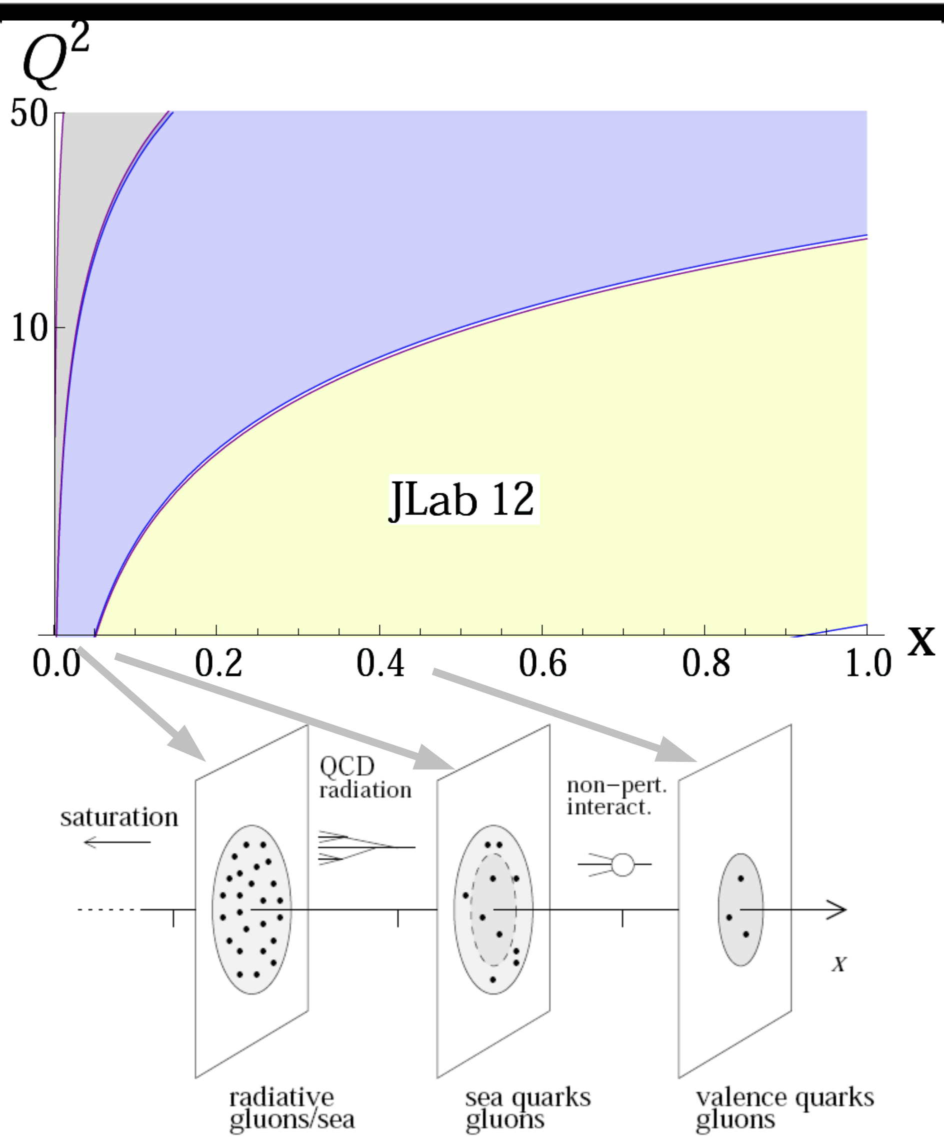
TMD EVOLUTION OF HELICITY AND TRANSVERSITY

Alexei Prokudin

May 9, 2013

Nucleon landscape



Nucleon is a many body dynamical system of quarks and gluons

Changing x we probe different aspects of nucleon wave function

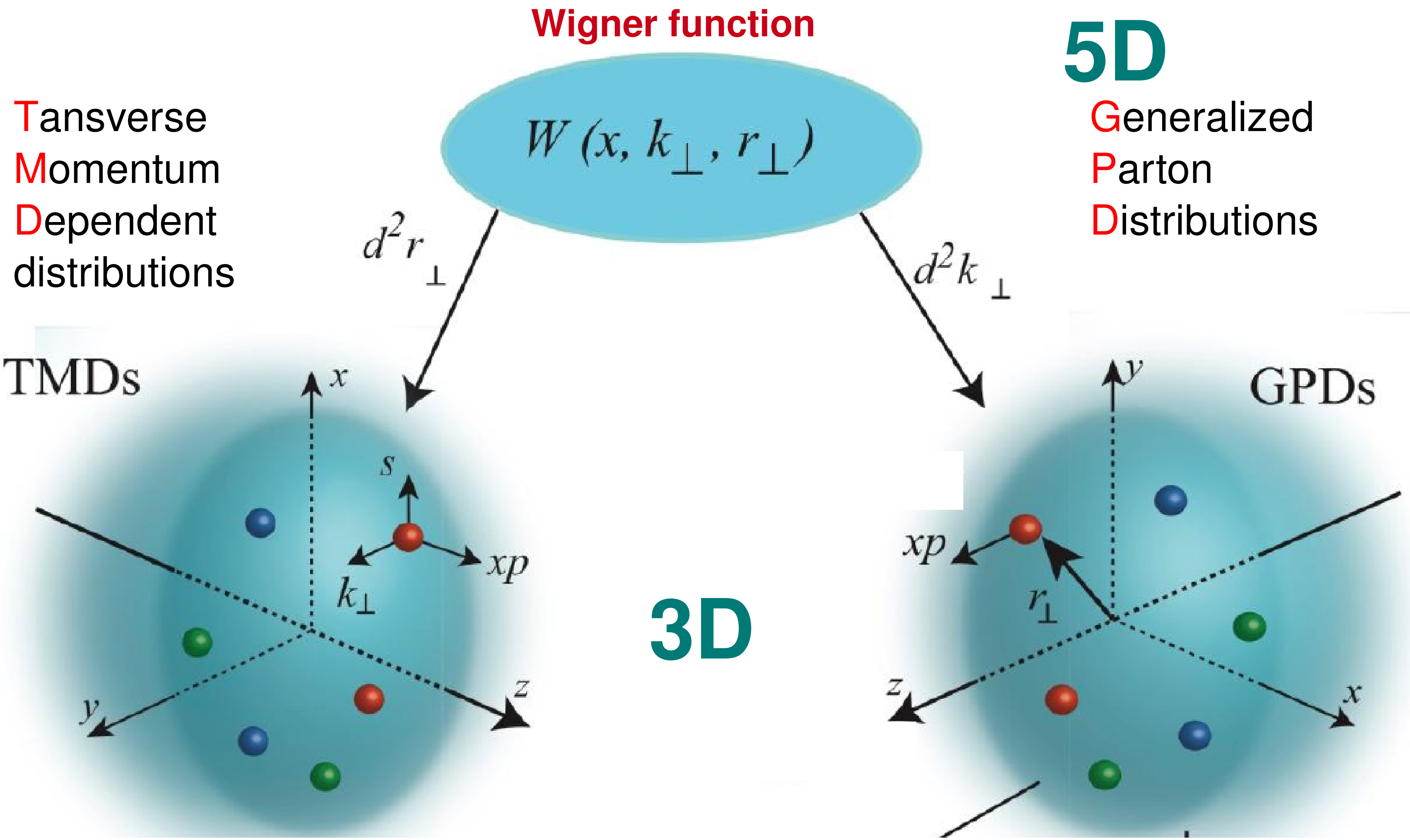
How **partons move** and how they are distributed in **space** is one of the future directions of development of nuclear physics

Technically such information is encoded into Generalised Parton Distributions and Transverse Momentum Dependent distributions

These distributions are also referred to as **3D (three-dimensional) distributions**

Unified View of Nucleon Structure

Tansverse
Momentum
Dependent
distributions



Generalized
Parton
Distributions

Belitsky, Ji, Yuan 2003

Unified View of Nucleon Structure

5D

Wigner function

$$W(x, \mathbf{k}_\perp, \mathbf{r}_\perp)$$

Tansverse Momentum Dependent distributions

$$\int d^2\mathbf{r}_\perp$$

$$\int d^2\mathbf{k}_\perp$$

Generalized Parton Distributions

3D

$$f(x, \mathbf{k}_\perp)$$

$$H(x, \xi, t)$$

$$\int d^2\mathbf{k}_\perp$$

Parton Distribution Functions

Form Factors

$$\int dx$$

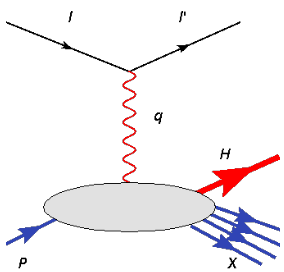
1D

$$f(x)$$

$$F(Q^2)$$

Unified View of Nucleon Structure

TMD

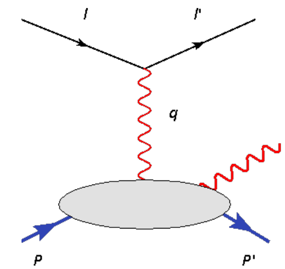


Wigner function

$$W(x, \mathbf{k}_\perp, \mathbf{r}_\perp)$$

5D

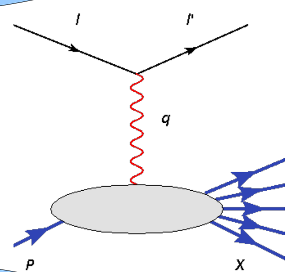
GPD

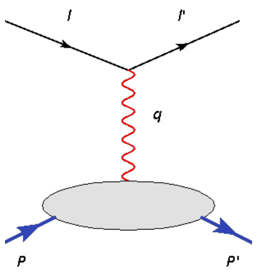


$$f(x, \mathbf{k}_\perp)$$

3D

$$H(x, \xi, t)$$





PDF

$$f(x)$$

1D

FF

$$F(Q^2)$$

Particular processes to study. Polarization is required!

Twist-2 collinear PDFs

Quark-quark correlator can be decomposed by means of
3 Parton Distributions Functions (PDF) in collinear case

$$\Phi(x;P,S) = \frac{1}{2}\left\{ f_1(x)\,\not{P} + S_L g_1(x)\gamma_5\,\not{P} + \frac{1}{2} h_1(x)\gamma_5[\,\not{S}_T,\not{P}] \right\}$$

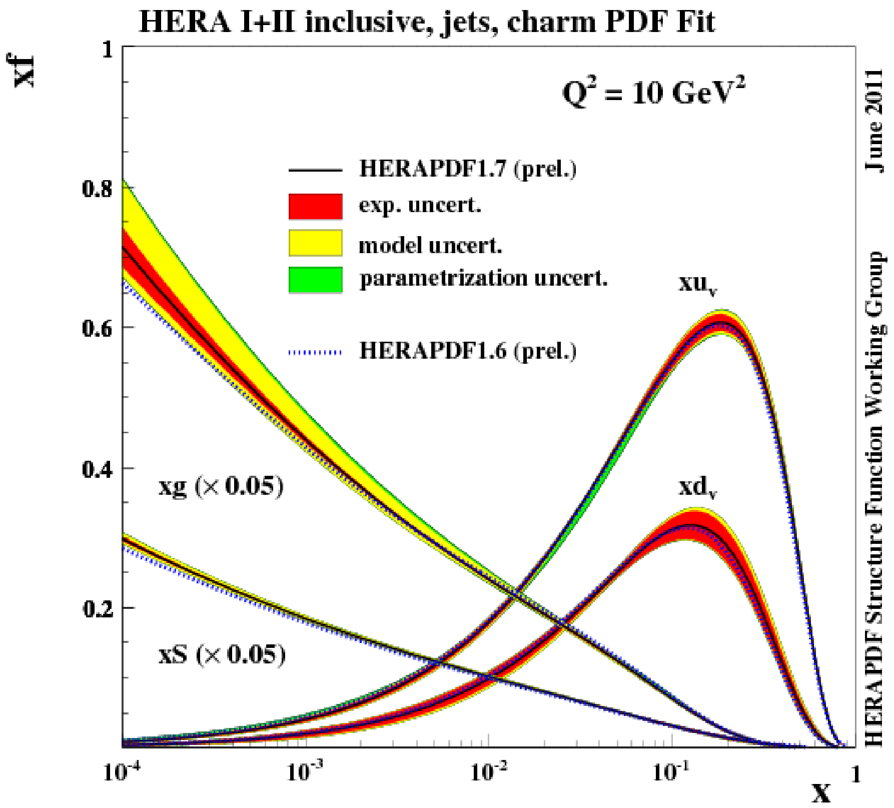
Unpolarised PDF

Helicity distribution

Transversity distribution

Unpolarised PDFs

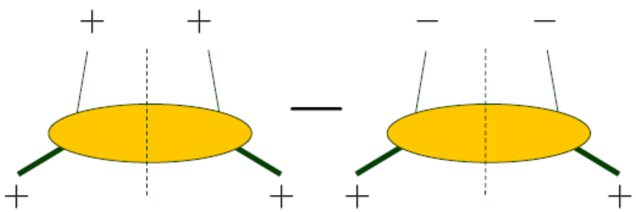
Good knowledge of unpolarised **P**arton **D**istribution **F**unctions is acquired using HERA data

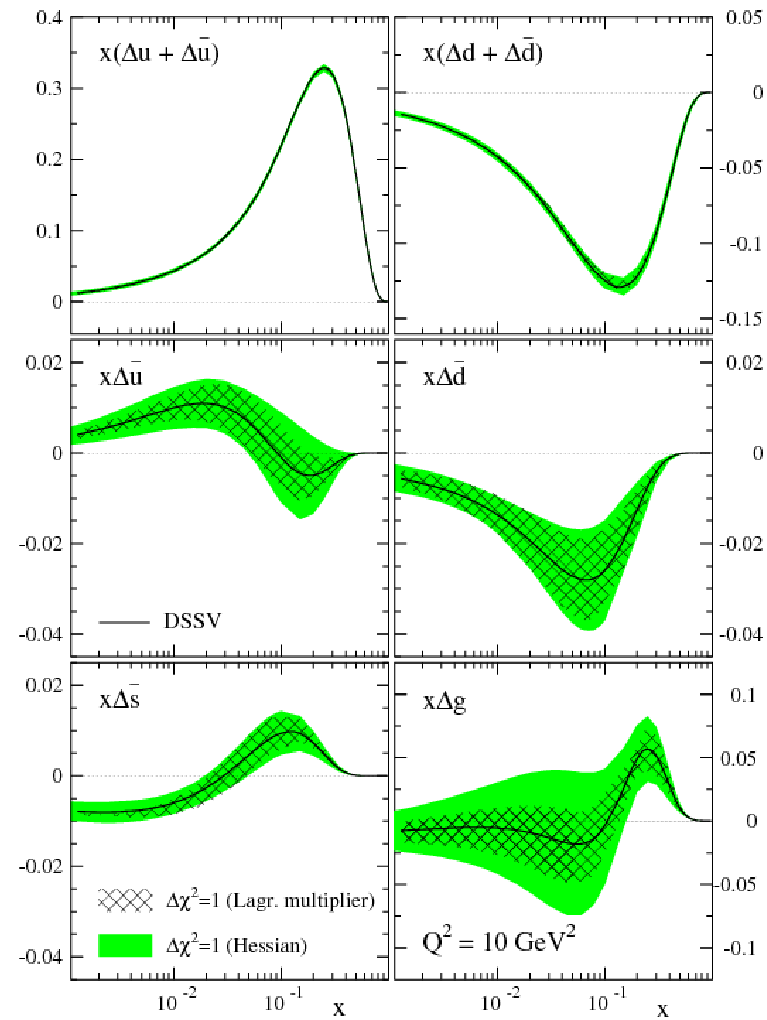


Plot from
Eram Rizvi et.al.,
JHEP 2009 (arXiv:0911.0884)

Helicity distributions

Helicity distributions are relatively well known



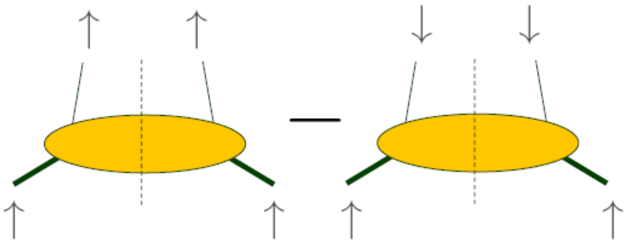


Plot from
DSSV PRD80 (2009) 034030

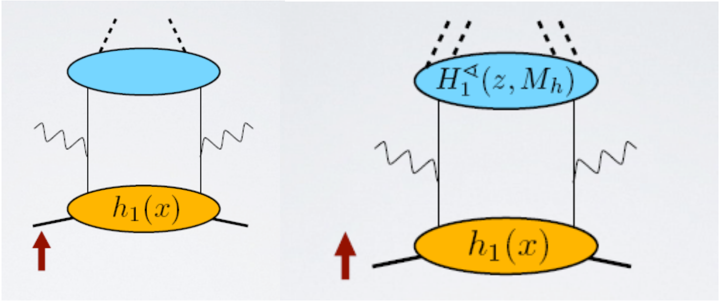
Jefferson Lab

Transversity distributions

Distribution of transversely polarised quarks inside transversely polarised Nucleon, chiral odd

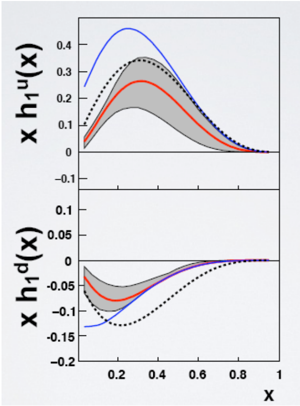


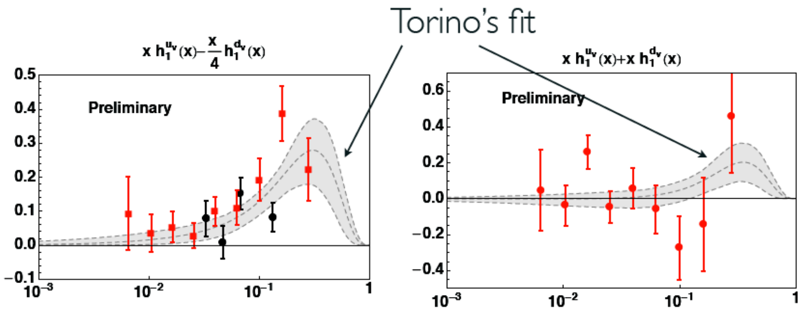
Can be stydied in SIDIS (COMPASS, HERMES, JLAB)



With Collins FF   With Dihadron FF

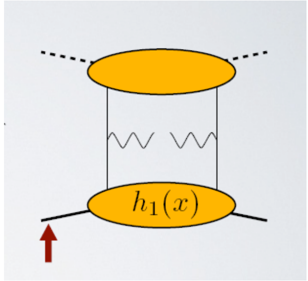
Extractions





Bacchetta et al.,arXiv:1206.1836

Drell-Yan



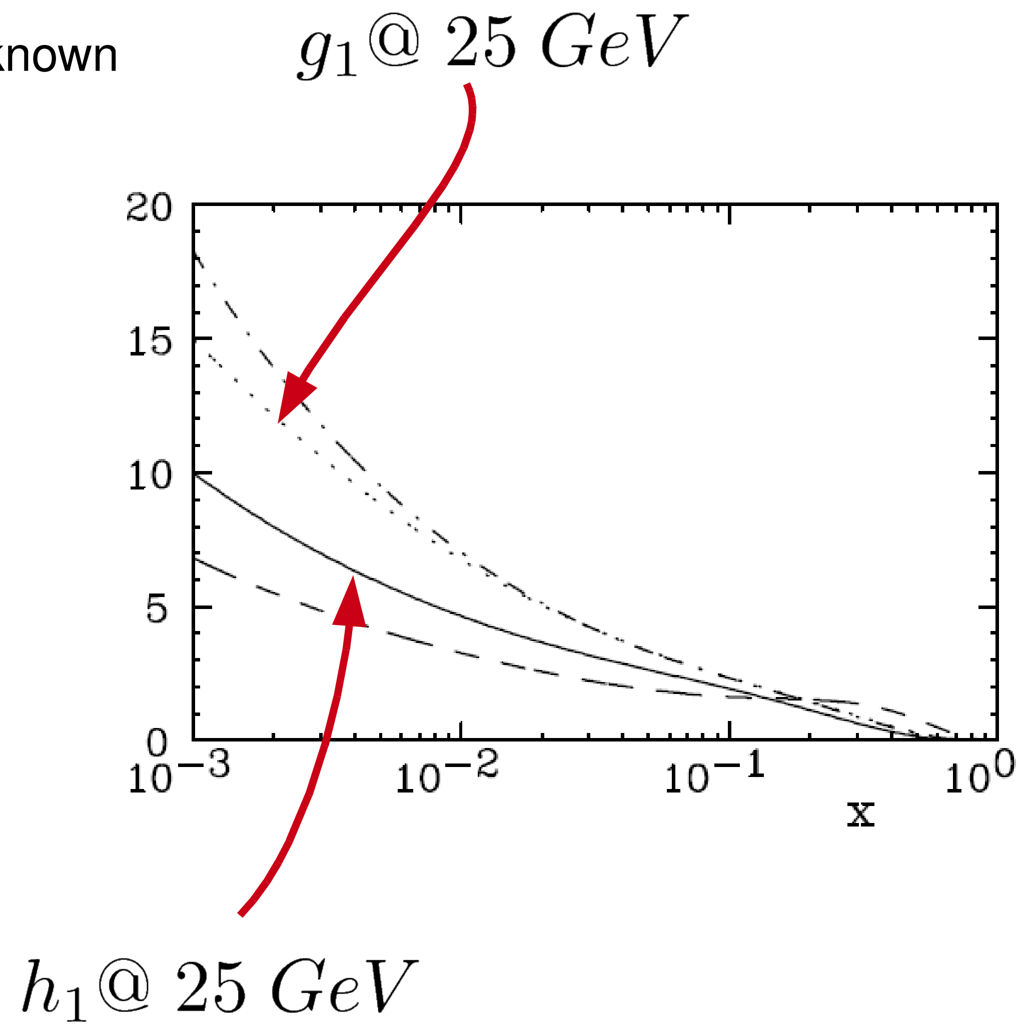
With transversity or Boer-Mulders

Anselmino et al 09, 13   See talks of Elena Boglione and Marco Radici

Evolution of transversity and helicity

Collinear (DGLAP) case is well known

Barone 1997
Vogelsang 1998



Evolution of Transverse Momentum Dependent transversity and helicity

Alessandro Bacchetta, Alexei Prokudin
arXiv:1303.2129

Definition of TMDs

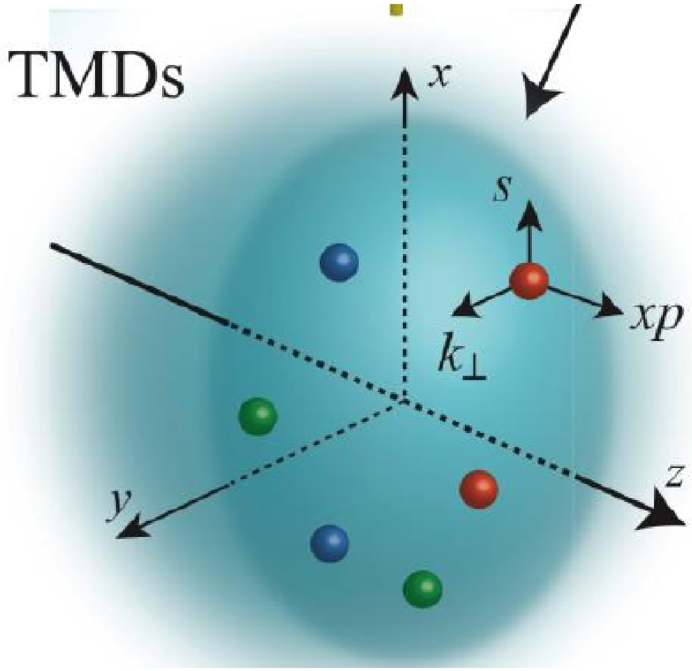
TMD functions describe processes where quark intrinsic motion is important

Historically TMD factorization is formulated as Collins-Soper-Sterman resummation
Collins, Soper, Sterman 1985

Proven for polarized case
Ji, Ma, Yuan 2004
Collins 2011

Alternative formulations
Cherednikov, Stefanis 2008
Echevarria, Idilbi, Scimemi 2011
Trentadue, Ceccoperi, 2008
Hautman, 2008

Equivalence with some approaches was shown in
Collins, Rogers 2012



See talks of John Collins, Ahmad Idilbi, Igor Cherednikov

Definition of TMDs

In the following we will use definition by

Collins 2011

TMD is defined in coordinate space

$$\tilde{f}(x, b_\perp; \mu, \zeta)$$

Fourier conjugate to $k_\perp$

Definition of TMDs

In the following we will use definition by

Collins 2011

TMD is defined in coordinate space

$$\tilde{f}(x, b_\perp; \mu, \zeta)$$

RG renormalization

Definition of TMDs

In the following we will use definition by

Collins 2011

TMD is defined in coordinate space

$$\tilde{f}(x, b_\perp; \mu, \zeta)$$

Additional parameter to cancel rapidity divergence

TMD evolution

$$\frac{\partial \ln \tilde{F}(x, b_\perp, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_\perp, \mu)$$

Collins, Soper, Sterman 1985
Idilbi, Ji, Ma, Yuan, 2004
Collins, 2011

$$\frac{d\tilde{K}(b_\perp, \mu)}{d \ln \mu} = -\gamma_K(g(\mu))$$

$$\frac{d \ln \tilde{F}(x, b_\perp, \mu, \zeta)}{d \ln \mu} = \gamma_F(g(\mu), \zeta)$$

TMD evolution:helicity and transversity

Solve evolution equations:

Collins Soper Sterman 1985
Collins 2011

$$\widetilde{f}_1^f(x,b_T;\mu,\zeta_F) = \sum_i \underbrace{(\tilde{C}_{f/i}\otimes f_1^i)(x,b_*;\mu_b)}_{\text{Expansion at small bt, Spin dependent}} \underbrace{e^{\tilde{S}(b_*;\mu_b,\mu,\zeta_F)} e^{g_K(b_T)\ln\frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{f0}}}}}_{\text{Contains gluon radiation, no spin dependence}} \hat{f}^q_{\mathrm{NP}}(x,b_T)$$

TMD evolution:helicity and transversity

Solve evolution equations:

Collins Soper Sterman 1985
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$$\widetilde{f}_1^f(x, b_T; \mu, \zeta_F) = \sum_i \underbrace{(\tilde{C}_{f/i} \otimes f_1^i)(x, b_*; \mu_b)}_{\text{Expansion at small bt, Spin dependent}} \underbrace{e^{\tilde{S}(b_*;\mu_b,\mu,\zeta_F)} e^{g_K(b_T)\ln\frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{f0}}}}}_{\text{Contains gluon radiation, no spin dependence}} \hat{f}_{\rm NP}^q(x, b_T)$$

$$\tilde{S}(b_*; \mu_b, \mu, \zeta_F) = \ln\frac{\sqrt{\zeta_F}}{\mu_b}\tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{\mu}\frac{d\mu'}{\mu'}\left[\gamma_F(g(\mu'); 1) - \ln\frac{\sqrt{\zeta_F}}{\mu'}\gamma_K(g(\mu'))\right]$$

See talk of John Collins

TMD evolution:helicity and transversity

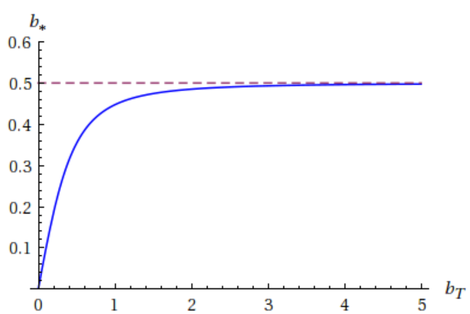
Collins Soper Sterman 1985
Collins 2011

Solve evolution equations:

$$\widetilde{f}_1^f(x,b_T;\mu,\zeta_F) = \sum_i \left(\tilde{C}_{f/i}\otimes f_1^i\right)(x,b_*;\mu_b)\, e^{\tilde{S}(b_*;\mu_b,\mu,\zeta_F)}\, e^{g_K(b_T)\ln\frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{f0}}}}\, \hat{f}^q_{\mathrm{NP}}(x,b_T)$$

Method to avoid Landau pole

$$b_*(b_T) = \frac{b_T}{\sqrt{1+b_T^2/b_{max}^2}}$$



$$\mu_b = 2e^{-\gamma_E}/b_*$$

is chosen to optimise convergenge of perturbative series

See talks of Daniel Boer and Feng Yuan

TMD evolution:helicity and transversity

Solve evolution equations:

Collins Soper Sterman 1985
Collins 2011

$$\tilde{f}_1^f(x,b_T;\mu,\zeta_F)=\sum_i(\tilde{C}_{f/i}\otimes f_1^i)(x,b_*;\mu_b)e^{\tilde{S}(b_*;\mu_b,\mu,\zeta_F)}e^{g_K(b_T)\ln\frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{f0}}}}\hat{f}^q_{\mathrm{NP}}(x,b_T)$$

Method to avoid Landau pole

Collins Soper 1982

Always perturbative

$$\tilde{K}(b_T,\mu)=\tilde{K}(b_*,\mu)+g_K(b_T)$$

Non perturbative

$$g_K(b_T)=-g_2 b_T^2$$ The choice is not unique

Brock, Landry, Nadolsky, Yuan 2003

See talks of Daniel Boer and Feng Yuan

TMD evolution:helicity and transversity

Collins Soper Sterman 1985
Collins 2011

Solve evolution equations:

$$\tilde{f}_1^f(x,b_T;\mu,\zeta_F) = \sum_i \left(\tilde{C}_{f/i} \otimes f_1^i\right)(x,b_*;\mu_b)\, e^{\tilde{S}(b_*;\mu_b,\mu,\zeta_F)}\, e^{g_K(b_T)\ln\frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{f0}}}}\, \hat{f}^q_{\mathrm{NP}}(x,b_T)$$

Calculate these in order to derive evolution of helicity and transversity

Bacchetta, Prokudin 2013

TMD evolution:helicity and transversity

Relation to collinear treatment:

Collins Soper Sterman 1985
Collins 2011

$$\tilde{f}_f(x, b_T, \mu, \zeta) = \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{j/f}\left(\frac{x}{\hat{x}}, b_T, \mu, \zeta\right) f_j(\hat{x}, \mu) + \mathcal{O}(\Lambda_{QCD} b_T)$$

Valid at small b_T. Lowest order:

$$\tilde{C}_{j/f}(\frac{x}{\hat{x}}, b_T, \mu, \zeta) = \delta_{jf}\delta\left(\frac{x}{\hat{x}} - 1\right) + \mathcal{O}(\alpha_s)$$

First order for TMD PDFs

Aybat Rogers 2011

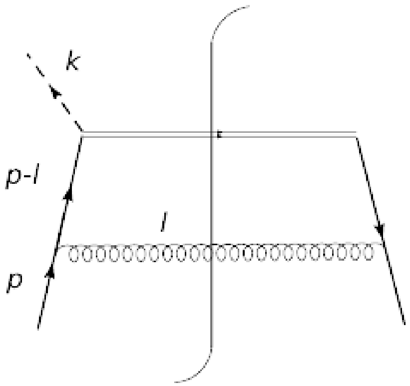
First order for Sivers function

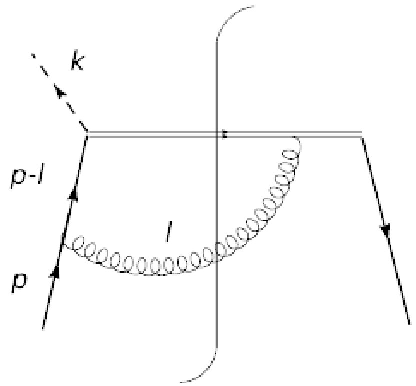
Kang, Xiao, Yuan 2011

See talks of Ivan Vitev, Daniel Boer, and Feng Yuan

TMD evolution:helicity and transversity

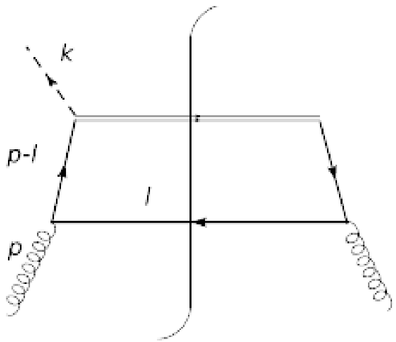
Diagrams:





Quark in quark coefficient function

$$\tilde{C}_{q/q}$$



Quark in a gluon Coefficient function

$$\tilde{C}_{q/g}$$

Projections: Unpolarised γ^+ Helicity $\gamma^+\gamma_5$ Transversity $\gamma^+\gamma_i\gamma_5$

TMD evolution:helicity and transversity

A. Bacchetta, AP, 2013

Calculate everything for quark:

$$\tilde{C}_{j/j'}(x, \boldsymbol{b}_T; \mu; \zeta_F/\mu^2) = \delta_{j'j}\delta(1-x) + \delta_{j'j}\frac{\alpha_s C_\mathrm{F}}{\pi}\left\{ \ln\left(\frac{2e^{-\gamma_\mathrm{E}}}{\mu b_T}\right)\frac{1+x^2}{(1-x)_+} + \frac{1}{2}(1-x) + \right.$$
$$\left. + \delta(1-x)\left[-\ln^2\left(\frac{2e^{-\gamma_\mathrm{E}}}{\mu b_T}\right) + \ln\left(\frac{2e^{-\gamma_\mathrm{E}}}{\mu b_T}\right)\ln\left(\frac{\zeta_F}{\mu^2}\right)\right]\right\} + \mathcal{O}(\alpha_s^2)$$

$$\Delta\tilde{C}_{j/j'}(x, \boldsymbol{b}_T; \mu; \zeta_F/\mu^2) = \delta_{j'j}\delta(1-x) + \delta_{j'j}\frac{\alpha_s C_\mathrm{F}}{\pi}\left\{ \ln\left(\frac{2e^{-\gamma_\mathrm{E}}}{\mu b_T}\right)\frac{1+x^2}{(1-x)_+} + \frac{1}{2}(1-x) + \right.$$
$$\left. + \delta(1-x)\left[-\ln^2\left(\frac{2e^{-\gamma_\mathrm{E}}}{\mu b_T}\right) + \ln\left(\frac{2e^{-\gamma_\mathrm{E}}}{\mu b_T}\right)\ln\left(\frac{\zeta_F}{\mu^2}\right)\right]\right\} + \mathcal{O}(\alpha_s^2)$$

$$\delta\tilde{C}_{j/j'}(x, \boldsymbol{b}_T; \mu; \zeta_F/\mu^2) = \delta_{j'j}\delta(1-x) + \delta_{j'j}\frac{\alpha_s C_\mathrm{F}}{\pi}\left\{ \ln\left(\frac{2e^{-\gamma_\mathrm{E}}}{\mu b_T}\right)\frac{2x}{(1-x)_+} + \right.$$
$$\left. + \delta(1-x)\left[-\ln^2\left(\frac{2e^{-\gamma_\mathrm{E}}}{\mu b_T}\right) + \ln\left(\frac{2e^{-\gamma_\mathrm{E}}}{\mu b_T}\right)\ln\left(\frac{\zeta_F}{\mu^2}\right)\right]\right\} + \mathcal{O}(\alpha_s^2)$$

TMD evolution:helicity and transversity

A. Bacchetta, AP, 2013

Calculate everything for quark:

$$\tilde{C}_{j/j'}(x, \boldsymbol{b}_T; \mu; \zeta_F/\mu^2) = \delta_{j'j}\delta(1-x) + \delta_{j'j}\frac{\alpha_s C_{\mathrm{F}}}{\pi}\left\{ \ln\left(\frac{2e^{-\gamma_{\mathrm{E}}}}{\mu b_T}\right) \frac{1+x^2}{(1-x)_+} + \frac{1}{2}(1-x) + \right.$$
$$\left. + \delta(1-x)\left[-\ln^2\left(\frac{2e^{-\gamma_{\mathrm{E}}}}{\mu b_T}\right) + \ln\left(\frac{2e^{-\gamma_{\mathrm{E}}}}{\mu b_T}\right)\ln\left(\frac{\zeta_F}{\mu^2}\right)\right]\right\} + \mathcal{O}(\alpha_s^2)$$

$$\Delta\tilde{C}_{j/j'}(x, \boldsymbol{b}_T; \mu; \zeta_F/\mu^2) = \delta_{j'j}\delta(1-x) + \delta_{j'j}\frac{\alpha_s C_{\mathrm{F}}}{\pi}\left\{ \ln\left(\frac{2e^{-\gamma_{\mathrm{E}}}}{\mu b_T}\right) \frac{1+x^2}{(1-x)_+} + \frac{1}{2}(1-x) + \right.$$
$$\left. + \delta(1-x)\left[-\ln^2\left(\frac{2e^{-\gamma_{\mathrm{E}}}}{\mu b_T}\right) + \ln\left(\frac{2e^{-\gamma_{\mathrm{E}}}}{\mu b_T}\right)\ln\left(\frac{\zeta_F}{\mu^2}\right)\right]\right\} + \mathcal{O}(\alpha_s^2)$$

$$\delta\tilde{C}_{j/j'}(x, \boldsymbol{b}_T; \mu; \zeta_F/\mu^2) = \delta_{j'j}\delta(1-x) + \delta_{j'j}\frac{\alpha_s C_{\mathrm{F}}}{\pi}\left\{ \ln\left(\frac{2e^{-\gamma_{\mathrm{E}}}}{\mu b_T}\right) \frac{2x}{(1-x)_+} + \right.$$
$$\left. + \delta(1-x)\left[-\ln^2\left(\frac{2e^{-\gamma_{\mathrm{E}}}}{\mu b_T}\right) + \ln\left(\frac{2e^{-\gamma_{\mathrm{E}}}}{\mu b_T}\right)\ln\left(\frac{\zeta_F}{\mu^2}\right)\right]\right\} + \mathcal{O}(\alpha_s^2)$$

Connection to DGLAP! Collinear splitting functions

TMD evolution:helicity and transversity

A. Bacchetta, AP, 2013

Calculate everything for gluon:

$$\tilde{C}_{j/g}(x, \boldsymbol{b}_T; \mu, \zeta_F/\mu^2) = \frac{\alpha_s T_f}{\pi} \left\{ \ln\left(\frac{2e^{-\gamma_E}}{\mu b_T}\right) \left(x^2 + (1-x)^2\right) + x(1-x) \right\} + \mathcal{O}(\alpha_s^2)\,,$$

$$\Delta\tilde{C}_{j/g}(x, \boldsymbol{b}_T; \mu, \zeta_F/\mu^2) = \frac{\alpha_s T_f}{\pi} \left\{ \ln\left(\frac{2e^{-\gamma_E}}{\mu b_T}\right) (2x-1) + (1-x) \right\} + \mathcal{O}(\alpha_s^2)\,,$$

$$\delta\tilde{C}_{j/g}(x, \boldsymbol{b}_T; \mu, \zeta_F/\mu^2) = 0\,,$$

True to all orders

TMD evolution:helicity and transversity

A. Bacchetta, AP, 2013

Simplify by choosing : $\mu_b = 2e^{-\gamma_E}/b_*$

$$\tilde{C}_{j'/j}(x, b_*; \mu_b) = \delta_{j'j}\delta(1-x) + \delta_{j'j}\frac{\alpha_s C_{\rm F}}{2\pi}(1-x) + \mathcal{O}(\alpha_s^2)\,,$$

$$\Delta\tilde{C}_{j'/j}(x, b_*; \mu_b) = \delta_{j'j}\delta(1-x) + \delta_{j'j}\frac{\alpha_s C_{\rm F}}{2\pi}(1-x) + \mathcal{O}(\alpha_s^2)\,,$$

$$\delta\tilde{C}_{j'/j}(x, b_*; \mu_b) = \delta_{j'j}\delta(1-x) + \mathcal{O}(\alpha_s^2).$$

$$\tilde{C}_{j/g}(x, b_*; \mu_b) = \frac{\alpha_s T_f}{\pi}\, x(1-x) + \mathcal{O}(\alpha_s^2)\,,$$

$$\Delta\tilde{C}_{j/g}(x, b_*; \mu_b) = \frac{\alpha_s T_f}{\pi}\,(1-x) + \mathcal{O}(\alpha_s^2)\,,$$

$$\delta\tilde{C}_{j/g}(x, b_*; \mu_b) = 0\,,$$

Result coincides with CSS:
Koike, Nagashima, Vogelsang 2006

Numerics

A. Bacchetta, AP, 2013

Choose initial conditions:

$$xu_0(x) = xd_0(x) \equiv x^{0.5}(1-x)^{0.5}, \qquad x\bar{u}_0(x) = x\bar{d}_0(x) \equiv 0, \qquad xg_0(x) \equiv x^{0.5}(1-x)^{0.5}.$$

Choose initial conditions for evolution:

$$\hat{f}^f_{\rm NP}(x, b_T) = \exp\left(-\frac{b_T^2 \langle k_T^2 \rangle}{4}\right) , \qquad g_K(b_T) = -g\, b_T^2 ,$$

where $\langle k_T^2 \rangle = 0.25$ (GeV^2), $g = 0.2$ (GeV^2). We also choose $b_{max} = 1$ (GeV^{-1}).

Numerics

A. Bacchetta, AP, 2013

Choose initial conditions, $Q_0 = 1GeV$:

$$xu_0(x) = xd_0(x) \equiv x^{0.5}(1-x)^{0.5}, \qquad x\bar{u}_0(x) = x\bar{d}_0(x) \equiv 0, \qquad xg_0(x) \equiv x^{0.5}(1-x)^{0.5}.$$

Choose initial conditions for evolution:

$$\hat{f}^f_{\mathrm{NP}}(x, b_T) = \exp\left(-\frac{b_T^2 \langle k_T^2 \rangle}{4}\right), \qquad g_K(b_T) = -g\, b_T^2\,,$$

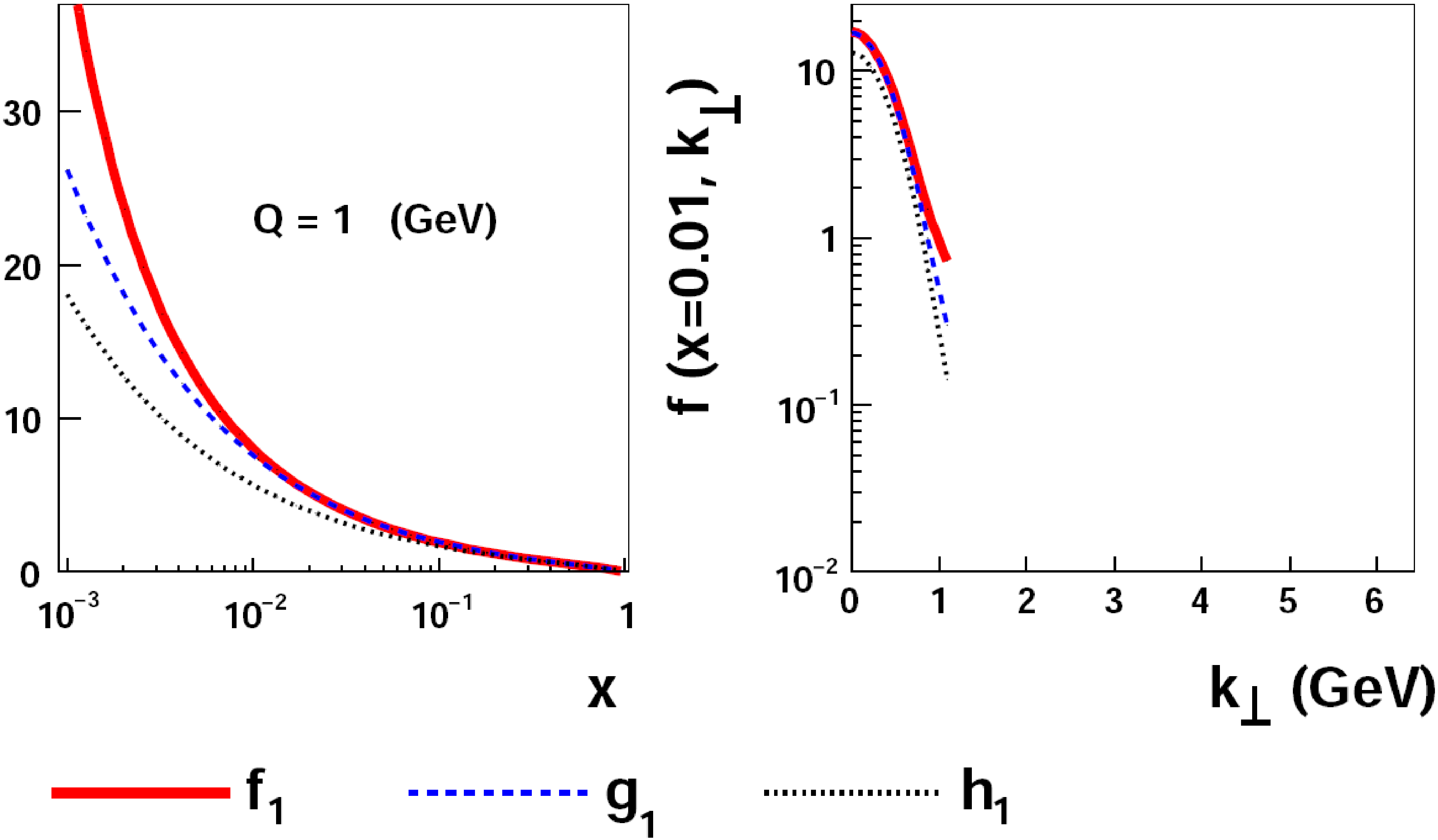
where $\langle k_T^2 \rangle = 0.25\ (\mathrm{GeV}^2)$, $\underline{g = 0.2\ (\mathrm{GeV}^2)}$. We also choose $\underline{b_{max} = 1\ (\mathrm{GeV}^{-1})}$.

Important!

See talks of Daniel Boer and Feng Yuan

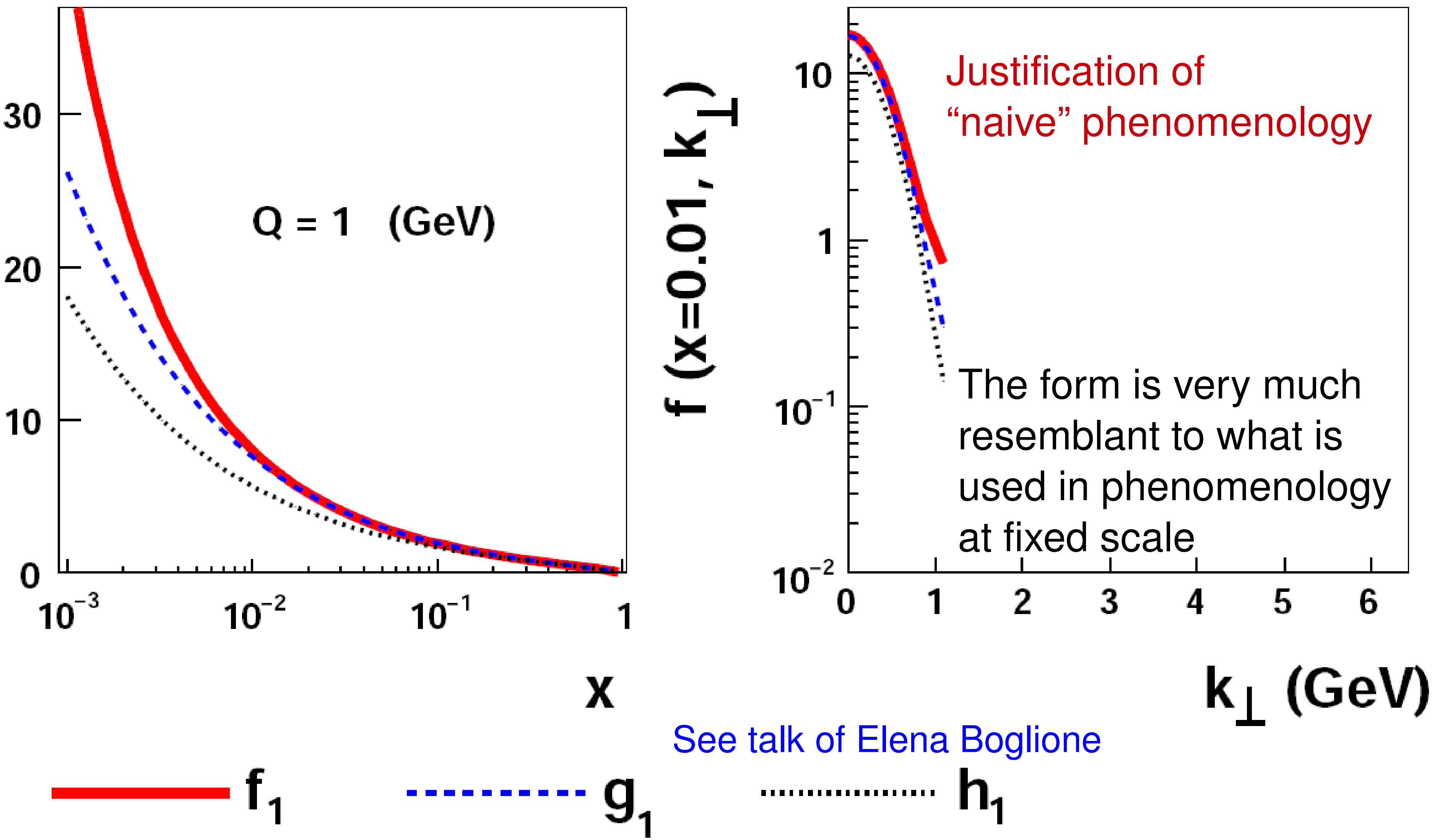
Numerics

A. Bacchetta, AP, 2013



Numerics

A. Bacchetta, AP, 2013



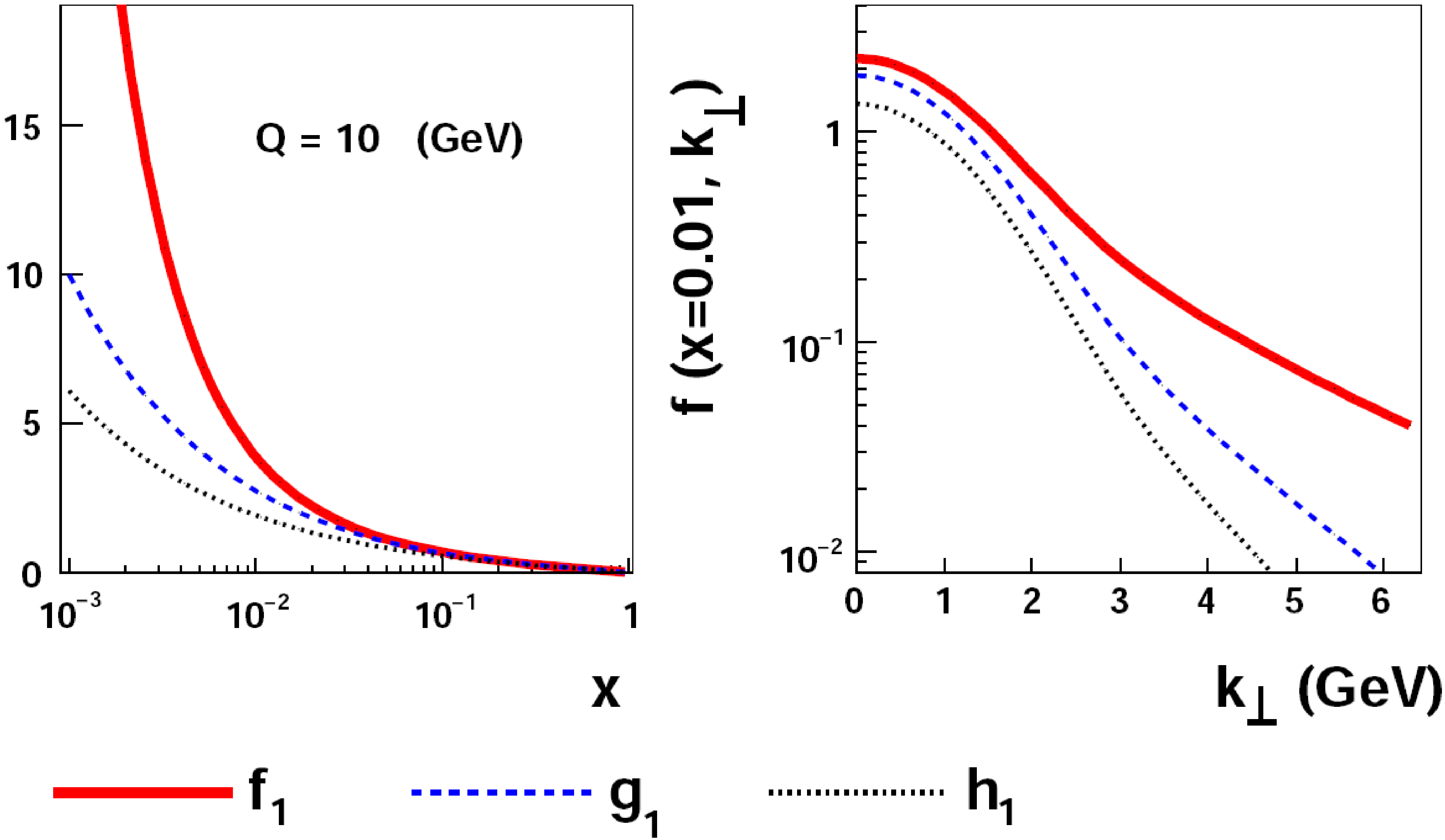
Justification of "naive" phenomenology

The form is very much resemblant to what is used in phenomenology at fixed scale

See talk of Elena Boglione

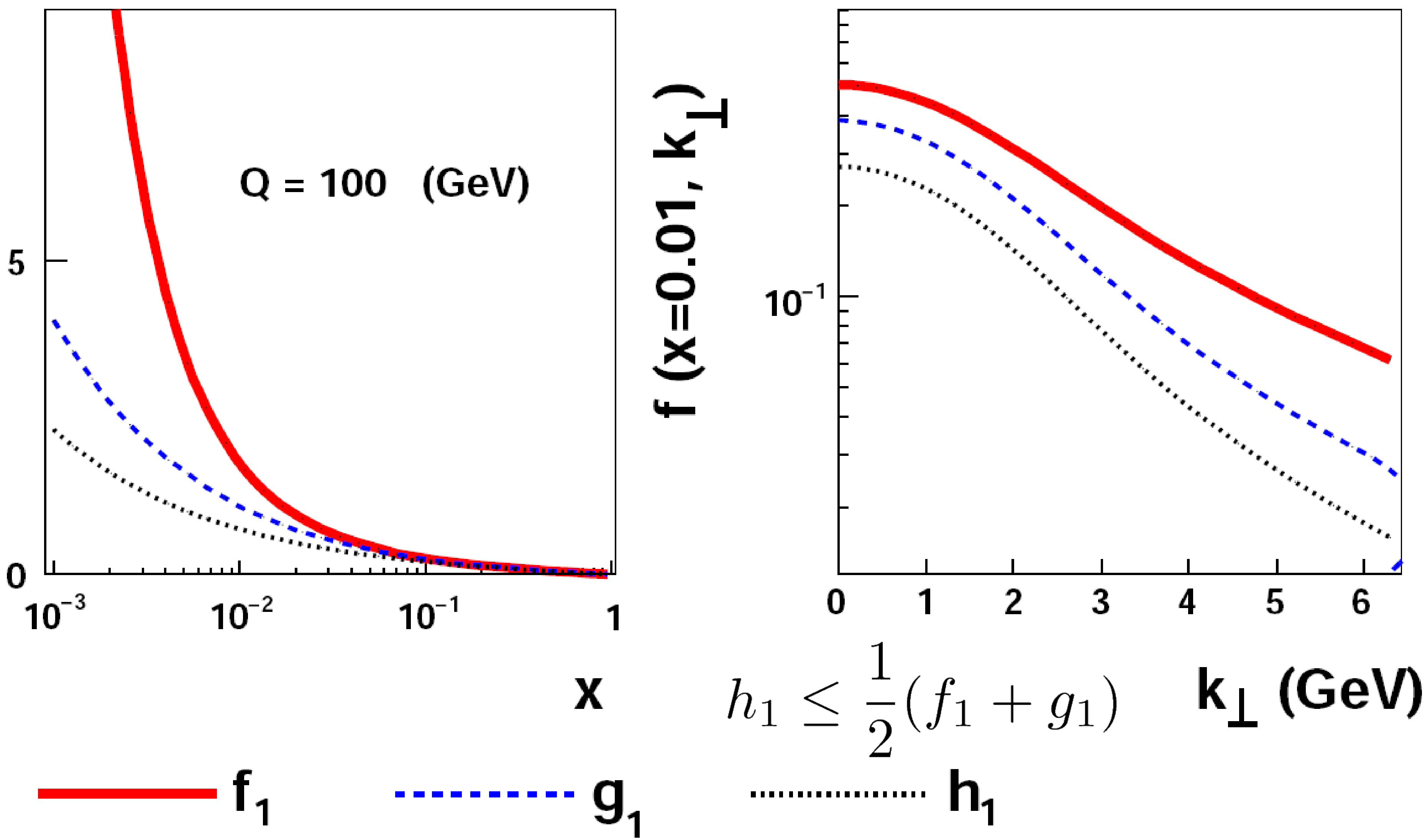
Numerics

A. Bacchetta, AP, 2013



Numerics

A. Bacchetta, AP, 2013



$$h_1 \leq \frac{1}{2}(f_1 + g_1)$$

Soffer bound on transversity is not violated numerically

Conclusions

- Coefficient functions for TMD evolution of transversity and helicity are calculated
- Results are compared with CSS formalism
- Soffer bound on transversity is not violated numerically