Generalized Parton Distributions in the Chiral Odd Sector & Their Role in Neutral Meson Leptoproduction

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These ideas were developed in Trento ECT*, INT, Jlab, DIS2011, Frascati INF, etc.
& in consultation with many of you

Exploring Transversity

- "I've been to the zoo! Do you know what I did before I went to the zoo today? I walked all the way up Fifth Avenue from Washington Square; all the way . . . I took the subway down to the Village so I could walk all the way up Fifth Avenue to the zoo. It's one of those things a person has to do; sometimes a person has to go a very long distance out of his way to come back a short distance correctly."

Zoo Story, Edward Albee (1958)

Outline

- Some history of Transversity
- "Flexible" parameterization for Chiral Even GPDs
  - Regge ✖ diquark spectator model
  - Satisfies all constraints
- Results for DVCS (transverse γ^* → transverse γ)
  - cross sections & asymmetries
- Extend to Chiral Odd GPDs via diquark spin relations
  - Some relations between Chiral even & odd helicity amps
- π^0, η, η' production data involve sizable $\gamma^*_{Transverse}$ (factorization shown at leading twist for $\gamma^*_{Longitudinal}$ [Collins, Franfurt, Strikman])
- γ^*_T *requires* chiral odd GPDs
- Q^2 dependence for π^0 depends on $\gamma^*+(\rho, b_1)\rightarrow\pi^0$
  - π^0 cross sections & asymmetries
- Transversity Amplitudes, GPDs & TMDs

Transversity

- Early history – 4 decades ago

SSA $\pi p\Uparrow \Rightarrow \pi p$ elastic – single ρ Regge pole exchange

MEASUREMENT OF POLARIZATION IN $\pi^- p \to \pi^0 n$ AND $\pi^- p \to \eta n$*

D. D. Drobnis, J. Lales, R. C. Lamb, R. A. Lundy, A. Moretti, R. C. Niemann, T. B. Novey, J. Simanton, A. Yokosawa, and D. D. Yovanovitch
Argonne National Laboratory, Argonne, Illinois
(Received 13 November 1967)

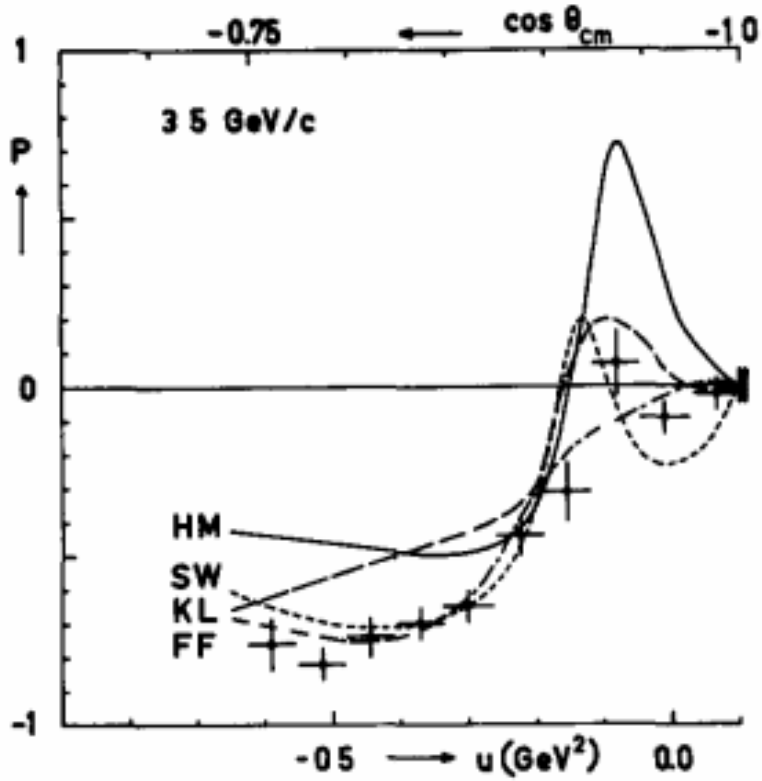
dynamical models of strong interactions. A Regge-pole model involving the exchange of a single 1^{-+} trajectory (the ρ) has been successful in fitting the differential cross-section data from 4 to 18 GeV/c.[1] This simple one-trajectory model clearly implies a polarization of zero.

But…

Polarization in $\pi^- p \to \pi^0 n$

2.07 GeV/c Momentum transfer interval $[\mathrm{GeV}/c]^2$	Polarization %	2.50 GeV/c Momentum transfer interval $[\mathrm{GeV}/c]^2$	Polarization %	2.72 GeV/c Momentum transfer interval $[\mathrm{GeV}/c]^2$	Polarization %
.039 to .094	3 ± 7	.036 to .095	36 ± 8	.034 to .070	29 ± 14
.094 to .174	37 ± 9	.095 to .180	10 ± 6	.070 to .119	33 ± 10
.174 to .241	68 ± 14	.180 to .294	31 ± 7	.119 to .181	-3 ± 11
.241 to .362	32 ± 16	.294 to .387	41 ± 15	.181 to .256	17 ± 11
background	1 ± 1	background	3 ± 1	.256 to .395	10 ± 15
				background	-2 ± 1

F. Bradamante et al., Elastic π^+p scattering



Nucl. Phys. B56 (1973)

Transversity

- Early history – 4 decades ago

SSA $\pi p\Uparrow \Rightarrow \pi p$ elastic – single ρ Regge pole exchange

Need interference & helicity flip

Regge + fsi !

- Spin is **not** an unnecessary complication

  Polarized target experiments $pp\Uparrow \Rightarrow \pi X$

  Polarized hyperon production $pp \Rightarrow \Lambda\Uparrow X$

- Amplitude analyses
- How to parameterize? Define **Transversity** Amplitudes

("Optimally Simple Connection Between the Reaction Matrix and the Observables".

G. R. Goldstein and M. J. Moravcsik, Ann. Phys. (N.Y.) 98, 128 (1976); 142, 219 (1982); 195, 213 (1989).)

- QCD & SSAs? Kane, Pumplin & Repko, PRL41, 1889 (1978).

  Pol or Asym ∝ $\alpha(\hat{s})m_q / \sqrt{\hat{s}}$

SSA's cont'd

- Several SSA directions
  - Λ (WGD Dharmaratna & GG,PRD 41, 1731 (1990); PRD 53, 1073 (1996)) & $\Lambda_{C,B}$
  - Heavy quark$\Uparrow$ fragmentation (A.Adamov and GG, PRD56, 7381 (1997).
- Jaffe & Ji PRL67,552 (1991) introduce $h_1(x)$ – transversity transfer

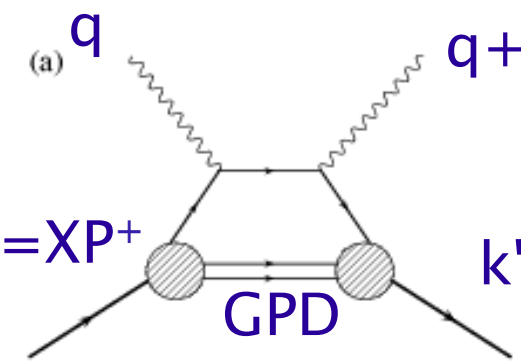
  "Chiral-Odd Parton Distributions and Polarized Drell-Yan Process"
  - How to measure chiral odd? DY or SIDIS
  - or Fragmentation version $\Lambda\Uparrow$ + anti$\Lambda\Uparrow$ (Chen, GG, Jaffe, Ji NPB 445, 380 (1995))

New Millennium

- Transversity $\rightarrow$ tensor current $\Psi^{bar\text{-}q}\,\sigma^{\mu\nu}\,\Psi^{q} \rightarrow \delta q$ (L.Gamberg & GG, PRL 87, 242001 (2001))
- Leptoproduction: SIDIS & TMDs (S.Liuti & GG . . .)
- Leptoproduction: Exclusives & GPDs (π^0 electroproduction)
- Λ in e p scattering, target fragmentation & fracture functions
- Λ in p+p at LHC

DVCS & DVMP $\gamma^*(Q^2)+P\rightarrow(\gamma \text{ or meson})+P'$

partonic picture



k_T
$k^+=XP^+$

$k'_T=k_T-\Delta$
$k'^+=(X-\zeta)P^+$

P^+

$P'^+=(1-\zeta)P^+$
$P'_T=-\Delta$

$\zeta\rightarrow 0$
Regge

Factorized
"handbag"
picture

Quark-spectator
quark+diquark

$X>\zeta$ DGLAP Δ_T -> b_T transverse spatial
$X<\zeta$ ERBL $x=(X-\zeta/2)/(1-\zeta/2)$; $x=\zeta/(2-\zeta)$

see Ahmad, GG, Liuti, PRD79, 054014, (2009) for first chiral odd GPD parameterization focused on pseudoscalar production

GPD definitions – 8 quark + 8 gluon

Momentum space nucleon matrix elements of quark field correlators

see, e.g. M. Diehl, Eur. Phys. J. C 19, 485 (2001).

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\,\bar\psi(-\tfrac{1}{2}z)\,\gamma^+\psi(\tfrac{1}{2}z)\,|p,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0}$$
$$= \frac{1}{2P^+}\bar u(p',\lambda')\left[H^q\gamma^+ + E^q\,\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}\right]u(p,\lambda),$$

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\,\bar\psi(-\tfrac{1}{2}z)\,\gamma^+\gamma_5\psi(\tfrac{1}{2}z)\,|p,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0}$$
$$= \frac{1}{2P^+}\bar u(p',\lambda')\left[\tilde H^q\gamma^+\gamma_5 + \tilde E^q\,\frac{\gamma_5\Delta^+}{2m}\right]u(p,\lambda),$$

Chiral even GPDs
-> Ji sum rule

$$\left\langle J_q^x\right\rangle = \tfrac{1}{2}\int dx\left[H(x,0,0)+E(x,0,0)\right]x$$

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\,\bar\psi(-\tfrac{1}{2}z)\,i\sigma^{+i}\psi(\tfrac{1}{2}z)\,|p,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0}$$
$$= \frac{1}{2P^+}\bar u(p',\lambda')\left[H_T^q\, i\sigma^{+i} + \tilde H_T^q\,\frac{P^+\Delta^i-\Delta^+P^i}{m^2}\right.$$
$$\left. + E_T^q\,\frac{\gamma^+\Delta^i-\Delta^+\gamma^i}{2m} + \tilde E_T^q\,\frac{\gamma^+P^i-P^+\gamma^i}{m}\right]u(p,\lambda).$$

Chiral odd GPDs
-> transversity

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How to determine GPDs?
Flexible Parameterization ➔ Recursive Fit

CHIRAL EVENS: O. Gonzalez Hernandez, G. G., S. Liuti, PRD84, 034007 (2011)

Constraints from Form Factors

$$\int_0^1 dx H(x,\xi,t) = F_1(t)$$

Dirac

$$\int_0^1 dx E(x,\xi,t) = F_2(t)$$

Pauli, etc. including Axial & Pseudoscalar

How can these be independent of ξ?

Constraints from Polynomiality

Result of Lorentz invariance & causality. Not necessarily built into models

$$\int_{-1}^{+1} dx\, x^n H(x,\xi,t) = \sum_{k=0,2,..}^{n} A_{n,k}(t)\xi^k + \frac{1-(-1)^n}{2} C_n(t)\xi^{n+1}$$

$$\int_{-1}^{1} dx\, x^n E(x,\xi,t) = \sum_{k=0,2,...}^{n} B_{n,k}(t)\xi^k - \frac{1-(-1)^n}{2} C_n(t)\xi^{n+1}$$

Chiral odd

Constraints from pdf's: $H(x,0,0)=f_1(x)$, $H\sim(x,0,0)=g_1(x)$, $H_T(x,0,0)=h_1(x)$

Helicity amps (q'+N->q+N') are linear combinations of GPDs

$$A_{+,+;+,+} = \sqrt{1-\xi^2}\left[\frac{H+\tilde{H}}{2} - \frac{\xi^2}{1-\xi^2}\frac{E+\tilde{E}}{2}\right]$$

$$A_{-,+;-,+} = \sqrt{1-\xi^2}\left[\frac{H-\tilde{H}}{2} - \frac{\xi^2}{1-\xi^2}\frac{E-\tilde{E}}{2}\right]$$

$$A_{+,+;-,+} = -\frac{\sqrt{t_0-t}}{4M}(E-\xi\tilde{E})$$

$$A_{-,+;+,+} = \frac{\sqrt{t_0-t}}{4M}(E+\xi\tilde{E})$$

for chiral even GPDs and

T-reversal at ξ =0

$$A_{+-,++} = -\frac{\sqrt{t_0-t}}{2M}\left[\widetilde{H}_T + \frac{1+\xi}{2}E_T - \frac{1+\xi}{2}\widetilde{E}_T\right]$$

$$A_{++,--} = \sqrt{1-\xi^2}\left[H_T + \frac{t_0-t}{4M^2}\widetilde{H}_T - \frac{\xi^2}{1-\xi^2}E_T + \frac{\xi}{1-\xi^2}\widetilde{E}_T\right]$$

$$A_{+-,-+} = -\sqrt{1-\xi^2}\,\frac{t_0-t}{4M^2}\,\widetilde{H}_T$$

$$A_{++,+-} = \frac{\sqrt{t_0-t}}{2M}\left[\widetilde{H}_T + \frac{1-\xi}{2}E_T + \frac{1-\xi}{2}\widetilde{E}_T\right],$$

for chiral odd GPDs, where for consistency with previous literature we have

In diquark spectator models $A_{++;++}$, etc. are calculated directly. Inverted -> GPDs

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Spectator inspired model of GPDs

- 2 directions –
  - 1. getting good parameterization of H, E & ~H, ~E
    satisfying many constraints
    (see O. Gonzalez-Hernandez, GG, S. Liuti - Phys.Rev. D84, 034007 (2011))
  - 2. getting 8 spin dependent GPDs
    - Chiral Odd GPDs π^0 production is testing ground (Ahmad, GG, Liuti, PRD79,054014 (2009), Gonzalez, GG, Liuti, arXiv:1201.6088 [hep-ph])
- Simple Spectator -- scalar diquark –
  helicity amps for P→q ::::: q'→P'
  Spin simplicity - P→q+diq and q'+diq→P' are spin disconnected
  => Chiral even related to Chiral odd GPDs
  H, E, . . ← helicity amp relations → H_T, E_T, . .
- Axial diquark: more complex linear relations
  & distinction between u & d flavors
- Small x & Regge behavior
- Bridge through GPD in helicity or transversity to TMDs?

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Invert to obtain model for GPDs

$A_{++,-+} = - A_{++,+-}$

$A_{-+,++} = - A_{+-,++}$

$A_{++,++} = A_{++,--}$

$$H(x,\xi,t) = \frac{1}{\sqrt{1-\xi^2}}(A_{+,+;+,+} + A_{-,+;-,+}) - \frac{2M\xi^2}{\Delta(1-\xi^2)}(A_{+,+;-,+} - A_{-,+;+,+})$$

$$E(x,\xi,t) = -\frac{2M}{\Delta}(A_{+,+,-,+} - A_{-,+;+,+})$$

$$\tilde{H}(x,\xi,t) = \frac{1}{\sqrt{1-\xi^2}}(A_{+,+;+,+} - A_{-,+;-,+}) + \frac{2M\xi}{\Delta(1-\xi^2)}(A_{+,+,-,+} + A_{-,+;+,+})$$

$$\tilde{E}(x,\xi,t) = \frac{2M}{\Delta\xi}(A_{+,+,-,+} + A_{-,+;+,+})$$

for chiral even GPDs and

$$H_T(x,\xi,t) = \frac{1}{\sqrt{1-\xi^2}}(A_{+,+;-,-} + A_{-,+;+,-}) + \frac{2M\xi}{\Delta(1-\xi^2)}(A_{+,+;+,-} - A_{-,+;-,-})$$

$$\xi E_T(x,\xi,t) - \tilde{E}_T(x,\xi,t) = \frac{2M}{\Delta}(A_{+,+;+,-} - A_{-,+;-,-})$$

$$E_T(x,\xi,t) + \tilde{E}_T(x,\xi,t) = \frac{\Delta}{2M(1-\xi)}[2A_{+,+;+,-} + \frac{4M}{\Delta\sqrt{1-\xi^2}}A_{-,+;+,-}]$$

double flip

$$\tilde{H}_T(x,\xi,t) = \frac{4M^2}{\Delta^2\sqrt{1-\xi^2}}A_{-,+;+,-}$$

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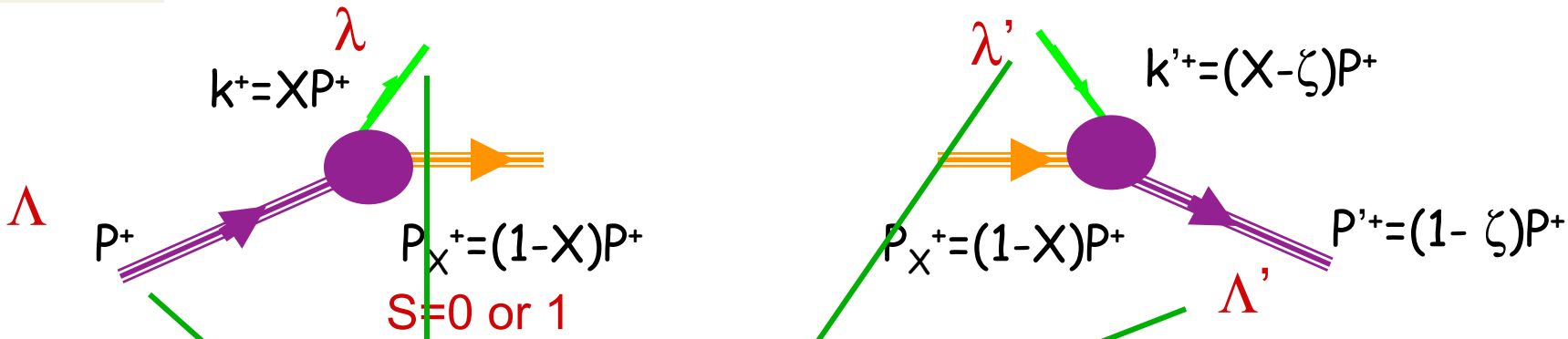
Vertex Structures



First focus e.g. on S=0 pure spectator

$$H \Rightarrow \varphi^*_{++}(k',P')\varphi_{++}(k,P) + \varphi^*_{-+}(k',P')\varphi_{-+}(k,P)$$

$$E \Rightarrow \varphi^*_{++}(k',P')\varphi_{+-}(k,P) + \varphi^*_{+-}(k',P')\varphi_{++}(k,P)$$

$$\tilde{H} \Rightarrow \varphi^*_{++}(k',P')\varphi_{++}(k,P) - \varphi^*_{-+}(k',P')\varphi_{-+}(k,P)$$

$$\tilde{E} \Rightarrow \varphi^*_{++}(k',P')\varphi_{+-}(k,P) - \varphi^*_{+-}(k',P')\varphi_{++}(k,P)$$

Vertex functions

$$\phi(k^2,\lambda) = \frac{k^2 - m^2}{|k^2 - \lambda^2|^2}.$$

Fitting Procedure e.g. for H and E

- Fit at $\zeta=0$, $t=0$ => $H_q(x,0,0)=q(X)$
  - 3 parameters per quark flavor (M_X^q, Λ_q, α_q) + initial Q_o^2
- Fit at $\zeta=0$, $t\neq0$ ⇒

$$\int_0^1 dX\, H^q(X,t) = F_1^q(t)$$

$$\int_0^1 dX\, E^q(X,t) = F_2^q(t),$$

  - 2 parameters per quark flavor (β, p)

$$R = X^{-[\alpha+\alpha'(1-X)^p t+\beta(\varsigma)t]}$$ – Regge factor

$$G^{\lambda}_{M_X}(X,t) = \mathcal{N}\frac{X}{1-X}\int d^2\mathbf{k}_\perp \frac{\phi(k^2,\lambda)}{D(X,\mathbf{k}_\perp)}\frac{\phi(k'^2,\lambda)}{D(X,\mathbf{k}_\perp+(1-X)\mathbf{\Delta}_\perp)}$$ Quark-Diquark

- Fit at $\zeta\neq0$, $t\neq0$ ⇒ DVCS, DVMP,… data (convolutions of GPDs with Wilson coefficient functions) + lattice results (Mellin Moments of GPDs)
- Note! This is a multivariable analysis ⇒ see e.g. Moutarde, Kumericki and D. Mueller, Guidal and Moutarde

Reggeized diquark mass formulation

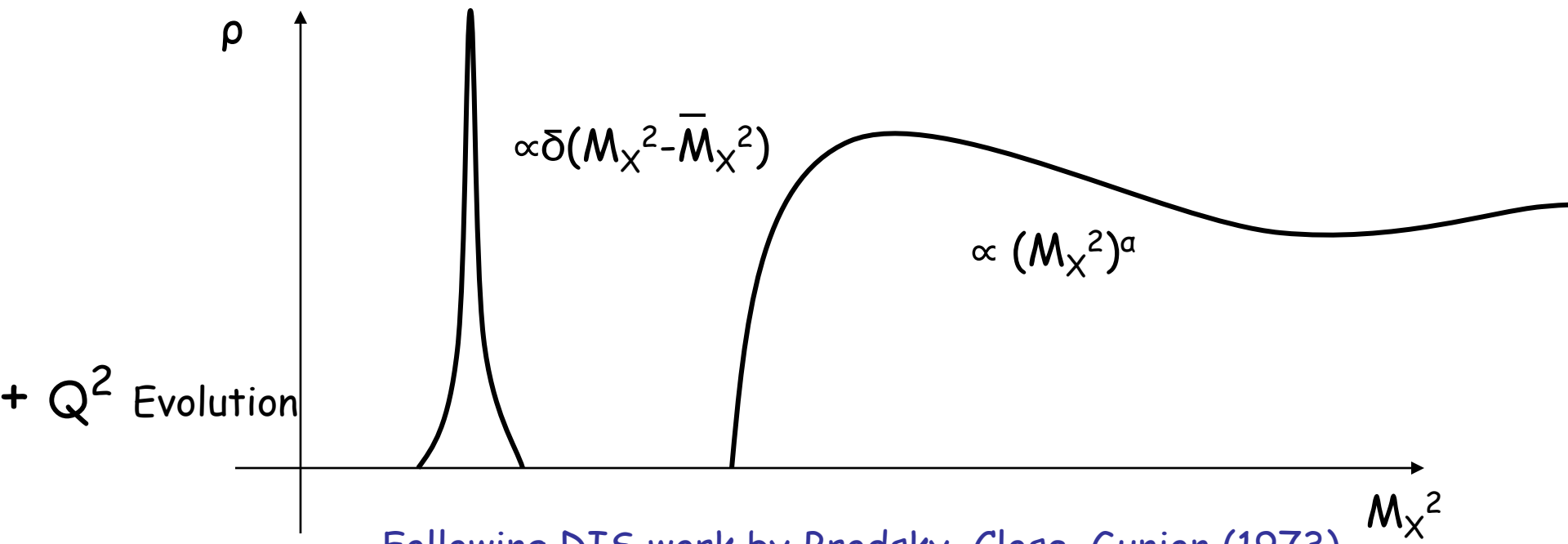
Where does the Regge behavior come from?

$$G^{\Lambda^2}_{M_X}(X,\zeta,t) = \int d^2\mathbf{k}_\perp \int dM_X^2\, \rho(M_X^2)\, \frac{\phi(k^2,\Lambda^2)}{k^2 - M_X^2}\, \frac{\phi(k'^2,\Lambda^2)}{k'^2 - m^2}\, \mathcal{N}_{\mathcal{H}(\mathcal{E},\tilde{\mathcal{H}},\tilde{\mathcal{E}})} \qquad \zeta \geq X$$

Diquark spectral function

$$F(X,\zeta,t) = \mathcal{N}\, G^{M_\Lambda}_{M_X,m}(X,\zeta,t)\, R_p^{\alpha,\alpha'}(X,\zeta,t)$$

"Regge"



Following DIS work by Brodsky, Close, Gunion (1973)

Chiral Even Parameterizations

Flexible Parametrization of Generalized Parton Distributions from Deeply Virtual Compton Scattering Observables
Gary R. Goldstein, J. Osvaldo Gonzalez Hernandez , Simonetta Liuti arXiv:1012.3776

Parameters	H	E	$\widetilde{H}$	$\widetilde{E}$
m_u (GeV)	0.420	0.420	2.624	2.624
M_X^u (GeV)	0.604	0.604	0.474	0.474
M_Λ^u (GeV)	1.018	1.018	0.971	0.971
α_u	0.210	0.210	0.219	0.219
α'_u	2.448 ± 0.0885	2.811 ± 0.765	1.543 ± 0.296	5.130 ± 0.101
p_u	0.620 ± 0.0725	0.863 ± 0.482	0.346 ± 0.248	3.507 ± 0.054
$\mathcal{N}_u$	2.043	1.803	0.0504	1.074
χ^2	0.773	0.664	0.116	1.98
m_d (GeV)	0.275	0.275	2.603	2.603
M_X^d (GeV)	0.913	0.913	0.704	0.704
M_Λ^d (GeV)	0.860	0.860	0.878	0.878
α_d	0.0317	0.0317	0.0348	0.0348
α'_d	2.209 ± 0.156	1.362 ± 0.585	1.298 ± 0.245	3.385 ± 0.145
p_d	0.658 ± 0.257	1.115 ± 1.150	0.974 ± 0.358	2.326 ± 0.137
$\mathcal{N}_d$	1.570	-2.800	-0.0262	-0.966
χ^2	0.822	0.688	0.110	1.00

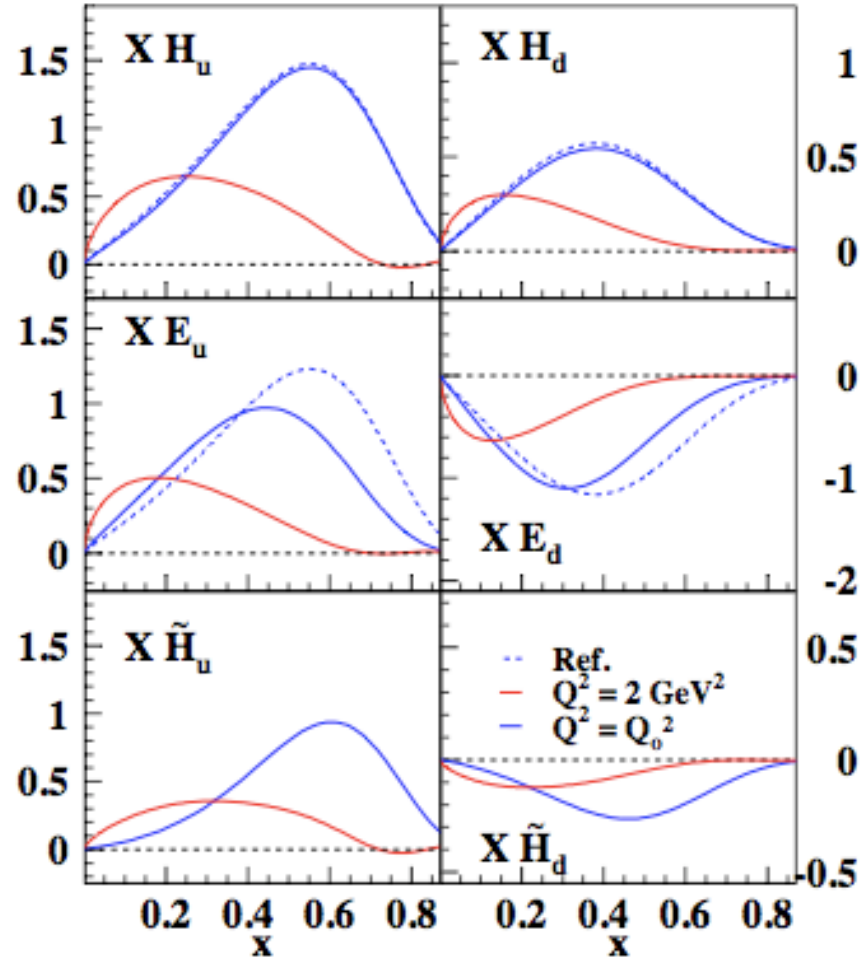


FIG. 6: (color online) GPDs $F_q(X,0,0) \equiv \{H_q, E_q, \widetilde{H}_q\}$, for $q = u$ (left) and $q = d$ (right), evaluated at the initial scale, $Q_o^2 = 0.0936$ GeV^2, and at $Q^2 = 2$ GeV^2, respectively. The dashed lines were calculated using the model in Refs. [24], [25] at the initial scale.

Compton Form Factors
→ Real & Imaginary Parts

$$\mathcal{H}_q(\zeta,t,Q^2) = \int_{-1+\zeta}^{+1} dX H_q(X,\zeta,t,Q^2) \times \left(\frac{1}{X-\zeta+i\epsilon} + \frac{1}{X-i\epsilon}\right)$$

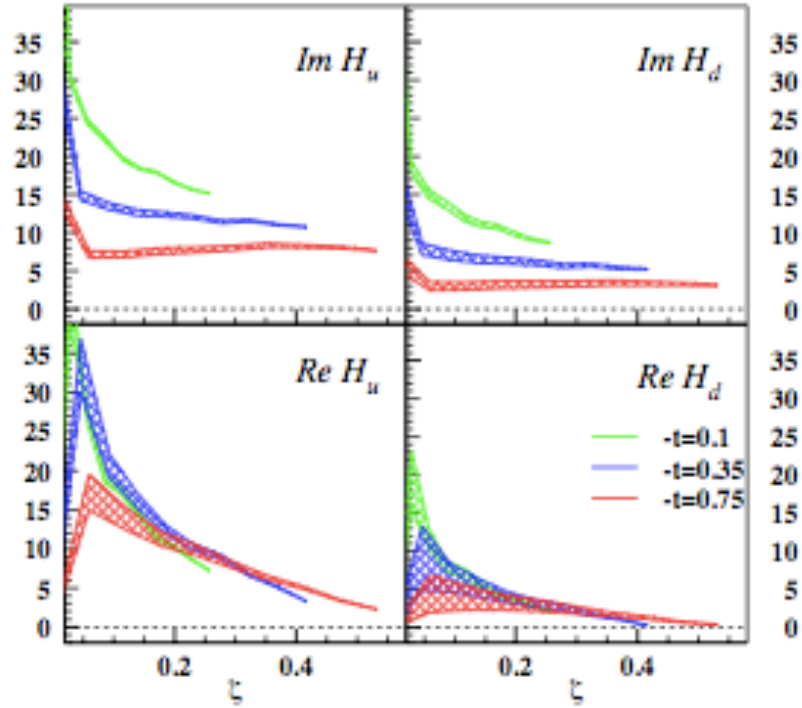


FIG. 9: (color online) Real and imaginary parts of the CFFs, $\mathcal{H}_i(\zeta,t)$, entering Eqs.(85). The CFFs are plotted vs. $x_{Bj} \equiv \zeta$, for different values of t, at $Q^2 = 2$ GeV^2. They are shown with the theoretical uncertainty from the parameters in TableI. Similar results are obtained for E and $\widetilde{H}$.

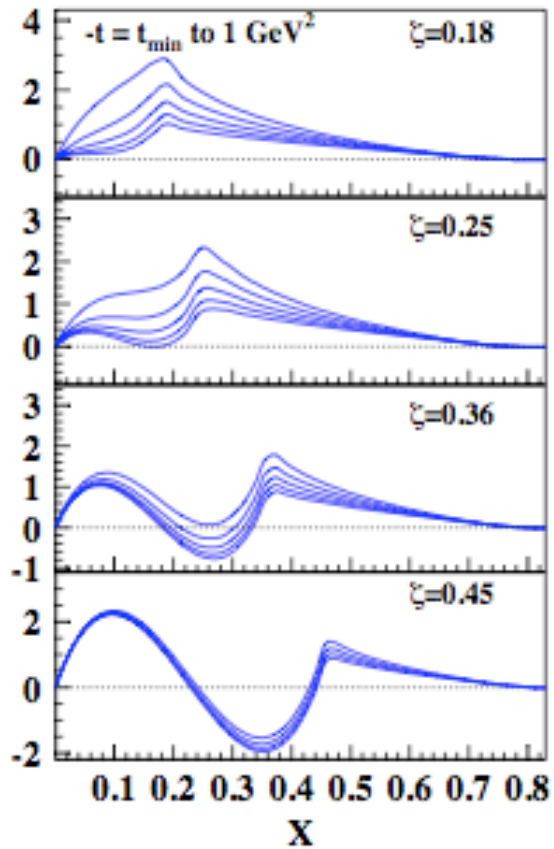


FIG. 7: $H_u(X, \zeta, t; Q^2)$ evaluated at $Q^2 = 2$ GeV^2. Each panel shows H_u plotted vs. X at different values of $\zeta = 0.18, 0.25, 0.36, 0.45$. For each value of ζ several curves are shown that correspond to a range of values in $-t$ from $t_{min} = -M^2\zeta^2/(1-\zeta)$ to 1 GeV^2.

Observables

$$A_{LU} = \frac{\frac{d^4\sigma^\uparrow}{d\Phi} - \frac{d^4\sigma^\downarrow}{d\Phi}}{\frac{d^4\sigma^\uparrow}{d\Phi} + \frac{d^4\sigma^\downarrow}{d\Phi}} = \frac{\Gamma(I^\uparrow - I^\downarrow)}{2\,d\sigma/d\Phi},$$

$$\frac{d\sigma}{d\Phi} = \frac{1}{2}\left[\frac{d^4\sigma^\uparrow}{d\Phi} + \frac{d^4\sigma^\downarrow}{d\Phi}\right] = \Gamma\left[\mid T_{BH}\mid^2 + \mid T_{DVCS}\mid^2 + \frac{I^\uparrow + I^\downarrow}{2}\right],$$

Note GPD & Bethe-Heitler separate amplitudes
-> interference linear in GPD

$$A_C = \frac{\frac{d^4\sigma^+}{d\Phi} - \frac{d^4\sigma^-}{d\Phi}}{\frac{d^4\sigma^+}{d\Phi} + \frac{d^4\sigma^-}{d\Phi}} = \frac{\Gamma(I^\uparrow + I^\downarrow)}{2\,d\sigma/d\Phi},$$

$$A_{UT}^{DVCS} = \frac{1}{S_\perp}\frac{\left(\frac{d^4\sigma^+_\Leftarrow}{d\Phi} - \frac{d^4\sigma^+_\Rightarrow}{d\Phi}\right) + \left(\frac{d^4\sigma^-_\Leftarrow}{d\Phi} - \frac{d^4\sigma^-_\Rightarrow}{d\Phi}\right)}{\frac{d^4\sigma^+}{d\Phi} + \frac{d^4\sigma^-}{d\Phi}} = \frac{\Gamma\mid T_{TP}^{DVCS}\mid^2}{2\,d\sigma/d\Phi},$$

$$A_{UT}^{I} = \frac{1}{S_\perp}\frac{\left(\frac{d^4\sigma^+_\Leftarrow}{d\Phi} - \frac{d^4\sigma^+_\Rightarrow}{d\Phi}\right) - \left(\frac{d^4\sigma^-_\Leftarrow}{d\Phi} - \frac{d^4\sigma^-_\Rightarrow}{d\Phi}\right)}{\frac{d^4\sigma^+}{d\Phi} + \frac{d^4\sigma^-}{d\Phi}} = \frac{\Gamma\mid T_{TP}^{I}\mid^2}{2\,d\sigma/d\Phi}$$

Hall B data
$A_{LU}(90^o)$

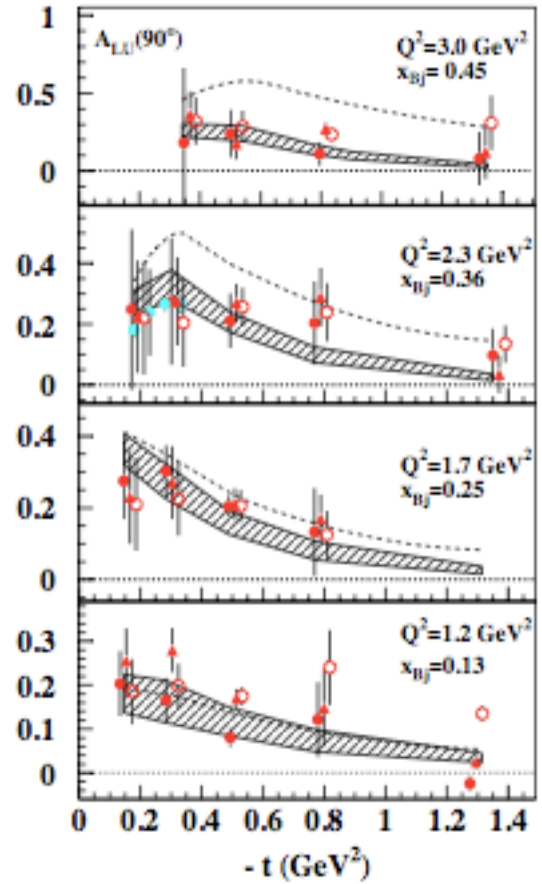


FIG. 10: Beam spin asymmetry, $A_{LU}(\phi = 90^o)$ in 12 of the x_{Bj} and Q^2 bins measured in Hall B [51]. The data points were extracted by fitting $A_{LU}(\phi)$, however they do not represent the uncertainties reported in the experimental analysis. The second panel from the top includes also data from Hall A [49]. The full circles, open circles, and triangles represent data in similar x_{Bj} and t bins, but at Q^2 values slightly displaced around the value reported in the legend. All curves were obtained at the kinematics displayed in the figure. Dashed lines: results from the fit using only the PDF and form factors constraints as from Table I. The full lines represent the effect of introducing the ζ dependent term, Eq.(34), in the numerator of the asymmetry only.

Hermes data

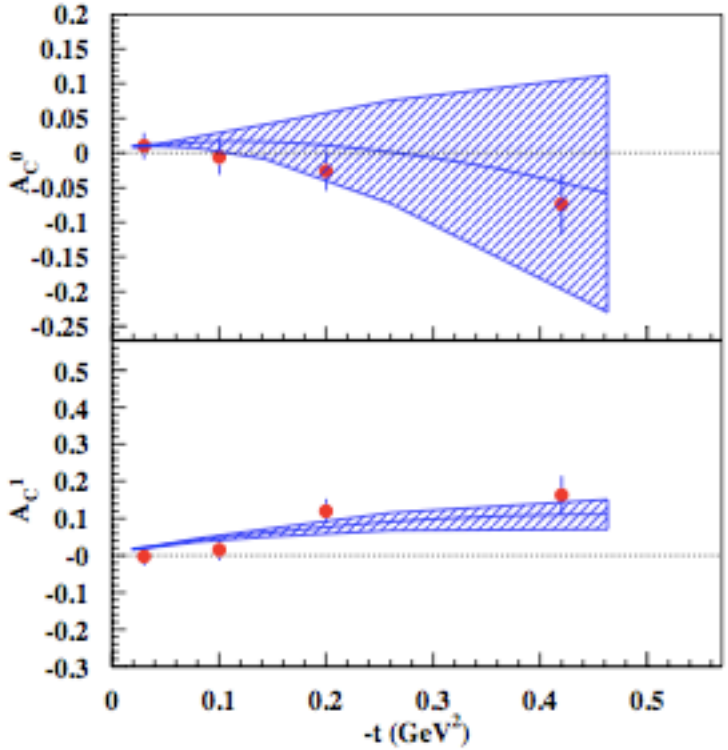


FIG. 20: Coefficients of the beam charge asymmetry, A_C, extracted from experiment [52, 53]. The lower panel is the coefficient for the $\cos\phi$ dependent term in Eq.(82), while the upper panel is the $\cos\phi$ independent term.

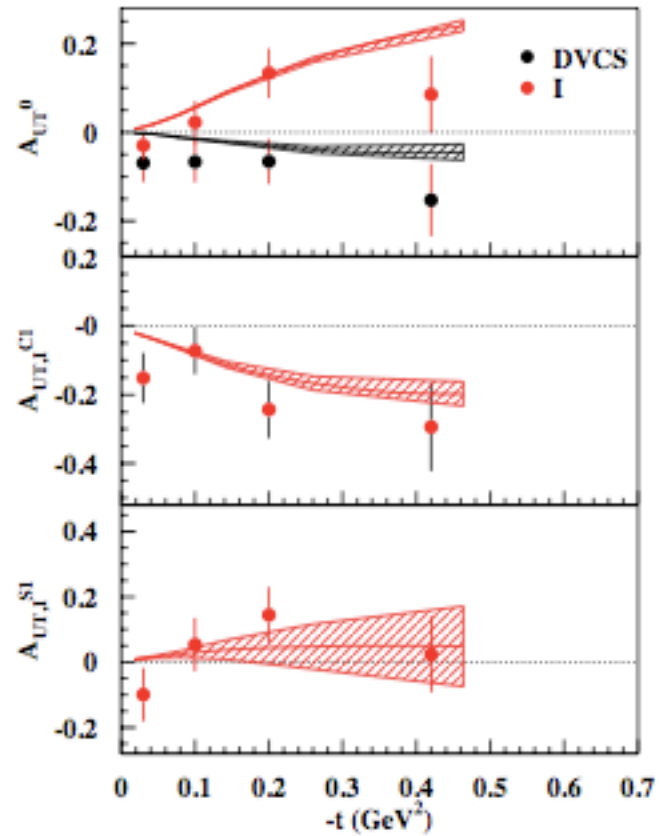
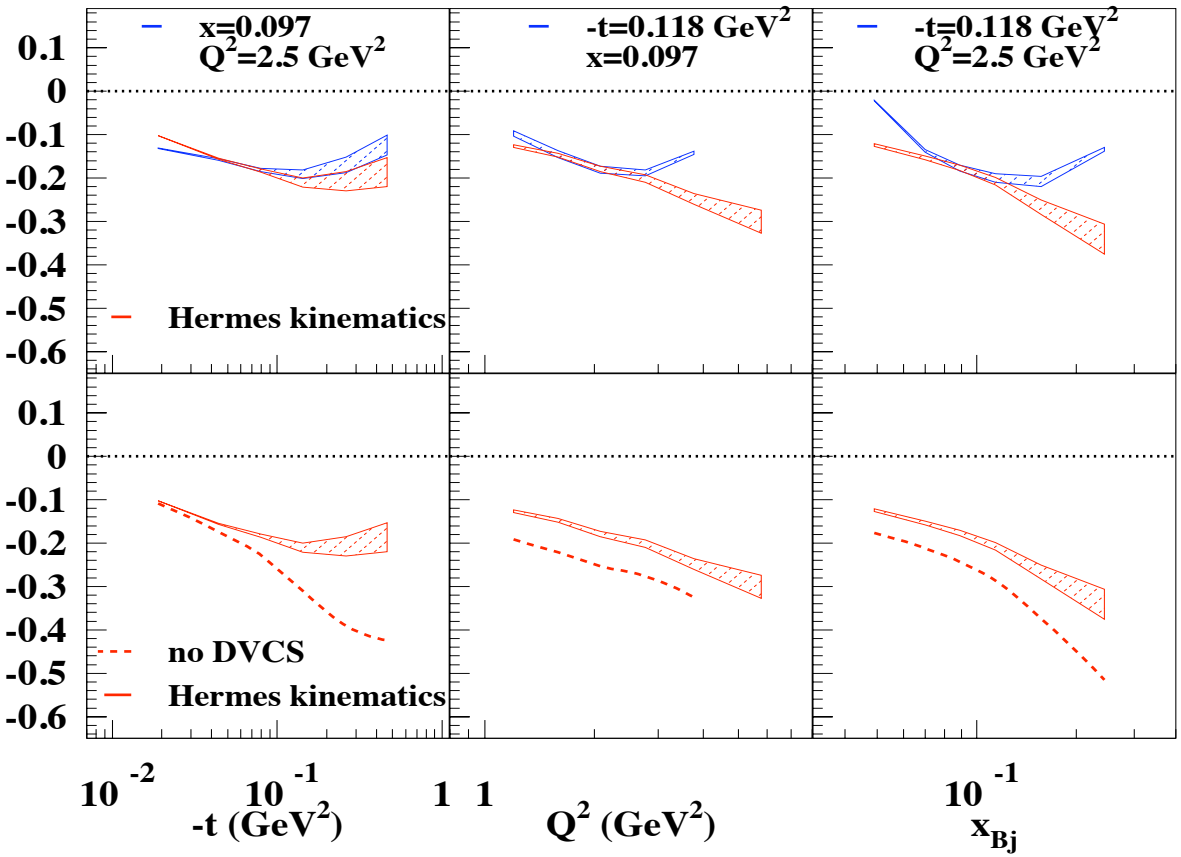


FIG. 21: Coefficients of the [redacted] A_{UT}, extracted from experiment [52, 53]. The upper panel shows the terms E and F from Eqs.(83) and (84), respectively; the middle panel shows G, and the lower panel H, both in Eq.(84). The curves are predictions obtained extending our quantitative fit of Jefferson lab data to the Hermes set of observables.

$A_{LU}(90^o)$


x=0.097
Q²=2.5 GeV²
-t=0.118 GeV²
x=0.097
-t=0.118 GeV²
Q²=2.5 GeV²
Hermes kinematics
no DVCS
Hermes kinematics
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
10 -2
10 -1
1
-t (GeV²)
1
Q² (GeV²)
10 -1
x_Bj

Interference contribution to cos(ϕ) term in DVCS cross section

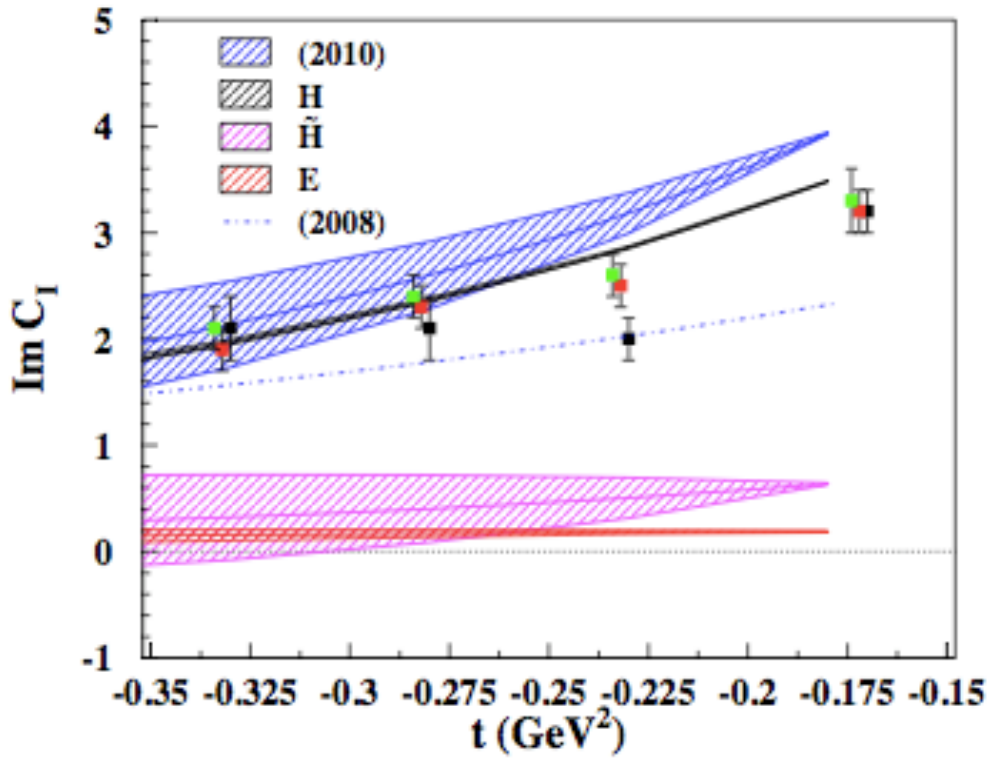


FIG. 17: Coefficient C, Eqs.(72) extracted from Hall A data [49]. Shown are the contributions from the GPDs, H, E and $\tilde{H}$. All curves include the term in Eq.(34). A comparison with a previous prediction based on a simplified diquark model, and including H only [25] is also shown.

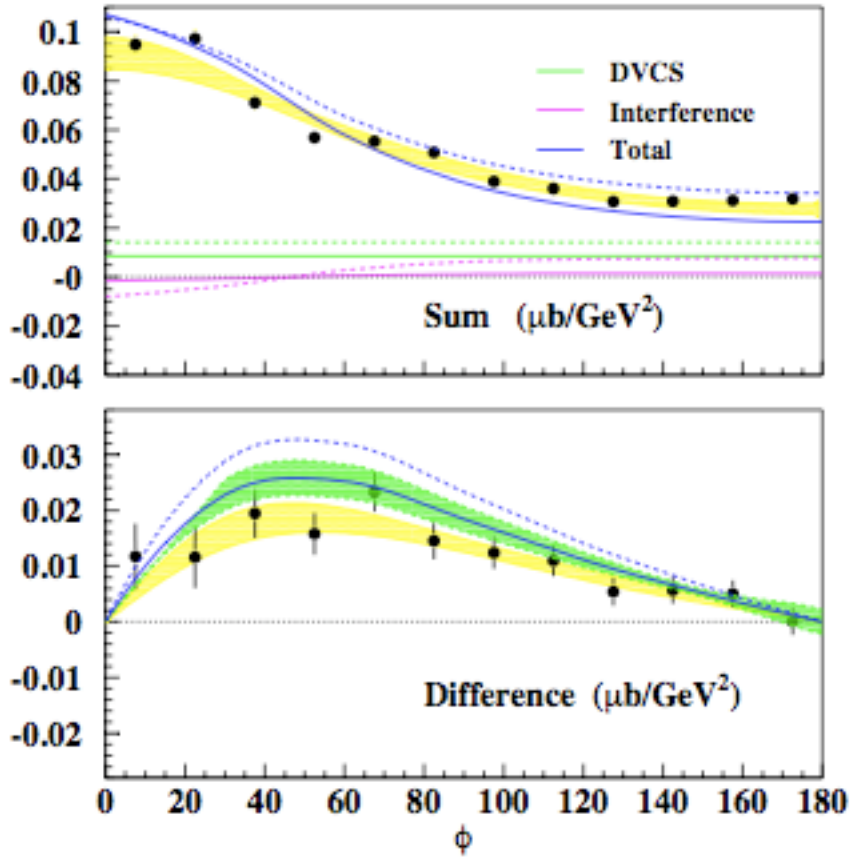
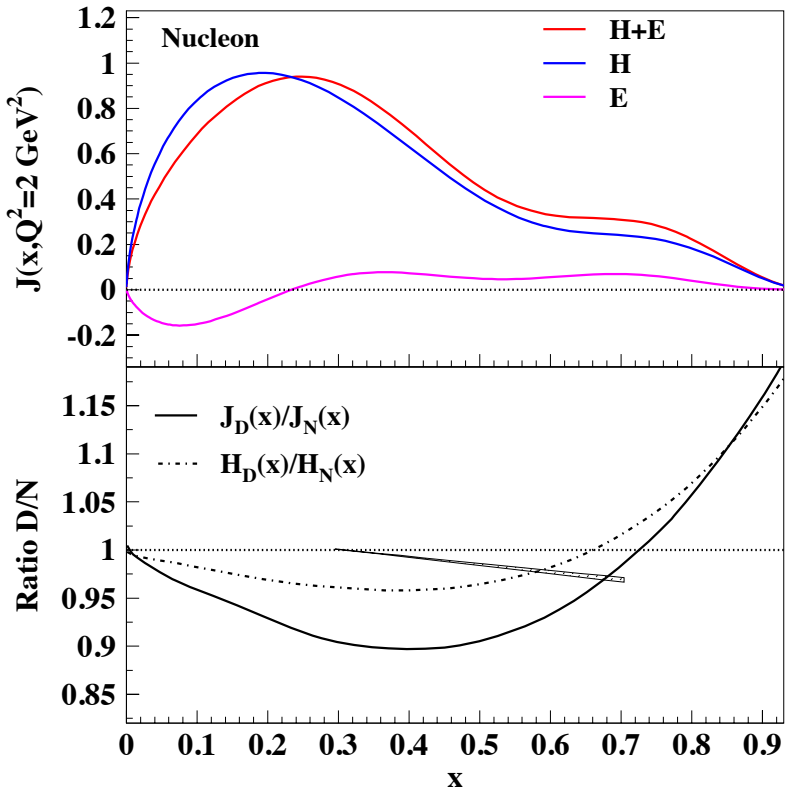
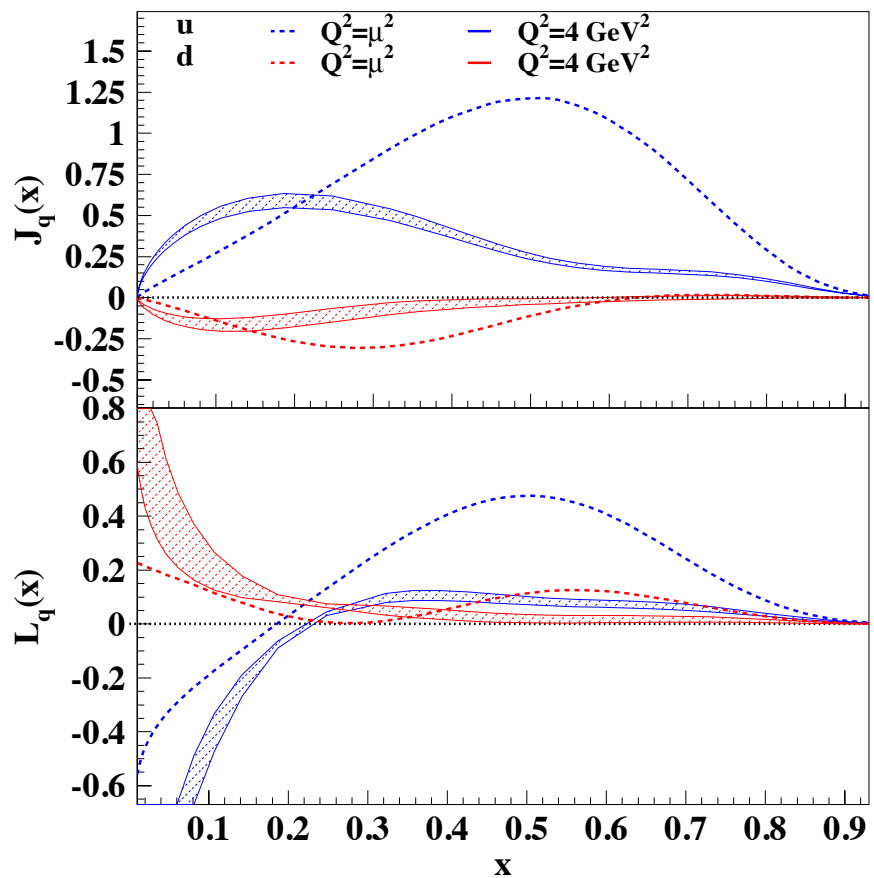


FIG. 11: HallA data [47] for the "sum" (upper panel) and "difference" (power panel) of the two electron beam polarizations. Shown are curves including the contribution of the ζ dependent factor from Eq.(33) (full lines), and neglecting it (dashed lines). All terms (DVCS, Interference and Total are shown for the "sum" graph.

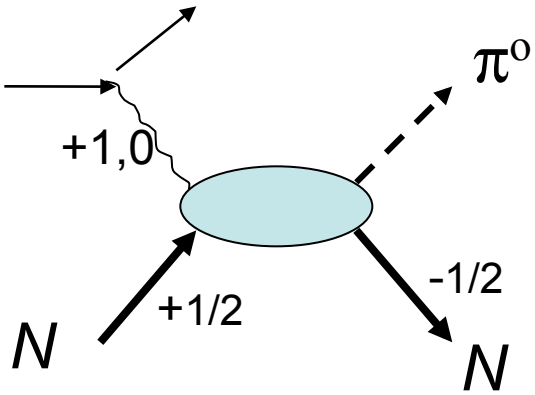




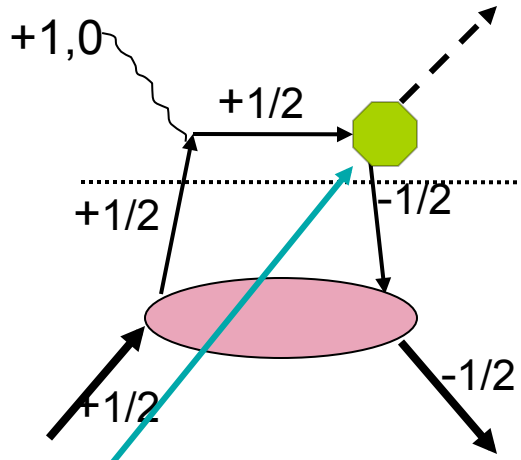
Deuteron Spin Sum Rule

Taneja, Katuria, Liuti, GG arXiV 2012

Exclusive Lepto-production of π^o or η, η'
to measure chiral odd GPDs



e.g. $f_{+1+,0-}(s,t,Q^2)$



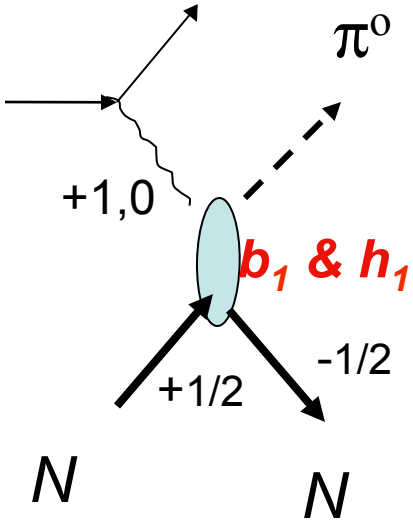
q nos of C-odd
1^{--} exchange
ρ & ω
1^{+-} exchange
b_1 & h_1

$g_{+1+,0-}$ $A_{++,--}$

H_T

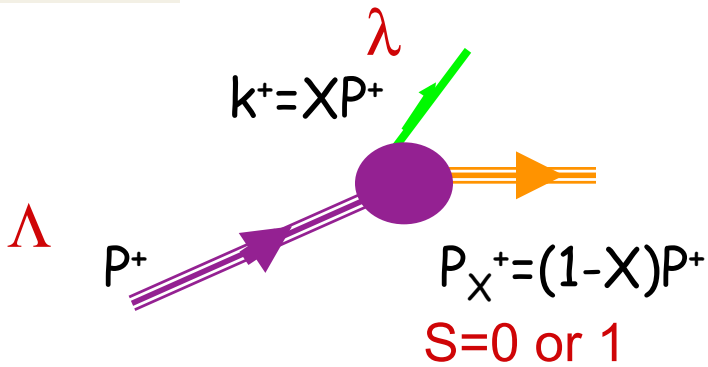
What about coupling of π to $q \to q'$? Assumed γ^5 vertex
Then for $m_{quark}=0$ has to flip helicity
for $q \to \pi + q'$ and $\mathbf{q} \times \mathbf{q}' \neq \mathbf{0}$.

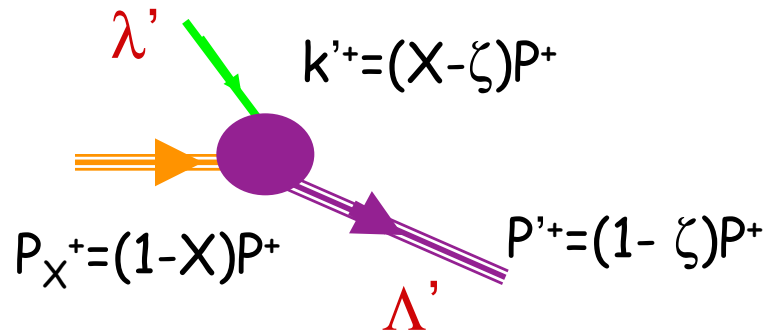
Naïve twist 3 $\psi_{bar}\, \gamma^5\, \psi$



Rather than $\gamma^\mu\gamma^5$ – does not flip ***twist 2.*** But $q'\ \gamma^\mu\gamma^5 q$ will not contribute to transverse γ^*. Differs from t-channel approach to Regge factorization

Vertex Structures





First focus e.g. on S=0 pure spectator

$$H \Rightarrow \varphi^*_{++}(k',P')\varphi_{++}(k,P) + \varphi^*_{-+}(k',P')\varphi_{-+}(k,P)$$

$$E \Rightarrow \varphi^*_{++}(k',P')\varphi_{+-}(k,P) + \varphi^*_{+-}(k',P')\varphi_{++}(k,P)$$

$$\tilde{H} \Rightarrow \varphi^*_{++}(k',P')\varphi_{++}(k,P) - \varphi^*_{-+}(k',P')\varphi_{-+}(k,P)$$

$$\tilde{E} \Rightarrow \varphi^*_{++}(k',P')\varphi_{+-}(k,P) - \varphi^*_{+-}(k',P')\varphi_{++}(k,P)$$

Note that by switching λ➔- λ & Λ ➔ -Λ (Parity) will have chiral evens go to ± chiral odds giving relations – <u>before k integrations</u>

A(Λ'λ';Λλ)➔ ±A(Λ',λ';-Λ,-λ)

Vertex function

$$\phi(k^2,\lambda) = \frac{k^2 - m^2}{|k^2 - \lambda^2|^2}.$$

but then (Λ'-λ')-(Λ-λ) ≠(Λ'-λ')+(Λ-λ) unless Λ=λ

S=0 Chiral even <-> odd

$$A^{(0)}_{++,--} = A^{(0)}_{++,++}$$
$$A^{(0)}_{++,+-} = -A^{(0)}_{++,-+}$$
$$A^{(0)}_{+-,++} = -A^{(0)}_{-+,++},$$

Invert to get GPDs

$$\widetilde{H}^0_T = -(1-\zeta)^2 \frac{M(1-x)}{m+Mx'}\left[E^0 - \frac{\zeta}{2}\widetilde{E}^0\right]$$

$$E^0_T = -\frac{(1-\zeta/2)^2}{1-\zeta}\left[2\widetilde{H}^0_T - E^0 + \left(\frac{\zeta/2}{1-\zeta/2}\right)^2 \widetilde{E}^0\right]$$

$$\widetilde{E}^0_T = -\frac{\zeta/2(1-\zeta/2)}{1-\zeta}\left[2\widetilde{H}^0_T - E^0 + \widetilde{E}^0\right]$$

$$H^0_T = \frac{H^0+\widetilde{H}^0}{2} - \frac{\zeta^2/4}{1-\zeta}\frac{E^0+\widetilde{E}^0}{2} - \frac{\zeta^2/4}{(1-\zeta/2)(1-\zeta)}E^0_T + \frac{\zeta/4(1-\zeta/2)}{1-\zeta}\widetilde{E}^0_T + \widetilde{H}^0_T,$$

$S=0$ double helicity flip amplitude was calculated directly from Eq.(16),

$$A^{(0)}_{+-,-+} = \frac{t_0-t}{4M}\frac{1}{\sqrt{1-\zeta}}\frac{1}{(1-\zeta/2)}\frac{\tilde{x}}{m+Mx'}\left[E^0 - \frac{\zeta}{2}\widetilde{E}^0\right].$$

S=1 Chiral even <-> odd

$$
\begin{aligned}
A^{(1)}_{++,--} &= -\frac{x+x'}{1+xx'}\, A^{(1)}_{++,++} \\
A^{(1)}_{+-,-+} &= 0 \\
A^{(1)}_{++,+-} &= -\sqrt{\frac{\langle \tilde{k}_\perp^2 \rangle}{x'^{\,2} + \langle \tilde{k}_\perp^2 \rangle / P^{+\,2}}}\, A^{(1)}_{++,-+} \\
A^{(1)}_{+-,++} &= -\sqrt{\frac{\langle k_\perp^2 \rangle}{x^2 + \langle k_\perp^2 \rangle / P^{+\,2}}}\, A^{(1)}_{-+,++},
\end{aligned}
$$

Invert to get GPDs

$$
\begin{aligned}
\widetilde{H}_T^{(1)} &= 0 \\
E_T^{(1)} &= \frac{1-\zeta/2}{1-\zeta}\left[\tilde{a}\left(E^{(1)} - \frac{\zeta/2}{1-\zeta/2}\widetilde{E}^{(1)}\right) + a\left(E^{(1)} + \frac{\zeta/2}{1-\zeta/2}\widetilde{E}^{(1)}\right)\right] \\
\widetilde{E}_T^{(1)} &= \frac{1-\zeta/2}{1-\zeta}\left[\tilde{a}\left(E^{(1)} - \frac{\zeta/2}{1-\zeta/2}\widetilde{E}^{(1)}\right) - a\left(E^{(1)} + \frac{\zeta/2}{1-\zeta/2}\widetilde{E}^{(1)}\right)\right] \\
H_T^{(1)} &= -\frac{x+x'}{1+xx'}\left[\frac{H^{(1)}+\widetilde{H}^{(1)}}{2} - \frac{\zeta^2/4}{1-\zeta}\frac{E^{(1)}+\widetilde{E}^{(1)}}{2}\right] - \frac{\zeta^2/4}{1-\zeta}E_T^{(1)} + \frac{\zeta/4}{1-\zeta}\widetilde{E}_T^{(1)}
\end{aligned}
$$

Chiral odd amplitudes

$$A_{+-,++} = -A_{-+,--} = \int d^2k_\perp \phi^*_{+-}(k',P')\phi_{++}(k,P)$$

$$= -\frac{\sqrt{t_0-t}}{2M}\left[\widetilde{H}_T + \frac{1+\xi}{2}E_T - \frac{1+\xi}{2}\widetilde{E}_T\right]$$

$$A_{++,--} = \int d^2k_\perp \phi^*_{++}(k',P')\phi_{--}(k,P)$$

$$= \sqrt{1-\xi^2}\left[H_T + \frac{t_0-t}{4M^2}\widetilde{H}_T - \frac{\xi^2}{1-\xi^2}E_T\right]$$

$$A_{+-,-+} = \int d^2k_\perp \phi^*_{+-}(k',P')\phi_{-+}(k,P)$$

$$= -\sqrt{1-\xi^2}\,\frac{t_0-t}{4M^2}\,\widetilde{H}_T$$

$$A_{++,+-} = \int d^2k_\perp \phi^*_{++}(k',P')\phi_{+-}(k,P)$$

$$= \frac{\sqrt{t_0-t}}{2M}\left[\widetilde{H}_T + \frac{1-\xi}{2}E_T + \frac{1-\xi}{2}\widetilde{E}_T\right]$$

Which GPDs involved in π^0

- Consider t-channel $\gamma^* \pi^0 \rightarrow$ N+antiN

S/L	0	1	2	3	4	...
0	0^{-+}	1^{+-}	2^{-+}	3^{+-}	4^{-+}	
1	1^{--}	0^{++}	1^{--}	2^{++}	3^{--}	
		1^{++}	2^{--}	3^{++}	4^{--}	
		2^{++}	3^{--}	4^{++}	5^{--}	

TABLE I: J^{PC} of the $N\bar{N}$ states.

Axial Vector operators (S;L)

n	$J^{PC}(S;L)$				
0	$0^{-+}(0;0)$	$1^{++}(1;1)$			
1	0^{--}	$1^{+-}(0;1)$	$2^{--}(1;2)$		
2	$0^{-+}(0;0)$	$1^{++}(1;1)$	$2^{-+}(0;2)$	$3^{++}(1;3)$	
3	0^{--}	$1^{+-}(0;1)$	$2^{--}(1;2)$	$3^{+-}(0;3)$	$4^{--}(1;4)$
...			...		

J^{PC} for chiral even GPDs

distribution	J^{PC}	
$H^q(x,\xi,t) - H^q(-x,\xi,t)$	$0^{++}, 2^{++}, \ldots$	S=1 crossing even
$E^q(x,\xi,t) - E^q(-x,\xi,t)$	$0^{++}, 2^{++}, \ldots$	
$\widetilde{H}^q(x,\xi,t) + \widetilde{H}^q(-x,\xi,t)$	$1^{++}, 3^{++}, \ldots$	←S=1 crossing odd
$\widetilde{E}^q(x,\xi,t) + \widetilde{E}^q(-x,\xi,t)$	$0^{-+}, 1^{++}, 2^{-+}, 3^{++}, \ldots$	S=0 crossing even & S=1 crossing odd
$H^q(x,\xi,t) + H^q(-x,\xi,t)$	$1^{--}, 3^{--}, \ldots$	S=1 crossing odd
$E^q(x,\xi,t) + E^q(-x,\xi,t)$	$1^{--}, 3^{--}, \ldots$	
$\widetilde{H}^q(x,\xi,t) - \widetilde{H}^q(-x,\xi,t)$	$2^{--}, 4^{--}, \ldots$	S=1 crossing even
$\widetilde{E}^q(x,\xi,t) - \widetilde{E}^q(-x,\xi,t)$	$1^{+-}, 2^{--}, 3^{+-}, 4^{--}, \ldots$	S=0 crossing odd & S=1 crossing even

Hence only *E*~ will enter in p^0 but will be suppressed by Dx or x^2.

Tables: See Lebed & Ji, PRD63,076005 (2001); Diehl & Ivanov, Eur. Phys. Jour. C52, 919 (2007)

J^{PC} for chiral odd GPDs

- 2 series for each GPD, space-space or time-space tensor from $\sigma^{\mu\nu}$. μ-> + in light front frame.
- Indices become (+,1) or (+,2), so mixtures.
- see P. Haegler, PLB 594 (2004) 164–170; Z.Chen & X.Ji, PRD 71, 016003 (2005)

n	σ^{0j}	$J^{PC}(S;L,L')$			σ^{jk}	$J^{PC}(S;L)$		
0	$1^{--}(1;0,2)$				$1^{+-}(0;1)$			
1	1^{-+}	$2^{++}(1;1,3)$			$1^{++}(1;1)$	$2^{-+}(0;2)$		
2	$1^{--}(1;0,2)$	2^{+-}	$3^{--}(1;2,4)$		$1^{+-}(0;1)$	$2^{--}(1;2)$	$3^{+-}(0;3)$	
3	1^{-+}	$2^{++}(1;1,3)$	3^{-+}	$4^{++}(1;3,5)$	$1^{++}(1;1)$	$2^{-+}(0;2)$	$3^{++}(1;3)$	$4^{-+}(0;4)$
...				...				...

TABLE III: J^{PC} of the tensor operators σ^{0j} and σ^{jk} with $(S;L)$ for the corresponding $N\bar{N}$ state.

	$L=0$	1	2	3	4	...
$\Lambda_\gamma = 0$	1^{+-}		$1,2,3^{+-}$		$3,4,5^{+-}$	
$\lvert\Lambda_\gamma\rvert = 1$	1^{+-}	$0,1,2^{--}$	$1,2,3^{+-}$	$2,3,4^{--}$	$3,4,5^{+-}$	

TABLE IV: J^{PC} of the $\gamma^*\pi^0$ states.

$H_T, E_T, \tilde{H}_T$

$\tilde{E}_T$

lowest J values have lowest L for N-Nbar states & are nearest meson singularities

GPDs & J^{PC}

- Even and odd under crossing

Chiral Even GPD	J^{PC}	
$H(x,\xi,t) - H(-x,\xi,t)$	$0^{++}, 2^{++}, \ldots$	$(S=1)$
$E(x,\xi,t) - E(-x,\xi,t)$	$0^{++}, 2^{++}, \ldots$	$(S=1)$
$\widetilde{H}(x,\xi,t) + \widetilde{H}(-x,\xi,t)$	$1^{++}, 3^{++}, \ldots$	$(S=1)$
$\widetilde{E}(x,\xi,t) + \widetilde{E}(-x,\xi,t)$	$0^{-+}, 1^{++}, 2^{-+}, 3^{++}, \ldots$	$(S=0,1)$
$H(x,\xi,t) + H(-x,\xi,t)$	$1^{--}, 3^{--}, \ldots$	$(S=1)$
$E(x,\xi,t) + E(-x,\xi,t)$	$1^{--}, 3^{--}, \ldots$	$(S=1)$
$\widetilde{H}(x,\xi,t) - \widetilde{H}(-x,\xi,t)$	$2^{--}, 4^{--}, \ldots$	$(S=1)$
$\widetilde{E}(x,\xi,t) - \widetilde{E}(-x,\xi,t)$	$1^{+-}, 2^{--}, 3^{+-}, 4^{--}, \ldots$	$(S=0,1)$

Chiral Odd GPD	J^{-C}		J^{+C}	
$H_T(x,\xi,t) - H_T(-x,\xi,t)$	$2^{-+}, 4^{-+}, \ldots$	$(S=0)$	$1^{++}, 3^{++} \ldots$	$(S=1)$
$E_T(x,\xi,t) - E_T(-x,\xi,t)$	$2^{-+}, 4^{-+}, \ldots$	$(S=0)$	$1^{++}, 3^{++} \ldots$	$(S=1)$
$\widetilde{H}_T(x,\xi,t) - \widetilde{H}_T(-x,\xi,t)$			$1^{++}, 3^{++}, \ldots$	$(S=1)$
$\widetilde{E}_T(x,\xi,t) - \widetilde{E}_T(-x,\xi,t)$	$2^{-+}, 4^{-+}, \ldots$	$(S=0)$	$3^{++}, 5^{++} \ldots$	$(S=1)$
$H_T(x,\xi,t) + H_T(-x,\xi,t)$	$1^{--}, 2^{--}, 3^{--} \ldots$	$(S=1)$	$1^{+-}, 3^{+-} \ldots$	(S=0)
$E_T(x,\xi,t) + E_T(-x,\xi,t)$	$1^{--}, 2^{--}, 3^{--} \ldots$	$(S=1)$	$1^{+-}, 3^{+-} \ldots$	(S=0)
$\widetilde{H}_T(x,\xi,t) + \widetilde{H}_T(-x,\xi,t)$	$1^{--}, 2^{--}, 3^{--} \ldots$	$(S=1)$		
$\widetilde{E}_T(x,\xi,t) + \widetilde{E}_T(-x,\xi,t)$	$2^{--}, 3^{--}, 4^{--} \ldots$	$(S=1)$	$3^{+-}, 5^{+-} \ldots$	(S=0)

Chiral even ⇔ odd relations

- Helicity amps from $\varphi^*_{q'N'} \times \varphi_{qN}$
  - » With $\varphi_{-q\,-N} = \pm\varphi^*_{qN}$

$S = 0$

$$\phi_{++}(k,P) = \mathcal{A}(m + Mx)$$
$$\phi_{+-}(k,P) = \mathcal{A}(k_1 - ik_2)$$

$S = 1$

$$\phi^+_{++}(k,P) = \mathcal{A}\,\frac{k_1 - ik_2}{1-x}$$
$$\phi^-_{++}(k,P) = -\mathcal{A}\,\frac{(k_1 + ik_2)X}{1-x}$$
$$\phi^+_{+-}(k,P) = 0$$
$$\phi^-_{+-}(k,P) = -\mathcal{A}(m + Mx)$$
$$\phi^+_{-+}(k,P) = -\mathcal{A}(m + Mx)$$
$$\phi^-_{-+}(k,P) = 0$$

$$\mathcal{A} = \frac{1}{\sqrt{x}}\frac{\Gamma(k)}{k^2 - m^2}.$$

$$A^{(0)}_{++,--} = A^{(0)}_{++,++}$$
$$A^{(0)}_{++,+-} = -A^{(0)}_{++,-+}$$
$$A^{(0)}_{+-,++} = -A^{(0)}_{-+,++},$$

$$A^{(1)}_{++,--} = -\frac{x + x'}{1 + xx'}\,A^{(1)}_{++,++}$$
$$A^{(1)}_{+-,-+} = 0$$
$$A^{(1)}_{++,+-} = -\sqrt{\frac{\langle \tilde{k}^2_\perp \rangle}{x'^2 + \langle \tilde{k}^2_\perp \rangle / P^{+2}}}\,A^{(1)}_{++,-+}$$
$$A^{(1)}_{+-,++} = -\sqrt{\frac{\langle k^2_\perp \rangle}{x^2 + \langle k^2_\perp \rangle / P^{+2}}}\,A^{(1)}_{-+,++},$$

$\mathcal{H}_T, \mathcal{E}_T, \tilde{\mathcal{E}}_T, \bar{\mathcal{E}}_T$

$$\frac{d\sigma_T}{dt} \approx \mathcal{N}^2[g_\pi^{odd}(Q)]^2 \frac{1}{(1+\xi)^4}\left[|\mathcal{H}_T|^2 + \tau\left(|\bar{\mathcal{E}}_T|^2 + |\tilde{\mathcal{E}}_T|^2\right)\right] \quad (1$$

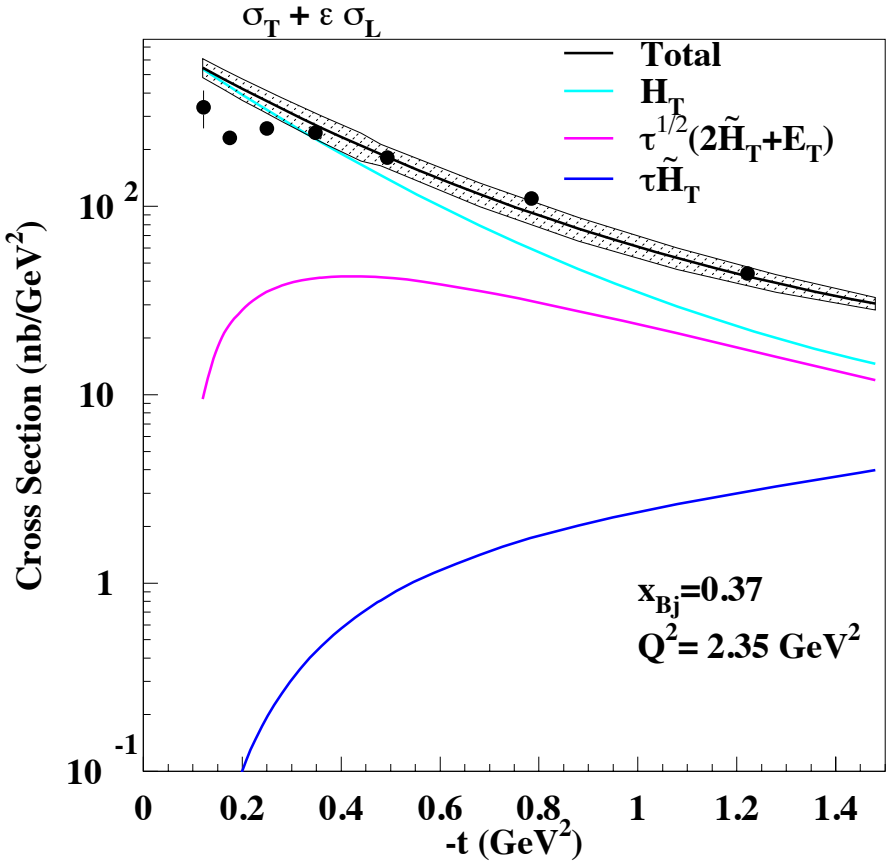
$$\frac{d\sigma_L}{dt} \approx \mathcal{N}^2[g_\pi^{odd}(Q)]^2 \frac{1}{(1+\xi)^4}\frac{2M^2\tau}{Q^2}|\mathcal{H}_T|^2 \quad (1$$

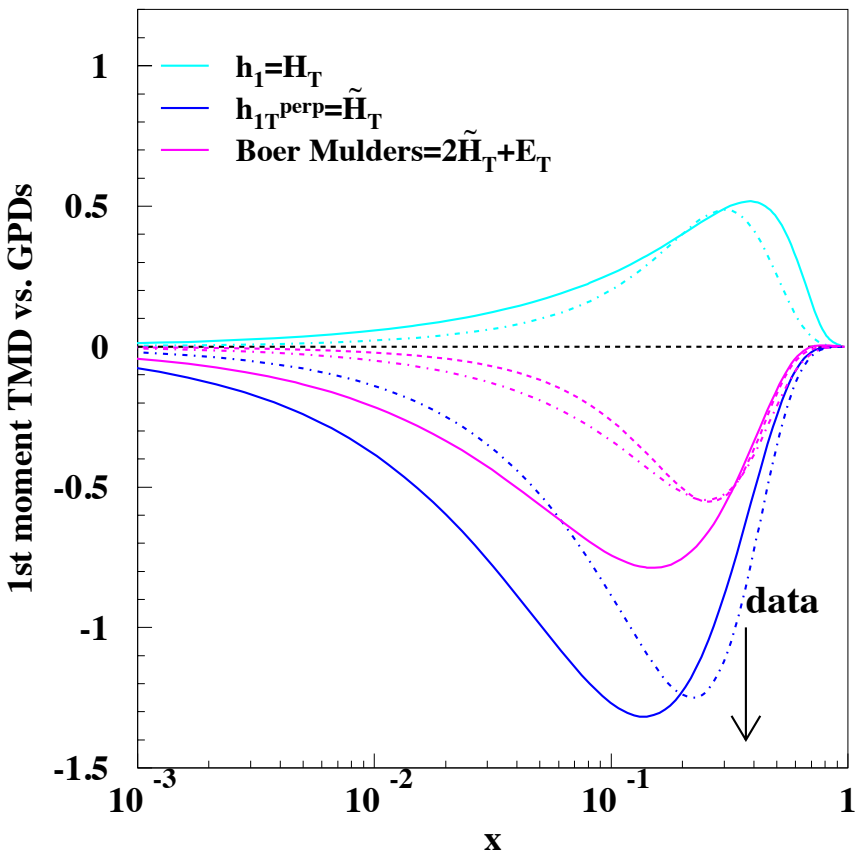
$$\frac{d\sigma_{TT}}{dt} \approx \mathcal{N}^2[g_\pi^{odd}(Q)]^2 \frac{1}{(1+\xi)^4}\tau\left[|\bar{\mathcal{E}}_T|^2 - |\tilde{\mathcal{E}}_T|^2 + \Re e\mathcal{H}_T\frac{\Re e(\bar{\mathcal{E}}_T - \mathcal{E}_T)}{2} + \Im m\mathcal{H}_T\frac{\Im m(\bar{\mathcal{E}}_T - \mathcal{E}_T)}{2}\right] \quad (1$$

$$\frac{d\sigma_{LT}}{dt} \approx \mathcal{N}^2[g_\pi^{odd}(Q)]^2 \frac{1}{(1+\xi)^4}\, 2\sqrt{\frac{2M^2\tau}{Q^2}}|\mathcal{H}_T|^2 \quad (1$$

$$\frac{d\sigma_{L'T}}{dt} \approx \mathcal{N}^2[g_\pi^{odd}(Q)]^2 \frac{1}{(1+\xi)^4}\tau\sqrt{\frac{2M^2\tau}{Q^2}}\left[\Re e\mathcal{H}_T\frac{\Im m(\bar{\mathcal{E}}_T - \mathcal{E}_T)}{2} - \Im m\mathcal{H}_T\frac{\Re e(\bar{\mathcal{E}}_T - \mathcal{E}_T)}{2}\right] \quad (1$$

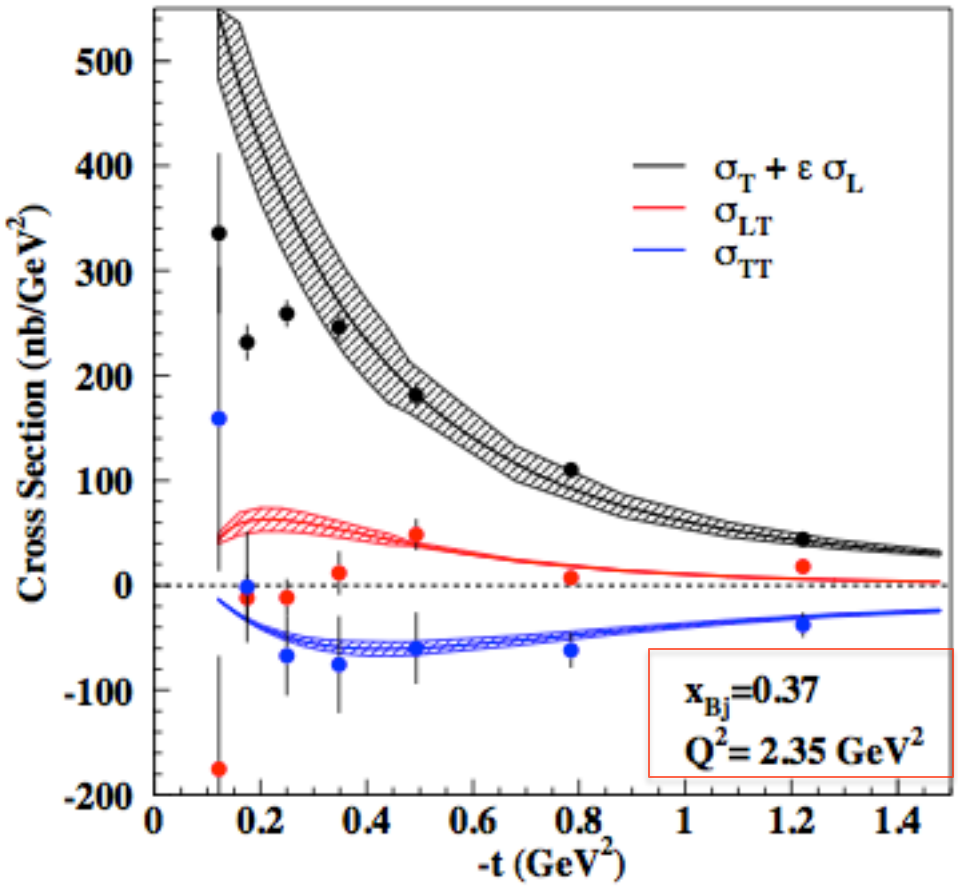
$\tau = (t_0 - t)/2M^2$





Hall B data, Kubarovsky& Stoler, PoS ICHEP 2010

How well do the parameters fixed with DVCS data reproduce π^0 electroproduction data?



Hall B data, Kubarovsky& Stoler, PoS ICHEP 2010

Transversity amplitudes

- $H_T(x,0,0) = h_1(x)$ "measures" transfer of transversity
- $|\, p,+(-)\rangle^{Ty} = [|p,+\rangle + (-)i\, |p,-\rangle]/\sqrt{2}$ (y-normal to scattering plane)
- Or $|\, p,+(-)\rangle^{Tx} = [|p,+\rangle + (-)|p,-\rangle]/\sqrt{2}$ (x-in plane)
- Or $|\, p,+(-)\rangle^{Tx} = [|p,+\rangle + (-)e^{i\phi}\, |p,-\rangle]/\sqrt{2}$ (in transverse plane)
- $A^{Ty}_{N',q';N,q}$ = linear combination of $A^{helicity}$
- $H_T \propto A^{Ty}_{++,++} - A^{ty}_{+-,+-} - A^{ty}_{-+,-+} + A^{ty}_{--,--}$
- Diagonal in transversities $\Rightarrow$ probabilistic interpretation

  w/o b-space
- Same *spin form* as TMD $h_{1T}(x,k_T^2)$

  $h_{1T}(x,k_T^2)$ compare $H_T(x,0,\Delta_T^2)$

  or unintegrated $H_T(x,0,\Delta_T^2,k)$

TMDs & GPDs

- $f_1 \propto A_{++,++} + A_{+-,+-} + A_{--,--} + A_{-+,-+}$
  $= A^{TY}_{++,++} + A^{TY}_{+-,+-} + A^{TY}_{--,--} + A^{TY}_{-+,-+} \sim H$
- $g_{1L} \propto A_{++,++} - A_{+-,+-} + A_{--,--} - A_{-+,-+}$
  $= A^{TY}_{++,--} + A^{TY}_{+-,+-} + A^{TY}_{--,++} + A^{TY}_{-+,-+} \sim H^{\sim}$
- $h_{1T}^{\perp} \propto A_{+-,-+} + A_{-+,+-} \sim H_T^{\sim}$ mixture of T_Y & T_X
- "T"-odd TMD vs. GPD
- $f_{1T}^{\perp} \propto A^{TY}_{++,++} + A^{TY}_{+-,+-} - A^{TY}_{--,--} - A^{TY}_{-+,-+} \sim E$
- $h_1^{\perp} \propto A^{TY}_{++,++} - A^{TY}_{+-,+-} + A^{TY}_{--,--} + A^{TY}_{-+,-+} \sim 2H_T^{\sim} + E_T$

$$\sum_{\Lambda} \Im m\, F_{\Lambda+,\Lambda-} \propto h_1^{\perp}(x,k_T^2)$$

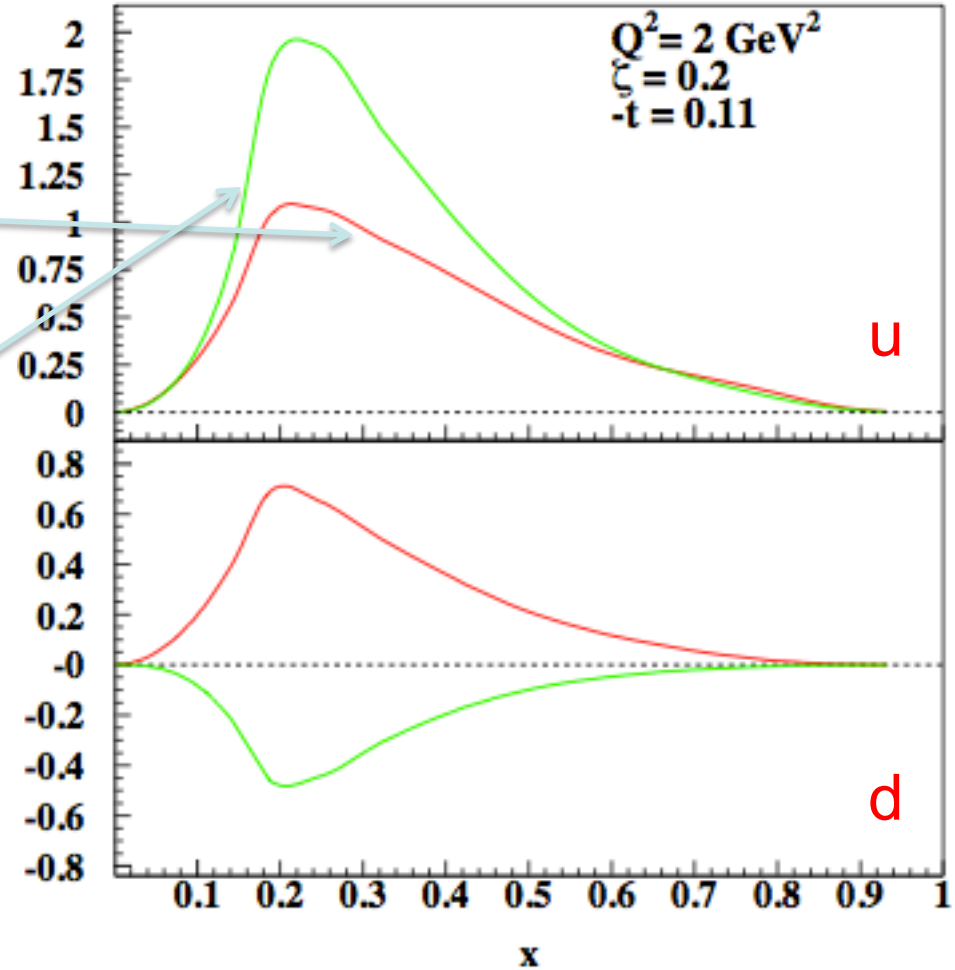
$$\sum_{\lambda} \Im m\, F_{+\lambda,-\lambda} \propto f_{1T}^{\perp}(x,k_T^2)$$

$$A_{++,+-} - A_{+-,++} \propto 2\tilde{H}_T + E_T$$

$$A_{++,-+} - A_{-+,++} \propto E$$

($h_1^{\perp}$) $2\tilde{H}_T + E_T$

$\tilde{E}_T$



Conclusions

- ☯ Flexible Model GPDs → phenomenology (DVCS & DVMP)
- ☯ Spectator models relate Chiral even to Chiral odd GPDs. How far broken? Regge behavior
- ☯ Exclusive π^0 electroproduction observables (depend on axial vector 1^{+-} exchange quantum numbers)
- ☯ GPD H_T yield values of δu & δd also have κ_T^u & κ_T^d .
- ☯ $d\sigma_T/dt$, $d\sigma_{TT}/dt$, A_{UT}, beam asymmetry, beam-target correlations, $d\sigma_L/dt$, $d\sigma_{LT}/dt$
- ☯ FUTURE: DVCS & π^0 along with η, ρ, ω production will narrow range of basic parameters of GPDs, transversity & hadronic spin.
- ☯ GPD ⇔ TMDs through transversity

backup slides

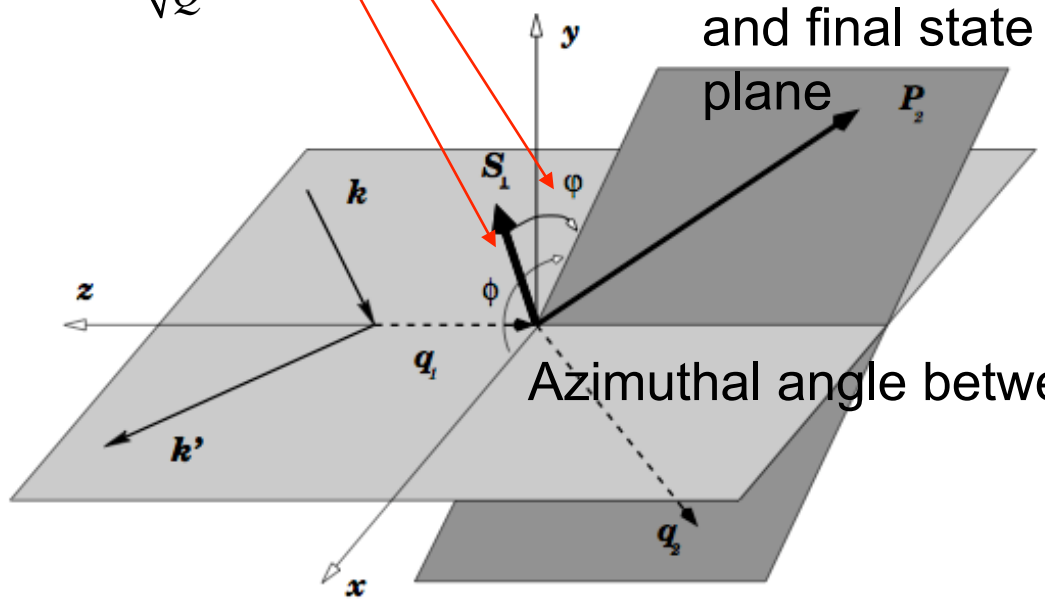
DVCS Cross Section (Belitsky, Kirchner, Muller, 2002)

$$\frac{d\sigma}{dx_B\, dy\, d|\Delta^2|\, d\phi\, d\varphi} = \frac{\alpha^3 x_B y}{16\pi^2 Q^2\sqrt{1+\epsilon^2}} \left|\frac{\mathcal{T}}{e^3}\right|^2$$

Amplitude

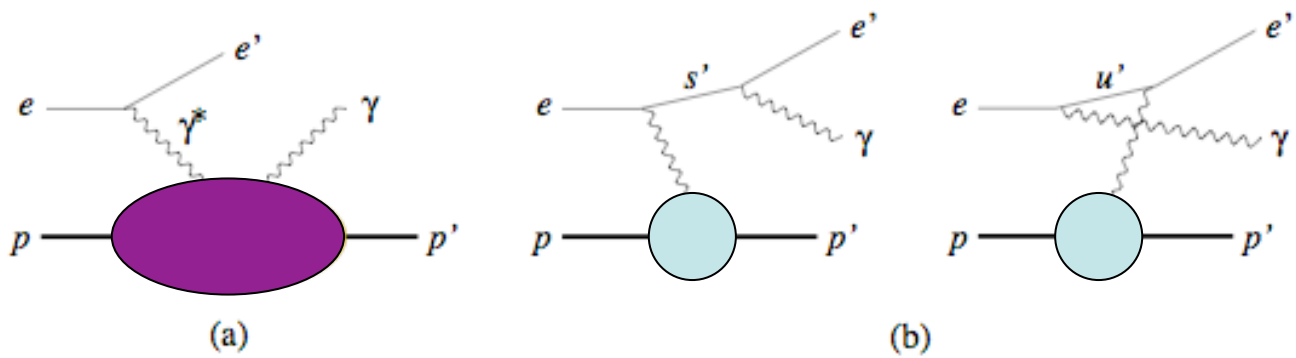
$$y = \frac{(P_1 q)}{(P_1 k_1)} \qquad \varepsilon = 2x_B \frac{M}{\sqrt{Q^2}}$$

Angle between transverse spin and final state plane



Azimuthal angle between planes

Comtpon Scattering and Bethe Heitler Processes



Dynamics

$$\mathcal{T}^2 = |\mathcal{T}_{\rm BH}|^2 + |\mathcal{T}_{\rm DVCS}|^2 + \mathcal{I}\,,$$

$$|\mathcal{T}_{\rm BH}|^2 = \frac{e^6}{x_{\rm B}^2 y^2 (1+\epsilon^2)^2 \Delta^2\, \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\rm BH} + \sum_{n=1}^{2} c_n^{\rm BH} \cos(n\phi) + s_1^{\rm BH} \sin(\phi) \right\}$$

$$|\mathcal{T}_{\rm DVCS}|^2 = \frac{e^6}{y^2 Q^2} \left\{ c_0^{\rm DVCS} + \sum_{n=1}^{2} \left[c_n^{\rm DVCS} \cos(n\phi) + s_n^{\rm DVCS} \sin(n\phi) \right] \right\}\,,$$

$$\mathcal{I} = \frac{\pm e^6}{x_{\rm B} y^3 \Delta^2 \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\mathcal{I}} + \sum_{n=1}^{3} \left[c_n^{\mathcal{I}} \cos(n\phi) + s_n^{\mathcal{I}} \sin(n\phi) \right] \right\}\,,$$

Look for instance at DVCS-BH Interference

$$c^{\mathcal{I}}_{0,\mathrm{unp}} = -8(2-y)\Re\mathrm{e}\left\{\frac{(2-y)^2}{1-y}K^2\mathcal{C}^{\mathcal{I}}_{\mathrm{unp}}(\mathcal{F}) + \frac{\Delta^2}{Q^2}(1-y)(2-x_{\mathrm{B}})\left(\mathcal{C}^{\mathcal{I}}_{\mathrm{unp}} + \Delta\mathcal{C}^{\mathcal{I}}_{\mathrm{unp}}\right)(\mathcal{F})\right\}$$

$$\begin{aligned}
\begin{Bmatrix} c^{\mathcal{I}}_{1,\mathrm{unp}} \\ s^{\mathcal{I}}_{1,\mathrm{unp}} \end{Bmatrix} &= 8K\begin{Bmatrix} -(2-2y+y^2) \\ \lambda y(2-y) \end{Bmatrix}\begin{Bmatrix} \Re\mathrm{e} \\ \Im\mathrm{m} \end{Bmatrix}\mathcal{C}^{\mathcal{I}}_{\mathrm{unp}}(\mathcal{F}) \,, \\
\begin{Bmatrix} c^{\mathcal{I}}_{2,\mathrm{unp}} \\ s^{\mathcal{I}}_{2,\mathrm{unp}} \end{Bmatrix} &= \frac{16K^2}{2-x_{\mathrm{B}}}\begin{Bmatrix} -(2-y) \\ \lambda y \end{Bmatrix}\begin{Bmatrix} \Re\mathrm{e} \\ \Im\mathrm{m} \end{Bmatrix}\mathcal{C}^{\mathcal{I}}_{\mathrm{unp}}\left(\mathcal{F}^{\mathrm{eff}}\right) \,, \\
c^{\mathcal{I}}_{3,\mathrm{unp}} &= -\frac{8Q^2K^3}{M^2(2-x_{\mathrm{B}})^2}\Re\mathrm{e}\,\mathcal{C}^{\mathcal{I}}_{T,\mathrm{unp}}(\mathcal{F}_T) \,.
\end{aligned}$$

$$\boxed{\mathcal{C}^{\mathcal{I}}_{\mathrm{unp}} = F_1\mathcal{H} + \frac{x_{\mathrm{B}}}{2-x_{\mathrm{B}}}(F_1+F_2)\widetilde{\mathcal{H}} - \frac{\Delta^2}{4M^2}F_2\mathcal{E}}$$

$$\begin{aligned}
\boxed{\frac{d^4\Sigma}{d^4\Phi} = \frac{1}{2}\left[\frac{d^4\sigma^+}{d^4\Phi} - \frac{d^4\sigma^-}{d^4\Phi}\right]} &= \frac{d^4\Sigma(|DVCS|^2)}{d^4\Phi} \\
&+ \sin(\phi_{\gamma\gamma})\Gamma_1^{\mathrm{Im}}\,\mathrm{Im}\left[\mathcal{C}^I(\mathcal{F})\right] \\
&- \sin(2\phi_{\gamma\gamma})\Gamma_2^{\mathrm{Im}}\,\mathrm{Im}\left[\mathcal{C}^I(\mathcal{F}^{\mathrm{eff}})\right] \,, \\
\boxed{\frac{d^4\sigma}{d^4\Phi} = \frac{1}{2}\left[\frac{d^4\sigma^+}{d^4\Phi} + \frac{d^4\sigma^-}{d^4\Phi}\right]} &= \frac{d^4\sigma(|DVCS|^2)}{d^4\Phi} \\
&+ \frac{d^4\sigma(|BH|^2)}{d^4\Phi} + \Gamma^{\mathrm{Re}}_{0,\Delta}\mathrm{Ree}\left[\mathcal{C}^I + \Delta\mathcal{C}^I\right](\mathcal{F}) \\
&+ \Gamma_0^{\mathrm{Re}}\mathrm{Ree}\left[\mathcal{C}^I(\mathcal{F})\right] - \cos(\phi_{\gamma\gamma})\Gamma_1^{\mathrm{Re}}\mathrm{Re}\left[\mathcal{C}^I(\mathcal{F})\right] \\
&+ \cos(2\phi_{\gamma\gamma})\Gamma_2^{\mathrm{Re}}\mathrm{Re}\left[\mathcal{C}^I(\mathcal{F}^{\mathrm{eff}})\right] \,,
\end{aligned}$$

Observables

$$\frac{d\sigma_T}{dt} = \mathcal{N}\left(|f_{1,+;0,+}|^2 + |f_{1,+;0,-}|^2 + |f_{1,-;0,+}|^2 + |f_{1,-;0,-}|^2\right)$$
$$= \mathcal{N}\left(|f_1|^2 + |f_2|^2 + |f_3|^2 + |f_4|^2\right) \quad (1)$$
$$\frac{d\sigma_L}{dt} = \mathcal{N}\left(|f_{0,+;0,+}|^2 + |f_{0,+;0,-}|^2\right)$$
$$= \mathcal{N}\left(|f_5|^2 + |f_6|^2\right), \quad (2)$$
$$\mathcal{N} = \left[M(s-M^2)^2\right]^{-1} G \quad (3)$$

$$\frac{d\sigma_{TT}}{dt} = -2\mathcal{N}\Re e\left(f^*_{1,+;0,+}f_{1,-;0,-} - f^*_{1,+;0,-}f_{1,-;0,+}\right)$$
$$= -2\mathcal{N}\,\Re e\left(f_1^* f_4 - f_2^* f_3\right). \quad (4)$$

$$\frac{d\sigma_{LT}}{dt} = 2\mathcal{N}\,\Re e\left[f^*_{0,+;0,-}(f_{1,+;0,-} + f_{1,-;0,+}) + f^*_{0,+;0,+}(f_{1,+;0,+} - f_{1,-;0,-})\right]$$
$$= 2\mathcal{N}\,\Re e\left[f_5^*(f_2+f_3) + f_6^*(f_1-f_4)\right]. \quad (5)$$

For the beam polarization, taking the virtual photon in the z-direction in the target rest frame,

$$\frac{d\sigma_{LT'}}{dt} = 2\mathcal{N}\,\Im m\left[f^*_{0,+;0,-}(f_{1,+;0,-} + f_{1,-;0,+}) + f^*_{0,+;0,+}(f_{1,+;0,+} - f_{1,-;0,-})\right]$$
$$= 2\mathcal{N}\,\Im m\left[f_5^*(f_2+f_3) + f_6^*(f_1-f_4)\right] \quad (6)$$

transversely polarized target asymmetry,

$$A_{UT} = \frac{2\Im m(f_1^* f_3 - f_4^* f_2)}{\frac{d\sigma_T}{dt}}. \quad (7)$$

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more observables

$$A_{UT} = \frac{2\Im m(f_1^* f_3 - f_4^* f_2)}{\frac{d\sigma_T}{dt}}. \tag{7}$$

This is also called the $sin(\phi - \phi_s)$ moment for purely transverse photons. When the azimuthal dependence of the hadron plane is averaged over $sin(\phi_s)$ moment

$$A_{UT'} = \sqrt{\epsilon(1+\epsilon)}\frac{2\Im m(f_1^* f_5 - f_2^* f_6)}{\frac{d\sigma_T}{dt}}. \tag{8}$$

The beam spin asymmetry,

$$\alpha = \frac{\sqrt{2\epsilon_L(1-\epsilon)}\,\frac{d\sigma_{LT'}}{dt}}{\frac{d\sigma_T}{dt} + \epsilon_L \frac{d\sigma_L}{dt}} \tag{9}$$

For the target polarized longitudinally to the virtual photon direction,

$$A_{UL} = \sqrt{\epsilon(1+\epsilon)}\Im m\left[(f_1 + f_4)^* f_6 + (f_2 - f_3)^* f_5\right] \tag{10}$$

where ϵ is the ratio of longitudinal to transverse virtual photon polarization. ers.

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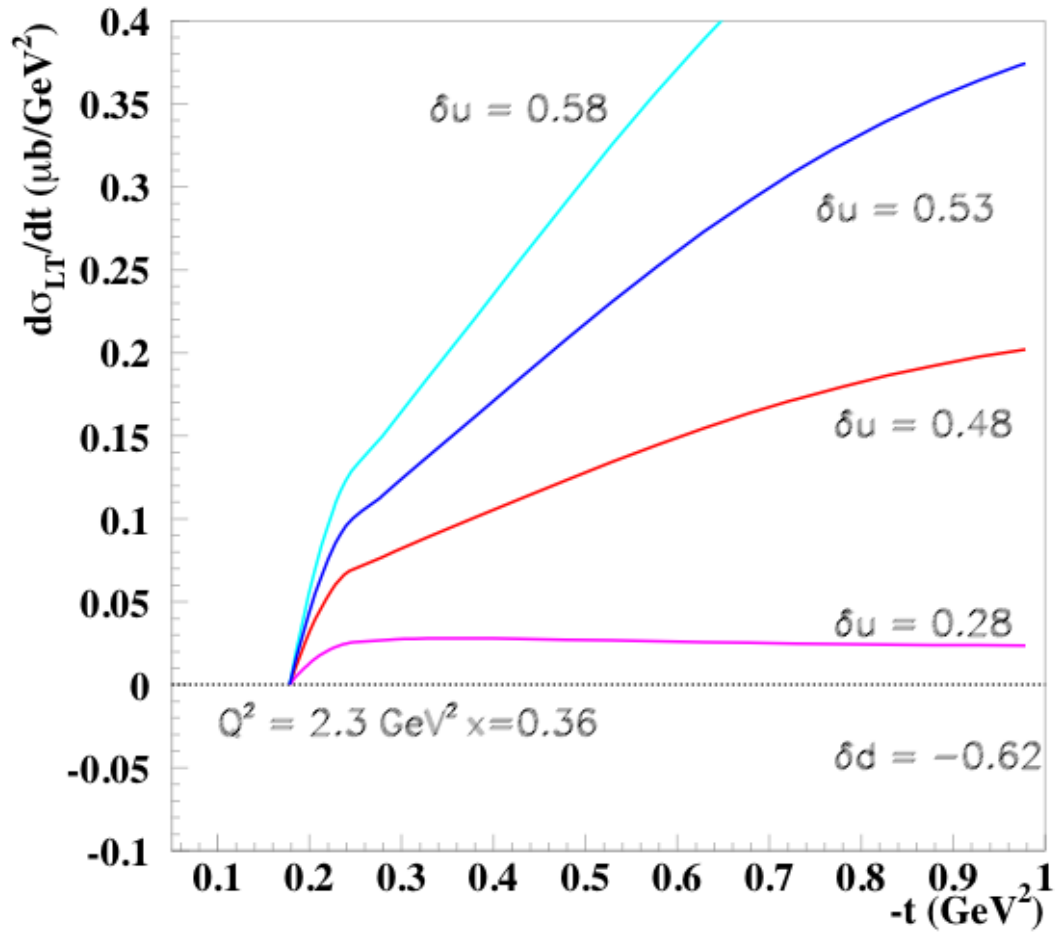
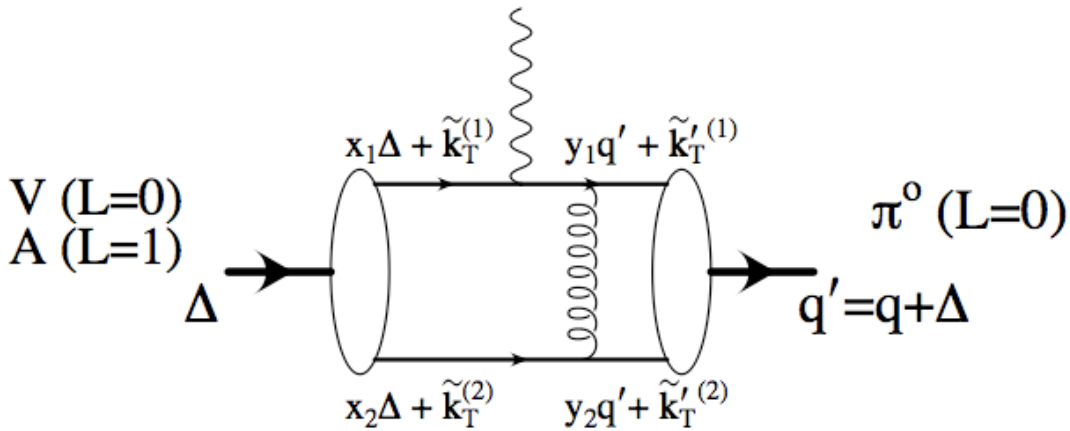


All GPDs

Ahmad, GRG, Liuti, PRD79, 054014 (2009)

Q^2 dependent form factors t-channel view

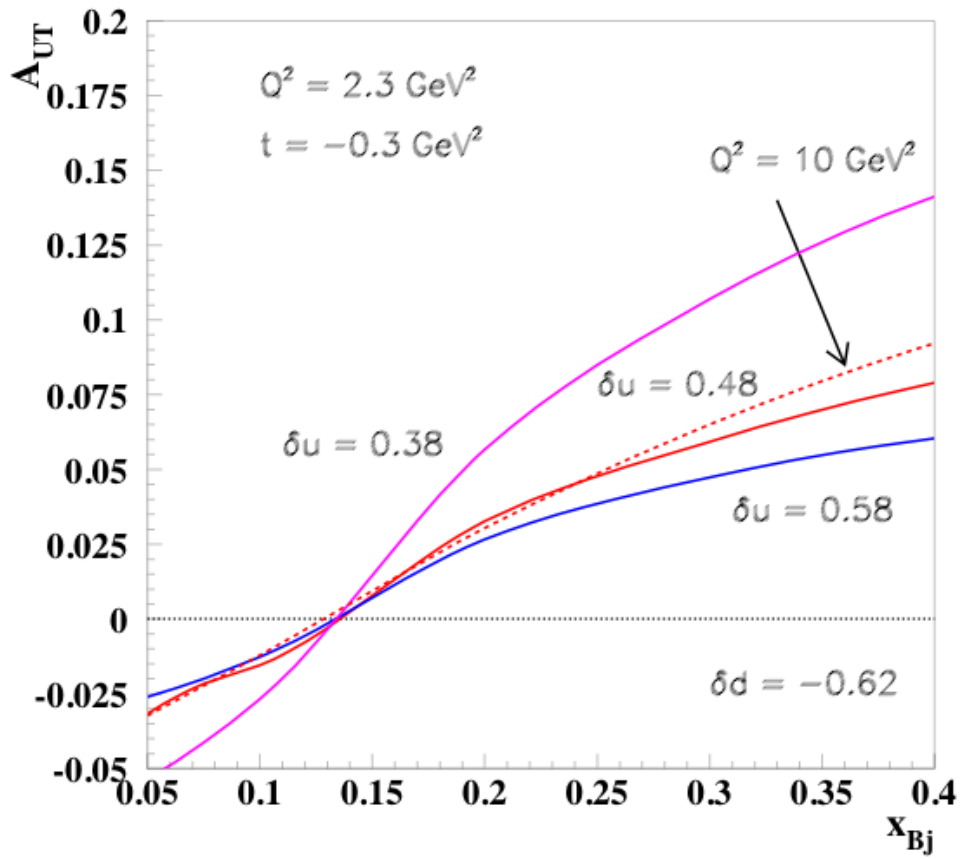


see Belitsky, Ji, Yuan (2007) & Ng (2007)

$$F_{\gamma^* A \pi^o} = \int dx_1 dy_1 \int d^2\mathbf{b}\, \psi_A^{(1)}(y_1, b) \mathcal{C} K_o(\sqrt{x_1(1-x_1)Q^2}\, b) \times \psi_{\pi^o}(x_1, b) \exp(-S), \qquad (50)$$

where now

$$\psi_A^{(1)}(y_1, b) = \int d^2 k_T J_1(y_1 b)\, \psi(y_1, k_T), \qquad (51)$$

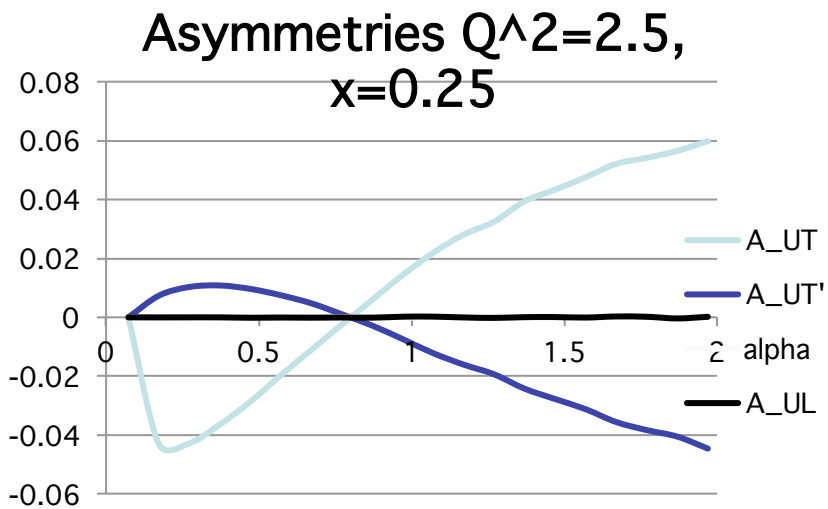


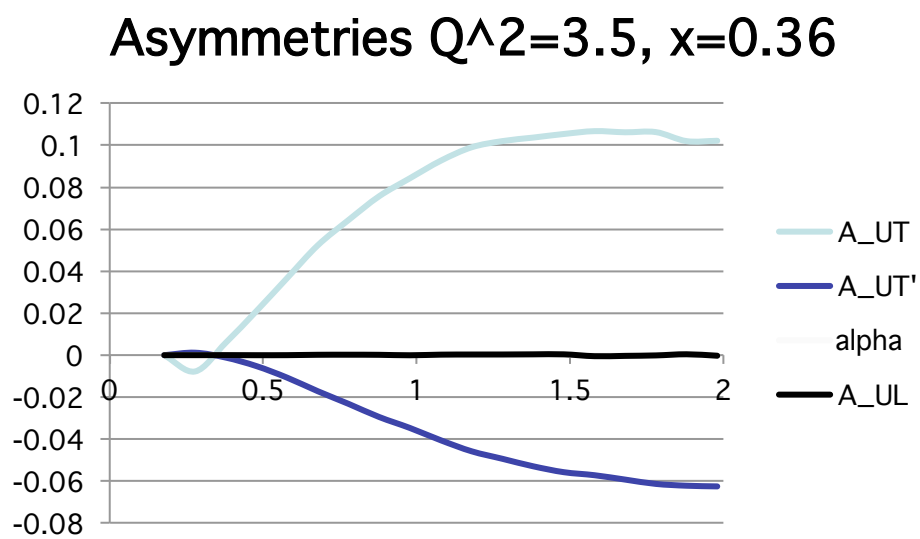
All GPDs

Ahmad, GRG, Liuti, PRD79, 054014 (2009)

Asymmetries

- delicate interplay among amps, GPDs & Compton Form factor phases. very sensitive to physical parameters – tensor charges, "anomalous transversity"
- A_{UT} $(\sin(\phi-\phi_s))$ $A_{UT'}$ $(\sin\phi_s)$ α (beam pol'z'n) A_{UL} ("long.")





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