Sivers Asymmetry in $e + p^{\uparrow} \rightarrow e + J/\psi + X$

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- Single spin asymmetry
- Model for J/ψ production
- Formalism for calculating the asymmetry
- Models for the Sivers function used
- Results
- Effect of TMD evolution
- Summary and conclusions

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Single Spin Asymmetry in J/ψ production

- Aim is to study the asymmetry in low virtuality electroproduction of charmonium by scattering unpolarized electrons off transversely polarized proton

- Transverse momentum dependent Sivers function describes the probability of finding an unpolarized parton inside a transversely polarized hadron : correlation between transverse momentum of the unpolarized quarks and gluons and the nucleon spin related to orbital angular momentum : Sivers asymmetry gives access to the orbital angular momentum of the partons

- SSAs involving the transverse momentum dependent pdfs and fragmentation functions: very often two or more of these functions contribute to the same physical observable

- In this process that we are considering, at LO, there is contribution only from a single partonic subprocess $\gamma g \longrightarrow c\bar{c}$: can be used as a clean probe of gluon Sivers function

- May throw some light on the charmonium production mechanism as well

Model for charmonium production that we consider

Color evaporation model : first proposed by

Halzen and Matsuda (1978), H. Fritsch (1977)

CEM predicts a cross section for the J/ψ production from the cross section of the $c\bar{c}$ pair

Statistical treatment of color : color can 'evaporate' by multiple soft gluon emission

Cross section for charmonium production is proportional to the rate of production of $c\bar{c}$ pair integrated over the mass range $2m_c$ to $2m_D$

$$\sigma = \frac{1}{9}\int_{2m_c}^{2m_D} dM_{c\bar{c}} \frac{d\sigma_{c\bar{c}}}{dM_{c\bar{c}}}$$

where m_c is the charm quark mass and $2m_D$ is the $D\bar{D}$ threshold; $1/9$ is the statistical probability for the production of a color singlet state; $M^2_{c\bar{c}}$ is the squared invariant mass of $c\bar{c}$

In later versions data are better fitted by the inclusion of a phenomenological factor in differential cross section formula, which depends on a Gaussian distribution of the transverse momentum of the charmonium

Eboli, Gregores, Halzen (2003)

Cross section for charmonium production

Cross section for low-virtuality electroproduction of J/ψ :

$$\sigma^{ep\to e+J/\psi+X} = \int_{4m_c^2}^{4m_D^2} dM_{c\bar{c}}^2 \int dy\, dx\, f_{\gamma/e}(y)\, f_{g/p}(x)\, \frac{d\hat{\sigma}^{\gamma g\to c\bar{c}}}{dM_{c\bar{c}}^2}.$$

The photon flux in the electron is approximated by the distribution (Weizsacker-Williams approximation)

$$f_{\gamma/e}(y,E) = \frac{\alpha}{\pi}\{\frac{1+(1-y)^2}{y}\left(ln\frac{E}{m}-\frac{1}{2}\right)+\frac{y}{2}\left[ln\left(\frac{2}{y}-2\right)+1\right] + \frac{(2-y)^2}{2y}ln\left(\frac{2-2y}{2-y}\right)\}.$$

Brodsky, Kinoshita,Terazawa (1971); Kniehl (1991)

where y is the energy fraction of electron carried by the photon, E is the energy of the electron and m is the mass

Single Spin Asymmetry

Generalization of CEM expression by taking into account the transverse momentum dependence

$$\frac{d\sigma^{e+p^\uparrow\to e+J/\psi+X}}{dM^2} = \int dx_\gamma\, dx_g\, [d^2\boldsymbol{k}_{\perp\gamma} d^2\boldsymbol{k}_{\perp g}]\, f_{g/p^\uparrow}(x_g, \boldsymbol{k}_{\perp g}) f_{\gamma/e}(x_\gamma, \boldsymbol{k}_{\perp\gamma}) \frac{d\hat{\sigma}^{\gamma g\to c\bar{c}}}{dM^2}$$

where $M^2 \equiv M^2_{c\bar{c}}$

Single spin asymmetry for a transversely polarized target is defined as

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

The numerator is of the form

$$d\sigma^\uparrow - d\sigma^\downarrow = \int dx_\gamma\, dx_g\, d^2\boldsymbol{k}_{\perp\gamma}\, d^2\boldsymbol{k}_{\perp g}\, \Delta^N f_{g/p^\uparrow}(x_g, \boldsymbol{k}_{\perp g})\, f_{\gamma/e}(x_\gamma, \boldsymbol{k}_{\perp\gamma})\, d\hat{\sigma}^{\gamma g\to c\bar{c}}$$

$d\hat{\sigma}$ is the elementary cross section for the process $\gamma g \to c\bar{c}$ given by

$$d\hat{\sigma} = \frac{1}{2\hat{s}}\,\frac{d^3p_c}{2E_c}\,\frac{d^3p_{\bar{c}}}{2E_{\bar{c}}}\,\frac{1}{(2\pi)^2}\,\delta^4(p_\gamma + p_g - p_c - p_{\bar{c}})\,\overline{\mid M_{\gamma g\to c\bar{c}}\mid^2}\,.$$

Changing the variable to $q = p_c + p_{\bar{c}}$ we get

$$\frac{d^3p_{\bar{c}}}{2E_{\bar{c}}} = d^4q\,\delta((q-p_c)^2 - m_c^2).$$

Using the expression for total partonic cross section

$$\hat{\sigma_0}^{\gamma g\to c\bar{c}}(M^2) = \frac{1}{2\hat{s}}\int\frac{d^3p_c}{2E_c}\,\frac{1}{(2\pi)^2}\,\delta((q-p_c)^2 - m_c^2)\,\overline{\mid M_{\gamma g\to c\bar{c}}\mid^2}$$

We also change the variables from q_0 and q_L to M^2 and rapidity y so that

$$dM^2 dy = 2dq_0 dq_L$$

Single Spin Asymmetry

We finally obtain

$$\frac{d^4\sigma^\uparrow}{dy\,dM^2d^2\boldsymbol{q}_T} - \frac{d^4\sigma^\downarrow}{dy\,dM^2d^2\boldsymbol{q}_T} = \frac{1}{2}\int [dx_\gamma d^2\boldsymbol{k}_{\perp\gamma} dx_g d^2\boldsymbol{k}_{\perp g}]\Delta^N f_{g/p^\uparrow}(x_g, \boldsymbol{k}_{\perp g})$$
$$\times f_{\gamma/e}(x_\gamma, \boldsymbol{k}_{\perp\gamma})\delta^4(p_g + p_\gamma - q)\,\hat{\sigma}_0^{\gamma g\to c\bar{c}}(M^2)$$

$\Delta^N f_{g/p^\uparrow}(x_g, \boldsymbol{k}_{\perp g})$ is related to the gluon Sivers function $\Delta^N f_{g/p^\uparrow}(x, k_{\perp g})$ by

$$\Delta^N f_{g/p^\uparrow}(x_g, \boldsymbol{k}_{\perp g}) = \Delta^N f_{g/p^\uparrow}(x_g, k_\perp)\,\hat{\boldsymbol{S}}\cdot(\hat{\boldsymbol{P}}\times\hat{\boldsymbol{k}}_{\perp g})$$

One can perform the x_γ and x_g integrations to obtain

$$\frac{d^4\sigma^\uparrow}{dy\,dM^2d^2\boldsymbol{q}_T} - \frac{d^4\sigma^\downarrow}{dy\,dM^2d^2\boldsymbol{q}_T} = \frac{1}{s}\int [d^2\boldsymbol{k}_{\perp\gamma} d^2\boldsymbol{k}_{\perp g}]\Delta^N f_{g/p^\uparrow}(x_g, \boldsymbol{k}_{\perp g}) f_{\gamma/e}(x_\gamma, \boldsymbol{k}_{\perp\gamma})$$
$$\times\,\delta^2(\boldsymbol{k}_{\perp\gamma} + \boldsymbol{k}_{\perp g} - \boldsymbol{q}_T)\hat{\sigma}_0^{\gamma g\to c\bar{c}}(M^2)$$

$$x_{g,\gamma} = \frac{M}{\sqrt{s}}\,e^{\pm y}.$$

And similarly for the denominator of the asymmetry

Sivers Asymmetry

Calculate the weighted asymmetry to extract the Sivers function

Vogelsang and Yuan (2005)

$$A_N^{\sin(\phi_{q_T}-\phi_S)} = \frac{\int d\phi_{q_T}[d\sigma^\uparrow - d\sigma^\downarrow]\sin(\phi_{q_T}-\phi_S)}{\int d\phi_{q_T}[d\sigma^\uparrow + d\sigma^\downarrow]}$$

ϕ_{q_T} and ϕ_S are the azimuthal angles of the J/ψ and proton spin respectively

Asymmetry in the rapidity distribution will involve

$$\begin{aligned} d\sigma^\uparrow - d\sigma^\downarrow = \int d\phi_{q_T} \int q_T\, dq_T \int_{4m_c^2}^{4m_D^2} [dM^2] \int [d^2\boldsymbol{k}_{\perp g}] \Delta^N f_{g/p^\uparrow}(x_g, \boldsymbol{k}_{\perp g}) \\ \times f_{\gamma/e}(x_\gamma, \boldsymbol{q}_T - \boldsymbol{k}_{\perp g})\, \hat{\sigma}_0(M^2)\, \sin(\phi_{q_T}-\phi_S) \end{aligned}$$

SSA depends on Weizsacker-Williams function, gluon distribution function and gluon Sivers function

Models for WW Function and Sivers function

We choose a kinematical configuration in which proton with momentum $\mathbf{P}$ is moving along z axis and is transversely polarized in y direction

$$\hat{\boldsymbol{S}} \cdot (\hat{\boldsymbol{P}} \times \hat{\boldsymbol{k}}_{\perp g}) = \hat{k}_{\perp g z} = \cos \phi_{k_\perp}$$

where, $\boldsymbol{k}_{\perp g} = k_\perp (\cos \phi_{k_\perp}, \sin \phi_{k_\perp}, 0)$

For $k_{\perp g}$ dependence of the unpolarized pdf's, we use a simple factorized and Gaussian form

$$f_{g/p}(x_g, k_\perp) = f_{g/p}(x_g) \frac{1}{\pi \langle k_{\perp g}^2 \rangle} e^{-k_{\perp g}^2 / \langle k_{\perp g}^2 \rangle}.$$

The WW distribution has $k_\perp$ dependence. We have used two choices for WW function for the photon

Models for WW Function and Sivers function

We have used two choices for WW function for the photon

1) A simple Gaussian form as above :

$$f_{\gamma/e}(x_\gamma, k_{\perp\gamma}) = f_{\gamma/e}(x_\gamma)\frac{1}{\pi\langle k^2_{\perp\gamma}\rangle} e^{-k^2_{\perp\gamma}/\langle k^2_{\perp\gamma}\rangle}$$

2) Second form :

$$f_{\gamma/e}(x_\gamma, k_{\perp\gamma}) = f_{\gamma/e}(x_\gamma)\ \frac{1}{2\pi}\ \frac{N}{k^2_{\perp\gamma} + {k_0}^2}$$

N is a normalization constant, which gets canceled in the asymmetry

For the Sivers function we use two models

M. Anselmino, M. Boglione, U. D'Alesio, S. Melis, F. Murgia and A. Prokudin (2009)
M. Anselmino, M. Boglione, U. D'Alesio, E. Leader and F. Murgia (2004)

Model I has been used in analysis of SSA in SIDIS and DY process Model II has been used for quark Sivers function to estimate SSA in D meson production at RHIC

Models for Sivers Function

Model I :

$$\Delta^N f_{g/p^\uparrow}(x_g, \boldsymbol{k}_{\perp g}) = \Delta^N f_{g/p^\uparrow}(x_g) \frac{1}{\pi \langle k_{\perp g}^2 \rangle} \, h(k_{\perp g}) \, e^{-k_{\perp g}^2/\langle k_{\perp g}^2 \rangle} \cos(\phi_{k_\perp})$$

where the gluon Sivers function, $\Delta^N f_{g/p^\uparrow}(x_g)$ is defined as

$$\Delta^N f_{g/p^\uparrow}(x_g) = 2 \, \mathcal{N}_g(x_g) \, f_{g/p}(x_g)$$

$\mathcal{N}_g(x_g)$ is an x-dependent normalization for gluon to be chosen so that the gluon Sivers function obeys the positivity bound

$$\frac{|\Delta^N f_{g/p^\uparrow}(x_g, \boldsymbol{k}_{\perp g})|}{2 \, \hat{f}_{g/p}(x_g, k_{\perp g})} \leq 1, \qquad \forall \, x_g, \, k_{\perp g}$$

Models for Sivers Function

$$h(k_{\perp g}) = \sqrt{2e}\,\frac{k_{\perp g}}{M_1}\,e^{-k_{\perp g}^2/M_1^2}$$

M_1 is parameter obtained by fitting the recent experimental data corresponding to pion and kaon production at HERMES and COMPASS

Parametrization for quark Sivers function

$$\mathcal{N}_f(x) = N_f x^{a_f}(1-x)^{b_f}\,\frac{(a_f+b_f)^{(a_f+b_f)}}{{a_f}^{a_f}{b_f}^{b_f}}\ .$$

where a_f, b_f, N_f for u and d quarks are free parameters obtained by fitting the data. However, there is no information available on $\mathcal{N}_g(x)$
We use (Boer and Vogelsang (2004))

(a) $\mathcal{N}_g(x) = (\mathcal{N}_u(x) + \mathcal{N}_d(x))/2$,

(b) $\mathcal{N}_g(x) = \mathcal{N}_d(x)$.

Models for Sivers Function

Model II :

$$\Delta^N f_{g/p^\uparrow}(x_g, \boldsymbol{k}_{\perp g}) = \Delta^N f_{g/p^\uparrow}(x_g) \frac{1}{\pi \langle k_{\perp g}^2 \rangle} e^{-k_{\perp g}^2/\langle k_{\perp g}^2 \rangle} \frac{2k_{\perp g} M_0}{k_{\perp g}^2 + M_0^2} \cos(\phi_{k_\perp}),$$

and $M_0 = \sqrt{\langle k_{\perp g}^2 \rangle}$ where the gluon Sivers function is given as in Model I

Best fit parameters of Sivers functions (Anselmino et al (2011)

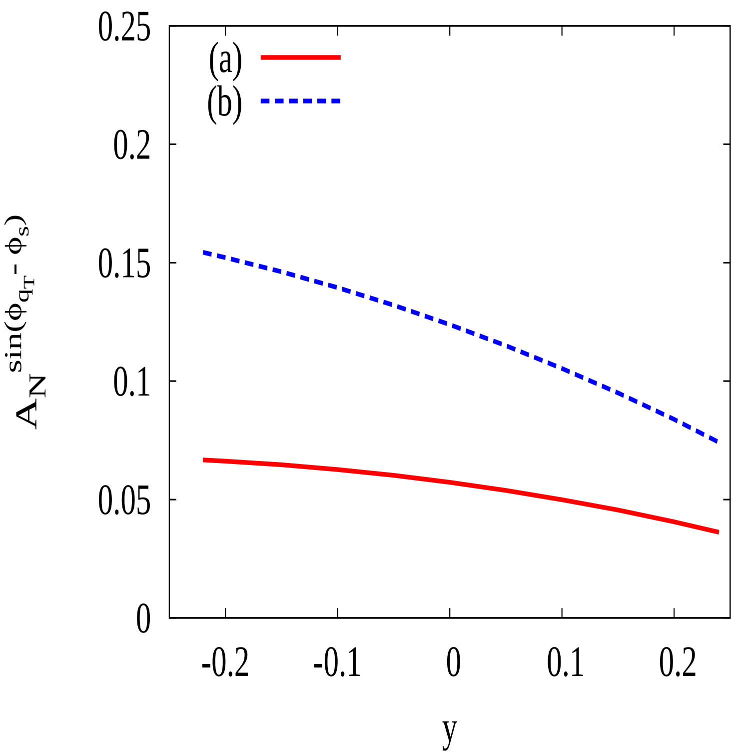
$$N_u = 0.40,\ a_u = 0.35,\ b_u = 2.6\ ,$$
$$N_d = -0.97,\ a_d = 0.44,\ b_d = 0.90\ ,$$
$$M_1^2 = 0.19\ GeV^2.$$

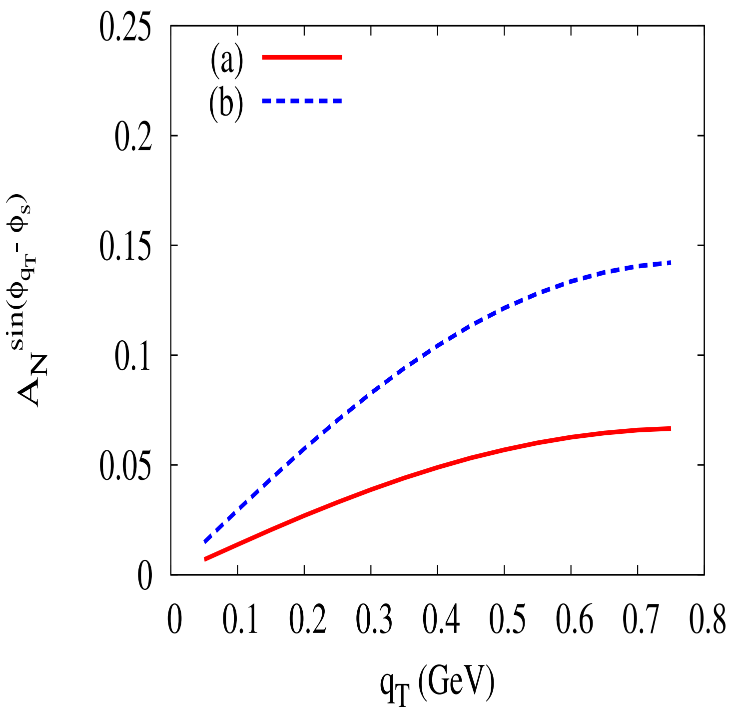
Other parameters we use are

$$\langle k_{\perp g}^2 \rangle = \langle k_{\perp \gamma}^2 \rangle = 0.25\ GeV^2.$$

Will comment on this choice later

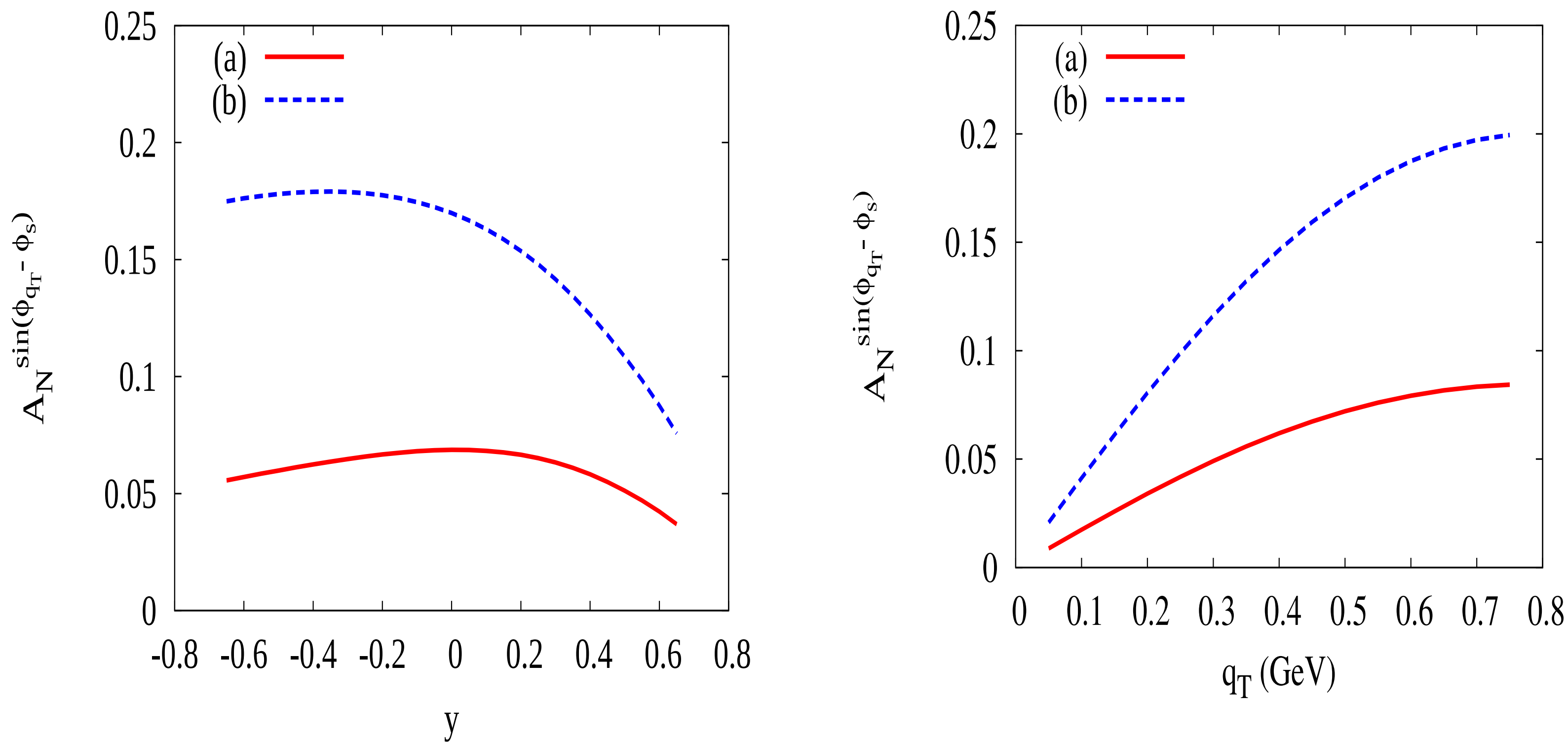
SSA for J/ψ production at JLab





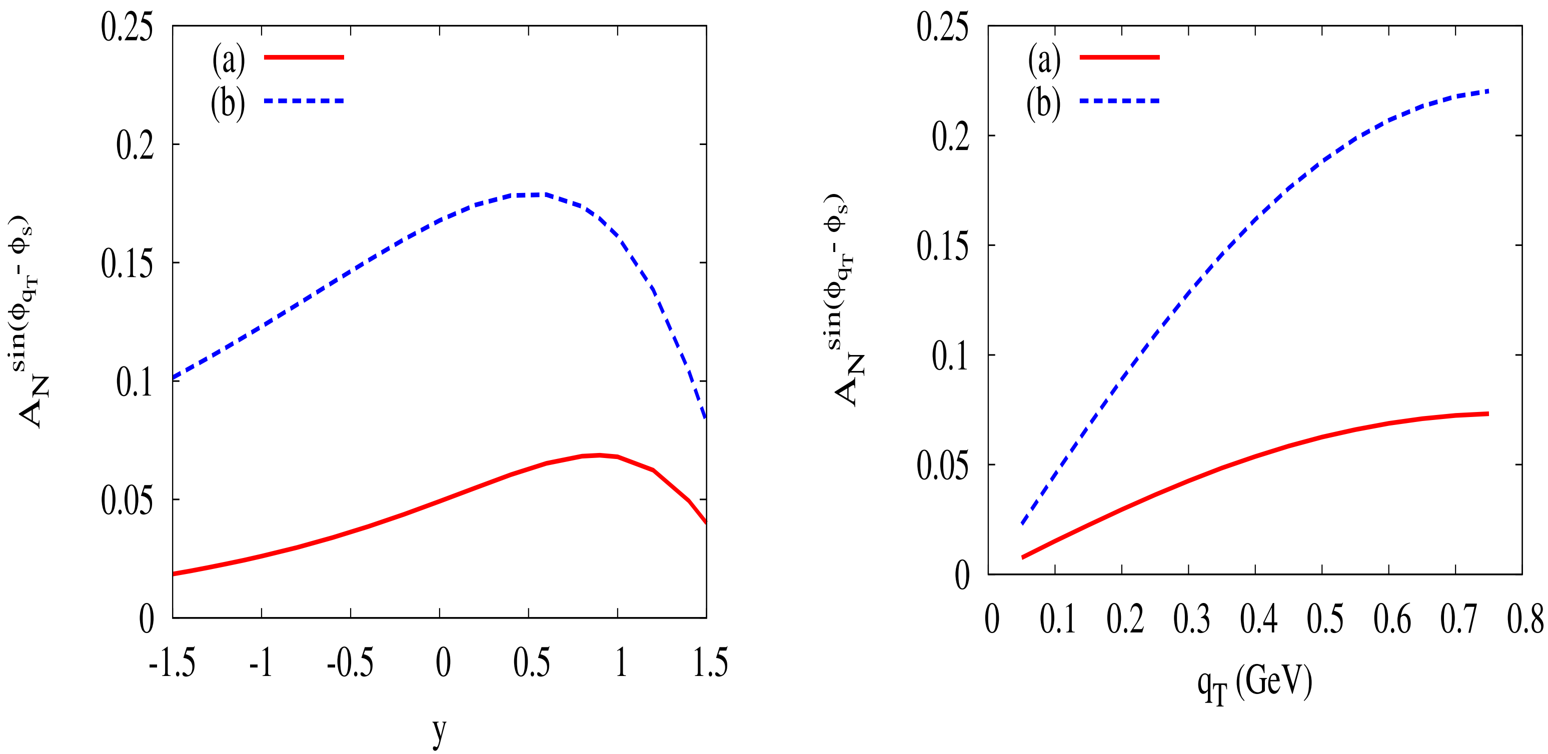
Single spin asymmetry at JLab as a function of y (left panel) and q_T (right panel). The plots are for model I with two parametrizations (a) (solid red line) and (b) (dashed blue line). The integration ranges are $(0 \leq q_T \leq 1)$ GeV and $(0 \leq y \leq 0.25)$. The results are given at $\sqrt{s} = 4.7$ GeV

SSA for J/ψ production at HERMES



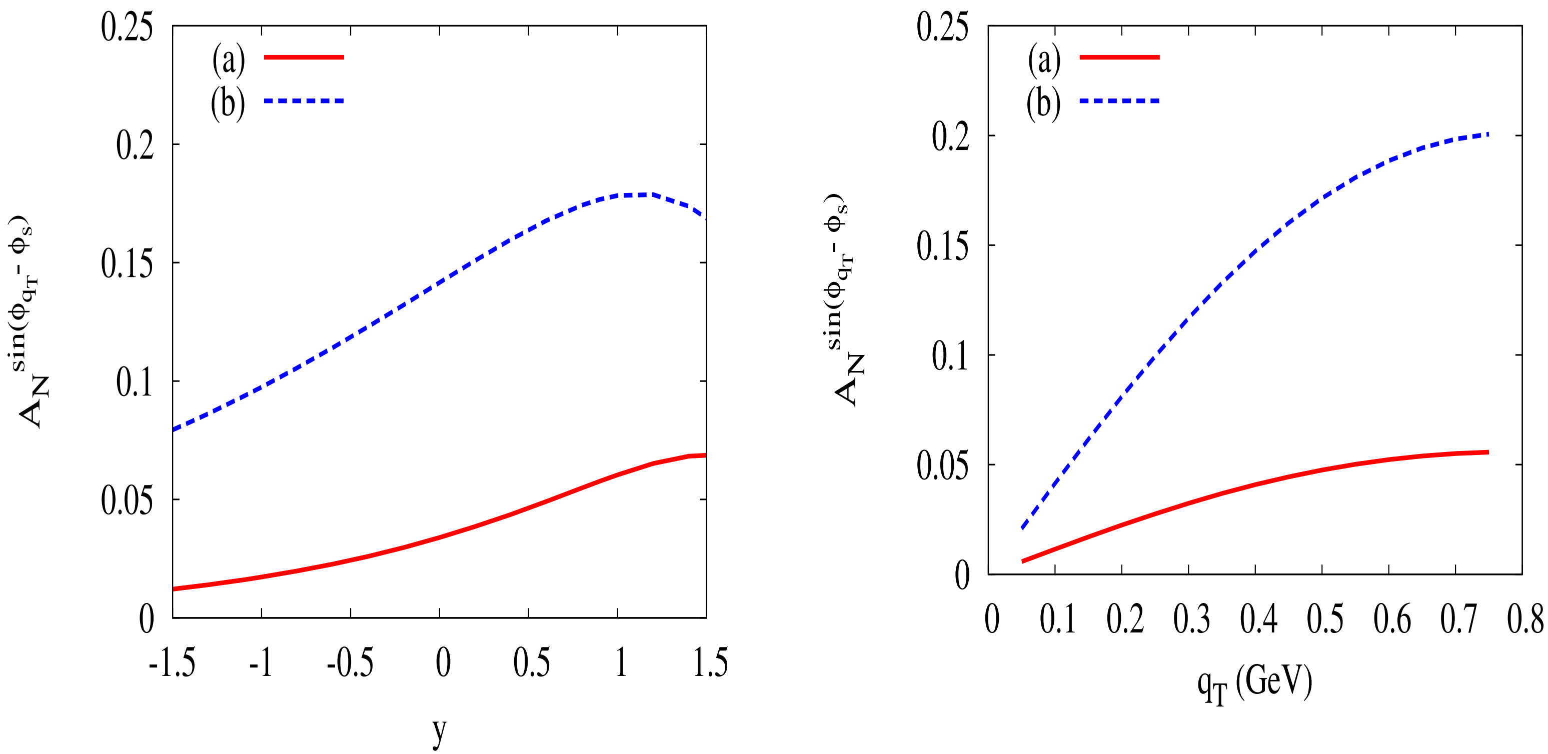
Single spin asymmetry at HERMES as a function of y (left panel) and q_T (right panel). The plots are for model I with two parametrizations (a) (solid red line) and (b) (dashed blue line). The integration ranges are $(0 \leq q_T \leq 1)$ GeV and $(0 \leq y \leq 0.6)$. The results are given at $\sqrt{s} = 7.2$ GeV

SSA for J/ψ production at COMPASS



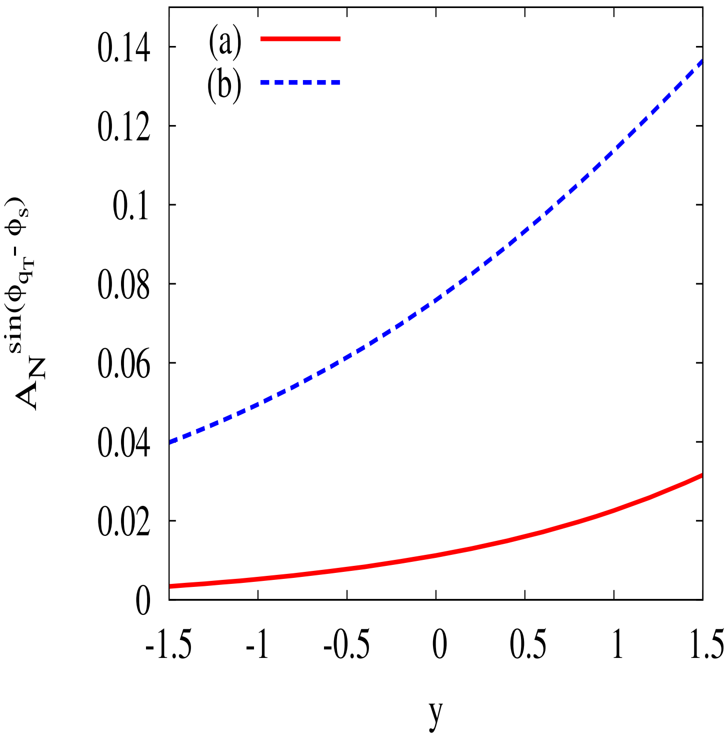
Single spin asymmetry at COMPASS as a function of y (left panel) and q_T (right panel). The plots are for model I with two parametrizations (a) (solid red line) and (b) (dashed blue line). The integration ranges are $(0 \leq q_T \leq 1)$ GeV and $(0 \leq y \leq 1)$. The results are given at $\sqrt{s} = 17.33$ GeV

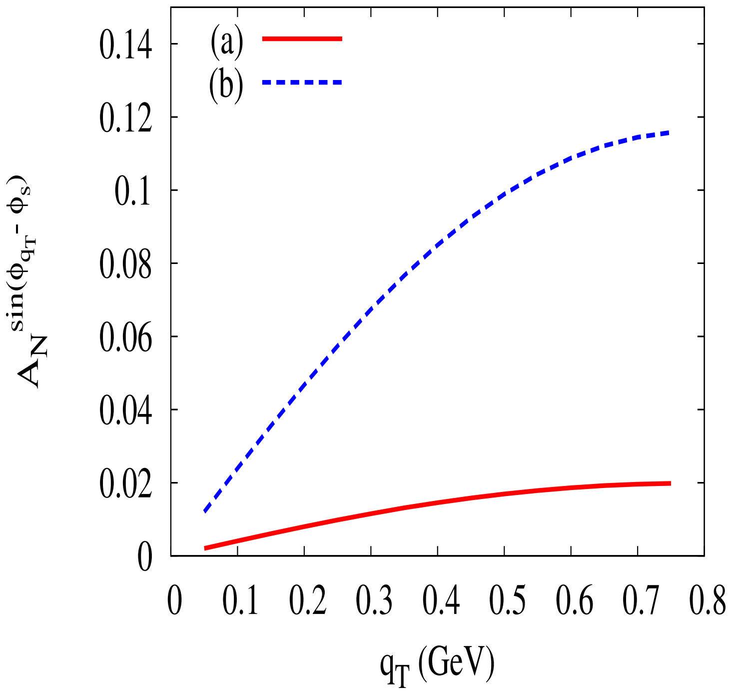
SSA for J/ψ production at eRHIC



Single spin asymmetry at eRHIC as a function of y (left panel) and q_T (right panel). The plots are for model I with two parametrizations (a) (solid red line) and (b) (dashed blue line). The integration ranges are $(0 \leq q_T \leq 1)$ GeV and $(0 \leq y \leq 1)$. The results are given at $\sqrt{s} = 31.6$ GeV

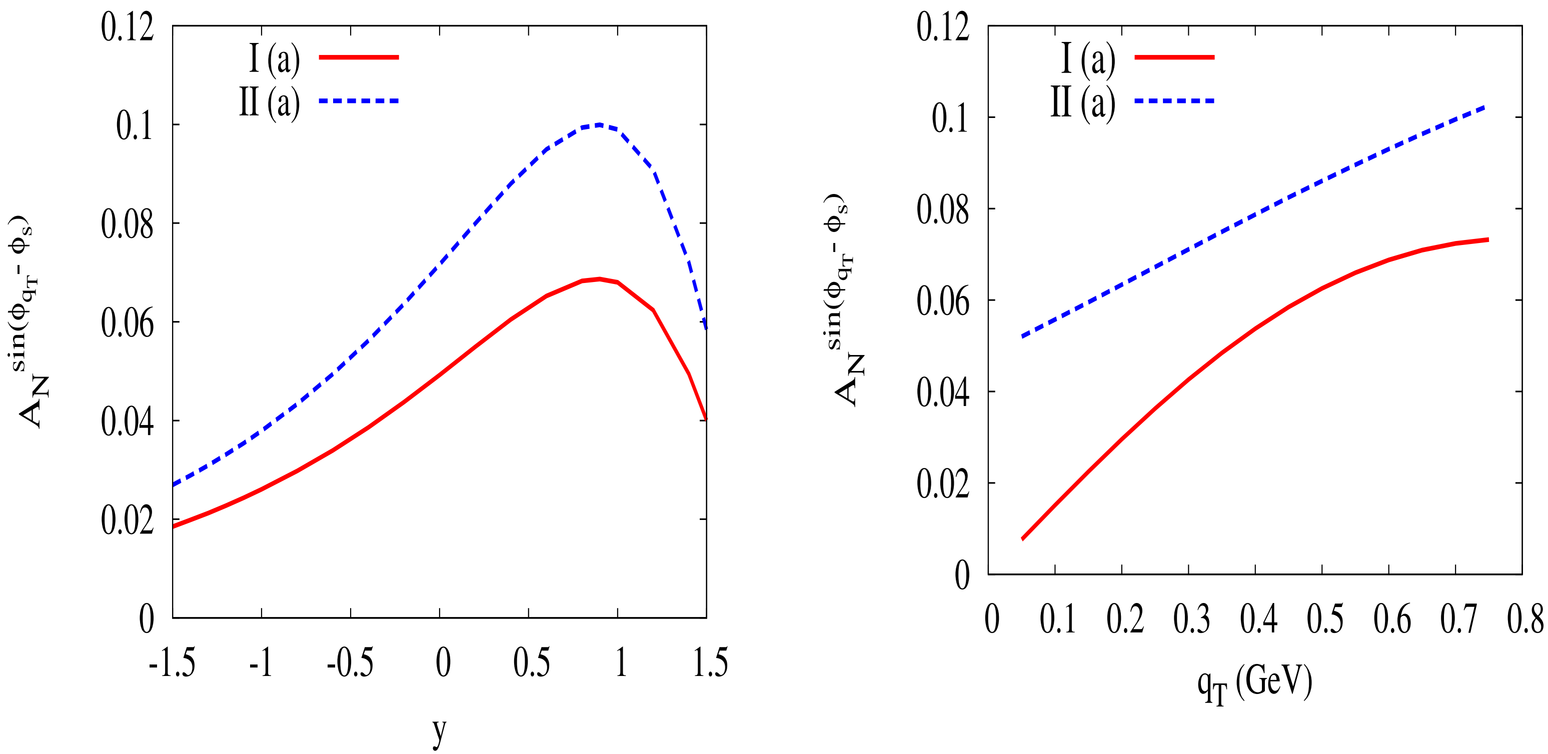
SSA for J/ψ production at eRHIC





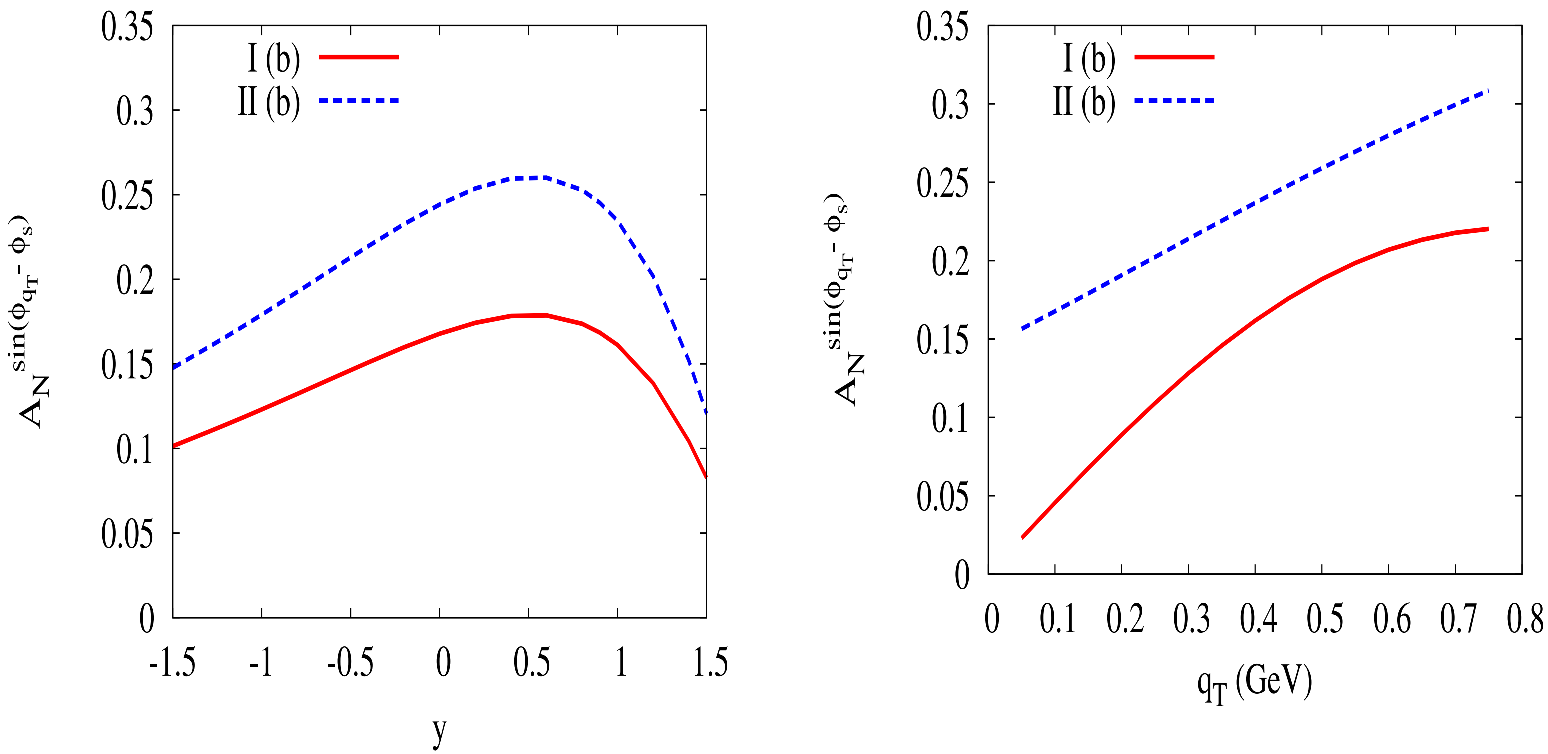
Single spin asymmetry at eRHIC as a function of y (left panel) and q_T (right panel). The plots are for model I with two parametrizations (a) (solid red line) and (b) (dashed blue line). The integration ranges are $(0 \leq q_T \leq 1)$ GeV and $(0 \leq y \leq 1)$. The results are given at $\sqrt{s} = 158.1$ GeV

SSA for J/ψ production at COMPASS



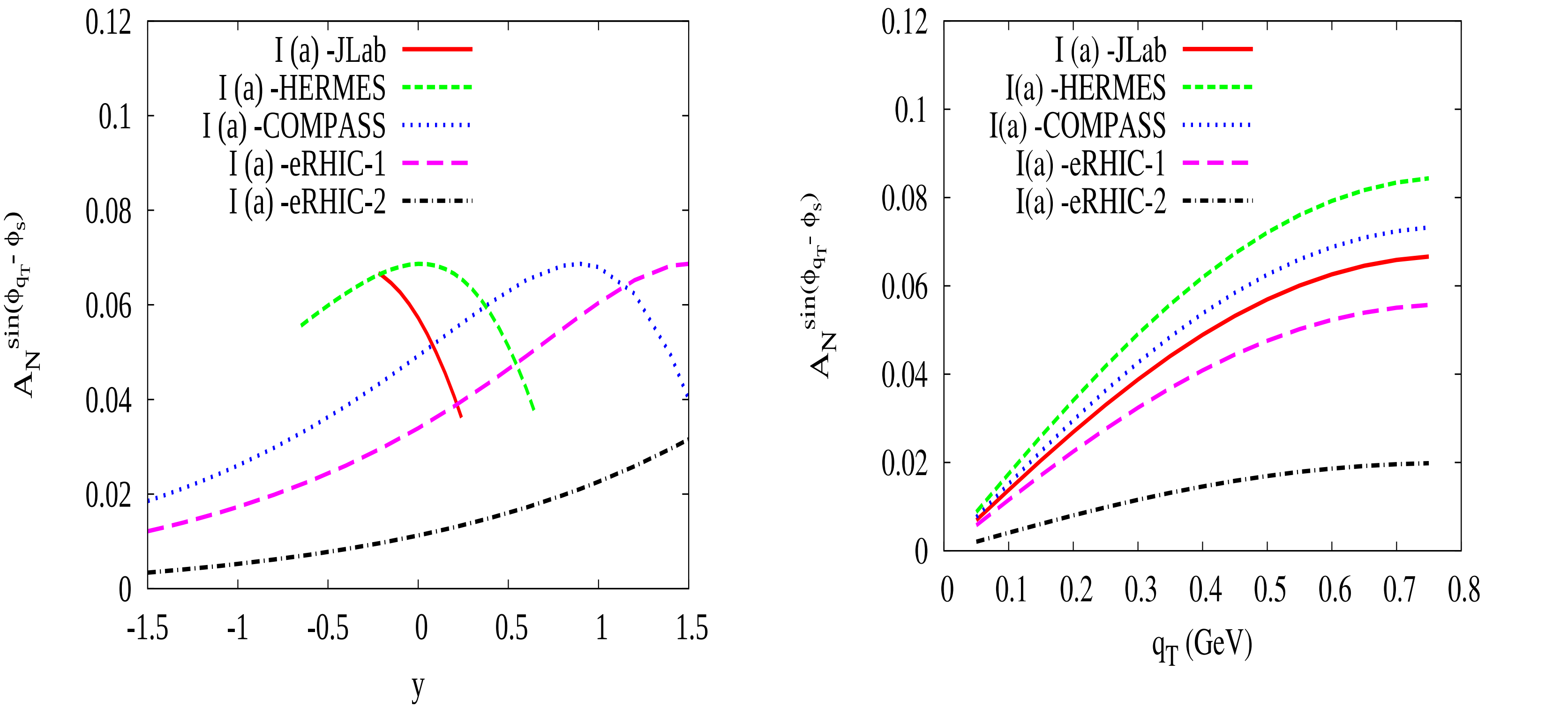
The single spin asymmetry at COMPASS as a function of y (left panel) and q_T (right panel). The plots are for two models I (solid red line) and II (dashed blue line) with parametrization(a). The integration ranges are $(0 \leq q_T \leq 1)$ GeV and $(0 \leq y \leq 1)$. The results are given at $\sqrt{s} = 17.33$ GeV

SSA for J/ψ production at COMPASS



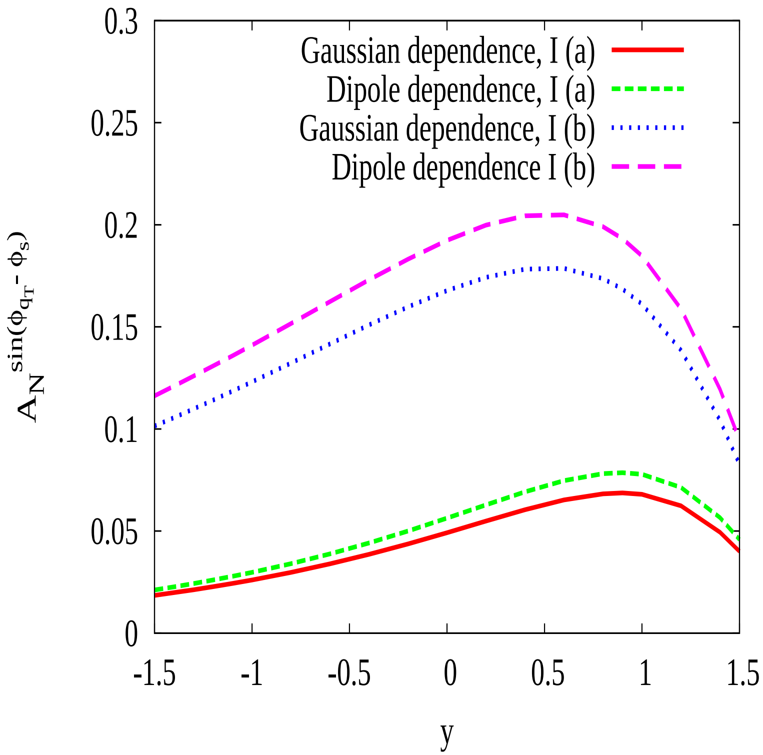
Single spin asymmetry at COMPASS as a function of y (left panel) and q_T (right panel). The plots are for two models I (solid red line) and II (dashed blue line) with parametrization (b). The integration ranges are $(0 \leq q_T \leq 1)$ GeV and $(0 \leq y \leq 1)$. The results are given at $\sqrt{s} = 17.33$ GeV

Compare



Asymmetry in model I with parameterization (a) compared for JLab ($\sqrt{s} = 4.7$ GeV) (solid red line), HERMES ($\sqrt{s} = 7.2$ GeV) (dashed green line), COMPASS ($\sqrt{s} = 17.33$ GeV) (dotted blue line), parametrization-1 ($\sqrt{s} = 31.6$ GeV) (long dashed pink line) and eRHIC-2 ($\sqrt{s} = 158.1$ GeV) (dot-dashed black line)

Dependence on the photon distribution



Single spin asymmetry $A_N^{\sin(\phi_{q_T}-\phi_S)}$ at COMPASS as a function of y : plots are for model I with two parametrization (a) (solid red line and dashed green line) and (b) (dotted blue line and long dashed pink line) compared for Gaussian and dipole $k_\perp$ dependence of WW function. The results are given at $\sqrt{s} = 17.33$ GeV

Scale Evolution of TMDs

- TMD evolution

Aybat, Rogers; Aybat, Collins, Qiu, Rogers, (2011)

- It has been found that TMD evolution affects the single spin asymmetry in SIDIS

Aybat, Prokudin, Rogers (2011); Anselmino, Boglione, Melis (2012)

- QCD scale evolution effectively changes the width of the gaussian in the parametrization

$$\tilde{f}(x, k_\perp, Q) = f(x, Q_0) R(Q, Q_0) \frac{e^{\frac{-k_\perp^2}{w^2}}}{\pi w^2}$$

$R(Q, Q_0)$: overall scale dependent factor, $f(x, Q_0)$: integrated pdf at scale Q_0

$$w^2(Q, Q_0) = < k_\perp^2 > + 2 g_2 log \frac{Q}{Q_0}$$

- TMD evolution is governed by the overall factor $R(Q, Q_0)$ and by the gaussian width and is faster than DGLAP evolution
- Also, the average value of $k_\perp$ changes with energy

Effect of scale evolution of TMDs

In the color evaporation model for charmonium production, the numerator and denominator of the asymmetry become

$$\frac{d^3\sigma^\uparrow}{dy d^2\boldsymbol{q}_T} - \frac{d^3\sigma^\downarrow}{dy d^2\boldsymbol{q}_T} = \frac{1}{s}\int_{4m_c^2}^{4m_D^2} dM^2 \Delta^N f_{g/p^\uparrow}(x_g) f_{\gamma/e}(x_\gamma)\sqrt{2e}\,\frac{q_T}{M_1}$$
$$\times \frac{\langle k_S^2\rangle^2 \exp[-q_T^2/(\langle k_S^2\rangle + \langle k_{\perp\gamma}^2\rangle)]}{\pi[\langle k_S^2\rangle + \langle k_{\perp\gamma}^2\rangle]^2\langle k_{\perp g}^2\rangle}\cos(\phi_{q_T})\,\hat{\sigma}_0^{\gamma g\to c\bar{c}}(M^2)$$

$$\frac{d^3\sigma^\uparrow}{dy d^2\boldsymbol{q}_T} + \frac{d^3\sigma^\downarrow}{dy d^2\boldsymbol{q}_T} = \frac{2}{s}\int_{4m_c^2}^{4m_D^2} dM^2 f_{g/p}(x_g) f_{\gamma/e}(x_\gamma)$$
$$\times \frac{\exp[-q_T^2/(\langle k_{\perp g}^2\rangle + \langle k_{\perp\gamma}^2\rangle)]}{\pi[\langle k_{\perp g}^2\rangle + \langle k_{\perp\gamma}^2\rangle]}\,\hat{\sigma}_0^{\gamma g\to c\bar{c}}(M^2)$$

$$\frac{1}{\langle k_S^2\rangle} = \frac{1}{M_1^2} + \frac{1}{\langle k_{\perp g}^2\rangle}\ .$$

Effect of scale evolution of TMDs

- Charmonium production in this model depends only on the scale $M^2 = \hat{s}$; which is the invariant mass squared of the $c\bar{c}$ pair

- Cross section is integrated over M^2 from $4m_c^2$ to $4m_D^2$, which is a rather narrow region

- The TMDs in the differential cross section can be sensitive to this scale : and this range of scale is independent of the CM energy of the experiment concerned

- So the parametrization of the Sivers function at a low scale can be used to predict the asymmetry at a high scale (for example eRHIC) for charmonium production in this model

- We take the average $k_\perp$ to be the same in all the experiments

Summary and Conclusion

- We have calculated SSA for J/ψ electroproduction using the simple color evaporation model : at leading order this involves only photon-gluon fusion and can give information on gluon Sivers function
- We use Weizsacker-Williams approximation for the photon distribution
- Introduce transverse momentum dependence in the distribution functions and use the existing parametrization of the TMDs : use gaussian $k_\perp$ dependence of the WW function
- The relevant scale for charmonium production in this model is M^2 which is the squared invariant mass of the $c\bar{c}$ pair and independent of the CM energy of the experiment : use of the parametrization justified
- Photoproduction : have to consider higher order and resolved photon contributions; also the role of gauge links will become important at higher orders
- Would be interesting to see how sensitive the SSA is on the charmonium production model and the effect of scale evolution of the TMDs
- SSA is sizable and does not depend much on the choice of the $k^\perp$ dependence of the photon distribution; can be used as a probe for the gluon Sivers function