

# **Transverse momentum-dependent parton distributions from lattice QCD**

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## Lattice theorists go shopping ...

Looking for:

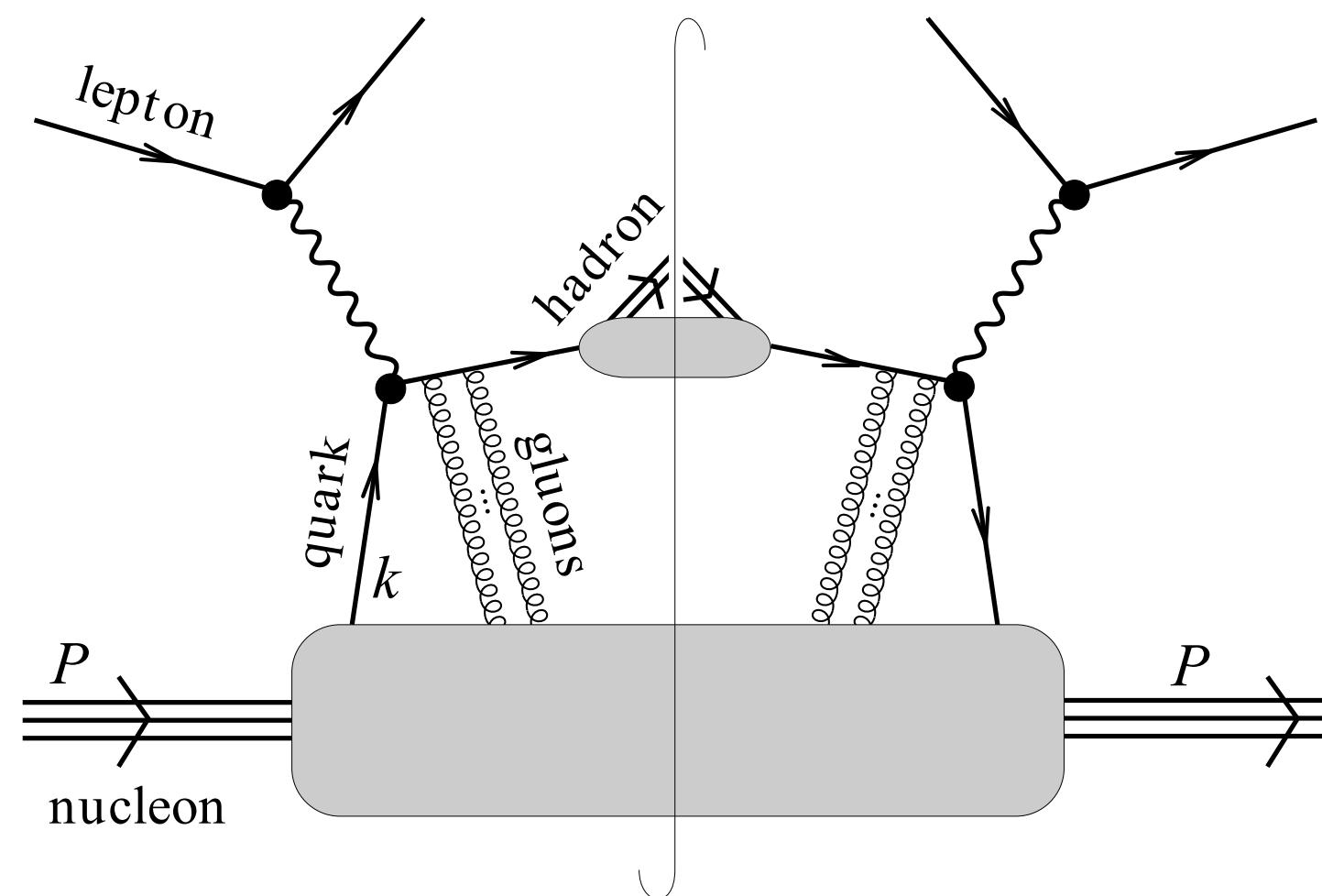
- Definition of TMDs in terms of matrix elements of a QCD operator
- There should exist a Lorentz frame in which the operator is at one fixed time —> separations in operator should be generically space-like (light cone approached as a limit)

... and find pictures of SIDIS (and DY) ...

suggesting such things can actually be defined,

Factorization?

Inclusion of final/initial state interactions?



Want to appropriately count quarks of momentum  $k$  ...

$$\begin{aligned}\Phi^{[\Gamma]}(x, k_T, P, S, \dots) &= \text{“} \int dk^- \langle P, S | \bar{q}(k) \Gamma q(k) | P, S \rangle \Big|_{k^+ \equiv x P^+} \text{”} \\ &= \int \frac{d^2 b_T}{(2\pi)^2} \int \frac{d(b \cdot P)}{(2\pi) P^+} \exp(ix(b \cdot P) - ib_T \cdot k_T) \frac{\bar{\Phi}_{\text{unsubtr.}}^{[\Gamma]}(b, P, S, \dots)}{\bar{\mathcal{S}}(b^2, \dots)} \Big|_{b^+ = 0}\end{aligned}$$

$$\bar{\Phi}_{\text{unsubtr.}}^{[\Gamma]}(b, P, S, \dots) \equiv \frac{1}{2} \langle P, S | \bar{q}(0) \Gamma \mathcal{U}[0, \dots, b] q(b) | P, S \rangle$$

- “Soft factor”  $\bar{\mathcal{S}}$  (typically a combination of vacuum expectation values of Wilson line structures) required to subtract divergences of Wilson line  $\mathcal{U}$
- Here, will consider only ratios in which soft factors cancel

## Decomposition into TMDs

All leading twist structures:

$$\Phi^{[\gamma^+]} = f_1 - \left[ \frac{\epsilon_{ij} k_i S_j}{m_N} f_{1T}^\perp \right]_{\text{odd}}$$

$$\Phi^{[\gamma^+ \gamma^5]} = \Lambda g_1 + \frac{k_T \cdot S_T}{m_N} g_{1T}$$

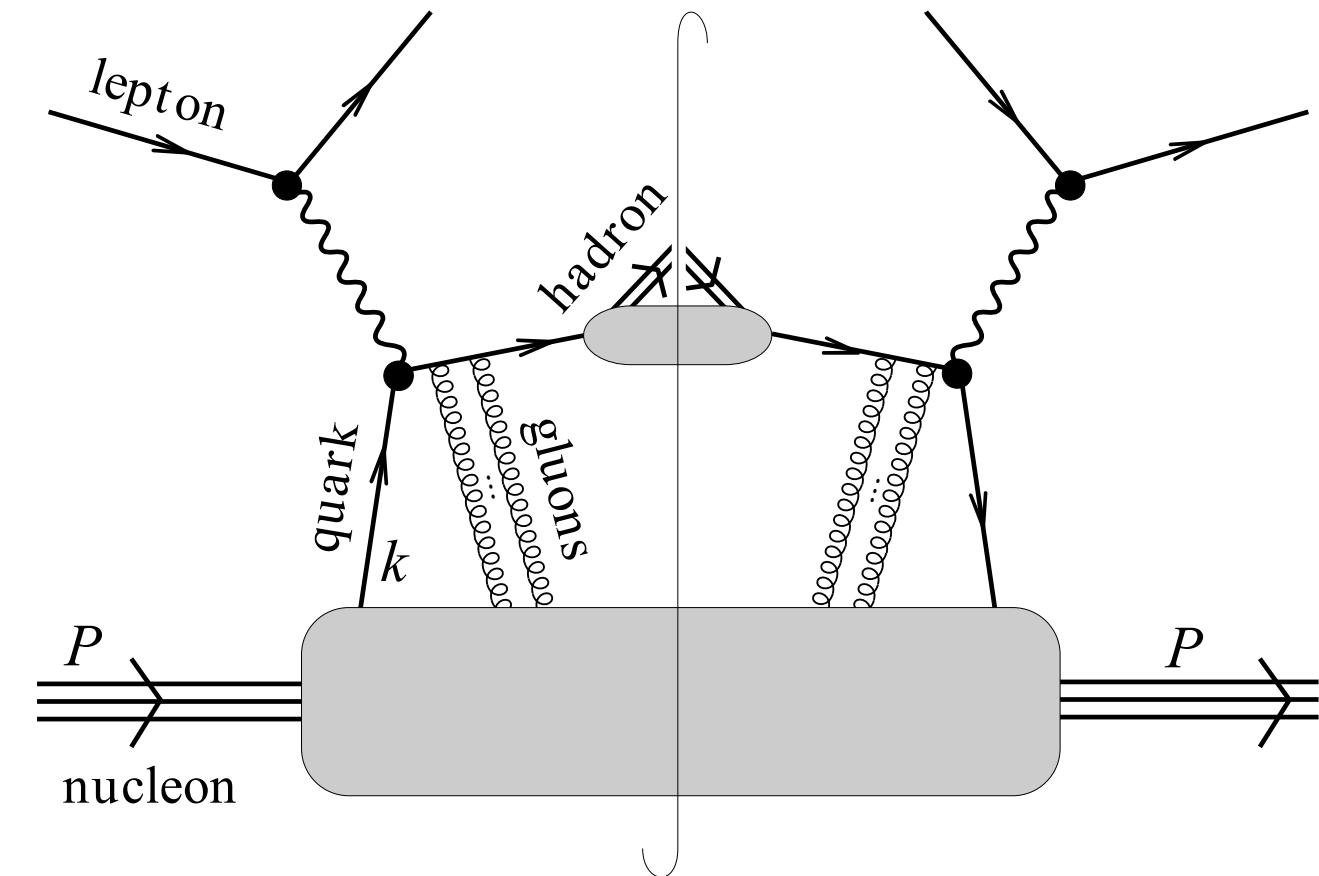
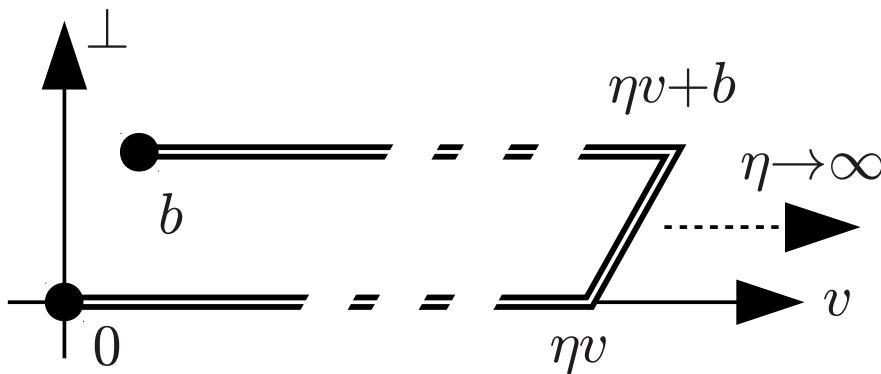
$$\Phi^{[i\sigma^{i+} \gamma^5]} = S_i h_1 + \frac{(2k_i k_j - k_T^2 \delta_{ij}) S_j}{2m_N^2} h_{1T}^\perp + \frac{\Lambda k_i}{m_N} h_{1L}^\perp + \left[ \frac{\epsilon_{ij} k_j}{m_N} h_1^\perp \right]_{\text{odd}}$$

## Gauge link structure ...

dictated by the physical process:

In matrix element  $\tilde{\Phi}_{\text{unsubtr.}}^{[\Gamma]}(b, P, S, \dots) \equiv$   
 $\frac{1}{2} \langle P, S | \bar{q}(0) \Gamma \mathcal{U}[0, \dots, b] q(b) | P, S \rangle$

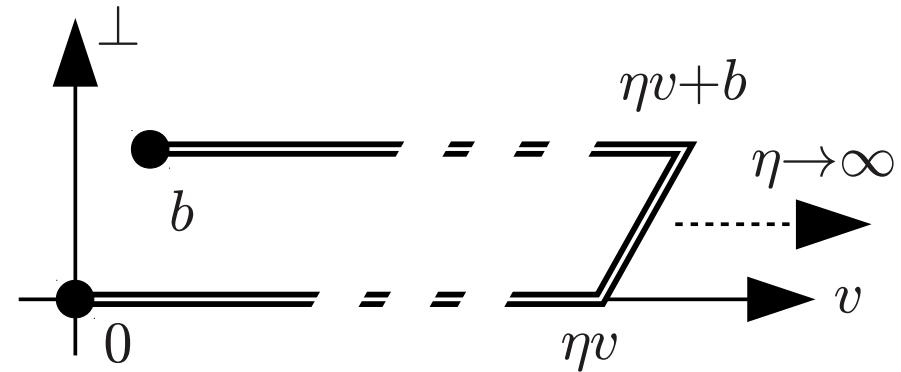
Staple-shaped gauge link  $\mathcal{U}[0, \eta v, \eta v + b, b]$



incorporates SIDIS final state effects

## Gauge link structure

- Gauge link roughly follows direction of ejected quark, (close to) light cone
- Effective resummed description of gluon exchanges between ejected quark and remainder of nucleon in evolving final state
- Take staple direction off light-cone into space-like region – cf. scheme advanced by Aybat, Collins, Qiu, Rogers in order to control rapidity divergences. Parametrize in terms of Collins-Soper-type parameter



$$\hat{\zeta} \equiv \frac{P \cdot v}{|P||v|}$$

Light-like staple for  $\hat{\zeta} \rightarrow \infty$ . Perturbative evolution equations for large  $\hat{\zeta}$ ?

This scheme appears to fit the requirements of a lattice calculation!

## Return to decomposition into TMDs (leading twist):

$$\Phi^{[\gamma^+]} = f_1 - \left[ \frac{\epsilon_{ij} k_i S_j}{m_N} f_{1T}^\perp \right]_{\text{odd}}$$

$$\Phi^{[\gamma^+ \gamma^5]} = \Lambda g_1 + \frac{k_T \cdot S_T}{m_N} g_{1T}$$

$$\Phi^{[i\sigma^{i+} \gamma^5]} = S_i h_1 + \frac{(2k_i k_j - k_T^2 \delta_{ij}) S_j}{2m_N^2} h_{1T}^\perp + \frac{\Lambda k_i}{m_N} h_{1L}^\perp + \left[ \frac{\epsilon_{ij} k_j}{m_N} h_1^\perp \right]_{\text{odd}}$$

- Without initial/final state effects, T-odd Sivers and Boer-Mulders functions  $f_{1T}^\perp$  and  $h_1^\perp$  would vanish
- “Modified universality”  $f^{\text{T-odd, SIDIS}} = -f^{\text{T-odd, DY}}$ . SIDIS:  $\eta v \cdot P \rightarrow \infty$ , DY:  $\eta v \cdot P \rightarrow -\infty$ .

## Decomposition into TMDs (leading twist)

$q \rightarrow$ N $\downarrow$	U	L	T
U	$f_1$		$h_1^\perp$
L		$g_1$	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}$	$h_1$ $h_{1T}^\perp$

↑  
Sivers (T-odd)

$\leftarrow$  Boer-Mulders  
(T-odd)

## Fourier-transformed TMDs

$$\tilde{f}(x, b_T^2, \dots) \equiv \int d^2 k_T \exp(ib_T \cdot k_T) f(x, k_T^2, \dots)$$

$$\tilde{f}^{(n)}(x, b_T^2, \dots) \equiv n! \left( -\frac{2}{m_N^2} \partial_{b_T^2} \right)^n \tilde{f}(x, b_T^2, \dots)$$

In limit  $|b_T| \rightarrow 0$ , formally recover  $k_T$ -moments:

$$\tilde{f}^{(n)}(x, 0, \dots) \equiv \int d^2 k_T \left( \frac{k_T^2}{2m_N^2} \right)^n f(x, k_T^2, \dots) \equiv f^{(n)}(x)$$

However,  $b_T \rightarrow 0$  limit delicate – will not attempt to extrapolate to  $b_T = 0$ , but give results at finite  $b_T$ ;  $b_T$  acts as regulator.

Also, we can only access limited range of  $b \cdot P$ , so cannot Fourier-transform to obtain  $x$ -dependence. Therefore, consider only first  $x$ -moments (accessible at  $b \cdot P = 0$ ):

$$f^{[1]}(k_T^2, \dots) \equiv \int_{-1}^1 dx f(x, k_T^2, \dots)$$

## Invariant amplitudes

Return to correlator

$$\bar{\Phi}_{\text{unsubtr.}}^{[\Gamma]}(b, P, S, \hat{\zeta}, \mu) \equiv \frac{1}{2} \langle P, S | \bar{q}(0) \Gamma \mathcal{U}[0, \eta v, \eta v + b, b] q(b) | P, S \rangle$$

Decompose in terms of invariant amplitudes; at leading twist,

$$\frac{1}{2P^+} \bar{\Phi}_{\text{unsubtr.}}^{[\gamma^+]} = \bar{A}_{2B} + im_N \epsilon_{ij} b_i S_j \bar{A}_{12B}$$

$$\frac{1}{2P^+} \bar{\Phi}_{\text{unsubtr.}}^{[\gamma^+ \gamma^5]} = -\Lambda \bar{A}_{6B} + i[(b \cdot P)\Lambda - m_N(b_T \cdot S_T)] \bar{A}_{7B}$$

$$\begin{aligned} \frac{1}{2P^+} \bar{\Phi}_{\text{unsubtr.}}^{[i\sigma^{i+} \gamma^5]} &= im_N \epsilon_{ij} b_j \bar{A}_{4B} - S_i \bar{A}_{9B} \\ &\quad - im_N \Lambda b_i \bar{A}_{10B} + m_N[(b \cdot P)\Lambda - m_N(b_T \cdot S_T)] b_i \bar{A}_{11B} \end{aligned}$$

## Invariant amplitudes and TMDs

Amplitudes related to TMDs via Fourier transform (showing just the ones relevant for Sivers, Boer-Mulders shifts):

$$\begin{aligned} f_1(x, k_T^2, \hat{\zeta}, \dots, \eta v \cdot P) &= 2 \int_{\mathcal{F}} \bar{A}_{2B} \\ f_{1T}^\perp(x, k_T^2, \hat{\zeta}, \dots, \eta v \cdot P) &= 4m_N^2 \partial_{k_T^2} \int_{\mathcal{F}} \bar{A}_{12B} \\ h_1^\perp(x, k_T^2, \hat{\zeta}, \dots, \eta v \cdot P) &= -4m_N^2 \partial_{k_T^2} \int_{\mathcal{F}} \bar{A}_{4B} \end{aligned}$$

where

$$\int_{\mathcal{F}} \bar{A}_i \equiv \int \frac{d^2 b_T}{(2\pi)^2} \exp(-ib_T \cdot k_T) \frac{1}{\bar{S}(b^2, \dots)} \int \frac{d(b \cdot P)}{2\pi} \exp(ix(b \cdot P)) \bar{A}_i(-b_T^2, b \cdot P, \hat{\zeta}, \eta v \cdot P)$$

## Invariant amplitudes and TMDs

Conversely, invariant amplitudes directly give selected  $x$ -integrated TMDs in Fourier ( $b_T$ ) space (again showing just the ones relevant for Sivers, Boer-Mulders shifts):

$$\tilde{f}_1^{[1](0)}(b_T^2, \hat{\zeta}, \dots, \eta v \cdot P) = 2\bar{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)/\bar{S}(b^2, \dots)$$

$$\tilde{f}_{1T}^{\perp[1](1)}(b_T^2, \hat{\zeta}, \dots, \eta v \cdot P) = -2\bar{A}_{12B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)/\bar{S}(b^2, \dots)$$

$$\tilde{h}_1^{\perp[1](1)}(b_T^2, \hat{\zeta}, \dots, \eta v \cdot P) = 2\bar{A}_{4B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)/\bar{S}(b^2, \dots)$$

## Generalized shifts

Form ratios in which soft factors, ( $\Gamma$ -independent) multiplicative renormalization factors cancel

Sivers shift:

$$\langle k_y \rangle_{TU} \equiv m_N \frac{f_{1T}^{\perp[1](1)}}{f_1^{[1](0)}} = \frac{\int dx \int d^2 k_T k_y \Phi^{[\gamma^+]}(x, k_T, S_T = (1, 0))}{\int dx \int d^2 k_T \Phi^{[\gamma^+]}(x, k_T, S_T = (1, 0))}$$

Average transverse momentum of unpolarized (“ $U$ ”) quarks orthogonal to the transverse (“ $T$ ”) spin of nucleon; normalized to the number of valence quarks.

Since  $b_T \rightarrow 0$  limit delicate, consider more generally ratio of Fourier-transformed TMDs at *nonzero*  $b_T^2$ ,

$$\langle k_y \rangle_{TU}(b_T^2, \dots) \equiv m_N \frac{\tilde{f}_{1T}^{\perp[1](1)}(b_T^2, \dots)}{\tilde{f}_1^{[1](0)}(b_T^2, \dots)}$$

(remember  $b_T \rightarrow 0$  limit corresponds to taking  $k_T$ -moment). “Generalized shift”.

## Generalized shifts from amplitudes

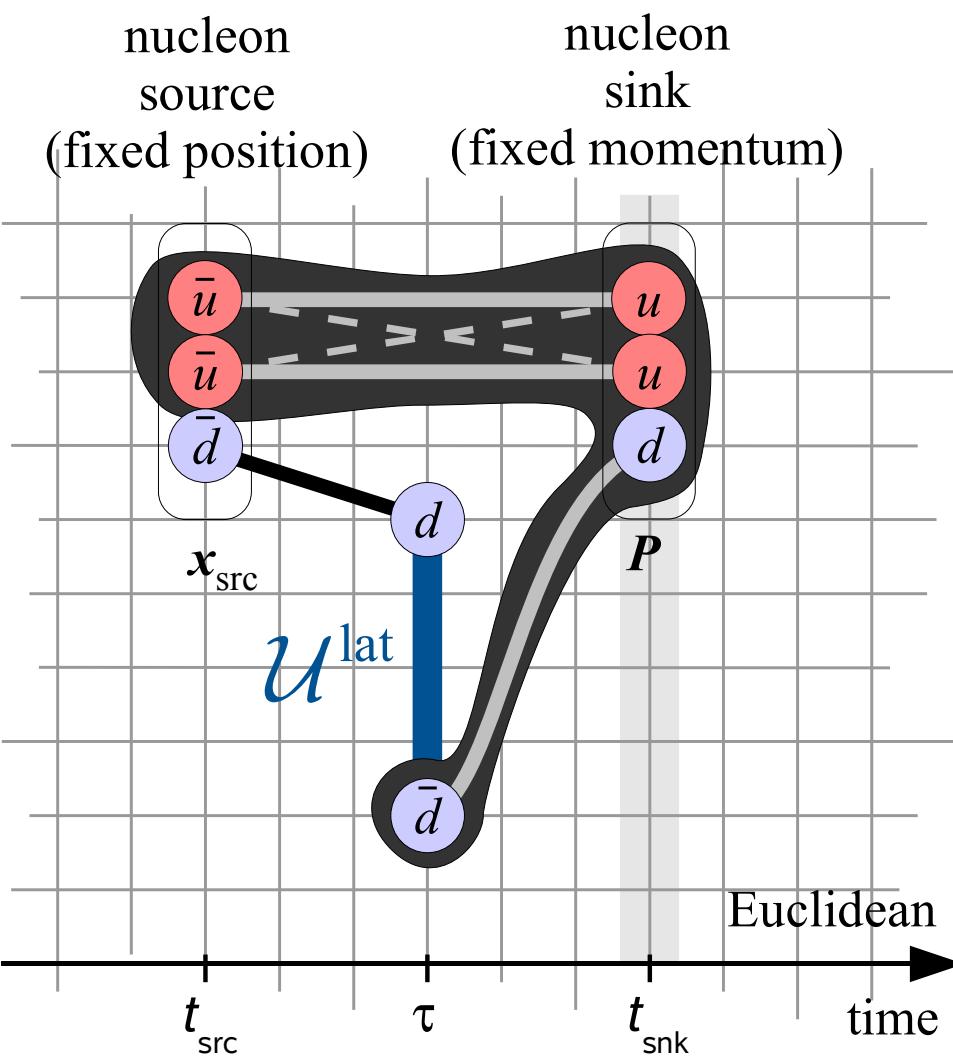
Now, can also express this in terms of invariant amplitudes:

$$\langle k_y \rangle_{TU}(b_T^2, \dots) \equiv m_N \frac{\tilde{f}_{1T}^{\perp[1](1)}(b_T^2, \dots)}{\tilde{f}_1^{[1](0)}(b_T^2, \dots)} = -m_N \frac{\bar{A}_{12B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\bar{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

Analogously, Boer-Mulders shift:

$$\langle k_y \rangle_{UT}(b_T^2, \dots) = m_N \frac{\bar{A}_{4B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\bar{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

## Lattice setup



- Evaluate directly  $\bar{\Phi}_{\text{unsubtr.}}^{[\Gamma]}(b, P, S, \hat{\zeta}, \mu)$   
 $\equiv \frac{1}{2} \langle P, S | \bar{q}(0) \Gamma \mathcal{U}[0, \eta v, \eta v + b, b] q(b) | P, S \rangle$
- Euclidean time: Place entire operator at one time slice, i.e.,  $b, \eta v$  purely spatial
- Since generic  $b, v$  space-like, no obstacle to boosting system to such a frame!
- Parametrization of correlator in terms of  $\tilde{A}_i$  invariants permits direct translation of results back to original frame
- Form desired ratios of  $\tilde{A}_i$  invariants
- Extrapolate  $\eta \rightarrow \infty, \hat{\zeta} \rightarrow \infty$  numerically

## Lattice setup

Use three MILC 2+1-flavor gauge ensembles with  $a \approx 0.12 \text{ fm}$ :

$m_\pi = 369 \text{ MeV}$ ;  $28^3 \times 64$ ; 2184 samples

$m_\pi = 369 \text{ MeV}$ ;  $20^3 \times 64$ ; 5264 samples

$m_\pi = 518 \text{ MeV}$ ;  $20^3 \times 64$ ; 3888 samples

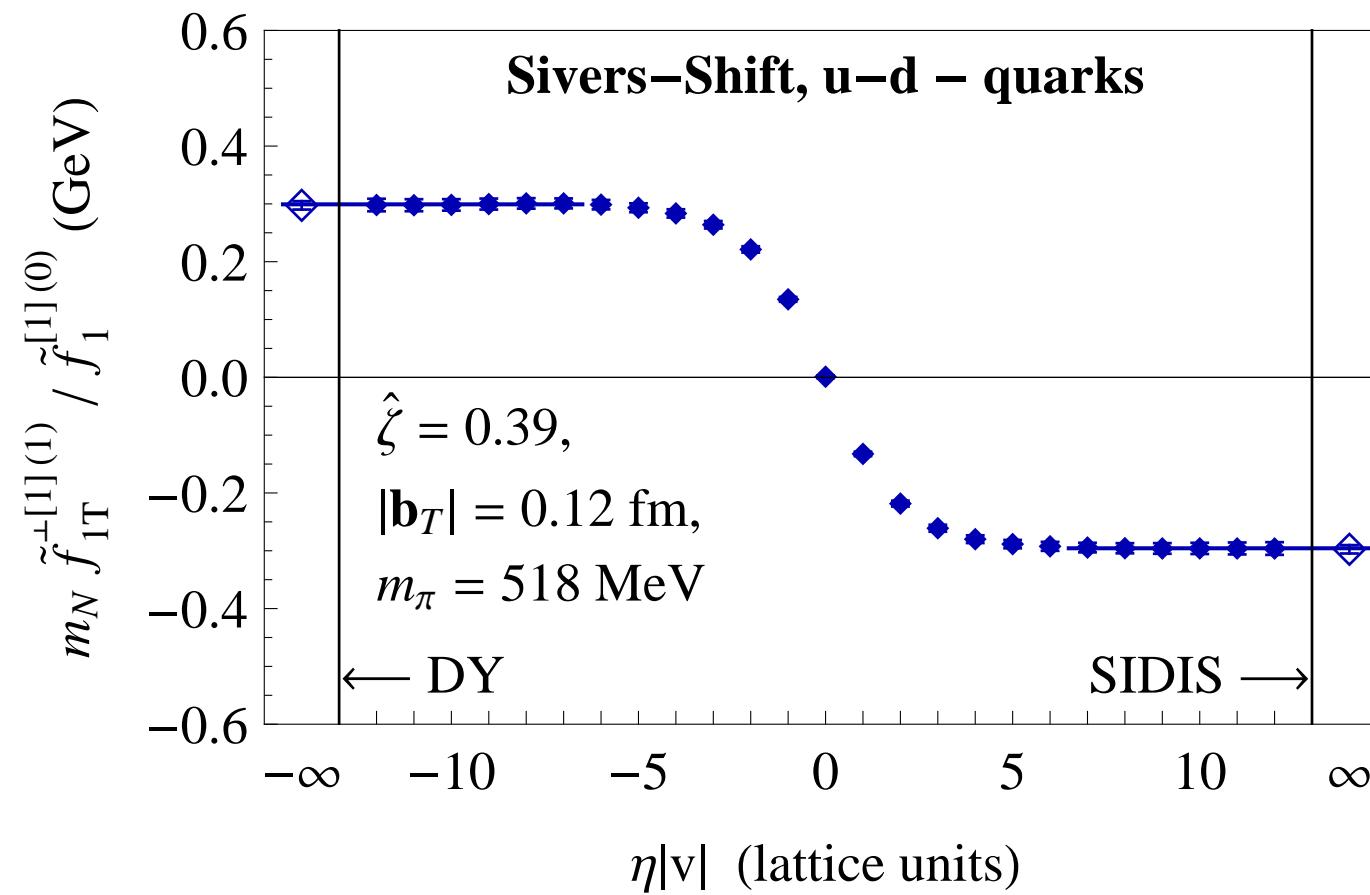
Sink momenta  $P$ :  $(0, 0, 0)$ ,  $(-1, 0, 0)$ ,  $(-2, 0, 0)$ ,  $(1, -1, 0)$

Variety of  $b$ ,  $\eta v$ ; note  $b \perp P$ ,  $b \perp v$  (lowest  $x$ -moment, kinematical choices/constraints)

Largest  $\hat{\zeta} = 0.78$

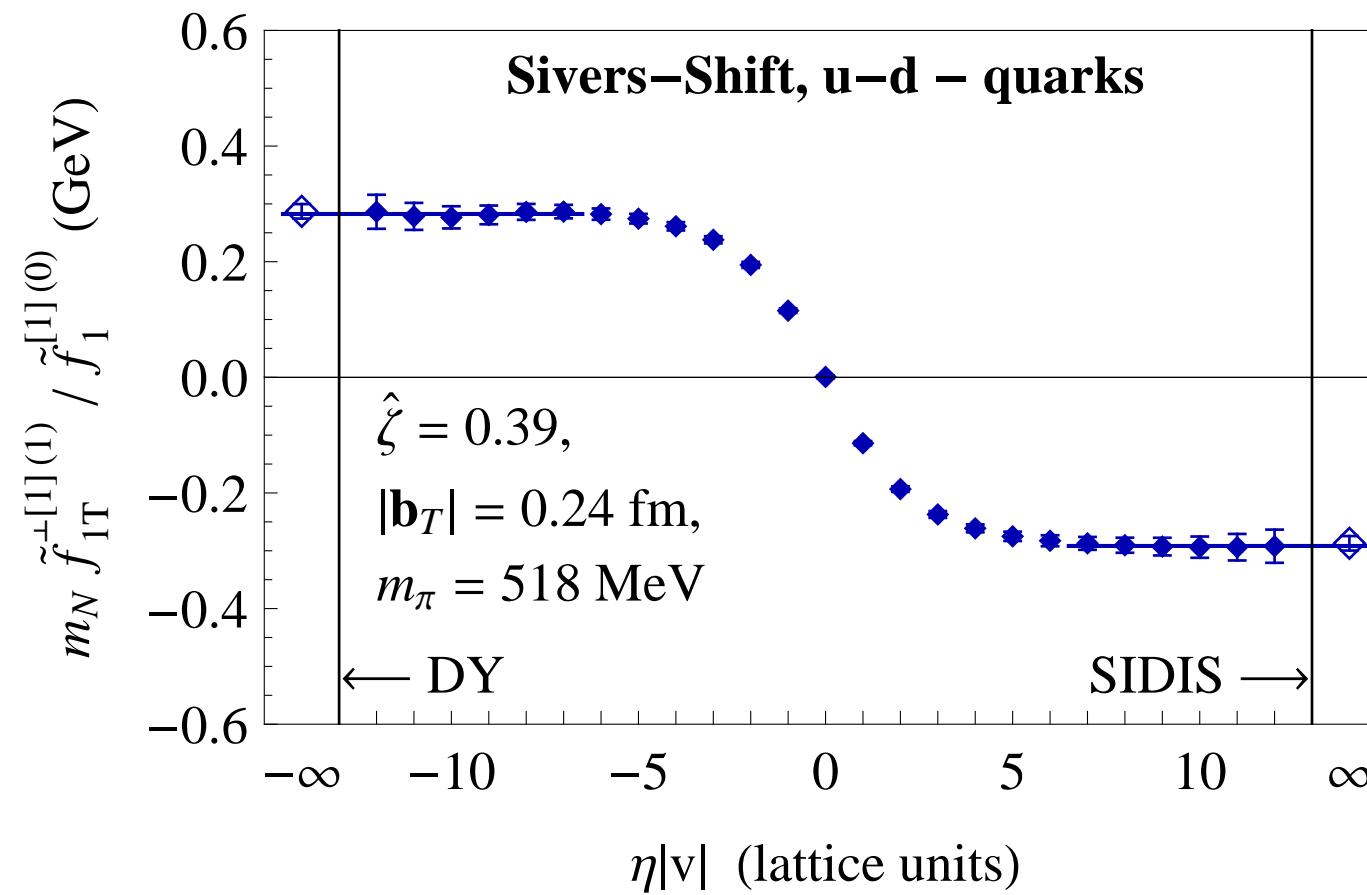
## Results: Sivers shift

Dependence on staple extent; sequence of panels at different  $|b_T|$



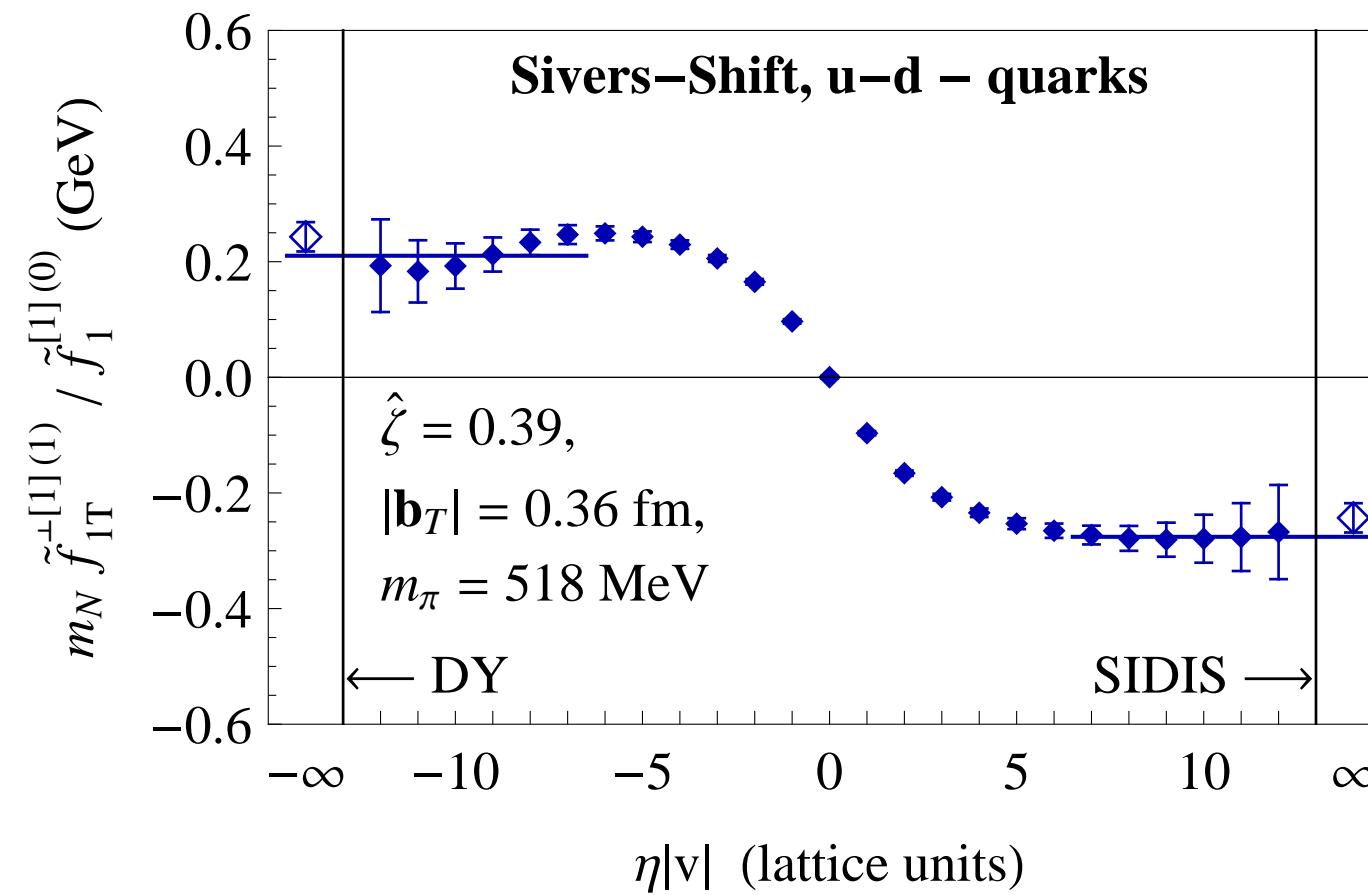
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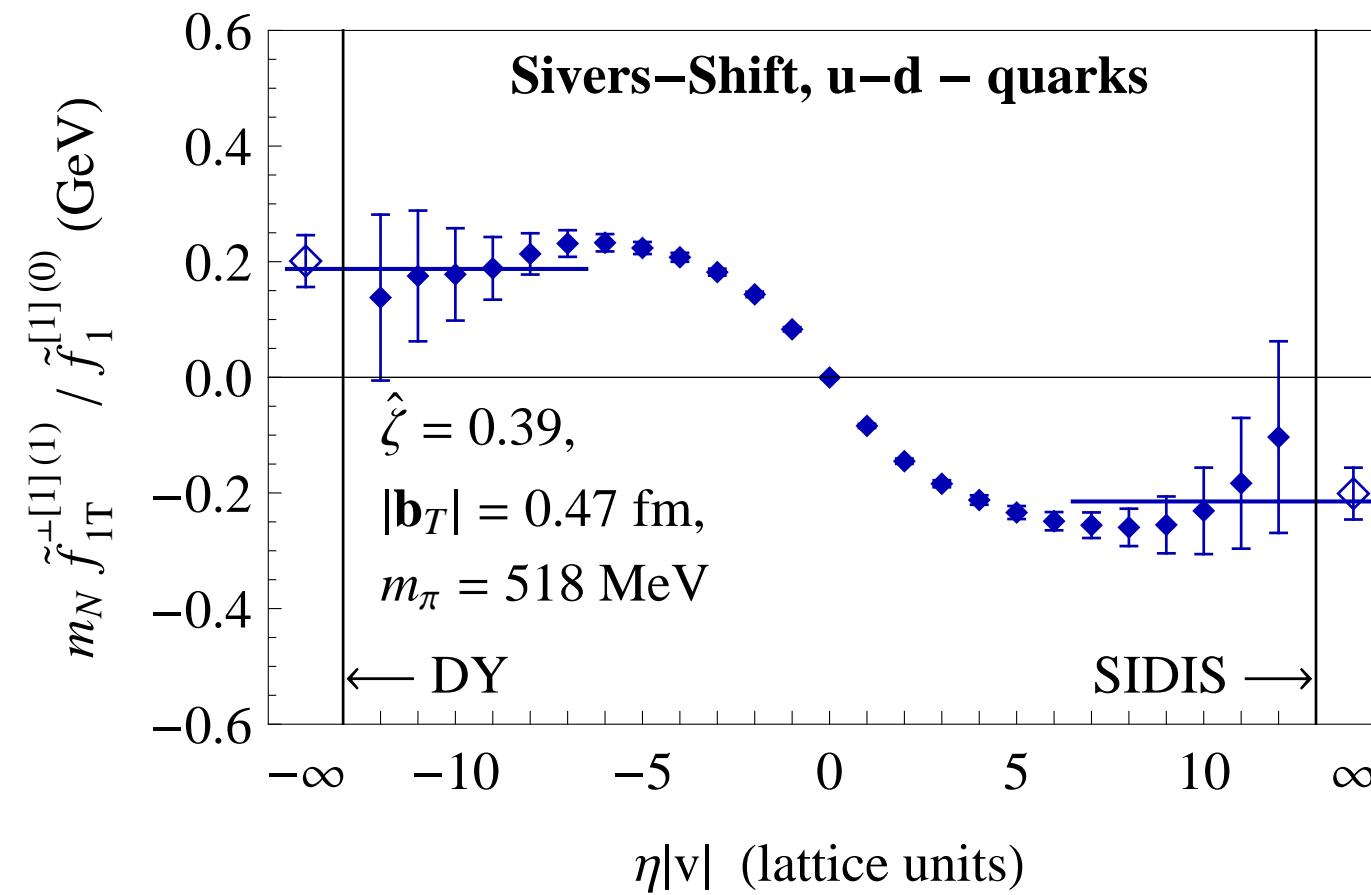
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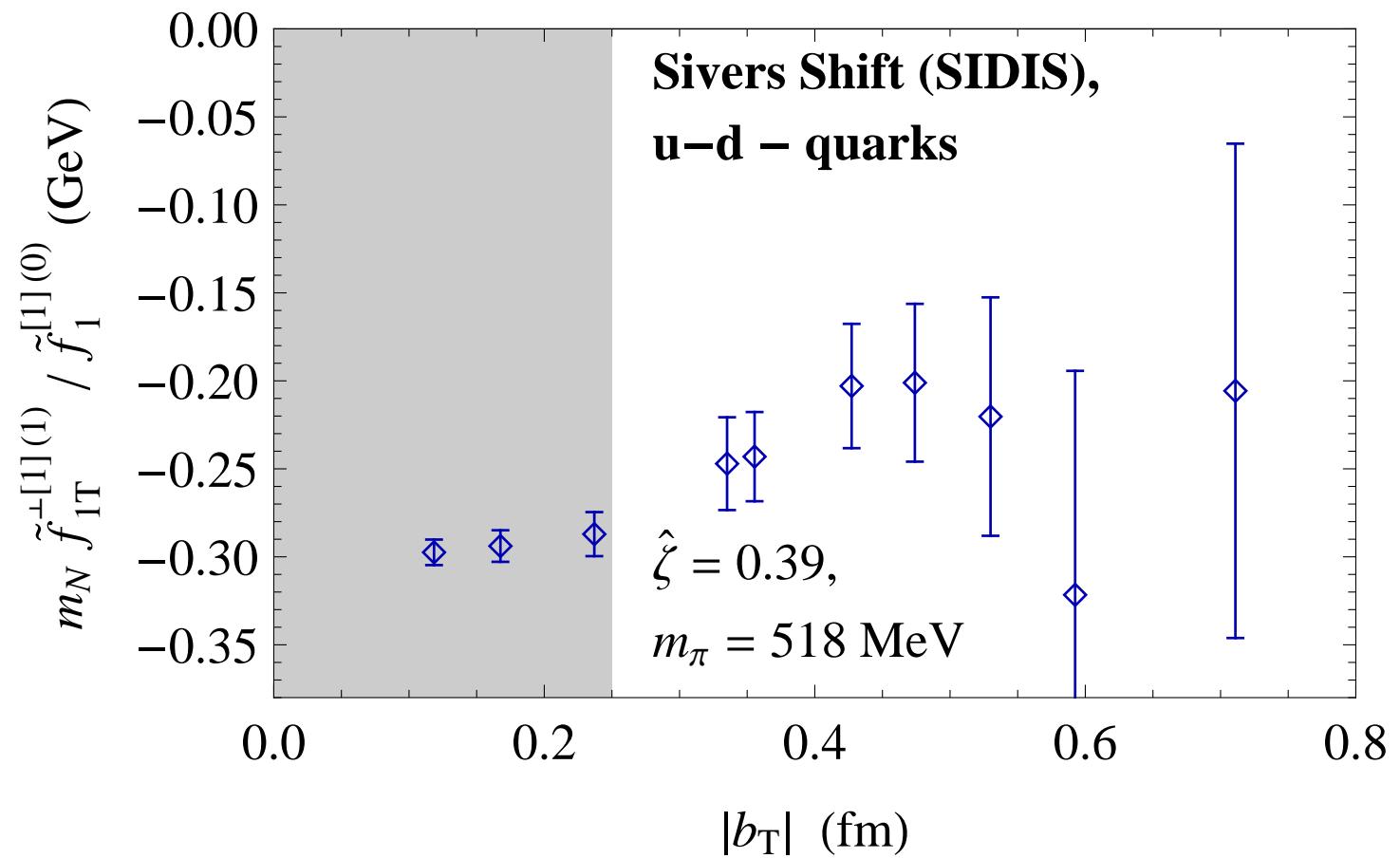
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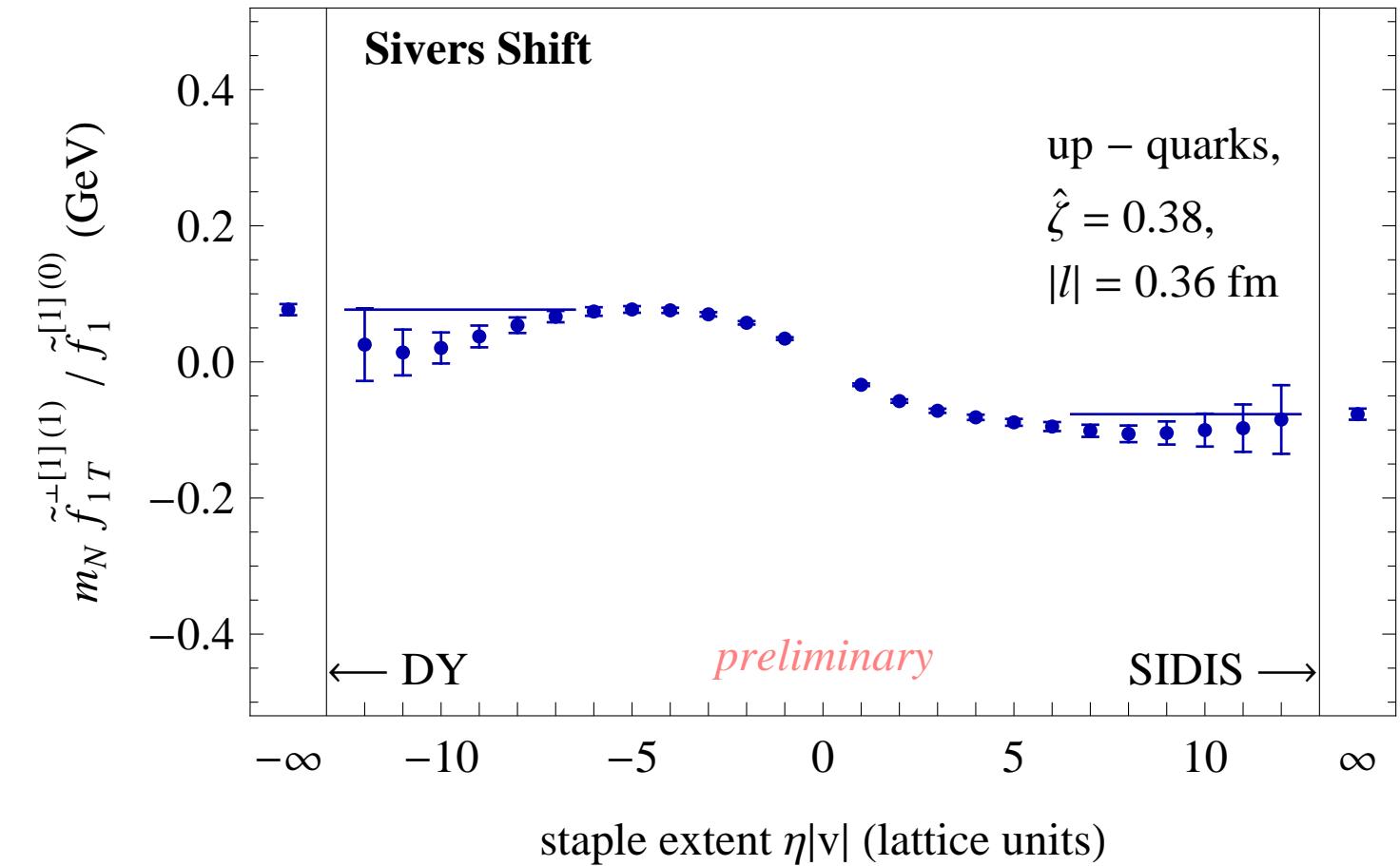
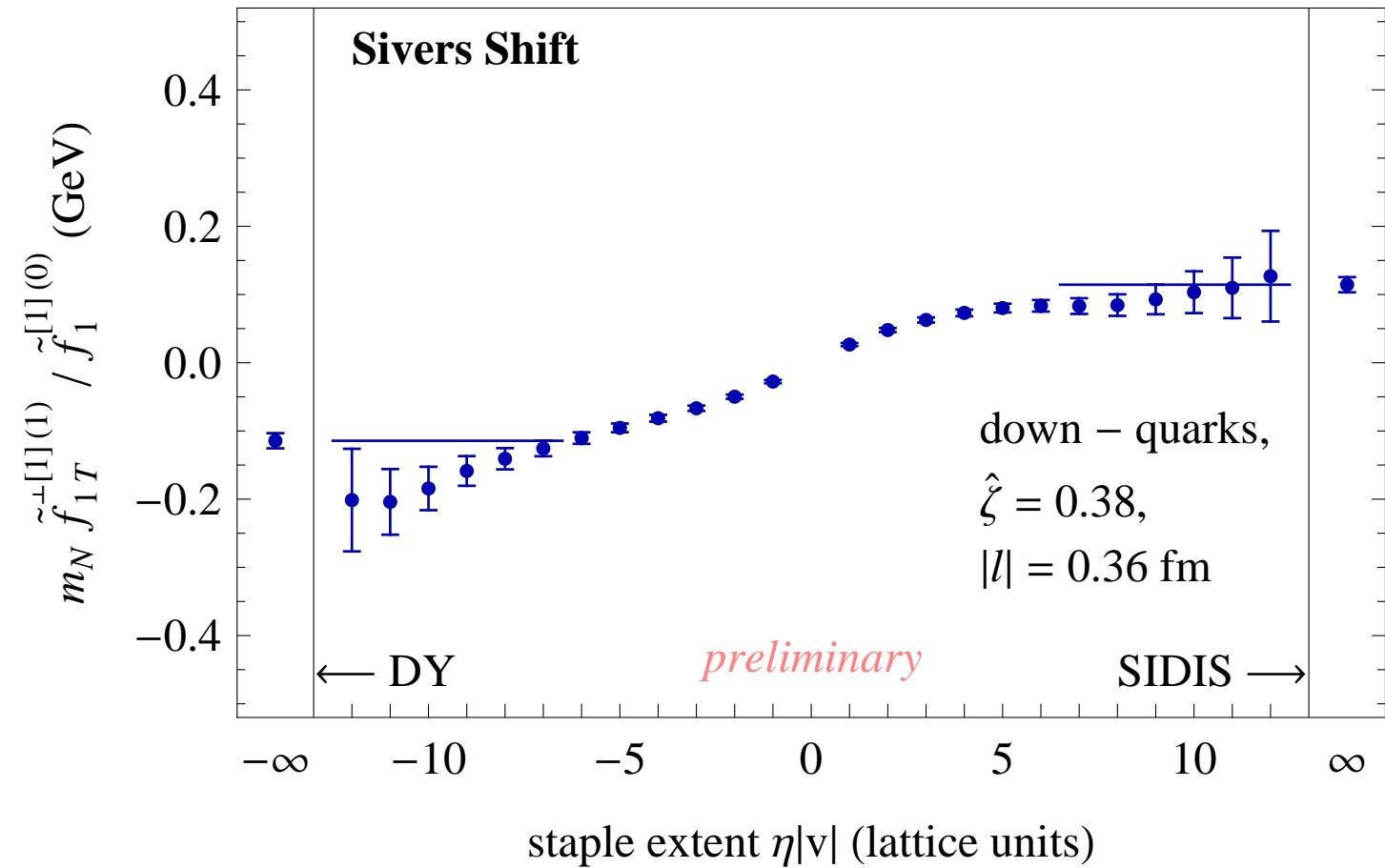
## Results: Sivers shift

Dependence of SIDIS limit on  $|b_T|$



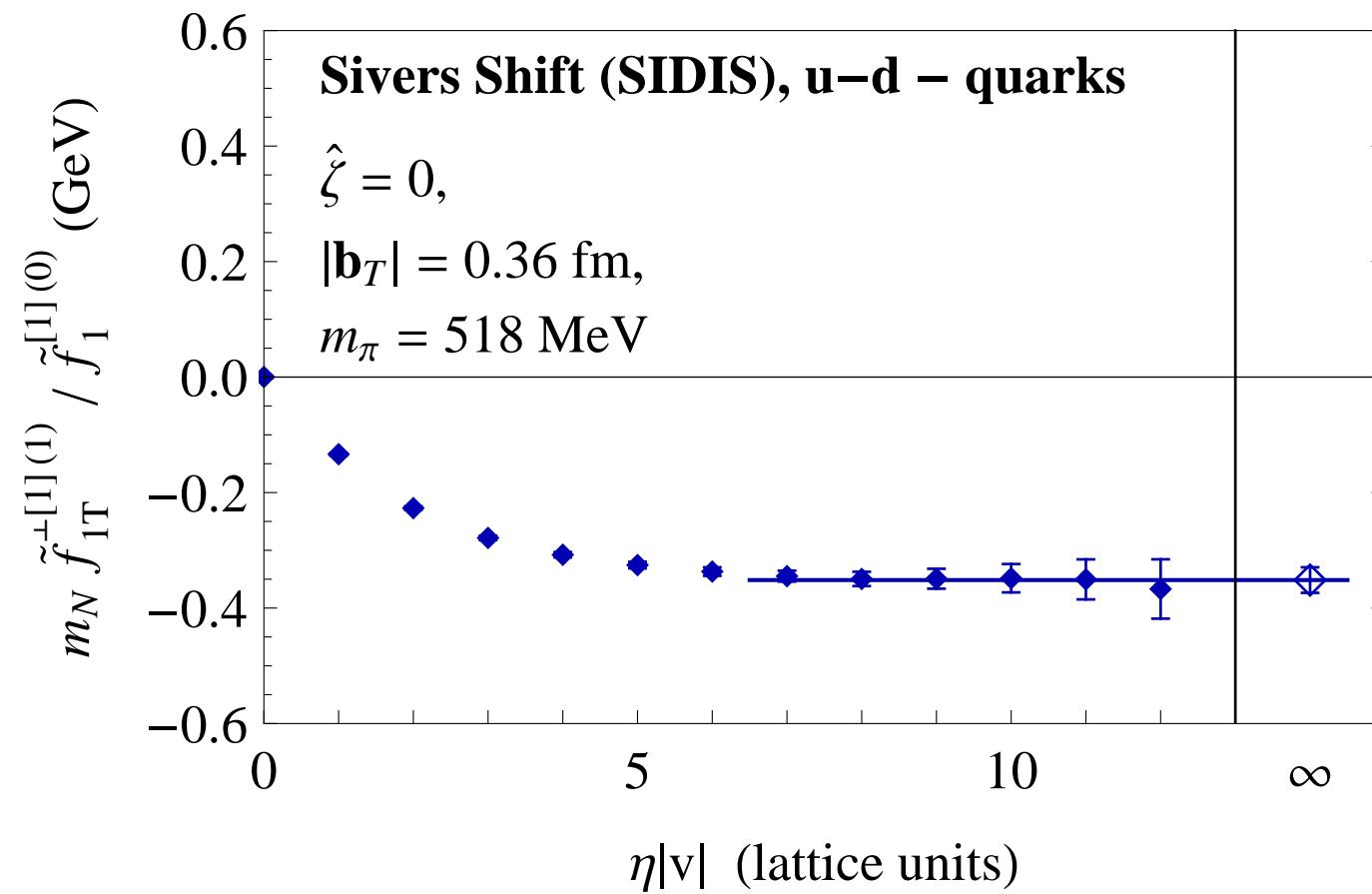
## Results: Sivers shift

Dependence on staple extent; flavor separated



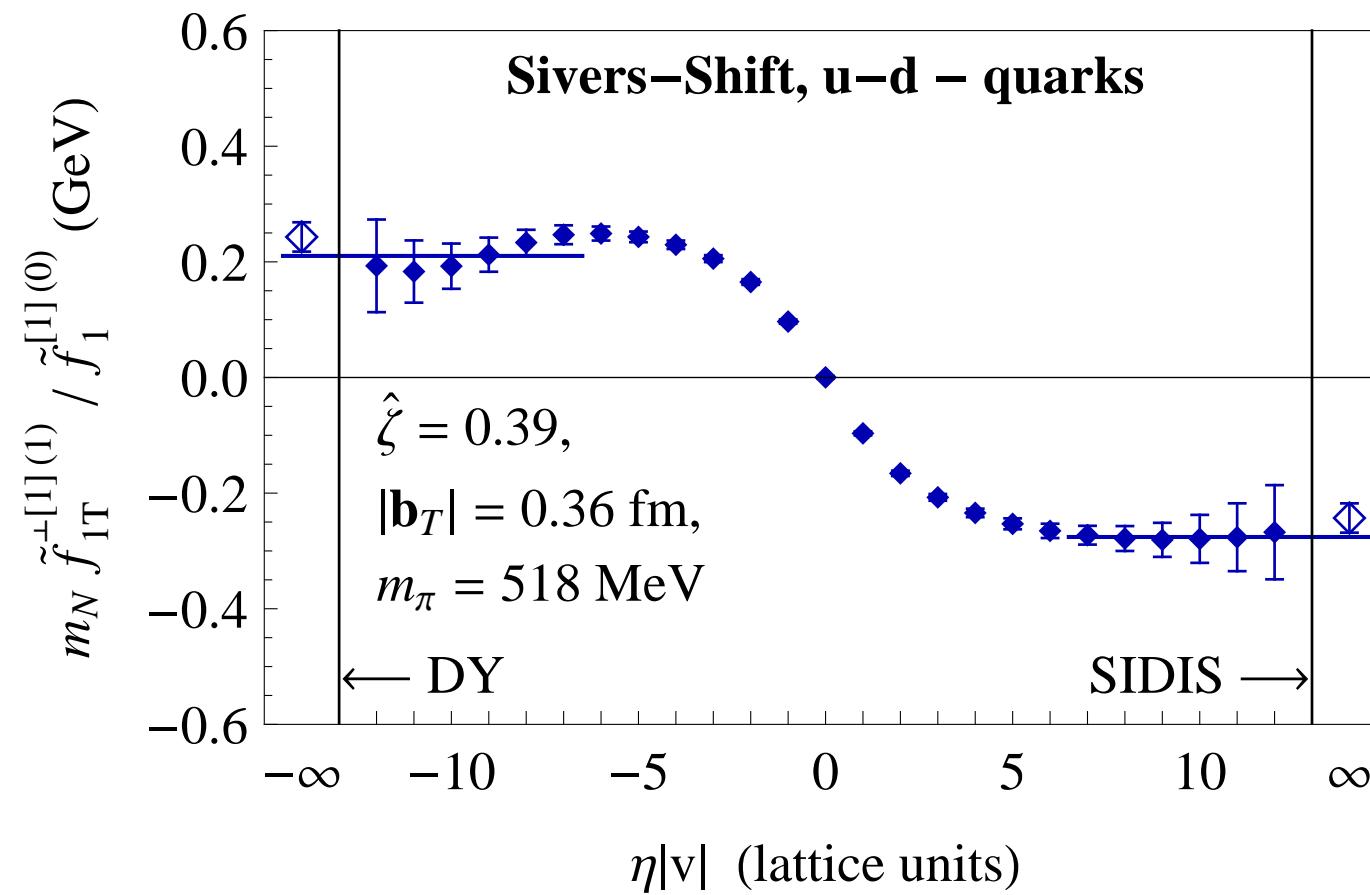
## Results: Sivers shift

Dependence on staple extent; sequence of panels at different  $\hat{\zeta}$



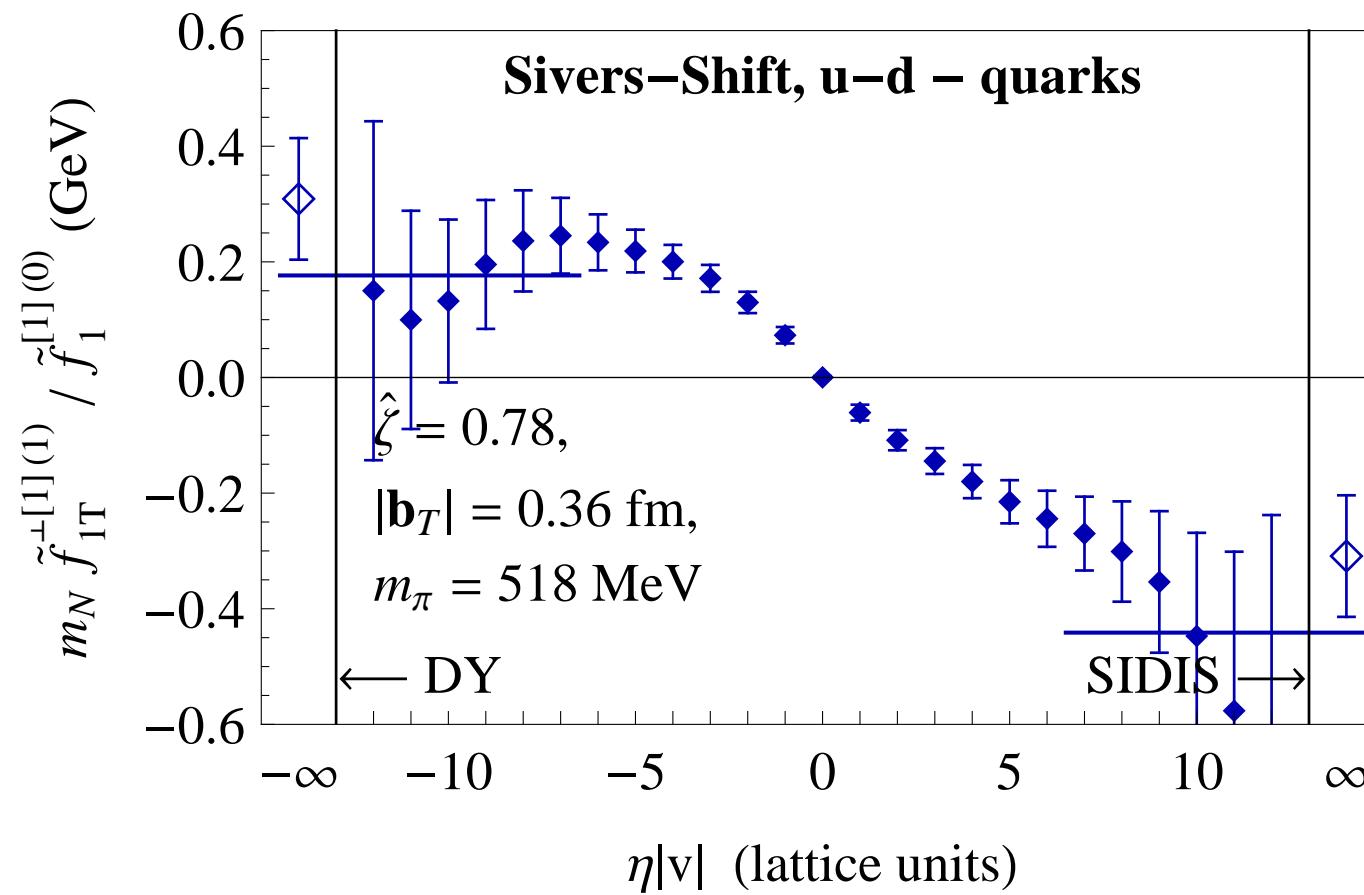
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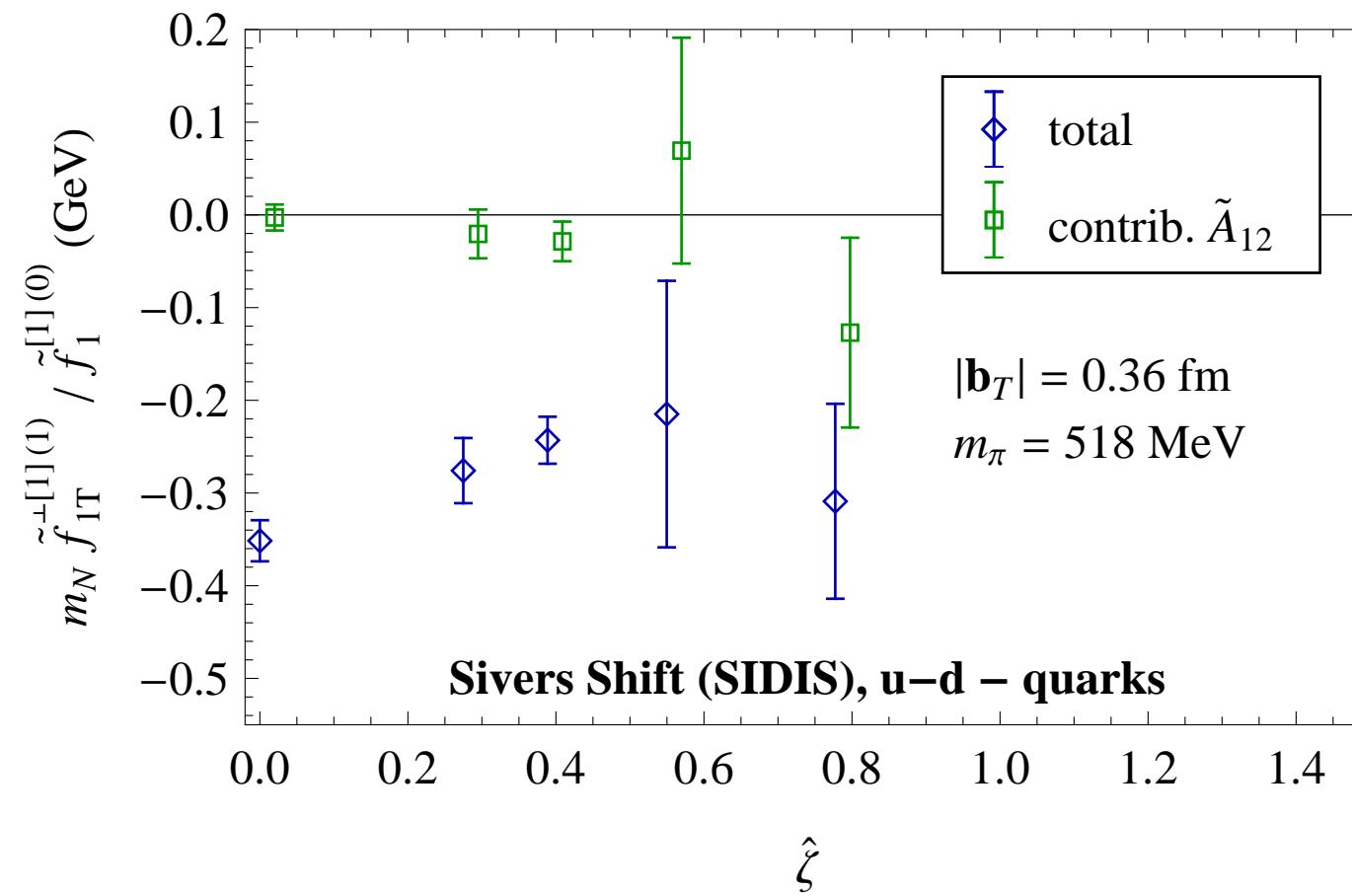
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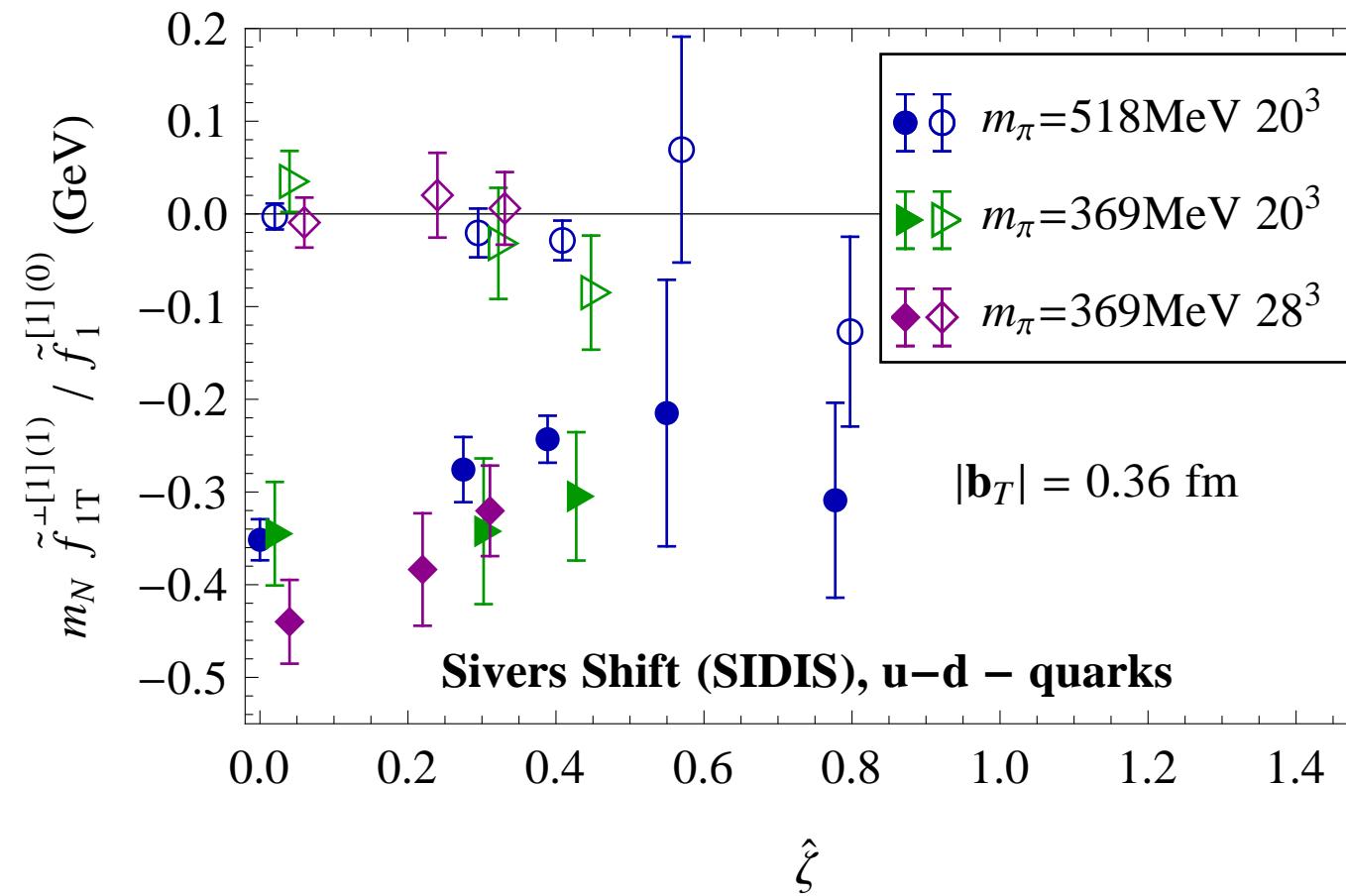
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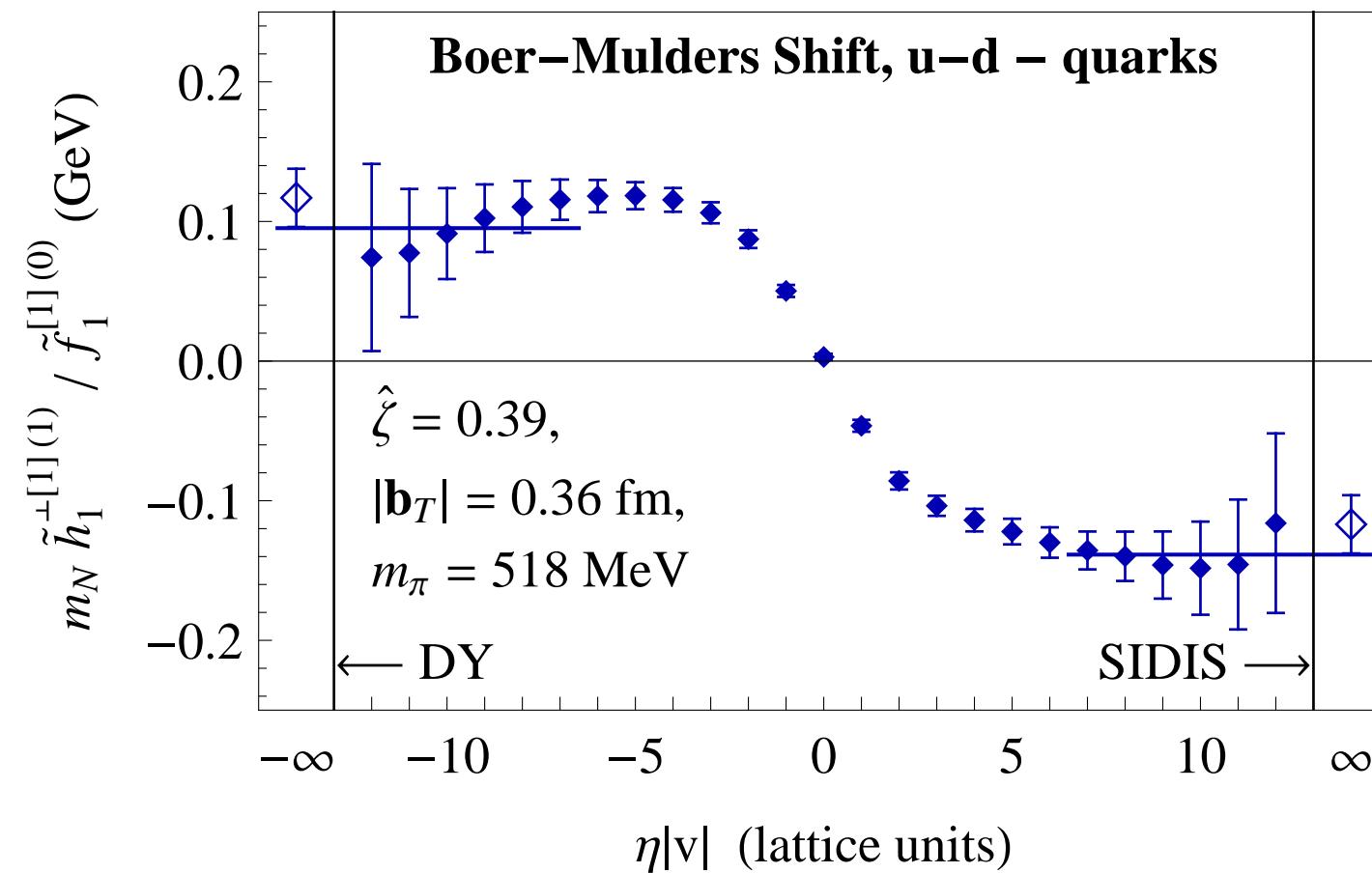
## Results: Sivers shift

Dependence of SIDIS limit on  $\hat{\zeta}$ , all three ensembles



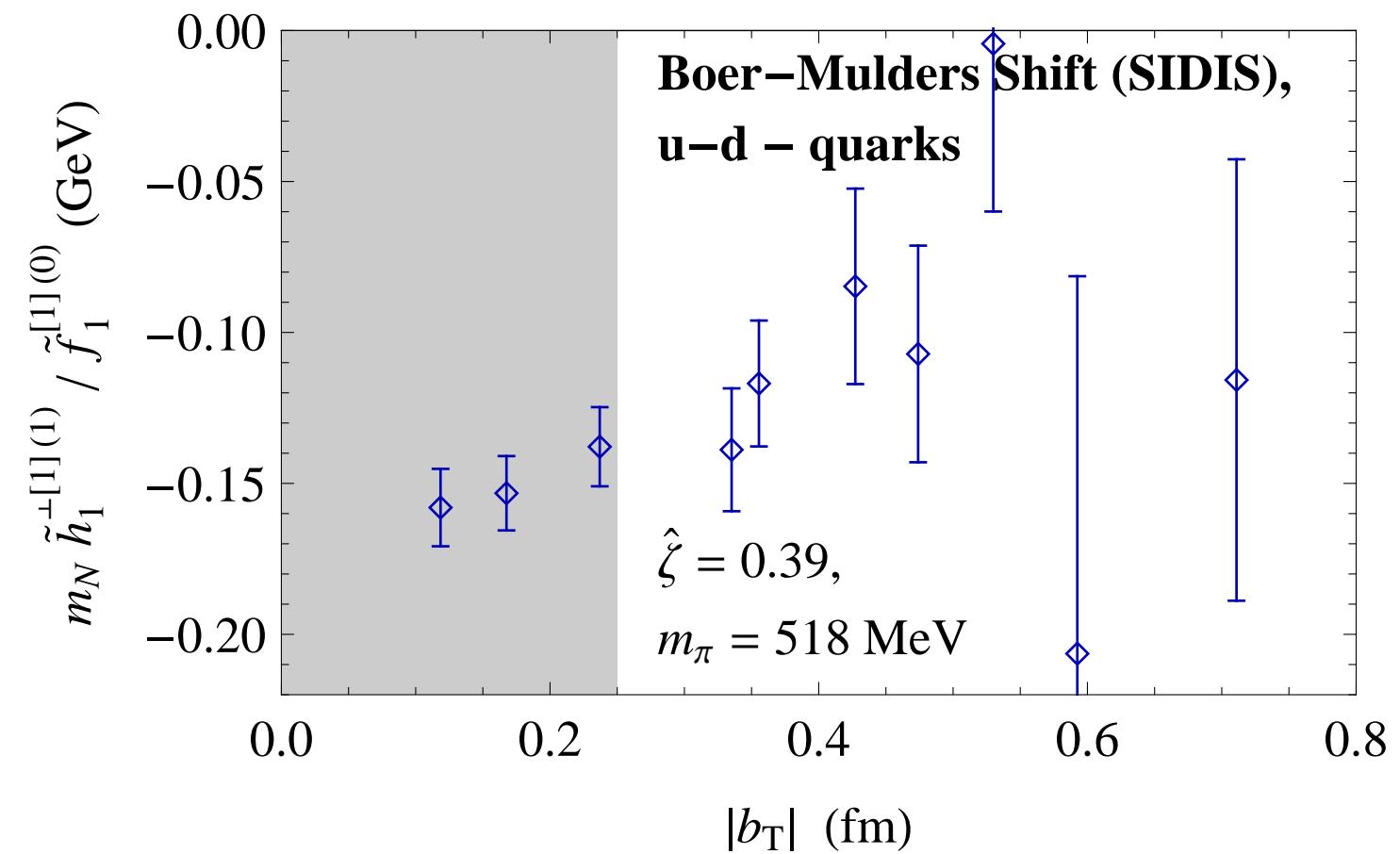
## Results: Boer-Mulders shift

Dependence on staple extent



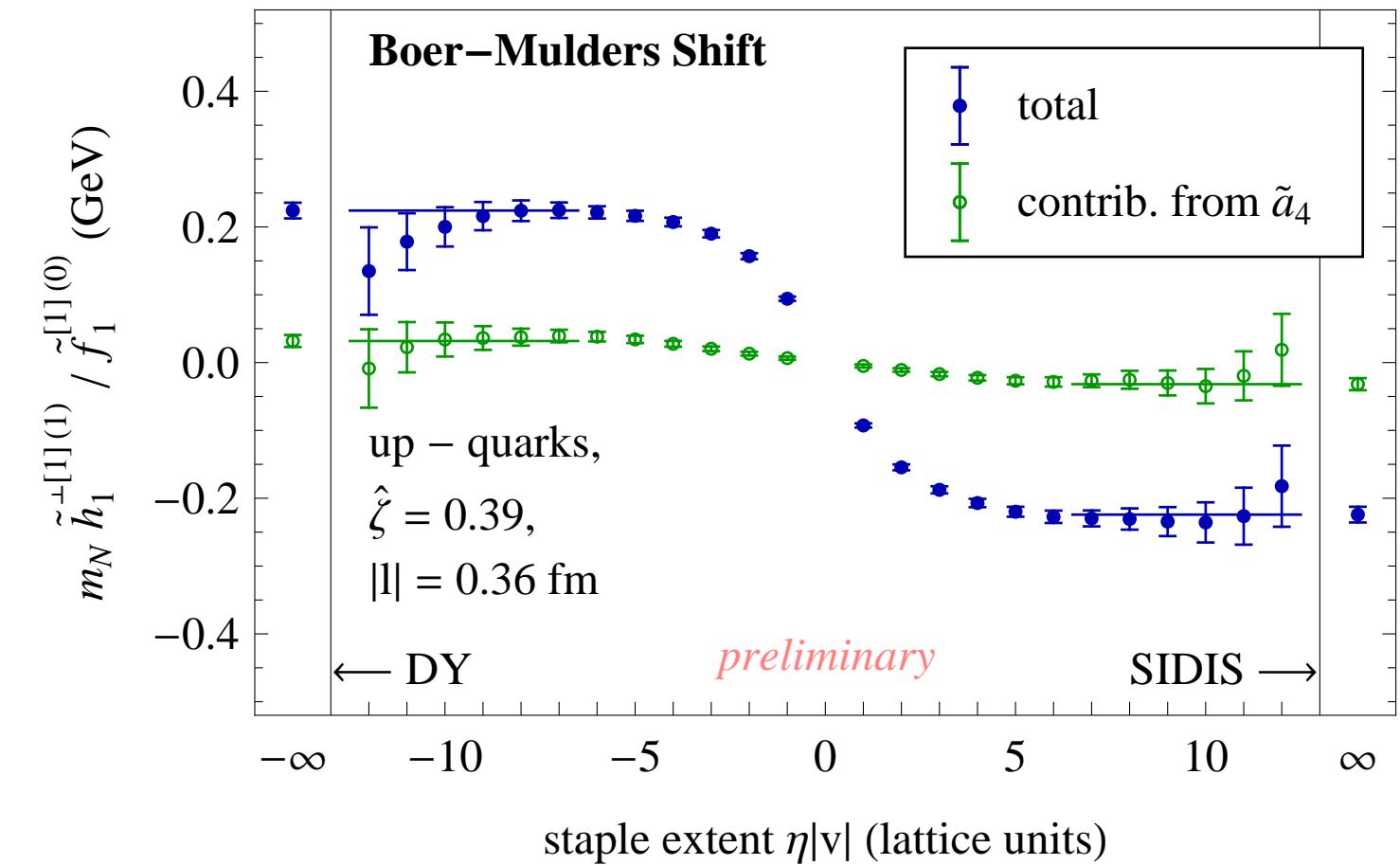
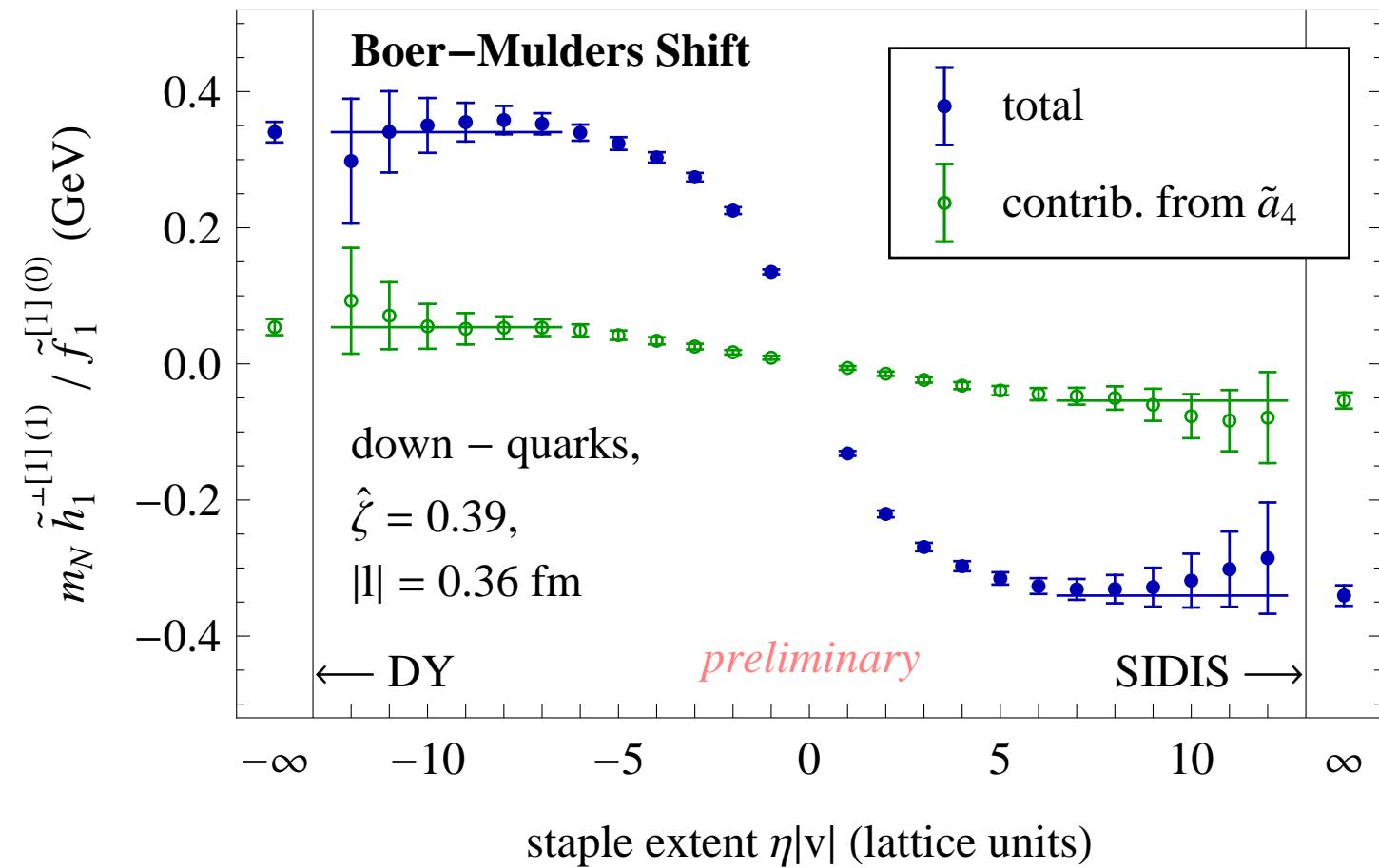
## Results: Boer-Mulders shift

Dependence of SIDIS limit on  $|b_T|$



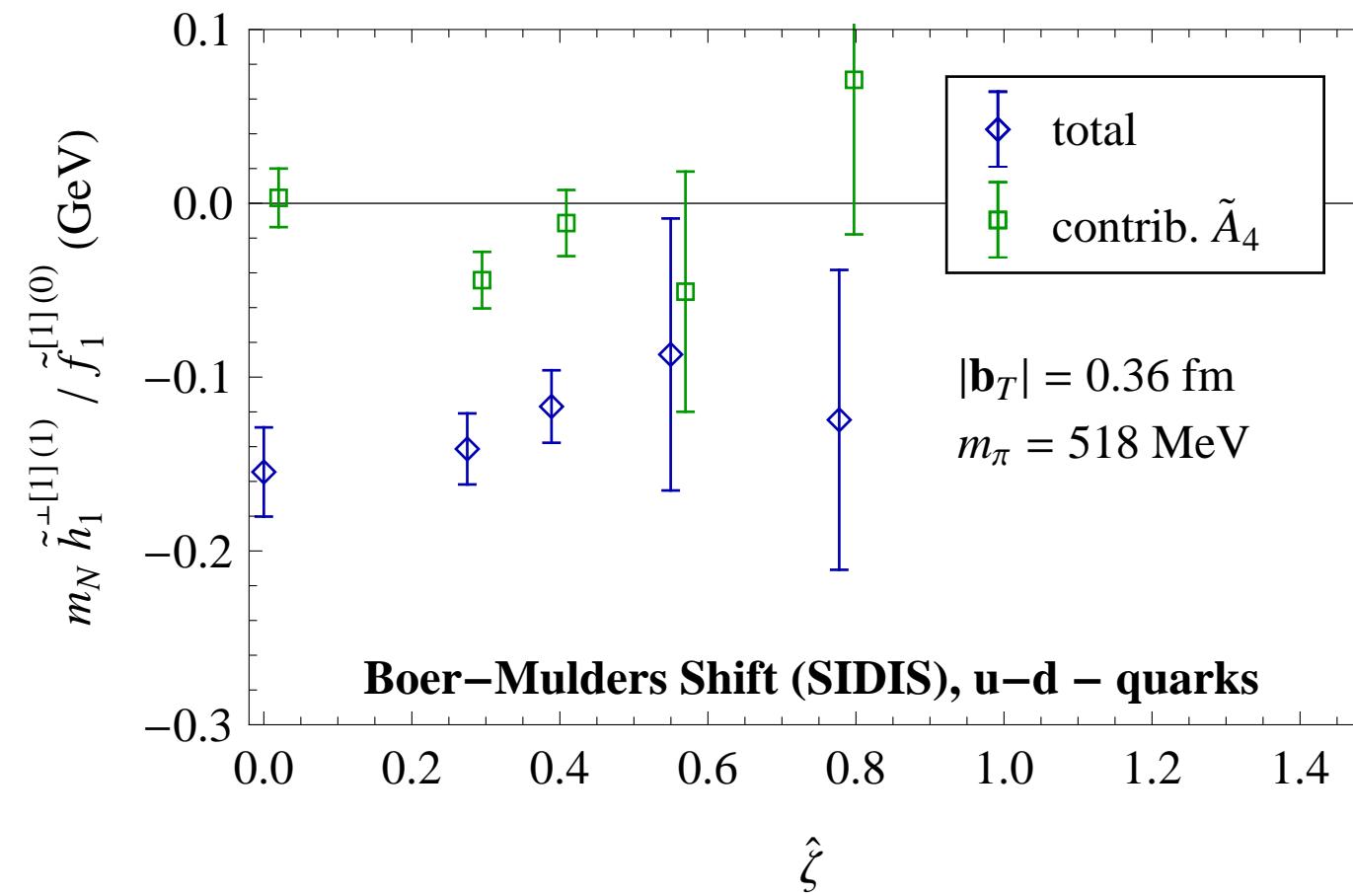
## Results: Boer-Mulders shift

Dependence on staple extent; flavor separated



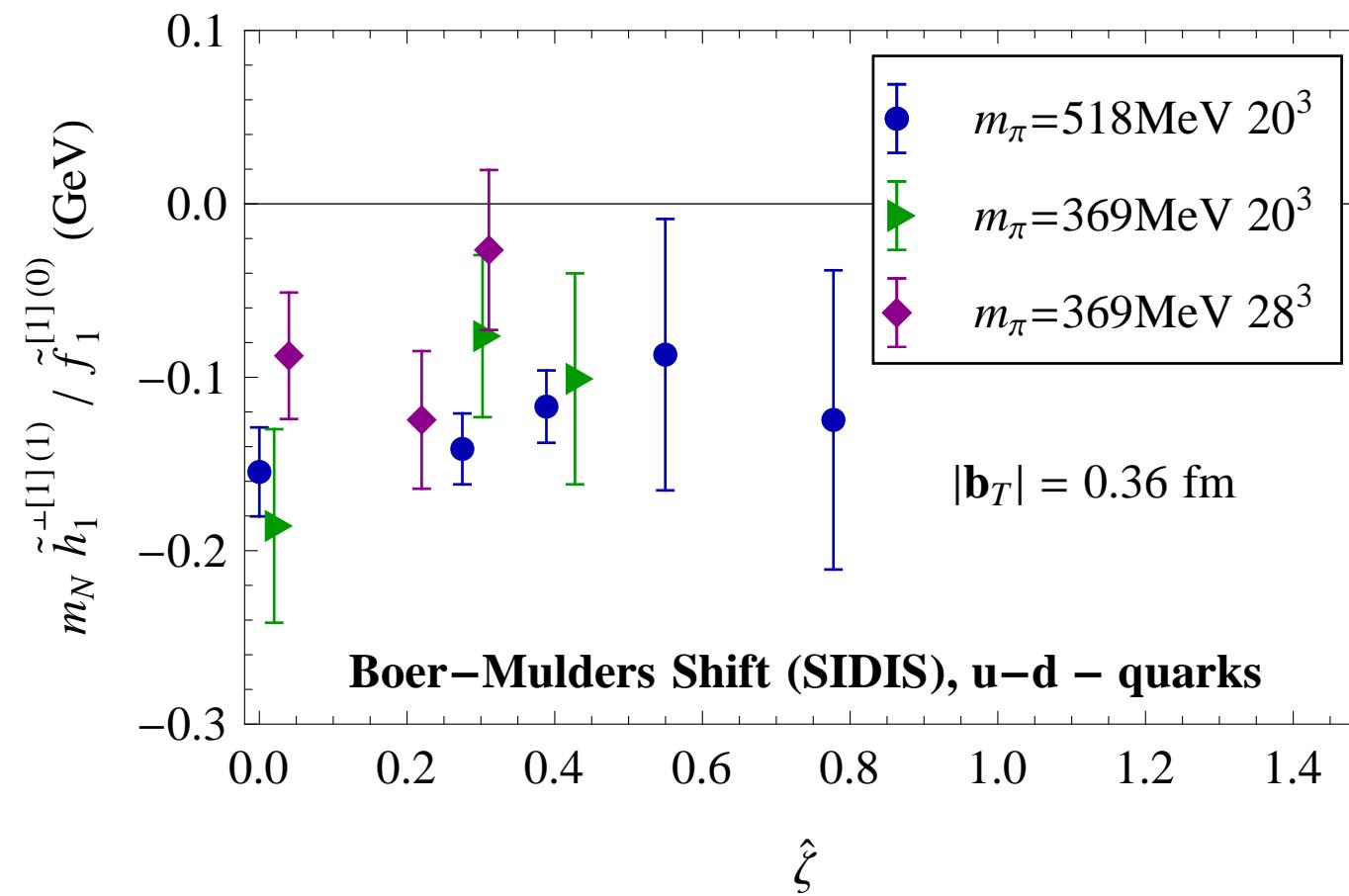
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## Results: Boer-Mulders shift

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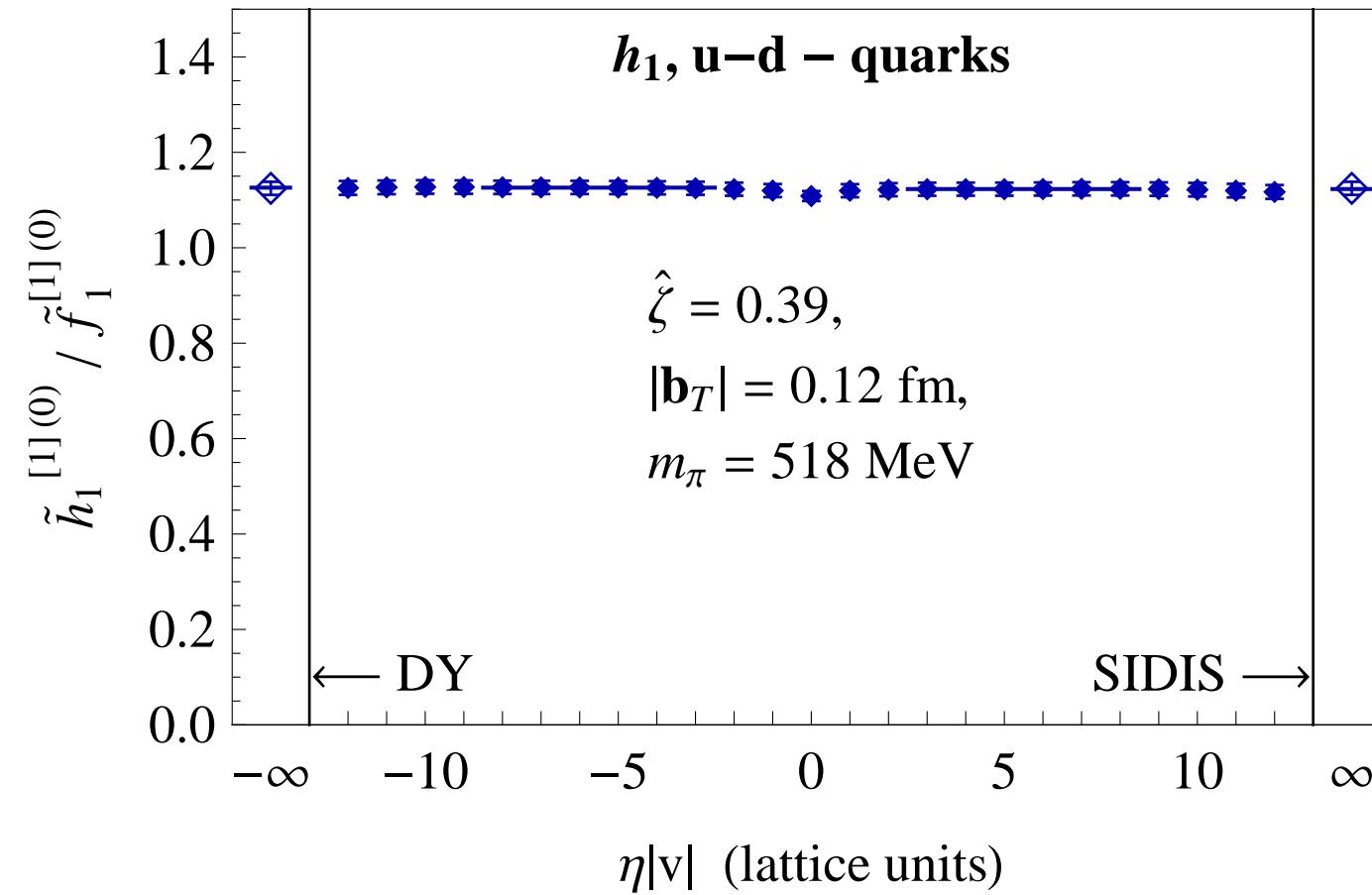


## Conclusions

- Exploratory study of TMDs using staple-shaped gauge link structures
- Accessed T-odd Sivers, Boer-Mulders observables; SIDIS, DY limits distinguished by sign of  $v \cdot P$ . For u-d quark combination, SIDIS Sivers and Boer-Mulders TMDs both sizeable and negative.
- To avoid soft factors, multiplicative renormalization constants, constructed appropriate ratios of Fourier-transformed TMDs (“shifts”).
- $v$  taken off light cone: Dependence on Collins-Soper parameter  $\hat{\zeta}$ . In addition to  $\eta v \rightarrow \infty$ , need to also consider  $\hat{\zeta} \rightarrow \infty$ .
- $\eta v \rightarrow \infty$  seems under good control; plateaux reached at moderate values.
- $\hat{\zeta} \rightarrow \infty$  remains a challenge. No clear trends seen in the data sets available. Need much larger  $\hat{\zeta}$ . Presently investigating pion with this in mind.
- No significant volume dependence, pion mass dependence detected within the limited set of (three) cases considered

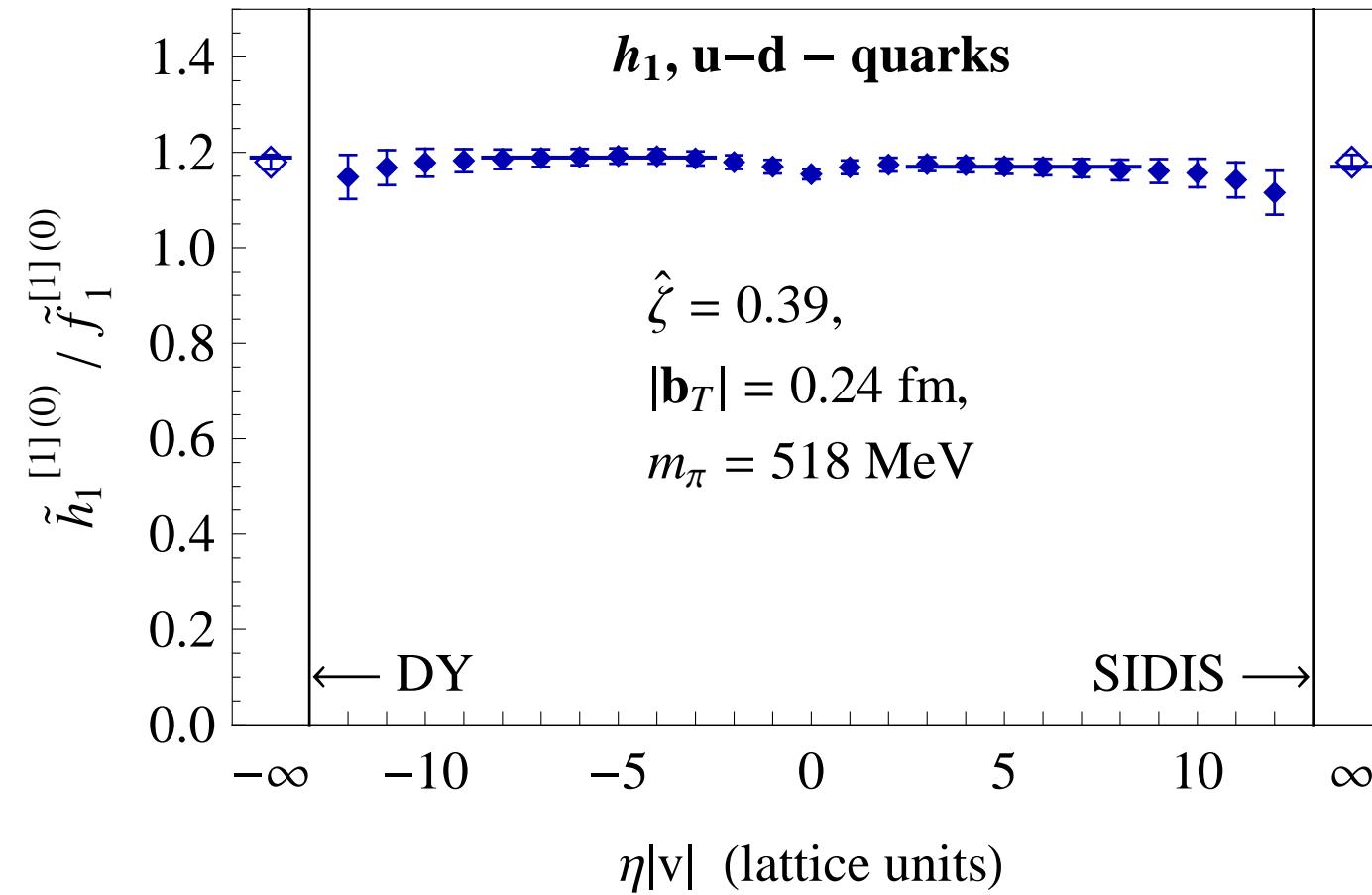
## Results: Transversity

Dependence on staple extent; sequence of panels at different  $|b_T|$



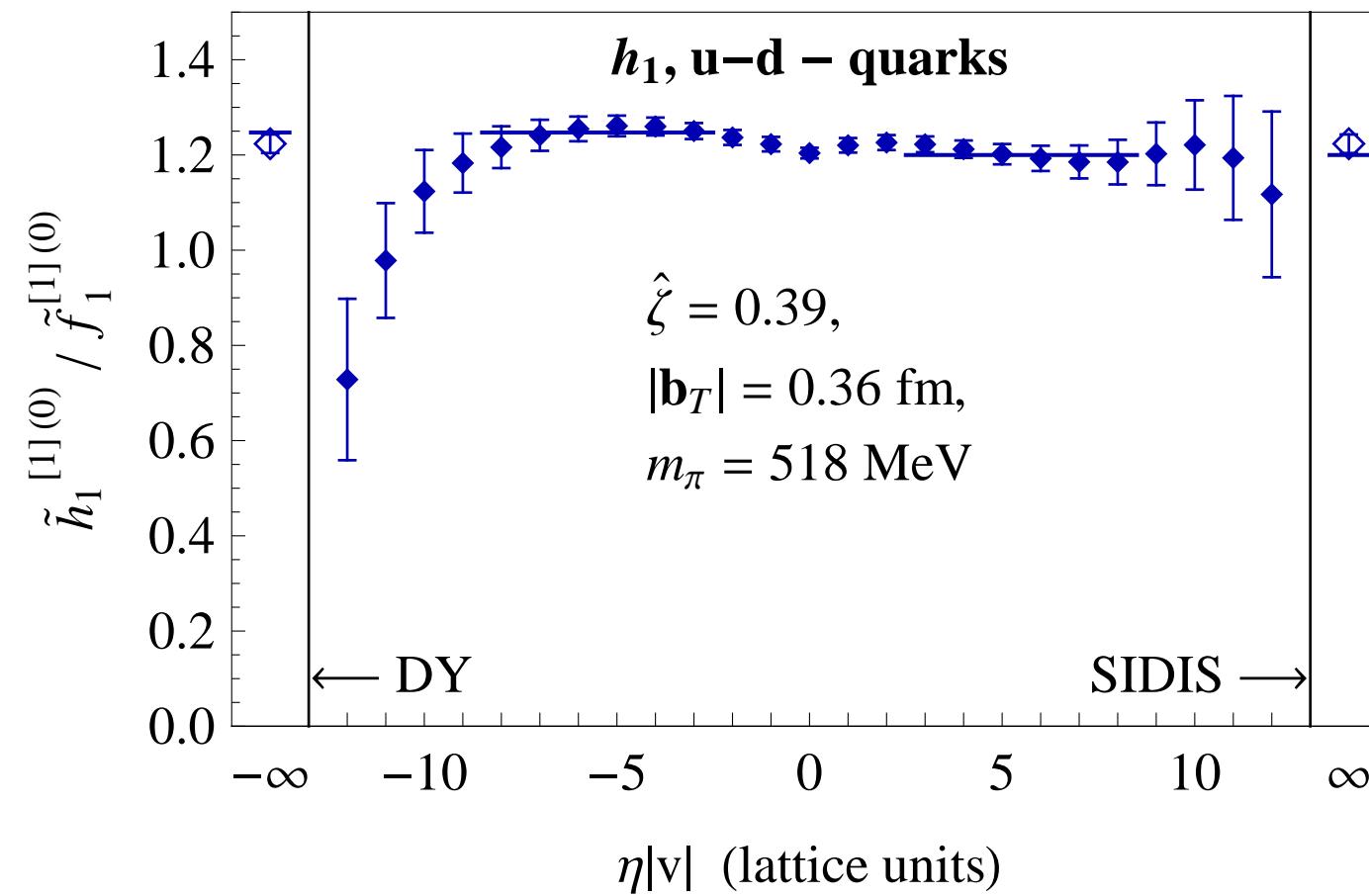
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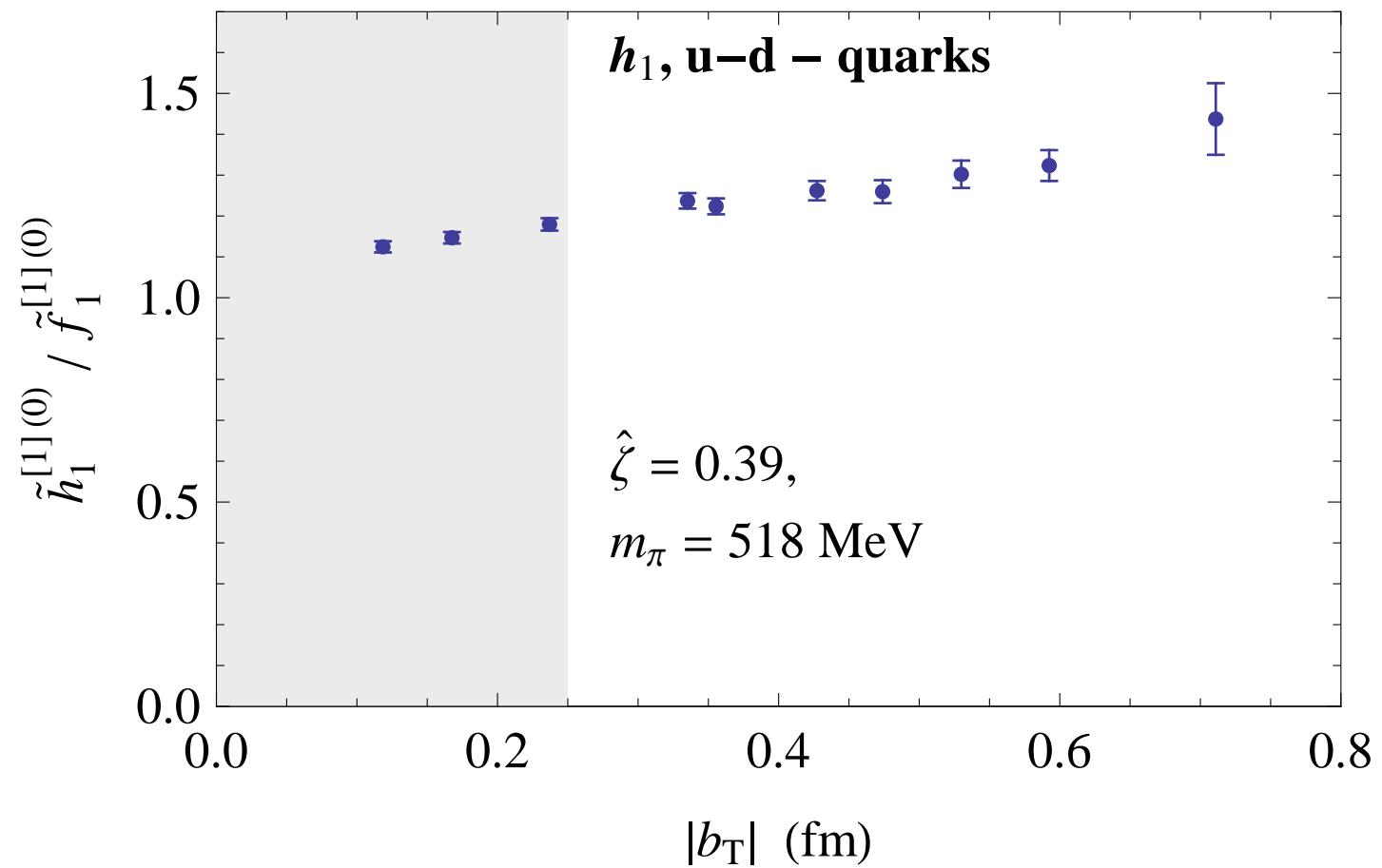
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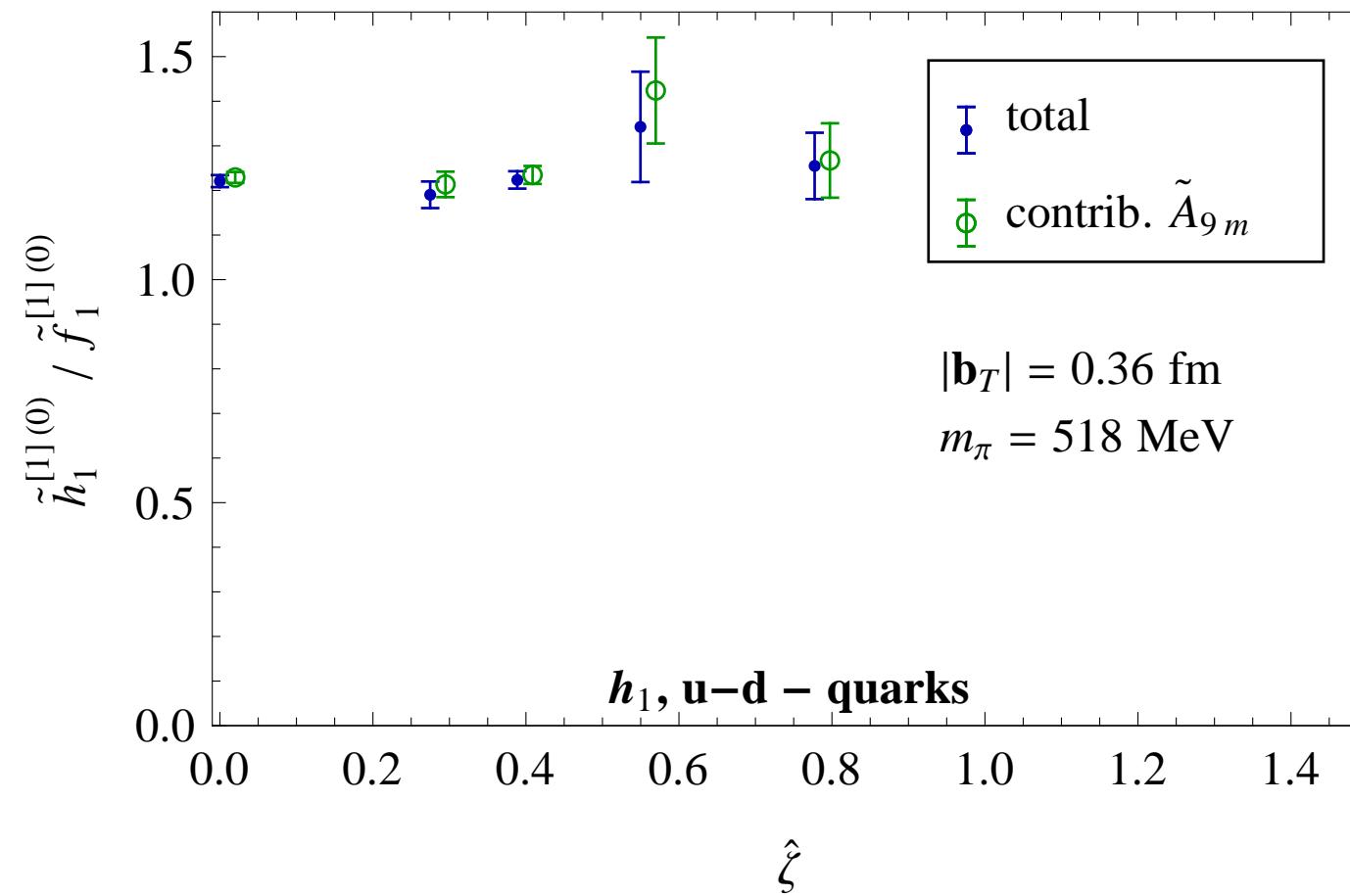
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Dependence of SIDIS/DY limit on  $|b_T|$



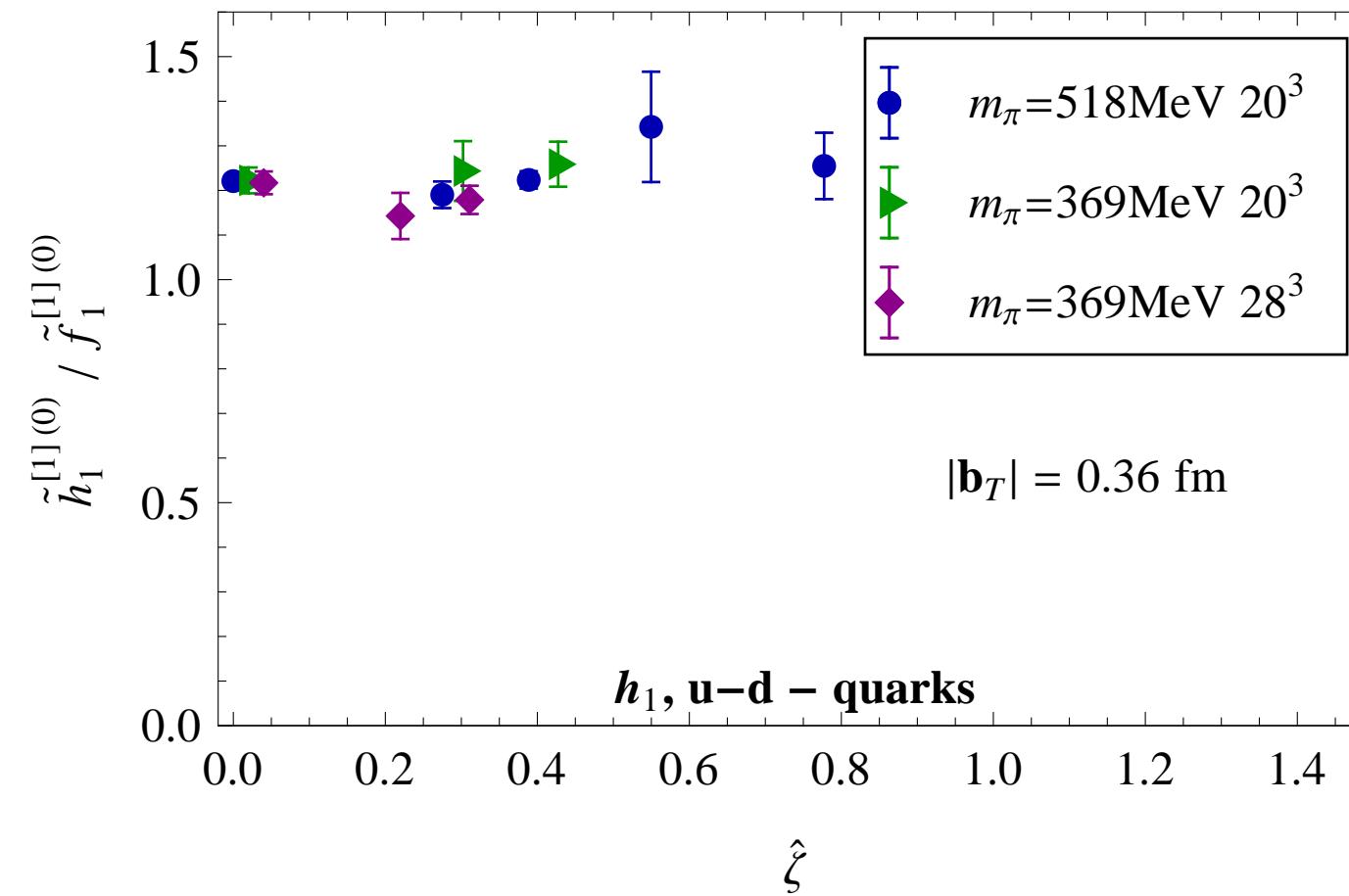
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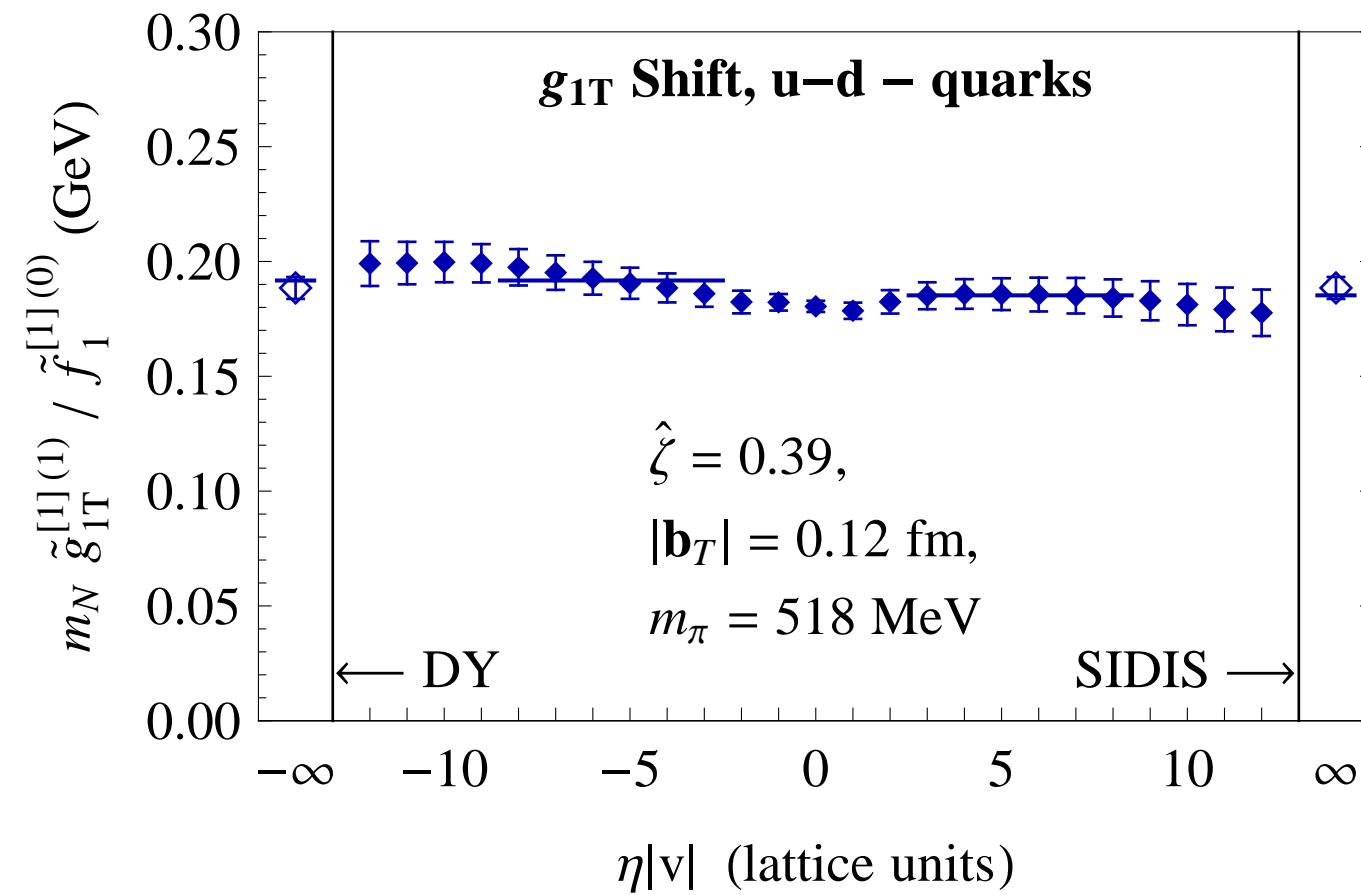
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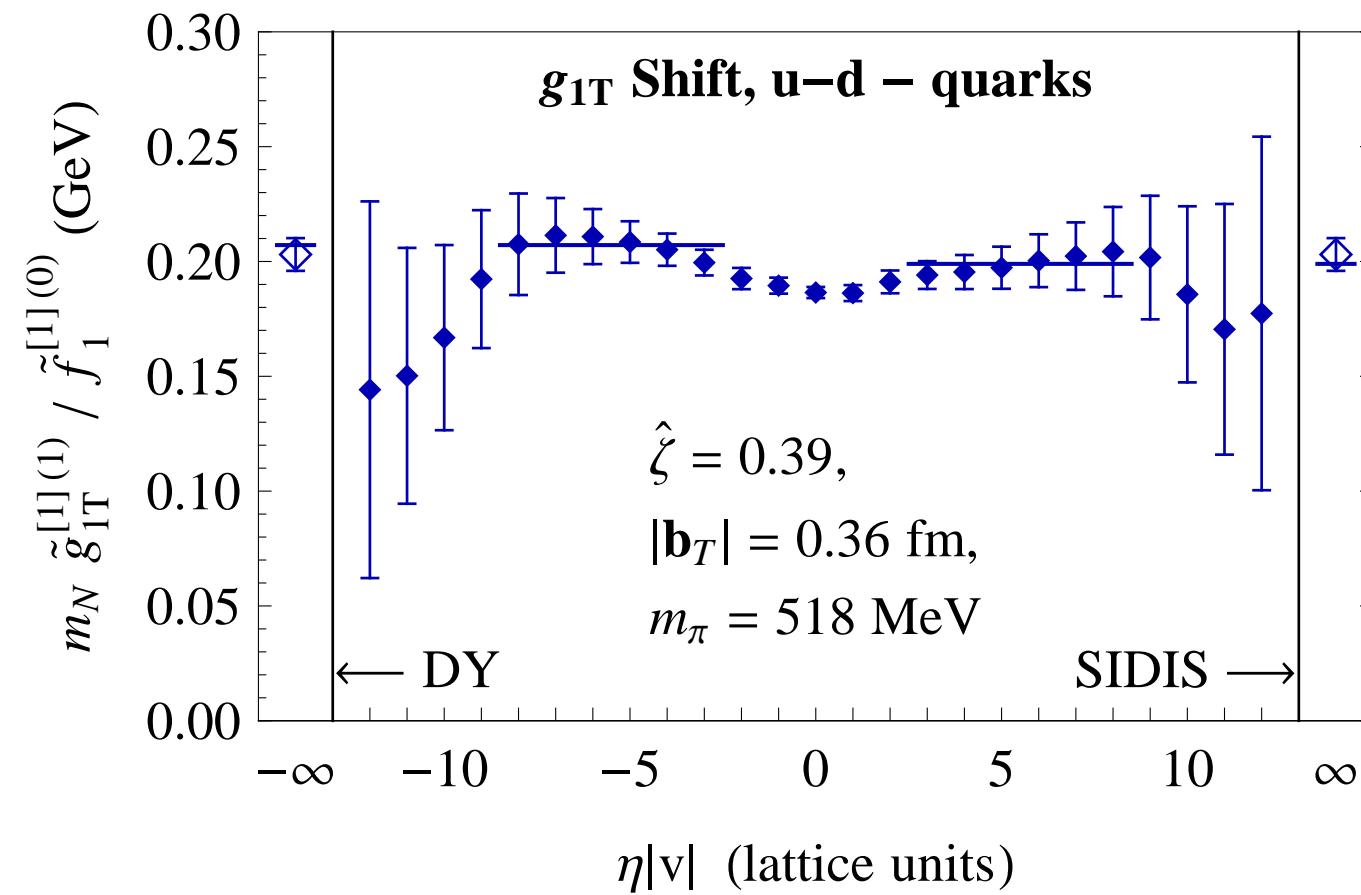
## Results: $g_{1T}$ worm gear shift

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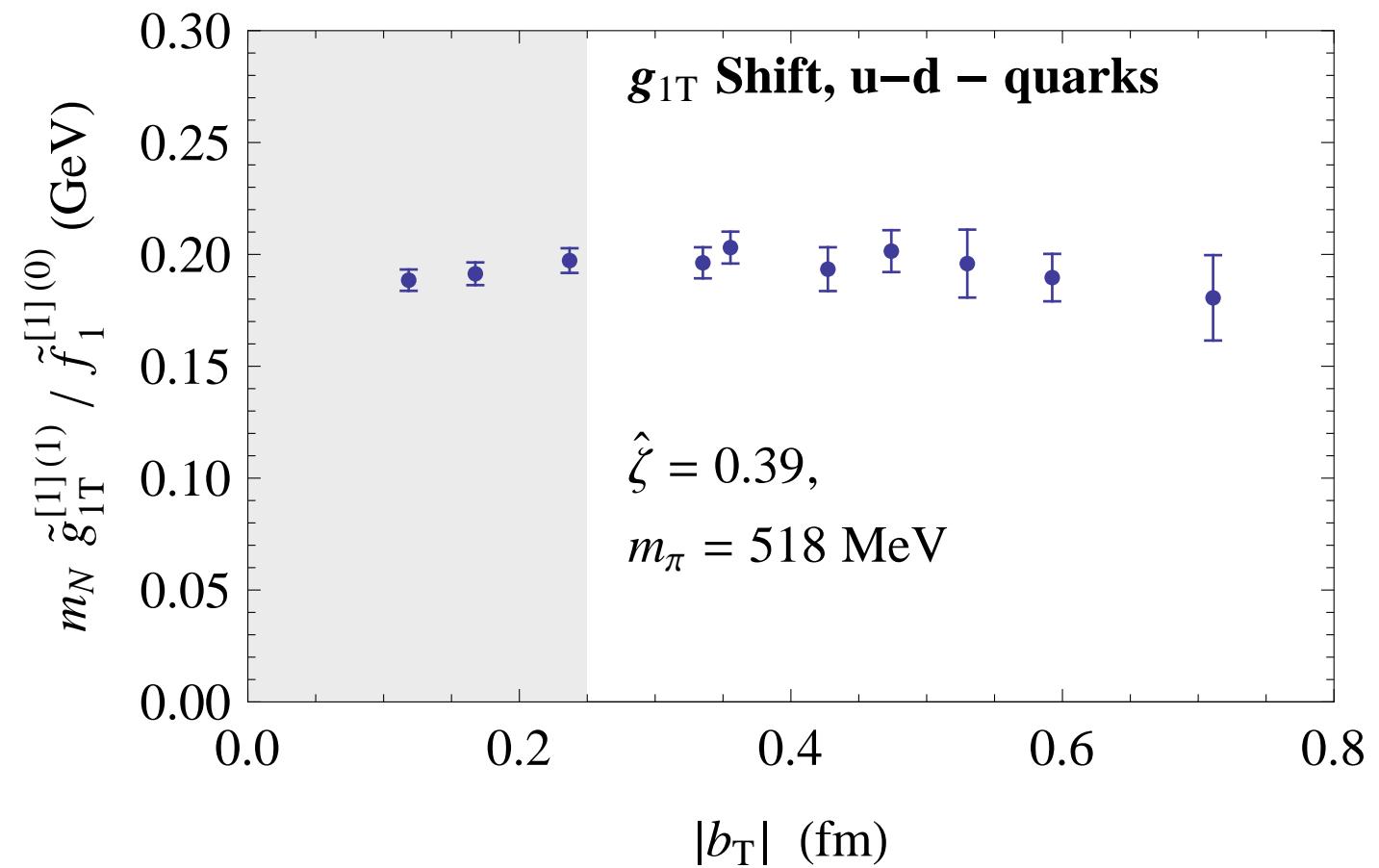
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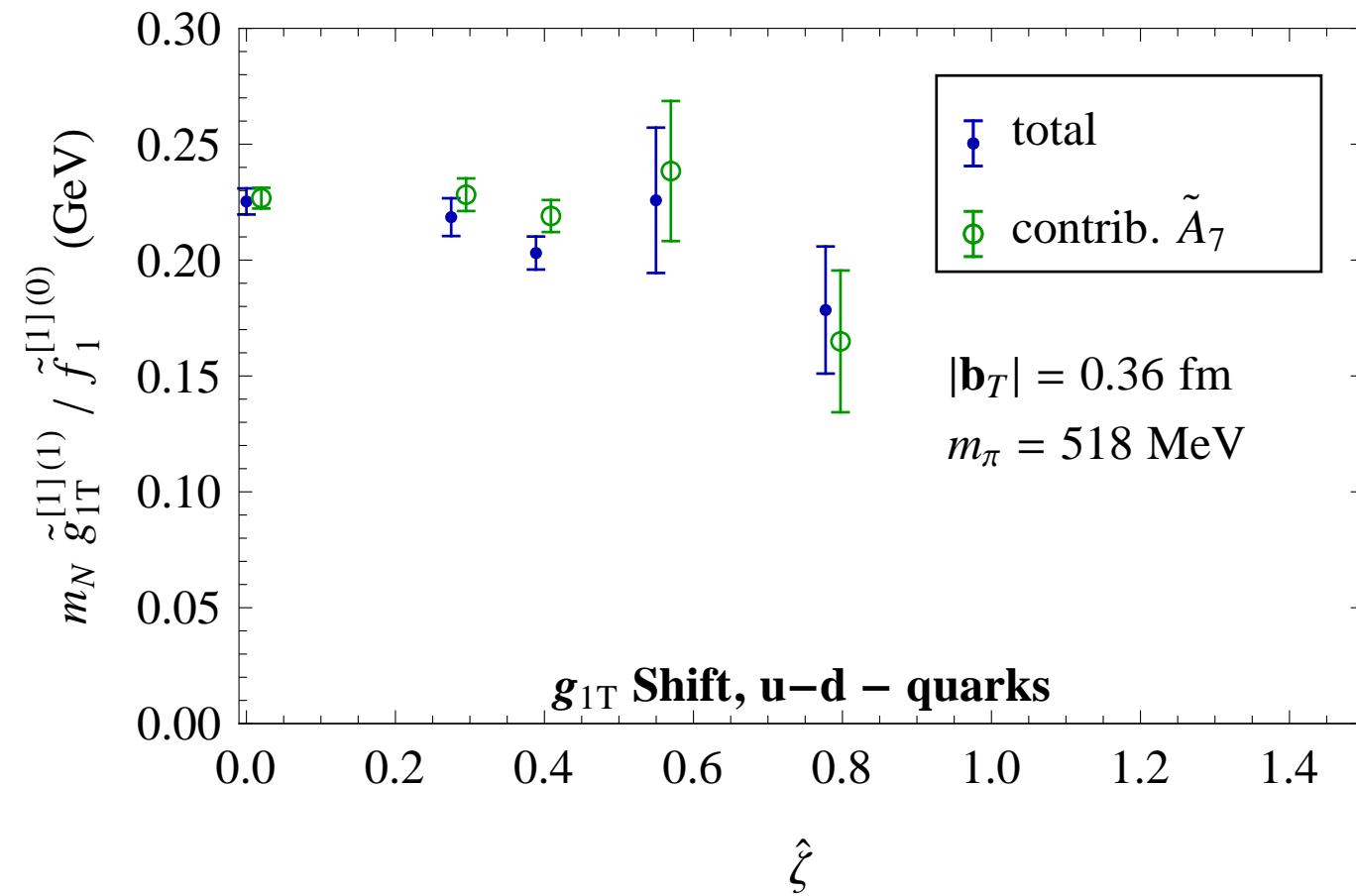
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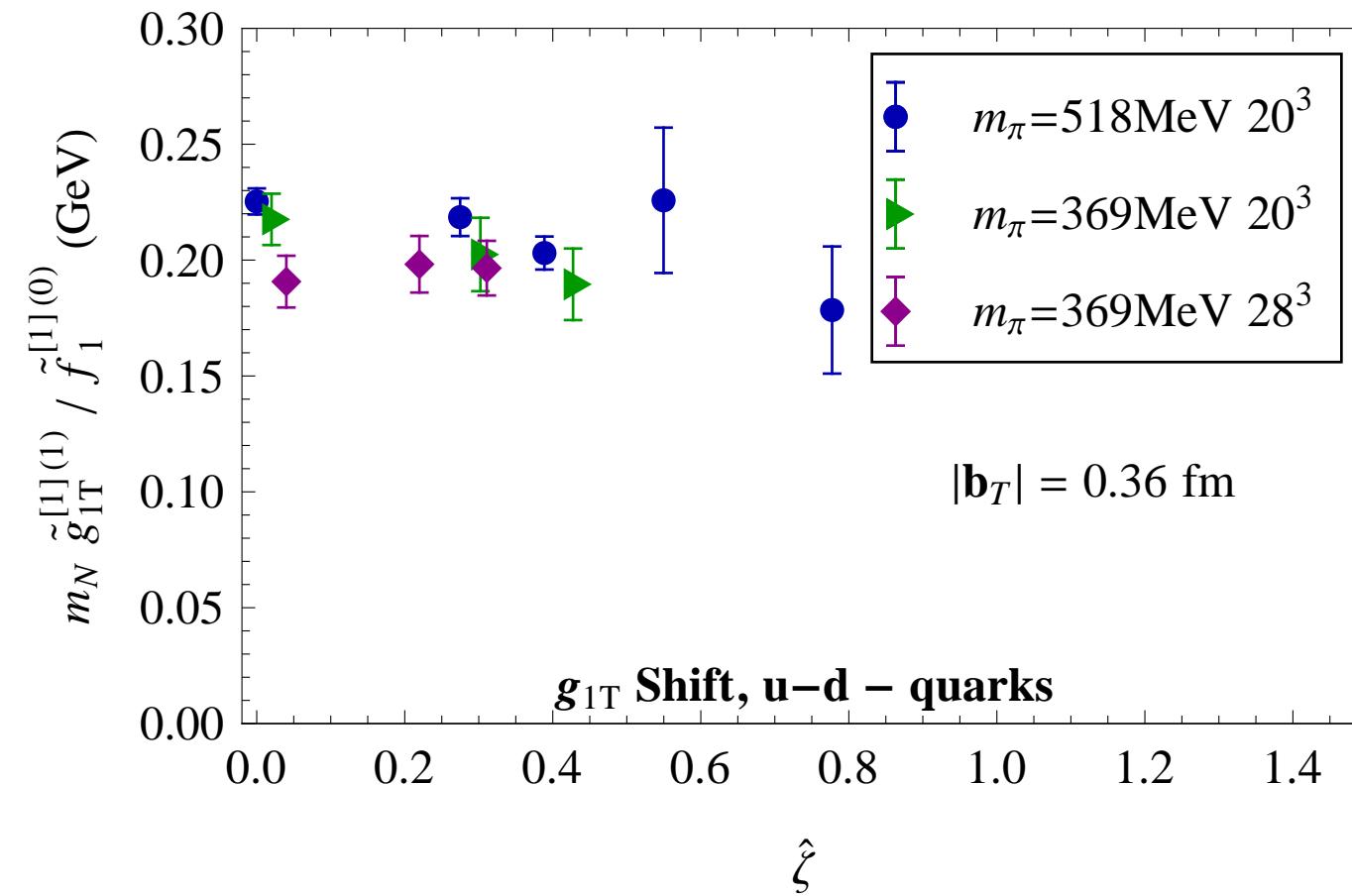
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