Are there infinitely many decompositions of the nucleon spin ?

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1. Introduction

current status and homework of nucleon spin problem

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma^Q + \Delta G + \text{Orbital Angular Momenta ?}$$

(1) $\Delta\Sigma^Q$: fairly precisely determined ! ($\sim 1/3$)

(2) ΔG : likely to be small , but large uncertainties

⇓

What carries the remaining 2 / 3 of nucleon spin ?

a fundamental question of QCD

⇓

To answer this question unambiguously, we cannot avoid to clarify

- What is a **precise (QCD) definition** of each term of the decomposition ?
- How can we extract individual term by means of **direct measurements** ?

nucleon spin decomposition problem

Importance of gauge-invariance in nucleon spin decomposition problem

Because QCD is a color SU(3) gauge theory, the **color gauge-invariance** plays a crucially important role in the nucleon spin decomposition problem.

For, the **general principle of physics** dictates that

gauge-invariance is a **necessary condition** of **observability** !

Unfortunately, it is quite a delicate problem, which is still under intense debate.

conflict

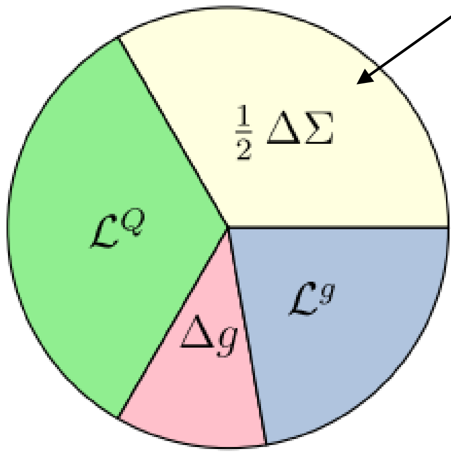
⇩

Interpretation of the meaning of gauge-invariance !

2. Controversies of nucleon spin decomposition problem ?

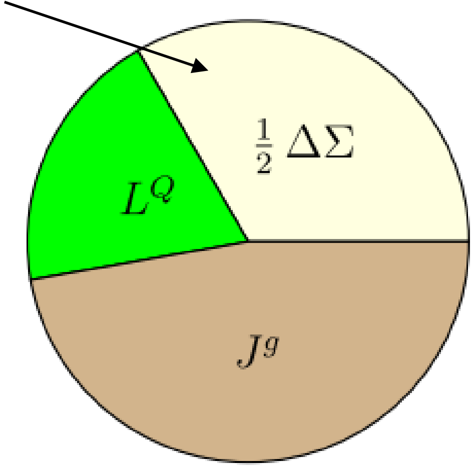
two popular decompositions of the nucleon spin

Jaffe-Manohar



common

Ji



$$
\begin{aligned}
\boldsymbol{J}_{QCD} \;=\; & \int \psi^\dagger \, \frac{1}{2} \, \boldsymbol{\Sigma} \, \psi \, d^3x \\
+ & \int \psi^\dagger \, \boldsymbol{x} \times \frac{1}{i} \, \nabla \, \psi \, d^3x \\
+ & \int \boldsymbol{E}^a \times \boldsymbol{A}^a \, d^3x \\
+ & \int E^{ai} \, \boldsymbol{x} \times \nabla \, A^{ai} \, d^3x
\end{aligned}
$$

Each term is not separately gauge-invariant !

$$
\begin{aligned}
\boldsymbol{J}_{QCD} \;=\; & \int \psi^\dagger \, \frac{1}{2} \, \boldsymbol{\Sigma} \, \psi \, d^3x \\
+ & \int \psi^\dagger \, \boldsymbol{x} \times \frac{1}{i} \, \boldsymbol{D} \, \psi \, d^3x \\
+ & \underbrace{\int \boldsymbol{x} \times (\boldsymbol{E}^a \times \boldsymbol{B}^a) \, d^3x}_{\boxed{J_g}}
\end{aligned}
$$

No further GI decomposition !

First, pay attention to the **difference** of **quark OAM parts**

$$L_Q(\mathrm{JM}) \ \sim \ \psi^\dagger \boldsymbol{x} \times \boldsymbol{p} \, \psi \qquad\qquad L_Q(\mathrm{Ji}) \ \sim \ \psi^\dagger \boldsymbol{x} \times (\boldsymbol{p} - g \, \boldsymbol{A}) \, \psi$$

canonical OAM | **dynamical OAM**

not gauge invariant ! | **gauge invariant** !

gauge principle

observables must be gauge-invariant !

- Observability of canonical OAM has long been questioned ?
- On the other hand, it has been shown that the dynamical quark OAM can be related to observables through GPDs. (X. Ji, 1997)

However, Chen et al. proposed a new gauge-invariant complete decomposition

X.-S. Chen et al., Phys. Rev. Lett. 103, 062001 (2009) ; 100, 232002 (2008).

basic idea

$$A^{\mu} = A^{\mu}_{phys} + A^{\mu}_{pure}$$

which is a sort of generalization of the decomposition of photon field in QED into the transverse and longitudinal components :

$$\boldsymbol{A}_{phys} \Leftrightarrow \boldsymbol{A}_{\perp}, \qquad \boldsymbol{A}_{pure} \Leftrightarrow \boldsymbol{A}_{||}$$

Their decomposition is given in the following form :

$$\begin{aligned} \boldsymbol{J}_{QCD} &= \int \psi^{\dagger} \frac{1}{2} \Sigma \psi \, d^3x + \int \psi^{\dagger} \boldsymbol{x} \times (\boldsymbol{p} - g \, \boldsymbol{A}_{pure}) \, \psi \, d^3x \\ &+ \int \boldsymbol{E}^a \times \boldsymbol{A}^a_{phys} \, d^3x + \int E^{aj} \, (\boldsymbol{x} \times \nabla) \, \boldsymbol{A}^{aj}_{phys} \, d^3x \\ &= \boldsymbol{S}'_q + \boldsymbol{L}'_q + \boldsymbol{S}'_G + \boldsymbol{L}'_G \end{aligned}$$

- Each term is separately gauge-invariant !
- It reduces to gauge-variant **Jaffe-Manohar decomposition** in a particular gauge !

$$\boldsymbol{A}_{pure} = 0, \quad \boldsymbol{A} = \boldsymbol{A}_{phys}$$

Soon after, we pointed out that the way of gauge-invariant decomposition of nucleon spin is not necessarily unique, and proposed another G.I. decomposition :

$$\boldsymbol{J}_{QCD} = \boldsymbol{S}_q + \boldsymbol{L}_q + \boldsymbol{S}_G + \boldsymbol{L}_G$$

where

$$\begin{aligned}
\boldsymbol{S}_q &= \int \psi^\dagger \frac{1}{2} \Sigma \psi \, d^3x \\
\boldsymbol{L}_q &= \int \psi \, \boldsymbol{x} \times (\boldsymbol{p} - g\, \boldsymbol{A}) \, \psi \, d^3x \;=\; \boldsymbol{L}_q \, (\mathtt{Ji}) \\
\boldsymbol{S}_G &= \int \boldsymbol{E}^a \times \boldsymbol{A}^a_{phys} \, d^3x \\
\boldsymbol{L}_G &= \int E^{aj} \, (\boldsymbol{x} \times \nabla) \, A^{aj}_{phys} \, d^3x + \underbrace{\boxed{\int \rho^a \, (\boldsymbol{x} \times \boldsymbol{A}^a_{phys}) \, d^3x}}_{\boldsymbol{L}_{pot}}
\end{aligned}$$

"**potential angular momentum**"

The QED correspondent of $\boldsymbol{L}_{pot}$ is the orbital angular momentum carried by electromagnetic potential, appearing in the famous Feynman paradox.

An **arbitrariness** of the spin decomposition arises, because this potential angular momentum term is solely gauge-invariant ! Shifting it to the quark OAM part

$$\begin{aligned}
\boldsymbol{L}_q &+ \boldsymbol{L}_{pot} = \boldsymbol{L}'_q \; (\text{Chen}) \\
\boldsymbol{L}_G &- \boldsymbol{L}_{pot} = \boldsymbol{L}'_G \; (\text{Chen})
\end{aligned}$$

Further, we found that we can make a **seemingly** **covariant extension** of the **2 gauge-invariant decompositions** of **QCD angular momentum tensor**.

Decomposition (I) & Decomposition (II)

[Remarks]

(1) The word "**seemingly**" is important here, because the decomposition

$$A^\mu(x) = A^\mu_{phys}(x) + A^\mu_{pure}(x),$$

which is a foundation of the above gauge-invariant decompositions, is **intrinsically non-covariant** or **frame-dependent**, as we shall see.

(2) Still, it is useful to find relations to high-energy DIS observables.

(3) It is useful also for perturbative calculations of Feynman diagrams.

Gauge-invariant decomposition (II) : "covariant" generalization of Chen et al's

$$M^{\mu\nu\lambda}_{QCD} = M'^{\mu\nu\lambda}_{q-spin} + M'^{\mu\nu\lambda}_{q-OAM} + M'^{\mu\nu\lambda}_{g-spin} + M'^{\mu\nu\lambda}_{g-OAM} + \text{boost} + \text{total divergence}$$

with

pure-gauge covariant derivative

$$M'^{\mu\nu\lambda}_{q-spin} = \frac{1}{2}\epsilon^{\mu\nu\lambda\sigma}\bar{\psi}\gamma_\sigma\gamma_5\psi$$

$$M'^{\mu\nu\lambda}_{q-OAM} = \bar{\psi}\gamma^\mu(x^\nu i D^\lambda_{pure} - x^\lambda i D^\nu_{pure})\psi$$

$$M'^{\mu\nu\lambda}_{g-spin} = 2\,\mathrm{Tr}\{F^{\mu\lambda}A^\nu_{phys} - F^{\mu\nu}A^\lambda_{phys}\}$$

$$M'^{\mu\nu\lambda}_{q-OAM} = 2\,\mathrm{Tr}\{F^{\mu\alpha}(x^\nu D^\lambda_{pure} - x^\lambda D^\nu_{pure})A^{phys}_\alpha\}$$

[Remark]

This decomposition reduces to any ones of Bashinsky-Jaffe, of Chen et al., and of Jaffe-Manohar, after an appropriate gauge-fixing in a suitable Lorentz frame.

Gauge-invariant decomposition (I) : "extension" of Ji's decomposition

The difference with the decomposition (II) resides in **OAM parts** !

$$M^{\mu\nu\lambda} = M^{\mu\nu\lambda}_{q-spin} + M^{\mu\nu\lambda}_{q-OAM} + M^{\mu\nu\lambda}_{g-spin} + M^{\mu\nu\lambda}_{g-OAM} + \text{boost} + \text{total divergence}$$

with

full covariant derivative

$$M^{\mu\nu\lambda}_{q-spin} = M'^{\mu\nu\lambda}$$

$$\boxed{M^{\mu\nu\lambda}_{q-OAM}} = \bar{\psi}\,\gamma^\mu\,(\,x^\nu\,i\,D^\lambda - x^\lambda\,i\,D^\nu\,)\,\psi \neq \boxed{M'^{\mu\nu\lambda}_{q-OAM}}$$

$$M^{\mu\nu\lambda}_{g-spin} = M'^{\mu\nu\lambda}_{g-spin}$$

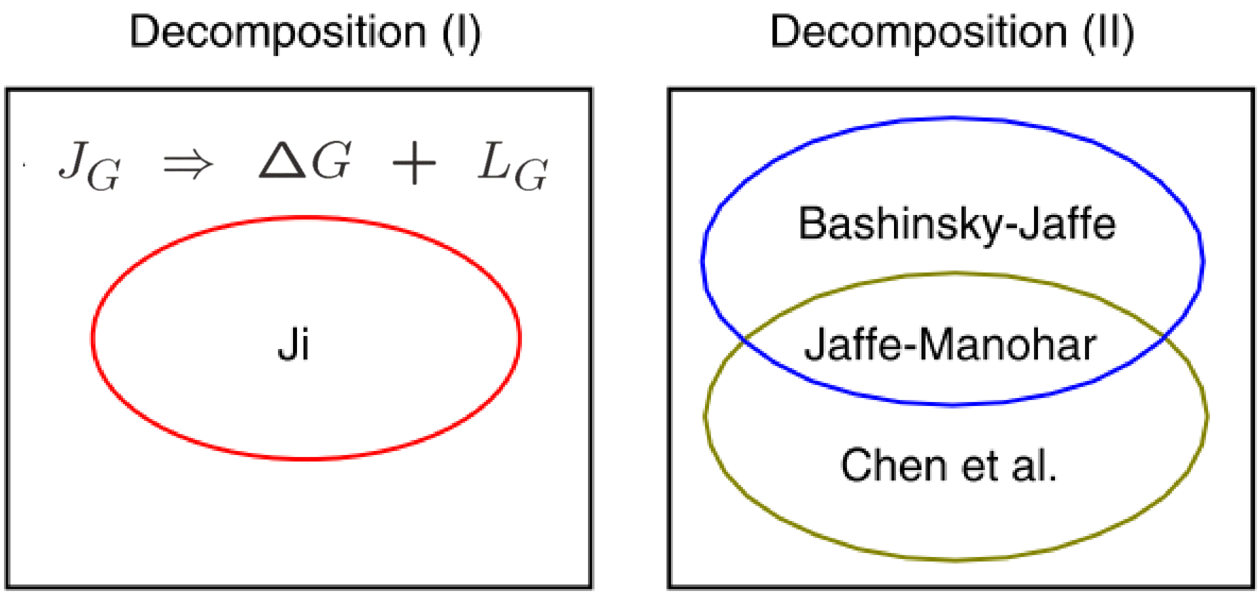
$$\boxed{M^{\mu\nu\lambda}_{g-OAM}} = \boxed{M'^{\mu\nu\lambda}_{g-OAM}} + 2\,\mathrm{Tr}\,[\,(\,D_\alpha\,F^{\alpha\mu}\,)\,(\,x^\nu\,A^\lambda_{phys} - x^\lambda\,A^\nu_{phys}\,)\,]$$

$\Uparrow$

"covariant" generalization of potential OAM !

[Our claim]

Basically, there are only 2 physically nonequivalent gauge-invariant decompositions (I) and (II) of the nucleon spin.



[Opposing claim]

Since the decomposition of the gauge field into its physical and pure-gauge components is not unique and there are infinitely many such decompositions, there are in principle infinitely many GI decompositions of the nucleon spin.

An argument in favor of the second claim :

- X. Ji, Y. Xu, and Y. Zhao, arXiv : 1205.0516 [hep-ph]

According to them, the **Chen decomposition** is a gauge-invariant extension (GIE) of the **Jaffe-Manohar decomposition** based on the Coulomb gauge, while the **Bashinsky-Jaffe decomposition** is a GIE of the **Jaffe-Manohar decomposition** based on the light-cone gauge.

Because the way of GIE with use of a path-dependent Wilson line is not unique, there is no need that the 2 decompositions give the same physical predictions.

This makes Ji reopen his longstanding claim that the gluon spin ΔG has a meaning only in the light-cone gauge, and it is not a gauge-invariant quantity in a true or traditional sense, although it is measurable in DIS scatterings.

$\Downarrow$

One should recognize a self-contradiction inherent in this claim.

In fact, first remember the **fundamental proposition of physics** :

"Observables must be gauge-invariant ! "

The **contraposition** of this proposition (it is always correct) is

"gauge-variant quantities cannot be observables ! "

This dictates that, if ΔG is observable, it must be gauge-invariant !

[Caution]

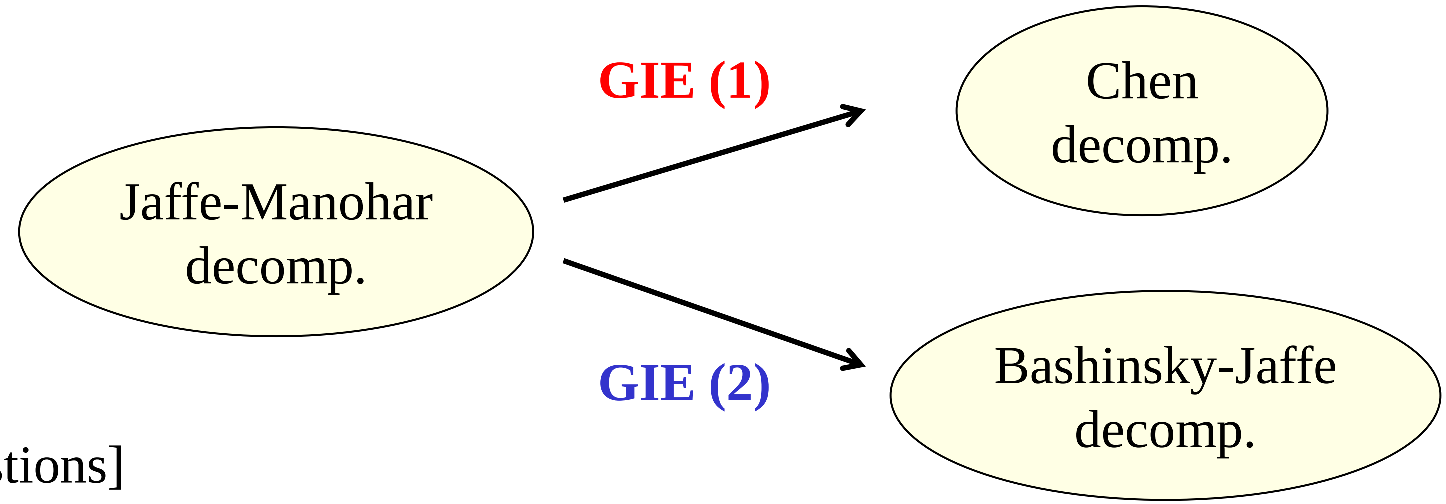
Here we are using the word "**observables**" in a strong sense : they must be quantities that can be extracted purely experimentally, or model-independently.

(Ex.) Genuine observables in DIS processes are structure functions, whereas the PDFs are not genuine observables in a strict sense.

Structure functions : **"true" observables**

PDFs, TMD PDFs, etc. : **"quasi" observables**

Assume that the **Chen decomposition** and the **Bashinsky-Jaffe decompositions** are 2 physically inequivalent GIEs of the **Jaffe-Manohar decomposition**.



[Questions]

What is the meaning of extended gauge symmetries ?

Are there plural color gauge symmetries in nature ?

Our (standard ?) viewpoint

- The color gauge symmetry is an intrinsic property of QCD, which is present from the beginning and in principle there is no need of extending it.

- The gauge symmetry is rather freedoms to be eliminated by gauge-fixing procedures rather than to be obtained by extension.

Another argument in favor of the existence of infinitely many decompositions of the nucleon spin was given in

- C. Lorcé, arXiv : 1205.6483 ; Phys. Lett. B719 (2013) 185.

According to him, the Chen decomposition is a **GIE** based on Stückelberg trick.

There is a hidden symmetry called the Stückelberg symmetry, under which

$$A_\mu^{pure}(x) \;\rightarrow\; A_\mu^{pure}(x) \;+\; \frac{i}{g}\, U_{pure}(x)\, U_0^{-1}(x)\, [\partial_\mu U_0(x)]\, U_{pure}^{-1}(x)$$
$$A_\mu^{phys}(x) \;\rightarrow\; A_\mu^{phys}(x) \;-\; \frac{i}{g}\, U_{pure}(x)\, U_0^{-1}(x)\, [\partial_\mu U_0(x)]\, U_{pure}^{-1}(x)$$

Since this transformation leaves $A_\mu(x) = A_\mu^{phys}(x) + A_\mu^{pure}(x)$ intact, there are infinitely many decompositions of $A_\mu(x)$ into $A_\mu^{phys}(x)$ and $A_\mu^{pure}(x)$ and consequently infinitely many decompositions of the nucleon spin.

We claim and in fact showed that, in the QED case, the Chen decomposition is not a GIE based on the Stückelberg symmetry. See sect. III of

- M.W. ,Phys. Rev. D85 (2012) 114039.

3. Chen decomposition is not a GIE a la Stückelberg

As is well-known, the vector potential $\boldsymbol{A}$ of the photon field can be decomposed into transverse and longitudinal components as

$$\boldsymbol{A} = \boldsymbol{A}_{\perp} + \boldsymbol{A}_{||}$$

with the divergence-free and irrotational conditions :

$$\nabla \cdot \boldsymbol{A}_{\perp} = 0, \qquad \nabla \times \boldsymbol{A}_{||} = 0$$

This transverse-longitudinal decomposition is **unique**, once the Lorentz frame of reference is specified. Under a general gauge-transformation given by

$$A^0 \rightarrow A'^0 = A^0 - \frac{\partial}{\partial t}\omega(x), \qquad \boldsymbol{A} \rightarrow \boldsymbol{A}' = \boldsymbol{A} + \nabla\omega(x)$$

the transverse and longitudinal components transform as

$$\boldsymbol{A}_{\perp} \rightarrow \boldsymbol{A}'_{\perp} = \boldsymbol{A}_{\perp}, \qquad \boldsymbol{A}_{||} \rightarrow \boldsymbol{A}'_{||} + \nabla\omega(x)$$

indicating that $\boldsymbol{A}_{||}$ carries **unphysical gauge degrees of freedom** !

Important remarks

Naturally, the transverse-longitudinal decomposition of the 3-vector potential is Lorentz-frame dependent. (Anyhow, the whole treatment above is non-covariant !)

It is true that a vector field that appears transverse in a certain Lorentz frame is not necessarily transverse in another Lorentz frame.

Nonetheless, the Lorentz-frame dependence of the transverse-longitudinal decomposition should not make any trouble, because one can start this decomposition in an arbitrarily chosen Lorentz frame.

There is nothing wrong with the non-covariant treatment in the last result !

After all, the gauge- and frame-independence of observables is the **core** of the celebrated **Maxwell's electrodynamics** as a **Lorentz-invariant gauge theory** !

This QED example dictates that, as long as we are working in a chosen Lorentz frame, there is no arbitrariness in the decomposition $A^\mu = A^\mu_{phys} + A^\mu_{pure}$, as arising from the Stückelberg-like transformation of Lorcé.

Stückelberg transformation (in the **abelian case**)

$$A_\mu^{pure}(x) \rightarrow A_\mu^{pure,g}(x) = A_\mu^{pure}(x) - \partial_\mu C(x)$$
$$A_\mu^{phys}(x) \rightarrow A_\mu^{phys,g}(x) = A_\mu^{phys}(x) + \partial_\mu C(x)$$

$C(x)$: arbitrary funcion of space-time

$$A_\mu^{phys}(x) + A_\mu^{pure}(x) \rightarrow \texttt{invariant}$$

infinitely many decompositions into $A_\mu^{phys}(x)$ and $A_\mu^{pure}(x)$

$\Downarrow$

contradicts the unique nature of longitudinal-transverse decomposition

$$\boldsymbol{A} = \boldsymbol{A}_{||} + \boldsymbol{A}_{\perp} \quad \text{with} \quad \nabla \times \boldsymbol{A}_{||} = 0, \quad \nabla \cdot \boldsymbol{A}_{\perp} = 0$$

In fact, under the Stückelberg

$$\begin{aligned} \boldsymbol{A}_{\parallel} &\rightarrow \boldsymbol{A}_{\parallel}^{g} = \boldsymbol{A}_{\parallel} - \nabla C(x) \\ \boldsymbol{A}_{\perp} &\rightarrow \boldsymbol{A}_{\perp}^{g} = \boldsymbol{A}_{\perp} + \nabla C(x) \end{aligned}$$

we find that

$$\nabla \times \boldsymbol{A}_{\parallel}^{g} = \nabla \times (\boldsymbol{A}_{\parallel} - \nabla C(x)) = \nabla \times \boldsymbol{A}_{\parallel} \quad \texttt{(O.K.)}$$

irrotational property of $\boldsymbol{A}_{\parallel}$ is preserved !

However, under it

$$\begin{aligned} \nabla \cdot \boldsymbol{A}_{\perp}^{g} &= \nabla \cdot (\boldsymbol{A}_{\perp} + \nabla C(x)) = \nabla \cdot \boldsymbol{A}_{\perp} + \Delta C(x) \\ &\neq \nabla \cdot \boldsymbol{A}_{\perp} \quad \texttt{unless} \ \Delta C(x) = 0 \end{aligned}$$

transversality condition is not preserved !

From $\Delta C(x) = 0$, we can take $C(x) = 0$ without loss of generality !

No arbitrariness of Stückelberg transformation !

Although A^{μ}_{pure} changes arbitrarily under the gauge-transformation, A^{μ}_{phys} is essentially a unique object, constrained by the transversality condition.

4. What is needed to settle the controversies

We recall that the main criticism from the GIE approach with use of the Wilson-line is that the following decomposition is not unique at all, and there are infinitely many such decompositions arising from infinitely many choices of paths.

$$A^{\mu}(x) = A^{\mu}_{phys}(x) + A^{\mu}_{pure}(x)$$

Note however that, from a physical viewpoint, the massless gauge field has only 2 physical or transverse degrees of freedom, and other components are not independent dynamical degrees of freedom.

The standard gauge-fixing procedure is essentially the process of projecting out the 2 transverse or physical components of gauge field.

Corresponding to the fact that there exist many gauge-fixing procedures, the expression of $A^{\mu}_{phys}(x)$ is not naturally unique.

Nevertheless, an important wisdom is that final physical predictions for gauge invariant quantities are independent of the choice of gauges !

[A hidden problem of the GIE approach]

DeWitt's gauge invariant formulation of QED

For a given set of electron and photon fields $(\psi(x), A_\mu(x))$, he constructed a gauge-invariant set of those $(\psi'(x), A'_\mu(x))$ by

$$\psi'(x) \equiv e^{i\Lambda(x)}\psi(x)$$
$$A'_\mu(x) \equiv A_\mu(x) + \partial_\mu \Lambda(x)$$

with

$$\Lambda(x) = -\int_{-\infty}^{0} A_\sigma(z) \frac{\partial z^\sigma}{\partial \xi} d\xi$$

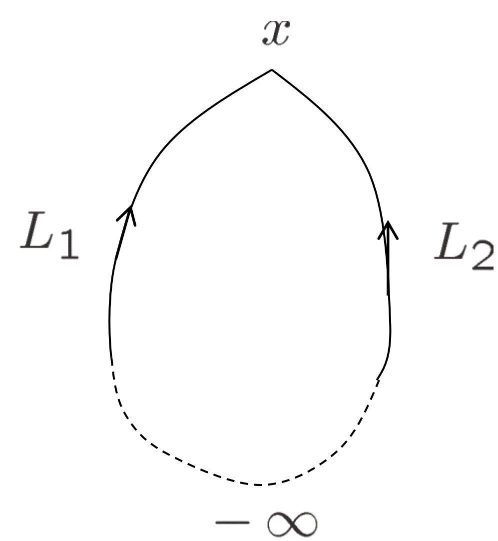
where $z^\mu(x,\xi)$ is a path satisfying the boundary conditions :

$$z^\mu(x,0) = x^\mu, \quad z^\mu(x,-\infty) = \texttt{spatial infinity}$$

The problem is that , while $(\psi'(x), A'_\mu(x))$ are gauge-invariant by construction, they are generally path-dependent !

The path-dependence can easily be convinced by considering the simplest case of constant-time paths, which amounts to taking

$$\Lambda(x) \;=\; - \int_{-\infty}^{x} \boldsymbol{A}(x^0, \boldsymbol{z}) \cdot d\boldsymbol{z}$$



GI electron fields corresponding to 2 choices of paths

$$\psi'(x\,;L_1) \;=\; \exp\left[-\,i\,e \int_{L_1}^{x} \boldsymbol{A}(x^0, \boldsymbol{z}) \cdot d\boldsymbol{z} \right] \psi(x)$$

$$\psi'(x\,;L_2) \;=\; \exp\left[-\,i\,e \int_{L_2}^{x} \boldsymbol{A}(x^0, \boldsymbol{z}) \cdot d\boldsymbol{z} \right] \psi(x)$$

The relation

$$\psi'(x\,;L_1) \;=\; \exp\left[i\,e \left(\int_{L_1}^{x} - \int_{L_2}^{x} \right) \boldsymbol{A}(x^0, \boldsymbol{z}) \right] \psi'(x\,;L_2)$$

Closing the path to a loop L by a connection at spatial infinity

$$\begin{aligned}
\psi'(x\,;L_1) \;&=\; \exp\left[i\,e \oint_{L} \boldsymbol{A}(x^0, \boldsymbol{z}) \cdot d\boldsymbol{z} \right] \psi'(x\,;L_2) \\
&=\; \exp\left[i\,e \int\!\!\int_{S} (\nabla_z \times \boldsymbol{A}(x^0, \boldsymbol{z})) \cdot d\boldsymbol{z} \right] \psi'(x\,;L_2) \\
&=\; \exp\left[i\,e \int\!\!\int_{S} \boldsymbol{B}(x^0, \boldsymbol{z}) \cdot d\boldsymbol{z} \right] \psi'(x\,;L_2)
\end{aligned}$$

Since the magnetic flux does not vanish in general, $\psi'(x)$ is path-dependent !

Why is the path-dependence a problem ?

path dependence $\Longrightarrow$ gauge dependence

Belinfante, Mandelstam, Rohrlich-Strocchi, ...

However, there are some interesting choices of the function $\Lambda(x)$, which leads to path-independent set of electron and photon fields $(\psi'(x), A'_\mu(x))$:

[Example 1]

$$\Lambda(x) \;=\; - \int_{-\infty}^{x} \boldsymbol{A}_{||}(x^0, \boldsymbol{z}) \cdot d\boldsymbol{z}$$

where $\boldsymbol{A}_{||}(x)$ is the longitudinal component of photon satisfying $\nabla \times \boldsymbol{A}_{||}(x) = 0$.

Since

$$\oint_L \boldsymbol{A}_{||}(x^0, \boldsymbol{z}) \cdot d\boldsymbol{z} \;=\; \int\int_S (\nabla_z \times \boldsymbol{A}_{||}(x^0, \boldsymbol{z})) \cdot d\boldsymbol{S} \;=\; 0$$

the electron field defined by

$$\psi'(x) \;=\; \exp\left[- i\, e \int_{-\infty}^{x} \boldsymbol{A}_{||}(x^0, \boldsymbol{z}) \cdot d\boldsymbol{z} \right] \psi(x)$$

is not only gauge-invariant but also path-independent !

If one remembers

$$\boldsymbol{A}_{\|}(x) \;=\; \nabla \frac{1}{\nabla^2} \nabla \cdot \boldsymbol{A}(x), \quad \boldsymbol{A}_{\perp}(x) \;=\; \boldsymbol{A} \;-\; \boldsymbol{A}_{\|}(x)$$

one can also write as

$$\begin{aligned}
\psi'(x) \;&=\; \exp\left[-e \int_{-\infty}^{x} \left(\nabla_z \frac{1}{\nabla_z^2} \nabla_z \cdot \boldsymbol{A}(x^0, \boldsymbol{z}) \right) \cdot d\boldsymbol{z} \right] \psi(x) \\
&=\; \exp\left[-i\, e \frac{\nabla \cdot \boldsymbol{A}}{\nabla^2}(x) \right] \psi(x)
\end{aligned}$$

path-independence is self-evident !

Note that $\psi'(x)$ is nothing but the GI physical electron introduced by Dirac !

Using the same function $\Lambda(x)$, the GI potential $A'_\mu(x)$ becomes

$$\begin{aligned}
\boldsymbol{A}'(x) \;&=\; \boldsymbol{A}_{\perp}(x) \\
A'^0(x) \;&=\; A^0(x) \;+\; \int_{-\infty}^{x} \boldsymbol{A}_{\|}(x^0, \boldsymbol{z}) \cdot d\boldsymbol{z}
\end{aligned}$$

One reconfirms that the physical component of the spatial part of the photon field is nothing but the familiar transverse component.

[Example 2]

Using a constant 4-vector n^{μ} , we introduce the following decomposition :

$$A_{\mu}(x) \;=\; A_{\mu}^{phys}(x) \;+\; A_{\mu}^{pure}(x) \;\equiv\; (\,P_{\mu\nu} \;+\; Q_{\mu\nu}\,)\,A^{\nu}(x)$$

with

$$P_{\mu\nu} \;=\; g_{\mu\nu} \;-\; \frac{\partial_{\mu}\,n_{\nu}}{n\cdot\partial}, \qquad Q_{\mu\nu} \;=\; \frac{\partial_{\mu}\,n_{\nu}}{n\cdot\partial}$$

These two components satisfy the important properties :

$$n^{\mu}\,A_{\mu}^{phys}(x) \;=\; 0, \qquad \partial_{\mu}\,A_{\nu}^{pure}(x) \;-\; \partial_{\nu}\,A_{\mu}^{pure} \;=\; 0$$

Now, we propose to take

$$\Lambda(x) \;=\; -\int_{-\infty}^{x} A_{\mu}^{pure}(z)\,dz^{\mu}$$

and define the GI electron and photon fields by

$$\psi'(x) \;\equiv\; e^{\,i\,\Lambda(x)}\,\psi(x)$$
$$A'_{\mu}(x) \;\equiv\; A_{\mu}(x) \;+\; \partial_{\mu}\Lambda(x)$$

Note that, by using the Stokes theorem in 4 space-time dimension, we have

$$\oint_L A_\mu^{pure}(z)\, dz^\mu \;=\; \frac{1}{2}\int\int_S \left(\partial_\mu A_\nu^{pure} \;-\; \partial_\nu A_\mu^{pure}\right) d\sigma^{\mu\nu} \;=\; 0$$

so that $\Lambda(x)$ turns out to be path-independent. In fact,

$$\begin{aligned} \Lambda(x) \;&=\; -\int_{-\infty}^{x} \frac{\partial_\mu^z\, n_\nu}{n \cdot \partial^z}\, A^\nu(z)\, dz^\mu \\ &=\; -\int_{-\infty}^{x} \partial_\mu^z \left\{ \frac{n \cdot A(z)}{n \cdot \partial^z} \right\} dz^\mu \;=\; \frac{n \cdot A(x)}{n \cdot \partial} \end{aligned}$$

The GI electron and photon fields are then reduced to

$$\begin{aligned} \psi'(x) \;&=\; e^{i\, e\, \frac{n \cdot A(x)}{n \cdot \partial}}\, \psi(x) \\ A'_\mu(x) \;&=\; \left(g_{\mu\nu} \;-\; \frac{\partial_\mu\, n_\nu}{n \cdot \partial} \right) A^\nu(x) \;=\; A_\mu^{phys}(x) \end{aligned}$$

$n^\mu A_\mu^{phys} \;=\; 0 \quad \Leftrightarrow$ gauge-fixing cond. in general axial gauge

[**Important remarks**]

These two examples show that the form of $A_\mu^{phys}(x)$ is not in fact unique. It is expressed in several different forms, which is not unrelated to the fact that there are many gauge-fixing procedures corresponding to different Lorentz frame choice.

Nevertheless, standard belief is that, as far as we handle the gauge- and Lorentz-invariant quantity in a standard sense, the final prediction should be the same !

[**Our central question**]

Is the gluon spin term appearing in the longitudinal nucleon spin sum rule such a quantity with standard GI or not ?

To answer this question, we must generalize the construction of $A_\mu^{phys}(x)$ to the nonabelian gauge theory.

We point out that, in the past, tremendous efforts have been made to figure out the 2 physical components of the gauge field.

Geometrical construction by Ivanov, Korchemsky, and Radyushkin (1985, 1986)

In their formulation, the gauge-covariant gluon field can be constructed in the form

$$\begin{aligned} A^g_\mu(x) &= A_\nu(x_0)\,\frac{\partial x_0^\nu}{\partial x^\mu} \\ &- \int_{x_0}^{x} dz^\nu\,\frac{\partial z^\rho}{\partial x^\mu}\,W_C(x_0,z)\,F_{\nu\rho}(z\,;A)\,W_C(z,x_0) \end{aligned}$$

where

$$W_C(x,x_0) \equiv P\,\exp\left[i\,g\int_{x_0}^{x} dz^\mu\,A_\mu(z)\right]$$

is a Wilson line with $z(s)$ being a path C in the 4-dimensional space-time with

$$z^\mu(s=1) = x^\mu, \qquad z^\mu(s=0) = x_0^\mu$$

One should clearly keep in mind the fact that $A^g_\mu(x)$ so constructed is generally dependent of the choice of path C.

However, these authors clearly recognize the fact that the choice of path in the geometrical formulation corresponds to the choice of gauge-fixing procedure.

It was also shown that, with some natural choices of paths, the above way of fixing the gauge is equivalent to taking gauges satisfying the condition :

$$W_C(x, x_0) \; = \; P \exp \left[i\, g \int_{x_0}^{x} dz^\mu \, A_\mu^g(z) \right] \; = \; 1$$

This class of gauge is called the contour gauge and it is shown to have an attractive feature that they are ghost-free.

contour gauge $\supset$ Fock-Schwinger, Hamilton, and axial gauges

The axial gauge corresponds to taking an infinitely long straight-path :

$$z^\mu(s) \; = \; x^\mu \; + \; s\, n^\mu \qquad (0 < s < \infty)$$

with n^μ being a constant 4-vector characterizing the direction of path.

This gives

$$A_\mu^g(x) \; = \; n^\nu \int_0^\infty W_C^\dagger(x + n\, s, \infty)\, F_{\mu\nu}(x + n\, s\, ; A)\, W_C(x + n\, s, \infty)$$

with

$$W_C(x, \infty) \; = \; P \exp \left(i\, g \int_0^\infty ds\, n^\mu \, A_\mu(x + n\, s) \right)$$

Using $F_{\nu\mu} = - F_{\mu\nu}$, one can easily convince that

$$n^{\mu} A^{g}_{\mu} = 0$$

This is nothing but the gauge-fixing condition in general axial gauge.

Since n^{μ} is an arbitrary constant 4-vector, it contains several popular gauges :

$$\begin{aligned} n^{\mu} &= (1,0,0,0) && \Rightarrow \ \texttt{temporal gauge} \\ n^{\mu} &= (1,0,0,1)\,/\,\sqrt{2} && \Rightarrow \ \texttt{light-cone gauge} \\ n^{\mu} &= (0,0,0,1) && \Rightarrow \ \texttt{spatial axial gauge} \end{aligned}$$

Since our main interest here is to show the traditional gauge-invariance of the evolution equation of the longitudinal gluon spin, we inspect the perturbative (lowest order) contents of the defining equation of $A^{phys}_{\mu}(x) \equiv A^{g}_{\mu}(x)$

$$A^{phys}_{\mu}(x) \simeq n^{\nu} \int_0^{\infty} ds \, \left(\partial_{\mu} A_{\nu}(x + n\, s) - \partial_{\nu} A_{\mu}(x + n\, s)\right)$$

Introducing the Fourier transform $\tilde{A}_{\mu}(k)$ of $A_{\mu}(x)$, we can show

$$\begin{aligned} A^{phys}_{\mu}(x) &\simeq n^{\nu} \int_0^{\infty} \int \frac{d^4 k}{(2\,\pi)^4} e^{\,i\, k \cdot (x + n\, s)} \left(i\, k_{\mu}\, \tilde{A}_{\nu}(k) - i\, k_{\nu}\, \tilde{A}_{\mu}(k) \right) \\ &= \int \frac{d^4 k}{(2\,\pi)^4} \left(g_{\mu\nu} - \frac{k_{\mu}\, n_{\nu}}{k \cdot n} \right) \tilde{A}^{\nu}(k) = \left(g_{\mu\nu} - \frac{\partial_{\mu}\, n_{\nu}}{n \cdot \partial} \right) A^{\nu}(x) \end{aligned}$$

This gives the lowest order expression for the physical gluon propagator

$$\langle T\,(A^{phys}_{\mu,a}(x)\,A^{phys}_{\nu,b}(y))\rangle^{(0)} \;=\; \int \frac{d^4k}{(2\,\pi)^4}\,e^{\,i\,k\,(x-y)}\,\frac{-\,i\delta_{ab}}{k^2+i\,\varepsilon}\,P_{\mu\nu}(k)$$

with

$$P_{\mu\nu}(k) \;=\; g_{\mu\nu} \;-\; \frac{k_\mu\,n_\nu + n_\mu\,k_\nu}{k\cdot n} \;+\; \frac{n^2\,k_\mu\,k_\nu}{(k\cdot n)^2}$$

free gluon propagator in general axial gauge

In this way, the path dependence or direction dependence in the geometric formulation is replaced by the gauge dependence in the general axial gauge.

In this setting, the gluon spin operator reduces to

$$M^{\lambda\mu\nu}_{G-spin} \;=\; 2\,\mathrm{Tr}\left[F^{\lambda\nu}\,A^\mu \;-\; F^{\lambda\mu}\,A^\nu\right]$$

where A^μ in this expression should be regarded as the physical gluon field satisfying the general axial gauge condition $n^\mu\,A_\mu = 0$.

5. Evolution equation for the quark and gluon spin in general axial gauge

The starting covariant relation

$$\langle Ps \,|\, M^{\lambda\mu\nu}(0) \,|\, Ps\rangle \;=\; J_N \,\frac{P_\rho\, s_\sigma}{M_N^2}\left[2\,P^\lambda\,\epsilon^{\nu\mu\rho\sigma} \;-\; P^\mu\,\epsilon^{\lambda\nu\rho\sigma} \;-\; P^\nu\,\epsilon^{\mu\lambda\rho\sigma}\right]$$

Without loss of generality, we can take as

$$P^\mu \;=\; (P^0, 0, 0, P^3) \;\texttt{and}\; s^\mu \;=\; (P^3, 0, 0, P^0) \;\texttt{with}\; P^0 \;=\; \sqrt{(P^3)^2 + M_N^2}$$

The longitudinal nucleon spin sum rule is obtained by setting $\mu = 1,\, \nu = 2$ and contracting with the constant 4-vector n_λ , which gives

$$J_N \;=\; \frac{1}{2} \;=\; \frac{\langle Ps \,|\, n_\lambda\, M^{\lambda 12}(0) \,|\, Ps\rangle}{2\,P\cdot n}$$

An important fact is that this last equation is no longer a covariant relation.

The quantity n_λ appearing in this equation should then be identified with the 4-vector that characterizes the Lorentz-frame, in which the gauge-fixing condition $n^\mu A_\mu = 0$ is imposed.

We have calculated the 1-loop anomalous dimension of the above gluon spin operator, and found that it reproduces the commonly-known answer :

$$\begin{pmatrix} \Delta\gamma_{qq} & \Delta\gamma_{qG} \\ \Delta\gamma_{Gq} & \Delta\gamma_{GG} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ \frac{\alpha_S}{2\pi}\cdot\frac{3}{2}C_F & \frac{\alpha_S}{2\pi}\left(\frac{11}{6}C_A - \frac{1}{3}n_f\right) \end{pmatrix}$$

irrespectively of the choice of n^μ !

Although this is a proof within a restricted class of gauge, i.e. the general axial gauge, characterized by a constant 4-vector n^μ, it strongly indicates that the gluon spin term in the longitudinal nucleon spin sum rule is a gauge-invariant quantity in a true or traditional sense.

This is a welcome conclusion, because it means that there is no conceptual discrepancy between the observability of the nucleon spin decomposition (I) and the general principle of gauge invariance.

The spatial axial gauge choice $n^\mu = (0,0,0,1)$ may be particularly useful for lattice QCD calculation of ΔG, since it contains no time component.

Possible basis of lattice QCD calculation

$$\Delta G \;=\; \frac{\langle Ps \,|\, n_\lambda \, M^{\lambda 12}_{G-spin}(0) \,|\, Ps \rangle}{2\, P \cdot n}$$

with

$$M^{\lambda\mu\nu}_{G-spin} \;=\; 2\,\mathrm{Tr}\left[F^{\lambda\nu} A^{\mu} \;-\; F^{\lambda\mu} A^{\nu} \right]$$

Here A^{μ} in this expression is the physical gluon field satisfying the spatial axial gauge condition

$$n^{\mu} A_{\mu} = 0, \qquad n^{\mu} = (0, 0, 0, 1)$$

with some appropriate boundary condition at

$$x^3 \;=\; \pm\infty$$

6. Conclusion

We have carried out a detailed comparison of the two fundamentally different approaches to the nucleon spin decomposition problems.

GIE approach $\Longleftrightarrow$ gauge-fixing approach

If both give the same answer, there is no practical problem. However, if they give different answers, one must stop and think it over.

In my opinion, conceptually legitimate is the latter approach. For, there is only 1 color gauge symmetry of QCD, which is present from the start.

This gauge symmetry is rather freedoms to be eliminated by gauge-fixing procedures rather than to be gained by extension.

This general consideration gives a support to our claim that there are only 2 physically inequivalent GI decompositions (I) and (II) of the nucleon spin.

[**Backup slides**]

Nontrivial problems in the Coulomb gauge calculation of evolution matrix

- Lorentz-frame dependence ?

- Role of instantaneous Coulomb interaction ?

- Coulomb-gauge Ward-identity requires ghost field !

- Ambiguous nature of loop-integral ?

  Sophisticated regularization method like split dimensional regularization of Leibbrandt ?

$$d^4 q \;=\; dq_4\, d^3\boldsymbol{q} \;\Rightarrow\; d^{2\sigma}\, d^{2\omega}\, \boldsymbol{q}\Big|^{\sigma \to (1/2)^+}_{\omega \to (3/2)^+}$$

- Might need some sophisticated limiting procedure ?

  Coulomb-gauge limit of interpolating gauge between the Coulomb gauge and the Landau gauge etc. ?

Important remark

interaction term ?

It is a wide-spread belief that, among the following two quantities :

$$\boldsymbol{L}_{can} = \boldsymbol{r} \times \boldsymbol{p} \quad \Longleftrightarrow \quad \boldsymbol{L}_{mech} = \boldsymbol{r} \times (\boldsymbol{p} - \boxed{e\,\boldsymbol{A}_{\perp}})$$

what is closer to physical image of orbital motion is the former, because the latter appears to contain an extra **interaction term** with the gauge field !

The fact is just opposite !

$$\begin{aligned} \boldsymbol{L}_{\text{“}can\text{”}} &= \boxed{\boldsymbol{L}_{mech}} + \sum_i \boldsymbol{r}_i \times q_i\,\boldsymbol{A}_{\perp}(\boldsymbol{r}_i) \\ &= \boxed{\sum_i m_i\,\boldsymbol{r}_i \times \dot{\boldsymbol{r}}_i} + \int d^3r\,\boldsymbol{r} \times (\boldsymbol{E}_{\parallel} \times \boldsymbol{B}_{\perp}) \end{aligned}$$

orbital motion !

- It is the “mechanical” angular momentum $\boldsymbol{L}_{mech}$ not the “canonical” angular momentum $\boldsymbol{L}_{\text{“}can\text{”}}$ that has a natural physical interpretation as orbital motion of particles !
- It may sound **paradoxical**, but what contains an extra interaction term is rather the “canonical” angular momentum than the “mechanical” angular momentum !

[Backup Slide] Nuclear spin decomposition problem

It is not a well-defined problem, because of the ambiguities of nuclear force.

$$H = \sum_i \frac{\boldsymbol{p}_i^2}{2\,m} + \sum_{i<j} V_{NN}(\boldsymbol{r}_i - \boldsymbol{r}_j)$$

To explain it, let us consider the deuteron, the simplest nucleus.

$$H\,\psi_d(\boldsymbol{r}) = E\,\psi_d(\boldsymbol{r})$$

$$\psi_d(\boldsymbol{r}) = \left[u(r) + \frac{S_{12}(\hat{\boldsymbol{r}})}{\sqrt{8}}\, w(r) \right] \frac{\chi}{\sqrt{4\,\pi}}$$

deuteron w.f. and S- and D-state probabilities

$$P_S = \int_0^\infty u^2(r)\, r^2\, dr, \quad P_D = \int_0^\infty w^2(r)\, r^2\, dr$$

angular momentum decomposition of deuteron spin

$$1 = \langle J_3 \rangle = \langle L_3 \rangle + \langle S_3 \rangle = \frac{3}{2} P_D + \left(P_S - \frac{1}{2} P_D \right)$$

We however know the fact that the D-state probability is not a direct observable !

- R.D. Amado, Phys. Rev. C20 (1979) 1473.
- J.L. Friar, Phys. Rev. C20 (1979) 325.

♣ 2-body unitary transformation arising in the theory of meson-exchange currents can change the D-state probability, while keeping the deuteron observables intact.

♣ The ultimate origin is arbitrariness of short range part of NN potential.

infinitely many phase-equivalent potential !

♣ The D-state probability, for instance, depends on the cutoff Λ of short range physics in an effective theory of 2-nucleon system.

- S.K. Bogner et al., Nucl. Phys. A784 (2007) 79.

⇩

See the figure in the next page !

Deuteron **D-state probability** in an effective theory (Bogner et al., 2007)

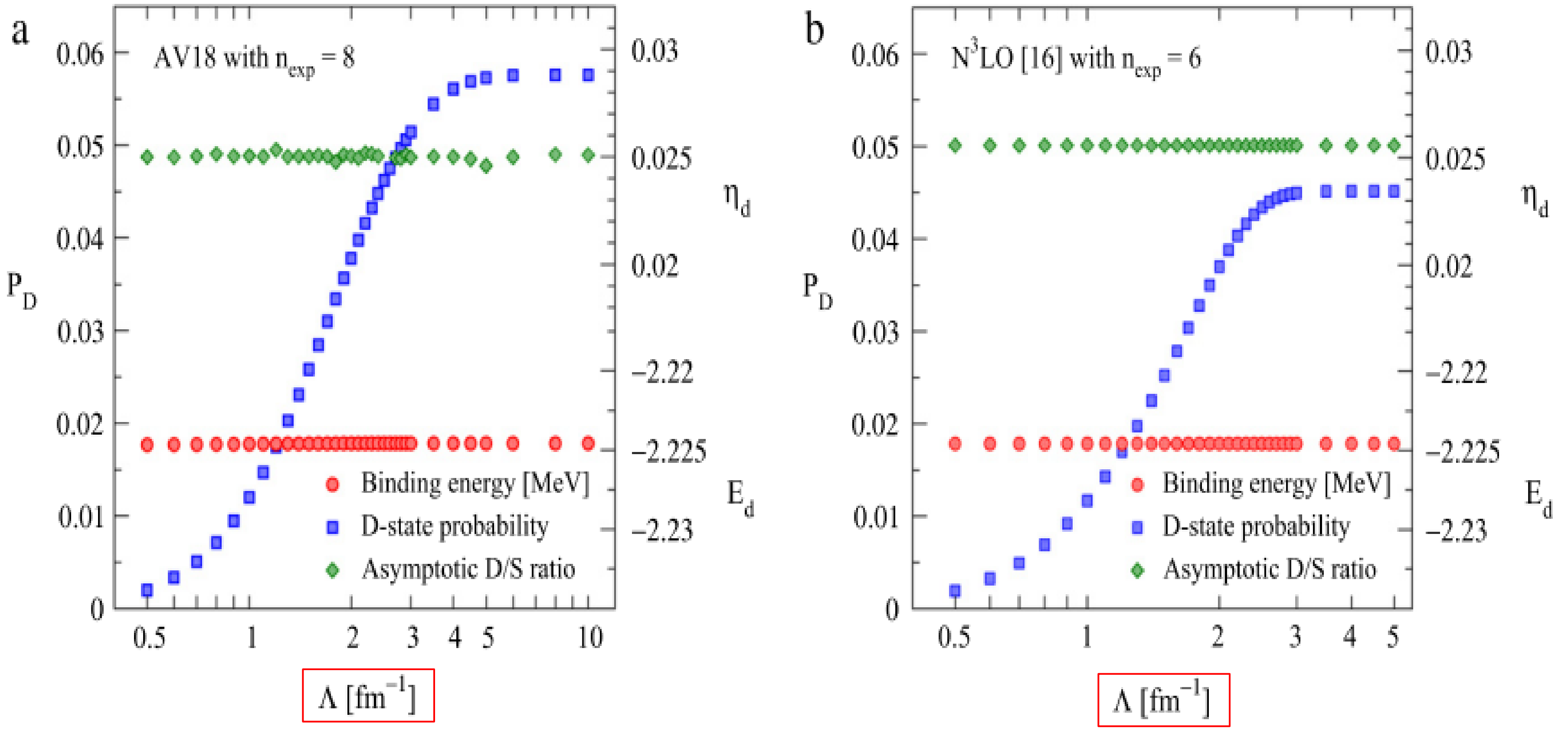


Fig. 57. D-state probability P_D (left axis), binding energy E_d (lower right axis), and asymptotic D/S-state ratio η_d (upper right axis) of the deuteron as a function of the cutoff [6], starting from (a) the Argonne v_{18} [18] and (b) the N^{3}LO NN potential of Ref. [20] using different smooth $V_{\mathrm{low}\,k}$ regulators. Similar results are found with SRG evolution.

Note that the asymptotic D/S ratio corresponds to observables, although the D-state probability not !