Conceptual Issues of GPDs

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QCD evolution

Jefferson Lab

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Review

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CONCEPTUAL ISSUES CONCERNING GENERALIZED PARTON DISTRIBUTIONS

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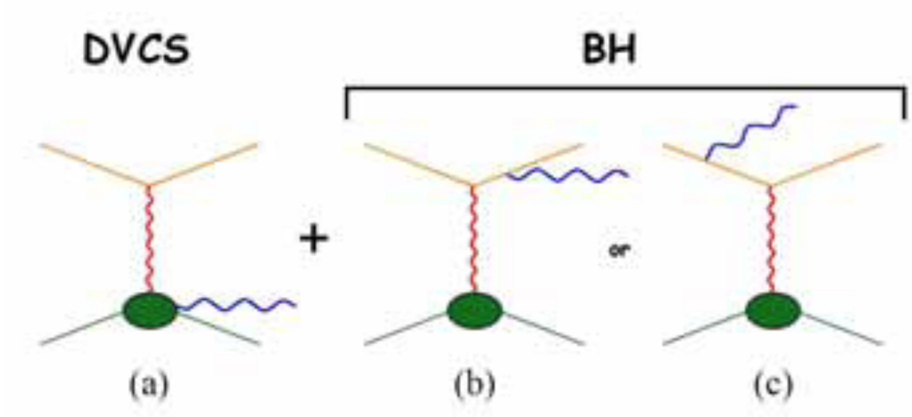
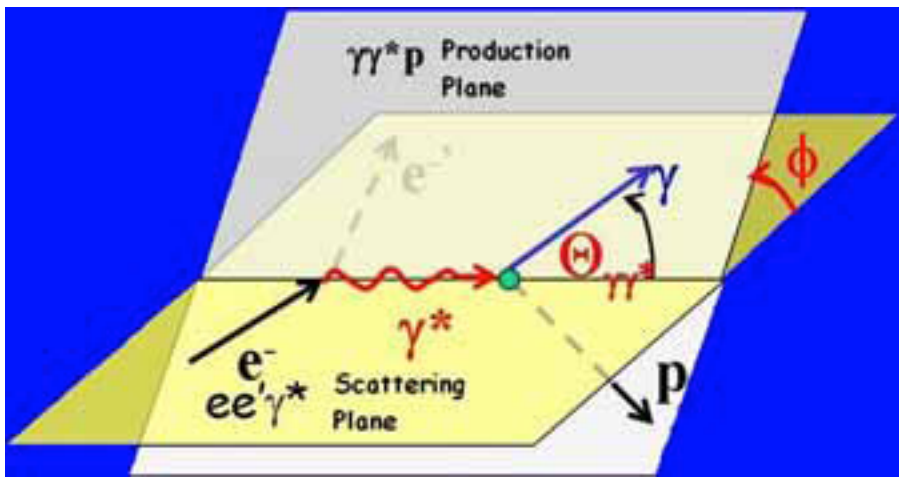
Outline

- JLab Kinematics ($t < -|t_{min}| \neq 0$)
- Original Formulation of DVCS with GPDs (Twist 2)
- Benchmark Barebone Calculation for JLab Kinematics (Exact Result vs. Reduced Result)
- Toward finding the Most General Hadronic Tensor Structure with CFFs (DNA method)
- Conclusion and Outlook

JLab Kinematics $t < -|t_{min}| \neq 0$





$$t = \Delta^2 = -\frac{\varsigma^2 M^2 + \Delta_\perp^2}{1-\varsigma} \;;\; \Delta^+ (\equiv \Delta^0 + \Delta^3) = -\varsigma\, p^+ \;;\; \Delta_\perp^2 > \Delta_{\perp\min}^2 \neq 0$$

Coincidence Experiment

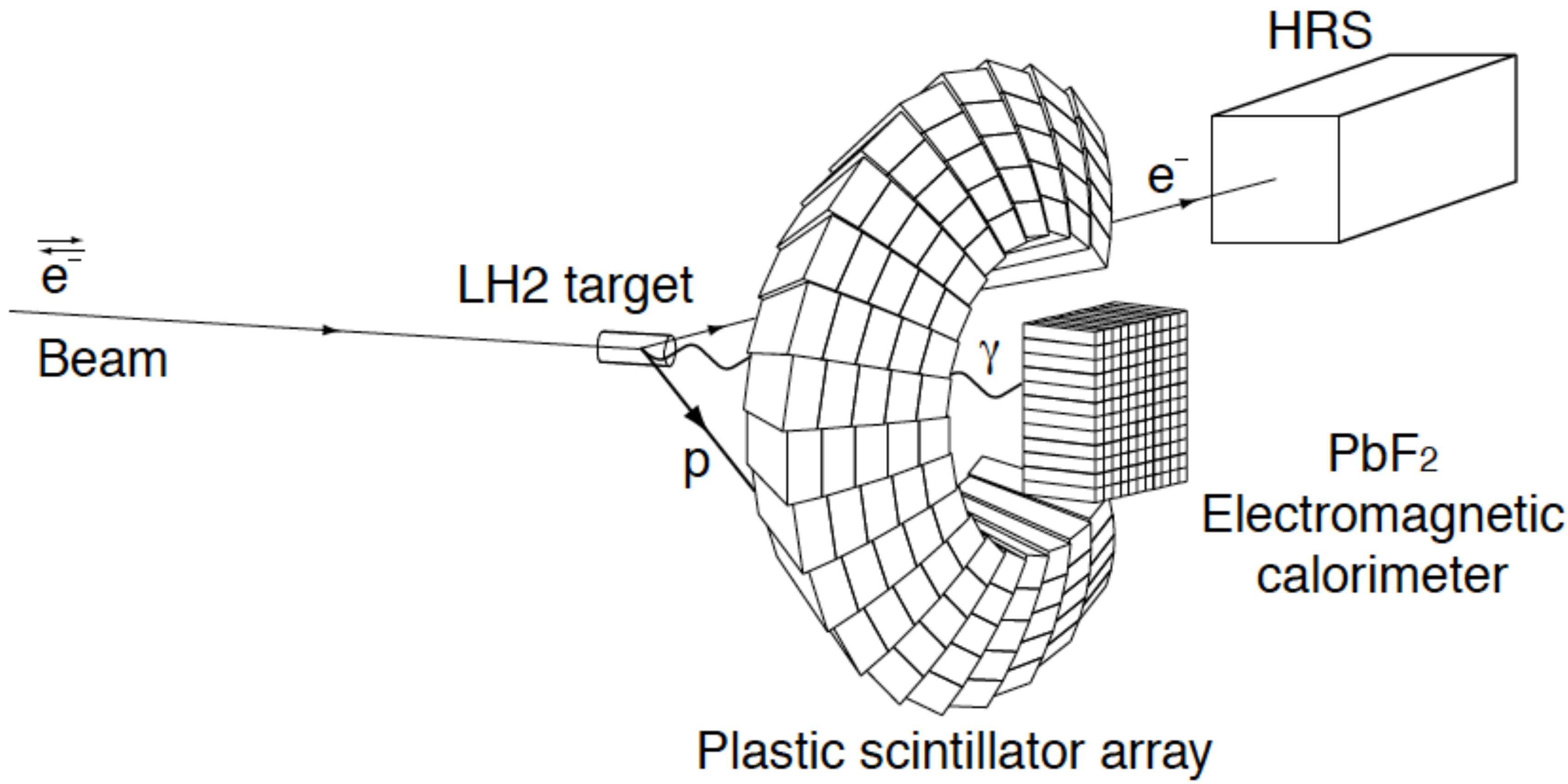


Figure 1.11: E00-110 schematic setup showing the three different detectors used to measured each of the particles in the final state. **Carlos Muñoz Camacho** Thesis('05)

In[277]:=

```
Flatten[{{{"Q^2", "xBj", "k", "k'", "the", "thq", "thq'", "thp'", "q'", "t/Q^2"}},
   Table[Block[{M = 0.938, Q = Sqrt[pars[[i, 1]]], xBj = pars[[i, 2]], Eb = pars[[i, 3]],
      the = pars[[i, 4]] Pi / 180, thetaC = 20 Pi / 180, thq = ArcCos[costhetaqT2]},
     {Q^2, xBj, Eb, PeT1, the 180 / Pi, thq 180 / Pi, ArcCos[costhqf] 180 / Pi, ArcCos[
        costhpf] 180 / Pi, qfT3mu[[1]], MandeltT2 / Q^2}], {i, 1, 12}]}, 1] // MatrixForm
```

Out[277]//MatrixForm=

Q^2	xBj	k	k'	the	thq	thq'	thp'	q'	t/Q^2
1.9	0.36	5.75	2.93669	19.3	18.0503	11.9997	69.6549	2.65582	-0.1555
3.	0.36	6.6	2.15792	26.5	11.6529	6.69292	66.0322	4.23202	-0.131356
4.	0.36	8.8	2.87723	22.9	10.3184	5.96439	64.4845	5.66645	-0.120212
4.55	0.36	11.	4.26285	17.9	10.6859	6.58293	64.1816	6.45567	-0.116055
3.1	0.5	6.6	3.2951	22.5	19.5262	14.4829	60.4087	3.0229	-0.170658
4.8	0.5	8.8	3.68273	22.2	14.4748	10.3331	57.1229	4.76891	-0.13615
6.3	0.5	11.	4.28358	21.1	12.4174	8.76649	55.3539	6.31422	-0.119765
7.2	0.5	11.	3.32409	25.6	10.1755	6.74662	53.737	7.24243	-0.112945
5.1	0.6	8.8	4.26908	21.2	17.7604	13.7928	51.4698	4.06193	-0.172513
6.	0.6	8.8	3.46951	25.6	14.8072	11.1302	49.5692	4.82846	-0.156969
7.7	0.6	11.	4.1592	23.6	13.0416	9.77439	48.0193	6.28128	-0.136317
9.	0.6	11.	3.00426	30.2	10.1946	7.16227	46.0515	7.39496	-0.125229

Table III in E12 - 06 - 114, Julie Roche et al.

Jlab 12 GeV Exclusive Kinematics

Nucleon GPDs in DVCS Amplitude

X.Ji,PRL78,610(1997): Eqs.(14) and (15)

$$p^\mu = \Lambda(\overset{ct}{1}, \overset{x}{0}, \overset{y}{0}, \overset{z}{1}) \quad ,$$

$$n^\mu = (\overset{ct}{1}, \overset{x}{0}, \overset{y}{0}, \overset{z}{-1})/(2\Lambda) \quad ,$$

$$\overline{P}^\mu = \frac{1}{2}(P+P')^\mu = p^\mu + \frac{M^2 - \Delta^2/4}{2} n^\mu ,$$

$$q^\mu = -\xi p^\mu + \frac{Q^2}{2\xi} n^\mu \quad , \quad \xi = \frac{Q^2}{2\overline{P}\cdot q} \quad ,$$

$$\Delta^\mu = -\xi \left[p^\mu - \frac{M^2 - \Delta^2/4}{2} n^\mu \right] + \Delta_\perp^\mu \ .$$

$$T^{\mu\nu}(p,q,\Delta) = -\frac{1}{2}(p^\mu n^\nu + p^\nu n^\mu - g^{\mu\nu}) \int_{-1}^{+1} dx \left(\frac{1}{x - \frac{\xi}{2} + i\varepsilon} + \frac{1}{x + \frac{\xi}{2} - i\varepsilon} \right)$$
$$\times \left[H(x,\Delta^2,\xi)\overline{U}(P')\not{n}U(P) + E(x,\Delta^2,\xi)\overline{U}(P')\frac{i\sigma^{\alpha\beta} n_\alpha \Delta_\beta}{2M} U(P) \right]$$
$$- \frac{i}{2}\varepsilon^{\mu\nu\alpha\beta} p_\alpha n_\beta \int_{-1}^{+1} dx \left(\frac{1}{x - \frac{\xi}{2} + i\varepsilon} - \frac{1}{x + \frac{\xi}{2} - i\varepsilon} \right)$$
$$\times \left[\tilde{H}(x,\Delta^2,\xi)\overline{U}(P')\not{n}\gamma_5 U(P) + \tilde{E}(x,\Delta^2,\xi)\frac{\Delta\cdot n}{2M}\overline{U}(P')\gamma_5 U(P) \right]$$

Just above Eq.(14),
``To calculate the scattering amplitude, it is convenient to define <u>a special system of coordinates</u>.”

Note here that $q'^2 = -\mathbf{\Delta}_\perp^2 = 0$.

Nucleon GPDs in DVCS Amplitude

A.V.Radyushkin, PRD56, 5524 (1997): Eq.(7.1)

$$q = q' - \zeta p \quad ,$$
$$\zeta = \frac{Q^2}{2p\cdot q'} \quad ,$$
$$r = p - p'$$

$$T^{\mu\nu}(p,q,q') = \frac{1}{2(p\cdot q')}\sum_a e_a^2 \Big[\left(-g^{\mu\nu} + \frac{1}{p\cdot q'}(p^\mu q'^\nu + p^\nu q'^\mu)\right)$$
$$\times\left\{\bar{u}(p')\not{q}'u(p)T_F^a(\zeta) + \frac{1}{2M}\bar{u}(p')(\not{q}'\not{r} - \not{r}\not{q}')u(p)T_K^a(\zeta)\right\}$$
$$+ i\varepsilon^{\mu\nu\alpha\beta}\frac{p_\alpha q'_\beta}{p\cdot q'}\left\{\bar{u}(p')\not{q}'\gamma_5 u(p)T_G^a(\zeta) + \frac{q'\cdot r}{2M}\bar{u}(p')\gamma_5 u(p)T_P^a(\zeta)\right\}\Big]$$

At the beginning of Section 2E (Nonforward distributions),
``Writing the momentum of the virtual photon as q=q' -ζp is equivalent to using the Sudakov decomposition in the light-cone `plus' (p) and `minus' (q') components in a situation <u>when there is no transverse momentum</u> ."

Note that here $(q-q')^2 = \Delta^2 = t = \zeta^2 M^2 > 0$

while $t < 0$ in DVCS.

Benchmark Calculation in JLab Kinematics

- To see the effect of taking t<0, we mimic the kinematics at JLab and compute bare bone VCS amplitudes neglecting masses.

$$k^\mu = \left(xp^+, 0, 0, 0\right), \quad k'^\mu = \left((x-\zeta_{eff})\,p^+, \mathbf{\Delta}_\perp, \frac{{\mathbf{\Delta}_\perp}^2}{2(x-\zeta_{eff})p^+}\right)$$

$$q^\mu = \left(-\zeta p^+, 0, 0, \frac{Q^2}{2\zeta p^+}\right), \quad q'^\mu = \left(\alpha\frac{{\mathbf{\Delta}_\perp}^2}{Q^2}p^+, -\mathbf{\Delta}_\perp, \frac{Q^2}{2\alpha p^+}\right)$$

Here, for $Q \to \infty$,

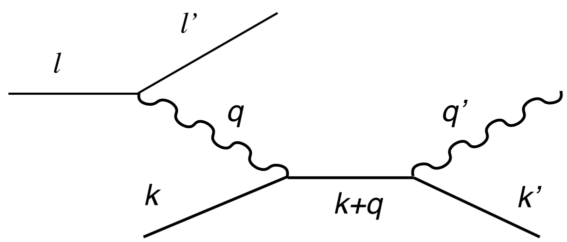
$$\zeta_{\rm eff} = \zeta + \alpha\frac{\mathbf{\Delta}_\perp^2}{Q^2} \to \zeta$$

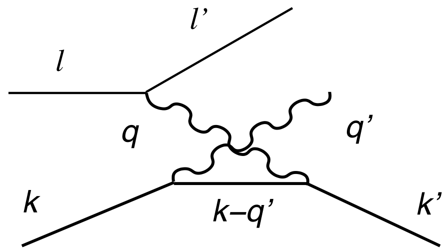
$$\alpha = \frac{(x-\zeta)Q^2}{2\mathbf{\Delta}_\perp^2}\left(1-\sqrt{1-\frac{4\zeta}{x-\zeta}\frac{\mathbf{\Delta}_\perp^2}{Q^2}}\right) \to \zeta$$

$$q'^- \to q^- = \frac{Q^2}{2\zeta p^+}$$

"Bare Bone" VCS Operators & Amplitudes

$S = (k+q)^2$





$U = (k-q')^2$

$$\mathcal{O}_s = \frac{\not{\epsilon}^*(q';h')(\not{k}+\not{q})\not{\epsilon}(q;h)}{(k+q)^2} \qquad \mathcal{O}_u = \frac{\not{\epsilon}(q;h)(\not{k}-\not{q}')\not{\epsilon}^*(q';h')}{(k-q')^2}$$

$$\mathcal{O}_s|_{Q\to\infty} = \frac{\not{\epsilon}^*(q';h')\gamma^+ \, q^- \not{\epsilon}(q;h)}{2\,(x-\zeta)p^+ \, q^-} \qquad \mathcal{O}_u|_{Q\to\infty} = \frac{\not{\epsilon}(q;h)\gamma^+ \, (-q'^-)\not{\epsilon}^*(q';h')}{2\,xp^+ \, (-q'^-)}$$

⇩ ⇩

$$\mathcal{O}_s|_{\mathrm{GPDRed}} = \frac{\not{\epsilon}^*(q';h')\gamma^+\not{\epsilon}(q;h)}{2p^+}\,\frac{1}{x-\zeta} \qquad \mathcal{O}_u|_{\mathrm{GPDRed}} = \frac{\not{\epsilon}(q;h)\gamma^+\not{\epsilon}^*(q';h')}{2p^+}\,\frac{1}{x}$$

$$\mathcal{H}(\{s',s\}\{h',h\}) = \bar{u}(k';s')(\mathcal{O}_s + \mathcal{O}_u)u(k;s)$$

$$\mathcal{L}(\{\lambda',\lambda\}h) = \bar{u}(\ell';\lambda')\not{\epsilon}^*(q;h)u(\ell;\lambda)$$

$$\mathcal{M} = \sum_h \mathcal{L}(\{\lambda',\lambda\}h)\frac{1}{q^2}\mathcal{H}(\{s',s\}\{h',h\})$$

Using the identity $\gamma^\mu \gamma^\alpha \gamma^\nu = g^{\mu\alpha}\gamma^\nu + g^{\alpha\nu}\gamma^\mu - g^{\mu\nu}\gamma^\alpha + i\varepsilon^{\mu\alpha\nu\beta}\gamma_\beta\gamma_5$
and Sudakov vectors $n(+)^\mu = (1,0,0,0),\ n(-)^\mu = (0,0,0,1)$

we compare the exact amplitude

$$\begin{aligned} T_s{}^{\mu\nu} &= \frac{1}{s}[(\{(k^+ + q^+)n^\mu(+) + q^- n^\mu(-) + q_\perp{}^\mu\}n^\nu(+) \\ &+\{(k^+ + q^+)n^\nu(+) + q^- n^\nu(-) + q_\perp{}^\nu\}n^\mu(+) - g^{\mu\nu}q^-) \\ &\times \bar{u}(k';s')\not{n}(-)u(k;s) \\ &-i\epsilon^{\mu\nu\alpha\beta}\{(k^+ + q^+)n_\alpha(+) + q^- n_\alpha(-) + q_{\perp\alpha}\}n_\beta(+) \\ &\times \bar{u}(k';s')\not{n}(-)\gamma_5 u(k;s)] \end{aligned}$$

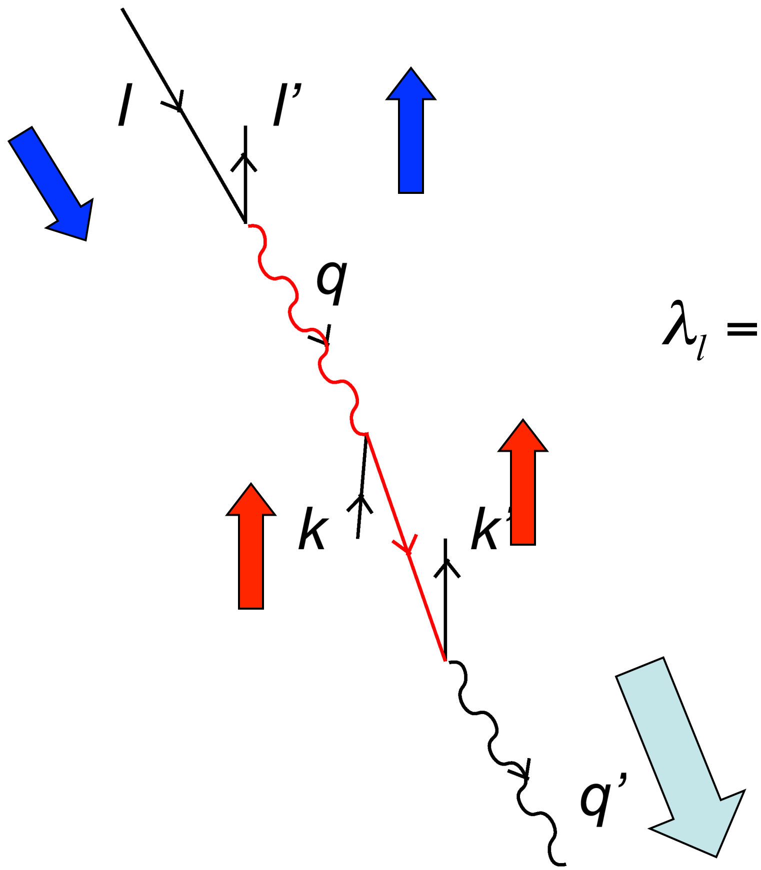
and the reduced amplitude that agrees in the DVCS limit

$$\begin{aligned} T_s{}^{\mu\nu} &= \frac{q^-}{s}[\{n^\mu(-)n^\nu(+) + n^\nu(-)n^\mu(+) - g^{\mu\nu}\} \\ &\times \bar{u}(k';s')\not{n}(-)u(k;s) \\ &-i\epsilon^{\mu\nu\alpha\beta}n_\alpha(-)n_\beta(+) \times \bar{u}(k';s')\not{n}(-)\gamma_5 u(k;s)] \end{aligned}$$

The tensor structure of the reduced amplitude is identical to the ones given by X. Ji and A.V. Radyushkin.

Sanity Checks of Amplitudes

- Gauge invariance of each and every polarized amplitude including the longitudinal polarization for the virtual photon.
- Klein-Nishina Formula in RCS.
- Angular Momentum Conservation.

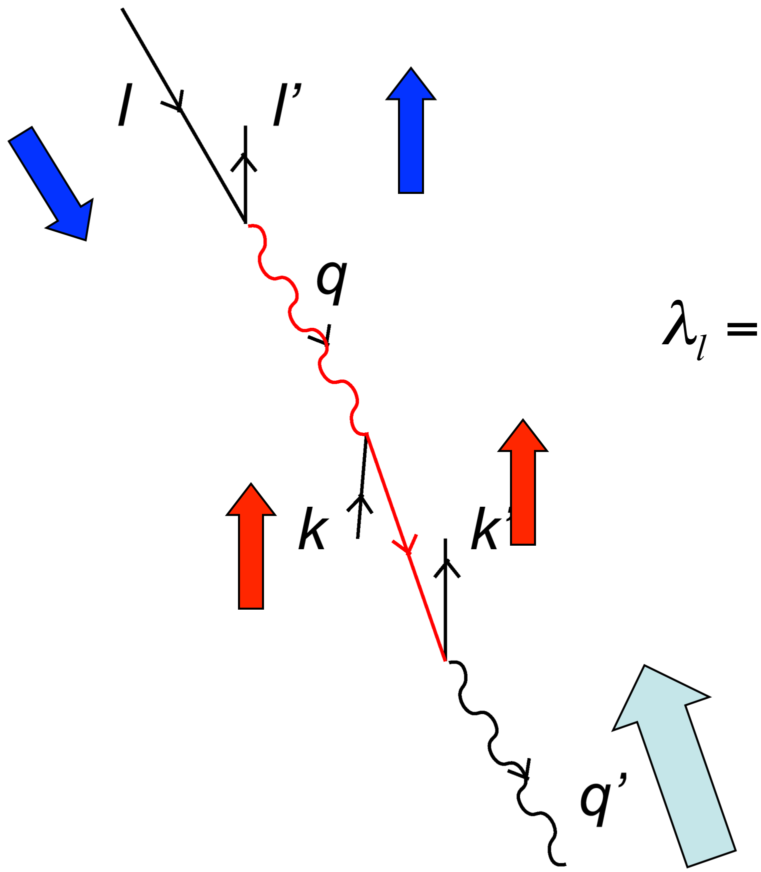


$$\lambda_l = \lambda_{l'} = +\frac{1}{2}\,,\, s_k = s_{k'} = +\frac{1}{2}\,,\, h_{q'} = +1\,;$$

Allowed !

Checking Amplitudes

- Gauge invariance of each and every polarized amplitude including the longitudinal polarization for the virtual photon.
- Klein-Nishina Formula in RCS.
- Angular Momentum Conservation.



$$\lambda_l = \lambda_{l'} = +\frac{1}{2}\,,\ s_k = s_{k'} = +\frac{1}{2}\,,\ h_{q'} = -1\,;$$

Prohibited !

Comparison

Complete DVCS amplitudes, $\sum_h \mathcal{L}(\{\lambda',\lambda\},h)\frac{1}{q^2}\mathcal{H}(\{h',h\}\{s',s\})$ in three approaches, ours, A.V. Radyushkin, and X. Ji. Because the hadrons and leptons are massless, $\lambda'=\lambda$ and $s'=s$.

λ h' s	this work	AVR	XJ
$\frac{1}{2}$ 1 $\frac{1}{2}$	$\frac{4}{Q}\sqrt{\frac{x}{x-\zeta}}\left(1+\frac{\zeta}{2(x-\zeta)}\frac{\mathbf{\Delta}_\perp^2}{Q^2}\right)$	$\frac{4}{Q}\sqrt{\frac{x}{x-\zeta}}\left(1-\frac{\zeta}{2(x-\zeta)}\frac{\mathbf{\Delta}_\perp^2}{Q^2}\right)$	0
$\frac{1}{2}$ 1 $-\frac{1}{2}$	$\frac{4}{Q}\sqrt{\frac{x-\zeta}{x}}\left(1-\frac{\zeta}{2(x-\zeta)}\frac{\mathbf{\Delta}_\perp^2}{Q^2}\right)$	$\frac{4}{Q}\sqrt{\frac{x-\zeta}{x}}\left(1+\frac{\zeta}{2(x-\zeta)}\frac{\mathbf{\Delta}_\perp^2}{Q^2}\right)$	0
$\frac{1}{2}$ -1 $\frac{1}{2}$	$-\frac{4}{Q^3}\frac{\zeta^2}{\sqrt{x(x-\zeta)}(x-\zeta)}\frac{\mathbf{\Delta}_\perp^2}{Q^2}$	0	$\frac{4}{Q}\sqrt{\frac{x}{x-\zeta}}\left(1-\frac{\zeta}{2(x-\zeta)}\frac{\mathbf{\Delta}_\perp^2}{Q^2}\right)$
$\frac{1}{2}$ -1 $-\frac{1}{2}$	0	0	$\frac{4}{Q}\sqrt{\frac{x-\zeta}{x}}\left(1+\frac{\zeta}{2(x-\zeta)}\frac{\mathbf{\Delta}_\perp^2}{Q^2}\right)$

AVR=XJ, taking into account the real photon helicity swap for the exact collinear kinematics vs. the nonlinear kinematicsin LFD:

$$q'^{\mu}=\left(\alpha\frac{\mathbf{\Delta}_\perp{}^2}{Q^2}p^+,-\mathbf{\Delta}_\perp,\frac{Q^2}{2\alpha p^+}\right)\leftrightarrow\left(0,0_\perp,\frac{Q^2}{2\zeta p^+}\right)$$

$$+h'\leftrightarrow -h'$$

C.Carlson and C.Ji, Phys.Rev.D67,116002 (2003);
B.Bakker and C.Ji, Phys.Rev.D83,091502(R) (2011).

For any orders in Q

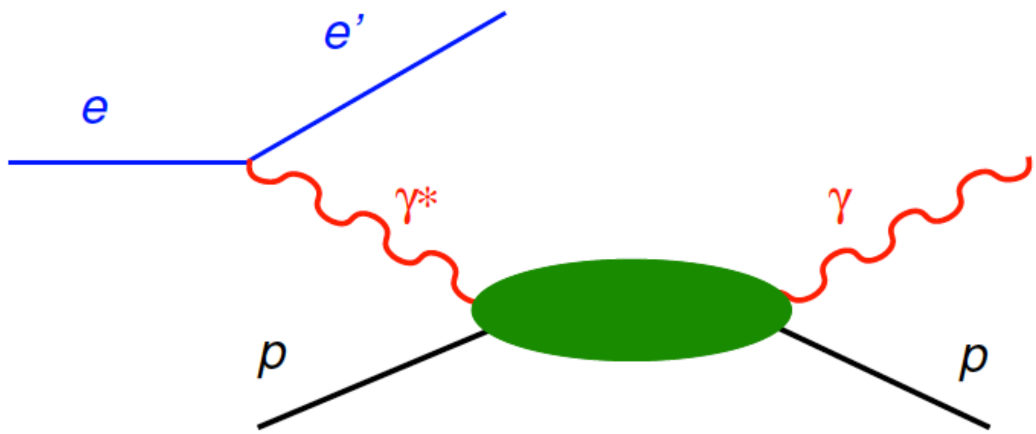
Exact | Reduced

λ	h'	s	$\mathcal{A} = \Sigma \mathcal{L} \frac{1}{q^2} \mathcal{H}$	$\mathcal{A}_{\rm red} = \Sigma \mathcal{L} \frac{1}{q^2} \mathcal{H}_{\rm red}$
$\frac{1}{2}$	1	$\frac{1}{2}$	$4 \sqrt{\frac{x}{(x-\zeta)D_+}} \frac{Q^3}{Q^4-4(\zeta p^+)^4}$	$-4(\zeta p^+)^2 \sqrt{\frac{x-\zeta}{xD_+}} \frac{4Q\Delta(\zeta p^+)^2-D_-Q^4}{\Delta(Q^4-4(\zeta p^+)^4)}$
$\frac{1}{2}$	1	$-\frac{1}{2}$	$2\frac{2Q\{Q^3(x-\zeta)-4\Delta\zeta(\zeta p^+)^2\}-D_-\{Q^4(x-\zeta)-4\zeta(\zeta p^+)^4\}}{\sqrt{x(x-\zeta)D_+}Q(Q^4-4(\zeta p^+)^4)}$	$-8 \sqrt{\frac{xD_+}{x-\zeta}} \frac{(\zeta p^+)^4}{Q(Q^4-4(\zeta p^+)^4)}$
$\frac{1}{2}$	-1	$\frac{1}{2}$	$2\frac{4(\zeta p^+)^2\{2Q\Delta\zeta-(\zeta p^+)^2(x-\zeta)D_+\}-D_-Q^4\zeta}{\sqrt{x(x-\zeta)D_+}Q(Q^4-4(\zeta p^+)^4)}$	$2 \sqrt{\frac{xD_+}{x-\zeta}} \frac{Q^3}{Q^4-4(\zeta p^+)^4)}$
$\frac{1}{2}$	-1	$-\frac{1}{2}$	$-16 \sqrt{\frac{x}{(x-\zeta)D_+}} \frac{(\zeta p^+)^4}{Q(Q^4-4(\zeta p^+)^4)}$	$4 \sqrt{\frac{x-\zeta}{xD_+}} \frac{Q^3\Delta-(\zeta p^+)^2D_-Q^2}{\Delta(Q^4-4(\zeta p^+)^4)}$

$$D = \frac{4\zeta\Delta^2}{(x-\zeta)Q^2}, \quad D_\pm = 1 \pm \sqrt{1-D}$$

Here, $\Delta = |\boldsymbol{\Delta}_\perp|$

Number of Independent Amplitudes in VCS



Nucleon Target

$$3 \times 2 \times 2 \times 2/2 = 12$$

12 independent tensor structures

M.Perrottet, Lett. Nuovo Cim. 7, 915 (1973);
R.Tarrach, Nuovo Cim. 28A, 409 (1975);
D.Drechsel et al.,PRC57,941(1998);
A.V.Belitsky, D.Mueller and A.Kirchner, NPB629, 323(2002);
A.V.Belitsky and D.Mueller, PRD82, 074010(2010)

DNA Method

$$\boxed{d^{\mu\nu\alpha\beta} = g^{\mu\nu}g^{\alpha\beta} - g^{\mu\beta}g^{\nu\alpha}}$$

$$G^{\mu\nu}(q'q) = q'_\alpha d^{\mu\nu\alpha\beta} q_\beta = q' \cdot q\, g^{\mu\nu} - q^\mu q'^\nu,$$
$$G^{\mu\nu}(qq) = q_\alpha d^{\mu\nu\alpha\beta} q_\beta = q^2\, g^{\mu\nu} - q^\mu q^\nu,$$
$$G^{\mu\nu}(q'q') = q'_\alpha d^{\mu\nu\alpha\beta} q'_\beta = q'^2\, g^{\mu\nu} - q'^\mu q'^\nu,$$
$$G^{\mu\nu}(\overline{P}q) = \overline{P}_\alpha d^{\mu\nu\alpha\beta} q_\beta = \overline{P} \cdot q\, g^{\mu\nu} - q^\mu \overline{P}^\nu,$$
$$G^{\mu\nu}(q'\overline{P}) = q'_\alpha d^{\mu\nu\alpha\beta} \overline{P}_\beta = \overline{P} \cdot q'\, g^{\mu\nu} - \overline{P}^\mu q'^\nu.$$

$$\begin{aligned}\tilde{T}^{\mu\nu}_{\rm DNA} := \sum_{i=1}^{5} \mathcal{S}_i\, \tilde{T}^{(i)\,\mu\nu}_{\rm DNA} &= \mathcal{S}_1\, G^{\mu\nu}(q'q) \\ &+ \mathcal{S}_2\, G^{\mu\lambda}(q'q')\, G_\lambda{}^\nu(qq) \\ &+ \mathcal{S}_3\, G^{\mu\lambda}(q'\overline{P})\, G_\lambda{}^\nu(\overline{P}q) \\ &+ \mathcal{S}_4\, [G^{\mu\lambda}(q'\overline{P})\, G_\lambda{}^\nu(qq) + G^{\mu\lambda}(q'q')\, G_\lambda{}^\nu(\overline{P}q)] \\ &+ \mathcal{S}_5\, G^{\mu\lambda}(q'q')\overline{P}_\lambda \overline{P}_{\lambda'} G^{\lambda\nu}(qq).\end{aligned}$$

Compton Form Factors (CFFs) : $\mathcal{S}_i,\ i = 1, 2, \ldots, 5$

B.Bakker and C.Ji,Few-Body Syst.58,1 (2017)

Most General Hadronic Tensor for Scalar Target

$$T^{\mu\nu} = G_{qq'}^{\mu\nu} S_1 + G_q^{\mu\lambda} G_{q'\lambda}{}^{\nu} S_2 + G_{q\bar{P}}^{\mu\lambda} G_{\bar{P}q'\lambda}{}^{\nu} S_3$$

$$+ (G_{q\bar{P}}^{\mu\lambda} G_{q'\lambda}{}^{\nu} + G_q^{\mu\lambda} G_{\bar{P}q'\lambda}{}^{\nu}) S_4 + G_q^{\mu\lambda} \bar{P}_\lambda \bar{P}_{\lambda'} G_{q'}^{\lambda'\nu} S_5$$

$$G_{qq'}^{\mu\nu} = g^{\mu\nu} q \cdot q' - q'^{\mu} q^{\nu}$$

$$G_q^{\mu\nu} = g^{\mu\nu} q^2 - q^{\mu} q^{\nu}$$

$$G_{q'}^{\mu\nu} = g^{\mu\nu} q'^2 - q'^{\mu} q'^{\nu}$$

$$G_{q\bar{P}}^{\mu\nu} = g^{\mu\nu} q \cdot \bar{P} - \bar{P}^{\mu} q^{\nu}$$

$$G_{\bar{P}q'}^{\mu\nu} = g^{\mu\nu} q' \cdot \bar{P} - q'^{\mu} \bar{P}^{\nu}$$

For $q'^2 = 0$, only S_1, S_2 and S_4 contribute.

Metz's approach $S_1 = -B_1, S_2 = B_3, S_3 = -B_2, S_4 = B_4, S_5 = B_{19}$

The method using the projectors introduces a kinematical singularity at $q' \cdot q = 0$. In Tarrach's paper a method is described to remove these kinematic poles. Here we give the final result of that algorithm as obtained in the thesis of Metz[3]. His CFFs are denoted as B_1, B_2, B_3, B_4, and B_{19}. They are implicitly given in terms of the elementary tensor by the following equations:

$$
\begin{aligned}
M^{\mu\nu} &= B_1 M_1^{\mu\nu} + B_2 M_2^{\mu\nu} + B_3 M_3^{\mu\nu} + B_4 M_4^{\mu\nu} + B_{19} M_{19}^{\mu\nu}, \\
M_1^{\mu\nu} &= -q' \cdot q\, g^{\mu\nu} + q^\mu q'^\nu, \\
M_2^{\mu\nu} &= -(\bar{P} \cdot q)^2 g^{\mu\nu} - q' \cdot q\, \bar{P}^\mu \bar{P}^\nu + \bar{P} \cdot q\, (\bar{P}^\mu q'^\nu + q^\mu \bar{P}^\nu), \\
M_3^{\mu\nu} &= q'^2 q^2 g^{\mu\nu} + q' \cdot q\, q'^\mu q^\nu - q^2 q'^\mu q'^\nu - q'^2 q^\mu q^\nu, \\
M_4^{\mu\nu} &= \bar{P} \cdot q\, (q'^2 + q^2)\, g^{\mu\nu} - \bar{P} \cdot q\, (q'^\mu q'^\nu + q^\mu q^\nu) \\
&\quad - q^2 \bar{P}^\mu q'^\nu - q'^2 q^\mu \bar{P}^\nu + q' \cdot q\, (\bar{P}^\mu q^\nu + q'^\mu \bar{P}^\nu), \\
M_{19}^{\mu\nu} &= (\bar{P} \cdot q)^2 q'^\mu q^\nu + q'^2 q^2 \bar{P}^\mu \bar{P}^\nu - \bar{P} \cdot q\, q^2 q'^\mu \bar{P}^\nu - \bar{P} \cdot q\, q'^2 \bar{P}^\mu q^\nu.
\end{aligned}
$$

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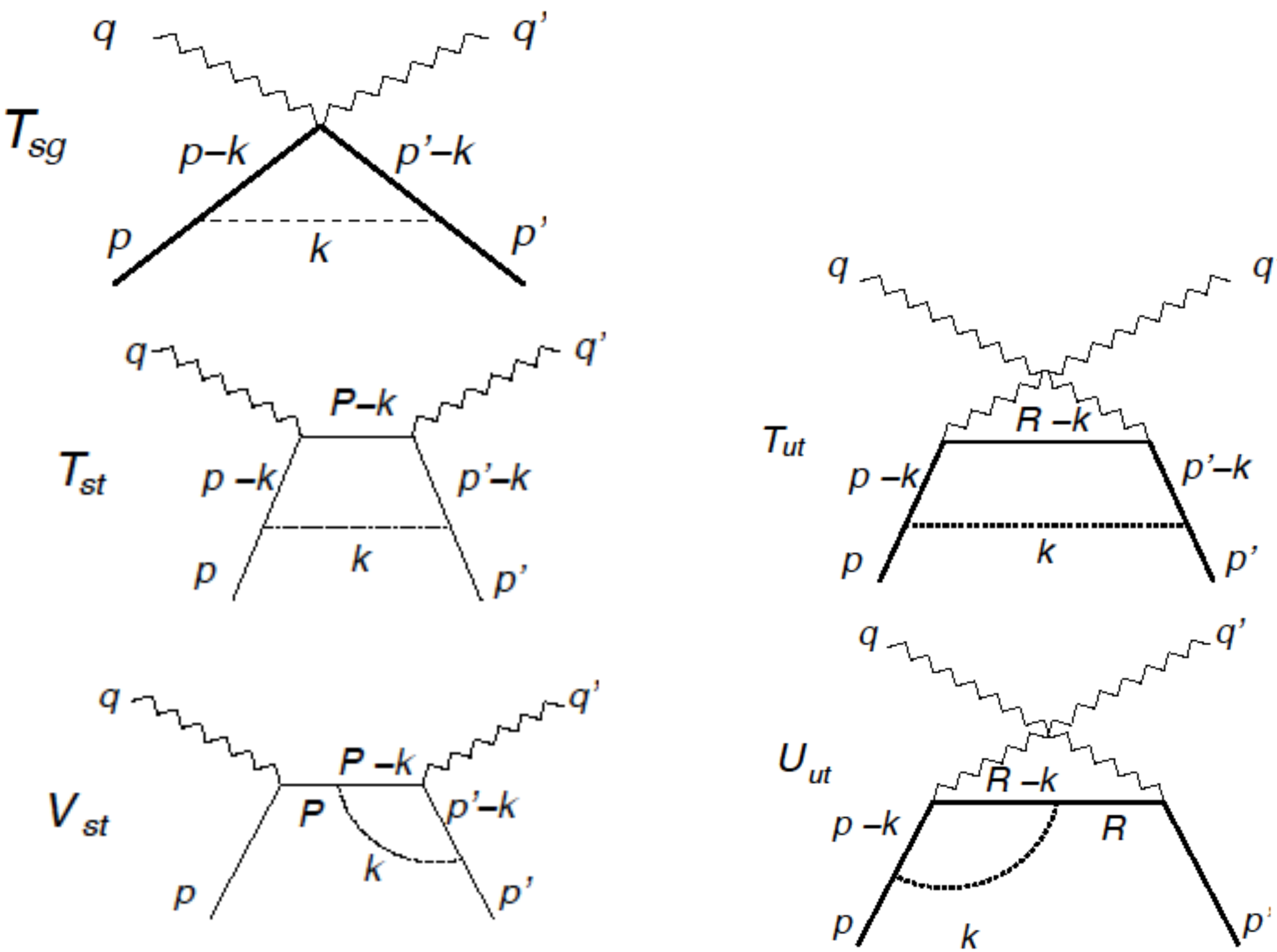
[3] A. Metz, *Virtuelle Comptonstreuung un die Polarisierbarkeiten de Nukleons* (in German), PhD thesis, Universität mainz, 1997.

Gauge invariance requires more than handbag amplitudes



Conclusion and Outlook

- Although the existing formulation meant already good progress, the realistic experimental setup requires the extension of the formalism to cover the broader kinematic regions of the DVCS experiments.
- The determination of most general hadron tensor structure is important not only for CFFs also for the discussion of GPDs.
- Maintaining EM gauge invariance is an important constraint.
- Most general hadronic tensor structure for scalar target is found and applicable to DVCS and DVMP off ^4H_e .