Effective Field Theory and/for TMD´s

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Introduction

Physical observables with non-vanishing (or un-integrated) transverse-momentum dependence are specially important at colliders, JLAB, RHIC, LHC, Tevatron, Compass, Hermes, Belle,…

- Higgs searches
- Interpretation of signals of New Physics
- Precision Physics
- Spin structure of the proton ..

An *exemplum* out of these processes is the semi-inclusive Drell-Yan (DY) cross-section.

We re-examine semi-inclusive DY in the region $\Lambda_{QCD} << q_T << M$

The essential problem is to write a factorization theorem for semi-inclusive processes, to define the correct non perturbative matrix elements, to resum logs.

This is a *battle field* for effective field theories!!

Issues treated in the talks

- SCET (Introduction)
- Gauge invariance
- Factorization theorem (on the light cone!)
- Rapidity and IR regulators
- Definition of the TMDPDF
- From TMDPDF to PDF
- Universality
- Resummation and exponentiation of logs
- Conclusions

Ignazio

Ahmad

The effective field theory: SCET

- SCET (soft collinear effective theory) is an effective theory of QCD
- SCET describes interactions between low energy, "soft" partonic fields and collinear fields (very energetic in one light-cone direction)
- SCET and QCD have the same infrared structure: matching is possible
- SCET helps in the proof of factorization theorems, identification of relevant scales and the resummation of logs.

SCET: Kinematics

Bauer, Fleming, Pirjol, Stewart, 'oo

SCET modes	
n-collinear (ξ_n, A_n)	$p_n^\mu \sim Q(\lambda^2, 1, \lambda)$
$\bar{n}$-collinear $(\xi_{\bar{n}}, A_{\bar{n}})$	$p_{\bar{n}}^\mu \sim Q(1, \lambda^2, \lambda)$
u-soft (ξ_{us}, A_{us})	$p_{us}^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$
soft (ξ_s, A_s)	$p_s^\mu \sim Q(\lambda, \lambda, \lambda)$

Light-cone coordinates

$$n = (1,0,0,1)$$

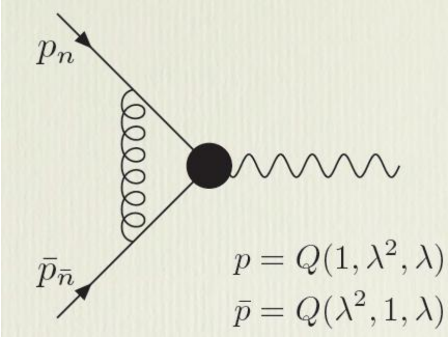
$$\bar{n} = (1,0,0,-1)$$

$$p^\mu = (\bar{n}p, np, p_\perp) = (+,-,\perp)$$

$$\bar{n}p \sim Q$$

$$p_\perp \sim \lambda Q$$

$$np \sim \lambda^2 Q$$



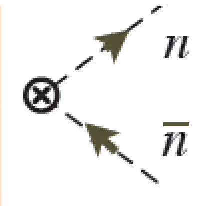
$$\psi(x) = \sum_n \sum_p e^{-ipx} \psi_{n,p}(x)$$

$$\psi = \left(\frac{\not{n}\not{\bar{n}}}{4} + \frac{\not{\bar{n}}\not{n}}{4} \right) \psi = \xi + \phi$$

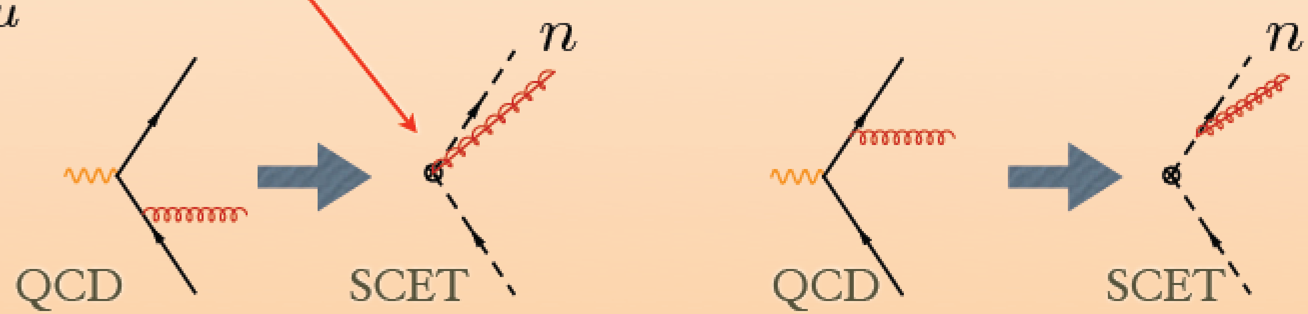
Integrated out with EOM

Soft modes $(\lambda, \lambda, \lambda)$
do not interact with
(anti) collinear or u-soft
In covariant gauge

SCET -I and –II, Pictorically



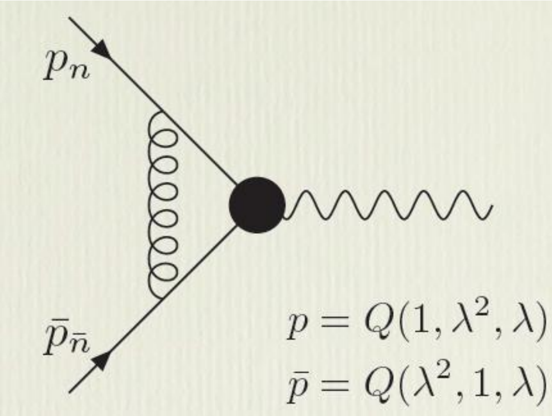
$$\underbrace{\bar{\psi}\,\Gamma^\mu\psi}_{\mathcal{J}_i^\mu} \to (\bar{\xi}_n W_n)_\omega\,\Gamma^\mu (W_{\bar{n}}^\dagger \xi_{\bar{n}})_{\bar{\omega}} = (\bar{\xi}_n W_n)_\omega\, Y_n^\dagger \Gamma^\mu Y_{\bar{n}} (W_{\bar{n}}^\dagger \xi_{\bar{n}})_{\bar{\omega}}$$



Theory	Modes	Virtuality
SCET-I	Collinear, ultra-soft	Λ_{QCD}^2
SCET-II	Collinear, soft	Λ_{QCD}
SCET-qT	Collinear, soft	qT

From QCD to SCET

The properties of QCD in the collinear limit can be understood directly from Feynman diagrams. In this way one obtains The collinear and IR divergencies of the theory.
SCET is just QCD in a kinematical limit!



$$V = \int \frac{d^d k}{(2\pi)^d} \frac{1}{[(p-k)^2 + i\varepsilon][(\bar{p}+k)^2 + i\varepsilon][k^2 + i0]}$$

$$\boxed{k = Q(1, \lambda^2, \lambda)} \quad \to \int \frac{d^d k}{(2\pi)^d} \frac{1}{[(p-k)^2 + i\varepsilon][\bar{p}^- k^+ + i\varepsilon][2k^+ k^- + k_\perp^2 + i0]}$$

Collinear divergence
$k^+ \to 0$

Soft divergence
$k^\mu \to 0\,, \forall \mu$

The Wilson line!

SCET Lagrangian

Bauer, Fleming, Pirjol, Stewart, 'oo

Leading order Lagrangian (n-collinear)

$$\mathcal{L}_{c,n} = \bar{\xi}_{n,p'} \left\{ i\, n\cdot D + g n\cdot A_{n,q} + \left(\not{\mathcal{P}}_\perp + g \not{A}^\perp_{n,q} \right) W \frac{1}{\bar{\mathcal{P}}} W^\dagger \left(\not{\mathcal{P}}_\perp + g \not{A}^\perp_{n,q'} \right) \right\} \frac{\not{\bar{n}}}{2} \xi_{n,p}$$

$$W_n(x) = \bar{P} \exp\left(-ig \int_0^\infty ds\ \bar{n}\cdot A_n(\bar{n}s + x) \right) \qquad Y_n(x) = \bar{P} \exp\left(-ig \int_0^\infty ds\ \bar{n}\cdot A_{us}(\bar{n}s + x) \right)$$

$$\boxed{\begin{aligned} iD_\mu &= i\partial_\mu + gA_{us} \\ inD &= Y_n^\dagger in\partial Y_n \end{aligned}}$$

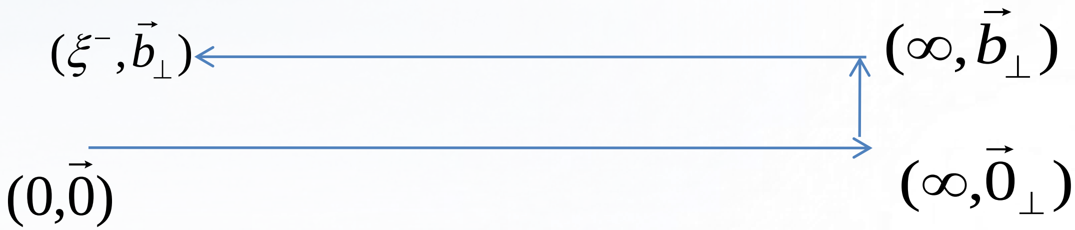
The new fields do not interact anymore with u-soft fields

$$\xi_n^{(0)} = Y_n W_n^\dagger \xi$$

All fields are Taylor expanded according to the power counting: Multipole expansion

The Wilson lines arise from the Lagrangian and are not ad hoc objects!

Transverse Gauge Link in QCD



Ji, Ma, Yuan
Ji, Yuan
Belitsky, Ji, Yuan
Cherednikov, Stefanis

- For gauges not vanishing at infinity [Singular Gauges] like the Light-Cone gauge (LC) one needs to introduce an additional Gauge Link which connects $(\infty,\vec{0}_\perp)$ with $(\infty,\vec{b}_\perp)$ to make it Gauge Invariant $A_\perp = -\frac{e}{2\pi}\theta(\xi^-)\nabla \ln(\mu\, r_\perp)$
- In LC Gauge This Gauge Link Is Built From The Transverse Component Of The Gluon Field:

$$\tilde{\mathcal{F}}_{i/h}(x,\boldsymbol{k}_\perp) = \frac{1}{2}\int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{2\pi(2\pi)^2} \mathrm{e}^{-ik^+\xi^- + i\boldsymbol{k}_\perp\boldsymbol{\xi}_\perp} \left\langle h|\bar{\psi}_i(\xi^-,\boldsymbol{\xi}_\perp)[\xi^-,\boldsymbol{\xi}_\perp;\infty^-,\boldsymbol{\xi}_\perp]^\dagger_{[n]}[\infty^-,\boldsymbol{\xi}_\perp;\infty^-,\boldsymbol{\infty}_\perp]^\dagger_{[\boldsymbol{l}]}\right.$$

$$\left. \times\gamma^+[\infty^-,\boldsymbol{\infty}_\perp;\infty^-,\boldsymbol{0}_\perp]_{[\boldsymbol{l}]}[\infty^-,\boldsymbol{0}_\perp;0^-,\boldsymbol{0}_\perp]_{[n]}\psi_i(0^-,\boldsymbol{0}_\perp)|h\right\rangle$$

$$[\infty^-,\boldsymbol{\infty}_\perp;\infty^-,\boldsymbol{\xi}_\perp]_{[\boldsymbol{l}]} \equiv \mathcal{P}\exp\left[ig\int_0^\infty d\tau\; \boldsymbol{l}\cdot\boldsymbol{A}_a t^a(\boldsymbol{\xi}_\perp + \boldsymbol{l}\tau)\right]$$

Gauge Invariant TMDPDF In SCET?

Are TMDPDF fundamental matrix elements in SCET?

Are SCET matrix elements gauge invariant?

Where are transverse gauge link in SCET?

The SCET Lagrangian is formed by gauge invariant building blocks. Gauge Transformations in covariant gauge for $W_n^+\xi$

$$\xi \to U\xi$$

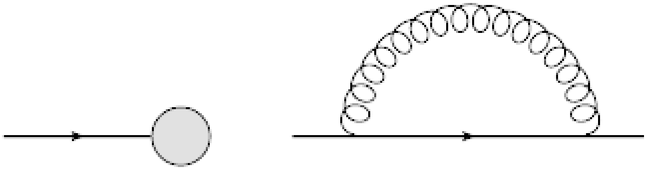
$$W_n^+ \to W_n^+ U^+$$

$$\boxed{W^\dagger \xi \xrightarrow{LC\ gauge} \xi}$$

Gauge invariance of SCET building blocks

We calculate $\langle 0 | W_{\bar{n}}^{\dagger} \xi_{\bar{n}} | q \rangle$ at one-loop in Feynman Gauge and in LC gauge

In LC Gauge



$$A^{+} = 0 \rightarrow W_{\bar{n}} = W_{\bar{n}}{}^{\dagger} = 1$$

$$\tilde{D}_{\mu\nu}(k) = \frac{-i}{k^2 + i0} \left(g_{\mu\nu} - \frac{n_\mu k_\nu + n_\nu k_\mu}{\left[k^{+} \right]} \right)$$

[Bassetto, Lazzizzera, Soldati] Canonical quantization imposes ML prescription

Prescription	$1/[k^+]$
$+i0$	$1/(k^+ + i0)$
$-i0$	$1/(k^+ - i0)$
PV	$1/2(1/(k^+ + i0) + 1/(k^+ - i0))$
ML	$1/(k^+ + i0\mathrm{Sgn}(k^-))$

Gauge invariance in SCET

The SCET matrix element $\langle 0|W_{\bar{n}}^{\dagger}\xi_{\bar{n}}\,|q\rangle$ is not gauge invariant. Using LC gauge we have different result (moreover even the result of the one-loop correction depends on the used prescription).

In order to restore gauge invariance we have to introduce a new Wilson line, T, in SCET matrix elements

$$T_{\bar{n}}^{\dagger}(x^{+},x_{\perp}) = P\exp\left[ig\int_{0}^{\infty} d\tau\,\mathbf{l}_{\perp}\cdot\mathbf{A}_{\perp}(\infty^{-},x^{+};\mathbf{l}_{\perp}\tau+\mathbf{x}_{\perp})\right]$$

And the new gauge invariant matrix element is $\boxed{\langle 0\,|\,T_{\bar{n}_{12}}^{\dagger}W_{\bar{n}}^{\dagger}\xi_{\bar{n}}\,|\,q\rangle}$

How do T´s arise in SCET?

$$A^{(\infty)}(x^+, x_\perp) \overset{def}{=} A(x^+, \infty^-, x_\perp)$$

$$\tilde{A}(x^+, x^-, x_\perp) \overset{def}{=} A(x^+, x^-, x_\perp) - A^{(\infty)}(x^+, x_\perp)$$

$$\left[A_\perp(x^+, \infty^-, x_\perp - \ell_\perp \tau), \tilde{A}_\perp(x^+, x^-, x_\perp) \right] = 0$$

$$iD^\mu{}_\perp = i\partial^\mu{}_\perp + gA^\mu{}_\perp = i\partial^\mu{}_\perp + g\,\tilde{A}^\mu{}_\perp + gA_\perp^{(\infty)\mu} \overset{def}{=} i\tilde{D}^\mu{}_\perp + gA_\perp^{(\infty)\mu}$$

$$\boxed{iD_\perp^\mu \gamma_\mu = T i\tilde{D}_\perp^\mu \gamma_\mu T^\dagger}$$

THIS ENSURES THAT T´S ARISE NATURALLY IN SCET!

$$T^\dagger(x^+, x_\perp) = P\exp\left[-ig\int_0^\infty d\tau\, \ell_\perp \cdot A_\perp^{(\infty)}(x^+, x_\perp - \ell_\perp \tau) \right]$$

The T-Wilson Lines in SCET-II

Now the degrees of freedom are just collinear and soft. In covariant gauges

$$(\bar{n}A_n, nA_n, A_{n\perp}) \sim Q(1, \boxed{\eta^2}, \eta); \quad (\bar{n}p_n, np_n, p_{n\perp}) \sim Q(1, \boxed{\eta^2}, \eta);$$

$$(\bar{n}A_s, nA_s, A_{s\perp}) \sim Q(\eta, \boxed{\eta}, \eta); \quad (\bar{n}p_s, np_s, p_{s\perp}) \sim Q(\eta, \boxed{\eta}, \eta);$$

No interaction is possible for on-shell states

$$\boxed{\mathcal{L}_{II} = \bar{\xi}_n \left(in\not{D}_n + i\not{D}_{n\perp} W_n \frac{1}{i\bar{n}\partial} W_n^\dagger i\not{D}_{n\perp} \right) \frac{\bar{\not{n}}}{2} \xi_n}$$

The T-Wilson Lines in SCET-II

$$(\bar{n}A_n, nA_n, A_{n\perp}) \sim Q(1, \eta^2, \eta);$$

$$(\bar{n}A_s, nA_s = 0, A_{s\perp}) \sim Q(\eta, \varnothing, \eta);$$

The gauge ghost acts only on some momentum components and couples with collinear

$$\prod_i \phi_n^i(x) A_{s\perp}^{\infty}(x^-, x^\perp) \to \prod_i \phi_n^i(x) A_{s\perp}^{\infty}(0, x^\perp)$$

Thus the covariant derivative is $iD^\mu = i\partial^\mu + gA_n^\mu(x) + gA_{s\perp}^{(\infty)\mu}(0^-, x_\perp)$

The decoupling of soft fields requires

$$A_n^{(0)\mu}(x) = T_{sn}(x_\perp) A_n^\mu(x) T_{sn}^\dagger(x_\perp)$$

$$T_{sn} = \bar{P} \exp\left[ig \int_0^\infty d\tau l_\perp \cdot A_{s\perp}^{(\infty)}(0^-, x_\perp - l_\perp \tau) \right]$$

The T-Wilson Lines in SCET-II

The new SCET-II Lagrangian is

$$\boxed{\mathcal{L}_{II} = \bar{\xi}_n^{(0)}\left(in D_n^{(0)} + i\not{D}_{n\perp}^{(0)} W_n^{T(0)} \frac{1}{i\bar{n}\partial} W_n^{T(0)\dagger} i\not{D}_{n\perp}^{(0)}\right)\frac{\not{\bar{n}}}{2}\xi_n^{(0)}}$$

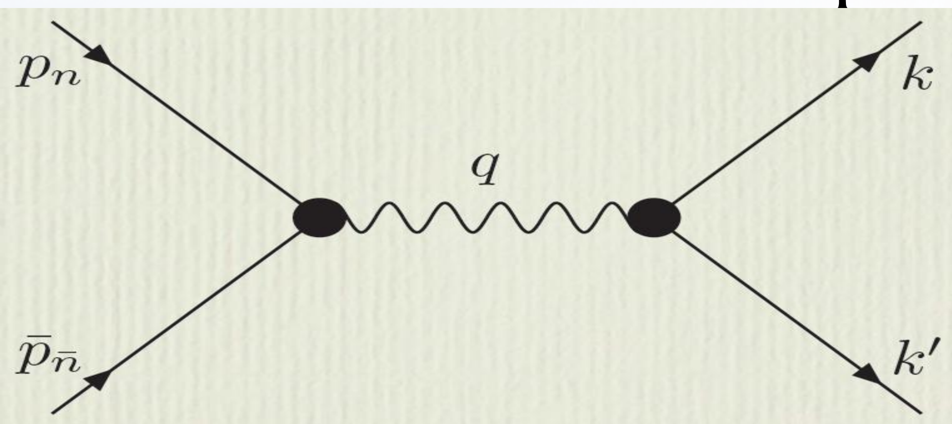
$$A_n^{(0)\mu}(x) = T_{sn}(x_\perp)A_n^\mu(x)T_{sn}^\dagger(x_\perp)$$

$$D_n^{(0)\mu} = i\partial^\mu + gA_n^{(0)\mu}$$

$$\xi_n^{(0)} = T_{sn}(x_\perp)\xi_n(x)$$

$$T_{sn} = \bar{P}\exp\left[ig\int_0^\infty d\tau l_\perp \cdot A_{s\perp}^{(\infty)}(0^-, x_\perp - l_\perp\tau)\right]$$

Drell-Yan at low qT



We have multiscale problem!

$$q^2 = Q^2 \gg Q_T^2 \gg \Lambda_{QCD}^2$$

$$\lambda_1 \sim \frac{Q_T}{Q} \; and \; \lambda_2 \sim \frac{\Lambda_{QCD}}{Q_T}$$

$$\sigma = LM$$

Main steps for the factorization:

$$M = |H(Q^2)|^2 \; F_n \otimes F_{\bar{n}} \otimes \Phi$$

$$M = |H(Q^2)|^2 \; \frac{f_n}{\phi} \otimes \frac{f_{\bar{n}}}{\phi} \otimes \phi$$

In PDF mixed UV-IR divergences cancel between virtual and real diagrams! This is not the case for TMD: there is no integration over pT

$$M = |H(Q^2)|^2 \; \hat{f}_n \otimes \hat{f}_{\bar{n}} \otimes \phi^{-1}$$

$$M = |H(Q^2)|^2 \; j_n \otimes j_{\bar{n}}$$

$$j_{n(\bar{n})} = \frac{\hat{f}_{n(\bar{n})}}{\sqrt{\phi}}$$

Drell-Yan

$$d\Sigma = \frac{4\pi\alpha}{3q^2 s}\frac{d^4 q}{(2\pi)^4}\frac{1}{4}\sum_{\sigma_1\sigma_2}\int d^4 y e^{-iqy}(-g_{\mu\nu})\langle N_1(P,\sigma_1)N_2(\bar{P},\sigma_2)|J^{\mu\dagger}(y)J^{\nu}(0)|N_1(P,\sigma_1)N_2(\bar{P},\sigma_2)\rangle$$

$$J^{\mu} = \sum_q e_q \bar{\psi}\gamma^{\mu}\psi \to J^{\mu} = H(Q^2,\mu)\sum_q e_q \bar{\chi}_{\bar{n}} S_{\bar{n}}^{T\dagger}\gamma^{\mu} S_n^T \chi_n$$

Collinear, anti-collinear and soft act on different Hilbert spaces!!

$$\boxed{d\Sigma = \frac{4\pi\alpha^2}{3N_c q^2 s}\frac{d^4 q}{(2\pi)^4}|H(Q^2,\mu)|^2\int d^4 y e^{-iq\cdot y}\sum_q e_q^2 F_n(y)F_{\bar{n}}(y)\Phi(y)}$$

$$F_n(y) = \frac{1}{2}\sum_{\sigma_1}\langle N_1(P,\sigma_1)|\bar{\chi}_n(y)\frac{\bar{n}_{\mu}\gamma^{\mu}}{2}\chi_n(0)|N_1(P,\sigma_1)\rangle$$

$$F_{\bar{n}}(y) = \frac{1}{2}\sum_{\sigma_1}\langle N_2(\bar{P},\sigma_2)|\bar{\chi}_{\bar{n}}(0)\frac{n_{\mu}\gamma_{\mu}}{2}\chi_{\bar{n}}(y)|N_2(\bar{P},\sigma_2)\rangle$$

$$\Phi(y) = \langle 0|\,\mathrm{Tr}\,\bar{\mathbf{T}}\left[S_n^{T\dagger}S_{\bar{n}}^T\right](y)\mathbf{T}\left[S_{\bar{n}}^{T\dagger}S_n^T\right](0)|0\rangle$$

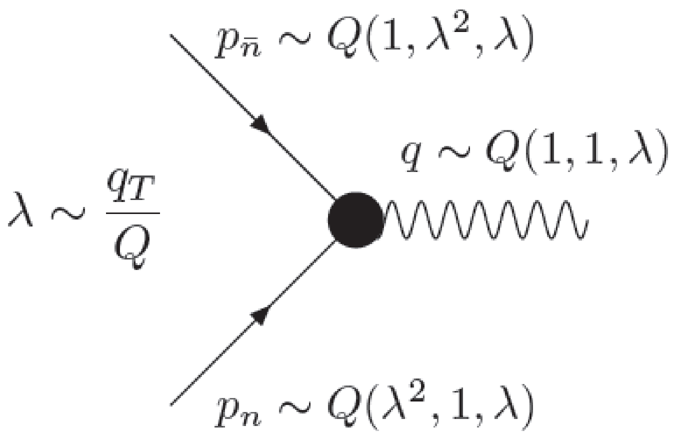
However this is not the end of the story…

Factorization (Taylor Expansion)

$$M = H(Q^2/\mu^2) \int d^4y\, e^{-iq\cdot y}\, F_n(y)\, F_{\bar{n}}(y)\, \Phi(y)$$

- This result includes subleading contributions (in powers of $1/Q^2$): Taylor Expansion.

The photon is hard:



$$y \sim \frac{1}{Q}(1, 1, \frac{1}{\lambda})$$

$$\left(\frac{\partial}{\partial y^-}, \frac{\partial}{\partial y^+}, \frac{\partial}{\partial y_\perp}\right) F_n(y) \sim Q(1, \lambda^2, \lambda)$$

$$\left(\frac{\partial}{\partial y^-}, \frac{\partial}{\partial y^+}, \frac{\partial}{\partial y_\perp}\right) F_{\bar{n}}(y) \sim Q(\lambda^2, 1, \lambda)$$

$$\left(\frac{\partial}{\partial y^-}, \frac{\partial}{\partial y^+}, \frac{\partial}{\partial y_\perp}\right) \Phi(y) \sim Q(\lambda, \lambda, \lambda)$$

$$M = H(Q^2/\mu^2) \int d^4y\, e^{-iq\cdot y} \boxed{F_n(0^+, y^-, \vec{y}_\perp)\, F_{\bar{n}}(y^+, 0^-, \vec{y}_\perp)} \boxed{\Phi(0^+, 0^-, \vec{y}_\perp)}$$

- Agreement with [Ji, Ma, Yuan '05], who first introduced this soft function with dependence ONLY on the transverse component [contrary to earlier works of J. C. Collins].

Factorization (problems...)

$$\boxed{M = H(Q^2/\mu^2) \int d^4y\, e^{-iq\cdot y}\, F_n(0^+, y^-, \vec{y}_\perp)\, F_{\bar{n}}(y^+, 0^-, \vec{y}_\perp)\, \Phi(0^+, 0^-, \vec{y}_\perp)}$$

- Individually F_n, $F_{\bar{n}}$ and Φ have mixed UV/IR divergencies: Renormalization and evolution becomes problematic

$$\frac{1}{\varepsilon_{\rm UV}\, \varepsilon_{\rm IR}}$$

- Individually F_n, $F_{\bar{n}}$ and Φ have non-regularized light-cone divergencies: Dimensional Regularization is not enough and we need another regulator.

$$\int_0^1 dt \frac{1}{t}$$

- Even with this regulator the mixed divergencies don't cancel between virtual and real.

F_n, $F_{\bar{n}}$ and Φ are ill-defined!!

- In QCD M has no mixed divergencies.
- The hadronic tensor can be calculated in pure Dimensional Regularization.
- F_n and $F_{\bar{n}}$ have analogous operator definition: same mixed divergencies.
- We can combine the matrix elements to build well-defined objects: TMDPDFs!!

Regulators and rapidity divergences

- DR: $\mu^{2\varepsilon}\int\frac{d^{4-2\varepsilon}k}{(2\pi)^{4-2\varepsilon}}$ per-se has problem with rapidity div. In real gluon exchange diagrams (it is good for the virtuals!!)
- Wilson lines off-the-light-cone (Collins):

$$n \to n = (1, e^{-2y_n}, 0_\perp)$$
$$\bar{n} \to \bar{n} = (-e^{2y_n}, 1, 0_\perp)$$

Works fine but it is impossible to recover the light cone limit, LCG?(Unphysical scales can be removed with our definition of TMD if it is used consitently in collinear and soft sectors)

- Δ-regulator (Chiu, Fuhrer, Hoang, Kelley,Manohar, see also Cherednikov, Stefanis):

Works like a mass term,
respects symmetries, fine in LCG

$$\frac{i}{(p+k)^2 + i\Delta^-} \to \frac{i}{k^- \pm i\delta^-}$$

$$\delta^- = \frac{\Delta^-}{p^+}$$

Regulators and rapidity divergences

- Analytic regulator (Becher Neubert): breaks SCET collinear/anticollinear symmetry, not usable for TMD's, it does not work in LCG (with ML!!)

$$\frac{1}{-(p-k)^2 - i\varepsilon} \to \frac{\nu^\alpha}{\left[-(p-k)^2 - i\varepsilon\right]^{1+\alpha}}$$

- Rapidity regulator (Chiu,Jain, Neill, Rothstein):

LCG? Transforms IR/rapidity divergences <u>into</u> UV divergences. Unphysical scales can be Removed with our definition of the TMD.

$$W_n = \sum_{perm} \exp\left[-\frac{g w^2}{\bar{n}\mathcal{P}} \frac{\left|\bar{n}\mathcal{P}_g\right|^{-\eta}}{\nu^{-\eta}} \bar{n}A_n \right]$$

We have proved the correctness of our TMD with all regulators which respect collinear/anticollinear symmetry .
We agree on the final result for the total cross section in DY with the ones that do break this symmetry.
Our definition of TMD is regulator independent! (No Rap. Div.)

Conclusions

- We have provided a factorization theorem for DY using an effective field theory of QCD!!
- Our definition can be used also **in Light Cone Gauge with the T-Wilson lines**
- We have a consistent definition of the TMDPDF **on-the-light-cone: no unphysical parameters**

- All the rest (and the best) in Ahmad´s talk!

Back up slides

The T-Wilson Lines in SCET-I

Are T´s compatible with SCET power counting?In SCET-I only collinear and u-soft fields.
The first step to obtain the SCET Lagrangian is integrating out energetic part of spinors

$$\mathcal{L} = \bar{\xi}_n \left(inD + i\not{D}_\perp \frac{1}{i\bar{n}D} i\not{D}_\perp \right) \frac{\not{\bar{n}}}{2} \xi_n$$

And then applying multipole expansion,

$$x_n \sim 1/Q(1, 1/\lambda^2, 1/\lambda)$$

$$x_{us} \sim 1/Q(1/\lambda^2, 1/\lambda^2, 1/\lambda^2)$$

$$\boxed{\mathcal{L}_{\mathcal{I}} = \bar{\xi}_n \left(in\not{D}_n + gnA_{us}(x^+) + i\not{D}_{n\perp} W_n^T \frac{1}{i\bar{n}\partial} W_n^{T\dagger} i\not{D}_{n\perp} \right) \frac{\not{\bar{n}}}{2} \xi_n}$$

Where $W_n^T = T_n W_n$

U-soft field does not give rise to any transverse gauge link!!
There are no transverse u-soft fields and they cannot depend
on transverse coordinates!!

Definition of the TMDPDF

Using the fact that all mixed divergences are the same in the collinear and anticollinear sectors and are canceled by the soft function

$$J_n(x;\vec{k}_{n\perp}) = \frac{1}{2}\int\frac{dr^- d^2\vec{r}_\perp}{(2\pi)^3} e^{-i(\frac{1}{2}r^- xP^+ - \vec{r}_\perp\cdot\vec{k}_{n\perp})} F_n(0^+, r^-, \vec{r}_\perp)\sqrt{\Phi(0^+,0^-,\vec{r}_\perp)}$$
$$J_{\bar{n}}(z;\vec{k}_{\bar{n}\perp}) = \frac{1}{2}\int\frac{dr^+ d^2\vec{r}_\perp}{(2\pi)^3} e^{-i(\frac{1}{2}r^+ z\bar{P}^- - \vec{r}_\perp\cdot\vec{k}_{\bar{n}\perp})} F_{\bar{n}}(r^+, 0^-, \vec{r}_\perp)\sqrt{\Phi(0^+,0^-,\vec{r}_\perp)}$$

We agree with Collins´11 in the square root, but we stay on the light cone and No rapidity cutoffs!

$$M = |H(Q^2,\mu)|^2 \sum_q e_q^2 \int d^2k_{n\perp} d^2k_{\bar{n}\perp}\, \delta^{(2)}(q_\perp - k_{n\perp} - k_{\bar{n}\perp})\, j_n(x;k_{n\perp};\mu)\, j_{\bar{n}}(z;k_{\bar{n}\perp};\mu)$$

$$j_n(x;k_{n\perp};\mu) = \frac{1}{2}\int\frac{dr^- d^2 r_\perp}{(2\pi)^3} e^{-i(\frac{1}{2}r^- xP^+ + r_\perp k_{n\perp})} \frac{\hat{f}_n(0^+, r^-, r_\perp)}{\sqrt{\phi(0^+,0^-,r_\perp)}}$$

And we have used the equivalence of zero-bin and soft function for the IR regulator that we used

Re-factorization

- The TMDPDF has perturbative content when q_T is perturbative.
- We can do an OPE of the TMDPDF onto the PDF in impact parameter space [integrating out the intermediate scale q_T^2.]

$$\boxed{\tilde{J}_n(x;\vec{b}_\perp,Q,\mu) = \int_x^1 \frac{dx'}{x'} \tilde{C}_n\left(\frac{x}{x'};\vec{b}_\perp,Q,\mu\right) \mathcal{Q}_n(x';\mu)}$$

- Properties of the C:
  - It is independent of the IR regulator (matching coefficient!!)
  - It is supposed to live at the intermediate scale. All logs should be cancelled by one choice for the intermediate scale (like at threshold).
  - BUT: it has a subtle Q^2-dependence [Becher, Neubert '11]

$$\begin{aligned}\tilde{C}_n(x;\vec{b}_\perp,Q,\mu) &= \delta(1-x) + \frac{\alpha_s C_F}{2\pi}\Big[-\mathcal{P}_{q/q} L_T + (1-x) \\ &\quad -\delta(1-x)\left(\frac{1}{2}L_T^2 - \frac{3}{2}L_T + \ln\frac{Q^2}{\mu^2}L_T + \frac{\pi^2}{12}\right)\Big]\end{aligned}$$

$$L_T = \ln\frac{\mu^2 b^2}{4e^{-2\gamma_E}}$$

- Accidentally, $\ln(Q^2/\mu^2)$ can be eliminated by choosing μ. Not at higher orders.
- The logs cannot be combined in one single log, like at threshold.

Re-factorization (Q^2 factor)

- Using the Δ-regulator, Lorentz invariance and dimensional analysis we can extract the Q-dependence from the TMDPDF to all orders.

$$\ln\tilde{j}_n = \ln\tilde{\hat{f}}_n - \frac{1}{2}\ln\tilde{\phi}$$

$$\ln\tilde{\hat{f}}_n = \mathcal{R}_n\left(x;\alpha_s, L_T, \ln\frac{\delta^+}{p^+} = \ln\frac{\Delta}{Q^2}\right)$$

$$\ln\tilde{\phi} = \mathcal{R}_\phi\left(\alpha_s, L_T, \ln\frac{\delta^+\delta^-}{\mu^2} = \ln\frac{\Delta^2}{Q^2\mu^2}\right)$$

$$\boxed{\ln\tilde{j}_n = \ln\tilde{j}_n^{sub} - D(\alpha_s, L_T)\left(\ln\frac{Q^2}{\mu^2} + L_T\right)}$$

$$\boxed{\tilde{J}_n(x;\vec{b}_\perp, Q, \mu) = \left(\frac{Q^2 b^2}{4e^{-2\gamma_E}}\right)^{-D(\alpha_s, L_T)} \tilde{C}_n(x;\vec{b}_\perp,\mu)\otimes\mathcal{Q}_n(x;\mu)}$$

$$\frac{dD(\alpha_s, L_T)}{d\ln\mu} = \Gamma_{cusp}(\alpha_s)$$

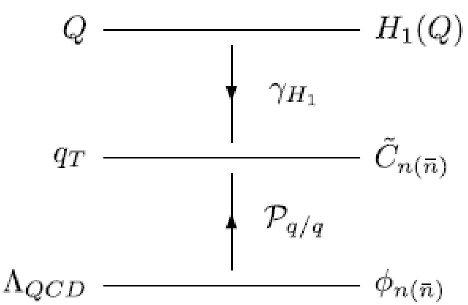
(Consistent with [Becher, Neubert '11])

- The Q^2-factor is extracted for each TMDPDF individually.
- Its origin is related to the appearance of the Soft function in the TMDPDF.

Factorization theorem (final)

$$M = \int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{b}_\perp} \int_x^1 \frac{dx'}{x'} \int_z^1 \frac{dz'}{z'} H\left(\frac{Q^2}{\mu^2}\right)$$

$$\boxed{\times\left[\left(\frac{Q^2 b^2 e^{2\gamma}}{4}\right)^{-D(\alpha_s, L_T)} \tilde{C}_n\left(\frac{x}{x'}; L_T, \mu\right) Q_n(x'; \mu)\right]}\left[\left(\frac{Q^2 b^2 e^{2\gamma}}{4}\right)^{-D(\alpha_s, L_T)} \tilde{C}_{\bar{n}}\left(\frac{z}{z'}; L_T, \mu\right) Q_{\bar{n}}(z'; \mu)\right]$$



The PDF are the only hadronic matrix element for intermediate Q_T

The Q-factor is resummed in 2 places H and J

The matching coefficients are extracted for each individual PDF!