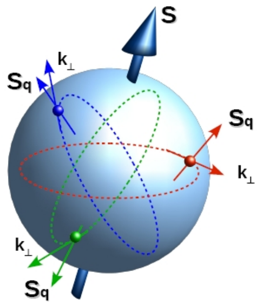
Phenomenology of TMDs



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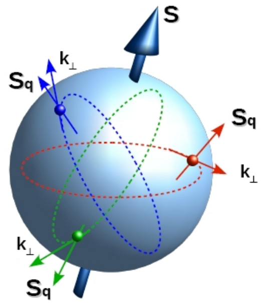
Fisica Torino

From a phenomenological point of view ...



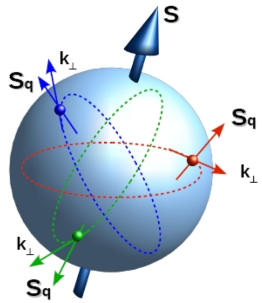
The exploration of the 3-dimensional structure of the nucleon, both in momentum and in configuration space, is one of the major issues in high energy hadron physics.

Information on the 3-dimensional structure of the nucleon is embedded in the Transverse Momentum Dependent distribution and fragmentation functions (TMDs).

Huge amount of experimental data on spin asymmetries in several different processes show that TMD distribution and fragmentation functions exist and are non zero.

Nicely introduced by Gunar Schnell

From a phenomenological point of view ...



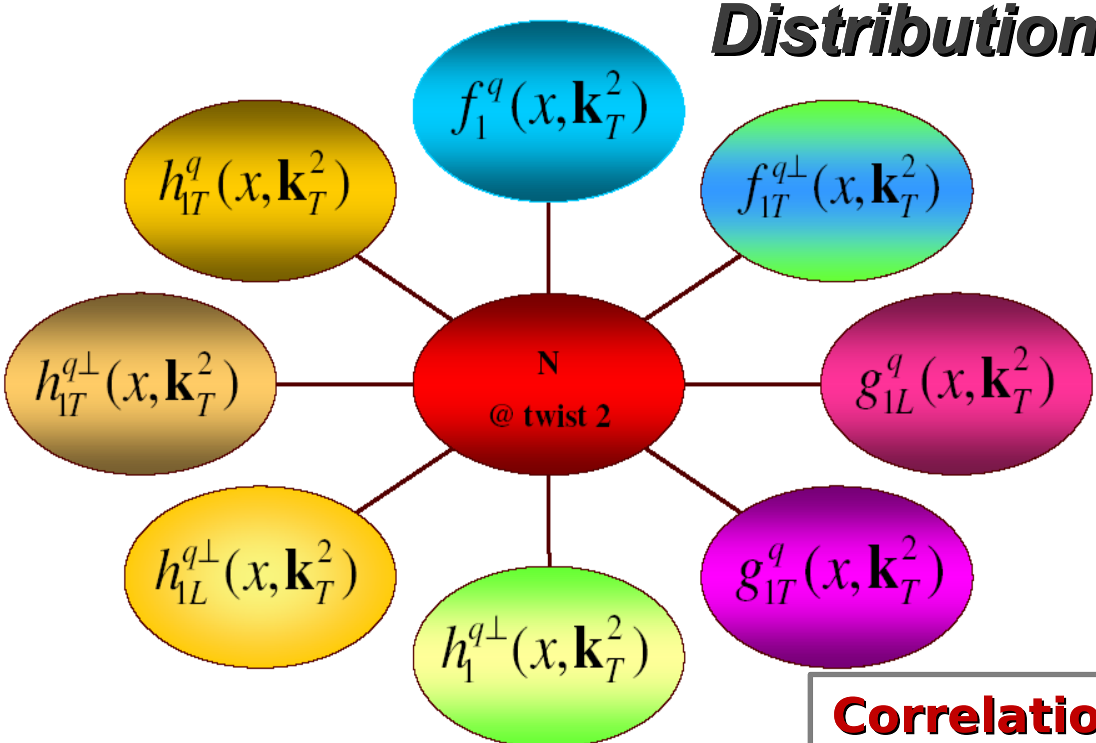
In a very simple phenomenological approach, cross sections and spin asymmetries are generated, in a factorized scheme, as convolutions of distribution and (or) fragmentation TMDs with elementary scattering cross sections.

see talk by Andreas Metz

This simple approach can successfully describe a wide range of experimental data.

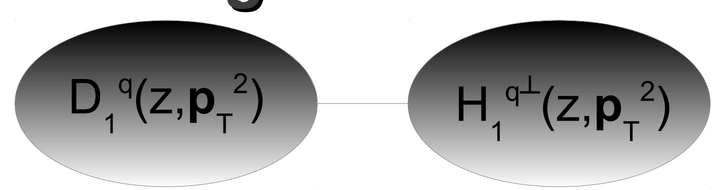
TMD distribution and fragmentation functions

Distribution



Correlations between spin and transverse momentum

Fragmentation

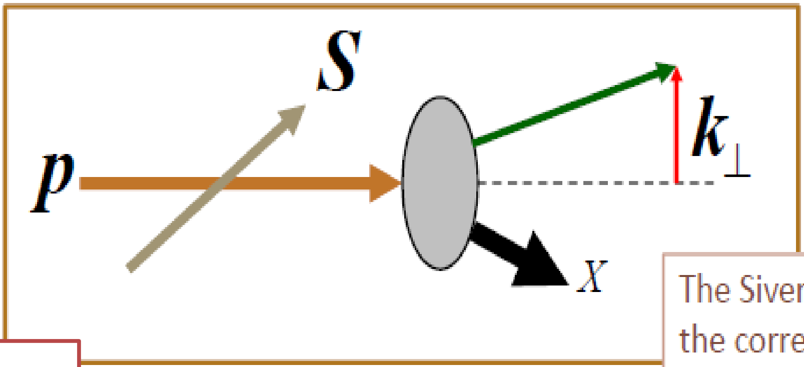


Sivers Distribution Function

$$f_{q/p,S}(x,k_\perp) = f_{q/p}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x,k_\perp)\, \boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

$$= f_{q/p}(x,k_\perp) - \frac{k_\perp}{M} f_{1T}^{\perp q}(x,k_\perp)\, \boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

The Sivers function is related to the probability of finding an unpolarized quark inside a transversely polarized proton



The Sivers function inbeds the correlation between the proton spin and the quark transverse momentum

The Sivers function, is particularly interesting, as it provides information on the partonic orbital angular momentum

see talk by Matthias Burkardt

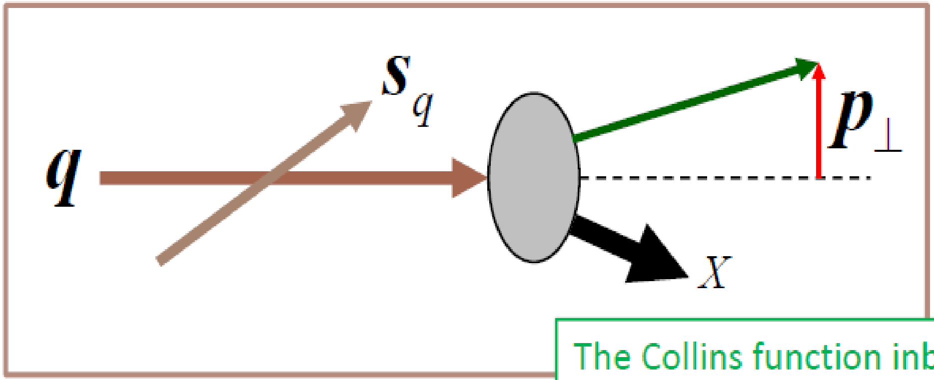
Collins Fragmentation Function

$$D_{h/q,s_q}(z,\boldsymbol{p}_\perp) = D_{h/q}(z,p_\perp) + \frac{1}{2}\Delta^N D_{h/q^\uparrow}(z,p_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}_q\times\hat{\boldsymbol{p}}_\perp)$$

$$= D_{h/q}(z,p_\perp) + \frac{p_\perp}{z\,M_h} H_1^{\perp q}(z,p_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}_q\times\hat{\boldsymbol{p}}_\perp)$$

The Collins function is related to the probability that a transversely polarized struck quark will fragment into a spinless hadron

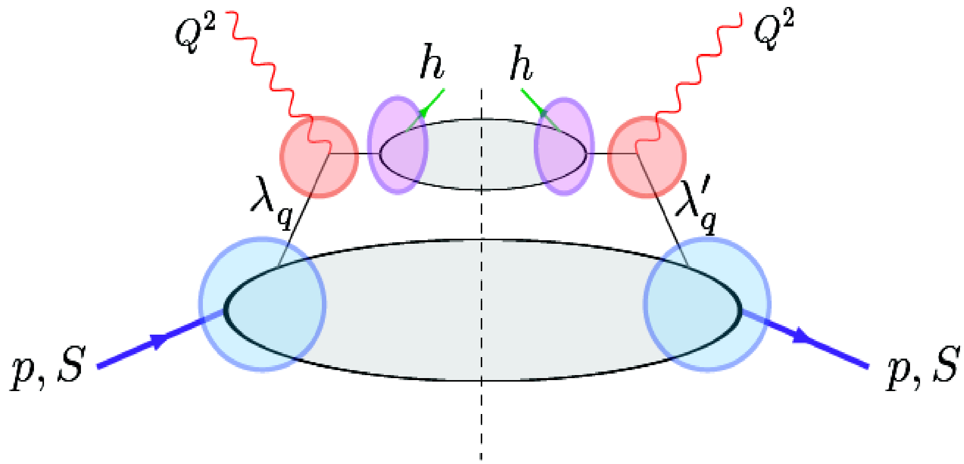
The Collins function is chirally odd



The Collins function inbeds the correlation between the fragmenting quark spin and the transverse momentum of the produced hadron

see talk by Isabella Garzia

Factorization in SIDIS



TMD factorization holds at large Q^2 and $P_T \approx k_\perp \approx \lambda_{QCD}$

Two scales: $P_T << Q^2$

(Collins, Soper, Ji, Ma, Yuan, Qiu, Vogelsang,Collins, Metz)

$$\frac{d\sigma^{\ell(S_\ell)+p(S)\to\ell'+h+X}}{dx_B\, dQ^2\, dz_h\, d^2\boldsymbol{P}_T\, d\phi_S} = \rho^{\ell,S_\ell}_{\lambda_\ell,\lambda'_\ell} \otimes \rho^{q/p,S}_{\lambda_q,\lambda'_q} \hat{f}_{q/p,S}(x,\boldsymbol{k}_\perp) \otimes \hat{M}_{\lambda_\ell,\lambda_q;\lambda_\ell,\lambda_q} \hat{M}^*_{\lambda'_\ell,\lambda'_q;\lambda'_\ell,\lambda'_q} \otimes \hat{D}^h_{\lambda_q,\lambda'_q}(z,\boldsymbol{p}_\perp)$$

TMD-PDF **hard scattering** **TMD-FF**

Historically ...

- *In the Torino-Cagliari standard approach, TMDs are parametrized in a form in which the x and $k_\perp$ dependences are factorized and only the collinear part evolves in Q*

Unpolarized TMD PDF

$$\boxed{\hat{f}_{q/p}(x, k_\perp; Q) = f_{q/p}(x; Q)\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle}}$$

Collinear PDF ꞁear PDF (DGLAP evolution)

Normalized gaussian (no evolution)

Unpolarized TMD FF

$$\boxed{D_{h/q}(z, p_\perp) = D_{h/q}(z)\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle},}$$

Collinear FF (DGLAP evolution)

Normalized gaussian (no evolution)

Historically ...

- *In the Torino-Cagliari standard approach TMDs are parametrized in a form in which the x and $k_\perp$ dependences are factorized and only the collinear part evolves in Q*

Sivers function

$$\boxed{\begin{aligned}\Delta^N \widehat{f}_{q/p^\uparrow}(x,k_\perp;Q) &= 2\mathcal{N}_q(x)h(k_\perp)\widehat{f}_{q/p}(x,k_\perp;Q)\\ &= 2\mathcal{N}_q(x)f_{q/p}(x;Q)\sqrt{2e}\,\frac{k_\perp}{M_1}\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_S}}{\pi\langle k_\perp^2\rangle}\end{aligned}}$$

Collinear PDF (DGLAP)

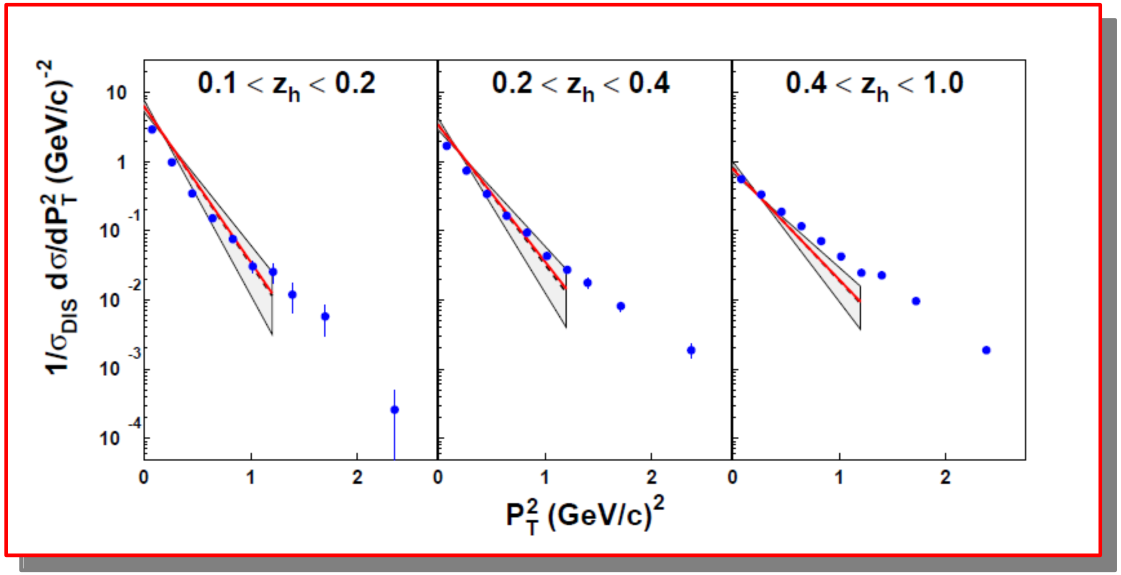
$$\mathcal{N}_q(x) = N_q\, x^{\alpha_q}(1-x)^{\beta_q}\,\frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\,\beta_q^{\beta_q}}$$

$$\langle k_\perp^2\rangle_S = \frac{M_1^2\,\langle k_\perp^2\rangle}{M_1^2+\langle k_\perp^2\rangle}$$

$$\boxed{\Delta^N \widehat{f}_{q/p^\uparrow}(x,k_\perp) = -\frac{2k_\perp}{m_p}\,f_{1T}^\perp(x,k_\perp)}$$

Extracting the unpolarized TMD gaussian widths from SIDIS data

Anselmino et al., Phys.Rev. D71 (2005) 074006

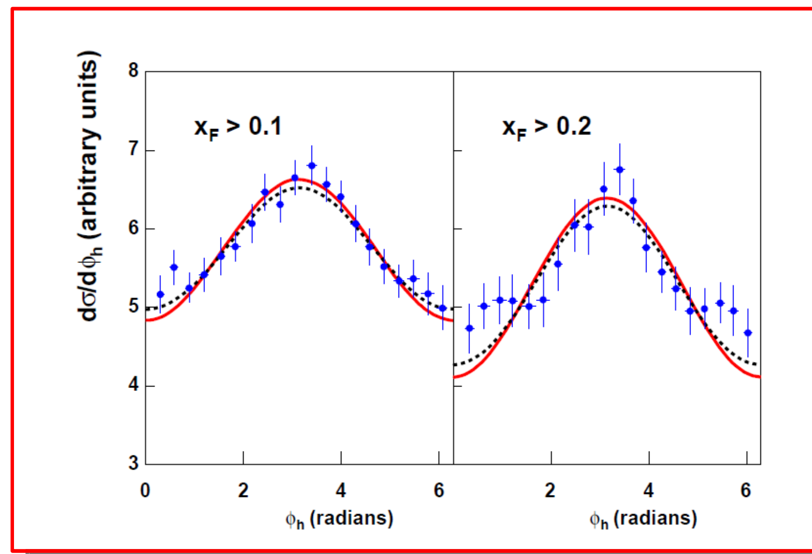


$$f_{q/p}(x, k_\perp) = f_{q/p}(x)\, \frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi \langle k_\perp^2\rangle},$$

$$D_{h/q}(z, p_\perp) = D_{h/q}(z)\, \frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi \langle p_\perp^2\rangle},$$

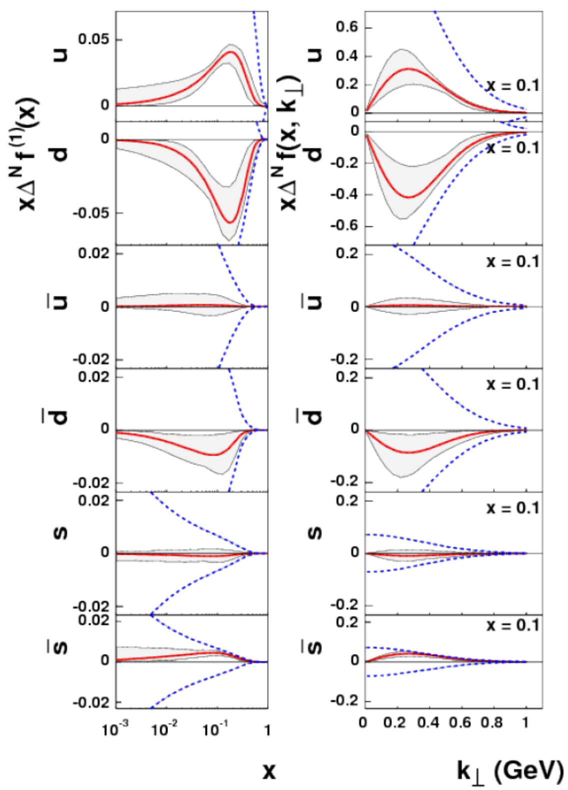
$$\langle k_\perp^2\rangle = 0.25\ (\mathrm{GeV}/c)^2$$

$$\langle p_\perp^2\rangle = 0.20\ (\mathrm{GeV}/c)^2$$



Extracting the Sivers function from SIDIS data

Anselmino et al, Eur. Phys. J. A39 (2009) 89



✔ Valence quark

- $\Delta^N f_{u/p^\uparrow} > 0 \quad \Rightarrow \quad f_{1T}^{\perp u} < 0$
- $\Delta^N f_{d/p^\uparrow} < 0 \quad \Rightarrow \quad f_{1T}^{\perp d} > 0$

✔ Sea quarks

- $\Delta^N f_{\bar{s}/p^\uparrow} > 0 \quad \Rightarrow \quad f_{1T}^{\perp \bar{s}} < 0$

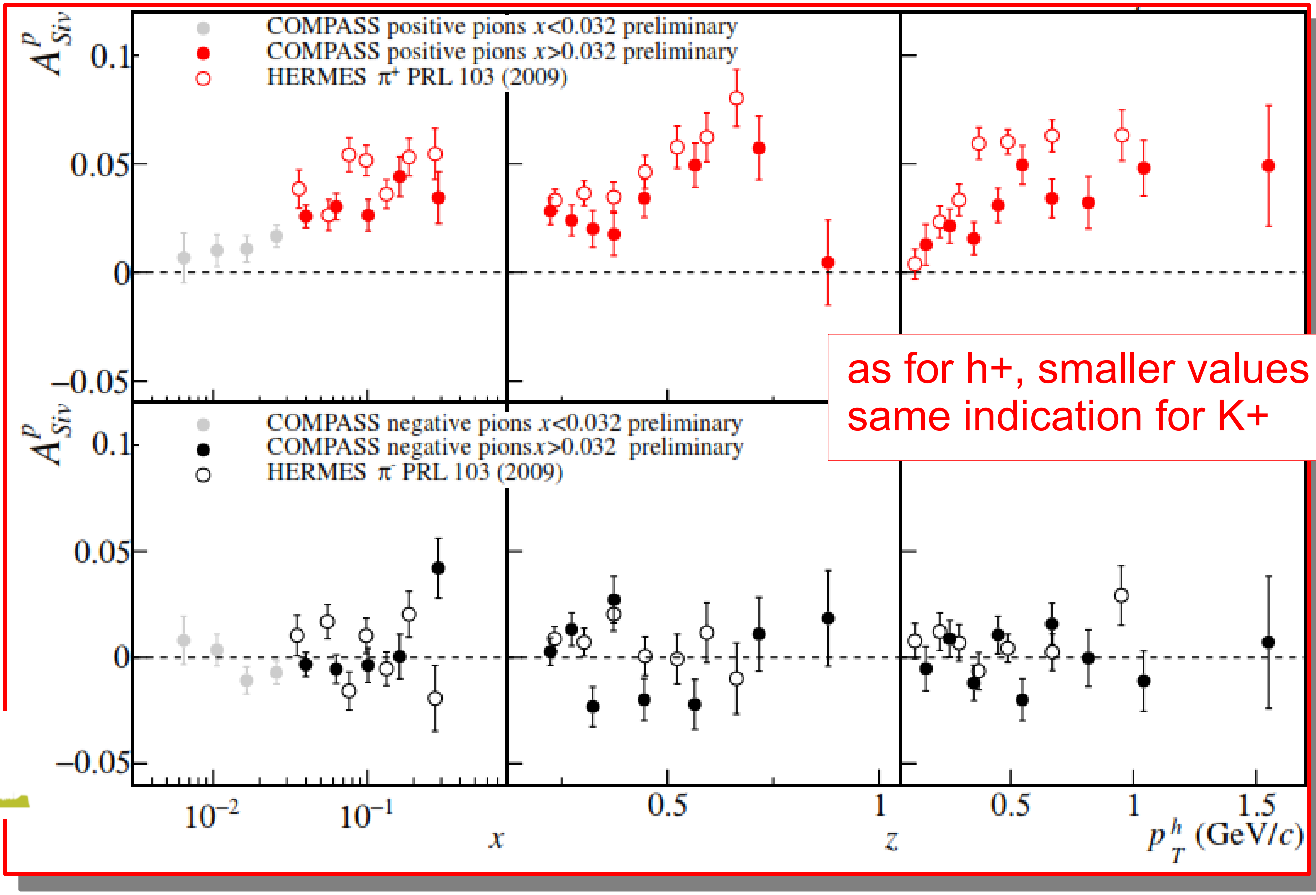
→ $\Delta^N f_q^{(1)}(x) \equiv \int d^2\boldsymbol{k}_\perp \, \frac{k_\perp}{4m_p} \, \Delta^N f_{q/p^\uparrow}(x, k_\perp) = -f_{1T}^{\perp(1)q}(x)$

SIDIS data from HERMES and COMPASS (deuterium target) available at that time could be successfully described in this simple scheme

Does most recent SIDIS data suggest TMD evolution ?

Sivers asymmetry on proton (x > 0.032)

Charged pions (and kaons), 2010 data
Comparison with HERMES results



as for h+, smaller values measured by COMPASS; same indication for K+

π+
COMPASS $\langle Q^2 \rangle = 3.2$ GeV^2
hermes $\langle Q^2 \rangle = 2.4$ GeV^2

π-
COMPASS $\langle Q^2 \rangle = 3.2$ GeV^2
hermes $\langle Q^2 \rangle = 2.4$ GeV^2

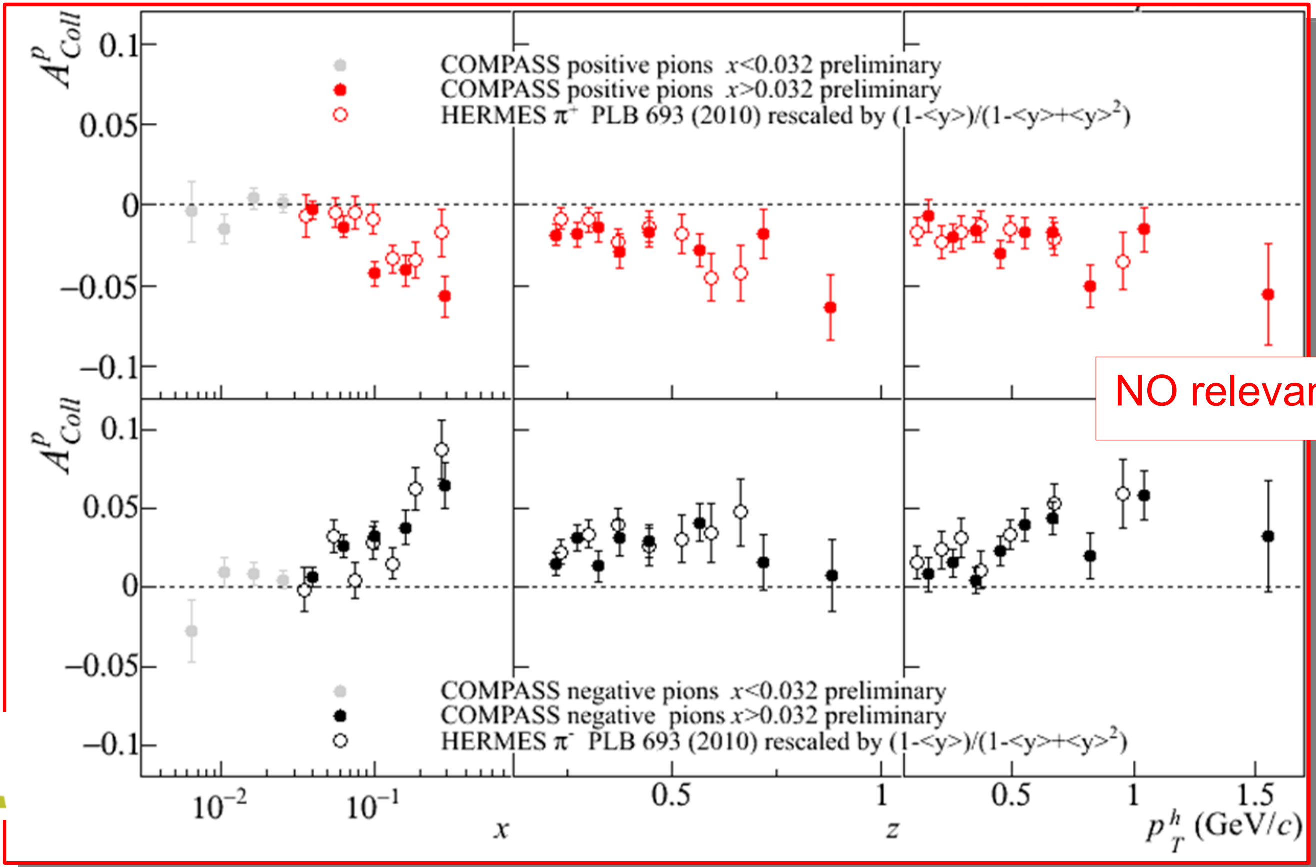
Anna Martin
DIS 2013

Does most recent SIDIS data suggest TMD evolution ?

Collins asymmetry on proton (x > 0.032)

Charged pions (and kaons), 2010 data

Comparison with HERMES results



$\pi+$ COMPASS $\langle Q^2 \rangle = 3.2\ GeV^2$

hermes $\langle Q^2 \rangle = 2.4\ GeV^2$

NO relevant hints of TMD evolution

$\pi-$ COMPASS $\langle Q^2 \rangle = 3.2\ GeV^2$

hermes $\langle Q^2 \rangle = 2.4\ GeV^2$

Anna Martin
DIS 2013

TMD evolution literature

- *Collins-Soper-Sterman resummation - Nucl. Phys. B250 (1985).*
- *Idilbi, Ji, Ma, Yuan - Phys.Lett. B 597, 299 (2004) - Phys. Rev. D70 (2004) 074021, Ji, Ma, Yuan - Phys. Rev. D71 (2005) 034005.*

- *John Collins, "Foundations of perturbative QCD" (2011), Cambridge monographs on particle physics, nuclear physics and cosmology.*
- *Aybat, Rogers, Phys. Rev. D83, 114042(2011).*
- *Aybat, Collins, Qiu, Rogers, Phys. Rev. D85, 034043 (2011).*
- *Echevarria, Idilbi, Schafer, Scimemi, arXiv:1208.1281*
  Echevarria, Idilbi, Scimemi, arXiv:1208.1281

- *Aybat, Prokudin, Rogers, Phys. Rev. Lett. 108, 242003*
- *Anselmino, Boglione, Melis, Phys. Rev. D86 (2012) 014028*

- *Godbole, Misra, Mukherjee, Raswoot, arXiv:1304:2584*
- *Sun, Yuan, arXiv:1304.5037*
- *Boer, arXiv:1304.5387*

TALKS BY

J. Collins
F. Yuan
A. Idilbi

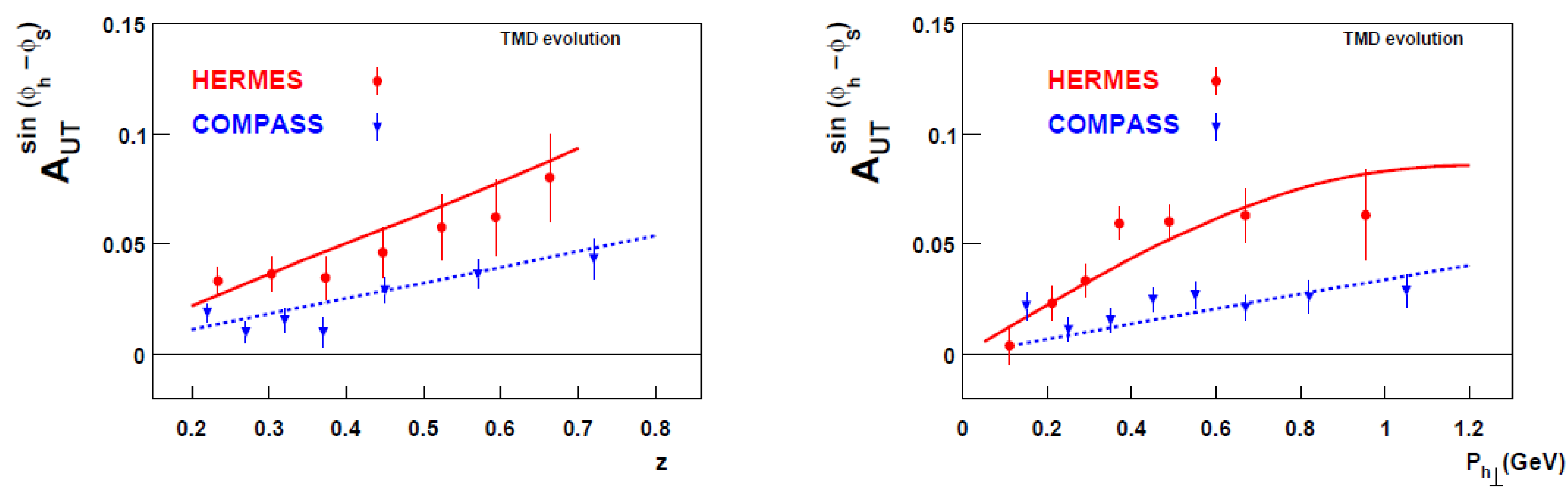
J. Collins
T. Rogers
M. Echevarria, A. Idilbi

A. Prokudin
M. Boglione
T. Rogers

A. Mukherjee
F. Yuan
D. Boer

Phenomenological results

Aybat, Prokudin, Rogers, Phys. Rev. Lett. 108, (2011) 242003



hermes

<Q²> = 2.4 GeV²

Q^2 in the range [1.3 – 6.2] GeV²

- No x dependence taken into account
- Sivers A_{UT} calculated at two fixed different values of $Q^{2:}$: 2.4 and 3.8 GeV²
- Evolution effects are then compared.

COMPASS

<Q²> = 3.8 GeV²

Q^2 in the range [1.3 – 20.5] GeV²

TMD evolution phenomenology

Anselmino, Boglione, Melis, Phys. Rev. D86 (2012) 014028

- *First step* : choose a TMD evolution scheme
- Let $\tilde{F}$ be either an unpolarized distribution or fragmentation function, or the first derivative of the Sivers distribution function, in the **impact parameter space**.
- In general terms, its TMD evolution equation can be written as

$$\boxed{\widetilde{F}(x,\boldsymbol{b}_T;Q) = \widetilde{F}(x,\boldsymbol{b}_T;Q_0)\,\widetilde{R}(Q,Q_0,b_T)\,\exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}}$$

Aybat, Collins, Qiu, Rogers

Having taken the renormalization scale μ^2 and the regulating parameters ζ_F and ζ_D to coincide with Q^2

Beautifully explained by John Collins

TMD evolution phenomenology

Anselmino, Boglione, Melis, Phys. Rev. D86 (2012) 014028

- Let F̃ be either an unpolarized distribution or fragmentation function, or the first derivative of the Sivers distribution function, in the **impact parameter space**.
- In general terms, its TMD evolution equation can be written as

$$\widetilde{F}(x,\boldsymbol{b}_T;Q) = \widetilde{F}(x,\boldsymbol{b}_T;Q_0)\,\boxed{\widetilde{R}(Q,Q_0,b_T)}\,\exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}$$

$$\widetilde{R}(Q,Q_0,b_T) \equiv \exp\left\{\ln\frac{Q}{Q_0}\int_{Q_0}^{\mu_b}\frac{\mathrm{d}\mu'}{\mu'}\gamma_K(\mu') + \int_{Q_0}^{Q}\frac{\mathrm{d}\mu}{\mu}\gamma_F\left(\mu,\frac{Q^2}{\mu^2}\right)\right\}$$

Beautifully explained by John Collins

$$\gamma_K(\mu) = \alpha_s(\mu)\,\frac{2\,C_F}{\pi}$$

$$\gamma_F(\mu;\frac{Q^2}{\mu^2}) = \alpha_s(\mu)\,\frac{C_F}{\pi}\left(\frac{3}{2}-\ln\frac{Q^2}{\mu^2}\right)$$

TMD evolution phenomenology

Anselmino, Boglione, Melis, Phys. Rev. D86 (2012) 014028

- Let $\tilde{F}$ be either an unpolarized distribution or fragmentation function, or the first derivative of the Sivers distribution function, in the **impact parameter space**.
- In general terms, its TMD evolution equation can be written as

$$\widetilde{F}(x,\boldsymbol{b}_T;Q) = \underbrace{\widetilde{F}(x,\boldsymbol{b}_T;Q_0)}_{\text{Input function}}\, \widetilde{R}(Q,Q_0,b_T)\, \exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}$$

$g_K(b_T)$: Unknown, but universal and scale Independent, input function

$$\widetilde{R}(Q,Q_0,b_T) \equiv \exp\left\{\ln\frac{Q}{Q_0}\int_{Q_0}^{\mu_b}\frac{d\mu'}{\mu'}\gamma_K(\mu') + \int_{Q_0}^{Q}\frac{d\mu}{\mu}\gamma_F\left(\mu,\frac{Q^2}{\mu^2}\right)\right\}$$

$$\gamma_K(\mu) = \alpha_s(\mu)\,\frac{2\,C_F}{\pi}$$

$$\gamma_F(\mu;\frac{Q^2}{\mu^2}) = \alpha_s(\mu)\,\frac{C_F}{\pi}\left(\frac{3}{2}-\ln\frac{Q^2}{\mu^2}\right)$$

TMD evolution phenomenology

The appropriate Fourier transform allows us to obtain the distribution and fragmentation functions $\tilde{F}$ in the **momentum space**

$$f_{q/p}(x,k_\perp;Q) = \frac{1}{2\pi}\int_0^\infty \mathrm{d}b_T\, b_T\, J_0(k_\perp b_T)\, \widetilde{f}_{q/p}(x,b_T;Q)\,,$$

$$D_{h/q}(z,p_\perp;Q) = \frac{1}{2\pi}\int_0^\infty \mathrm{d}b_T\, b_T\, J_0(\mathrm{k}_T b_T)\, \widetilde{D}_{h/q}(z,b_T;Q)\,,$$

$$f_{1T}^{\perp f}(x,k_\perp;Q) = \frac{-1}{2\pi k_\perp}\int_0^\infty \mathrm{d}b_T\, b_T\, J_1(k_\perp b_T)\, \widetilde{f}'^{\perp q}_{1T}(x,b_T;Q)\,,$$

$$\begin{aligned} f_{q/p^\uparrow}(x,\boldsymbol{k}_\perp,\boldsymbol{S};Q) &= f_{q/p}(x,k_\perp;Q) - f_{1T}^{\perp q}(x,k_\perp;Q)\frac{\epsilon_{ij}\,k_\perp^i\,S^j}{M_p} \\ &= f_{q/p}(x,k_\perp;Q) + \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x,k_\perp;Q)\frac{\epsilon_{ij}\,k_\perp^i\,S^j}{k_\perp} \end{aligned}$$

Parameterization of unknown functions

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\ \exp\left\{ -g_K(b_T) \ln \frac{Q}{Q_0} \right\}$$

$$g_K(b_T) = \frac{1}{2} g_2 b_T^2 \quad \text{with} \quad g_2 = 0.68$$

$$\widetilde{f}_{q/p}(x, b_T; Q_0) = f_{q/p}(x, Q_0) \exp\left\{-\alpha^2 b_T^2\right\}$$

$$f_{q/p}(x, k_\perp; Q_0) = f_{q/p}(x, Q_0) \frac{1}{\pi \langle k_\perp^2 \rangle} e^{-k_\perp^2/\langle k_\perp^2 \rangle}$$

$$\alpha^2 = \langle k_\perp^2 \rangle / 4$$

g_2 alert !

- g_2 controls the b_T gaussian width and its spreading as b_T varies.

$$\boxed{g_K(b_T) = \frac{1}{2}\, g_2\, b_T^2 \qquad \text{with} \qquad \boxed{g_2 = 0.68}}$$

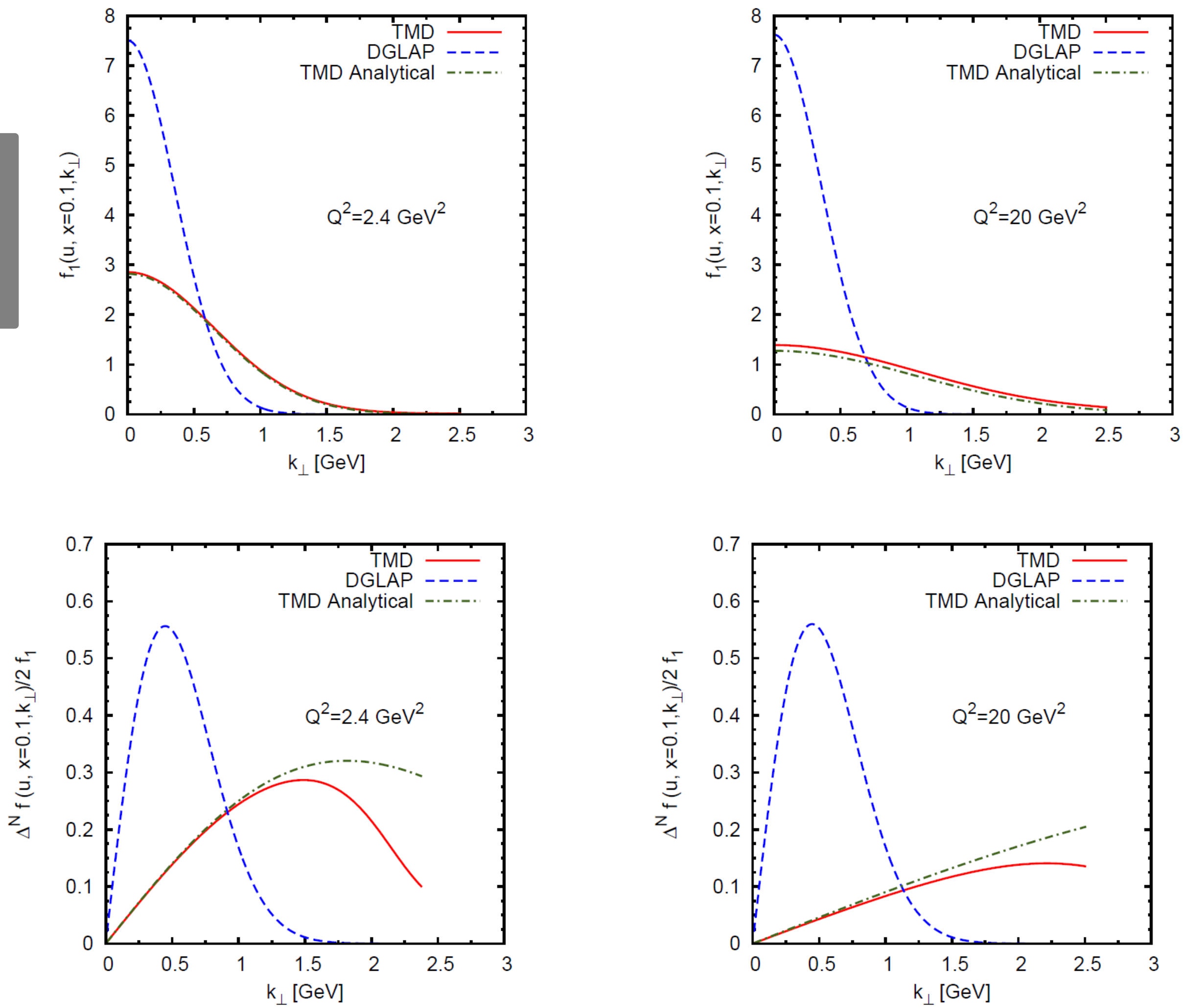
$$\boxed{b_{\mathrm{max}} = 0.5\ \mathrm{GeV}^{-1}}$$

- *We do not extract the value of g_2 from our fit*
- *We use a fixed value previously, determined in a fit of D-Y data. Landry, Brock, Nadolsky, Yuan, Phys. Rev. D67(2003) 073016 We could have done it, and probably got a smaller value, but it is important to remember that* ***SIDIS data are very little sensitive to the precise value of g_2.***
- *D-Y data, instead, are extremely sensitive to it: this requires a new, careful, global analysis on all SIDIS and D-Y, re-starting from unpolarized cross sections.*

Phenomenological results

DGLAP evolution is extremely slow in this Q^2 range

TMD evolution Very rapidly widens and dilutes the functions



Sivers function from HERMES and COMPASS SIDIS data

- **2 different fits:**
- TMD-fit (computing TMD evolution equations numerically)
- DGLAP evolution equation for the collinear part of the TMD)

hermes

$\langle Q^2 \rangle = 2.4$ GeV^2

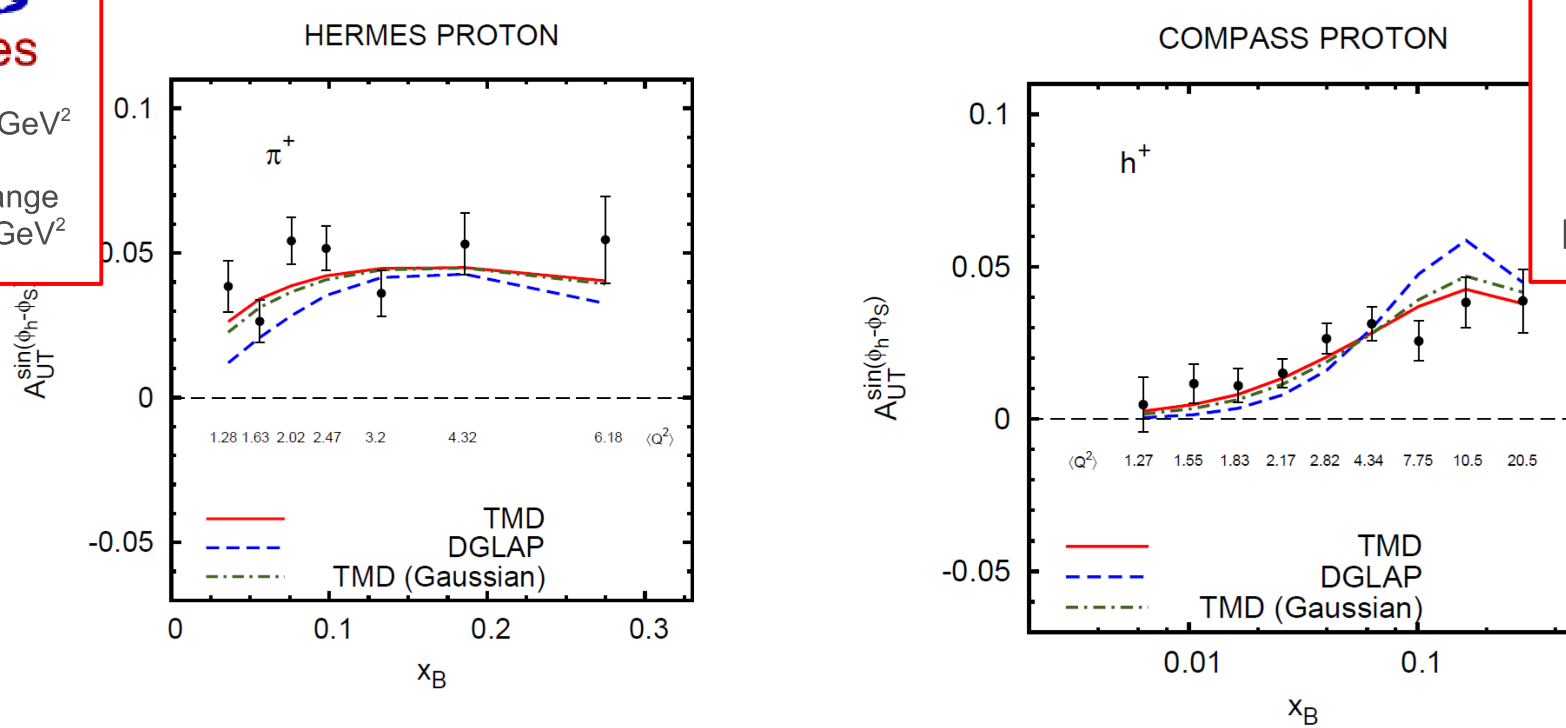
Q^2 in the range [1.3 – 6.2] GeV^2

Anselmino, Boglione, Melis, Phys. Rev. D86 (2012) 014028

COMPASS

$\langle Q^2 \rangle = 3.8$ GeV^2

Q^2 in the range [1.3 – 20.5] GeV^2



A. Airapetian et al., Phys. Rev. Lett. 103, (2009) 152002

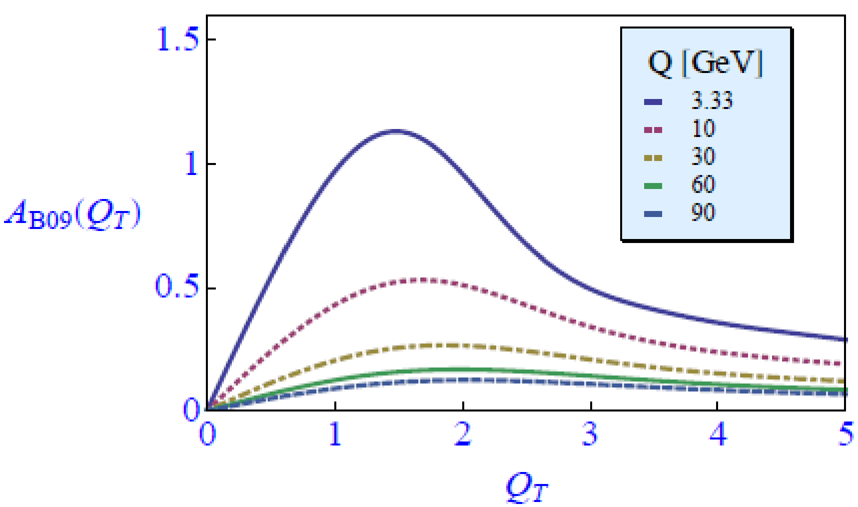
C. Adolph et al., Phys. Lett. B717 (2012) 383

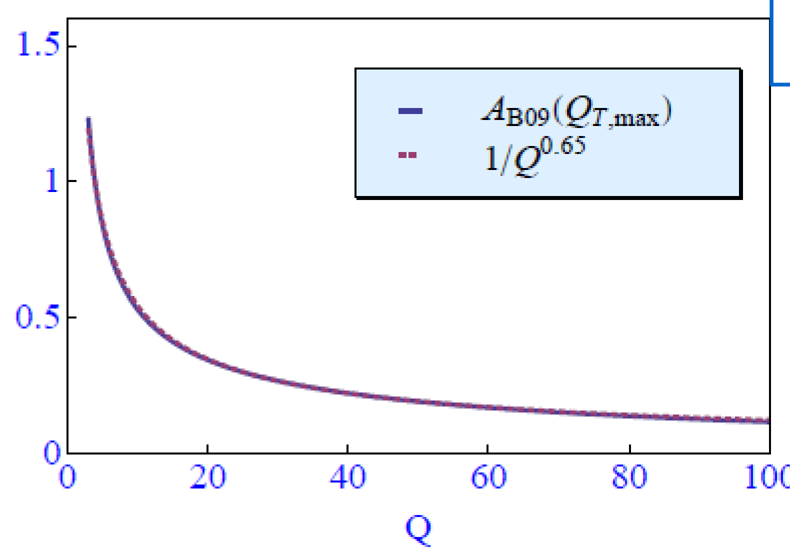
TMD evolution of the Sivers asymmetry

D. Boer, arXiv:1304.5387

- *Most recently D. Boer has performed a similar study of the energy scale dependence of the Sivers asymmetry in SIDIS, although on a larger Q range (3 - 100 GeV). He finds that peak of the Sivers asymmetry falls off with Q like $(1/Q)^{0.7}$, quite faster than found within the CSS evolution schemes.*
- *Moreover, the peak of the asymmetry is located at ~ Q_0 and moves rather slowly towards higher transverse momentum values as Q increases, which may be due to the absence of perturbative tails of the TMDs.*

See talk by Daniel Boer





TMD evolution phenomenology of the Collins fragmentation function

- *The Collins fragmentation function is chiral odd, so it always appears coupled with another chiral odd function. In SIDIS it is observed in the Collins effect, coupled to transversity.*

- *TMD evolution equations for the Collins functions where formulated by Zhong-Bo Kang in Phys.Rev. D83 (2011) 036006 while for transversity a recent study has been performed by Bacchetta and Prokudin in arXiv:1303.2129*

See talks by Prokudin and Kang

- *While for phenomenologically testing the TMD evolution of the Sivers function we have difficulties in finding data which span a sufficiently large range of Q^2 values, for the Collins fragmentation function we have data at large Q as well, from e^+e^- scattering experiments (BELLE and BaBar, Q^2=100 GeV^2), where a convolution of two Collins functions appears.*

See talk by Isabella Garzia

- *First experimental data from COMPASS and HERMES seem to hint to a very slow evolution, however a combined phenomenological study of TMD evolution in SIDIS and e^+e^- scattering would be highly desirable and certainly very interesting.*

SIDIS vs Drell-Yan

- *In these first attempts to perform TMD evolution phenomenology, implications of different evolution schemes have only been tested on a restricted range of Q^2 values*
- *To perform a serious phenomenological study of TMD evolution we would need SIDIS data (not only for spin asymmetries but also, very importantly, on unpolarized cross sections) in a much larger range of Q^2 values*

  → *We need an Electron - Ion Collider.*
- *Meanwhile, we can look at Drell-Yan unpolarized data, which cover a much larger range of Q^2 values*
- *However, if we apply the parameters extracted from fitting SIDIS spin asymmetries data to the computation of Drell-Yan cross sections, we find a very strong suppression due to the fast $k_\perp$ broadening effects induced by the TMD evolution within the Aybat, Collins, Qiu, Rogers scheme with a large g_2.*
- *Notice that this reduction has also been found by A. Mukherjee and her collaborators when computing the Sivers spin asymmetry for J/ψ electroproduction.*

  Godbole, Misra, Mukherjee, Raswoot, arXiv:1304:2584

See talk by Asmita Mukherjee

SIDIS vs Drell-Yan

- Numerator of the asymmetry in analytical approximation for a DY process

$$N_{DY} \propto \Delta^N f(x_1,Q_0) f(x_2,Q_0) \sqrt{2e}\,\frac{P_T}{M_1}\,\frac{\langle k_\perp^2\rangle^2_{Siv}}{\langle k_{\perp 1}^2\rangle \langle P_T^2\rangle^2_{Siv}}\, e^{-P_T^2/\langle P_T^2\rangle_{Siv}}$$

$$\boxed{\langle P_T^2\rangle^{DY}_{Siv} = \omega^2_{Siv} + \omega^2_2}$$

$$\boxed{w_S^2(Q,Q_0) = \langle k_\perp^2\rangle_S + 2g_2 \ln\frac{Q}{Q_0}}$$

$$\boxed{w^2(Q,Q_0) = \langle k_\perp^2\rangle + 2\,g_2 \ln\frac{Q}{Q_0}}$$

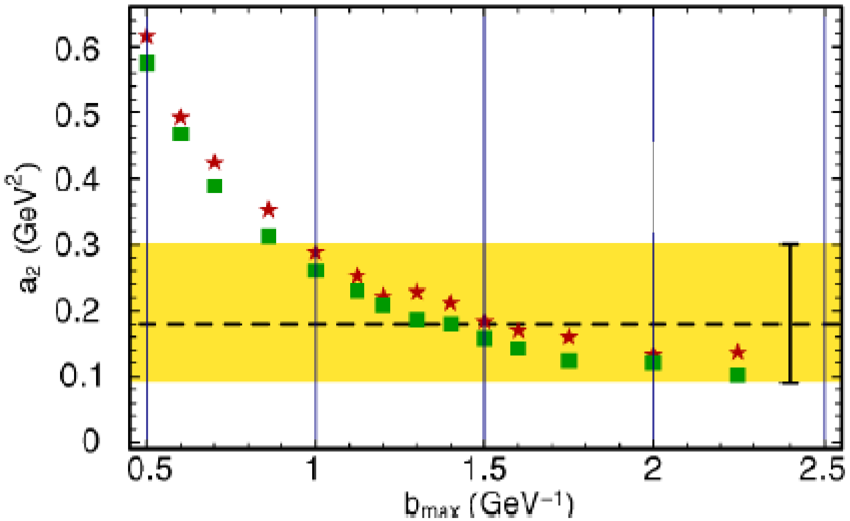
- Here it is squared, strongly suppresses the asymmetry as it becomes larger and larger

- g_2 is more crucial for DY processes than for the present SIDIS data because of the larger range spanned by Q

SIDIS vs Drell-Yan

Konychev, Nadolsky, Phys. Lett. B633 (2006) 710

- g_2 depends on the prescription for the separation of the perturbative region from the non -perturbative one. Depends also on the "order" at which you stop in the perturbative expansion.



a_2=g_2, stars correspond to the choice C1=2 exp(-γ_e), squares to C1=4 exp(-γ_e)

Low-Q Drell-Yan experiments (E288,E605 and R209) show a preference for b_{max} larger than 0.5 GeV^{-1} (around 1.5), while higher Q data are not very sensitive to this value.

SIDIS vs Drell-Yan

Konychev, Nadolsky, Phys. Lett. B633 (2006) 710

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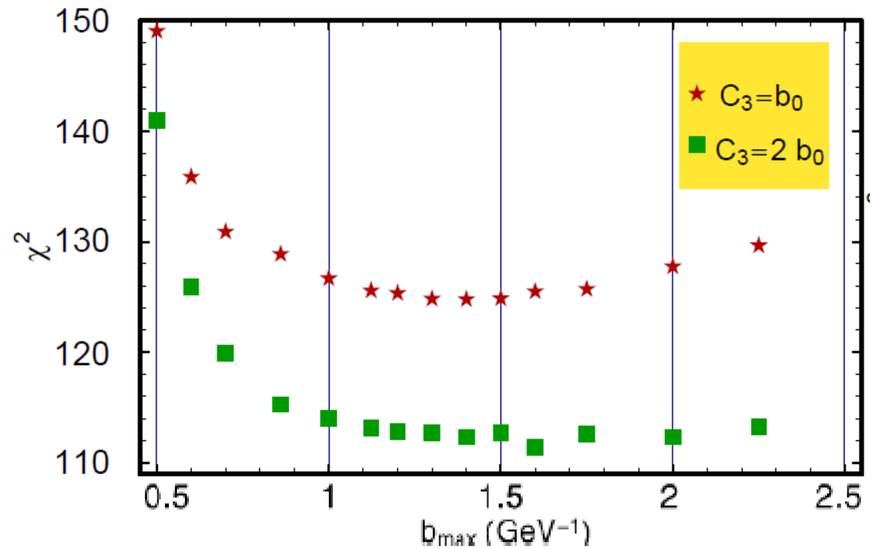

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TMD evolution phenomenology

Sun and Yuan, arXiv:1304.5037 [hep-ph]

- *Pen Sun and Feng Yuan have recently applied the CSS original evolution scheme at one loop (with some approximations which hold in the moderate Q and Q_0) to account for TMD evolution of the unpolarized TMD PDFs, and extended this formalism to the Sivers function as well.*

$$\widetilde{F}^{\alpha}_{\mathrm{sivers}}(Q;b) = \widetilde{F}^{\alpha}_{\mathrm{sivers}}(Q_0;b)e^{-\mathcal{S}_{Sud}(Q,Q_0,b)}$$

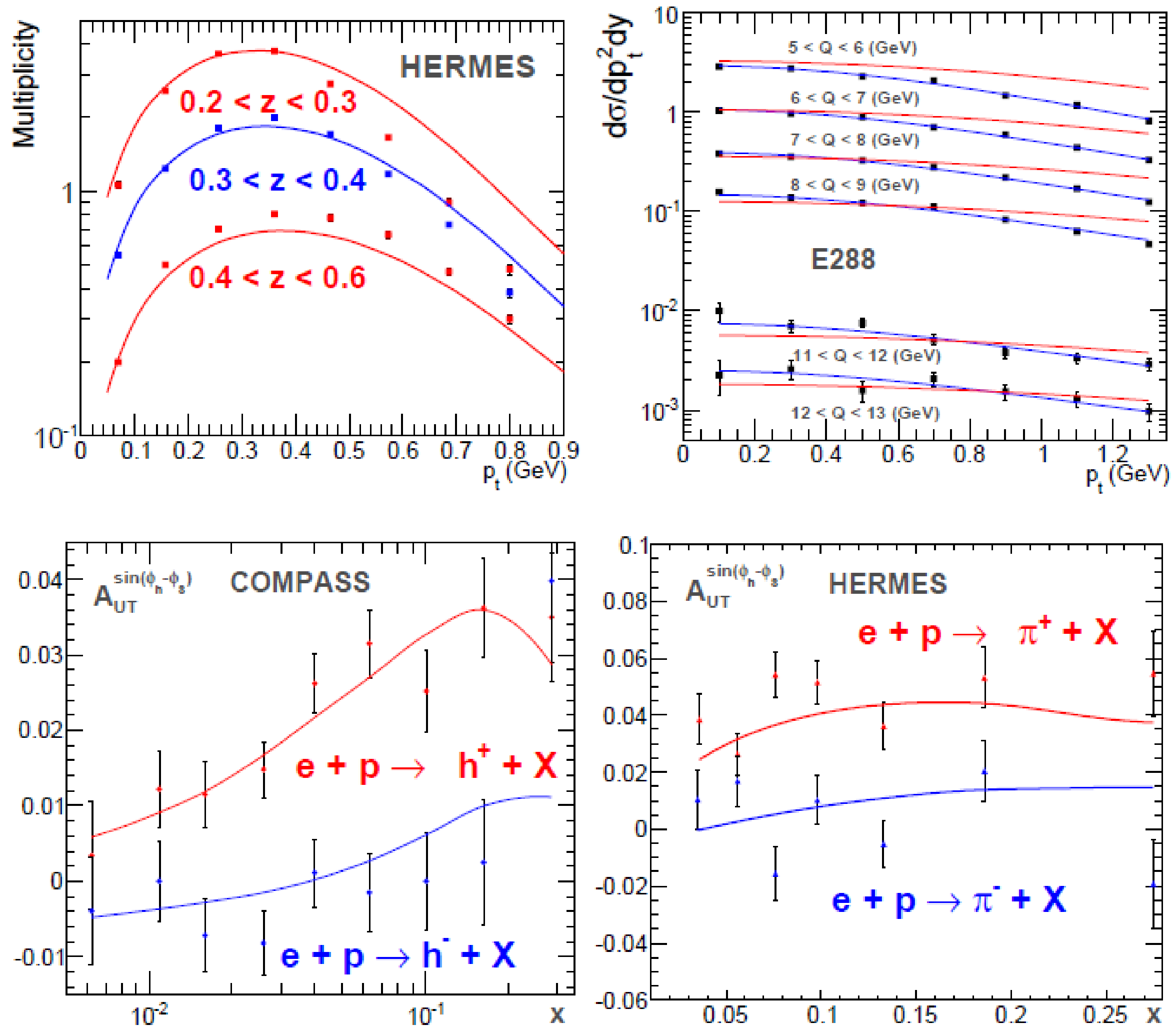
$$\mathcal{S}_{Sud} = 2C_F \int_{Q_0}^{Q} \frac{d\bar{\mu}}{\bar{\mu}} \frac{\alpha_s(\bar{\mu})}{\pi} \left[\ln\left(\frac{Q^2}{\bar{\mu}^2}\right) + \ln\frac{Q_0^2 b^2}{c_0^2} - \frac{3}{2}\right]$$

See talk by Feng Yuan

- *In this scheme the evolution does not produce such a strong (and worrying) suppression of the Drell-Yan asymmetries.*
- *They then perform a phenomenological study on a selection of Drell-Yan and SIDIS data, showing that the evolution scheme they propose can satisfactorily describe most of them.*

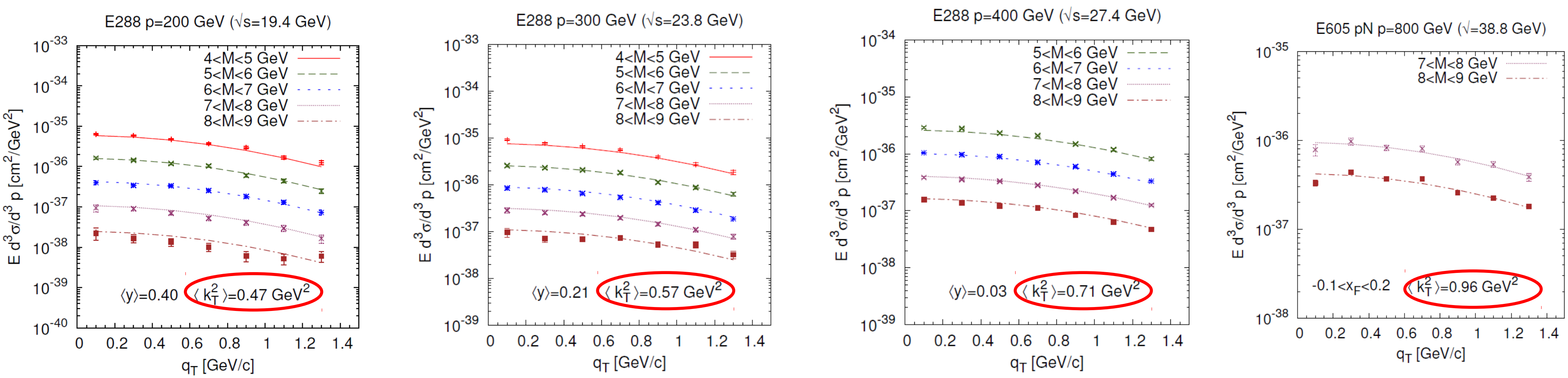
TMD evolution phenomenology

Sun and Yuan, arXiv:1304.5037 [hep-ph]



Drell-Yan phenomenology

Stefano Melis preliminary studies



The fit on E288 and E605 Drell-Yan data is performed by assuming a gaussian $k_\perp$ dependence with a DGLAP evolution of the factorized

$$\widehat{f}_{q/p}(x, k_\perp; Q) = f_{q/p}(x; Q)\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle}$$

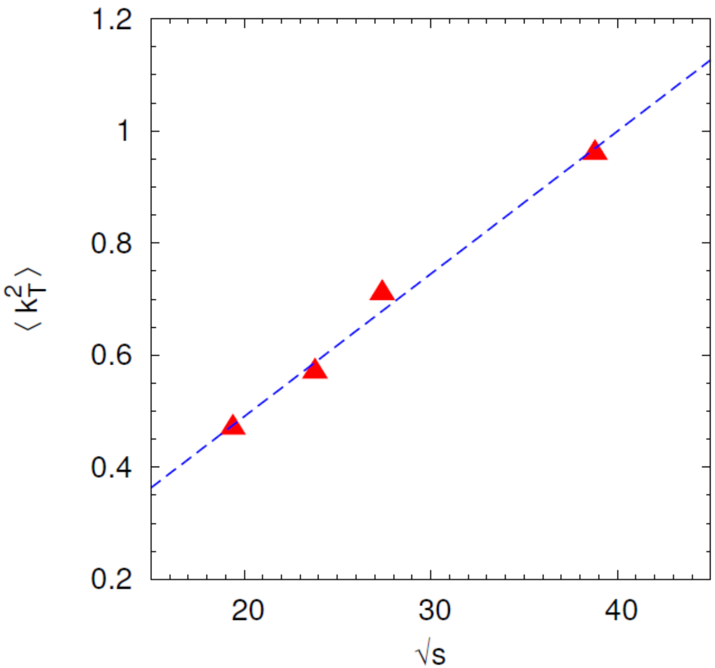
The gaussian width is fitted independently for each different energy data set.

Notice that $<k_\perp^2>$ grows as energy grows

Schweitzer, Teckentrup, Metz, Phys.Rev. D81 (2010) 094019
D'Alesio, Murgia, Phys. Rev. D70 (2004) 074009

Drell-Yan phenomenology

Stefano Melis preliminary studies



The dependence of $<k_\perp^2>$ on the energy is roughly linear

Loads of room for phenomenology !

Outlook and conclusions

- *As far as TMD evolution is concerned we have recently come a long way.*
- *We now have evolution schemes and some first attempts to the phenomenological study of the unpolarized distribution and fragmentation TMDs, of the TMD transversity and of the Sivers functions.*
- *These are very preliminary studies, which need to be refined and re-thought in a more consistent and appropriate way, especially as far as the parametrization of unknown phenomenological quantities are concerned.*
- *From the experimental side, we need more SIDIS (polarized and unpolarized) data at larger values of x (Jlab 12) and spanning a larger Q2 range (EIC) as well as more (and more precise) Drell-Yan data, for which many beautiful experiments are being planned (COMPASS, RHIC, Fermilab, NICA, JPARK).*
- *Hadron-hadron scattering processes represent a very interesting and infinitely challenging field where to "sharpen our tools".*

See talks by L. Bland, L. Gamberg, A. Metz

Outlook and conclusions

- *With the new experimental data on SIDIS multiplicities coming in, we have to go back one step, re-think and re-perform a solid, global analysis of Drell-Yan as well as SIDIS unpolarized cross sections, to determine the basic parameters for the phenomenological quantities needed for the implementation of the TMD evolution schemes.*
- *Afterwards, we can proceed on a firm footing to perform the same analysis for the Sivers, transversity and Collins TMD functions, keeping in mind the importance of finding phenomenological framework suitable for all processes.*