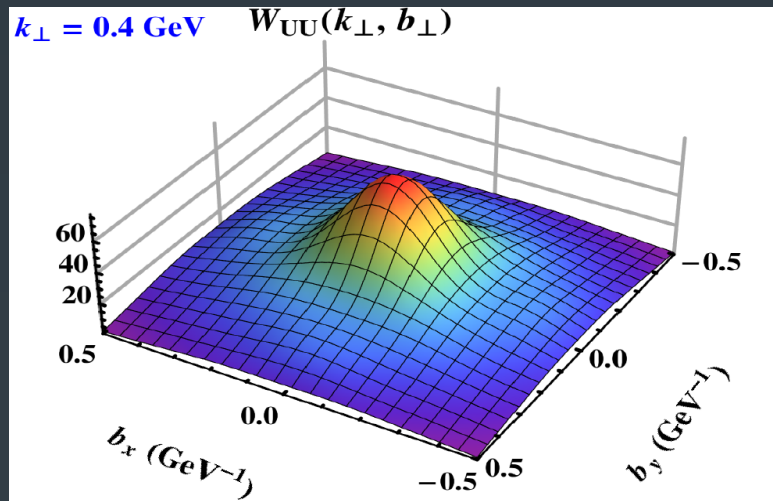
$$\Delta_{max} = 20 \text{ GeV}$$

For smaller values of the  $\Delta_{max}$  results for the unpolarized and longitudinally polarized gluon agree with the calculation presented in

AM, Nair, Ojha, PRD 91, 054018 (2015)

Integrate over  $x$ , to study the transverse Wigner distributions

# Numerical Results : $W_{UU}$



Depending on gauge link, Weizsacker-Williams type (++) or dipole type (+-)

Related to the same gluon OAM

Hatta, Nakagawa, Yuan, Zhao;  
1612.02445 [hep-ph]

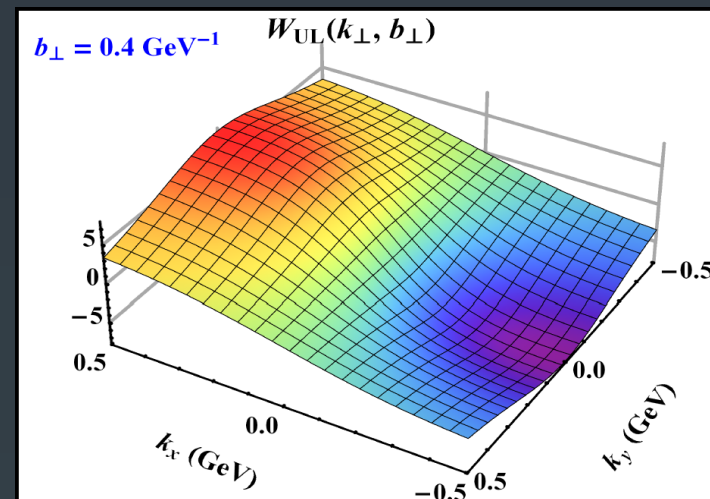
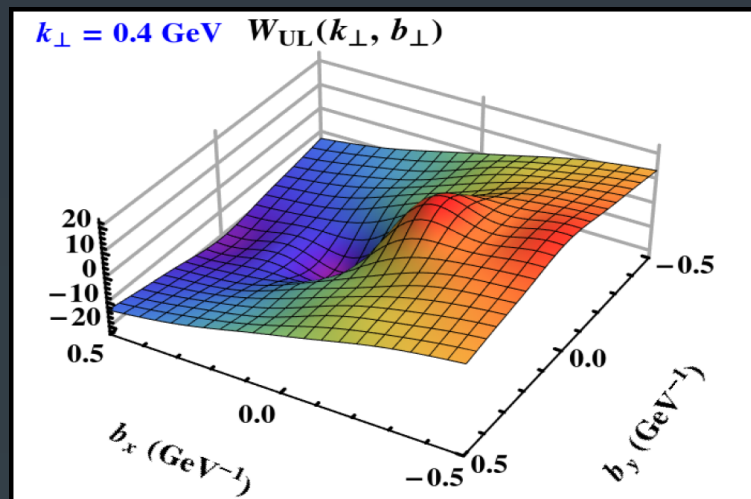
Plots in  $b$  space for fixed value of  $k_{\perp}$  in  $y$  direction, and in  $k$  space for fixed  $b$  in  $y$  direction

$k_{\perp}$  in  $y$  direction, and in  $k$  space

$m=0.33 \text{ GeV}$ , result independent of cutoff

More, AM, Nair, work in progress

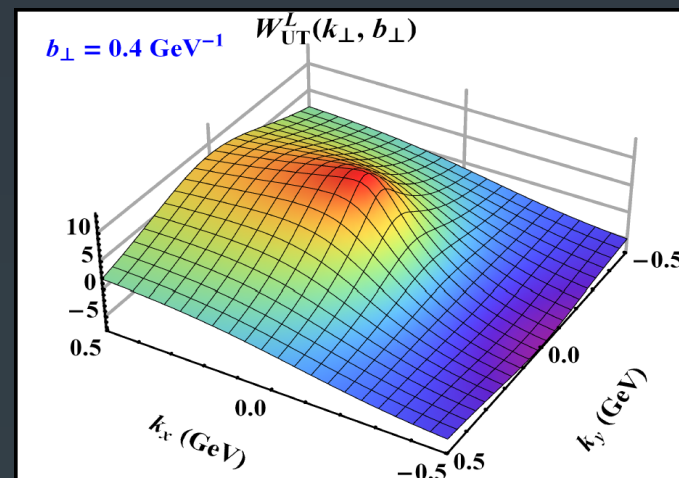
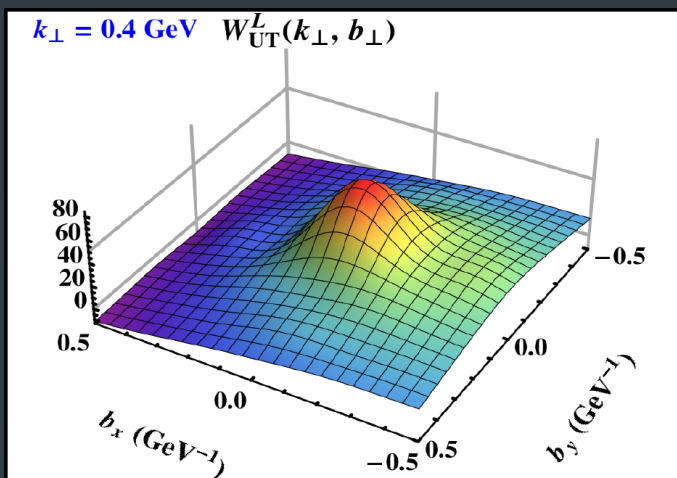
# Numerical Results : $W_{UL}$



Dipole structure both in  $b$  space and in  $k$  space

More, AM, Nair, work in progress

# Numerical Results : $W_{UT}^L$



Dipole structure in  $k$  space

Polarity opposite in  $W_{UT}^R$

More, AM, Nair, work in progress

# Summary and Conclusion

We calculated the Wigner distribution of quarks and gluons taking the state to be a quark dressed at one loop with a gluon, using overlaps of light-front wave functions

This is a simple composite spin  $\frac{1}{2}$  system, having a gluonic degree of freedom

We calculated the Wigner distributions for different polarization of the target and the quark : compared with other model calculations

In general behaviour in  $b$  space similar to other models

Numerical integration with better convergence : removal of the cutoff dependence of an earlier calculation

Work in progress : Wigner distribution for gluons