QCD evolution and resummation for transverse momentum distribution

Zhongbo Kang

Theoretical Division, Group T-2
Los Alamos National Laboratory

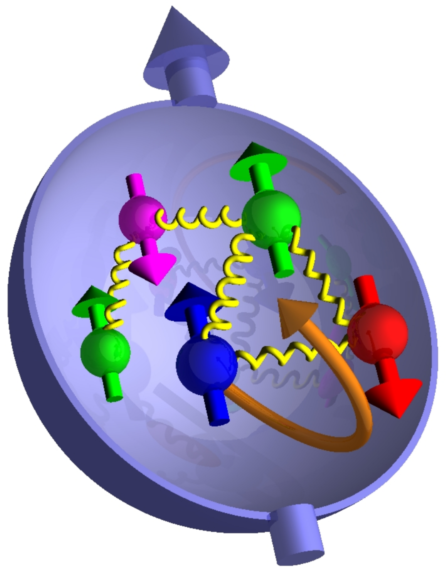
QCD Evolution Workshop
Jefferson Lab, Newport News, VA
May 14, 2012

Outline: whenever there is a evolution, there is a resummation

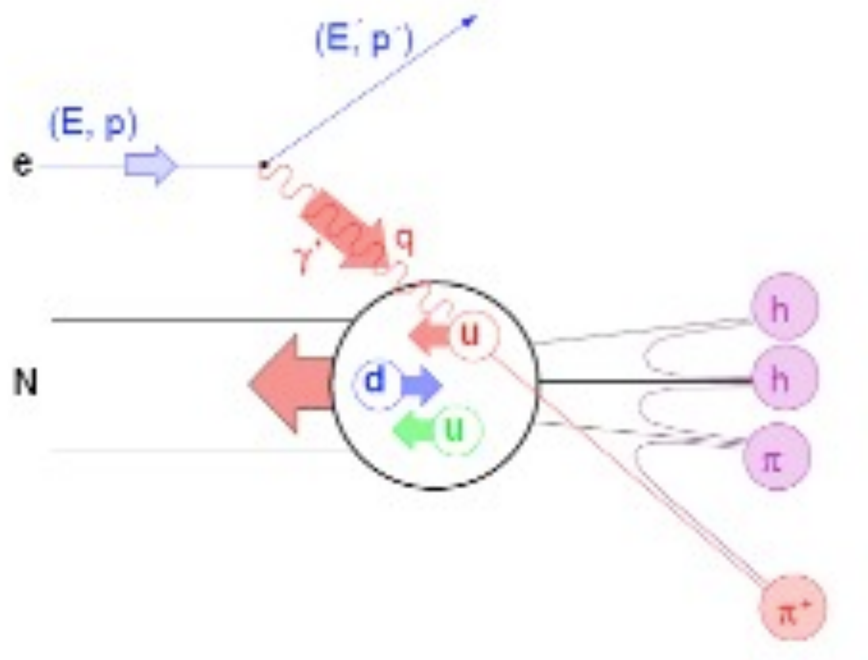
- QCD factorization: collinear .vs. TMD
  - Concepts: evolution, resummation, and their connection
  - Collinear factorization: DGLAP evolution = resummation of single logarithms
- TMD factorization: QCD evolution and resummation
  - Evolution of TMDs = resummation of double logarithms
  - Illustration of unpolarized TMDs
- Evolution of Sivers function
  - Difference between SIDIS and DY regarding sign change
  - Perturbative Y-term
- Phenomenology (preliminary)
- Summary

QCD factorization: a way to probe hadron structure

- We want to understand hadron structure in terms of quarks and gluons
  - longitudinal momentum distribution: collinear PDFs
  - transverse momentum distribution: TMDs



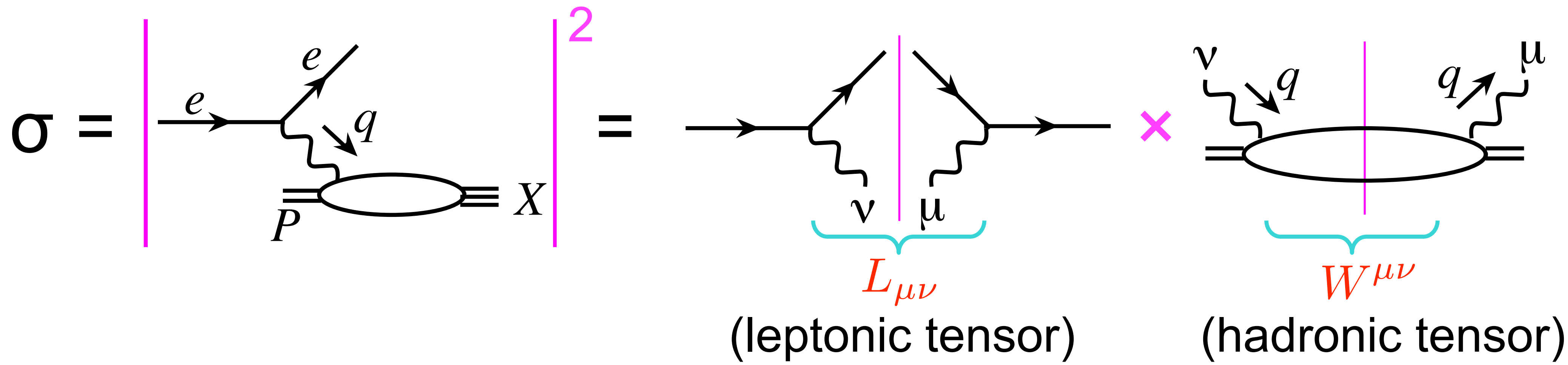
- To extract information on hadron structure, we send a probe and measure the outcome of the collisions
  - in order to trace back what's inside hadron from the outcome of the collisions, we rely on QCD factorization



- QCD factorization
  - collinear factorization: $pp \rightarrow h+X$ at high pt
  - TMD factorization: SIDIS, DY, $e^+e^- \rightarrow h_1h_2+X$
  - They are closely related to each other

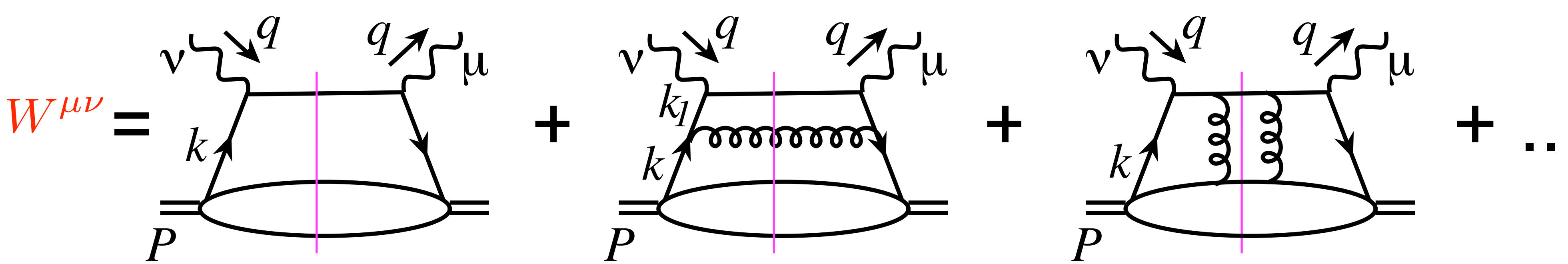
DIS as an example

- Deep Inelastic Scattering (DIS)

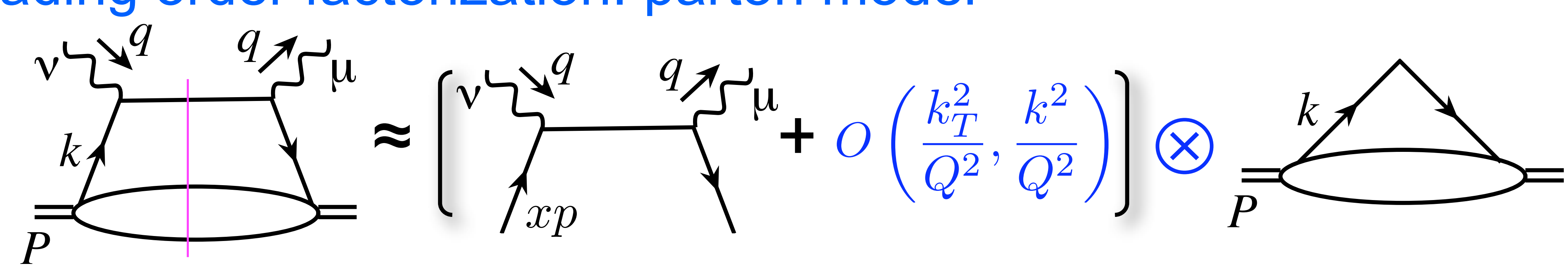


All the interesting physics (QCD dynamics) is contained in $W^{\mu\nu}$

- Hadronic tensor in perturbative expansion

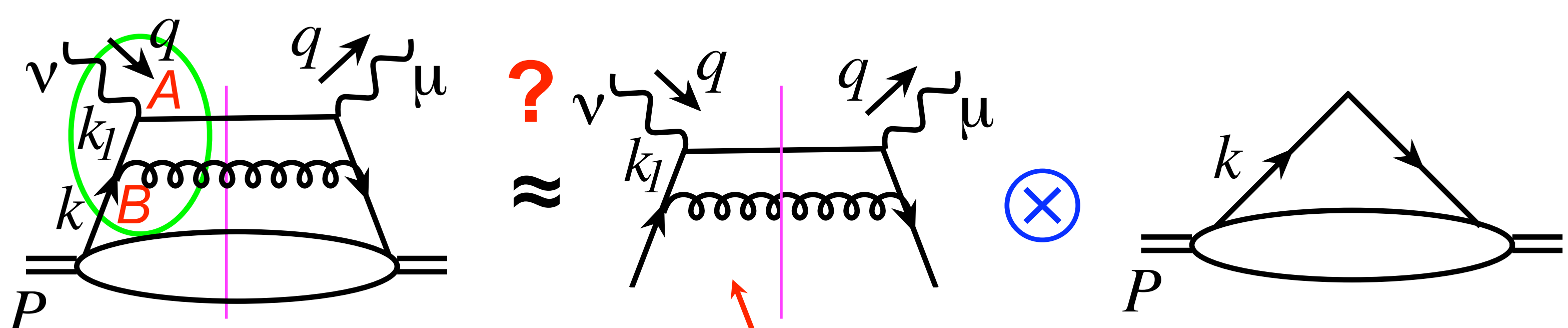


- Leading order factorization: parton model



QCD dynamics beyond leading order

- Radiative corrections



Collinear divergence!!! (from $k_1^2 \sim 0$)

$$\Rightarrow \int d^4k_1 \frac{i}{k_1^2+i\epsilon}\frac{-i}{k_1^2-i\epsilon} \Rightarrow \infty$$

❖ $k_1^2 \sim 0$ intermediate quark is on-shell

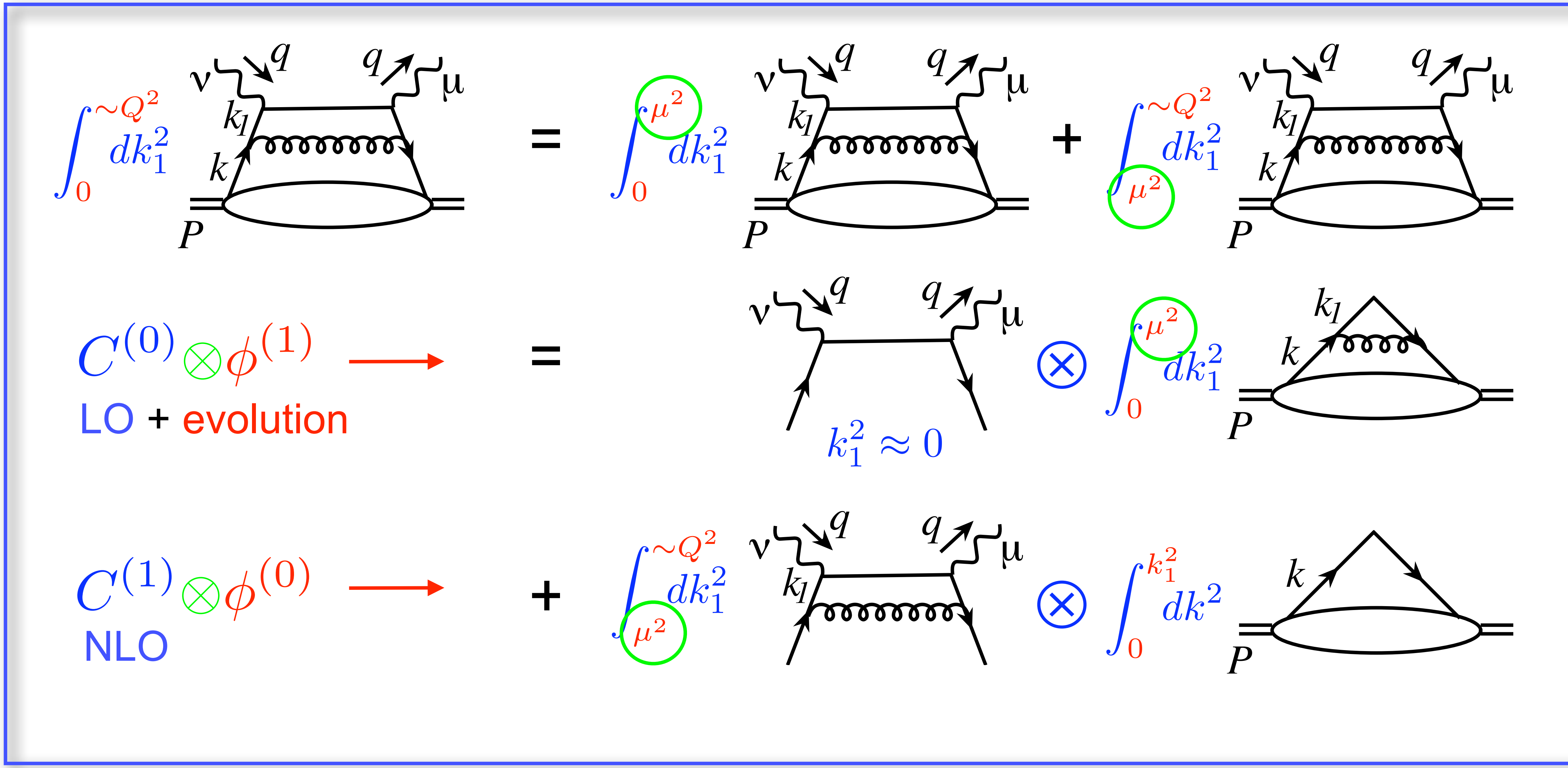
$t_{AB} \rightarrow \infty$

❖ gluon radiation takes place long before the photon-quark interaction
$\Longrightarrow$ a part of PDF

Partonic diagram has both long- and short-distance physics

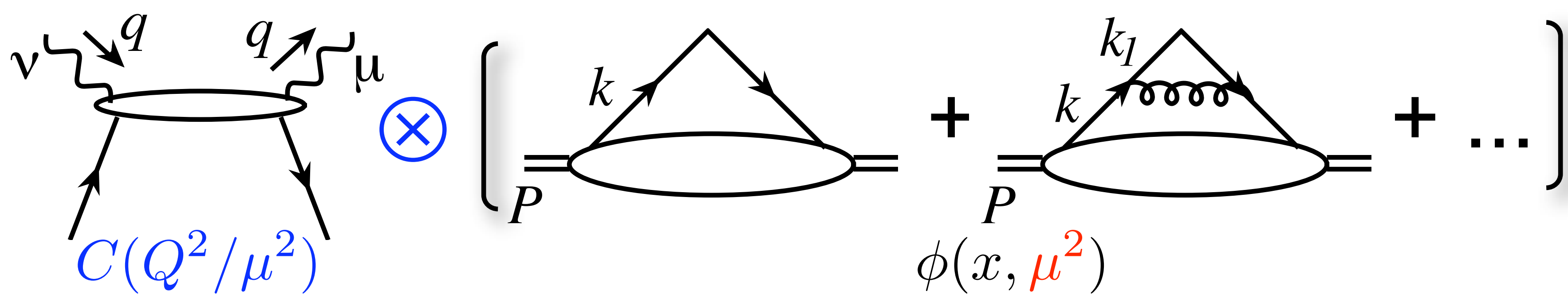
Factorization: separation of short- from long-distance

- Systematically remove all the long-distance physics into PDFs

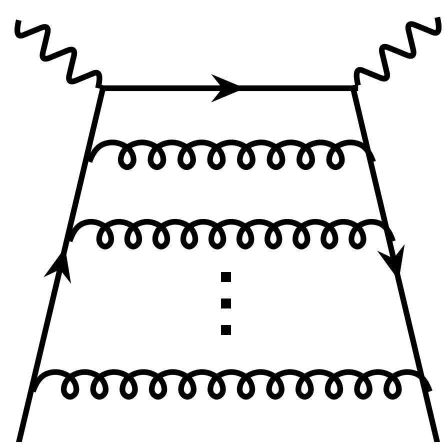


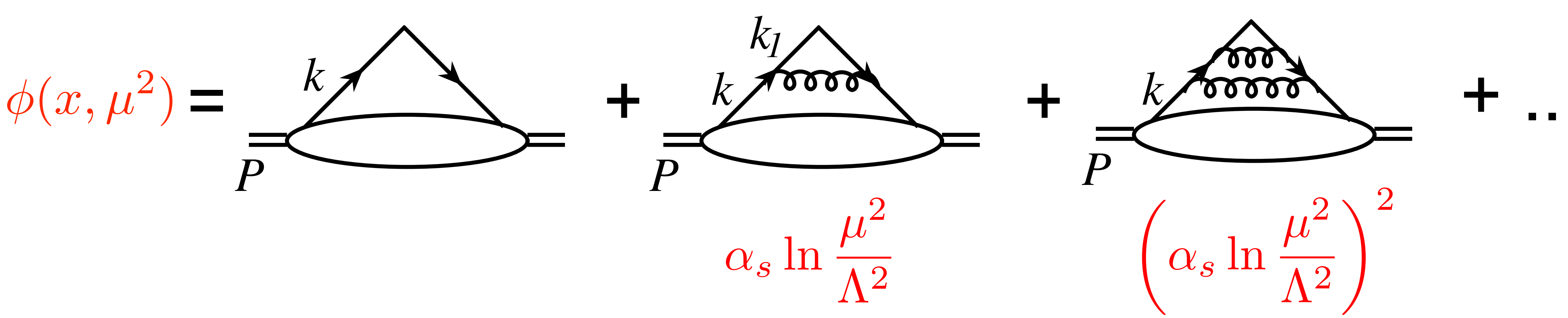
Scale-dependence of PDFs

- Logarithmic contributions into parton distributions



- Going to even higher orders: QCD resummation of single logs

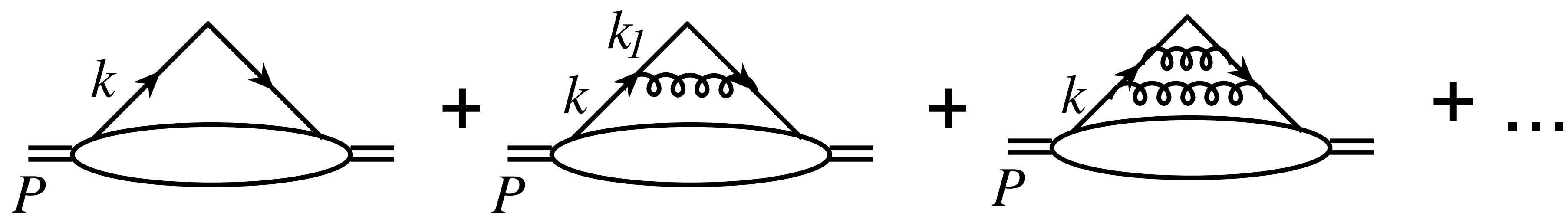




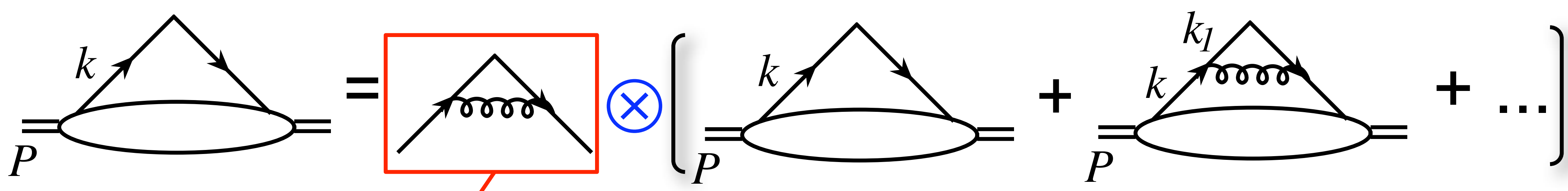
DGLAP evolution = resummation of single logs

- Evolution = Resum all the gluon radiation

$\phi(x, \mu^2)$ =



$\phi(x, \mu^2)$ -



DGLAP Equation

Evolution kernel
splitting function

$$\frac{\partial}{\partial \ln \mu^2}\phi_i(x,\mu^2) = \sum_j \boxed{P_{ij}(\frac{x}{x'})} \otimes \phi_j(x',\mu^2)$$

- By solving the evolution equation, one resums all the single logarithms of $\left(\alpha_s \ln \frac{\mu^2}{\Lambda^2}\right)^n$

Similar single logs for evolution of twist-3 correlation functions

- Qiu-Sterman function: first kt-moment of Sivers function

splitting kernel for unpolarized PDFs

$$\frac{\partial T_{q,F}(x,x,\mu)}{\partial \ln \mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\Big\{P_{qq}(z)T_{q,F}(\xi,\xi,\mu) + \frac{N_c}{2}\left[\frac{1+z^2}{1-z}\left(T_{q,F}(\xi,x,\mu)-T_{q,F}(\xi,\xi,\mu)\right)+zT_{q,F}(\xi,x,\mu)+T_{\Delta q,F}(x,\xi,\mu)\right] - N_c\delta(1-z)\,T_{q,F}(x,x,\mu)+\frac{1}{2N_c}\left[(1-2z)T_{q,F}(x,x-\xi,\mu)+T_{\Delta q,F}(x,x-\xi,\mu)\right]\Big\}$$

Kang-Qiu, arXiv: 1205.1019

- Another twist-3 correlation function: first kt-moment of Boer-Mulders function

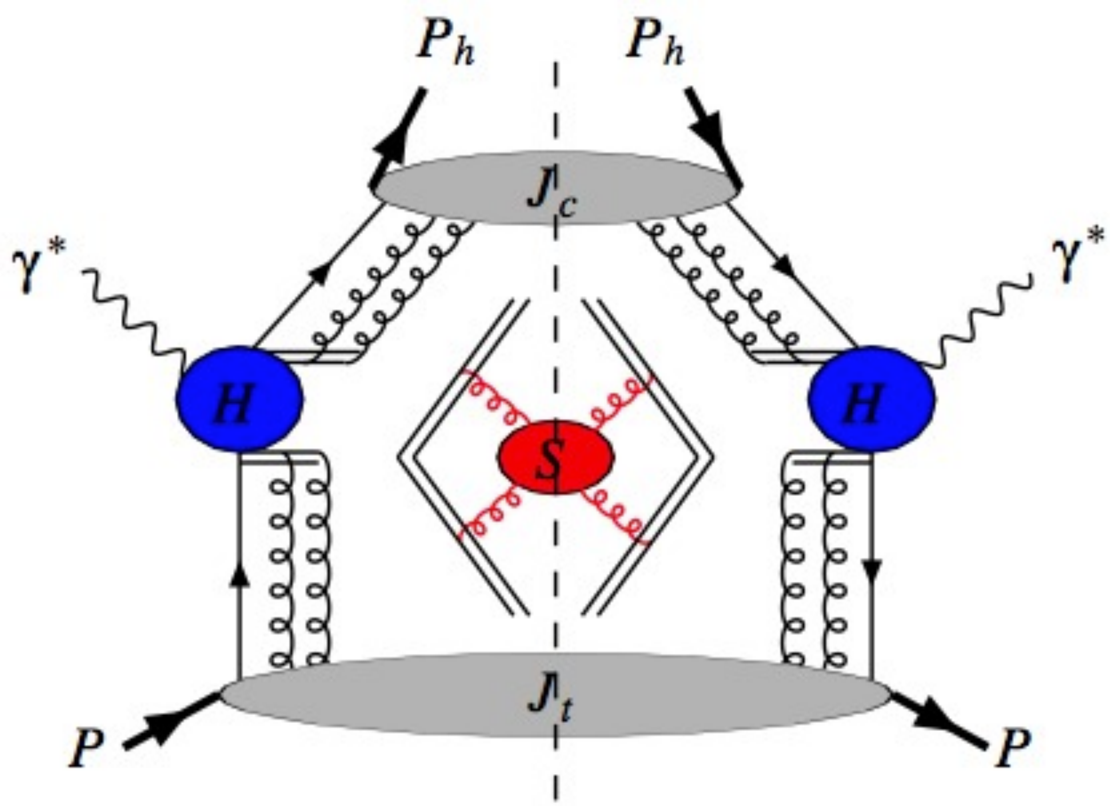
splitting kernel for transversity

$$\frac{\partial T^{(\sigma)}_{q,F}(x,x,\mu)}{\partial \ln \mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\Big\{\Delta_T P_{qq}(z)T^{(\sigma)}_{q,F}(\xi,\xi,\mu) + \frac{N_c}{2}\left[\frac{2T^{(\sigma)}_{q,F}(\xi,x,\mu)-2z\,T^{(\sigma)}_{q,F}(\xi,\xi,\mu)}{1-z}\right] - N_c\delta(1-z)\,T^{(\sigma)}_{q,F}(x,x,\mu)+\frac{1}{2N_c}\left[2(1-z)T^{(\sigma)}_{q,F}(x,x-\xi,\mu)\right]\Big\},$$

TMD factorization

- Example: SIDIS (two scales - Q and qt)

Ji-Ma-Yuan, 2005



$$F(x_B, z_h, P_{h\perp}, Q^2) = \sum_{q=u,d,s,\ldots} e_q^2 \int d^2\vec{k}_\perp d^2\vec{p}_\perp d^2\vec{\ell}_\perp$$
$$\times q\left(x_B, k_\perp, \mu^2, x_B\zeta, \rho\right) \hat{q}_h\left(z_h, p_\perp, \mu^2, \hat{\zeta}/z_h, \rho\right) S(\vec{\ell}_\perp, \mu^2, \rho)$$
$$\times H\left(Q^2, \mu^2, \rho\right) \delta^2(z_h\vec{k}_\perp + \vec{p}_\perp + \vec{\ell}_\perp - \vec{P}_{h\perp}) \ ,$$

Evolution of TMDs follow Collins-Soper evolution

- Evolution of collinear PDFs follow the usual DGLAP-type evolution equation, which is equivalent to resum the single-logarithmic contributions to all order

$$\left(\alpha_s \ln \frac{Q^2}{\mu^2}\right)^n$$

- Evolution of TMDs follow Collins-Soper-type evolution equation, which is equivalent to resum the double-logarithmic contributions to all order, which is usually more difficult

$$\left(\alpha_s \ln^2 \frac{Q^2}{q_T^2}\right)^n$$

SIDIS cross section

- in momentum space:

$$F(x_B, z_h, P_{h\perp}, Q^2) = \sum_{q=u,d,s,\ldots} e_q^2 \int d^2\vec{k}_\perp d^2\vec{p}_\perp d^2\vec{\ell}_\perp$$
$$\times q\left(x_B, k_\perp, \mu^2, x_B\zeta, \rho\right) \hat{q}_h\left(z_h, p_\perp, \mu^2, \hat{\zeta}/z_h, \rho\right) S(\vec{\ell}_\perp, \mu^2, \rho)$$
$$\times H\left(Q^2, \mu^2, \rho\right) \delta^2(z_h\vec{k}_\perp + \vec{p}_\perp + \vec{\ell}_\perp - \vec{P}_{h\perp}) \ ,$$

- in b-space:

$$F(x_B, z_h, b, Q^2) = \sum_{q=u,d,s,\ldots} e_q^2 q\left(x_B, z_h b, \mu^2, x_B\zeta, \rho\right) \hat{q}\left(z_h, b, \mu^2, \hat{\zeta}/z_h, \rho\right)$$
$$\times S(b, \mu^2, \rho) H\left(Q^2, \mu^2, \rho\right) \ ,$$

  - TMD quark distribution and fragmentation functions contain double logarithms, others contain only single logarithms

Evolution of TMDs

- Since one now needs to resum double logarithms, typically it involves two steps: Idilbi-Ji-Ma-Yuan, 2004
  - Energy evolution of the unpolarized PDFs

$$\zeta\frac{\partial}{\partial\zeta}q(x,b,\mu^2,x\zeta,\rho)=\left(K(\mu,b)+G(\mu,x\zeta)\right)q(x,b,\mu^2,x\zeta,\rho)$$

  - Since it contains double logarithms, the kernel still contains single logarithms

$$\mu\frac{d}{d\mu}K(\mu,b)=-\gamma_K=-\mu\frac{d}{d\mu}G(\mu,\zeta)$$

- Solving these two equations, equivalently one resums the double logs
  - First for the evolution equation of K and G

$$K(b,\mu)+G(x\zeta,\mu)=K(b,\mu_L)+G(x\zeta,\mu_H)-\int_{\mu_L}^{\mu_H}\frac{d\tilde{\mu}}{\tilde{\mu}}\gamma_K(\alpha(\tilde{\mu}))$$

  - Then feed the solution back to the energy evolution equation

$$\begin{aligned} q(x,b,\mu,x\zeta,\rho) &= \exp\left\{-\int_{\mu_L}^{C_2x\zeta}\frac{d\mu}{\mu}\left[\ln\left(\frac{C_2x\zeta}{\mu}\right)\gamma_K(\alpha(\mu))-K(b,\mu_L)-G(\mu/C_2,\mu)\right]\right\} \\ &\quad\times q(x,b,\mu,x\zeta_0=\mu_L/C_2,\rho)\ , \end{aligned}$$

The formalism contains all the evolutions

- Similar for the unpolarized fragmentation function

$$\hat{q}(z,b,\mu,\hat{\zeta}/z,\rho) = \exp\left\{-\int_{\mu_L}^{C_2\hat{\zeta}/z}\frac{d\mu}{\mu}\left[\ln\left(\frac{C_2\hat{\zeta}}{z\mu}\right)\gamma_K(\alpha(\mu)) - K(b,\mu_L) - G(\mu/C_2,\mu)\right]\right\} \times\hat{q}(z,b,\mu,\hat{\zeta}_0/z=\mu_L/C_2,\rho)\ .$$

- Hard function and Soft function contain only single logs

$$\mu\frac{\partial S(\vec{b}_\perp,\mu^2,\rho)}{\partial\mu} = \gamma_S(\rho)S(\vec{b}_\perp,\mu^2,\rho)$$

$$\mu\frac{dH(Q^2/\mu^2,\rho)}{d\mu} = -\left(4\gamma_F - \gamma_S(\rho)\right)H(Q^2/\mu^2,\rho)$$

- Eventually collect all the terms

$$F(x_B,z_h,b,Q^2) = q\left(x_B,z_hb,\mu_L^2,\mu_L/C_2,\rho\right)\hat{q}\left(z_h,b,\mu_L^2,\mu_L/C_2,\rho\right)S(b,\mu_L^2,\rho)H\left(1/C_2^2\rho,\rho\right) \times\exp\left\{-2\int_{\mu_L}^{C_2Q\sqrt{\rho}}\frac{d\mu}{\mu}\left[\ln\left(\frac{C_2Q\sqrt{\rho}}{\mu}\right)\gamma_K(\alpha(\mu)) - K(b,\mu_L) - G(\mu/C_2,\mu) - 2\gamma_F + \frac{1}{2}\gamma_S(\rho)\right]\right\}\ ,$$

Final step

- Once all the logs are resummed, the rest of b-dependent PDFs and FFs can be expanded as collinear PDFs and FFs

$$\frac{d\sigma}{dx_B dy dz_h d^2P_{h\perp}} = \sigma_0 F_{UU}$$

$$F_{UU} = \int \frac{d^2b}{(2\pi)^2} e^{i\vec{q_\perp}\cdot\vec{b}} W_{UU}(b, Q, x_B, z_h)$$

$$W_{UU}(b, Q, x_B, z_h) = e^{-S(b,Q)} \sum_q e_q^2 \left(C_{q/i} \otimes f_{i/A}\right)(x_B, \mu = \frac{c}{b})$$
$$\times \left(D_{B/j} \otimes \tilde{C}_{j/q}\right)(z_h, \mu = \frac{c}{b})$$

- All the large logarithms are resummed to the Sudakov exponential term

$$S(b, Q) = \int_{c^2/b^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left[A \ln(Q^2/\mu^2) + B\right] \qquad A = \sum_{n=1} A^{(n)} \left(\frac{\alpha_s}{\pi}\right)^n$$

Evolution of Sivers function

- The Collins-Soper energy evolution is really for the whole correlator

$$\mathcal{Q}(x,k_\perp,\mu,x\zeta)=\frac{1}{2}\int\frac{d\xi^- d^2\vec{b}_\perp}{(2\pi)^3}e^{-ix\xi^-P^+ + i\vec{b}_\perp\cdot\vec{k}_\perp}\langle P|\overline{\psi}_q(\xi^-,\vec{b}_\perp)\mathcal{L}_v^\dagger\gamma^+\mathcal{L}_v\psi_q(0)|P\rangle.$$

$$=\frac{1}{2}\Big[q(x,k_\perp)\not{p}+\frac{1}{M}\delta q(x,k_\perp)\sigma^{\mu\nu}k_\mu p_\nu+\Delta q_L(x,k_\perp)\lambda\gamma_5\not{p}+\frac{1}{M}\Delta q_T(x,k_\perp)\gamma_5 p(\vec{k}_\perp\cdot\vec{S}_\perp)$$
$$+\frac{1}{M}\delta q_L(x,k_\perp)\lambda i\sigma_{\mu\nu}\gamma_5 p^\mu k_\perp^\nu+\delta q_T(x,k_\perp)i\sigma_{\mu\nu}\gamma_5 p^\mu S_\perp^\nu+\frac{1}{M^2}\delta q_{T'}(x,k_\perp)i\sigma_{\mu\nu}\gamma_5 p^\mu(\vec{k}_\perp\cdot\vec{S}_\perp k_\perp^\nu-\frac{1}{2}\vec{k}_\perp^2 S_\perp^\nu)$$
$$+\frac{1}{M}q_T(x,k_\perp)\epsilon^{\mu\nu\alpha\beta}\gamma_\mu p_\nu k_\alpha S_\beta\Big],$$

- So for Sivers function, it really is $k_\perp^\alpha f_{1T}^\perp(x,k_\perp^2)$ that evolves as a whole
  - in b-space, it is

Kang-Xiao-Yuan, PRL, 2011

$$\tilde{f}_{1T}^{(\perp\alpha)}(x,b,\mu,\zeta)=\frac{1}{M}\int d^2k_\perp e^{-i\vec{k}_\perp\cdot\vec{b}_\perp}k_\perp^\alpha\, f_{1T}^\perp(x,k_\perp,\mu,\zeta)$$

  - it follows the same energy evolution equation

$$\zeta\frac{\partial}{\partial\zeta}\tilde{f}_{1T}^{(\perp\alpha)}(x,b,\mu,\zeta)=[K(\mu,b)+G(\mu,\zeta)]\,\tilde{f}_{1T}^{(\perp\alpha)}(x,b,\mu,\zeta)$$

  - Thus one should get a very similar resummation formalism

The Sivers effect for SIDIS

- The resummation formalism (consistent with experimental convention)

$$\frac{d\sigma}{dx_B dy dz_h d^2P_{h\perp}} = \sigma_0 \left[F_{UU} + |s_\perp| \sin(\phi_h - \phi_s) F_{UT}^{\sin(\phi_h-\phi_s)}\right]$$

  - spin-dependent structure function

$$F_{UT}^{\sin(\phi_h-\phi_s)} = -\frac{1}{4\pi}\int_0^\infty db\, b^2 J_1(q_\perp b) W_{UT}(b, Q, x_B, z_h)$$

$$W_{UT}(b, Q, x_B, z_h) = e^{-S(b,Q)} \sum_q (\Delta C_{q/i}^T \otimes T_{i,F})(x_B, \mu = \frac{c}{b})$$
$$\times (D_{B/j} \otimes \tilde{C}_{j/q})(z_h, \mu = \frac{c}{b})$$

  - only soft-gluonic pole Qiu-Sterman function appears in this part

$$(\Delta C_{q/i}^T \otimes T_{i,F})(x_B, \mu) = \int_{x_B}^1 \frac{dx}{x} \Delta C_{q/i}^T(\frac{x_B}{x}, \mu) T_{i,F}(x, x, \mu)$$

Similar form for DY production

- Unpolarized DY production at low qt

$$\frac{d\sigma}{dQ^2 dy d^2 q_\perp} = \frac{\sigma_0}{2\pi}\int_0^\infty db\, b J_0(q_\perp b) W_{UU}(b, Q, x_A, x_B)$$

$$W_{UU}(b, Q, x_A, x_B) = e^{-S(b,Q)} \sum_q e_q^2 (C_{q/i} \otimes f_{i/A})(x_A, \mu = \frac{c}{b})$$
$$\times (C_{\bar{q}/j} \otimes f_{j/B})(x_B, \mu = \frac{c}{b})$$

- Single transverse spin dependent DY production at low qt

$$\frac{d\sigma}{dQ^2 dy d^2 q_\perp} = \sigma_0 \epsilon_\perp^{\alpha\beta} S_\perp^\beta \hat{q}_\perp^\beta \frac{1}{4\pi}\int_0^\infty db\, b^2 J_1(q_\perp b) W_{UT}(b, Q, x_A, x_B)$$

$$W_{UT}(b, Q, x_A, x_B) = e^{-S(b,Q)} \sum_q e_q^2 (\Delta C^T_{q/i} \otimes T_{i,F})(x_A, \mu = \frac{c}{b})$$
$$\times (C_{\bar{q}/j} \otimes f_{j/B})(x_B, \mu = \frac{c}{b})$$

Comments on SIDIS and DY

- The only difference comes from so-called coefficient function
  - leading order

$$\Delta C_{i/j}^{T(0)}(z, \mu = \frac{c}{b}) = \delta_{ij}\delta(1-z) \qquad \text{DY}$$

$$\Delta C_{i/j}^{T(0)}(z, \mu = \frac{c}{b}) = -\delta_{ij}\delta(1-z) \qquad \text{SIDIS}$$

  - at next-leading-order: well-known difference due to $Q^2>0$ (<0)

$$\Delta C_{i/j}^{T(1)}(z, \mu = \frac{c}{b}) = \delta_{ij}\left[-\frac{1}{4N_c} + \frac{C_F}{2}(\frac{\pi^2}{2} - 4)\delta(1-z)\right] \qquad \text{DY}$$

$$\Delta C_{i/j}^{T(1)}(z, \mu = \frac{c}{b}) = -\delta_{ij}\left[-\frac{1}{4N_c} + \frac{C_F}{2}(-4)\delta(1-z)\right] \qquad \text{SIDIS}$$

  - Thus in the full perturbative QCD region, Sivers between SIDIS and DY is not just a sign: it is interesting to study the consequence

Coefficients for Sivers and unpolarized PDFs is different

- When expanded Sivers function in terms of Qiu-Sterman function at small b, only soft-gluon pole contributes to the coefficient function (ε-term in dimensional regularization)

$$\Delta C^{T(1)}_{i/j}(z,\mu=\frac{c}{b}) = \delta_{ij}\left[-\frac{1}{4N_c}+\frac{C_F}{2}(\frac{\pi^2}{2}-4)\delta(1-z)\right]$$ Sivers function

$$C^{(1)}_{i/j}(z,\mu=\frac{c}{b}) = \delta_{ij}\left[\frac{C_F}{2}+\frac{C_F}{2}(\frac{\pi^2}{2}-4)\delta(1-z)\right]$$ Unpolarized PDFs

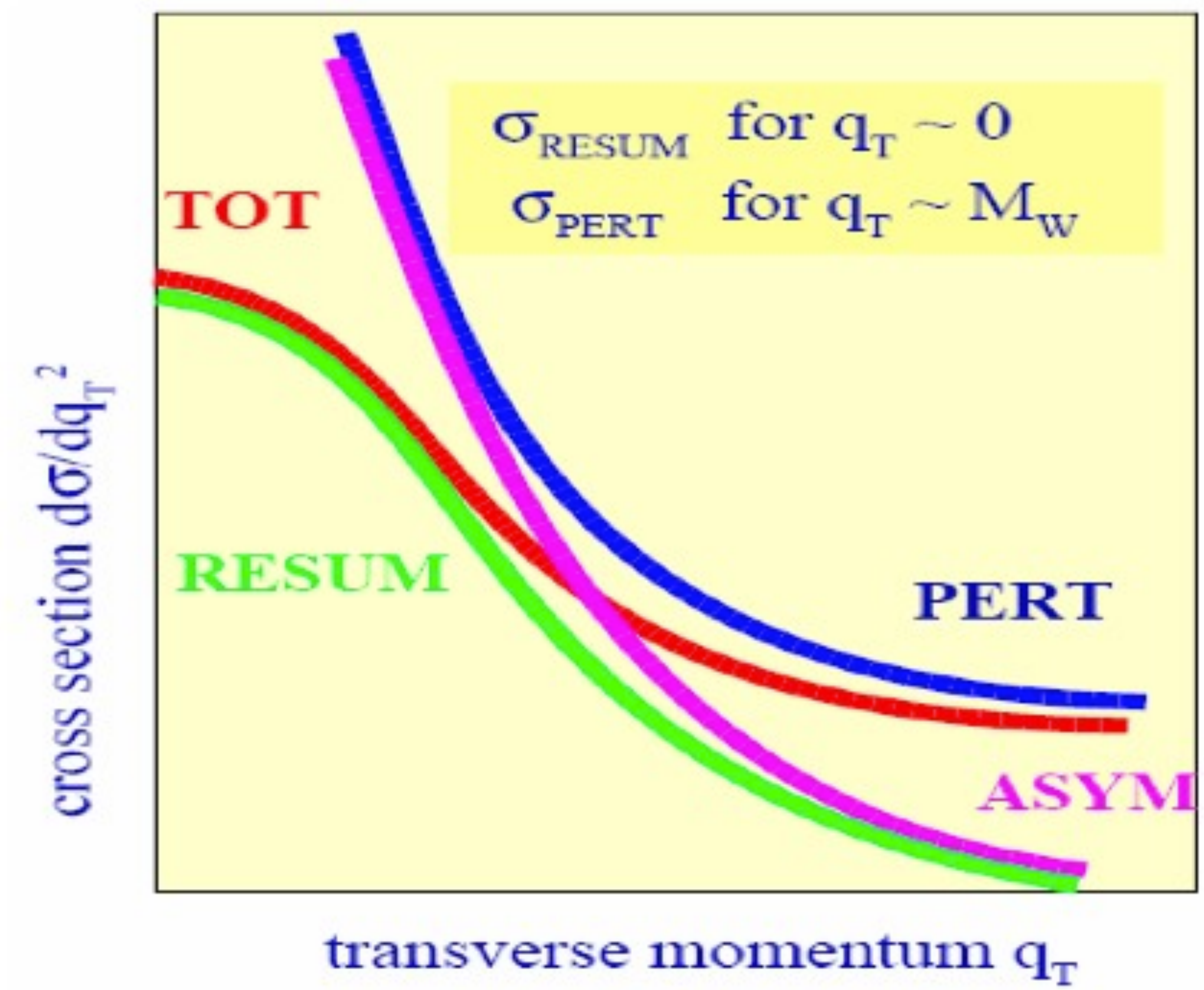
- In b-space, we found a extra collinear divergence which is supposed to absorbed into the evolution of Qiu-Sterman function. We know there is this -N_c term issue.

What is Y-term?

- So-far concentrate on the resumed term, which is most relevant when qt<<Q. When qt gets relatively large, the conventional NLO perturbative contribution becomes important

Y-term

$$\sigma_{TOT} = \sigma_{RESUM} + \sigma_{PERT} - \sigma_{ASYM}$$



Y term can be easily extracted

- Y term can be easily extracted/derived from existing calculations
  - For the DY production, in the paper of unified picture of Sivers effect

Ji-Qiu-Vogelsang-Yuan, PRD73, 2006

  - Perturbative term:

$$\frac{d^4\Delta\sigma(S_\perp)}{dQ^2dyd^2q_\perp} = \sigma_0\epsilon^{\alpha\beta}S_{\perp\alpha}q_{\perp\beta}\frac{\alpha_s}{2\pi^2}\int\frac{dx}{x}\frac{dx'}{x'}\sum_q e_q^2\left[\left(H_q^s+H_q^h\right)\bar{q}(x')+\left(H_g^s+H_g^h\right)g(x')\right]$$
$$\times\,\delta(\hat{s}+\hat{t}+\hat{u}-Q^2)\ ,$$

  - Asymptotic term (take the limit qt<<Q):

$$\frac{d^4\Delta\sigma^{q\bar{q}\to\gamma^*g}(S_\perp)}{dQ^2dyd^2q_\perp} = \sigma_0\,\epsilon^{\alpha\beta}S_{\perp\alpha}\frac{q_{\perp\beta}}{(q_\perp^2)^2}\frac{\alpha_s}{2\pi^2}\int\frac{dx}{x}\frac{dx'}{x'}\bar{q}(x')\left\{\delta(\xi_2-1)A+\delta(\xi_1-1)B\right\}$$

$$\frac{d^4\Delta\sigma^{qg\to\gamma^*q}(S_\perp)}{dQ^2dyd^2q_\perp} = \sigma_0\,\epsilon^{\alpha\beta}S_{\perp\alpha}\frac{q_{\perp\beta}}{(q_\perp^2)^2}\sum_q e_q^2T_F(z_1,z_1)$$
$$\times\frac{\alpha_s}{2\pi^2}T_R\int\frac{dx_2}{x_2}g(x_2)\left[\xi_2^2+(1-\xi_2)^2\right]$$

  - Y-term = perturbative-term - Asymptotic-term

Phenomenological studies

- Only at small b-region (corresponds to large momentum), one can calculate the relevant coefficients perturbatively.

$$\frac{d\sigma}{dQ^2 dy d^2 q_\perp} = \frac{\sigma_0}{2\pi}\int_0^\infty db\, b J_0(q_\perp b) W_{UU}(b, Q, x_A, x_B)$$

$$W_{UU}(b, Q, x_A, x_B) = e^{-S(b,Q)} \sum_q e_q^2 (C_{q/i} \otimes f_{i/A})(x_A, \mu = \frac{c}{b})$$
$$\times (C_{\bar{q}/j} \otimes f_{j/B})(x_B, \mu = \frac{c}{b})$$

- However, in order to Fourier transform back to qt-space, we need the whole b-region. Since large b-region will be non-perturbative, we need a non-perturbative input. This part should be universal if QCD factorization holds for the process.

Phenomenological study: concentrate on small qt part

- For small qt region, we could use the resumed formalism. Don't need to worry about Y-term.

- Only at small b-region (corresponds to large momentum), one can calculate the relevant coefficients perturbatively.

$$\frac{d\sigma}{dQ^2 dy d^2 q_\perp} = \frac{\sigma_0}{2\pi}\int_0^\infty db\, b J_0(q_\perp b) W_{UU}(b, Q, x_A, x_B)$$

$$W_{UU}(b, Q, x_A, x_B) = e^{-S(b,Q)} \sum_q e_q^2 (C_{q/i} \otimes f_{i/A})(x_A, \mu = \frac{c}{b})$$
$$\times (C_{\bar{q}/j} \otimes f_{j/B})(x_B, \mu = \frac{c}{b})$$

- However, in order to Fourier transform back to qt-space, we need the whole b-region. Since large b-region will be non-perturbative, we need a non-perturbative input. This part should be universal if QCD factorization holds for the process.

The parametrizations for the non-perturbative function

- Different approaches for the non-perturbative functions

$$\frac{d\sigma}{dQ^2 dy d^2 q_\perp} = \frac{\sigma_0}{2\pi} \int_0^\infty db\, b J_0(q_\perp b) W_{UU}(b, Q, x_A, x_B)$$

$$W_{UU}^{pert}(b, Q, x_A, x_B) = e^{-S(b,Q)} \sum_q e_q^2 (C_{q/i} \otimes f_{i/A})(x_A, \mu = \frac{c}{b}) \times (C_{\bar{q}/j} \otimes f_{j/B})(x_B, \mu = \frac{c}{b})$$

- Parametrize the full b-space function

$$W_{UU}(b, Q, x_A, x_B) = W_{UU}^{pert}(b, Q, x_A, x_B) F^{NP}(b, Q, x_A, x_B)$$

  - function form (through extrapolation): Qiu-Zhang, 2001
  - fitted form directly from experiments: Brock-Landry-Nadolsky-Yuan, 2003

$$W_{UU}(b, Q, x_A, x_B) = W_{UU}^{pert}(b_*, Q, x_A, x_B) F^{NP}(b, Q, x_A, x_B) \quad b_* = \frac{b}{\sqrt{1 + (b/b_{max})^2}}$$

$$F^{NP}(b, Q, x_A, x_B) = \exp\left\{ -\left[g_1(1 + g_3 \ln(100 x_A x_B)) + g_2 \ln\left(\frac{Q}{2Q_0}\right)\right] b^2 \right\}$$

The Sudakov knows to $A^{(1)}$ and $B^{(1)}$

- For Sivers effect, so far we only calculate $A^{(1)}$ and $B^{(1)}$ for the Sudakov exponent

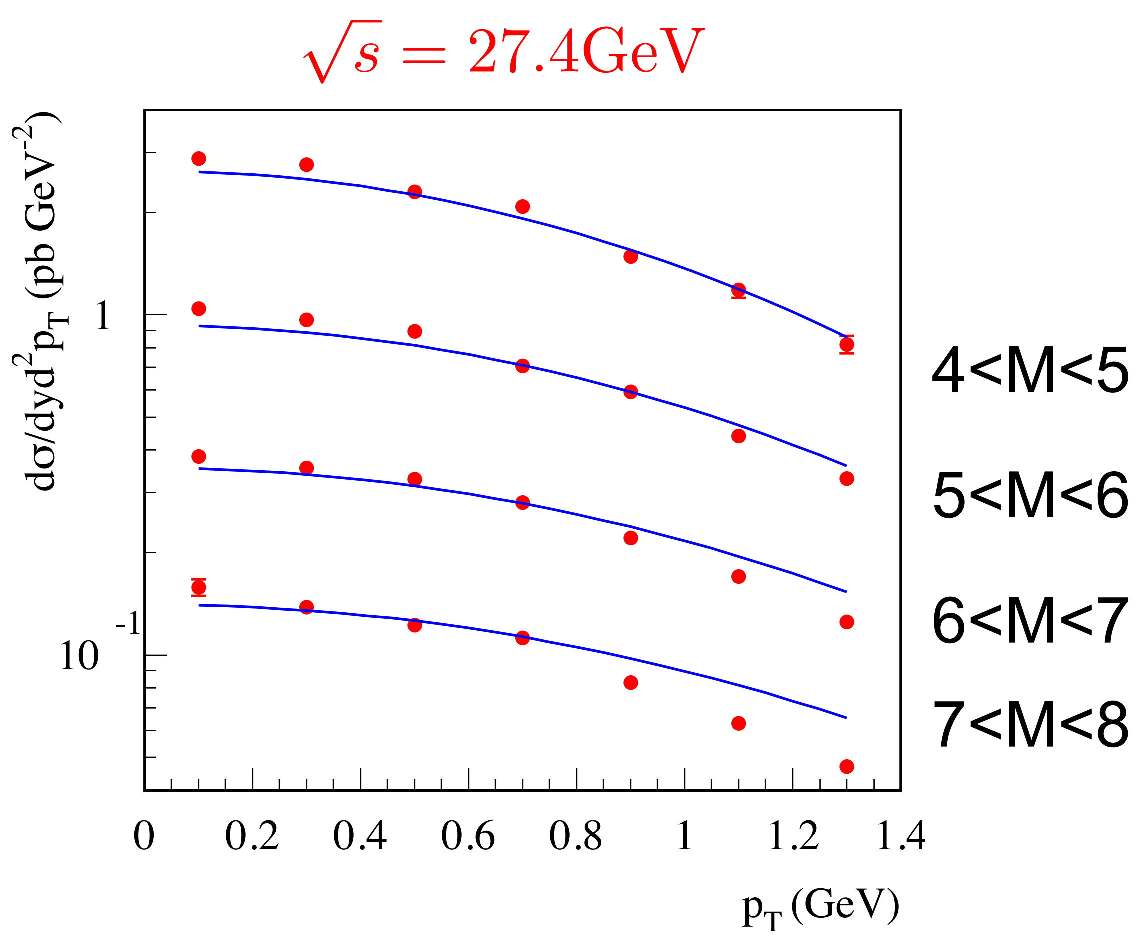
$$S(b,Q)=\int_{c^2/b^2}^{Q^2}\frac{d\mu^2}{\mu^2}\left[A\ln(Q^2/\mu^2)+B\right] \qquad A=\sum_{n=1}A^{(n)}\left(\frac{\alpha_s}{\pi}\right)^n$$

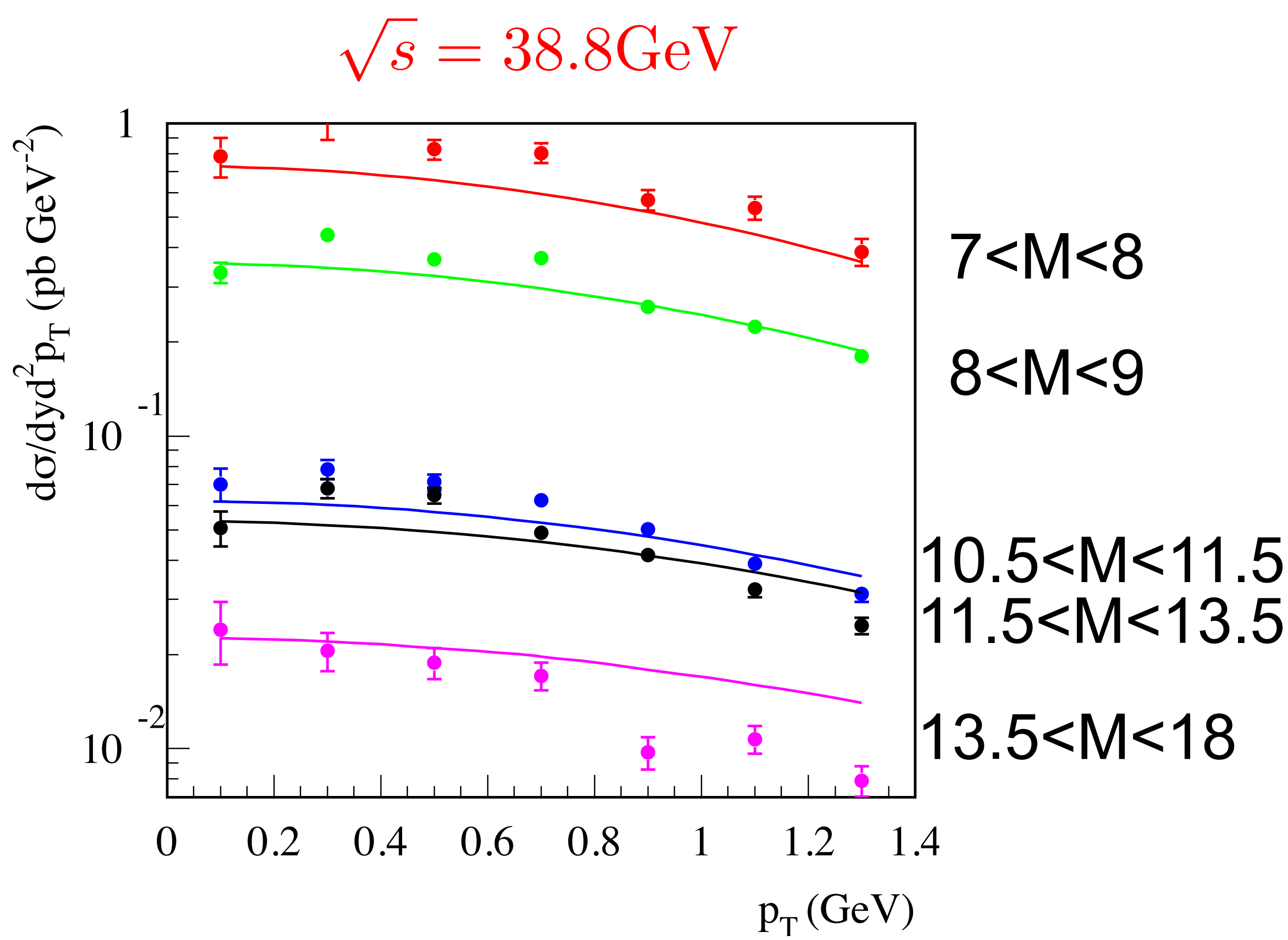
$$A^{(1)}=C_F \qquad B^{(1)}=-\frac{3}{2}C_F$$

  - They are exactly the same as those in the unpolarized pp collision
  - Thus for consistency, we will also use $A^{(1)}$ and $B^{(1)}$ for the unpolarized DY production
- As the whole perturbative Sudakov term (up to the order we have calculated) is the same, we will only assume the non-perturbative function is the same for the Sivers effect

Using $A^{(1)}$ and $B^{(1)}$ still describes data reasonably well

- E288 and E605

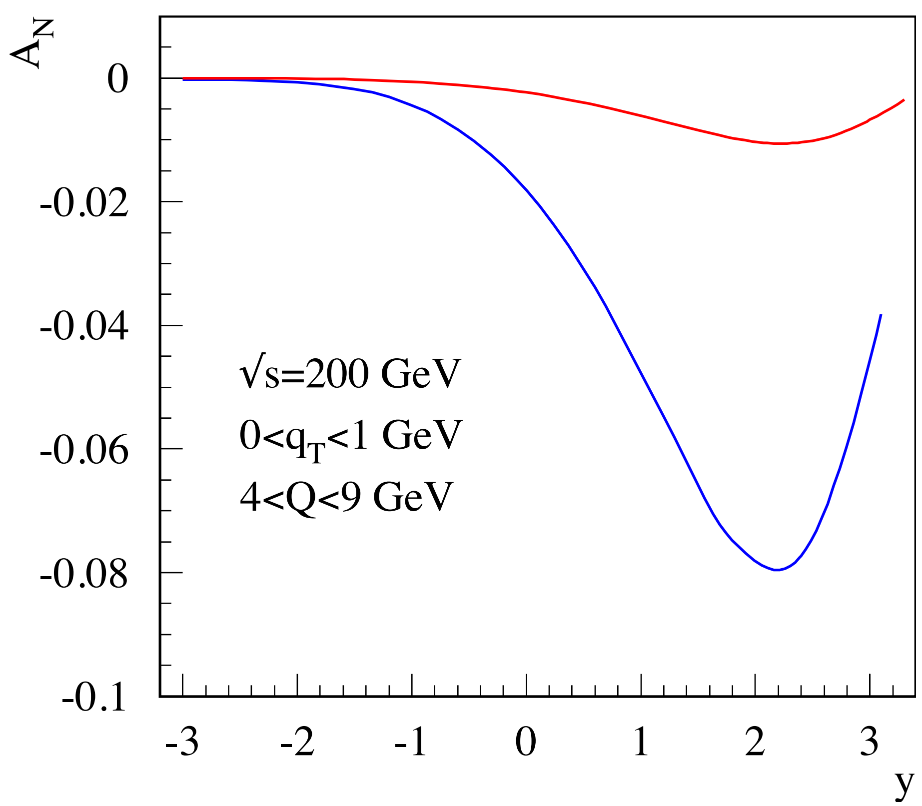




Sivers effect of DY production at RHIC

- Blue curve: bare parton model (using Torino TMD with Gaussian ansatz from SIDIS)
- Red curve: resummed formalism (using Torino TMD to calculate $T_F(x, x)$ as the initial input function, then evolve)

$$gT_{q,F}(x,x) = -\int d^2k_\perp \frac{|k_\perp|^2}{M} f_{1T}^{\perp q}(x,k_\perp^2)|_{\mathrm{SIDIS}}$$



- caution: non-perturbative part could be different for Sivers asymmetry

Summary

- QCD factorization is a useful tool to probe and understand hadron structure
- QCD evolution and resummation is closely related to each other
  - whenever there is an evolution, there is a resummation
- Scaling violation (QCD evolution of collinear unpolarized PDFs) has played a very important role in establishing QCD factorization formalism
- QCD evolution for collinear twist-3 function and TMDs will be extremely important in understanding hadron structure
  - Evolution makes Sivers asymmetry smaller

Summary

- QCD factorization is a useful tool to probe and understand hadron structure
- QCD evolution and resummation is closely related to each other
  - whenever there is an evolution, there is a resummation
- Scaling violation (QCD evolution of collinear unpolarized PDFs) has played a very important role in establishing QCD factorization formalism
- QCD evolution for collinear twist-3 function and TMDs will be extremely important in understanding hadron structure
  - Evolution makes Sivers asymmetry smaller

Thank you