Quark and Gluon Spin Dependent GPDs in a Flexible Spectator Model for Deeply Virtual Lepton Scattering Processes

Gary R. Goldstein
Tufts University

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Jlab, Newprt News, VA

Collaborators: Simonetta Liuti[2],
Osvaldo Gonzalez Hernandez[3],
Tracy McAskill[4]
Jon Poage[1]
Abha Rajan[2]
1) Tufts University, U.S. Dep't of Transportation
2) University of Virginia
3) Old Dominion/JLab
4) Wellesley College

- GRG, O. Gonzalez-Hernandez, S.Liuti, PRD(2015) arXiv:1311.0483
- GRG, O. Gonzalez-Hernandez, S.Liuti, arXiv:1401.0438
- Ahmad, GRG, Liuti, PRD79, 054014, (2009)
- Gonzalez, GRG, Liuti PRD84, 034007 (2011)
- GRG, Gonzalez, Liuti, PRD91, 114013 (2015)
- GRG, Gonzalez Hernandez, Liuti, J. Phys. G: Nucl. Part. Phys. **39** 115001 (2012)
- Gonzalez Hernandez, Liuti, GRG, Kathuria, PRC88, 065206 (2013)
- GRG, Liuti, PoS DIS2016 (2016) 238
- GRG, Liuti, EPJ Web Conf. 112 (2016) 01009
- T. McAskill, Tufts U. dissertation (2014)
- GRG, Gonzalez Hernandez, Liuti, McAskill, Proc. NuFAct09, 0911.0495
- J.Poage, Tufts U. dissertation (2016)

OUTLINE

- GPDs, Model– Reggeized spectator "flexible parameterization"
- Recall Valence quarks: Chiral Even & Odd
- Neutrinos & π production - extend "flexible"
- Gluon & Sea GPDs
- Preliminary results
- Some Observable quantities

GPD definitions – 8 quark + 8 gluon (twist 2)

Momentum space nucleon matrix elements of quark field correlators

see, e.g. M. Diehl, Eur. Phys. J. C 19, 485 (2001).

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\, \bar\psi(-\tfrac{1}{2}z)\, \gamma^+ \psi(\tfrac{1}{2}z)\; |p,\lambda\rangle \Big|_{z^+=0,\, \mathbf{z}_T=0}$$

$$= \frac{1}{2P^+} \bar u(p',\lambda') \left[H^q \gamma^+ + E^q \frac{i\sigma^{+\alpha}\Delta_\alpha}{2m} \right] u(p,\lambda),$$

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\, \bar\psi(-\tfrac{1}{2}z)\, \gamma^+\gamma_5\, \psi(\tfrac{1}{2}z)\; |p,\lambda\rangle \Big|_{z^+=0,\, \mathbf{z}_T=0}$$

$$= \frac{1}{2P^+} \bar u(p',\lambda') \left[\tilde H^q \gamma^+\gamma_5 + \tilde E^q \frac{\gamma_5 \Delta^+}{2m} \right] u(p,\lambda),$$

Chiral even GPDs
-> Ji sum rule

$$\left\langle J_q^x \right\rangle = \tfrac{1}{2}\int dx \left[H(x,0,0) + E(x,0,0) \right] x$$

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\, \bar\psi(-\tfrac{1}{2}z)\, i\sigma^{+i}\, \psi(\tfrac{1}{2}z)\; |p,\lambda\rangle \Big|_{z^+=0,\, \mathbf{z}_T=0}$$

$$= \frac{1}{2P^+} \bar u(p',\lambda') \left[H_T^q\, i\sigma^{+i} + \tilde H_T^q \frac{P^+\Delta^i - \Delta^+ P^i}{m^2} + E_T^q \frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2m} + \tilde E_T^q \frac{\gamma^+ P^i - P^+\gamma^i}{m} \right] u(p,\lambda$$

Chiral odd GPDs
-> transversity
How to measure and/or parameterize them?

Normalizing & constraining quark GPDs – Chiral even

Form factor, Forward limit

$$\int_0^1 H_q(x,\xi,t)\,dx = F_1^q(t), \quad H_q(x,0,0) = q(x)$$

Integrates to charge

$$\int_0^1 E_q(x,\xi,t)\,dx = F_2^q(t)$$

→Anomalous magnetic moments

$$\int_0^1 \tilde{H}_q(x,\xi,t)\,dx = g_A^q(t), \quad \tilde{H}_q(x,0,0) = \Delta q(x) = q_{\Rightarrow}^{\rightarrow}(x) - q_{\Rightarrow}^{\leftarrow}(x)$$

Integrates to axial charge

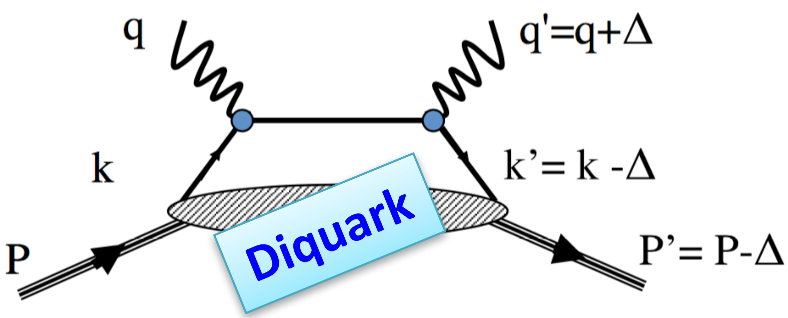
$$\int_0^1 \tilde{E}_q(x,\xi,t)\,dx = g_P^q(t)$$

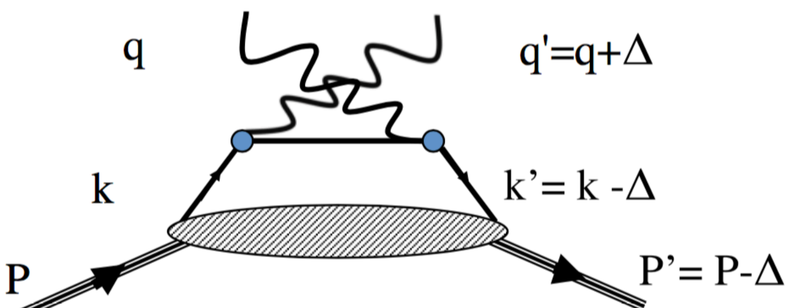
The Model for valence quarks– Reggeized Diquarks

The Model – first for Chiral Even –
Reggeized Diquark Spectator
Diquark: Color anti-3, scalar & axial vector

"Flexible" covariant model

Gonzalez, GG, Liuti PRD84, 034007 (2011)





$$H = \mathcal{N}\frac{1-\zeta/2}{1-X} \times \int d^2k_\perp \frac{[(m+MX)(m+M\frac{X-\zeta}{1-\zeta}) + \mathbf{k}_\perp\cdot\tilde{\mathbf{k}}_\perp]}{(k^2-M_\Lambda^2)^2(k'^2-M_\Lambda^2)^2} + \frac{\zeta^2}{4(1-\zeta)}E, \quad (27)$$

$$E = \mathcal{N}\frac{1-\zeta/2}{1-X}\int d^2k_\perp \times \frac{-2M(1-\zeta)[(m+MX)\frac{\tilde{k}\cdot\Delta}{\Delta_\perp^2} - (m+M\frac{X-\zeta}{1-\zeta})\frac{k_\perp\cdot\Delta}{\Delta_\perp^2}]}{(k^2-M_\Lambda^2)^2(k'^2-M_\Lambda^2)^2}, \quad (28)$$

$$\tilde{H} = \mathcal{N}\frac{1-\zeta/2}{1-X} \times \int d^2k_\perp \frac{[(m+MX)(m+M\frac{X-\zeta}{1-\zeta}) - \mathbf{k}_\perp\cdot\tilde{\mathbf{k}}_\perp]}{(k^2-M_\Lambda^2)^2(k'^2-M_\Lambda^2)^2} + \frac{\zeta^2}{4(1-\zeta)}\tilde{E}, \quad (29)$$

$$\tilde{E} = \mathcal{N}\frac{1-\zeta/2}{1-X}\int d^2k_\perp \times \frac{-\frac{4M(1-\zeta)}{\zeta}[(m+MX)\frac{\tilde{k}\cdot\Delta}{\Delta_\perp^2} + (m+M\frac{X-\zeta}{1-\zeta})\frac{k_\perp\cdot\Delta}{\Delta_\perp^2}]}{(k^2-M_\Lambda^2)^2(k'^2-M_\Lambda^2)^2}, \quad (30)$$

Reggeizing diquark model

Diquark+quark & fixed masses (e.g. at ξ=0)

$$H^{M^q_\Lambda}_{M^q_X,m_q} = \mathcal{N}_q \int \frac{d^2k_\perp}{1-x} \frac{[(m_q+Mx)(m_q+Mx)+\mathbf{k}_\perp\cdot\tilde{\mathbf{k}}_\perp]}{[\mathcal{M}_q^2(x)-k_\perp^2/(1-x)]^2[\mathcal{M}_q^2(x)-\tilde{k}_\perp^2/(1-x)]^2},$$

$$E^{M^q_\Lambda}_{M^q_X,m_q} = \mathcal{N}_q \int \frac{d^2k_\perp}{1-x} \frac{-2M/\Delta_\perp^2[(m_q+Mx)\tilde{\mathbf{k}}_\perp\cdot\mathbf{\Delta}_\perp-(m_q+Mx)\mathbf{k}_\perp\cdot\mathbf{\Delta}_\perp]}{[\mathcal{M}_q^2(x)-k_\perp^2/(1-x)]^2[\mathcal{M}_q^2(x)-\tilde{k}_\perp^2/(1-x)]^2},$$

Diquark mass “spectrum”
as in Brodsky, Close & Gunion
Phys. Rev. D8, 3678 (1973)

$$F_T^q(X,\zeta,t) = \mathcal{N}_q \int_0^\infty dM_X^2\, \rho(M_X^2)\, F_T^{(m_q,M_\Lambda^q)}(X,\zeta,t;M_X).$$

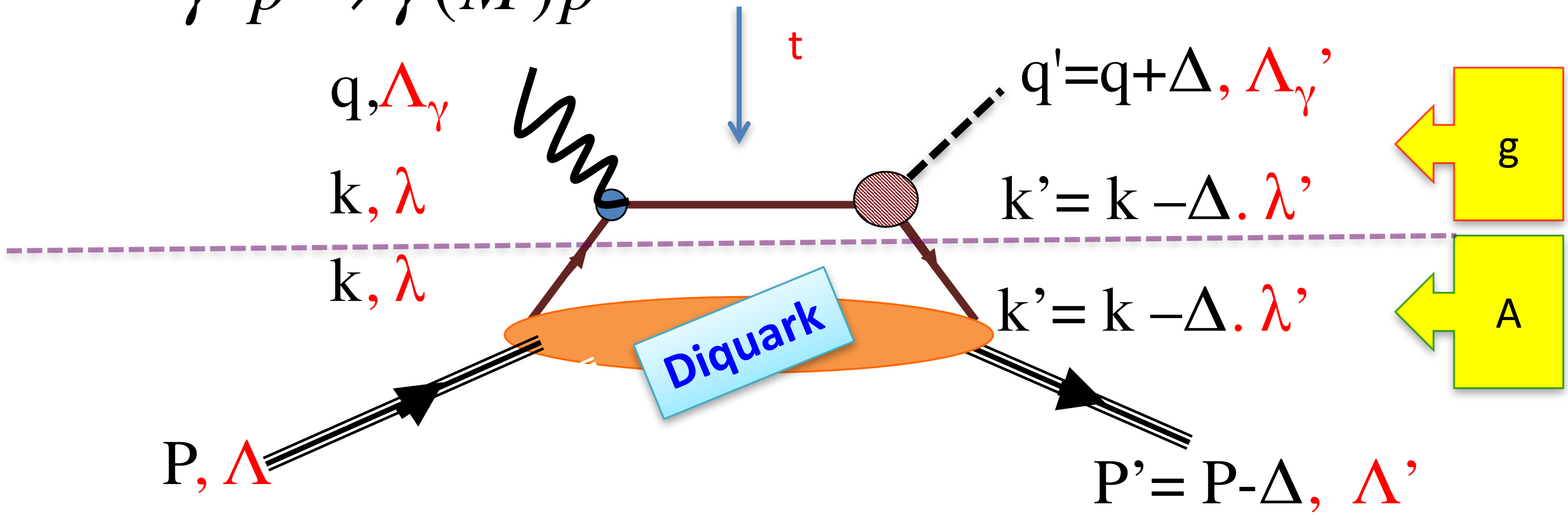
$$\rho(M_X^2) \approx \begin{cases} (M_X^2)^\alpha & M_X^2 \to \infty \\ \delta(M_X^2-\overline{M}_X^2) & M_X^2 \text{ few GeV}^2 \end{cases}$$

$$F_T^q(X,\zeta,t) \approx \mathcal{N}_q X^{-\alpha_q+\alpha_q'(X)t} F_T^{(m_q,M_\Lambda^q)}(X,\zeta,t;\overline{M}_X) = R_{p_q}^{\alpha_q,\alpha_q'}(X,\zeta,t)\, G^{M_\Lambda}_{M_X,m}(X,\zeta,t)$$

R✖Dq

Connecting to exclusive processes (DVCS, DVMP…)

$$\gamma^* p \to \gamma(M) p'$$



Convolution of "hard part" with quark-proton **Helicity** amplitudes

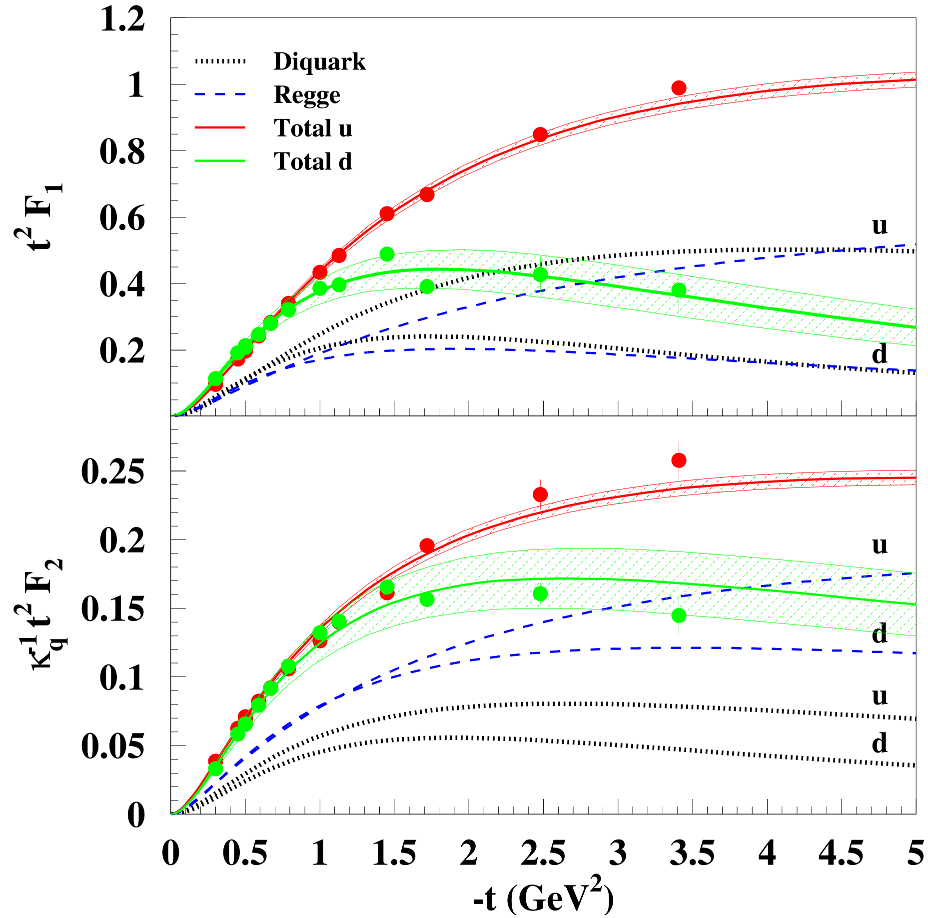
$$f_{\Lambda_\gamma,\Lambda;\Lambda'_\gamma,\Lambda'} = \sum_{\lambda,\lambda'} g_{\lambda,\lambda'}^{\Lambda_\gamma,\Lambda'_{\gamma(M)}}(x,k_T,\zeta,t;Q^2) \otimes A_{\Lambda',\lambda';\Lambda,\lambda}(x,k_T,\zeta,t),$$

$\lambda = +(-)\ \lambda'$ chiral even (odd)

See Ahmad, et al. PRD75, 0904003 (2007); ibid, EPJC63, 407 (2009) .

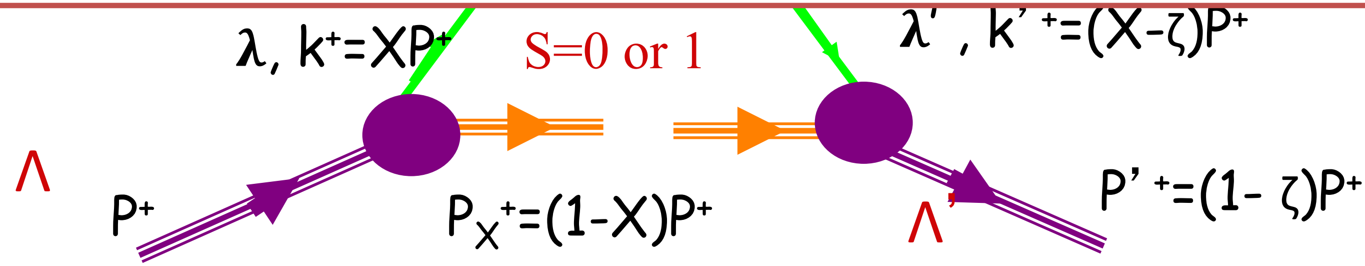
see Ahmad, GG, Liuti, PRD79, 054014, (2009)
for first chiral odd GPD parameterization
Gonzalez, GG, Liuti PRD84, 034007 (2011) chiral even GPD

EM Form Factors (t dependence)
precision mesurements→tighter parameters



O.Gonzalez, GRG, S.Liuti, K.Kathuria PRC88, 065206(2013)
data: G.D. Cates, et al. PRL106,252003 (2011).

Procedure to construct **Chiral Even GPDs** & observables Spectator color anti-3 diquark model & Reggeization



Product of diquark l.c.w.f.'s $\rightarrow A_{\Lambda\lambda;\Lambda'\lambda'=\lambda}$

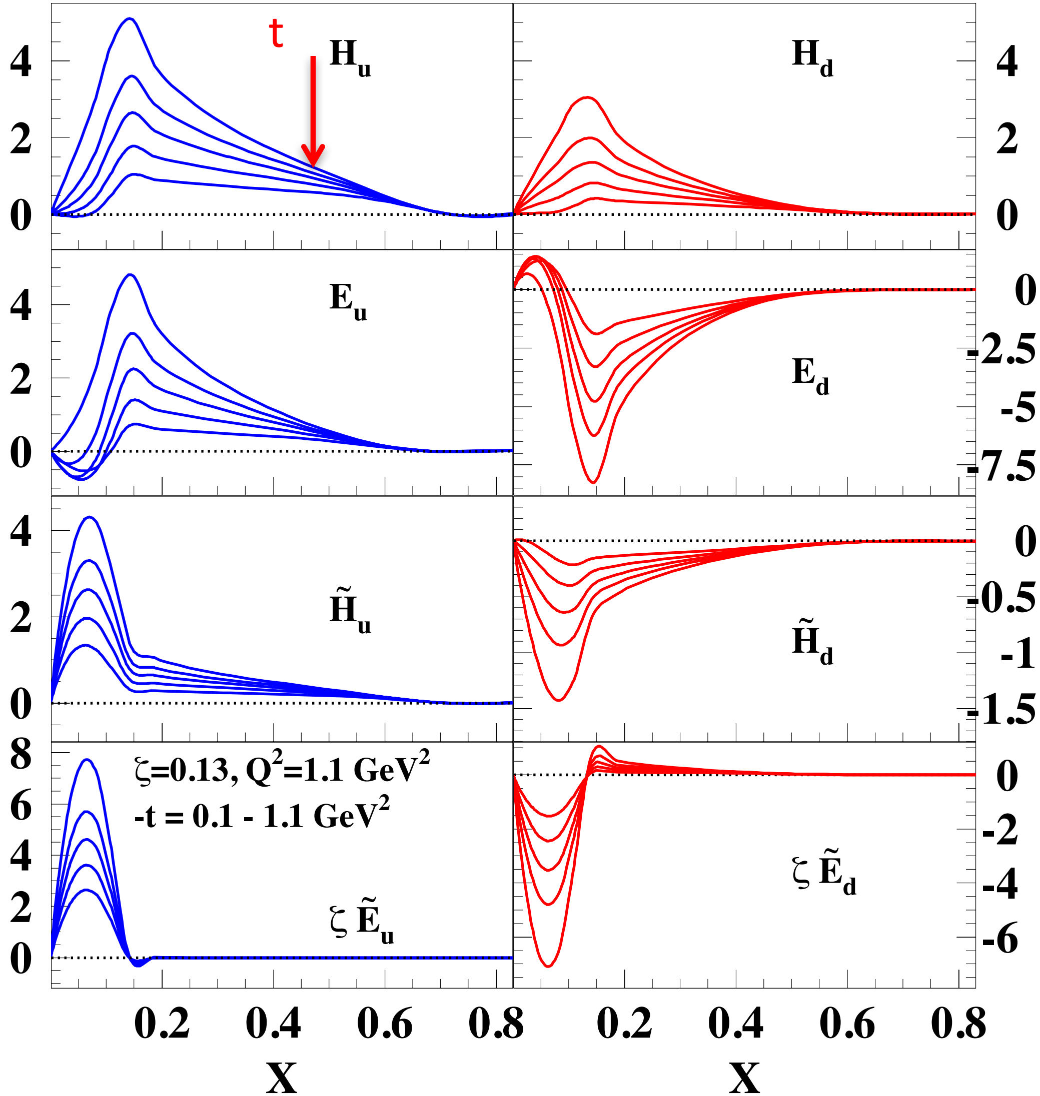
$A_{\Lambda\lambda;\Lambda'\lambda}$ → chiral even GPDs + Evolution

g $\otimes$ A $\rightarrow$ exclusive process helicity amps

pdf's, FF's, $d\sigma/d\Omega$ & Asymmetries: parameters & predictions

$A_{\Lambda\lambda;\Lambda'\lambda}$→ all chiral even GPDs→DVCS, DVMP

Chiral even GPDs



From GPDs with evolution to Compton Form Factors

↓

CFFs to helicity amps

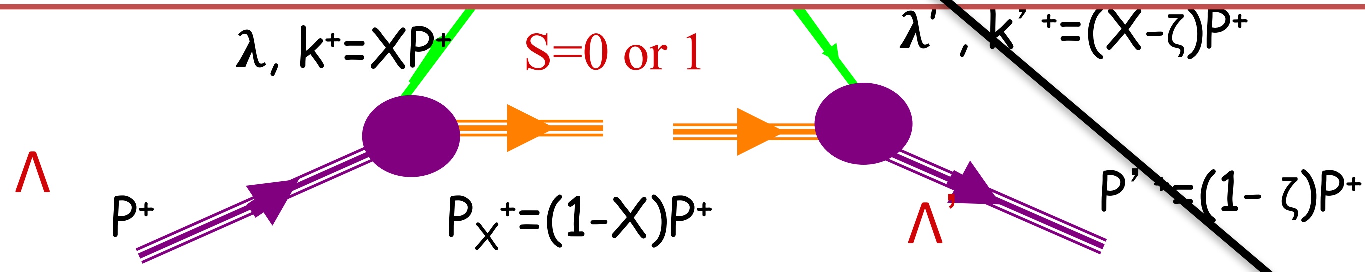
↓

helicity amps to observables

↓

<-> parameters

Procedure to construct **Chiral Even GPDs** & observables
Spectator color anti-3 diquark model & Reggeization



Product of diquark l.c.w.f.'s $\rightarrow$ $A_{\Lambda\,\lambda;\,\Lambda'\,\lambda'=-\lambda}$ — **Odd**

$A_{\Lambda\,\lambda;\,\Lambda'\,\lambda'=-\lambda}$ → chiral **Odd** GPDs + Evolution

g ⊗ A → exclusive process helicity amps

pdf's, FF's, dσ/dΩ & Asymmetries: parameters & predictions

$A_{\Lambda\,\lambda;\,\Lambda'\,\lambda'=-\lambda}$ → all chiral **Odd** GPDs→DVCS, DVMP

Chiral odd quark GPDs

One question is: how do we normalize chiral-odd GPDs?

The only Physical constraints on the various chiral-odd GPDs are

Forward limit

$$H_T(x,0,0) = q_{\Uparrow}^{\uparrow}(x) - q_{\Uparrow}^{\downarrow}(x) = h_1(x)$$

Transversity

Form Factors

Integrates to tensor charge δ_q

$$\int H_T^q(x,\xi,t)\,dx = \delta q(t)$$

$$\int \bar{E}_T^q(x,\xi,t)\,dx = \int\left(2\tilde{H}_T^q + E_T^q\right)dx = \kappa_T^q(t)$$

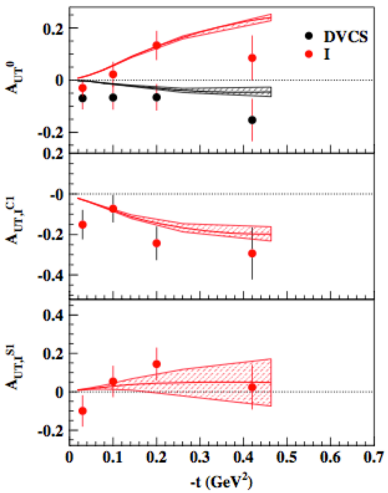
"transverse moment" κ_T^q

$$\int \tilde{E}_T(x,\xi,t)\,dx = 0$$

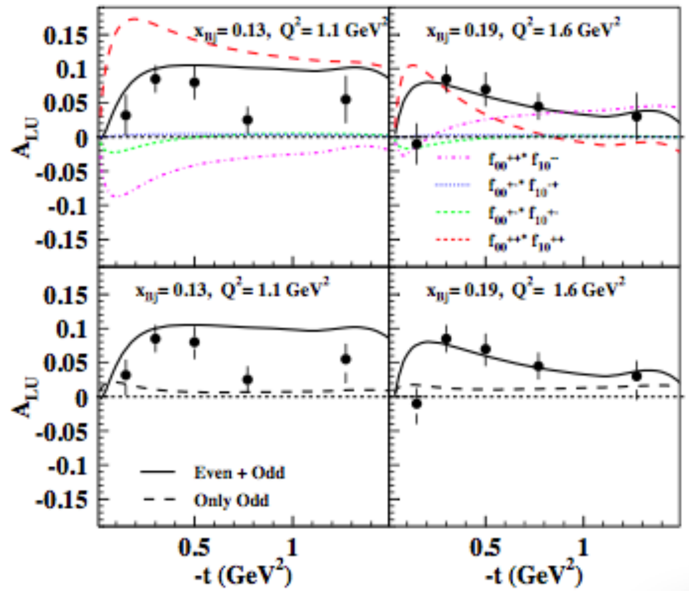
No direct interpretation of E_T .

Some results GG, Gonzalez Hernandez, Liuti, PRD84, 034007 (2011) . . .(2016);

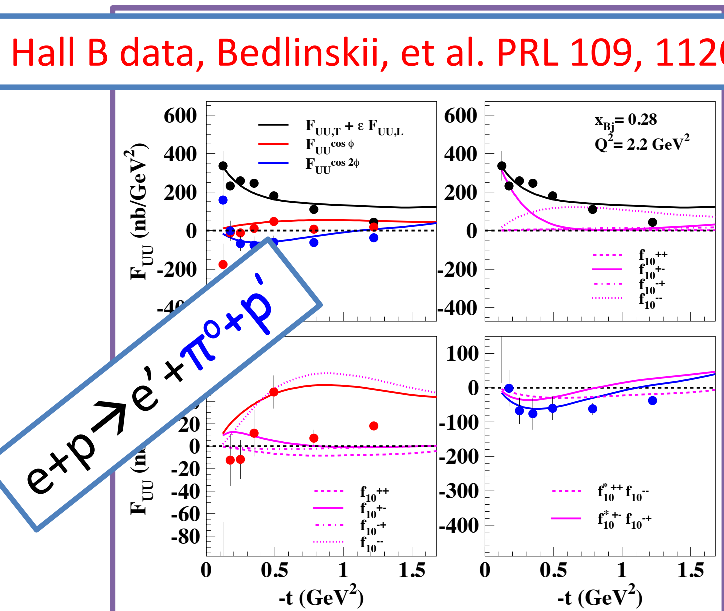
Beam charge asymmetry
HERMES A_{UT} coefficients

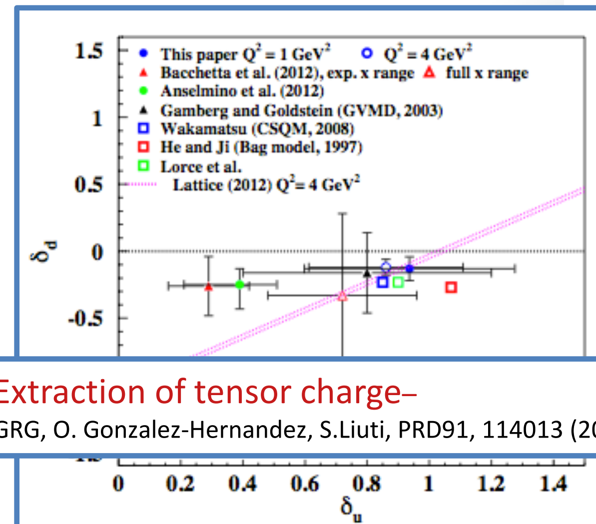


Beam spin asymmetry
(CLAS data –DeMasi, et al.)



Hall B data, Bedlinskii, et al. PRL 109, 112001 (2012)



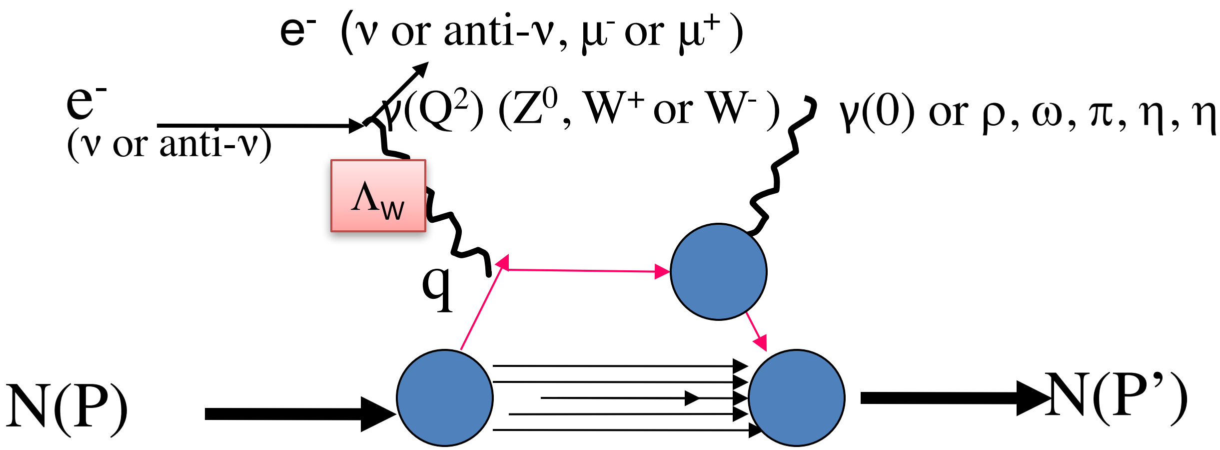


Extraction of tensor charge–
GRG, O. Gonzalez-Hernandez, S.Liuti, PRD91, 114013 (2015)

Extending Phenomenology of Flexible Model

Neutrino Production - What Processes? How to Measure?

From exclusive electroproduction to neutrino-production



$$f_{\Lambda_w,\Lambda;0,\Lambda'} = \sum_{\lambda,\lambda'} g_{\Lambda_w,\lambda;0,\lambda'}(X,\zeta,t,Q^2) \otimes A_{\Lambda',\lambda';\Lambda,\lambda}(X,\zeta,t)$$

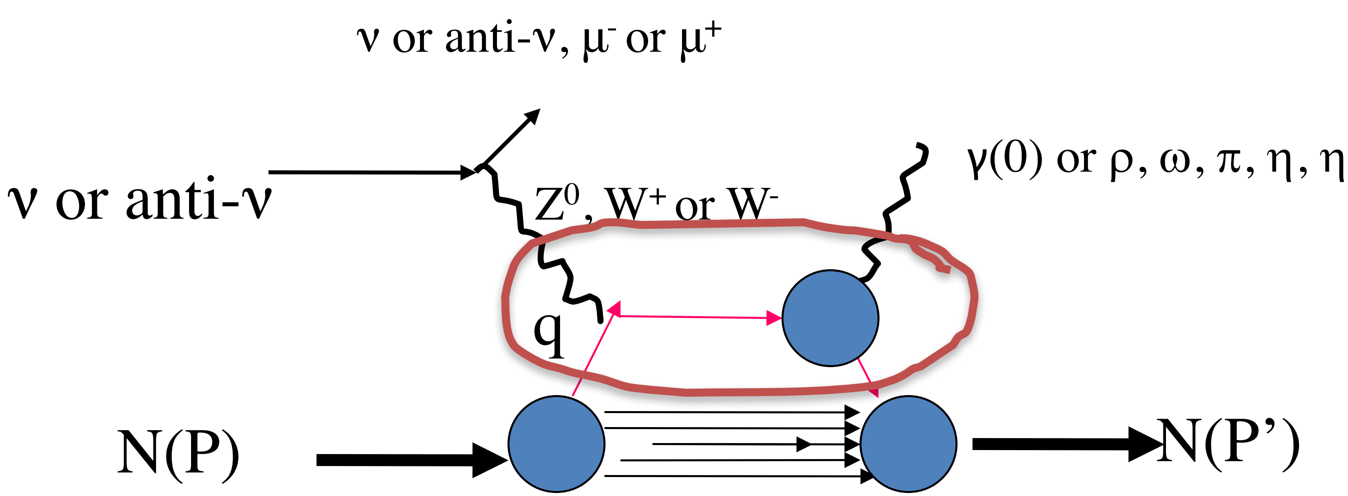
Λ_W Λ_W

$g_{\Lambda W,\lambda;0,\lambda'}$ = same helicity structure as W+p → π+p’

with $W^\mu \sim c_V\,\gamma^\mu - c_A\,\gamma^\mu\gamma^5$ & $\pi \sim \varphi_A q'_\nu \gamma^\nu\gamma^5 + \varphi_P \gamma^5$

$A_{\Lambda',\lambda';\Lambda,\lambda}$ → 2 x 6 helicity amps $\supset$ GPDs

Hard subprocesses



Consider π production $V^* + q \rightarrow \pi + q'$ with DA for $\pi \supset q'_\mu \gamma^\mu \gamma^5$ twist2 & γ^5 twist 3

Helicity amps: **Longitudinal** with non-flip quark – leading ~ Q/M: $q'_\mu \gamma^\mu \gamma^5$

Transverse with flip quark ~ constant (or Δ^2/Q^2 cross channel) : γ^5

Transverse non-flip ~ Δ/M : : $q'_\mu \gamma^\mu \gamma^5$ (Δ by angular momentum conservation)

Longitudinal with flip ~ Δ/Q (Δ by angular momentum conservation)

Question: Does Twist 3 dominate ν production of π as in electroproduction?

i.e. will measurements be sensitive to this twist 3 DA π mechanism?

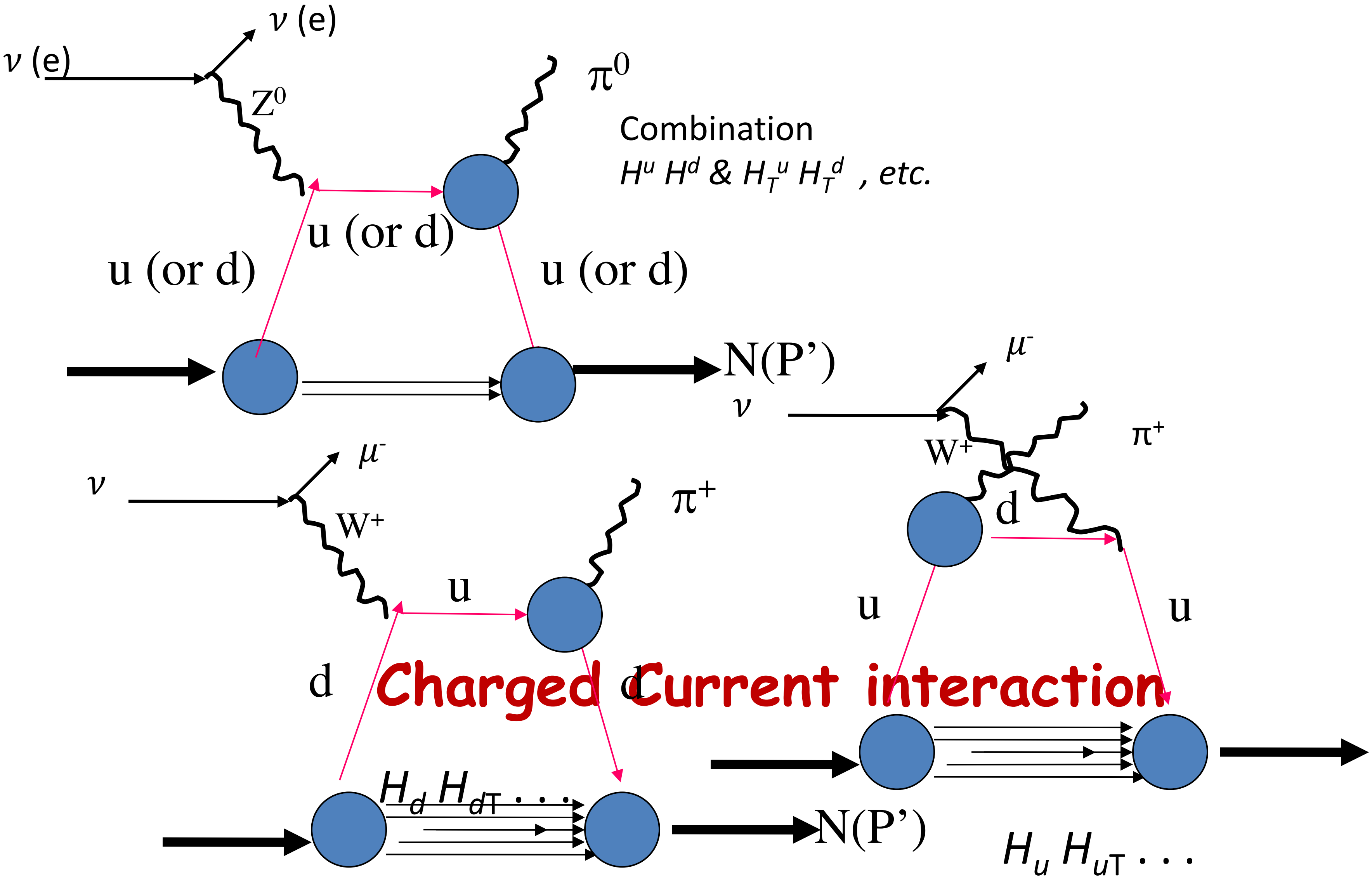
Will Transversity be important?

See Kopeliovich, Schmidt, Siddikov: Twist 2 & longitudinal dominate.

See Pire & Szyminowski: Twist 3 dominates for heavy quarks.

Neutral Current interaction

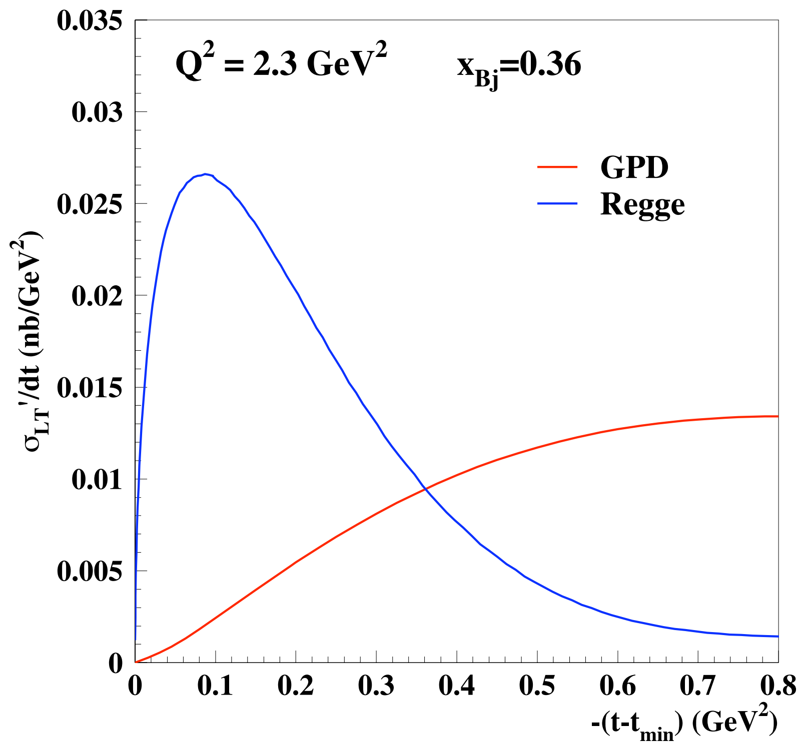
- NC & parity violating electroproduction



Charged Current interaction

To Measure Chiral Even-Odd Interference

Contribution to π^0 exclusive ν production cross section,
$d\sigma_{L'T}/dt$ from model GPDs (c.f. Regge description).
Few GeV region accessible in current & future neutrino experiments.



GRG, Gonzalez-Hernandez, Liuti, McAskill, Proc. NuFAct09, 0911.0495; in preparation 2017.

Gluon GPDs

Gluon GPDs

$$\frac{1}{\bar{P}^+}\int \frac{dz^-}{2\pi} e^{ix\bar{P}^+z^-}\langle P',\Lambda'|G^{+i}(-\frac{1}{2}z)G^{+i}(\frac{1}{2}z)|P,\Lambda\rangle\Big|_{z^+=0,\vec{z}_T=0} =$$

$$\frac{1}{2\bar{P}^+}\bar{U}(P',\Lambda')[H^g(x,\xi,t)\gamma^+ + E^g(x,\xi,t)\frac{i\sigma^{+\alpha}(-\Delta_\alpha)}{2M}]U(P,\Lambda)$$

Even t-channel parity & Gluon helicity conserving

$$\frac{-i}{\bar{P}^+}\int \frac{dz^-}{2\pi} e^{ix\bar{P}^+z^-}\langle P',\Lambda'|G^{+i}(-\frac{1}{2}z)\tilde{G}^{+i}(\frac{1}{2}z)|P,\Lambda\rangle\Big|_{z^+=0,\vec{z}_T=0} =$$

$$\frac{1}{2\bar{P}^+}\bar{U}(P',\Lambda')[\tilde{H}^g(x,\xi,t)\gamma^+\gamma_5 + \tilde{E}^g(x,\xi,t)\frac{\gamma_5(-\Delta^+)}{2M}]U(P,\Lambda)$$

Odd t-channel parity & Gluon helicity conserving

Gluon & Sea quark distributions Spectator Model

– generalize Regge-diquark spectator model

- N→ ***g*** + "color octet N" spectator ($8 \otimes 8 \supset 1$)

  (could be spin ½ or 3/2)
- (N→ ***anti-u*** + color 3 "tetraquark"uuud)

- How to normalize?

$$H_g(x,\xi,t)_{Q^2} \rightarrow H_g(x,0,0)_{Q^2} = xG(x)_{Q^2}$$

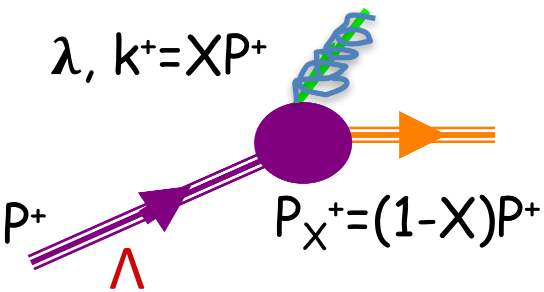
Evolution & small x phenomenology

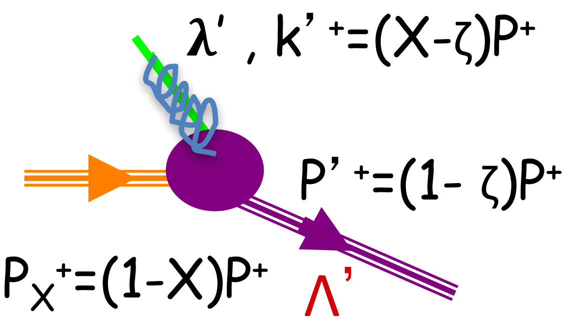
- Sea quark distributions $H_{anti\text{-}u}(x,0,0)$ . . .

- Use pdf's to fix x dependence

- Small x ~ Pomeron

Gluon 'vertex functions' $\mathcal{G}_{\Lambda x\,;\,\Lambda g,\,\Lambda}$





$\mathcal{G}_{+++}(x,\vec{k}_T^2)$	$-\frac{2}{\sqrt{2(1-X)}}\frac{(k_x-ik_y)}{X}$
$\mathcal{G}_{-++}(x,\vec{k}_T^2)$	$-\frac{2}{\sqrt{2(1-X)}}(M(1-X)-M_x)$
$\mathcal{G}_{++-}(x,\vec{k}_T^2)$	0
$\mathcal{G}_{-+-}(x,\vec{k}_T^2)$	$-\frac{2}{\sqrt{2(1-X)}}(1-X)\frac{(k_x-ik_y)}{X}$
$\mathcal{G}^*_{+++}(x,\vec{k}_T'^2)$	$-\frac{2}{\sqrt{2(1-X')}}\frac{(\tilde{k}_x+i\tilde{k}_y)}{X'}$
$\mathcal{G}^*_{-++}(x,\vec{k}_T'^2)$	$-\frac{2}{\sqrt{2(1-X')}}(M(1-X')-M_x)$
$\mathcal{G}^*_{++-}(x,\vec{k}_T'^2)$	0
$\mathcal{G}^*_{-+-}(x,\vec{k}_T'^2))$	$-\frac{2}{\sqrt{2(1-X')}}(1-X')\frac{(\tilde{k}_x+i\tilde{k}_y))}{X'}$

$$X' = \frac{X-\zeta}{1-\zeta},\ \tilde{k}_{i=1,2} = k_i - \frac{1-X}{1-\zeta}\Delta_i;$$

GG, Gonzalez Hernandez, Liuti, Poage, in progress

Construct Gluon helicity amps from Spectator Model, then GPDs

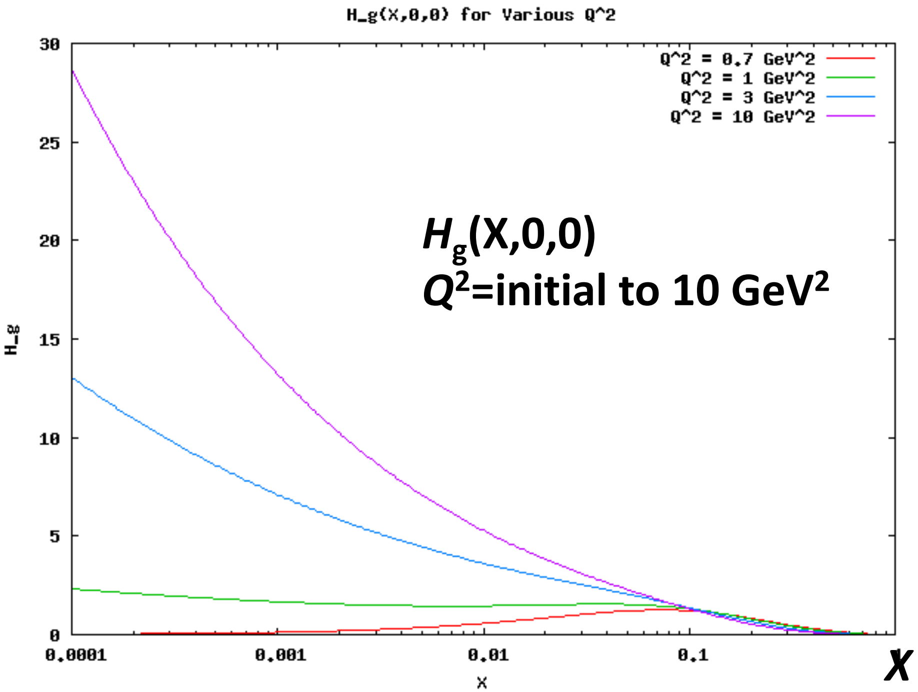
$$
\begin{aligned}
A^0_{++++} &= \mathcal{A}\Big(\frac{\vec{k}_\perp \cdot \vec{\tilde{k}}_\perp}{XX'} + ((1-X)M - M_X)((1-X')M - M_X)\Big) \\
A^0_{-+-+} &= \mathcal{A}\frac{\vec{k}_\perp \cdot \vec{\tilde{k}}_\perp}{XX'}((1-X)(1-X')) \\
A^0_{-+++} &= \mathcal{A}((1-X')((1-X)M - M_X)\frac{\tilde{k}_x + i\tilde{k}_y}{X'} \\
A^0_{++-+} &= \mathcal{A}((1-X)((1-X')M - M_X)\frac{(k_x - ik_y)}{X} \\
\text{with}\,\mathcal{A} &= \mathcal{N}\frac{1}{\sqrt{1-X}\sqrt{1-X'}(1-X)}\frac{1}{(k^2-M_\Lambda^2)^2(k'^2-M_\Lambda^2)^2} = \frac{\sqrt{1-\zeta}}{(1-X)^2}\frac{1}{(k^2-M_\Lambda^2)^2(k'^2-M_\Lambda^2)^2}
\end{aligned}
$$

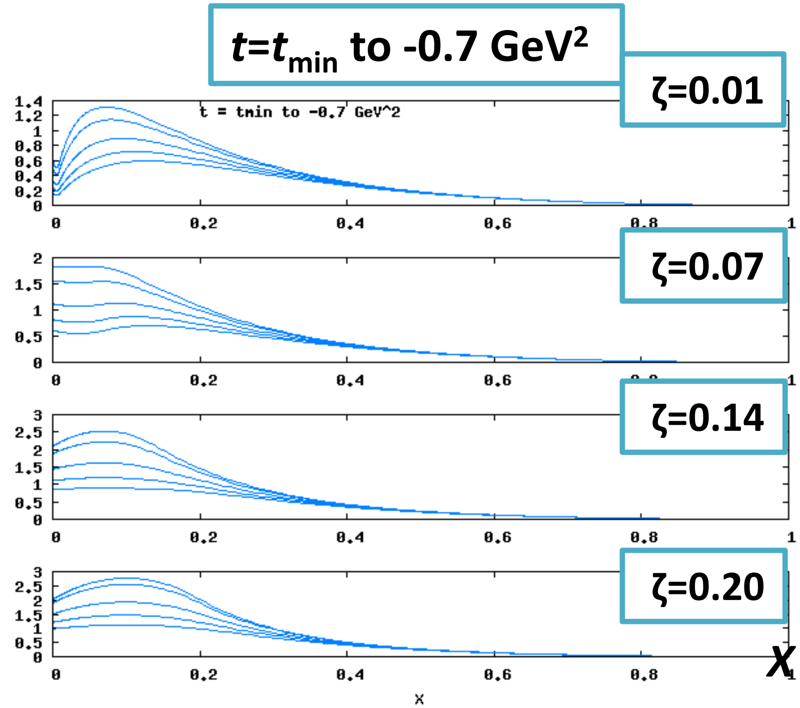
A^0: Unintegrated (k_T) Helicity Amps not yet GPDs

Invert Helicity amps to obtain GPDs

$$
\begin{aligned}
A_{++,++} &= \sqrt{1-\xi^2}\Big(\frac{H^g + \tilde{H}^g}{2} - \frac{\xi^2}{1-\xi^2}\frac{E^g + \tilde{E}^g}{2}\Big) \\
A_{-+,-+} &= \sqrt{1-\xi^2}\Big(\frac{H^g - \tilde{H}^g}{2} - \frac{\xi^2}{1-\xi^2}\frac{E^g - \tilde{E}^g}{2}\Big) \\
A_{++,-+} &= -e^{-i\phi}\frac{\sqrt{t_0 - t}}{2M}\Big(\frac{E^g - \xi\tilde{E}^g}{2}\Big) \\
A_{-+,++} &= e^{i\phi}\frac{\sqrt{t_0 - t}}{2M}\Big(\frac{E^g + \xi\tilde{E}^g}{2}\Big),
\end{aligned}
$$

After pdf's vs. Q^2 → fix x dependence
Regge behavior determines *t* dependence
Spectator determines ζ dependence

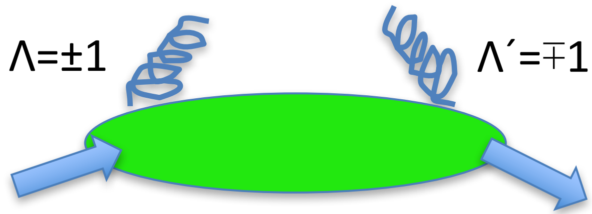




$H_g(X,\zeta,t)$
Q^2=initial

from J. Poage

Extension to Gluon "Transversity"



$$
\begin{aligned}
-\frac{1}{P^+}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\, \mathbf{S} F^{+i}(-\tfrac{1}{2}z)\, F^{+j}(\tfrac{1}{2}z)\, |p,\lambda\rangle \Big|_{z^+=0,\, \mathbf{z}_T=0} \\
= \quad \mathbf{S}\, \frac{1}{2P^+}\, \frac{P^+\Delta^j - \Delta^+P^j}{2mP^+} \\
\times \quad \bar{u}(p',\lambda') \Big[H_T^g\, i\sigma^{+i} + \tilde{H}_T^g\, \frac{P^+\Delta^i - \Delta^+P^i}{m^2} \\
+ E_T^g\, \frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2m} + \tilde{E}_T^g\, \frac{\gamma^+P^i - P^+\gamma^i}{m} \Big] u(p,\lambda).
\end{aligned}
$$

4 GPDs: M.Diehl, EPJC19, 485 (2001)

What about Gluon "transversity"? Double helicity flip *does not mix* with quark distributions

Transversity for on-shell gluons or photons : no |0> helicity

$$|+1)_{trans} = \{|+1> + |-1>\}/2 = |-1)_{trans}$$

$$|0\,)_{trans} = \{|+1> - |-1>\}/\sqrt{2}$$

$$\text{helicity } |\pm 1> = \{-/+x - iy\}/\sqrt{2}$$

$$x = -|0)_{trans} = P_{parallel}$$

Linear polarization in the plane

$$y = i\sqrt{2}\,|+1)_{trans} = P_{normal}$$

Linear polarization normal to the plane

GG&M.J.Moravcsik, Ann.Phys.195,213(1989).

Construct Gluon helicity flip amps Spectator Model, then GPDs

$$
\begin{aligned}
A^g_{++,+-} &= \int d^2k_\perp \, \mathcal{A}^g \left[(-1)(1-X) \frac{(k_1+ik_2)(\tilde{k}_1+i\tilde{k}_2)}{XX'} \right] \\
A^g_{-+,--} &= \int d^2k_\perp \, \mathcal{A}^g \left[(-1)(1-X') \frac{(k_1+ik_2)(\tilde{k}_1+i\tilde{k}_2)}{XX'} \right] \\
A^g_{++,--} &= \int d^2k_\perp \, \mathcal{A}^g \left[\Big((1-X)M - M_X\Big) \frac{(\tilde{k}_1+i\tilde{k}_2)}{X'} - \Big((1-X')M - M_X\Big) \frac{(k_1+ik_2)}{X} \right] \\
A^g_{-+,+-} &= 0 \qquad (5.15)
\end{aligned}
$$

Invert Helicity amps to obtain GPDs

$$
\begin{aligned}
A_{++,+-} &= \sqrt{1-\xi^2}\, \frac{t_0-t}{4M^2} \left(\tilde{H}^g_T + (1-\xi) \frac{E^g_T + \tilde{E}^g_T}{2} \right) \\
A_{-+,--} &= \sqrt{1-\xi^2}\, \frac{t_0-t}{4M^2} \left(\tilde{H}^g_T + (1+\xi) \frac{E^g_T - \tilde{E}^g_T}{2} \right) \\
A_{++,--} &= +e^{-i\phi}(1-\xi^2) \frac{\sqrt{t_0-t}}{2M} \left(H^g_T + \frac{t_0-t}{M^2} \tilde{H}^g_T - \frac{\xi^2}{1-\xi^2} E^g_T + \frac{\xi}{1-\xi^2} \tilde{E}^g_T \right) \\
A_{-+,+-} &= -e^{i\phi}(1-\xi^2) \frac{\sqrt{t_0-t}^3}{8M^3} \tilde{H}^g_T,
\end{aligned}
$$

Construct helicity flip amps Spectator Model, then GPDs

$$A_{++,+-} = \sqrt{1-\xi^2}\,\frac{t_0-t}{4M^2}\left(\tilde{H}_T^g + (1-\xi)\frac{E_T^g+\tilde{E}_T^g}{2}\right)$$

$$A_{-+,--} = \sqrt{1-\xi^2}\,\frac{t_0-t}{4M^2}\left(\tilde{H}_T^g + (1+\xi)\frac{E_T^g-\tilde{E}_T^g}{2}\right)$$

$$A_{++,--} = +e^{-i\phi}(1-\xi^2)\frac{\sqrt{t_0-t}}{2M}\left(H_T^g + \frac{t_0-t}{M^2}\tilde{H}_T^g - \frac{\xi^2}{1-\xi^2}E_T^g + \frac{\xi}{1-\xi^2}\tilde{E}_T^g\right)$$

$$A_{-+,+-} = -e^{i\phi}(1-\xi^2)\frac{\sqrt{t_0-t}^3}{8M^3}\tilde{H}_T^g,$$

Compare to spectator model results

$$\tilde{H}_T^g = 0$$

$$(1-X)A^0_{-+.--} = (1-X')A^0_{++,+-}$$

$$\tilde{E}_T^g = 0.$$

As in Hoodbhoy & Ji, PRD58, 054006 (1998)

Using the Reggeized Spectators Model

How to Measure? What Processes? Long standing question.

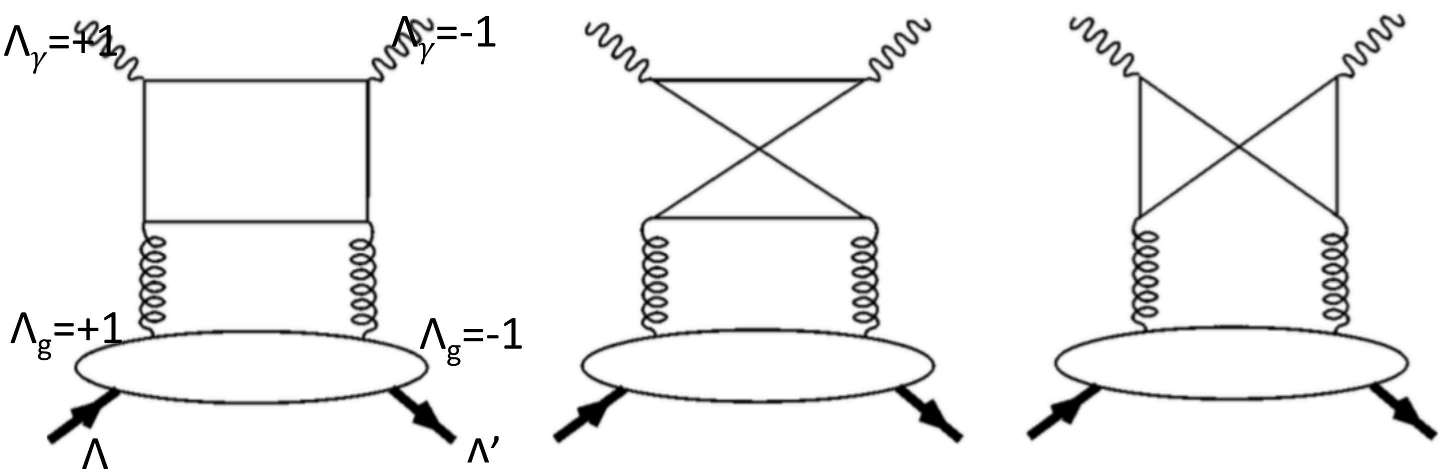
M. Diehl, T. Gousset, B. Pire, and J. P. Ralston, Phys. Lett. B411, 193 (1997).
X. Ji and J. Osborne, UMD PP#98-074, hep-ph/9801260.
P. Kroll, M. Schurmann, and P. A. M. Guichon, Nucl. Phys. A598, 435 (1996).
P. Hoodbhoy & X. Ji, PRD58, 054006 (1998)

$A_{\Lambda',-1;\,\Lambda,+1}$ **Gluon Transversity** contributes to DVCS ~ α_S

$$M_{\Lambda',\Lambda'\gamma=-1;\Lambda,\Lambda\gamma=+1} = -\frac{\alpha s}{2\pi}\sum_q e_q^2 \int_{-1}^{+1} dx \frac{A_{\Lambda',\Lambda' g=-1;\Lambda,\Lambda g=+1}(x,\xi,t)}{(\xi - x - i\varepsilon)(\xi + x - i\varepsilon)}\; C'(x,\xi,Q^2)$$

DVCS cross sections: $d\sigma/dt \propto \sum |M_{\Lambda'\ldots}|^2$

***Interference with Bethe-Heitler contains *cos*3φ modulation to distinguish from (leading twist) quark contribution ***



$$T^{\mu\nu} = \frac{\alpha_s}{2\pi}\left(\sum_q e_q^2\right)\int_{-1}^{1} dx \frac{x}{x^2-\xi^2}\left[1+\frac{x_B^2-\xi^2}{x^2-\xi^2}\ln\left(\frac{x_B^2-x^2}{x_B^2-\xi^2}\right)\right] n_\alpha n_\beta T^{\mu\nu\alpha\beta}\,,$$

e,

See Hoodbhoy & Ji, PRD58, 054006 (1998)

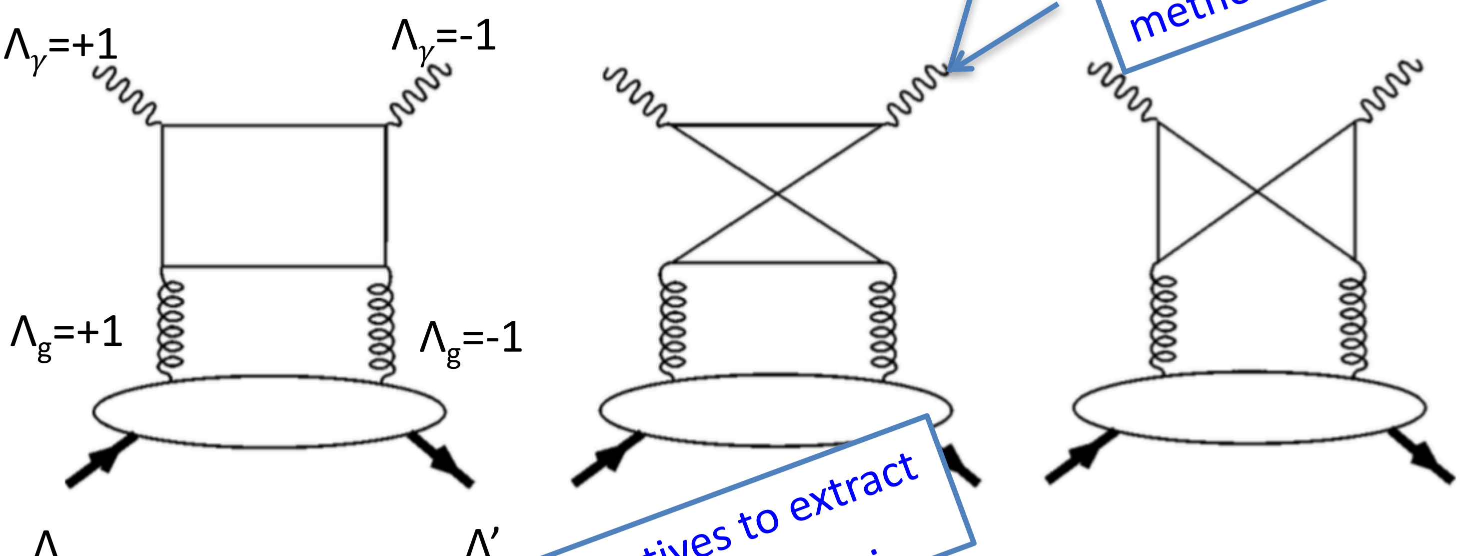
$$T^{\mu\nu\alpha\beta} = \frac{1}{x}\int \frac{d\lambda}{2\pi} e^{i\lambda x}\langle P'S'|F^{(\mu\alpha}(-\frac{\lambda}{2}n)F^{\nu\beta)}(\frac{\lambda}{2}n)|PS\rangle\,.$$

$A_{\Lambda',-1;\,\Lambda,+1}$ contributes to DVCS ~ α_S

$$M_{\Lambda',\Lambda'\gamma=-1;\Lambda,\Lambda\gamma=+1} = -\frac{\alpha s}{2\pi}\sum_q e_q^2 \int_{-1}^{+1} dx \frac{A_{\Lambda',\Lambda' g=-1;\Lambda,\Lambda g=+1}(x,\xi,t)}{(\xi - x - i\varepsilon)(\xi + x - i\varepsilon)} \; C'(x,\xi,Q^2)$$

Interference with Bethe-Heitler contains *cos*3φ modulation to distinguish from (leading twist) quark contribution

$\mu^- \mu^+$ alternative method to extract



other alternatives to extract J/Ψ production; jets . . . .

See Hoodbhoy & Ji, PRD58, 054006 (1998)

Measuring Gluons in Nucleons

DVCS

$$\frac{d^5\sigma}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} = \frac{\alpha^3}{16\pi^2(s-M^2)^2\sqrt{1+\gamma^2}}|T|^2$$

$$T(k,p,k',q',p') = T_{DVCS}(k,p,k',q',p') + T_{BH}(k,p,k',q',p'),$$

$$|T|^2 = |T_{\rm BH} + T_{\rm DVCS}|^2 = |T_{\rm BH}|^2 + |T_{\rm DVCS}|^2 + \mathcal{I}\,.$$

$$\mathcal{I} = T^*_{BH}T_{DVCS} + T^*_{DVCS}T_{BH}.$$

For unpolarized e+p→e'+γ+p' cross section depends on azimuthal angle ϕ.
cos3ϕ term in interference dσ measures gluon transversity GPDs (CFF's)

$$\frac{\sqrt{t_0-t}^3}{8M^3}\left[H_T^g F_2 - E_T^g F_1 - 2\tilde{H}_T^g\left(F_1 + \frac{t}{4M^2}F_2\right)\right]\cos 3\phi$$

$\mathcal{H}_T^g \sim \int dx\, H^g_T/(x-\xi)(x+\xi)$ CFF's

See Diehl, *et al*. PLB411, 193 (1997);
Diehl, EPJC25, 223 (2002);
Belitsky, Mueller, PLB486, 369 (2000).

Summary

- Flexible parameterization for chiral even valence quarks from form factors, pdfs & DVCS **R✖**Dq
- Extended **R✖**Dq to **R✖**Spectator
- Extend phenomenology to *v production*
- **New Extension to** gluons **& the sea**
- Considered Gluon sector
  - Helicity conserving & Helicity →gluon ***Transversity***
- Measurements?
- **More phenomenology to come**

Backup & extra slides

Gluon GPDs: Helicity flipping Gluon "Transversity"

$$
-\frac{1}{\bar{P}^+}\int\frac{dz^-}{2\pi}e^{ix\bar{P}^+z^-}\langle P',\Lambda'|\mathbf{S}G^{+j}(-\frac{1}{2}z)G^{+k}(\frac{1}{2}z)|P,\Lambda\rangle\Big|_{z^+=0,\vec{z}_T=0}
$$

$$
= \mathbf{S}\frac{1}{2\bar{P}^+}\frac{\bar{P}^+\Delta^j-\Delta^+\bar{P}^j}{2M\bar{P}^+}
$$

$$
\times\bar{U}(P',\Lambda')\Big[H_T^g(x,\xi,t)i\sigma^{+k}+\tilde{H}_T^g\frac{\bar{P}^+\Delta^k-\Delta^+\bar{P}^k}{M^2}
$$

$$
+E_T^g(x,\xi,t)\frac{\gamma^+\Delta^k-\Delta^+\gamma^k}{2M}+\tilde{E}_T^g\frac{\gamma^+\bar{P}^k-\bar{P}^+\gamma^k}{M}\Big]U(P,\Lambda)
$$

Some results – GG, Gonzalez Hernandez, Liuti, PRD84, 034007 (2011)

GOLDSTEIN, HERNANDEZ, AND LIUTI

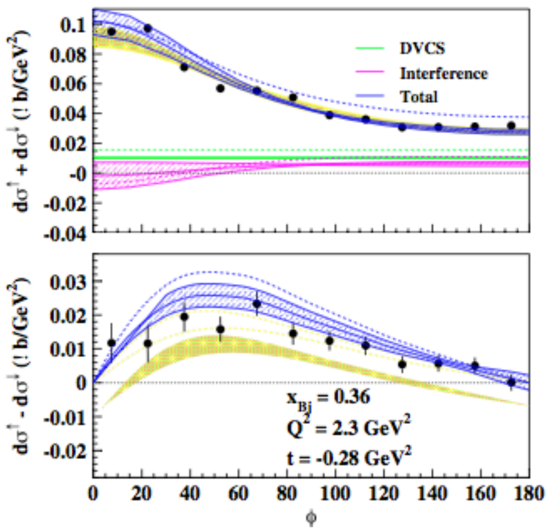


FIG. 16 (color online). Hall A data [49] for the "sum" (upper panel) and "difference" (lower panel) of the two electron beam polarizations. Shown are curves, including the contribution of the ζ-dependent factor from Eq. (34) (solid lines) and neglecting it (dashed lines). All terms (DVCS, Interference, and Total) are shown for the sum graph. The wide yellow bands in both panels represent the error of the data fit. The green band in the asymmetry graph is the theoretical error from our parametrization.

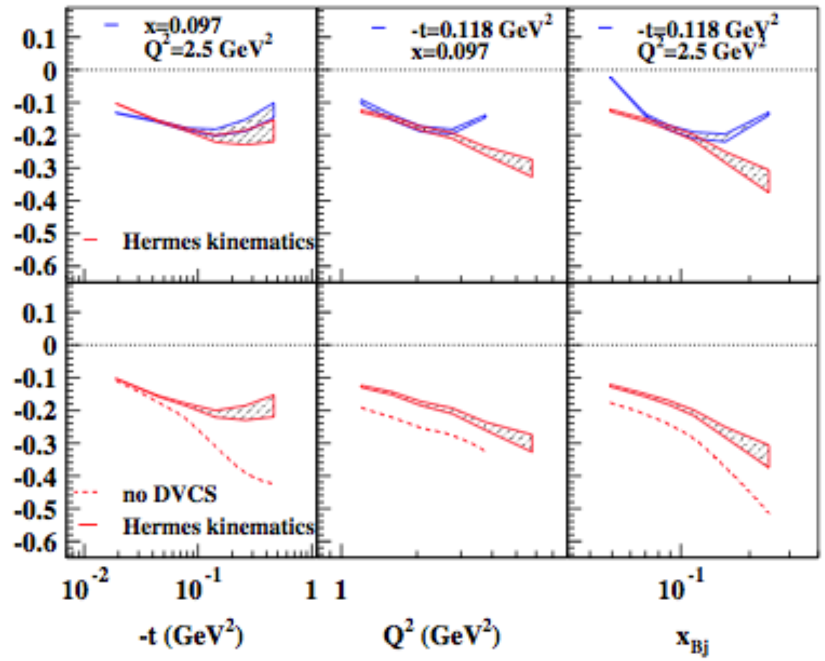
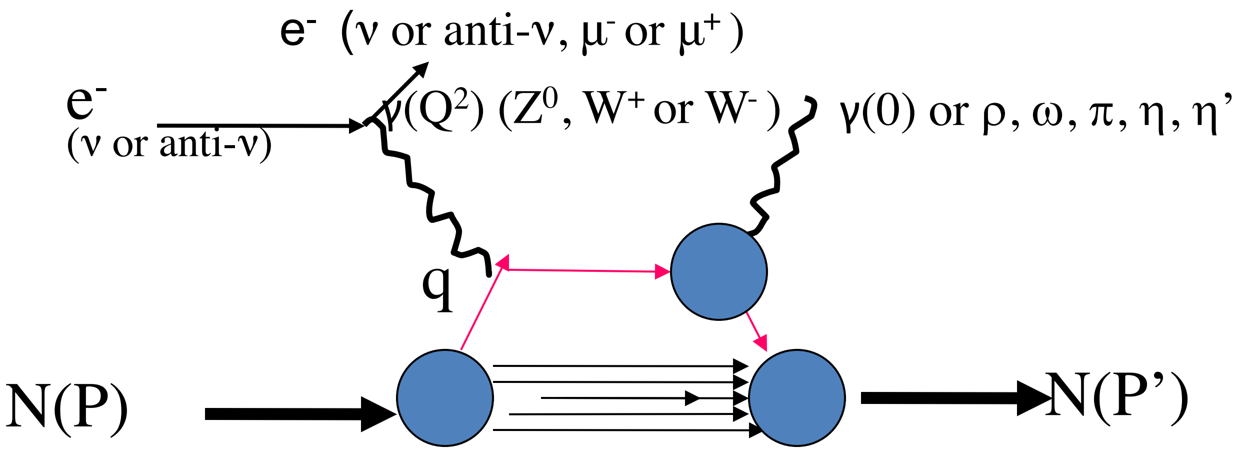


FIG. 18 (color online). Calculations at Hermes kinematics [52,53,56]. Shown is $A_{LU}(90°)$ vs $-t$, Q^2, and x_{BJ}, respectively, calculated at each kinematical bin provided by Hermes [56] (curve denoted as "Hermes kinematics") and at the nominal average values presented in each panel. It is interesting to notice that, due to the correlation between x_{Bj} and Q^2 in the data, different features arise when using the average bin values. In the lower panels, we also show the effect of disregarding the DVCS term in the denominator (dashed curves).

Hadrons to quarks



$g_{\Lambda w,\lambda;0,\lambda'}$ = *same helicity structure as W+p→π+p'*

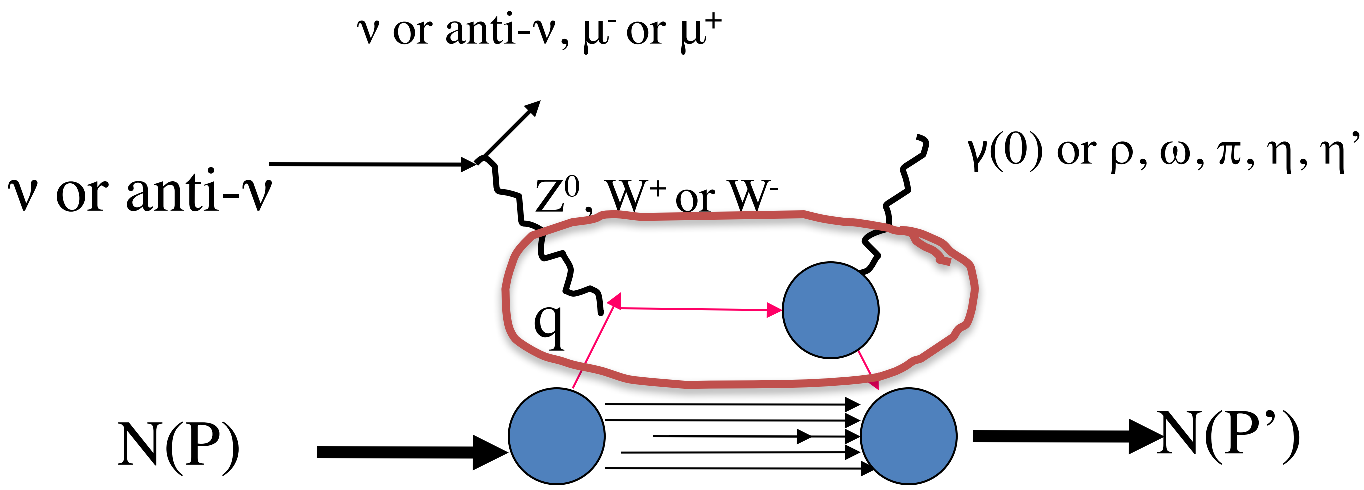
with $W^{\mu} \sim c_V\,\gamma^{\mu} - c_A\,\gamma^{\mu}\gamma^5$ & $\pi \sim \varphi_A q'_{\nu}\gamma^{\nu}\gamma^5 + \varphi_P\gamma^5$

$A_{\Lambda',\lambda';\Lambda,\lambda} \supset$ *GPDs*

One of 4 terms: $$\frac{-i}{2}\bar{u}(k')i\gamma_5\left[\frac{i\left((k+q)^{\mu}\gamma_{\mu}+m_3\right)}{(k+q)^2-m_3^2}\varepsilon^{\nu}\gamma_{\nu}(c_V-c_A\gamma_5)\right]u(k)$$

+ u-channel

Hard subprocess & Q



Consider π production $V^* + q \rightarrow \pi + q'$ with DA for $\pi \supset q'_\mu \gamma^\mu \gamma^5$ twist2 & γ^5 twist 3

Helicity amps:

- **Longitudinal** with non-flip quark – leading ~ Q/M: $q'_\mu \gamma^\mu \gamma^5$
- **Transverse** with flip quark ~ constant (or Δ^2/Q^2 cross channel) : γ^5
- **Transverse** non-flip ~ Δ/M : : $q'_\mu \gamma^\mu \gamma^5$ (Δ by angular momentum conservation)
- **Longitudinal** with flip ~ Δ/Q (Δ by angular momentum conservation)

Differential cross sections

Neutral Current:

$$\frac{d^4\sigma}{dx_{Bj}dQ^2d|t|d\phi} = \frac{G_F^2 x_{Bj}^2}{32\pi^3\cos^4(\theta_W)Q^2(1+Q^2/M_Z^2)^2(1+\gamma^2)^{3/2}}|T|^2$$

Charge Current: replace M_Z with M_W and set θ_W=0.

Separating the Vector boson + nucleon scattering:

$$\frac{d\sigma_Z}{dt} = \frac{\sqrt{2}G_F M_Z^2 x_{Bj}^2 \mid T \mid^2}{16\pi Q^4(1+\gamma^2)}.$$

$$\frac{d\sigma_W}{dt} = \frac{G_F M_Z^2 x_{Bj}^2 \mid T \mid^2}{16\pi\sqrt{2}Q^4(1+\gamma^2)}.$$

Azimuthal correlations for neutrinos to separate *P* violating parts

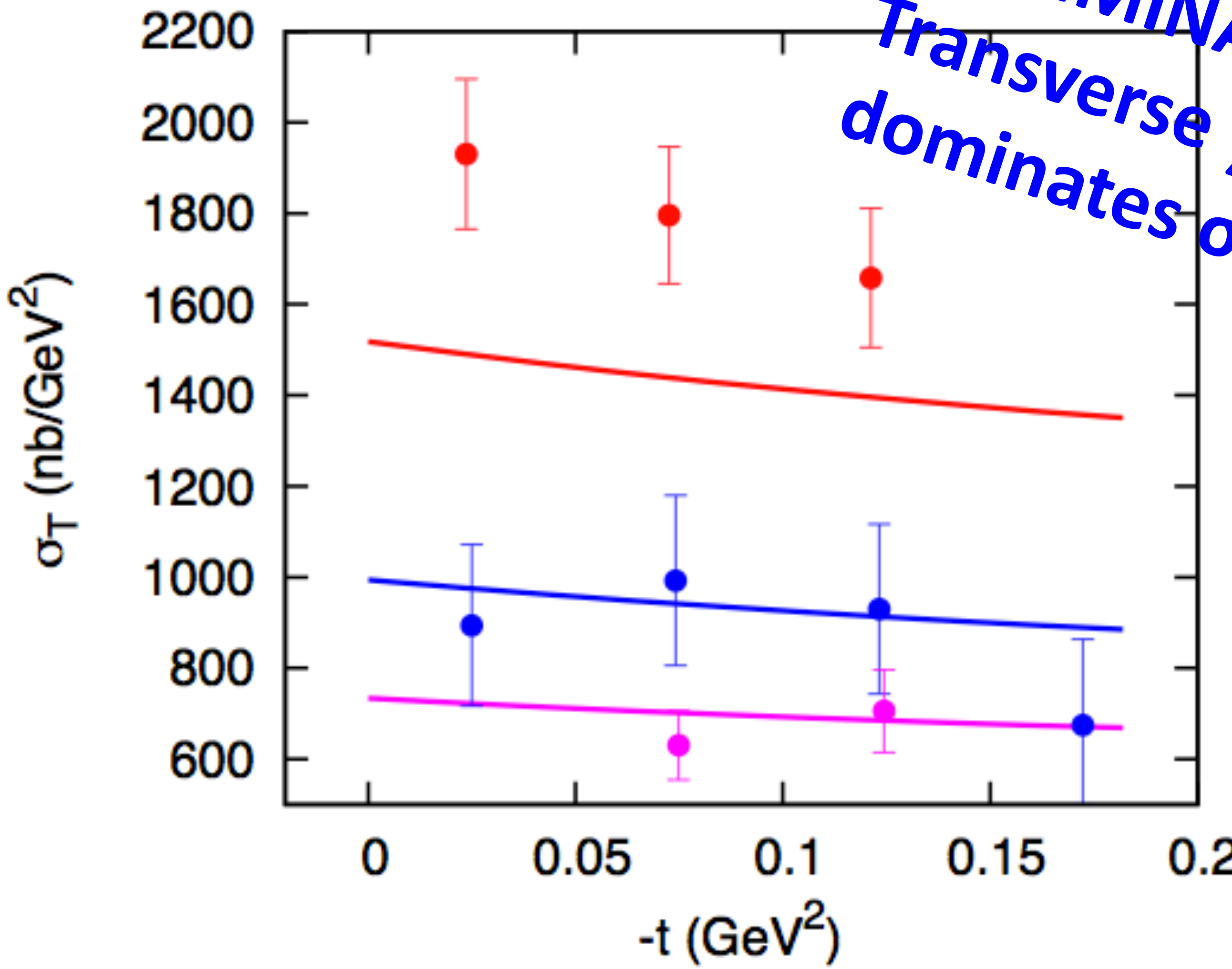
$$
\begin{aligned}
\frac{d^4\sigma}{d\Omega d\varepsilon_2 d\phi dt} &= \Gamma\left[\frac{d\sigma_T}{dt} + \varepsilon_L \frac{d\sigma_L}{dt} + \varepsilon \cos 2\phi \frac{d\sigma_{TT}}{dt}\right. \\
&+ \sqrt{2\varepsilon_L(\varepsilon+1)}\cos\phi \frac{d\sigma_{LT}}{dt} \\
&+ \varepsilon \sin 2\phi \frac{d\sigma_{T'T}}{dt} \\
&\pm \left.\sqrt{2\varepsilon_L(\varepsilon+1)}\sin\phi \frac{d\sigma_{L'T}}{dt}\right] \qquad (
\end{aligned}
$$

Hall A data x_B=0.36

courtesy F. Sabatie & M. Defurne

PRELIMINARY

Transverse γ^* dominates σ

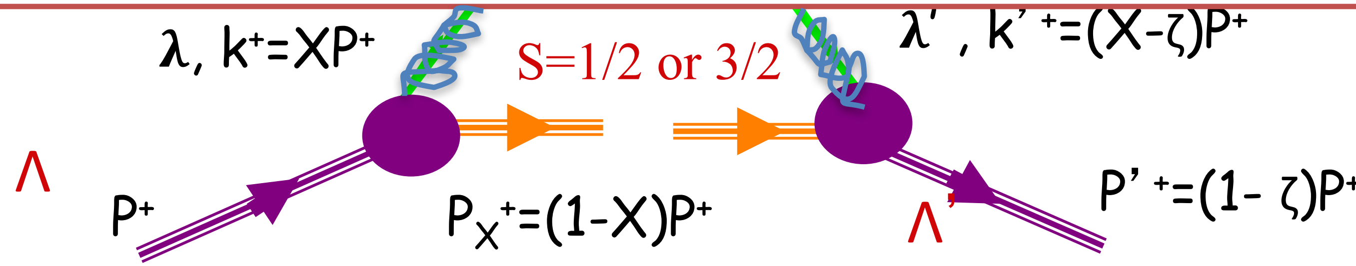


Q^2=1.5 GeV^2

Q^2=1.75 GeV^2

Q^2=2.0 GeV^2

Procedure to construct **Gluon** GPDs & observables Spectator color octet "nucleon" model & Reggeization



Product of baryon l.c.w.f.'s $\rightarrow A_{\Lambda\lambda;\Lambda'\lambda'=\lambda}$

↓

$A_{\Lambda\lambda;\Lambda'\lambda}$ → gluon GPDs

↓

g $\otimes$ A → exclusive process helicity amps

↓

pdf's, FF's, $d\sigma/d\Omega$ & Asymmetries: parameters & predictions

↓

vertex parity?$\rightarrow A_{\Lambda\lambda;-\Lambda'-\lambda'}$ → "transversity" gluon GPDs ?

pdf's fix x dependence

Gluon

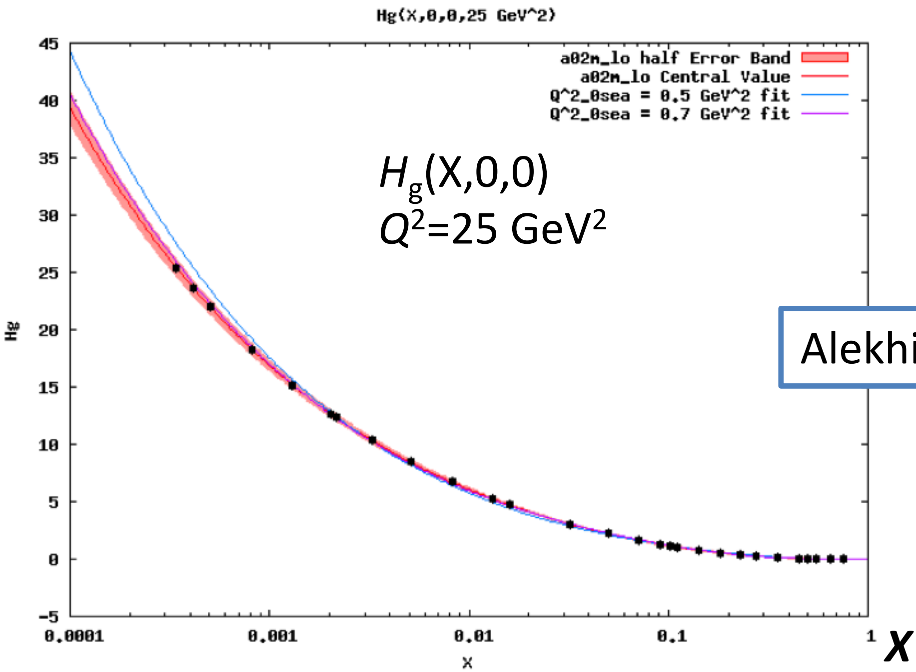


Figure 6: The plot above shows the distribution $H_g(X,0,0,25\ \mathrm{GeV}^2)$ for the two fits. Alekhin's distribution Xg(X) used in the fit procedure is included with an error band of one half of the error for the set a02m_lo. The X points used in the fit procedure are indicated by black dots.

Alekhin pdf fit

anti-u

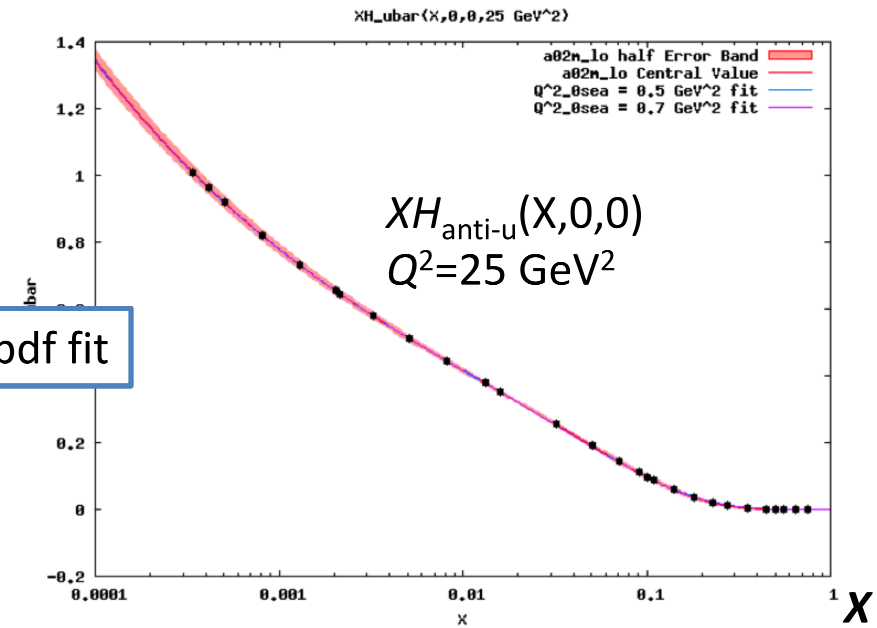


Figure 3: The plot above shows the distribution $XH_{\bar{u}}(X,0,0,25\ \mathrm{GeV}^2)$ for the two fits. Alekhin's distribution $X\bar{u}(X)$ used in the fit procedure is included with an error band of one half of the error for the set a02m_lo. The X points used in the fit procedure are indicated by black dots.

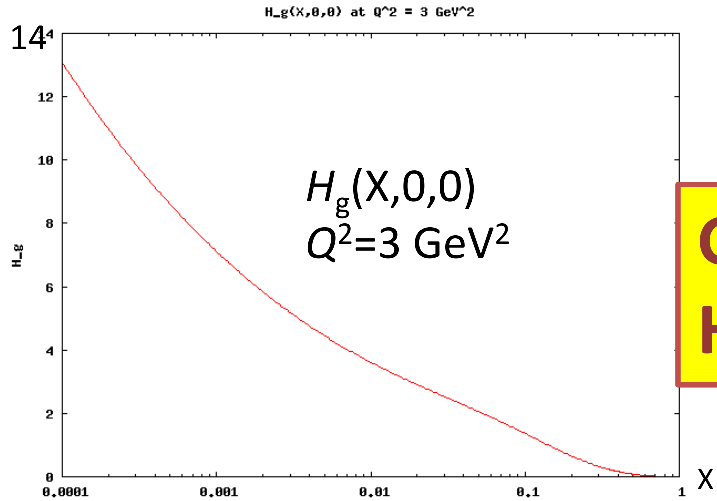
Single Q^2 value shown --- fit known pdf's all Q^2

from J. Poage

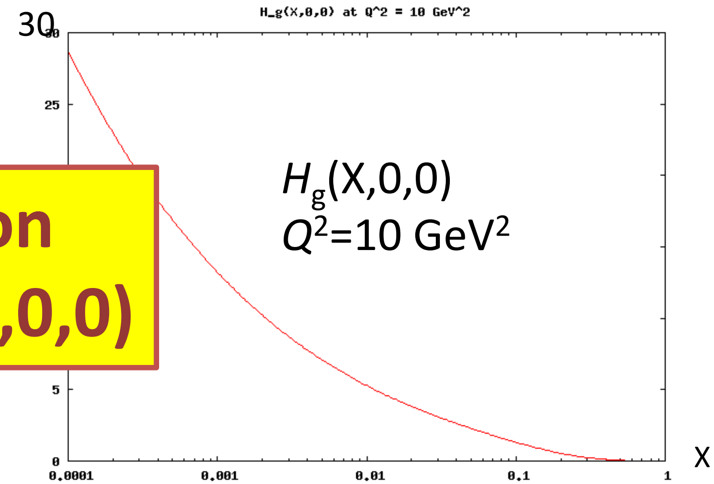
GG, Gonzalez Hernandez, Liuti, Poage, in progress

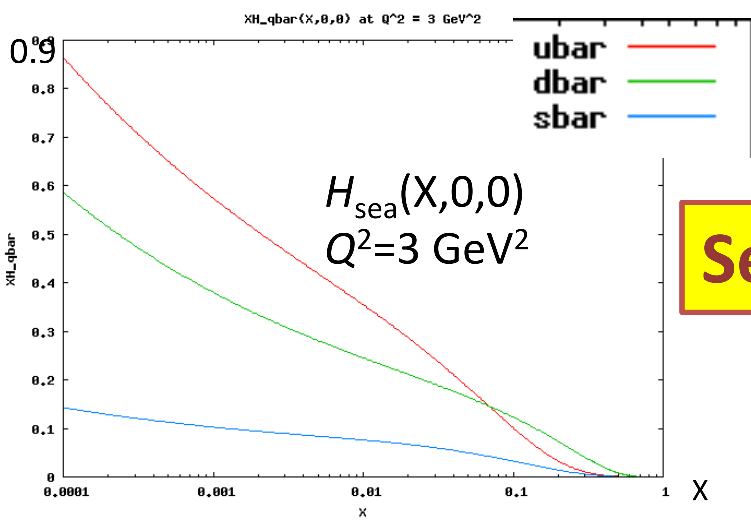
Gluon & sea distributions

J. Poage

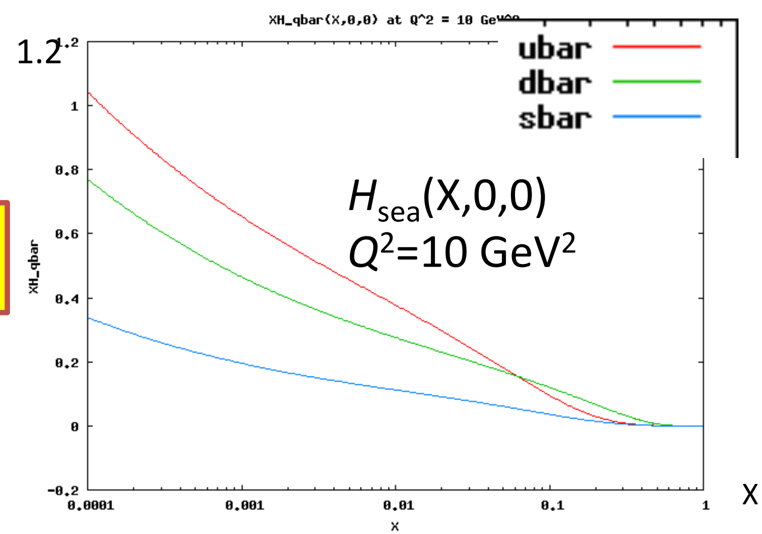


Gluon
$H_g(x,0,0)$





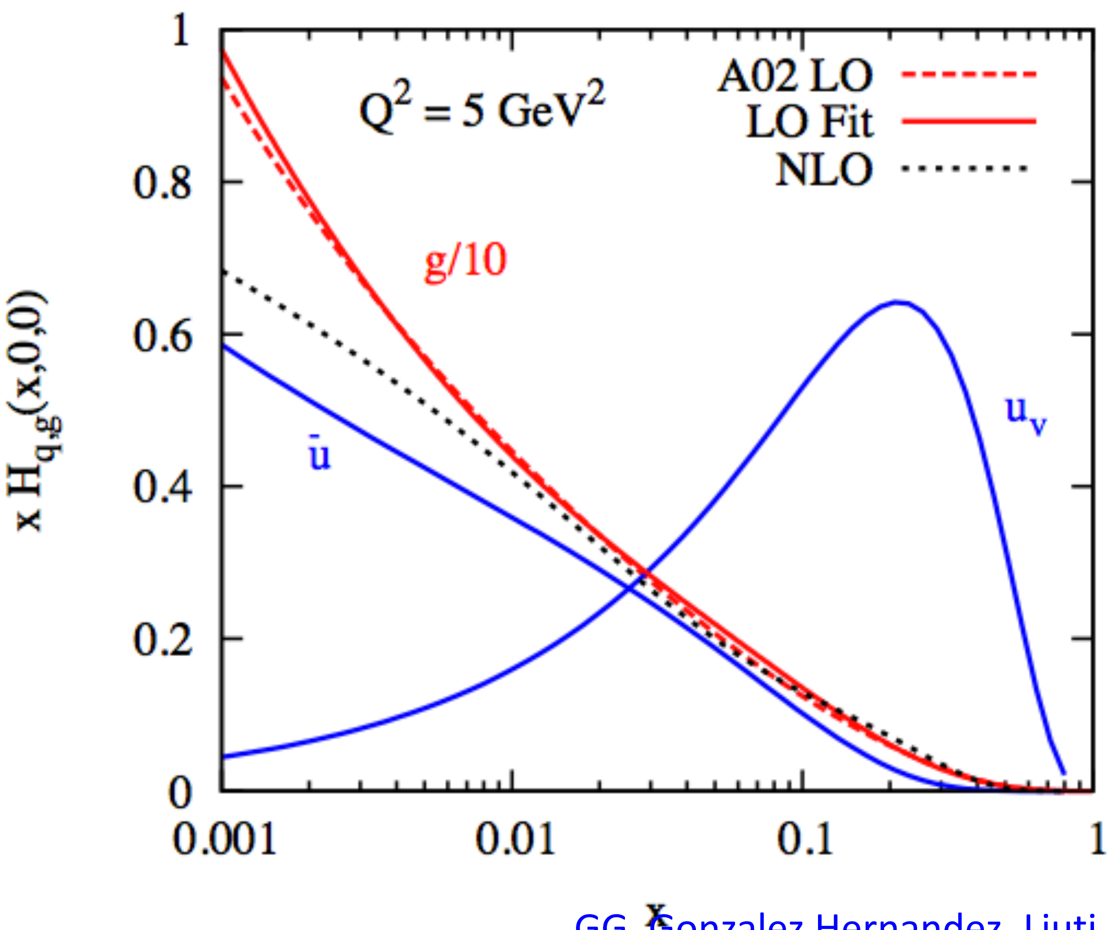
Sea



GG, Gonzalez Hernandez, Liuti, Poage, in progress

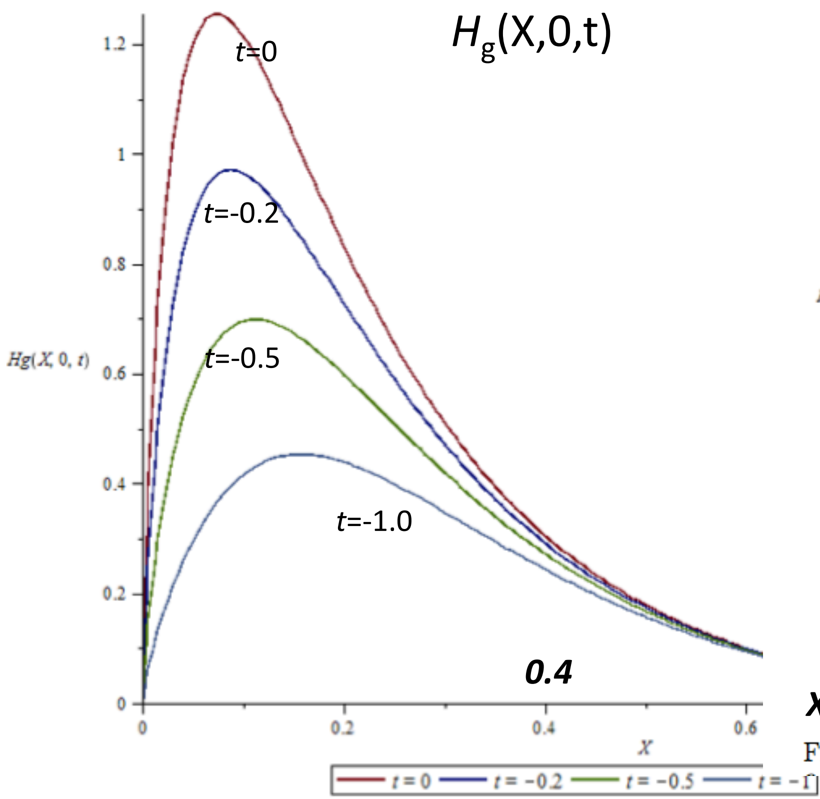
Fitting gluon pdf's

c.f. Alekhin, .. etc.



GG, Gonzalez Hernandez, Liuti, Poage, in progress

Preliminary: x and t dependence of $H_g(x, 0, t)$ for input scale



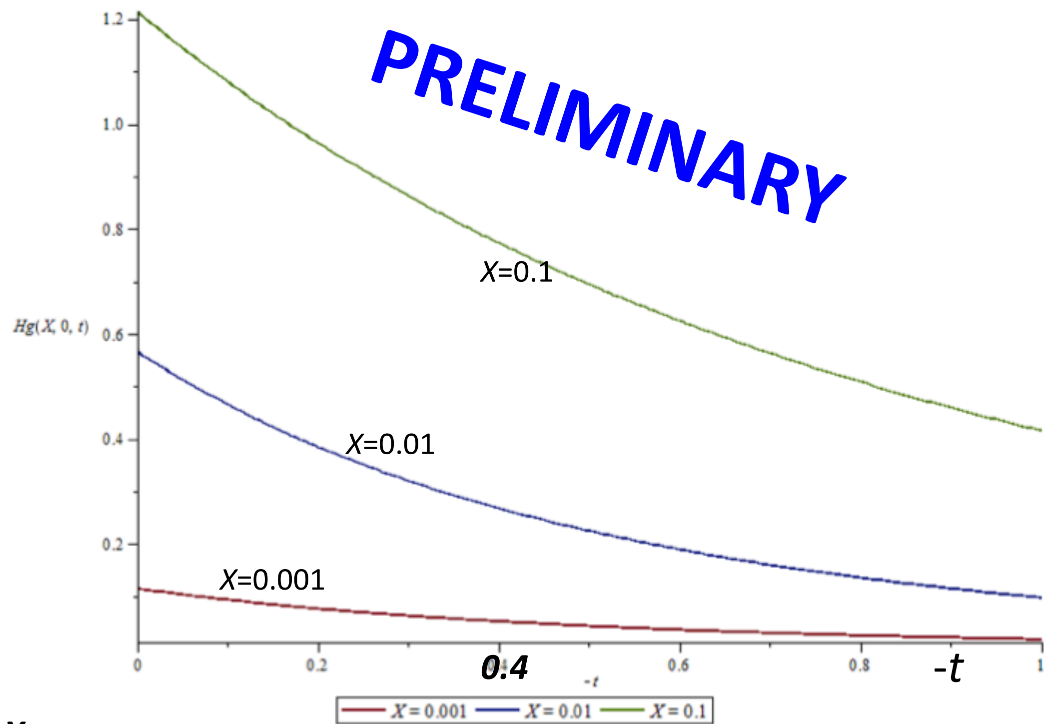


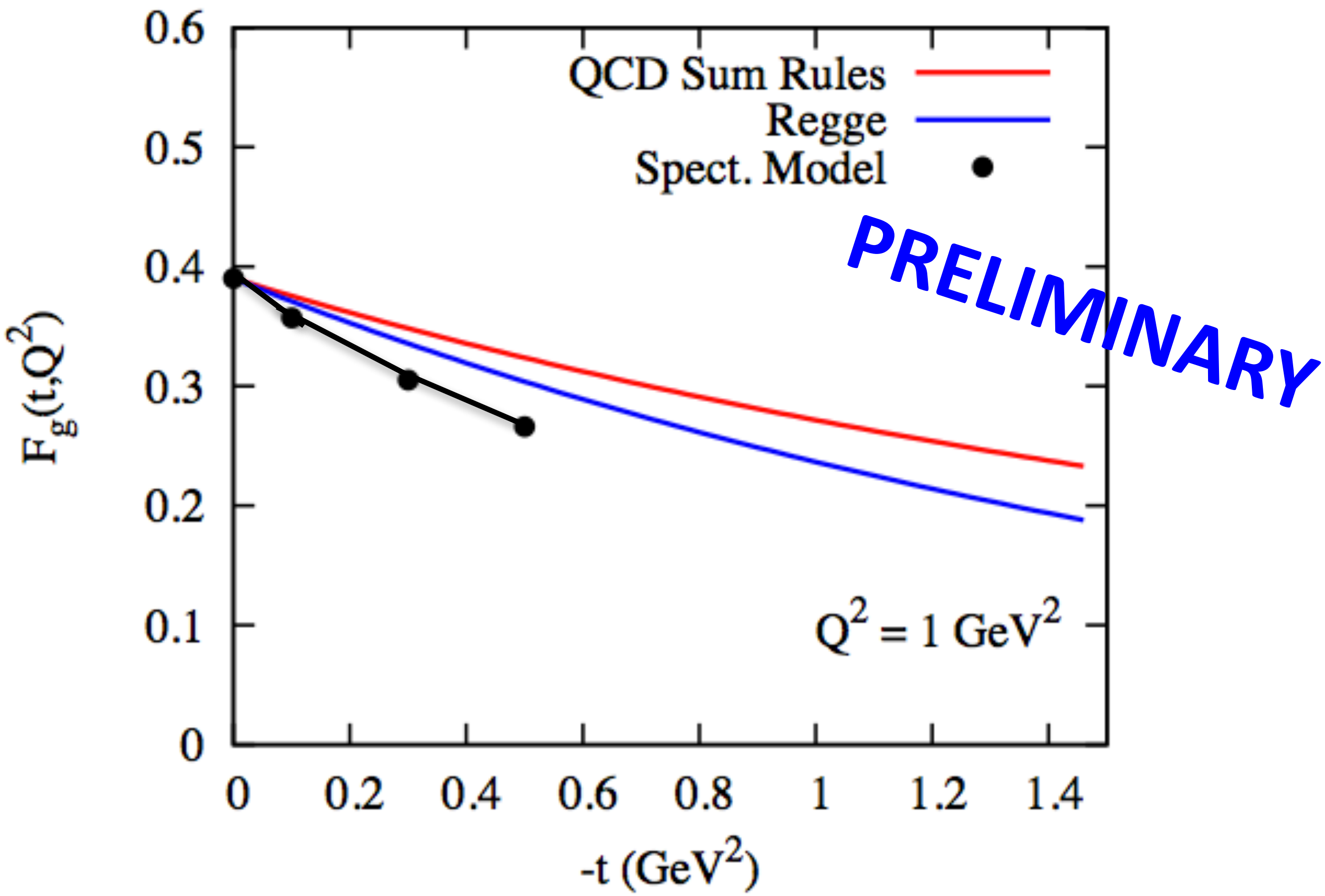
Figure 10: The plot above displays the distribution H_g(X,0,t) as a function of t, for several

'igure 9: The plot above displays the distribution H_g(X,0,t) for a range of t values.

GG, Gonzalez Hernandez, Liuti, Poage, in progress

$H_g(x,0,t)$ What constrains t-dependence?

Spectator t-dependence w/o Regge small x behavior:
& hybrid Regge-Spectator model combines



Compare Gluon form factor via QCD sum rules

Braun, Gornicki, Mankiewicz, Schaefer, Phys.Lett.B302, 291 (1993)

GG, Gonzalez Hernandez, Liuti, Poage, in progress

Abstract

- Chiral Even and Odd Generalized Parton Distributions for valence quarks were obtained in a "flexible" spectator model based on the covariant scattering matrix approach. The parametrization of the GPDs was constrained by nucleon form factors, PDFs and some deeply virtual Compton scattering data. The model is extended to sea quarks and gluons. A broad range of measured and measurable electron, muon and neutrino processes, including cross sections and polarization asymmetries, are compared with existing data. Predictions are made for processes sensitive to the newly parameterized GPDs.