NLO BFKL and anomalous dimensions of light-ray operators

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Outline

- Regge limit in the coordinate space
- "BFKL" representation of 4-point correlation function in $\mathcal{N} = 4$ SYM.
- Light-ray operators.
- "DGLAP" representation of 4-point correlation function.
- Anomalous dimensions from DGAP vs BFKL representations.

Problem

Gluon operators of leading twist

$$\mathcal{O}_n^g \equiv F^a_{-i} \nabla_-^{n-2} F^{ai}_-$$

Anomalous dimension (in gluodynamics)

$$\gamma_n = N_c \Big[-\frac{4}{j-1} + \frac{4}{j} - \frac{4}{j+1} + \frac{4}{j+2} + 4[\psi(j+1) + C] - \frac{11}{3} \Big] + O(\alpha_s^2)$$

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BFKL gives $\gamma(j\alpha_s)$ at the non-physical point $j\to 1$

$$\gamma_j = \Big[A_n^{\text{LO BFKL}}+(j-1)B_n^{\text{NLO BFKL}}+...\Big]\Big(\frac{\alpha_s N_c}{\pi(j-1)}\Big)^n$$

$\omega = j-1\to 0$ corresponds to

$$F^a_{-i}\nabla_-^{\omega-1}F^{ai}_-$$

- non-local light-ray operator

Conformal four-point amplitude in $\mathcal{N}=4$ SYM

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\left(D^{\mu}\Phi_I^a\right)\left(D_{\mu}\Phi_I^a\right) - \frac{1}{4}g^2 f^{abc}f^{lmc}\Phi_I^a\Phi_J^b\Phi_I^l\Phi_J^m$$
$$+ \bar{\lambda}^a_{\dot{\alpha}A}\sigma_{\mu}^{\dot{\alpha}\beta}\mathcal{D}^{\mu}\lambda_{\beta}^{aA} - i\,\lambda_a^{\alpha A}\bar{\Sigma}^s_{AB}\Phi_b^s\lambda^B_{\alpha k}f^{abc} + i\,\bar{\lambda}^a_{\dot{\alpha}A}\Sigma^{sAB}\Phi_b^s\bar{\lambda}^{\dot{\alpha}}_{Bc}f^{abc}$$

4-point correlation function (CF)

$$A(x,y,x',y') = [(x-y)^2(x'-y')^2]^{2+\gamma_k}N_c^2\langle\mathcal{O}(x)\mathcal{O}(y)\mathcal{O}(x')\mathcal{O}(y')\rangle$$

$\mathcal{O} = \mathrm{Tr}\{\bar{\phi}^I\phi^I\}$ - Konishi operator (γ_k - anomalous dimension)

In a conformal theory the amplitude is a function of two conformal ratios

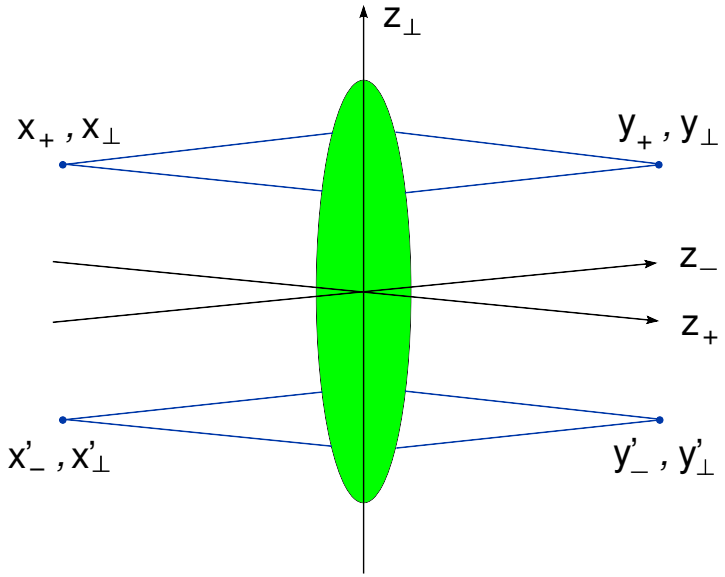
$$A = F(R,R')$$
$$R = \frac{(x-y)^2(x'-y')^2}{(x-x')^2(y-y')^2}, \qquad R' = \frac{(x-y)^2(x'-y')^2}{(x-y')^2(x'-y)^2}$$

At large N_c

$$A(x,y,x',y') = A(g^2N_c) \qquad g^2N_c = \lambda \; - \; \text{'t Hooft coupling}$$

Regge limit in the coordinate space

Regge limit: $x_+ \to \rho x_+,\ x'_+ \to \rho x'_+, y_- \to \rho' y_-,\ y'_- \to \rho' y-_- \qquad \rho, \rho' \to \infty$



Full 4-dim conformal group: $A = F(R, r)$

$$R = \frac{(x-y)^2(x'-y')^2}{(x-x')^2(y-y')^2} \to \frac{\rho^2\rho'^2 x_+ x'_+ y_- y'_-}{(x-x')^2_\perp (y-y')^2_\perp} \to \infty$$

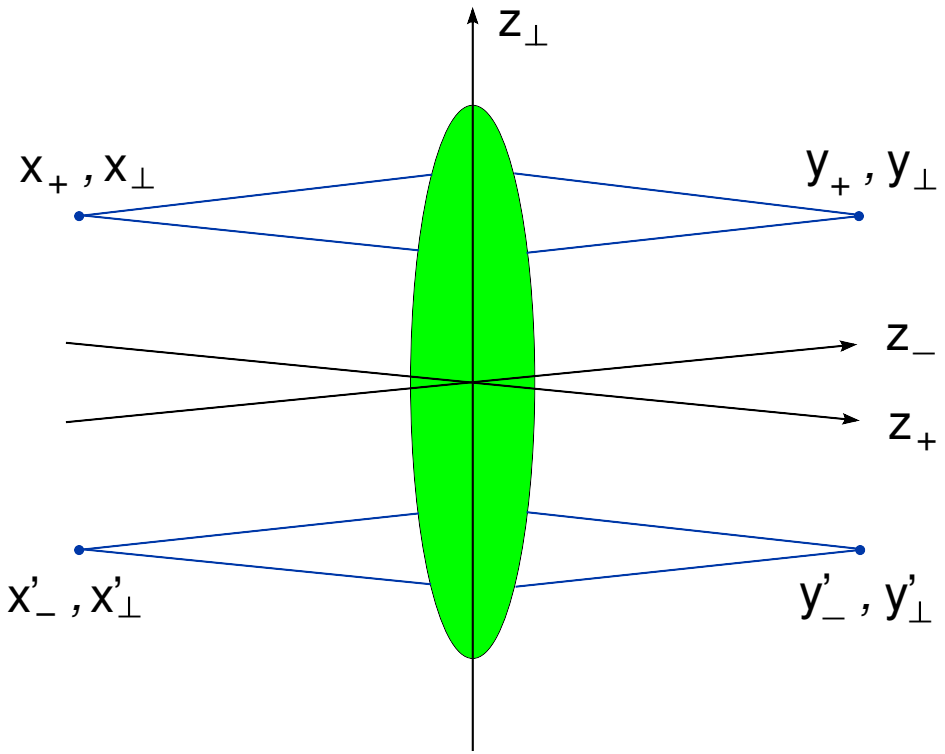
$$r = \frac{[(x-y)^2(x'-y')^2 - (x'-y)^2(x-y')^2]^2}{(x-x')^2(y-y')^2(x-y)^2(x'-y')^2}$$

$$\to \frac{[(x'-y')^2_\perp x_+ y_- + x'_+ y'_- (x-y)^2_\perp + x_+ y'_- (x'-y)^2_\perp + x'_+ y_- (x-y')^2_\perp]^2}{(x-x')^2_\perp (y-y')^2_\perp x_+ x'_+ y_- y'_-}$$

4-dim conformal group versus $SL(2,C)$

Regge limit: $x_+ \to \rho x_+,\ x'_+ \to \rho x'_+,\ y_- \to \rho' y_-,\ y'_- \to \rho' y-_-$

$\rho, \rho' \to \infty$



Regge limit symmetry: 2-dim conformal group $SL(2,C)$ formed from P_1, P_2, M^{12}, D, K_1 and K_2 which leave the plane $(0,0,z_\perp)$ invariant.

Pomeron in a conformal theory

$$A(x,y;x',y') \overset{s\to\infty}{=} \frac{i}{2}\int d\nu\, f_+(\aleph(\lambda,\nu))F(\lambda,\nu)\Omega(r,\nu)R^{\aleph(\lambda,\nu)/2}$$

L. Cornalba (2007)

$f_+(\omega) = \frac{e^{i\pi\omega}-1}{\sin\pi\omega}$ - signature factor

$$\Omega(r,\nu) = \frac{\sin\nu\rho}{\sinh\rho}, \qquad \cosh\rho = \frac{\sqrt{r}}{2}$$

- solution of the eqn $(\Box_{H_3} + \nu^2 + 1)\Omega(r,\nu) = 0$

The dynamics is described by:
$\aleph(\lambda,\nu)$ - pomeron intercept,
and
$F(\lambda,\nu)$ - "pomeron residue".

Pomeron in the conformal theory

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Pomeron intercept:

$$\aleph(\nu,\lambda) \;=\; \frac{\lambda}{4\pi^2}\left[\chi(\nu)+\frac{\lambda}{16\pi^2}\delta(\nu)\right]+O(\lambda^3)$$

$\chi(\nu) = 2\psi(1)-\psi(\frac{1}{2}+i\nu)-\psi(\frac{1}{2}-i\nu) = -S_1(-\frac{1}{2}+i\nu)+\text{c.c.}$ - BFKL intercept,

$$\delta(\nu) = -\frac{3}{2}\zeta(3)+\pi^2\ln 2+\frac{\pi^2}{3}S_1(-\frac{1}{2}+i\nu)+2S_3(-\frac{1}{2}+i\nu)+\pi^2 S_{-1}(-\frac{1}{2}+i\nu)-4S_{-2,1}(-\frac{1}{2}+i\nu)+\text{c.c.}$$

- NLO BFKL intercept $S_i(x)$ - harmonic sums Lipatov, Kotikov (2000)

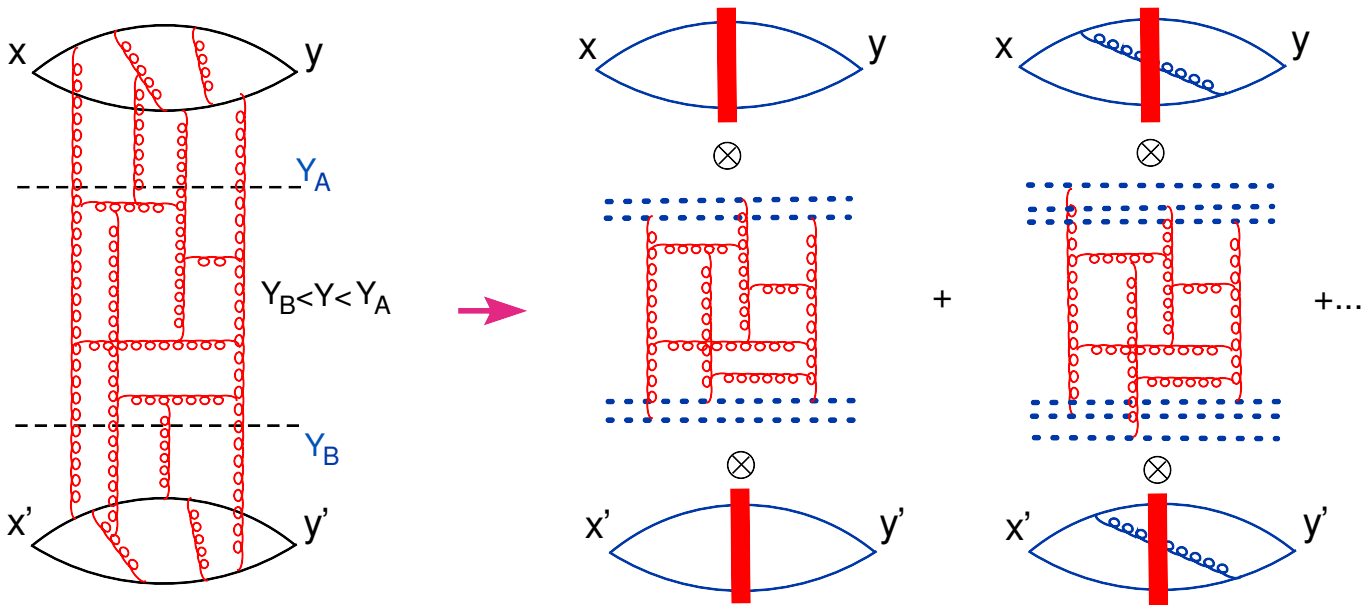
"Pomeron residue" $F(\nu,\lambda)$:

$$F(\nu,\lambda) \;=\; \lambda^2 F_0(\nu)+\lambda^3 F_1(\nu)+\ldots$$

$F_0(\nu) = \frac{\pi\sinh\pi\nu}{4\nu\cosh^3\pi\nu}$ Cornalba, Costa, Penedones (2007)

$F_1(\nu)$ = see below G. Chirilli and I.B. (2009)

NLO Amplitude in $\mathcal{N}$=4 SYM theory: factorization in rapidity

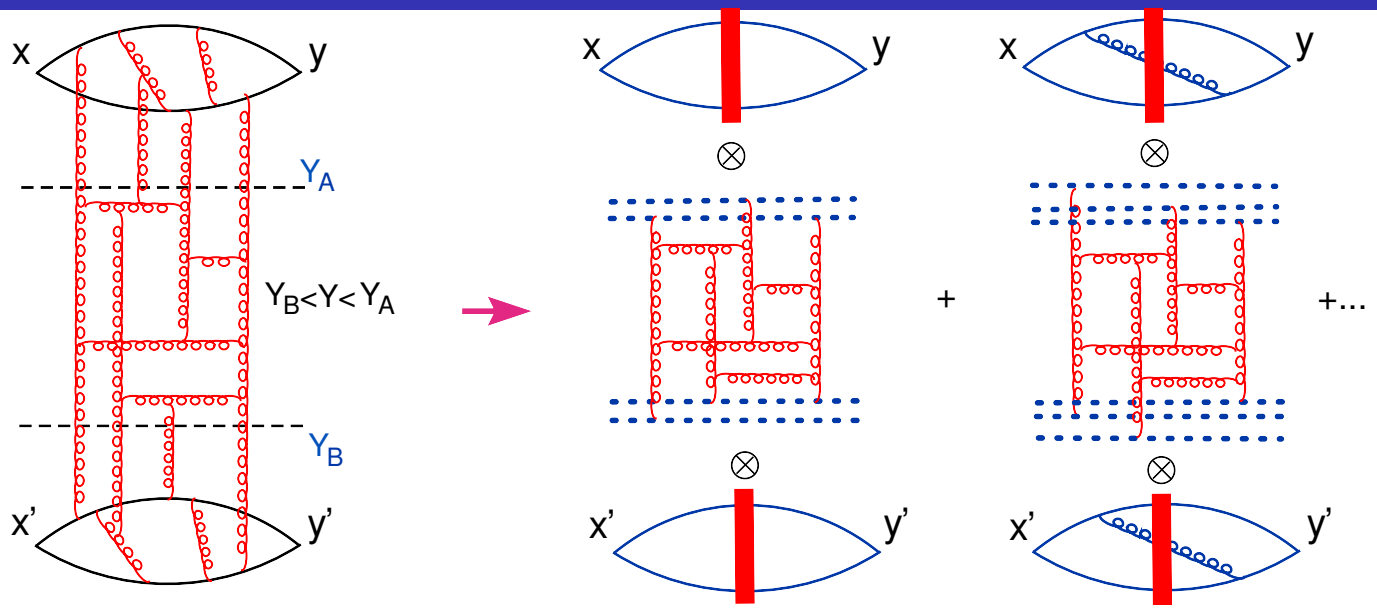


$$(x-y)^4(x'-y')^4\langle T\{\hat{\mathcal{O}}(x)\hat{\mathcal{O}}^\dagger(y)\hat{\mathcal{O}}(x')\hat{\mathcal{O}}^\dagger(y')\}\rangle$$
$$= \int d^2z_{1\perp}d^2z_{2\perp}d^2z'_{1\perp}d^2z'_{2\perp}\,\mathrm{IF}^{a_0}(x,y;z_1,z_2)[\mathrm{DD}]^{a_0,b_0}(z_1,z_2;z'_1,z'_2)\mathrm{IF}^{b_0}(x',y';z'_1,z'_2)$$

$a_0 = \frac{x_+y_+}{(x-y)^2}$, $b_0 = \frac{x'_-y'_-}{(x'-y')^2}$ $\Leftrightarrow$ impact factors do not scale with energy

$\Rightarrow$ all energy dependence is contained in $[\mathrm{DD}]^{a_0,b_0}$ $(a_0b_0 = R)$

NLO Amplitude in $\mathcal{N}$=4 SYM theory: factorization in rapidity



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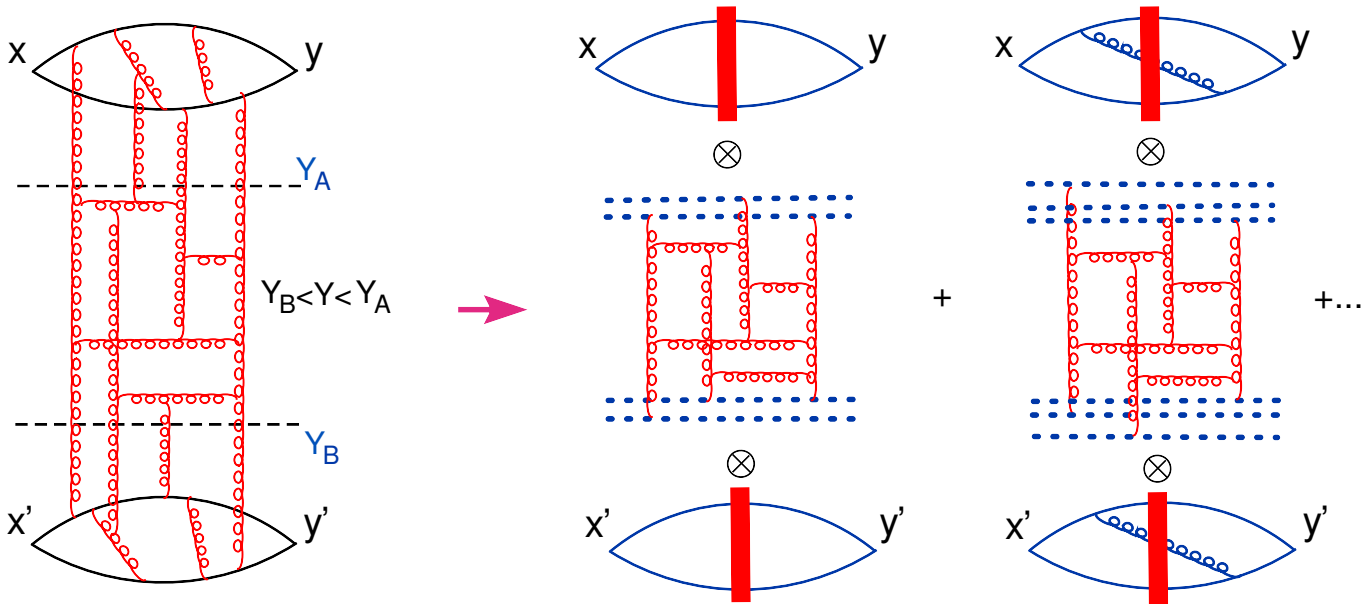
Dipole-dipole scattering

$$\chi(\gamma) \equiv 2C - \psi(\gamma) - \psi(1-\gamma)$$

$$[DD] = \int d\nu \int dz_0 \left(\frac{z_{12}^2}{z_{10}^2 z_{20}^2}\right)^{\frac{1}{2}+i\nu}\left(\frac{z_{12}^2}{z_{10}^2 z_{20}^2}\right)^{\frac{1}{2}-i\nu} D(\frac{1}{2}+i\nu;\lambda)R^{\omega(\nu)/2}$$

$$D(\gamma;\lambda) = \frac{\Gamma(-\gamma)\Gamma(\gamma-1)}{\Gamma(1+\gamma)\Gamma(2-\gamma)}\left\{1-\frac{\lambda}{4\pi^2}\left[\frac{\chi(\gamma)}{\gamma(1-\gamma)}-\frac{\pi^2}{3}\right]+O(\lambda^2)\right\}$$

NLO Amplitude in $\mathcal{N}$=4 SYM theory: factorization in rapidity



$$(x-y)^2(x'-y')^4\langle T\{\hat{\mathcal{O}}(x)\hat{\mathcal{O}}^\dagger(y)\hat{\mathcal{O}}(x')\hat{\mathcal{O}}^\dagger(y')\}\rangle$$

$$= \int d^2z_{1\perp}d^2z_{2\perp}d^2z'_{1\perp}d^2z'_{2\perp}\,\mathrm{IF}^{a_0}(x,y;z_1,z_2)[\mathrm{DD}]^{a_0,b_0}(z_1,z_2;z'_1,z'_2)\mathrm{IF}^{b_0}(x',y';z'_1,z'_2)$$

Result : (G.A. Chirilli and I.B.)

$$F(\nu) = \frac{N_c^2}{N_c^2-1}\frac{4\pi^4\alpha_s^2}{\cosh^2\pi\nu}\left\{1+\frac{\alpha_s N_c}{\pi}\left[\frac{\pi^2}{2}-\frac{2\pi^2}{\cosh^2\pi\nu}-\frac{8}{1+4\nu^2}\right]+O(\alpha_s^2)\right\}$$

Light-cone OPE in Cornalba's formula

$$[x_{12}^2 x_{34}^2]^{2+\gamma_K} \langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle = \frac{i}{2}\int d\nu \, \frac{\tanh \pi\nu}{\nu} F(\nu)\Omega(r,\nu) R^{\frac{1}{2}\aleph(\nu)} \tilde{f}_+(\aleph(\nu))$$

$\aleph(\nu)$ - pomeron intercept, $\tilde{f}_+(\omega) = (e^{i\pi\omega} - 1)/\sin\pi\omega$ - signature factor

$$\Omega(r,\nu) = \frac{\nu}{2\pi^2}\frac{\sin 2\nu\rho}{\sinh\rho}, \qquad \cosh\rho = \frac{\sqrt{r}}{2}$$

In the double limit (Regge) + $(x_{12}^2 \to 0)$

$$R = \frac{x_{13}^2 x_{24}^2}{x_{12}^2 x_{34}^2} \to \frac{x_{1_+} x_{2_+} x_{3_-} x_{4_-}}{x_{12_\perp}^2 x_{34_\perp}^2}$$

$$r \to \frac{x_{12_+}^2 (x_{3_-} x_{14_\perp}^2 - x_{4_-} x_{13_\perp}^2)^2}{x_{1_+} x_{2_+} x_{3_-} x_{4_-} x_{12_\perp}^2 x_{34_\perp}^2}$$

$$\Omega(r,\nu) \to \frac{\nu}{2\pi^2 i}\left(r^{-\frac{1}{2}+i\nu} - r^{-\frac{1}{2}-i\nu}\right)$$

Light-cone limit of Cornalba's formula

Cornalba's formula as $x_{12\perp}^2 \to 0$

$$[x_{12}^2 x_{34}^2]^{2+\gamma_K} \int dx_{2_+} dx_{3_-} \ \langle \mathcal{O}(L_1 + x_{2_+}, x_{1\perp}) \mathcal{O}(x_{2_+}, x_{2\perp}) \mathcal{O}(L_2 + x_{3_-}, x_{2\perp}) \mathcal{O}(x_{3_-}, x_{3\perp}) \rangle$$

$$= \frac{i\alpha_s^2}{8} \pi^2 L_1 L_2 \int_0^1 du dv \int d\nu \frac{\tanh \pi\nu}{\nu \cosh^2 \pi\nu} \left(\frac{x_{12\perp}^2 x_{34\perp}^2 \bar{u} u \bar{v} v}{[x_{13\perp}^2 v + x_{14}^2 \bar{v}]^2} \right)^{\frac{1}{2}+i\nu} \left(\frac{L_1^2 L_2^2 \bar{u} u \bar{v} v}{x_{12\perp}^2 x_{34\perp}^2} \right)^{\aleph(\nu)/2} f_+$$

$$= \frac{i\alpha_s^2}{8} \pi^2 \int_0^1 dv \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} d\xi \, f\big(\aleph(\xi)\big) \qquad \xi \equiv \frac{1}{2} + i\nu$$

$$\times \frac{(\bar{v}v)^{1-\xi+\frac{\aleph(\xi)}{2}} \cos \pi\xi}{(\xi - \frac{1}{2}) \sin^3 \pi\xi} \frac{B\big(2 - \xi + \frac{\aleph(\xi)}{2}\big)}{[x_{13\perp}^2 v + x_{14}^2 \bar{v}]^{2+\aleph(\xi)}} \left(\frac{x_{12\perp}^2 x_{34\perp}^2}{[x_{13\perp}^2 v + x_{14}^2 \bar{v}]^2} \right)^{-\xi - \frac{\aleph(\xi)}{2}} (L_1 L_2)^{1+\aleph(\xi)}$$

= "BFKL" representation of the amplitude in the (Regge) + ($x_{12}^2 \to 0$) limit

Light-ray operators and parton densities

Gluon light-ray (LR) operator of twist 2

$$F^a_{-i}(x'_+ + x_\perp)[x'_+, x_+]^{ab} F^{b\ i}_-(x_+ + x_\perp)$$

Forward matrix element - gluon parton density

$$z^\mu z^\nu \langle p|F^a_{\mu\xi}(z)[z,0]^{ab} F^{b\xi}_\nu(0)|p\rangle^\mu \overset{z^2=0}{=} 2(pz)^2 \int_0^1 dx_B\ x_B D_g(x_B,\mu)\cos(pz)x_B$$

Evolution equation (in gluodynamics)

$$\mu^2 \frac{d}{d\mu^2} F^a_{-i}(x'_+ + x_\perp)[x'_+, x_+]^{ab} F^{b\ i}_-(x_+ + x_\perp)$$

$$= \int_{x_+}^{x'_+} dz'_+ \int_{x_+}^{z'_+} dz_+\ K(x'_+, x_+; z'_+, z_+; \alpha_s) F^a_{-i}(z'_+ + x_\perp)[z'_+, z_+]^{ab} F^{b\ i}_-(z_+ + x_\perp)$$

"Forward" LR operator

$$F^\mu(L_+, x_\perp) = \int dx_+ \left(F^a_{-i}(L_+ + x_+ + x_\perp)[L_+ + x_+, x_+]^{ab} F^{bi}_-(x_+ + x_\perp)\right)^\mu$$

Light-ray operators and parton densities

Expansion in local operators

$$F^\mu(L_+, x_\perp) = \sum_{n=2}^{\infty} \frac{L_+^{n-2}}{(n-2)!} \mathcal{O}_n^g(x_+, x_\perp), \quad \mathcal{O}_n^g \equiv F^a_{-i} \nabla_-^{n-2} F^{ai}_-$$

Evolution equation for $F^\mu(L_+, x_\perp)$

$$\mu^2 \frac{d}{d\mu^2} F^\mu(L_+, x_\perp) = \int_0^1 du\, K_{gg}(u, \alpha_s) F^\mu(uL_+, x_\perp)$$

$$\Rightarrow \quad \gamma_n(\alpha_s) = -\frac{1}{2} \int_0^1 du\, u^{n-2} K_{gg}(u, \alpha_s) \qquad \mu \frac{d}{d\mu} \mathcal{O}_n^g = -\gamma_n(\alpha_s) \mathcal{O}_n^g$$

uK_{gg} - DGLAP kernel

$$uK_{gg}(u) = \frac{\alpha_s N_c}{\pi} \left(\bar{u}u + \left[\frac{1}{\bar{u}u} \right]_+ - \frac{1}{12_c} \delta(\bar{u}) \right) + \text{higher orders in } \alpha_s$$

Light-ray operators

Conformal LR operator ($j = \frac{1}{2} + i\nu$)

$$F^\mu(L_+, x_\perp) = \int_{-\infty}^{\infty} \frac{d\nu}{2\pi} (L_+)^{-\frac{3}{2}+i\nu} \mathcal{F}^\mu_{\frac{1}{2}+i\nu}(x_\perp)$$

$$\mathcal{F}^\mu_j(x_\perp) = \int_0^\infty dL_+ \, L_+^{1-j} F^\mu(L_+, x_\perp)$$

Evolution equation for "forward" conformal light-ray operators

$$\Rightarrow \mu^2 \frac{d}{d\mu^2} \mathcal{F}^\mu_j(z_\perp) = \int_0^1 du \, K_{gg}(u, \alpha_s) u^{j-2} \mathcal{F}^\mu_j(z_\perp)$$

$\Rightarrow \gamma_j(\alpha_s)$ is an analytical continuation of γ_n

Supermultiplet of twist-2 operators

SU_4 singlet operators. (Korchemsky et al)

$$
\begin{aligned}
S^{\mu}_{1n}(z) &= \mathcal{O}^n_g(z) + \frac{n-1}{24}\mathcal{O}^n_\lambda(z) + \frac{n(n-1)}{24}\mathcal{O}^n_\phi(z) \\
S^{\mu}_{2n}(z) &= \mathcal{O}^n_g(z) - \frac{1}{24}\mathcal{O}^n_\lambda(z) - \frac{n(n+1)}{72}\mathcal{O}^n_\phi(z) \\
S^{\mu}_{3n}(z) &= \mathcal{O}^n_g(z) - \frac{n+2}{12}\mathcal{O}^n_\lambda(z) + \frac{(n+1)(n+2)}{24}\mathcal{O}^n_\phi(z)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{O}^n_\lambda(z) &= \bar{\lambda}^a \nabla^{n-1} \lambda^a(z) + \text{descendants} \\
\mathcal{O}^n_\phi(z) &= \bar{\phi}^{a,I} \nabla^n \phi^{a,I}(z) + \text{descendants}
\end{aligned}
$$

All operators have the same anomalous dimension

$\gamma_n(\alpha_s) = \frac{2\alpha_s}{\pi} N_c [\psi(n-1) + C] + ...$

Supermultiplet of LR operators

Gluino and scalar LR operators

$$\Lambda^\mu(L_+,x_\perp) = \int dx'_+ \left(\bar\lambda^a(L_+ + x_+ + x_\perp)[x'_+ + x_+, x_+]^{ab}\lambda^b(x_+ + x_\perp)\right)^\mu$$

$$\Lambda^\mu_j(x_\perp) = \int_0^\infty dL_+ \, L_+^{-j}\Lambda^\mu(L_+,x_\perp)$$

$$\Phi^\mu(L_+,x_\perp) = \int dx'_+ \left(\bar\phi^{a,I}(L_+ + x_+ + x_\perp)[x'_+ + x_+, x_+]^{ab}\phi^{b,I}(x_+ + x_\perp)\right)^\mu$$

$$\Phi^\mu_j(x_\perp) = \int_0^\infty dL_+ \, L_+^{-j-1}\Phi^\mu(L_+,x_\perp)$$

SU_4 singlet LR operators.

$$S^\mu_{1j}(x_\perp) = F^\mu_j(x_\perp) + \frac{j-1}{24}\Lambda^\mu_j(x_\perp) + \frac{j(j-1)}{24}\Phi^\mu_j(x_\perp)$$

$$S^\mu_{2j}(x_\perp) = F^\mu_j(x_\perp) - \frac{1}{24}\Lambda^\mu_j(x_\perp) - \frac{j(j+1)}{72}\Phi^\mu_j(x_\perp)$$

$$S^\mu_{3j}(x_\perp) = F^\mu_j(x_\perp) - \frac{j+2}{12}\Lambda^\mu_j(x_\perp) + \frac{(j+1)(j+2)}{24}\Phi^\mu_j(x_\perp)$$

All operators have the same anomalous dimension

$\gamma_j(\alpha_s) = \frac{2\alpha_s}{\pi}N_c[\psi(j-1)+C] + \ldots$

Light-cone OPE

Three-point correlator of local operators ($\mathcal{O} = \mathrm{Tr}\{\bar{\phi}^I \phi^I\}$ - Konishi operator)

$$(\mu^2 z_{23}^2)^{-\gamma_K} \langle S^\mu_{1n}(z_1)\mathcal{O}(z_2)\mathcal{O}(z_3)\rangle = \sum \frac{c_n[1+(-1)^n]}{z_{12}^2 z_{13}^2 z_{23}^2}\left(\frac{z_{12-}}{z_{12}^2} - \frac{z_{13-}}{z_{13}^2}\right)^n \left(\frac{\mu^{-2} z_{23}^2}{z_{12}^2 z_{13}^2}\right)^{\frac{1}{2}\gamma(n,\alpha_s)}$$

$\Rightarrow$

$$x_{23_\perp}^2 (\mu^2 x_{23_\perp}^2)^{\gamma_K} \int dx_{1_+} dx_{3_-} S^\mu_{1n}(x_{1_+}, x_{1_\perp})\mathcal{O}(L_{2_-} + x_{3_-}, x_{2_\perp})\mathcal{O}(x_{3_-}, x_{3_\perp})$$

$$= -2\pi i \frac{s}{4}\int_0^1 du\, \frac{c_n[1+(-1)^n](L_{2_-})^n(\bar{u}u)^n}{[x_{12_\perp}^2 u + x_{13_\perp}^2 \bar{u}]^{1+n}} \frac{\Gamma(1+2n+\gamma_n)}{\Gamma^2(1+n+\frac{1}{2}\gamma_n)} \left(\frac{\bar{u}u\mu^{-2} x_{23_\perp}^2}{[x_{12_\perp}^2 u + x_{13_\perp}^2 \bar{u}]^2}\right)^{\frac{\gamma_n}{2}}$$

CF of LR operator and two local operators

$$x_{23_\perp}^2 (\mu^2 x_{23_\perp}^2)^{\gamma_K} \int dx_{3_-} \langle S_j^\mu(x_{1_\perp})\mathcal{O}(L_- + x_{3_-}, x_{2_\perp})\mathcal{O}(x_{3_-}, x_{3_\perp})\rangle$$

$$= \int_0^1 du\, \frac{c(j,\alpha_s)[1+e^{i\pi j}]L_-^j(\bar{u}u)^j}{[x_{12_\perp}^2 u + x_{13_\perp}^2 \bar{u}]^{1+j}} \left(\frac{\bar{u}u\mu^{-2} x_{23_\perp}^2}{[x_{12_\perp}^2 u + x_{13_\perp}^2 \bar{u}]^2}\right)^{\frac{1}{2}\gamma(j,\alpha_s)}$$

Light-cone OPE

Limit $x_{23_\perp} \equiv \Delta \to 0$

$$\Delta^2(\mu^2\Delta^2)^{\gamma_K}\int dx_{3_-}\,\langle S_j^+(x_{1_\perp})\mathcal{O}(L_- + x_{3_-}, x_{2_\perp})\mathcal{O}(x_{3_-}, x_{3_\perp})\rangle$$

$$= \frac{\Gamma^2\left(1+j+\frac{\gamma}{2}\right)}{\Gamma(2+2j+\gamma)}\,\frac{c(j,\alpha_s)[1+e^{i\pi j}]L_-^j}{(x^2_{12_\perp})^{1+j}(\mu^2 x^2_{12_\perp})^{-\gamma(j,\alpha_s)}}(\mu\Delta)^{\gamma(j,\alpha_s)}$$

CF of two LRs : $\quad \langle S^+_{j=\frac{3}{2}+i\nu}(x_{1_\perp})S^-_{j'=\frac{3}{2}+i\nu'}(x_{3_\perp})\rangle = \dfrac{\delta(\nu-\nu')a(j,\alpha_s)}{(x^2_{13_\perp})^{j+1}(x^2_{13_\perp}\mu^2)^{\gamma(j,\alpha_s)}}$

$\Rightarrow$ Light-cone OPE

$$\Delta^2(\mu^2\Delta^2)^{\gamma_K}\int dx_{3_-}\;\mathcal{O}(L_- + x_{3_-}, x_{2_\perp})\mathcal{O}(x_{3_-}, x_{3_\perp})$$

$$= \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} dj\; b(j,\alpha_s)[1+e^{i\pi j}]L^j(\mu^2 x^2_{23_\perp})^{\gamma(j,\alpha_s)}S_j^-(x_{3_\perp})$$

"DGLAP" represenation of 4-point CFs

$\Rightarrow$ Light-cone OPE

$$x_{12_\perp}^2(\mu^2 x_{12_\perp}^2)^{\gamma_K}\int dx_{1_+}\ \mathcal{O}(L_1+x_{2_+},x_{1_\perp})\mathcal{O}(x_{2_+},x_{2_\perp})$$

$$=\int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} dj\ c(j,\alpha_s)[1+e^{i\pi j}]L_1^j(\mu^2 x_{12_\perp}^2)^{\gamma(j,\alpha_s)}S_j^+(x_{1_\perp})$$

"DGLAP" result (at $x_{12}^2\to 0$)

$$\mu^{-4}(\mu^4 x_{12}^2 x_{34}^2)^{1+\gamma_k}\int dx_{2_+}dx_{3_-}\ \langle\mathcal{O}(L_1+x_{2_+},x_{1_\perp})\mathcal{O}(x_{2_+},x_{2_\perp})\mathcal{O}(L_2+x_{4_-},x_{3_\perp})\mathcal{O}(x_{4_-},x_{4_\perp})\rangle$$

$$=\int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} dj\ C(j,\alpha_s)[1+e^{i\pi j}]L_1^j(\mu^2 x_{12_\perp}^2)^{\frac{1}{2}\gamma(j,\alpha_s)}\langle S_j^\mu(x_{2_\perp})\mathcal{O}(L_2+x_{4_-},x_{3_\perp})\mathcal{O}(x_{4_-},x_{4_\perp})\rangle$$

$$=\int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} dj\ C(j,\alpha_s)\int_0^1 dv\ \frac{[1+e^{i\pi j}](L_1L_2)^j}{[x_{12_\perp}^2 v+x_{13_\perp}^2\bar v]^{1+j}}\left(\frac{x_{12}^2x_{34}^2}{[x_{13_\perp}^2 v+x_{14_\perp}^2\bar v]^2}\right)^{\frac{1}{2}\gamma(j,\alpha_s)}(\bar v v)^{j+\frac{1}{2}\gamma(j,\alpha_s)}$$

DGLAP *vs* BFKL for 4-point CFs

"DGLAP" result (leading twist)

$$\mu^{-4}(\mu^4 x_{12}^2 x_{34}^2)^{1+\gamma_k}\int dx_{2_+}dx_{3_-}\ \langle \mathcal{O}(L_1+x_{2_+},x_{1_\perp})\mathcal{O}(x_{2_+},x_{2_\perp})\mathcal{O}(L_2+x_{4_-},x_{3_\perp})\mathcal{O}(x_{4_-},x_{4_\perp})\rangle$$

$$= \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} dj\ C(j,\alpha_s)\int_0^1 dv\ \frac{[1+e^{i\pi j}](L_1L_2)^j}{\left[x_{12_\perp}^2 v + x_{13_\perp}^2 \bar{v}\right]^{1+j}}\left(\frac{x_{12}^2 x_{34}^2}{\left[x_{13_\perp}^2 v + x_{14_\perp}^2 \bar{v}\right]^2}\right)^{\frac{1}{2}\gamma(j,\alpha_s)}(\bar{v}v)^{j+\frac{1}{2}\gamma(j,\alpha_s)}$$

DGLAP *vs* BFKL for 4-point CFs

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$$= \int_{\frac12-i\infty}^{\frac12+i\infty} dj\ C(j,\alpha_s)\int_0^1 dv\ \frac{[1+e^{i\pi j}](L_1L_2)^j}{[x_{12_\perp}^2 v + x_{13_\perp}^2\bar v]^{1+j}}\left(\frac{x_{12}^2x_{34}^2}{[x_{13_\perp}^2 v + x_{14_\perp}^2\bar v]^2}\right)^{\frac12\gamma(j,\alpha_s)}(\bar v v)^{j+\frac12\gamma(j,\alpha_s)}$$

BFKL result ($\aleph(\xi,\alpha_s)=\frac{\alpha_s N_c}{\pi}\chi(\xi)+\ldots$ - pomeron intercept)

$$[x_{12}^2x_{34}^2]^{2+\gamma_K}\int dx_{2_+}dx_{3_-}\ \langle \mathcal{O}(L_1+x_{2_+},x_{1\perp})\mathcal{O}(x_{2_+},x_{2\perp})\mathcal{O}(L_2+x_{3_-},x_{2\perp})\mathcal{O}(x_{3_-},x_{3\perp})\rangle$$

$$= \frac{i\alpha_s^2}{8}\pi^2\int_0^1 dv\int_{\frac12-i\infty}^{\frac12+i\infty} d\xi\, f\big(\aleph(\xi)\big) \qquad \xi\equiv\frac12+i\nu$$

$$\times\ \frac{(\bar v v)^{1-\xi+\frac{\aleph(\xi)}{2}}\cos\pi\xi}{(\xi-\frac12)\sin^3\pi\xi}\frac{B\big(2-\xi+\frac{\aleph(\xi)}{2}\big)}{[x_{13\perp}^2 v + x_{14}^2\bar v]^{2+\aleph(\xi)}}\left(\frac{x_{12\perp}^2x_{34\perp}^2}{[x_{13\perp}^2 v + x_{14}^2\bar v]^2}\right)^{-\xi-\frac{\aleph(\xi)}{2}}(L_1L_2)^{1+\aleph(\xi)}$$

DGLAP *vs* BFKL for 4-point CFs

"DGLAP" result (leading twist)

$$\mu^{-4}(\mu^4 x_{12}^2 x_{34}^2)^{1+\gamma_k}\int dx_{2_+}dx_{3_-}\ \langle \mathcal{O}(L_1+x_{2_+},x_{1_\perp})\mathcal{O}(x_{2_+},x_{2_\perp})\mathcal{O}(L_2+x_{4_-},x_{3_\perp})\mathcal{O}(x_{4_-},x_{4_\perp})\rangle$$

$$= \int_{\frac12-i\infty}^{\frac12+i\infty} dj\ C(j,\alpha_s)\int_0^1 dv\ \frac{[1+e^{i\pi j}](L_1L_2)^j}{[x_{12_\perp}^2 v + x_{13_\perp}^2\bar v]^{1+j}}\left(\frac{x_{12}^2x_{34}^2}{[x_{13_\perp}^2 v + x_{14_\perp}^2\bar v]^2}\right)^{\frac12\gamma(j,\alpha_s)}(\bar v v)^{j+\frac12\gamma(j,\alpha_s)}$$

BFKL result ($\aleph(\xi,\alpha_s)=\frac{\alpha_s N_c}{\pi}\chi(\xi)+\ldots$ - pomeron intercept)

$$[x_{12}^2x_{34}^2]^{2+\gamma_K}\int dx_{2_+}dx_{3_-}\ \langle \mathcal{O}(L_1+x_{2_+},x_{1\perp})\mathcal{O}(x_{2_+},x_{2\perp})\mathcal{O}(L_2+x_{3_-},x_{2\perp})\mathcal{O}(x_{3_-},x_{3\perp})\rangle$$

$$= \frac{i\alpha_s^2}{8}\pi^2\int_0^1 dv\int_{\frac12-i\infty}^{\frac12+i\infty} d\xi\, f\big(\aleph(\xi)\big) \qquad \xi\equiv\frac12+i\nu$$

$$\times\ \frac{(\bar v v)^{1-\xi+\frac{\aleph(\xi)}{2}}\cos\pi\xi}{(\xi-\frac12)\sin^3\pi\xi}\frac{B\big(2-\xi+\frac{\aleph(\xi)}{2}\big)}{[x_{13\perp}^2 v + x_{14}^2\bar v]^{2+\aleph(\xi)}}\left(\frac{x_{12\perp}^2x_{34\perp}^2}{[x_{13\perp}^2 v + x_{14}^2\bar v]^2}\right)^{-\xi-\frac{\aleph(\xi)}{2}}(L_1L_2)^{1+\aleph(\xi)}$$

We compare these results around $\xi\sim j-1\sim 0$

$$\Rightarrow\ 1+\aleph(\xi,\alpha_s)=j\ \text{ and }\ \gamma(j,\alpha_s)=-2\xi-\aleph(\xi)$$

$$\aleph(\xi)-2\aleph(\xi)\aleph'(\xi)\ \simeq\ \frac{\alpha_s N_c}{\pi\xi}+\frac{\zeta(3)\alpha_s^2}{\xi}+\ldots\ \rightarrow\ \gamma_j=-2\frac{\alpha_s N_c}{\pi(j-1)}\ +\ [0+\zeta(3)(j-1)]\Big(\frac{\alpha_s N_c}{\pi(j-1)}\Big)^3+\ldots$$

Result

$$\gamma_j = -2\frac{\alpha_s N_c}{\pi(j-1)} + [0 + \zeta(3)(j-1)]\Big(\frac{\alpha_s N_c}{\pi(j-1)}\Big)^3 + ...$$

1. Conclusions:
   - NLO BFKL gives anomalous dimensions of light-ray operators $F_{+i}\nabla^{\omega-1}F_{+}{}^{i}$ at $\omega \to 0$ (in all orders in α_s)

DGLAP *vs* Conclusions and Outlook

1. Conclusions:
   - NLO BFKL gives anomalous dimensions of light-ray operators $F_{+i}\nabla^{\omega-1}F_{+}{}^{i}$ at $\omega \to 0$ (in all orders in α_s)
2. Outlook:
   - Do this in QCD using Chirilli-Kovhegov $O(\alpha_s)$ NLO BFKL solution