QCD evolution of naïve-time-reversal-odd parton distribution functions

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Based on work with Z. Kang, PRD2009, arXiv:1205.1019

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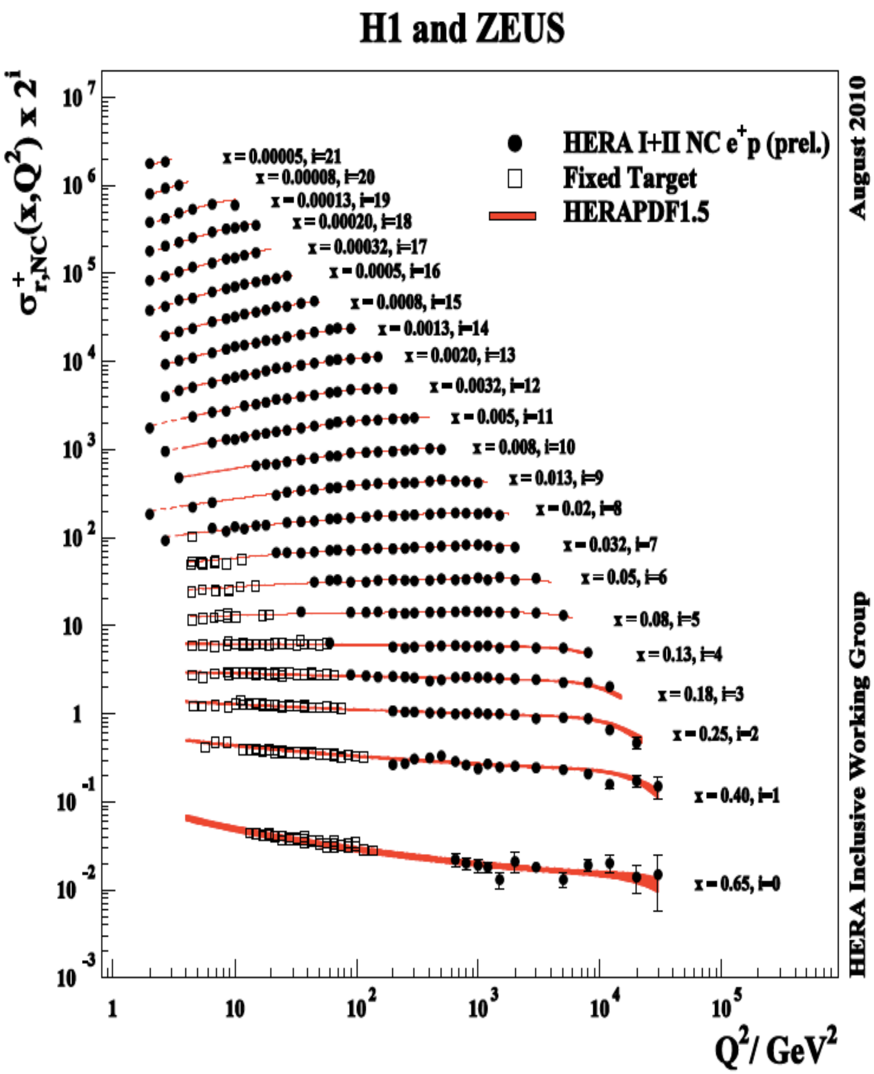
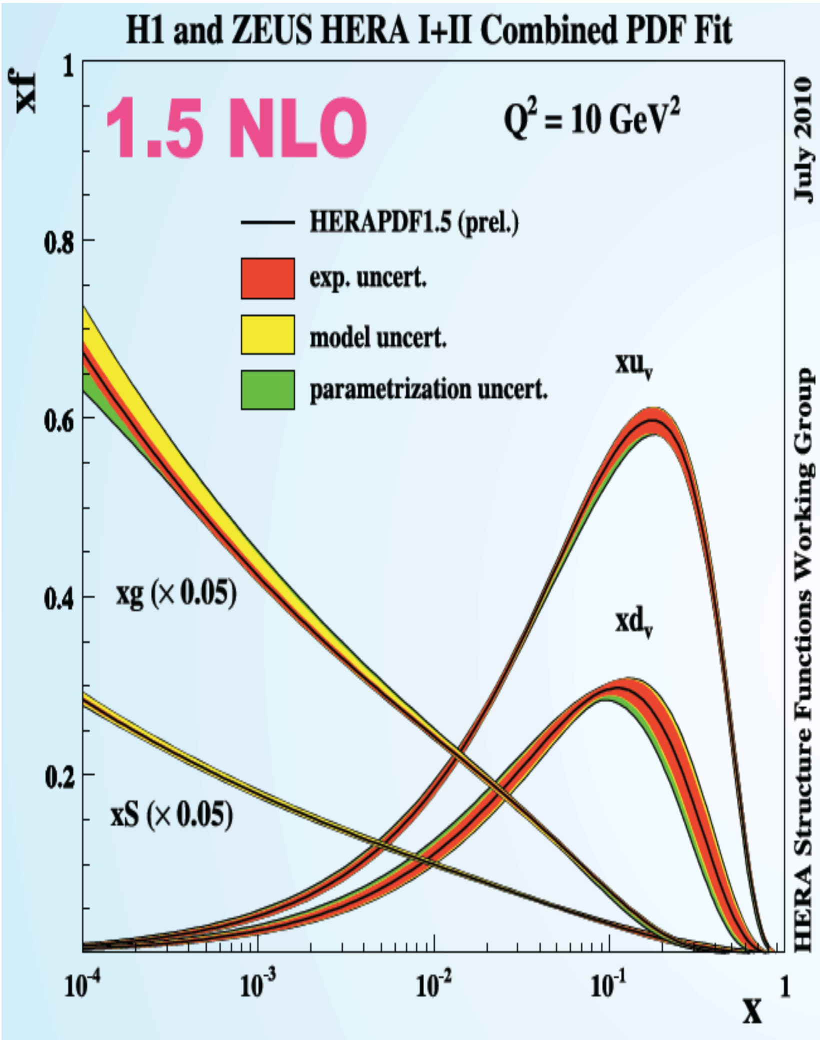
QCD and hadron structure



DATA

PDFs



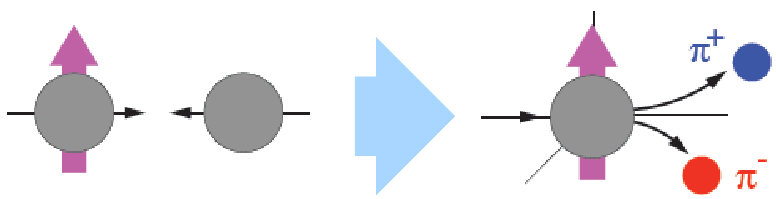
Go beyond 1D collinear PDFs?

3D TMDs, 1+2D GPDs, 1D multi-parton correlations, …

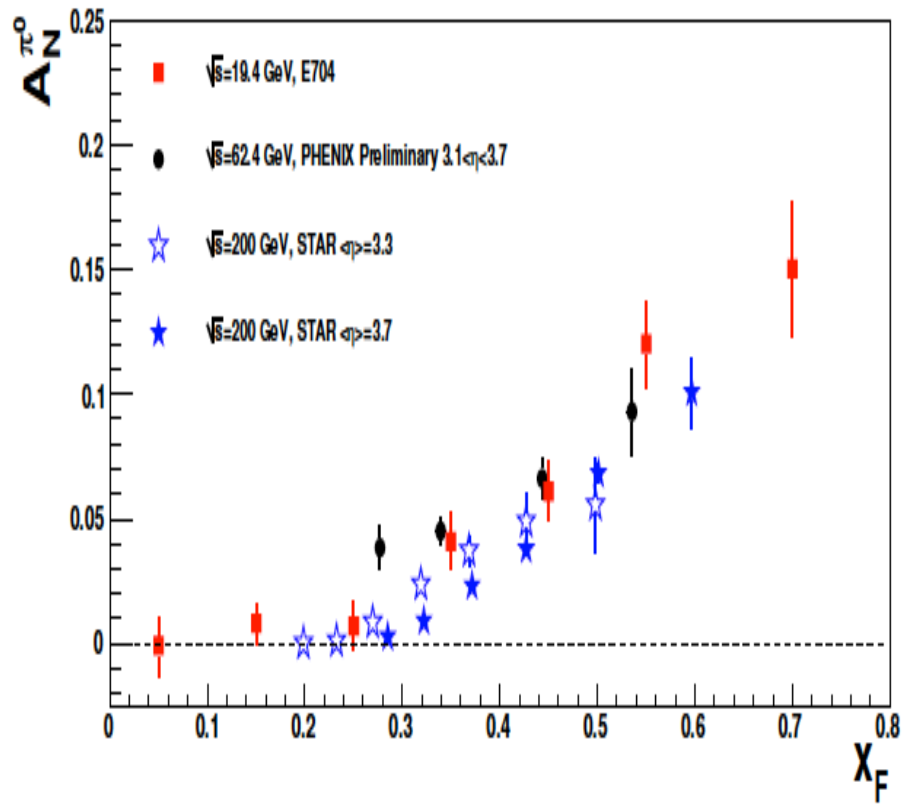
Transverse single-spin phenomena in QCD

- ❑ **Left-right asymmetry:**

$$A(p_A, s_\uparrow) + B(p_B) \to \pi(p) + X$$



$$A(\ell, \vec{s}) \equiv \frac{\Delta\sigma(\ell, \vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell, \vec{s}) - \sigma(\ell, -\vec{s})}{\sigma(\ell, \vec{s}) + \sigma(\ell, -\vec{s})}$$



- ❑ **Vanish without parton's transverse motion:**

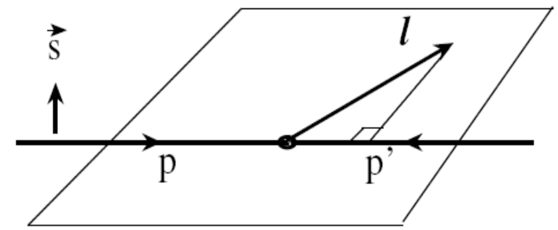
➡ **A direct probe for parton's transverse motion**

A direct probe of QCD quantum interference

Single transverse spin asymmetry

- SSA corresponds to a naively T-odd triple product:

$$A_N \propto i\,\vec{s}_p \cdot (\vec{p} \times \vec{\ell}) \;\Rightarrow\; i\,\epsilon^{\mu\nu\alpha\beta}\, p_\mu s_\nu \ell_\alpha p'_\beta$$

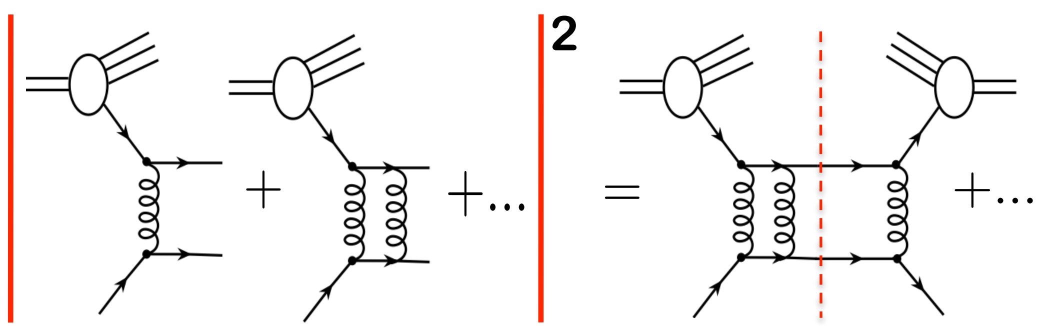


➡ **Novanish A_N requires a phase, enough vectors to fix a scattering plan, and a spin flip at the partonic scattering**

- Leading power in QCD:

Kane, Pumplin, Repko, PRL, 1978

$$\sigma_{AB}(p_T, \vec{s}) \propto$$



$$\propto \alpha_s \frac{m_q}{p_T}$$

➡ **A_N connects to parton's transverse motion!**

Collinear vs TMD factorization

❑ **Cover two different kinematic regions:**

Collinear: $Q_1 \ldots Q_n \gg \Lambda_{QCD}$

TMD: $Q_1 \gg Q_2 > \Lambda_{QCD}$

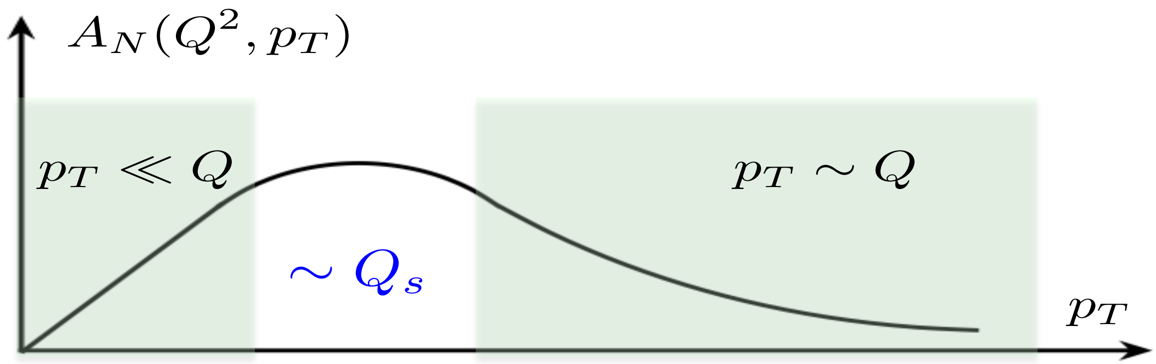
Twist-3 correlation functions: Integrated effect of parton k_T

$$\frac{1}{M_p}\int d^2\vec{k}_\perp \vec{k}_\perp^2 q_T(x,k_\perp) + \mathrm{UVCT}(\mu^2) = T_F(x,x,\mu^2)$$

TMDs: direct information on parton k_T

– more interesting if we can measure them

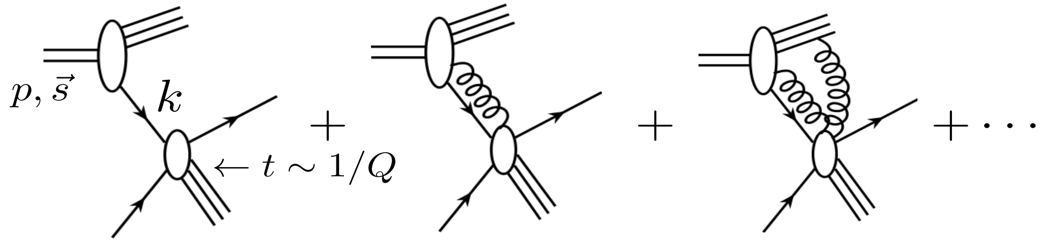
❑ **Consistent in the overlap (perturbative) region:**



Ji,Qiu,Vogelsang,Yuan,
Koike, Vogelsang, Yuan
…

Cross section with one large momentum transfer

- **Collinear factorization – power expansion: Q >> Λ_{QCD}**

$\sigma(Q,\vec{s}) \propto \Big|$  $\Big|^2$

$$= \sigma^{\mathrm{LP}}(Q,\vec{s}) + \frac{Q_s}{Q}\sigma^{\mathrm{NLP}}(Q,\vec{s}) + \ldots \qquad Q_s^2 \sim \langle k^2\rangle, \langle k_T^2\rangle$$

$$\approx H^{\mathrm{LP}} \otimes f_2 \otimes f_2 + \frac{Q_s}{Q} H^{\mathrm{NLP}} \otimes f_2 \otimes f_3 + \ldots$$ **pQCD factorization**

- **Single transverse spin asymmetry:**

$$\begin{aligned} A_N &\propto \sigma(Q,S_\perp) - \sigma(Q,-S_\perp) \\ &\propto H(Q)\,[\,\langle p,S_\perp|\mathcal{O}(\psi,A^\mu)|p,S_\perp\rangle - \langle p,-S_\perp|\mathcal{O}(\psi,A^\mu)|p,-S_\perp\rangle\,] \end{aligned}$$

- **Parity and time-reversal invariance:**

$$\langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle = \langle p,-\vec{s}|\mathcal{P}\mathcal{T}\mathcal{O}^\dagger(\psi,A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1}|p,-\vec{s}\rangle$$

Not all operators contribute to SSA!

Inclusive DIS

- **Inclusive DIS cross section:** $\sigma(\vec{s}_\perp) \propto L^{\mu\nu}\, W_{\mu\nu}(\vec{s}_\perp)$
- **Leptonic tensor is symmetric:** $L^{\mu\nu} = L^{\nu\mu}$
- **Hadronic tensor:** $W_{\mu\nu}(\vec{s}_\perp) \propto \langle P, \vec{s}_\perp |\, j^\dagger_\mu(0)\, j_\nu(y)\, | P, \vec{s}_\perp \rangle$
- **The difference of two cross sections:**

$$\Delta\sigma(\vec{s}_\perp) \propto L^{\mu\nu}\, \left[W_{\mu\nu}(\vec{s}_\perp) - W_{\mu\nu}(-\vec{s}_\perp)\right]$$

- **P and T invariance:**

$$\langle P, \vec{s}_\perp |\, j^\dagger_\mu(0) j_\nu(y)\, | P, \vec{s}_\perp \rangle = \langle P, -\vec{s}_\perp |\, j^\dagger_\nu(0) j_\mu(y)\, | P, -\vec{s}_\perp \rangle$$

$$\Rightarrow \quad W_{\mu\nu}(\vec{s}_\perp) = W_{\nu\mu}(-\vec{s}_\perp) \quad \Longleftrightarrow \quad A_N = 0$$

Inclusive single hadron production

- **One large scale:** $A(p_A, S_\perp) + B(p_B) \to h(p) + X$ **with $p_T >> \Lambda_{QCD}$**

  Three identified hadrons: $A(p_A, S_\perp),\ B(p_B),\ h(p)$

- **QCD collinear factorization:** Qiu, Sterman, 1991, 1998, …

$$
\begin{aligned}
A_N &\propto \sigma(p_T, S_\perp) - \sigma(p_T, -S_\perp) \\
&= T^{(3)}_{a/A}(x, x, S_\perp) \otimes \phi_{b/B}(x') \otimes \hat{\sigma}^T_{ab\to c} \otimes D_{h/c}(z) \\
&+ \delta q_{a/A}(x, S_\perp) \otimes T^{(3\sigma)}_{b/B}(x', x') \otimes \hat{\sigma}^\phi_{ab\to c} \otimes D_{h/c}(z) \\
&+ \delta q_{a/A}(x, S_\perp) \otimes \phi_{b/B}(x', x') \otimes \hat{\sigma}^D_{ab\to c} \otimes D^{(3)}_{h/c}(z, z)
\end{aligned}
$$

  Leading power contribution to cross section cancels!

  Only one twist-3 distribution at each term!

- **Three-type contributions:**

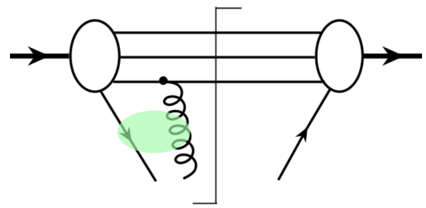
  Spin-flip: Twist-3 correlation functions, transversity distributions

  Phase: Interference between the real part and imaginary part of the scattering amplitude

Twist-3 correlation functions

❑ **Twist-3 polarized correlation functions:**

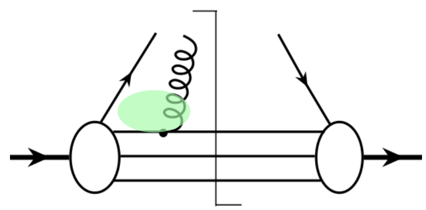
Efremov, Teryaev, 1982, …
Qiu, Sterman, 1991, …

$T^{(3)}(x, x, S_\perp) \propto$ 

Moment of Sivers function

❑ **Twist-3 unpolarized correlation functions:**

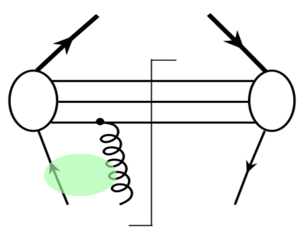
Kanazawa, Koike 2000, …

$T^{(3\sigma)}(x', x') \propto$ 

Moment of Boer-Mulders function

❑ **Twist-3 fragmentation functions:**

Kang, Yuan, Zhou, 2010

$D^{(3)}(z, z) \propto$ 

Moment of Collins function?

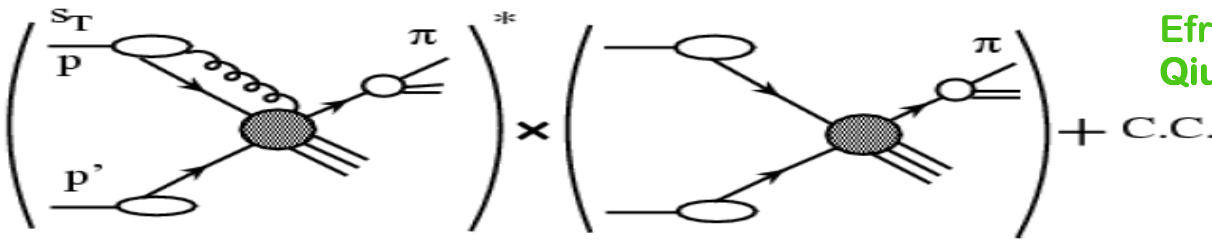
All these correlation functions have No probability interpretation!

Quantum interference between a single and a composite state

SSAs generated by twist-3 PDFs

- First non-vanish contribution – interference:

Efremov, Teryaev, 1982, …
Qiu, Sterman, 1991, …



- Dominated by the derivative term – forward region:

Qiu, Sterman, 1998, …

$$E\frac{d\Delta\sigma}{d^3\ell} \propto \epsilon^{\ell_T s_T n \bar{n}}\, D_{c\to\pi}(z) \otimes \left[-x\frac{\partial}{\partial x}T_F(x,x)\right] \otimes \frac{1}{-\hat{u}}\left[G(x')\otimes\Delta\hat{\sigma}_{qg\to c} + \sum_{q'} q'(x')\otimes\Delta\hat{\sigma}_{qq'\to c}\right]$$

$$\boxed{A_N \propto \left(\frac{\ell_\perp}{-\hat{u}}\right)\frac{n}{1-x} \quad \text{if } T_F(x,x)\propto q(x)\propto(1-x)^n}$$

Kouvaris, Qiu, Vogelsang, Yuan, 2006

- Complete leading order contribution:

$$E_\ell\frac{d^3\Delta\sigma(\vec{s}_T)}{d^3\ell} = \frac{\alpha_s^2}{S}\sum_{a,b,c}\int_{z_{\min}}^1\frac{dz}{z^2}D_{c\to h}(z)\int_{x'_{\min}}^1\frac{dx'}{x'}\frac{1}{x'S+T/z}\phi_{b/B}(x')\sqrt{4\pi\alpha_s}\left(\frac{\epsilon^{\ell s_T n\bar{n}}}{z\hat{u}}\right) \times\frac{1}{x}\left[T_{a,F}(x,x) - x\left(\frac{d}{dx}T_{a,F}(x,x)\right)\right]H_{ab\to c}(\hat{s},\hat{t},\hat{u})$$

Twist-3 parton distribution functions

Kang, Qiu, PRD, 2009

❑ Twist-2 parton distributions:

✧ **Unpolarized PDFs:**

$$q(x) \propto \langle P|\overline{\psi}_q(0)\frac{\gamma^+}{2}\psi_q(y)|P\rangle$$

$$G(x) \propto \langle P|F^{+\mu}(0)F^{+\nu}(y)|P\rangle(-g_{\mu\nu})$$

✧ **Polarized PDFs:**

$$\Delta q(x) \propto \langle P, S_\parallel|\overline{\psi}_q(0)\frac{\gamma^+\gamma^5}{2}\psi_q(y)|P, S_\parallel\rangle$$

$$\Delta G(x) \propto \langle P, S_\parallel|F^{+\mu}(0)F^{+\nu}(y)|P, S_\parallel\rangle(i\epsilon_{\perp\mu\nu})$$

❑ Two-sets Twist-3 parton distribution functions:

$$\widetilde{T}_{q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \langle P, s_T|\overline{\psi}_q(0)\, \frac{\gamma^+}{2}\left[\epsilon^{s_T\sigma n\bar{n}}\, F_\sigma{}^+(y_2^-)\right]\psi_q(y_1^-)|P, s_T\rangle$$

$$\widetilde{T}^{(f,d)}_{G,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \frac{1}{P+}\langle P, s_T|F^{+\rho}(0)\left[\epsilon^{s_T\sigma n\bar{n}}\, F_\sigma{}^+(y_2^-)\right]F^{+\lambda}(y_1^-)|P, s_T\rangle(-g_{\rho\lambda})$$

$$\widetilde{T}_{\Delta q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \langle P, s_T|\overline{\psi}_q(0)\, \frac{\gamma^+\gamma^5}{2}\left[i\, s_T^\sigma\, F_\sigma{}^+(y_2^-)\right]\psi_q(y_1^-)|P, s_T\rangle$$

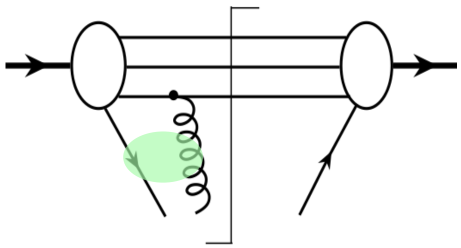
$$\widetilde{T}^{(f,d)}_{\Delta G,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \frac{1}{P+}\langle P, s_T|F^{+\rho}(0)\left[i\, s_T^\sigma\, F_\sigma{}^+(y_2^-)\right]F^{+\lambda}(y_1^-)|P, s_T\rangle\,(i\epsilon_{\perp\rho\lambda})$$

"Interpretation" of twist-3 correlation functions

- **Measurement of direct QCD quantum interference:**

Qiu, Sterman, 1991, …

$$T^{(3)}(x, x, S_\perp) \;\propto$$



Interference between a single active parton state and an active two-parton composite state

- **"Expectation value" of QCD operators:**

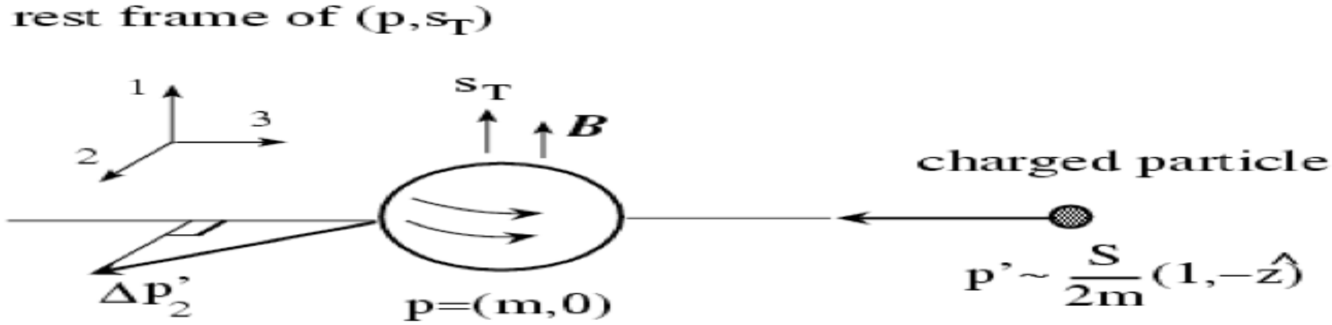
$$\langle P, s|\overline{\psi}(0)\gamma^+\psi(y^-)|P, s\rangle \;\longrightarrow\; \langle P, s|\overline{\psi}(0)\gamma^+\left[\epsilon_\perp^{\alpha\beta}s_{T\alpha}\int dy_2^- F_\beta^{\ +}(y_2^-)\right]\psi(y^-)|P, s\rangle$$

$$\langle P, s|\overline{\psi}(0)\gamma^+\gamma_5\psi(y^-)|P, s\rangle \;\longrightarrow\; \langle P, s|\overline{\psi}(0)\gamma^+\left[i\, g_\perp^{\alpha\beta}s_{T\alpha}\int dy_2^- F_\beta^{\ +}(y_2^-)\right]\psi(y^-)|P, s\rangle$$

How to interpret the "expectation value" of the operators in RED?

A simple example

- The operator in Red – a classical Abelian case:



- Change of transverse momentum:

$$\frac{d}{dt}p_2' = e(\vec{v}' \times \vec{B})_2 = -ev_3B_1 = ev_3F_{23}$$

- In the c.m. frame:

$$(m,\vec{0}) \to \bar{n} = (1,0,0_T), \quad (1,-\hat{z}) \to n = (0,1,0_T)$$

$$\Longrightarrow \quad \frac{d}{dt}p_2' = e\,\epsilon^{s_T\sigma n\bar{n}}\,F_\sigma{}^+$$

- The total change:

$$\boxed{\Delta p_2' = e\int dy^-\,\epsilon^{s_T\sigma n\bar{n}}\,F_\sigma{}^+(y^-)}$$

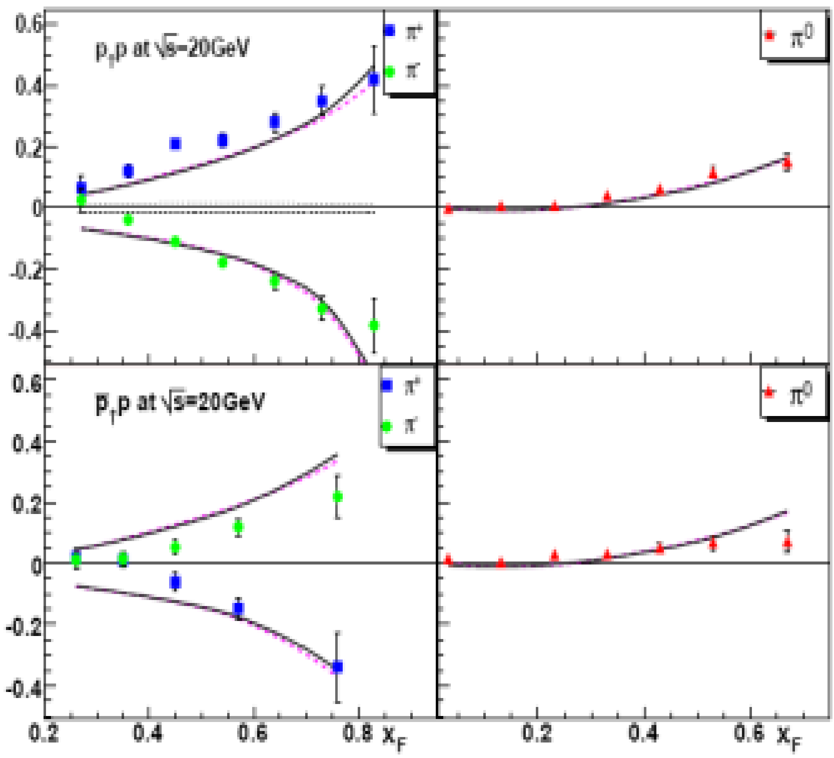
Net quark transverse momentum imbalance caused by color Lorentz force inside a transversely polarized proton

QCD global fitting at twist-3

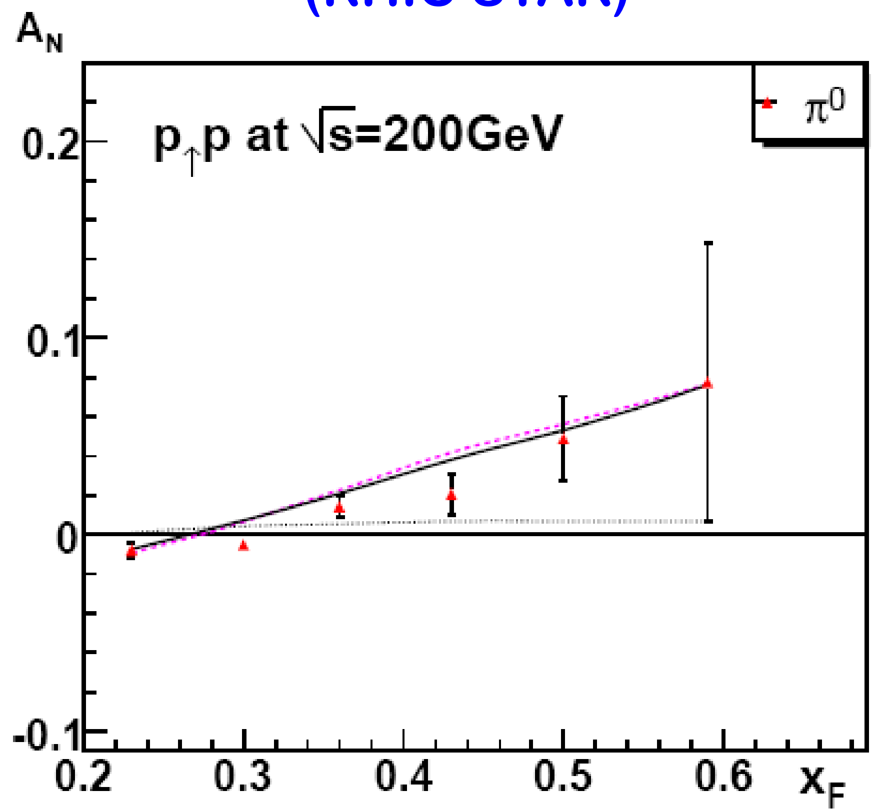
Kouvaris, Qiu, Vogelsang, Yuan, 2006

❑ **Leading order global fit:**

(FermiLab E704)



(RHIC STAR)



Evolution or scale dependence of twist-3 distributions?
Role of NLO contribution?

Evolution equations and kernels

- Evolution equation is a consequence of factorization:

Factorization: $$\Delta\sigma(Q, s_T) = (1/Q)H_1(Q/\mu_F, \alpha_s) \otimes f_2(\mu_F) \otimes f_3(\mu_F)$$

DGLAP for f_2: $$\frac{\partial}{\partial \ln(\mu_F)} f_2(\mu_F) = P_2 \otimes f_2(\mu_F)$$

Evolution for f_3: $$\frac{\partial}{\partial \ln(\mu_F)} f_3 = \left(\frac{\partial}{\partial \ln(\mu_F)} H_1^{(1)} - P_2^{(1)}\right) \otimes f_3$$

- Evolution kernel is process independent – prediction of pQCD:

  - ✧ Calculate directly from the variation of process independent twist-3 distributions

    Kang, Qiu, 2009, 2012, Yuan, Zhou, 2009
    Ma, Sang, 2011, Schafer, Zhou 2012, …

  - ✧ Extract from the scale dependence of the NLO hard part of any physical process

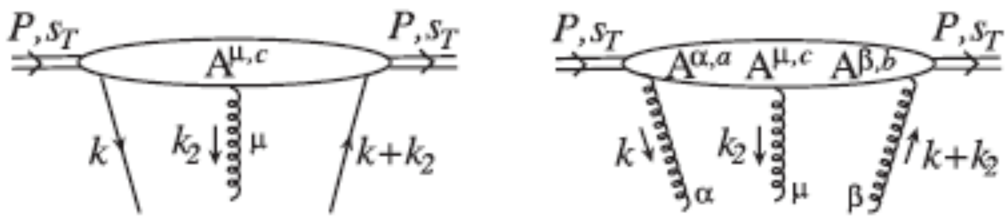
    Vogelsang, Yuan, 2009

  - ✧ Renormalization of the twist-3 operators

    Braun et al, 2009

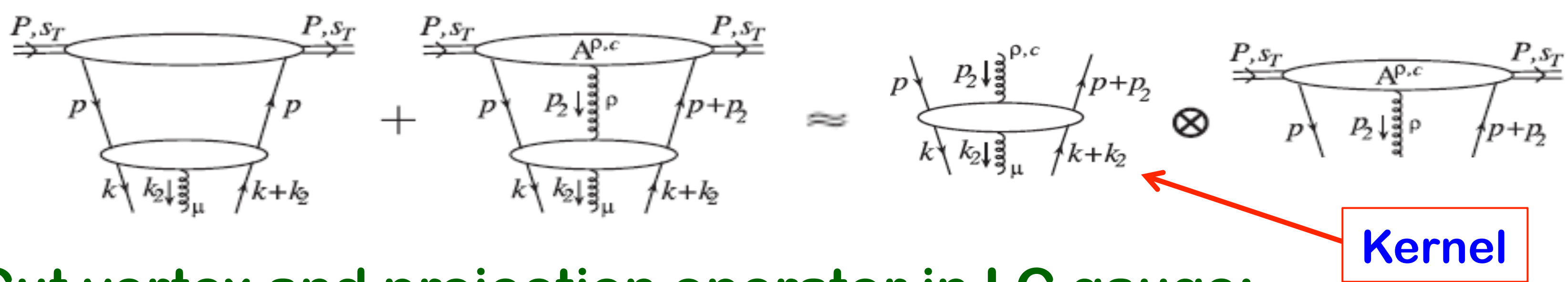
The Feynman diagram representation

- **Feynman diagram representation of twist-3 distributions:**



Different twist-3 distributions ⇔ diagrams with different cut vertices

- **Collinear factorization of twist-3 distributions:**



- **Cut vertex and projection operator in LC gauge:**

$$\mathcal{V}^{\,\mathrm{LC}}_{q,F} = \frac{\gamma^+}{2P^+}\delta\!\left(x - \frac{k^+}{P^+}\right)x_2\delta\!\left(x_2 - \frac{k_2^+}{P^+}\right)(i\epsilon^{s_T\sigma n\bar{n}})[-g_{\sigma\mu}]\mathcal{C}_q$$

$$\mathcal{P}^{\,(\mathrm{LC})}_{q,F} = \frac{1}{2}\gamma\cdot P\left(\frac{-1}{\xi_2}\right)(i\epsilon^{s_T\rho n\bar{n}})\tilde{\mathcal{C}}_q$$

Variation of twist-3 correlation functions

- Closed set of evolution equations (spin-dependent):

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}_{q,F}(x, x+x_2, \mu_F, s_T) = \int d\xi d\xi_2 [\tilde{\mathcal{T}}_{q,F}(\xi, \xi+\xi_2, \mu_F, s_T) K_{qq}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)$$

$$+ \tilde{\mathcal{T}}_{\Delta q,F}(\xi, \xi+\xi_2, \mu_F, s_T) K_{q\Delta q}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)]$$

$$+ \sum_{i=f,d} \int d\xi d\xi_2 [\tilde{\mathcal{T}}^{(i)}_{G,F}(\xi, \xi+\xi_2, \mu_F, s_T) K^{(i)}_{qg}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)$$

$$+ \tilde{\mathcal{T}}^{(i)}_{\Delta G,F}(\xi, \xi+\xi_2, \mu_F, s_T) K^{(i)}_{q\Delta g}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)].$$

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}^{(i)}_{G,F}(x, x+x_2, \mu_F, s_T) = \sum_{j=f,d} \int d\xi d\xi_2 [\tilde{\mathcal{T}}^{(j)}_{G,F}(\xi, \xi+\xi_2, \mu_F, s_T) K^{(ji)}_{gg}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)$$

$$+ \tilde{\mathcal{T}}^{(j)}_{\Delta G,F}(\xi, \xi+\xi_2, \mu_F, s_T) K^{(ji)}_{g\Delta g}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)]$$

$$+ \sum_{q} \int d\xi d\xi_2 [\tilde{\mathcal{T}}_{q,F}(\xi, \xi+\xi_2, \mu_F, s_T) K^{(i)}_{gq}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)$$

$$+ \tilde{\mathcal{T}}_{\Delta q,F}(\xi, \xi+\xi_2, \mu_F, s_T) K^{(i)}_{g\Delta q}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)],$$

Plus two more equations for:

$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}_{\Delta q,F}(x, x+x_2, \mu_F, s_T)$ and $\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}^{(i)}_{\Delta G,F}(x, x+x_2, \mu_F, s_T)$

Evolution equations

❑ Distributions relevant to SSA:

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \mathcal{T}_{q,F}(x, x+x_2, \mu_F) = \frac{1}{2}\left[\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}_{q,F}(x, x+x_2, \mu_F, s_T) + \mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}_{q,F}(x+x_2, x, \mu_F, s_T)\right],$$

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \mathcal{T}^{(i)}_{G,F}(x, x+x_2, \mu_F) = \frac{1}{2}\left[\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}^{(i)}_{G,F}(x, x+x_2, \mu_F, s_T) + \mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}^{(i)}_{G,F}(x+x_2, x, \mu_F, s_T)\right],$$

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \mathcal{T}_{\Delta q,F}(x, x+x_2, \mu_F) = \frac{1}{2}\left[\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}_{\Delta q,F}(x, x+x_2, \mu_F, s_T) - \mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}_{\Delta q,F}(x+x_2, x, \mu_F, s_T)\right],$$

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \mathcal{T}^{(i)}_{\Delta G,F}(x, x+x_2, \mu_F) = \frac{1}{2}\left[\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}^{(i)}_{\Delta G,F}(x, x+x_2, \mu_F, s_T) - \mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}^{(i)}_{\Delta G,F}(x+x_2, x, \mu_F, s_T)\right].$$

❑ Important symmetry property:

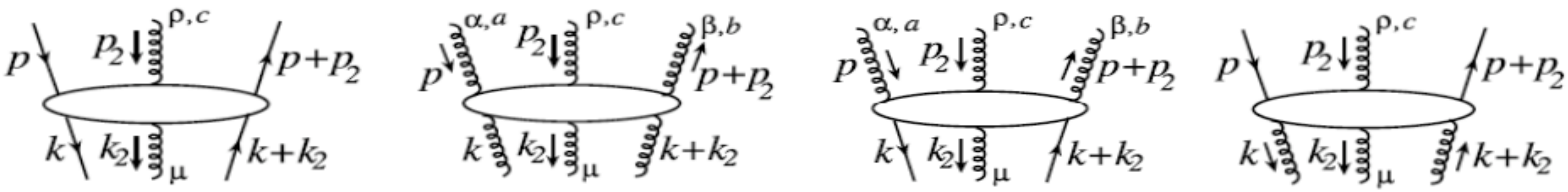
$$T_{\Delta q,F}(x, x, \mu_F) \equiv \int dx_2 [2\pi\delta(x_2)] \mathcal{T}_{\Delta q,F}(x, x+x_2, \mu_F) = 0,$$

$$T^{(f,d)}_{\Delta G,F}(x, x, \mu_F) \equiv \int dx_2 [2\pi\delta(x_2)] \left(\frac{1}{x}\right) \mathcal{T}^{(f,d)}_{\Delta G}(x, x+x_2, \mu_F) = 0.$$

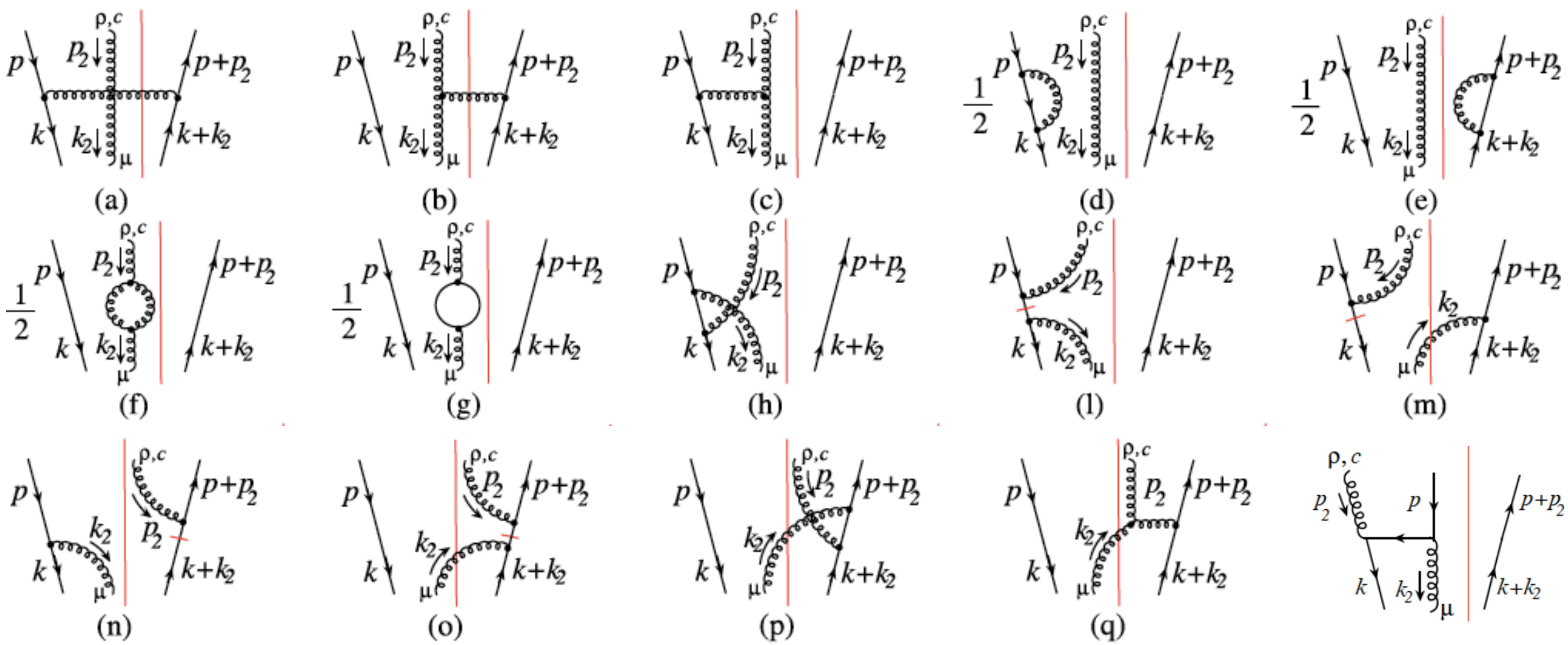
These two correlation functions do not give the gluonic pole contribution directly

Evolution kernels

- ❑ **Feynman diagrams:**



- ❑ **LO for flavor non-singlet channel:**

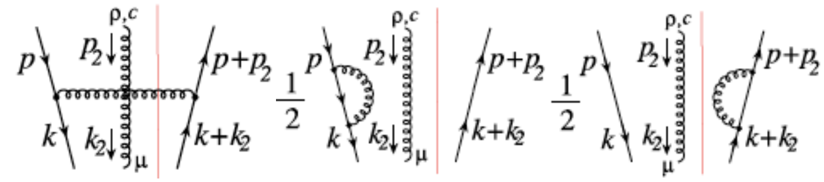


Evolution of quark-gluon distributions

❑ Evolution equations:

Kang, Qiu, PRD, 2009, 2012
Braun et al. 2009

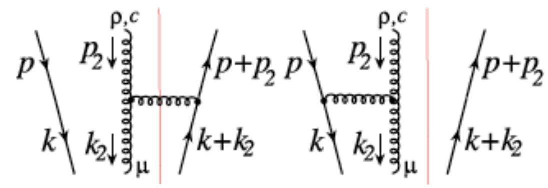
$$\frac{\partial T_{q,F}(x,x,\mu)}{\partial \ln \mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\Big\{ P_{qq}(z)\, T_{q,F}(\xi,\xi,\mu)$$



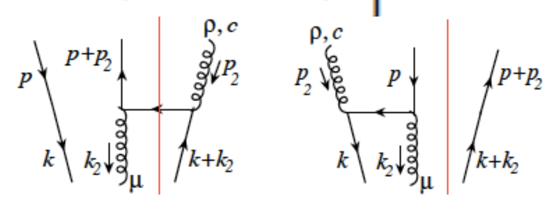
$$+\frac{N_c}{2}\left[\frac{1+z^2}{1-z}\left(T_{q,F}(\xi,x,\mu)-T_{q,F}(\xi,\xi,\mu)\right)+z\,T_{q,F}(\xi,x,\mu)+T_{\Delta q,F}(x,\xi,\mu)\right]$$

Missed in original paper

$$-N_c\,\delta(1-z)\,T_{q,F}(x,x,\mu)$$



$$+\frac{1}{2N_c}\left[(1-2z)T_{q,F}(x,x-\xi,\mu)+T_{\Delta q,F}(x,x-\xi,\mu)\right]\Big\}$$



$$+\,\text{tri-gluon contributions}$$

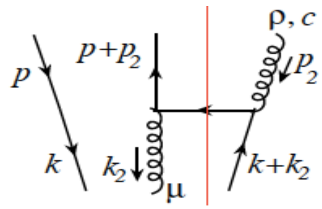
✧ **All kernels are infrared safe**

✧ **Diagonal contribution is the same as that of DGLAP**

Subtleties in deriving the kernel

❑ Quantum interference:

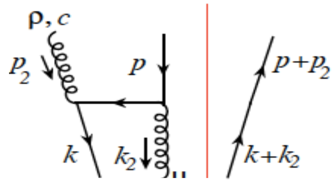
Kang, Qiu, 2012



Interference between spin-1 states

Interference between spin-1/2 states

$$\left.\frac{\partial T_{q,F}(x,x,\mu)}{\partial \ln \mu^2}\right|_{\text{Fig.}} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\frac{1}{2N_c}\left(\frac{1}{2}\right)\left[(1-2z)T_{q,F}(x,x-\xi,\mu)+T_{\Delta q,F}(x,x-\xi,\mu)\right]$$

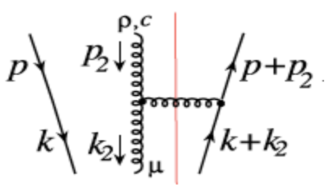


Interference between spin-1/2 states

Sign - $\epsilon^{0123} = \pm 1$

$$\left.\frac{\partial T_{q,F}(x,x,\mu)}{\partial \ln \mu^2}\right|_{\text{Fig.}} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\frac{1}{2N_c}\left(\frac{1}{2}\right)\left[(1-2z)T_{q,F}(x,x-\xi,\mu)+T_{\Delta q,F}(x,x-\xi,\mu)\right]$$

❑ 0/0 contribution for the gluonic pole:



$$\left.\frac{\partial T_{q,F}(x,x+x_2,\mu)}{\partial \ln \mu^2}\right|_{\text{Fig.}} = \frac{N_c}{2}\int_{x_2}^{1-x} d\xi_2\, T_{q,F}(x,x+\xi_2,\mu)\left[-\frac{x_2}{\xi_2^2}\right]+\cdots$$

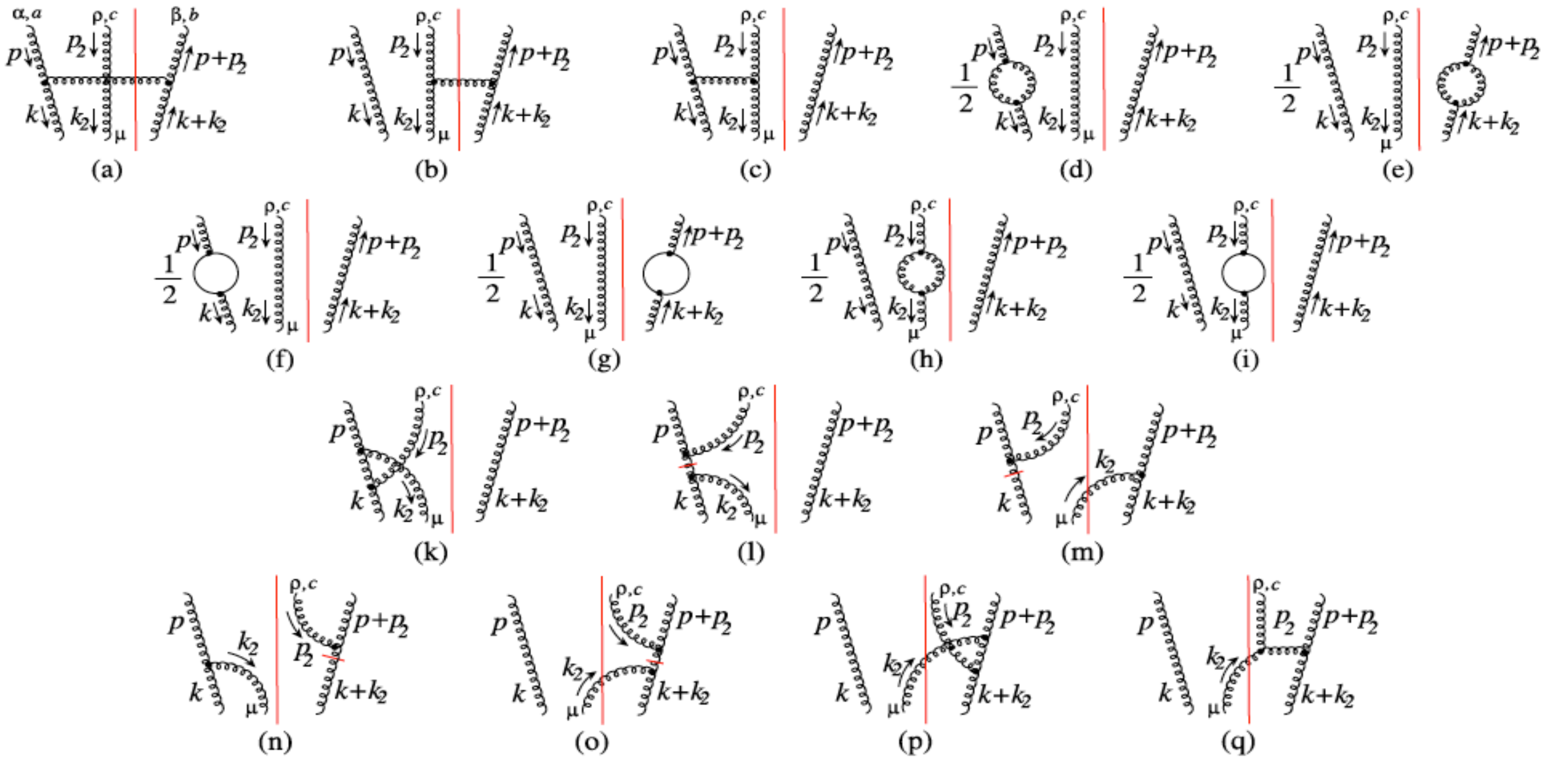
$$\left.\frac{\partial T_{q,F}(x,x,\mu)}{\partial \ln \mu^2}\right|_{\text{Fig.}} = \lim_{x_2\to 0}\left.\frac{\partial T_{q,F}(x,x_2,\mu)}{\partial \ln \mu^2}\right|_{\text{Fig.}} = -\frac{N_c}{2}T_{q,F}(x,x,\mu)$$

Three-gluon correlation and evolution

- **Two possible color contributions:**

$$d^{abc}, \;\; if^{abc} \quad \Longrightarrow \quad T_{G,F}^{(d)}(x_1, x_2), \quad T_{G,F}^{(f)}(x_1, x_2)$$

- **LO diagrams:**



✧ **Kernels are also infrared safe**

✧ **diagonal contribution is the same as that of DGLAP**

Numerical solution

- Evolution equations for diagonal correlation functions are not closed!

- "Model" for the off-diagonal correlation functions:

For the symmetric correlation functions:

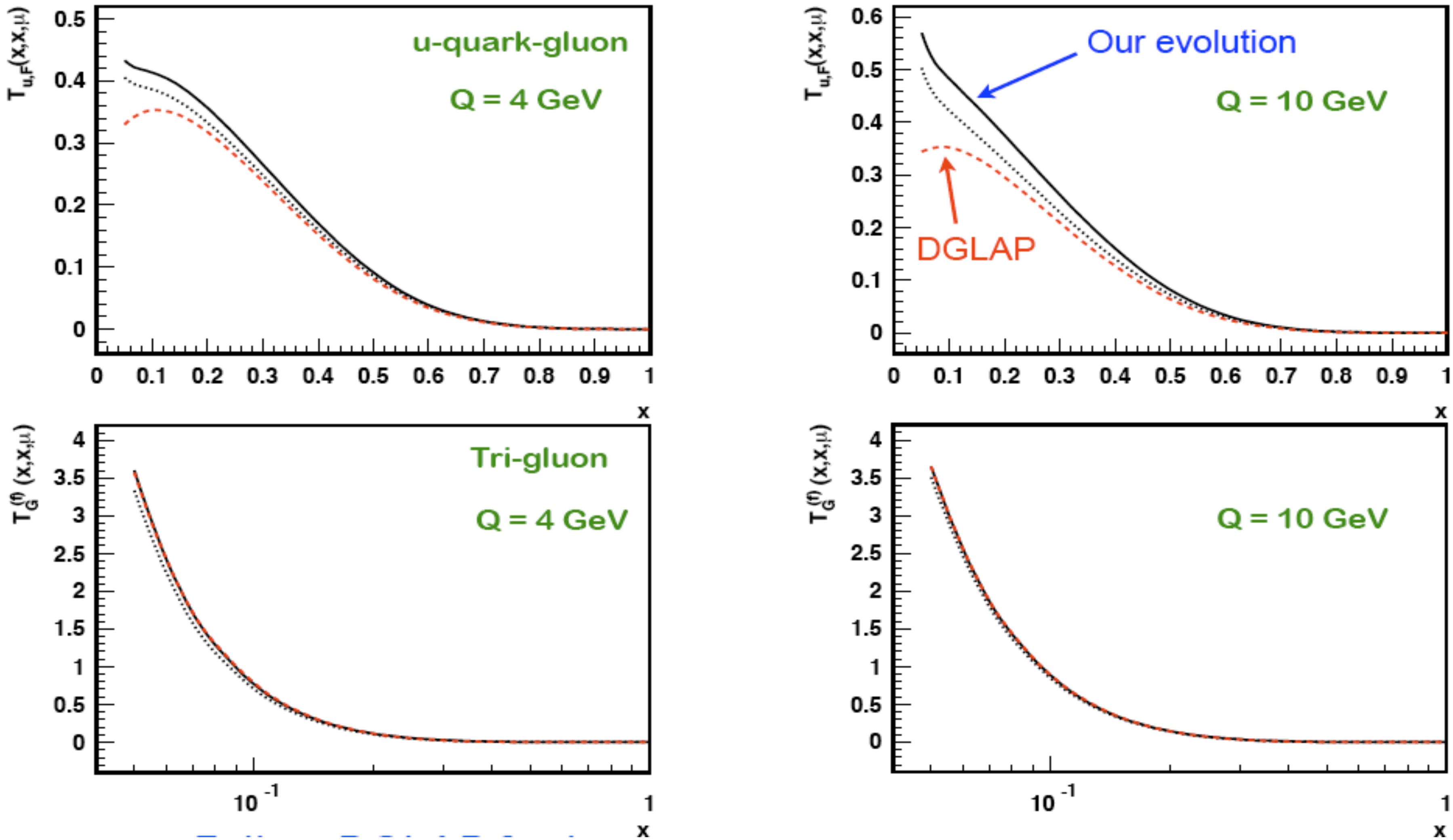
$$T_{q,F}(x_1, x_2, \mu_F) = \tfrac{1}{2}[T_{q,F}(x_1, x_1, \mu_F) + T_{q,F}(x_2, x_2, \mu_F)]e^{-[(x_1-x_2)^2/2\sigma^2]},$$

$$\mathcal{T}^{(f,d)}_{G,F}(x_1, x_2, \mu_F) = \tfrac{1}{2}[\mathcal{T}^{(f,d)}_{G,F}(x_1, x_1, \mu_F) + \mathcal{T}^{(f,d)}_{G,F}(x_2, x_2, \mu_F)]e^{-[(x_1-x_2)^2/2\sigma^2]},$$

$$\Longrightarrow$$

$$T^{(f,d)}_{G,F}(x_1, x_2, \mu_F) = \frac{1}{2}\left[T^{(f,d)}_{G,F}(x_1, x_1, \mu_F) + \frac{x_2}{x_1}T^{(f,d)}_{G,F}(x_2, x_2, \mu_F)\right]e^{-[(x_1-x_2)^2/2\sigma^2]}.$$

Scale dependence



✧ Follow DGLAP at large x

✧ Large deviation at low x (stronger correlation)

Chiral-odd contribution to SSAs

- **Role of the transversity:**

$$A_N \propto \sigma(p_T, S_\perp) - \sigma(p_T, -S_\perp)$$
$$= T^{(3)}_{a/A}(x, x, S_\perp) \otimes \phi_{b/B}(x') \otimes \hat{\sigma}^T_{ab\to c} \otimes D_{h/c}(z)$$
$$+ \delta q_{a/A}(x, S_\perp) \otimes T^{(3\sigma)}_{b/B}(x', x') \otimes \hat{\sigma}^\phi_{ab\to c} \otimes D_{h/c}(z)$$
$$+ \delta q_{a/A}(x, S_\perp) \otimes \phi_{b/B}(x', x') \otimes \hat{\sigma}^D_{ab\to c} \otimes D^{(3)}_{h/c}(z, z)$$

Transversity to flip spin

- **Chiral-odd correlation function:**

$$T^{(\sigma)}_{q,F}(x,x) = \int \frac{dy_1^- dy_2^-}{4\pi} e^{ixP^+y_1^-} \frac{1}{2} \sum_{s_T} \langle P, s_T | \bar{\psi}_q(0) \left[\sigma^{\alpha+} F_\alpha{}^+(y_2^-)\right] \psi_q(y_1^-) | P, s_T \rangle$$

Moment of Boer-Mulders function

- **Leading order contribution to SSAs:** Kanazawa, Koike 2000, …

- **Evolution equation:** Zhou, Yuan, Liang, 2008

$$\frac{\partial}{\partial \ln\mu^2} T_F^{(\sigma)}(x_B, \mu^2) = \frac{\alpha_s}{2\pi} \int \frac{dx}{x} \left[C_F \left\{ \frac{2z}{(1-z)_+} + 2\delta(1-z) \right\} T_F^{(\sigma)}(x,x) + \frac{C_A}{2} \left\{ \frac{2}{1-z} T_F^{(\sigma)}(xz, x) - \frac{2z}{1-z} T_F^{(\sigma)}(x,x) \right\} \right]$$

Revised evolution kernel

Kang, Qiu, 2012

❑ Evolution equation:

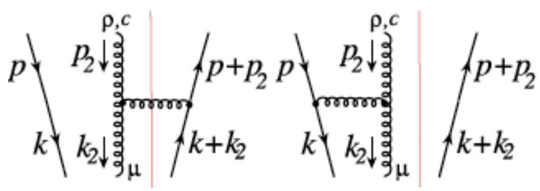
$$\frac{\partial T_{q,F}^{(\sigma)}(x,x,\mu)}{\partial \ln \mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\Bigg\{\Delta_T P_{qq}(z)\, T_{q,F}^{(\sigma)}(\xi,\xi,\mu) + \frac{N_c}{2}\left[\frac{2\,T_{q,F}^{(\sigma)}(\xi,x,\mu) - 2z\,T_{q,F}^{(\sigma)}(\xi,\xi,\mu)}{1-z}\right]$$

$$-N_c\,\delta(1-z)\,T_{q,F}^{(\sigma)}(x,x,\mu) + \frac{1}{2N_c}\left[2(1-z)T_{q,F}^{(\sigma)}(x,x-\xi,\mu)\right]\Bigg\},$$

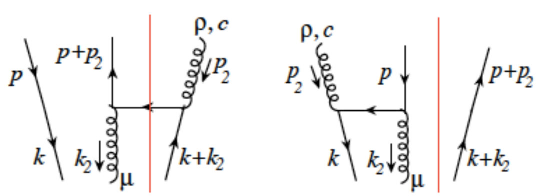
with DGLAP kernel for transversity:

$$\Delta_T P_{qq}(z) = C_F\left[\frac{2\,z}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right]$$

❑ The second line of the evolution equation above is new:



$$\longrightarrow \quad -N_c\,\delta(1-z)\,T_{q,F}^{(\sigma)}(x,x,\mu)$$



$$\longrightarrow \quad +\frac{1}{2N_c}\left[2(1-z)T_{q,F}^{(\sigma)}(x,x-\xi,\mu)\right]$$

QCD global analysis of SSAs

- **Factorization for physical observables:**

$$A(p_A, S_\perp) + B(p_B) \to h(p) + X$$

$$A(p_A, S_\perp) + B(p_B) \to \mathrm{jet}(p) + X$$

$$A(p_A, S_\perp) + B(p_B) \to \gamma(p) + X$$

$$\ldots$$

- **Urgently needed – NLO hard parts:**

Beyond LO!

$$A_N \propto \Delta\sigma(Q, S_\perp) \propto T_f^{(3)}(x,x) \otimes \hat{H}_f \otimes \ldots$$

Only NLO calculation – SSA for p_T weighted Drell-Yan Vogelsang, Yuan, 2009

- **A completely new domain to test QCD!**

From paton's transverse motion to direct QCD quantum interference

Summary

- QCD factorization/calculation has been very successful in interpreting HEP scattering data

- Single transverse-spin asymmetry is an ideal observable for probing partonic dynamics inside a hadron beyond the PDFs

- Experiments with a transversely polarized hadron beam open up new ways to test QCD and to study hadron structure

  Parton's transverse motion and transverse structure

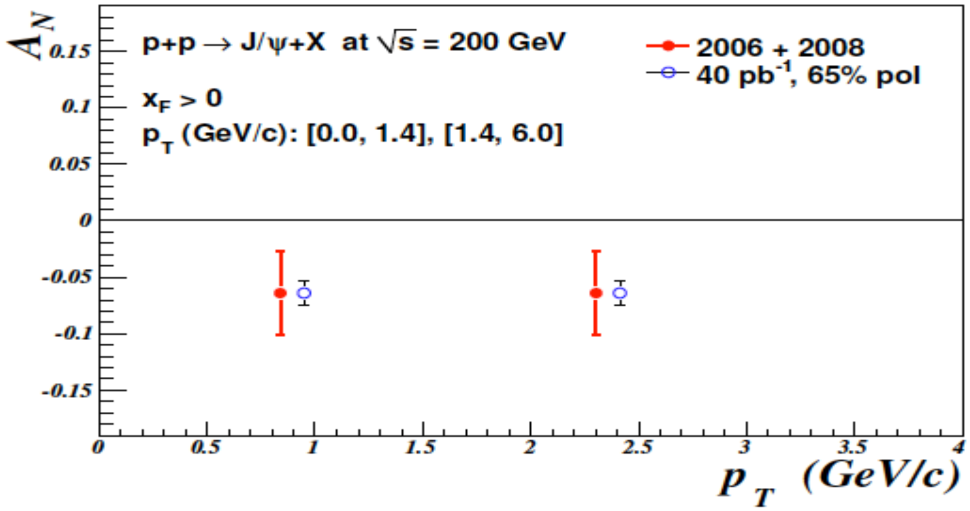
- Collinear and TMD factorization give complementary descriptions of QCD dynamics

Thank you!

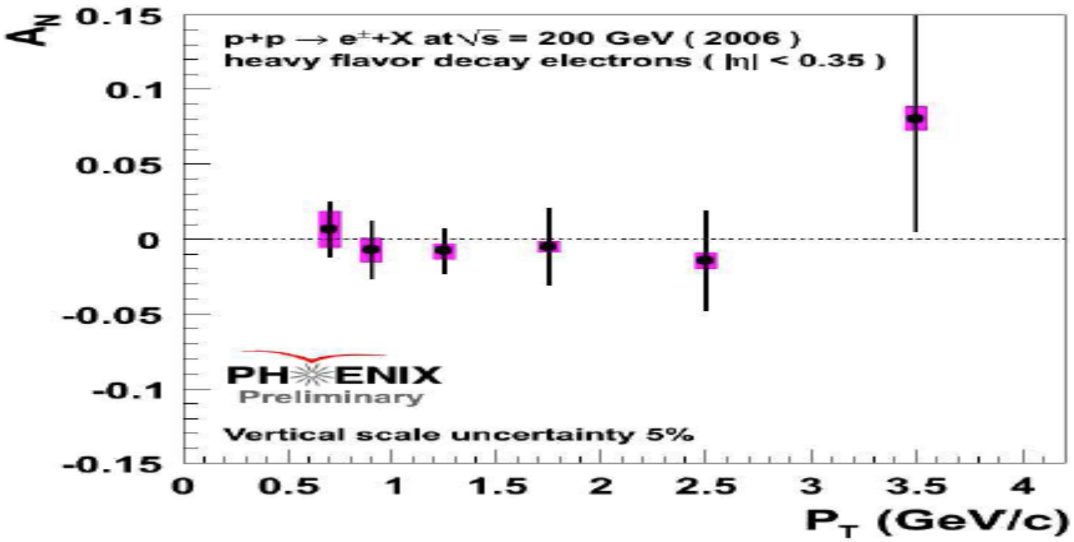
Backup slices

First hint of tri-gluon correlation

❑ PHENIX data on J/psi:



❑ PHENIX data on open charm:



Collinear factorization:

✧ tri-gluon correlation – direct quantum interference

Challenges:

✧ J/psi production mechanism

✧ Initial- vs final-state effect

✧ Connection to Gluon Sivers function

Collins, Qiu, Vogelsang, Yuan, Rogers, Mulder, …