Virtuality Distributions and $\gamma\gamma^* \to \pi^0$ Transition at Handbag Level

A.V. Radyushkin

Physics Department, Old Dominion University
&
Theory Center, Jefferson Lab

May 16, 2014,
QCD Evolution 2014, Santa Fe

Transverse Momentum Distributions

- Longitudinal Momentum Distribution

$$\int_0^1 f(x)\, x^N\, dx = \langle p|\phi(0)\partial_+^N \phi(0)|p\rangle$$

- Transverse Momentum Distribution

$$\int_0^1 f(x, k_T)\, x^N (k_T^2)^l\, dx = \langle p|\phi(0)\partial_+^N (\partial_\perp^2)^l \phi(0)|p\rangle$$

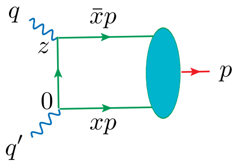
- Operators with $(\partial_\perp^2)^l$: higher twists
- Usual twist decomposition of $\phi(0)\partial^{\mu_1} \ldots \partial^{\mu_n} \phi(0)$ involves matrix elements

$$\langle p|\phi(0)\partial_+^N (\partial^2)^l \phi(0)|p\rangle$$

containing ∂^2 rather than $\partial_\perp^2$

- ∂^2 is related to parton virtuality
- Relate virtuality distributions with TMDs

$\gamma^*\gamma \to \pi^0$ transition amplitude at twist 2



$$\gamma(q')\gamma^*(q) \to \pi^0(p)$$

$$q'^2 = 0\ ,\ \ q^2 = -Q^2$$

- Twist-2 distribution amplitude:

$$\langle p|\phi(0)\phi(z)|0\rangle = \int_0^1 \varphi(x)\ e^{i\bar{x}(pz)}\ dx + \mathcal{O}(z^2)$$

- Twist-2 transition amplitude (for $p^2 = 0$ and $(q' - p)^2 = -Q^2$)

$$T(p,q) = \int_0^1 dx \int d^4 z\, e^{i(qz) - i\bar{x}(pz)}\, D^c(z)$$

$$= \int_0^1 \frac{\varphi(x)}{-(q' - xp)^2}\, dx = \int_0^1 \frac{\varphi(x)}{xQ^2}\, dx$$

Twist decomposition of bilocal operator

- Taylor expansion in bilocal operator $\phi(0)\phi(z)$

$$\phi(z) = \sum_{n=0}^{\infty} \frac{1}{n!} z_{\mu_1} \dots z_{\mu_n} \, \partial^{\mu_1} \dots \partial^{\mu_n} \phi(0)$$

- Twist expansion (with $\{z\partial\}^n \equiv \{z_{\mu_1} \dots z_{\mu_n}\} \, \partial^{\mu_1} \dots \partial^{\mu_n}$)

$$\phi(z) = \sum_{l=0}^{\infty} \left(\frac{z^2}{4}\right)^l \sum_{N=0}^{\infty} \frac{N+1}{l!(N+l+1)!} \{z\partial\}^N (\partial^2)^l \phi(0)$$

- Virtuality-dependent matrix elements ($p^2 = 0$)

$$\langle p|\phi(0)\{z\partial\}^k \left(\partial^2\right)^l \phi(0)|0\rangle \equiv [i(zp)]^k \Lambda^{2l} A_{kl}$$

- Treating A_{kl} as x^k moments of higher-twist DAs $\varphi_l(x)$

$$\langle p|\phi(0)\phi(z)|0\rangle = \sum_{l=0}^{\infty} \left(\frac{\Lambda^2 z^2}{4}\right)^l \int_0^1 \varphi_l(x) \, e^{i\bar{x}(pz)} \, dx$$

$$\equiv \int_0^1 B(x, z^2/4) \, e^{i\bar{x}(pz)} \, dx$$

- Bilocal function $B(x, z^2/4)$ accumulates information about parton virtuality

Virtuality distributions

- In fact, no assumptions about finiteness of matrix elements $\langle p|\phi(0)\{z\partial\}^k \left(\partial^2\right)^l \phi(0)|0\rangle$ are necessary!
- Schwinger alpha-representation for any contributing diagram

$$\langle p|\phi(0)\phi(z)|0\rangle = \text{const} \int_0^\infty \prod_{j=1}^{l} d\alpha_j [A(\alpha)+B(\alpha)]^{-d/2}$$
$$\times \exp\left\{-i\frac{z^2/4}{A(\alpha)+B(\alpha)} + i(pz)\frac{B(\alpha)}{A(\alpha)+B(\alpha)}\right\}$$
$$\times \exp\left\{ip^2 C(\alpha) - i\sum_j \alpha_j(m_j^2 - i\epsilon)\right\}$$

with positive $A(\alpha),\ B(\alpha),\ C(\alpha)$

- Representation through virtuality distribution amplitude (VDA) $\Phi(x,\sigma)$

$$\langle p|\phi(0)\phi(z)|0\rangle = \int_0^\infty d\sigma \int_0^1 dx\, \Phi(x,\sigma)\ e^{i\bar{x}(pz)-i\sigma(z^2-i\epsilon)/4}$$

- Note: dependence on $z^2 - i\epsilon$, support $\sigma \geq 0$, and in general $p^2 \neq 0$

Transverse momentum dependent DAs

- Pion momentum is defined to have no transverse components
- Projection on $z^+ = 0$ interval $z = (z^-, z_\perp)$

$$\langle p|\phi(0)\phi(z)|0\rangle|_{z^+=0, p_\perp=0} = \int_0^1 dx\, \varphi(x, z_\perp)\, e^{i\bar{x}(pz^-)}$$

- Impact parameter distribution amplitude (IDA) $\varphi(x, z_\perp) \equiv B(x, -z_\perp^2/4)$
- Transverse momentum dependent distribution amplitude

$$\varphi(x, z_\perp) = \int \Psi(x, k_\perp)\, e^{i(k_\perp z_\perp)}\, d^2k_\perp = \int_0^\infty d\sigma \Phi(x, \sigma)\; e^{i\sigma(z_\perp^2 + i\epsilon)/4}$$

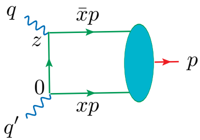
- TMDA can be written in terms of VDA (valid "always")

$$\Psi(x, k_\perp) = \frac{i}{\pi} \int_0^\infty \frac{d\sigma}{\sigma}\, \Phi(x, \sigma)\; e^{-i(k_\perp^2 - i\epsilon)/\sigma}$$

- Relation for moments (valid for soft functions)

$$\int \Psi(x, k_\perp)\, k_\perp^{2n}\, d^2k_\perp = \frac{n!}{i^n} \int_0^\infty \sigma^n\, \Phi(x, \sigma)\, d\sigma$$

Handbag diagram in VDA representation



- Starting expression

$$T(p,q) = \int_0^1 dx \int d^4z \, e^{i(q'z) - ix(pz)} \, D^c(z) \; B(x, z^2/4)$$

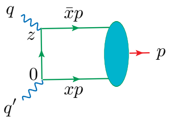
- Using VDA representation

$$T(Q^2) = \int_0^1 \frac{dx}{xQ^2} \int_0^\infty d\sigma \, \Phi(x,\sigma) \left\{ 1 - e^{-[ixQ^2+\epsilon]/\sigma} \right\}$$

- First term: twist-2 approximation
- Integral of VDA over σ may be written as integral of TMDA over $k_\perp$:

$$T(Q^2) = \int_0^1 \frac{dx}{xQ^2} \int_{k_\perp^2 \le xQ^2} \Psi(x, k_\perp) \, d^2k_\perp$$

Spin-1/2 quarks



- Handbag contribution

$$\int d^4z\, e^{-i(qz)} \langle p|\bar{\psi}(0)\gamma^{\nu}\, S^c(-z)\, \gamma^{\mu}\, \psi(z)|0\rangle = i\epsilon^{\mu\nu\alpha\beta} p_{\alpha} q_{\beta} F(Q^2)$$

- Antisymmetric part of $\gamma^{\nu} \not{z}\, \gamma^{\mu}$ is $iz_{\beta}\epsilon^{\mu\nu\alpha\beta}\gamma_5\gamma_{\alpha}$, and we need

$$\langle p|\bar{\psi}(0)\, \gamma_5\gamma_{\alpha}\, \psi(z)|0\rangle = ip_{\alpha} \int_0^{\infty} d\sigma \int_0^1 dx\, \Phi(x,\sigma)\; e^{i\bar{x}(pz) - i\sigma(z^2 - i\epsilon)/4}$$

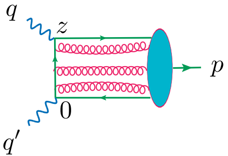
- Result in terms of VDA (based on $S^c(-z) \sim \not{z}/(z^2)^2$

$$F(Q^2) = \int_0^{\infty} d\sigma \int_0^1 \Phi(x,\sigma) \frac{dx}{xQ^2} \left\{ 1 + \frac{i\sigma}{xQ^2} \left[1 - e^{-[ixQ^2+\epsilon]/\sigma} \right] \right\}$$

- Result in terms of TMDA

$$F(Q^2) = \int_0^1 \frac{dx}{xQ^2} \int_0^{xQ^2} \frac{dk_{\perp}^2}{xQ^2} \int_{k'^2_{\perp} \le k_{\perp}^2} \Psi(x, k'_{\perp})\, d^2k'_{\perp}$$

Gauge theories



- In a covariant gauge handbag should be complemented by $\bar{\psi}(0)\ldots \not{A}(z_i)\ldots\psi(z)$ insertions of twist-0 gluonic field $A_{\mu_i}(z_i)$
- Can be organized into path-ordered exponential of zero-twist field A^μ

$$E(0,z;A) \equiv P\exp\left[ig\, z_\mu \int_0^1 dt\, A^\mu(tz)\right]$$

- and insertions of non-zero twist gluon field

$$\mathfrak{A}^\mu(z) = z_\nu \int_0^1 G^{\mu\nu}(sz)\, s\, ds\ ,$$

- which is the vector potential in the Fock-Schwinger gauge

VDA representation in gauge theories

- Two-body $\bar{q}q$ Fock component is given by gauge-invariant bilocal operator

$$\mathcal{O}^{\alpha}(0,z;A) \equiv \bar{\psi}(0)\,\gamma_5\gamma^{\alpha}\,E(0,z;A)\psi(z)$$

- Taylor expansion involves covariant derivatives $D^{\mu} = \partial^{\mu} - igA^{\mu}$

$$E(0,z;A)\,\psi(z) = \sum_{n=0}^{\infty} \frac{1}{n!}\,z_{\mu_1}\dots z_{\mu_n}\;D^{\mu_1}\dots D^{\mu_n}\psi(0)$$

- We can introduce VDA parametrization

$$\langle p|\,\mathcal{O}^{\alpha}(0,z;A)|0\rangle = ip^{\alpha}\int_0^{\infty} d\sigma \int_0^1 dx\,\Phi(x,\sigma)\;e^{i\bar{x}(pz)-i\sigma(z^2-i\epsilon)/4}$$

- and proceed as in non-gauge case

Modeling TMDAs

- Generic VDA representation treats (pz) and z^2 as independent variables

$$\langle p|\phi(0)\phi(z)|0\rangle \equiv F((pz), z^2) = \int_0^\infty d\sigma \int_0^1 dx\, \Phi(x,\sigma)\; e^{i\bar{x}(pz) - i\sigma(z^2 - i\epsilon)/4}$$

- Lorentz invariance is fully incorporated already
  $\Rightarrow$ no *a priori* correlation of x and σ dependence in VDA is expected
- Simplest example: factorized models for VDA

$$\Phi(x,\sigma) = \varphi(x)\,\Phi(\sigma)$$

- Factorized models for TMDA

$$\Psi(x, k_\perp) = \varphi(x)\,\psi(k_\perp^2)/\pi$$

Modeling soft TMDAs

- Gaussian dependence on $k_\perp$

$$\Psi_G(x,k_\perp) = \frac{\varphi(x)}{\pi\Lambda^2} e^{-k_\perp^2/\Lambda^2}$$

- Impact parameter Gaussian DA

$$\varphi_G(x,z_\perp) = \varphi(x)\, e^{-z_\perp^2\Lambda^2/4}$$

- Faster fall-off at large $z_\perp$ compared to $\sim e^{-|z_\perp|m}$ of massive propagator

$$D^c(z,m) = \frac{1}{16\pi^2}\int_0^\infty e^{-i\sigma z^2/4 - i(m^2-i\epsilon)/\sigma} d\sigma$$

- But we need $\langle p|\phi(0)\phi(z)|0\rangle$ finite at $z^2=0$
- Add a constant term $(-4/\Lambda^2)$ to z^2 in the VDA representation, i.e. take

$$\Phi_m(x,\sigma;\Lambda) = \varphi(x)\frac{e^{i\sigma/\Lambda^2 - im^2/\sigma - \epsilon\sigma}}{2im\Lambda K_1(2m/\Lambda)}\ ;\ \Psi_m(x,k_\perp) = \varphi(x)\frac{K_0\left(2\sqrt{k_\perp^2+m^2}/\Lambda\right)}{\pi m\Lambda K_1(2m/\Lambda)}$$

- Concentrating on finite-size effects: take $m=0$ model

$$\Phi_{m=0}(x,\sigma;\Lambda) = \varphi(x)\frac{e^{i\sigma/\Lambda^2-\epsilon\sigma}}{i\Lambda^2}\ ;\ \varphi_{m=0}(x,z_\perp) = \frac{\varphi(x)}{1+z_\perp^2\Lambda^2/4}$$

Modeling transition form factor

- Gaussian model

$$F_G(Q^2) = \int_0^1 \frac{dx}{xQ^2}\, \varphi(x) \left[1 - \frac{\Lambda^2}{xQ^2}\left(1 - e^{-xQ^2/\Lambda^2}\right)\right]$$

- Power-like (under x-integral) twist-4 contribution
- Formal $Q^2 \to 0$ limit is finite:

$$F_G(Q^2=0) = \frac{f_\pi}{2\Lambda^2} \quad ; \quad f_\pi \equiv \int_0^1 \varphi(x)\, dx$$

- Note: $F(Q^2)$ is finite for $Q^2 = 0$ in any model with finite $\Psi(x, k_\perp = 0)$

$$F(Q^2=0) = \frac{\pi}{2} \int_0^1 \Psi(x, k_\perp = 0)\, dx$$

- Non-Gaussian $m = 0$ model

$$F(Q^2) = \int_0^1 \frac{dx}{xQ^2}\, \varphi(x) \left[1 - \frac{\Lambda^2}{xQ^2} + 2K_2(2\sqrt{x}Q/\Lambda)\right]$$

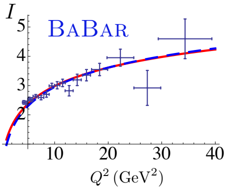
- Size of twist-4 term is governed by confinement scale Λ

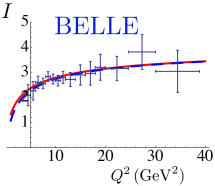
Comparison with data

- In leading-order perturbative QCD

$$F^{\mathrm{LOpQCD}}(Q^2) = \int_0^1 \frac{dx}{xQ^2}\,\varphi(x) \equiv I(Q^2) f_\pi / Q^2$$

- For DAs $\varphi_r(x) \sim (x\bar{x})^r$, one has $I_r^{\mathrm{LOpQCD}}(Q^2) = 1 + 2/r$
- $I^{\mathrm{as}}(Q^2) = 3$ for "asymptotic" wave function $\varphi^{\mathrm{as}}(x) = 6 f_\pi x\bar{x}$
- Recent experimental data from BaBar and Belle do not show flattening yet

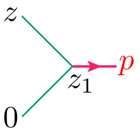




- Curves for BaBar data with flat DA $\varphi(x) = f_\pi$ and $\Lambda_G^2 = 0.35\,\mathrm{GeV}^2$ or $\Lambda_{m=0}^2 = 0.6\,\mathrm{GeV}^2$
- Curves for Belle data with $\varphi(x) \sim f_\pi (x\bar{x})^{0.4}$ and $\Lambda_G^2 = 0.3\,\mathrm{GeV}^2$ or $\Lambda_{m=0}^2 = 0.4\,\mathrm{GeV}^2$

Modeling hard tail, scalar case

- Quarks generated from local current

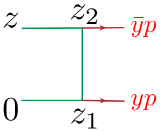


$$\Phi^{\text{point}}(x,\sigma) = \frac{1}{\sigma}\, e^{i(x\bar{x}p^2 - m^2)/\sigma}$$

$$\Psi^{\text{point}}(x,k_\perp) = \frac{1}{\pi}\,\frac{1}{k_\perp^2 + m^2 - x\bar{x}p^2}$$

$$\varphi^{\text{point}}(x,z_\perp) = 2K_0(z_\perp\sqrt{m^2 - x\bar{x}p^2})$$

- logarithmic singularity for $z_\perp = 0$
- Hard exchange model



$$\Phi^{\text{exch}}(x,\sigma;y) = ig^2\,\frac{e^{i(x\bar{x}p^2 - m^2)/\sigma}}{16\pi^2\,\sigma^2}$$

$$\times \int_0^{\min\left\{\frac{x}{y},\,\frac{\bar{x}}{\bar{y}}\right\}} e^{-iy\bar{y}\beta\, p^2/\sigma}\, d\beta$$

- For $p^2 = 0$, β-integral gives part of ERBL evolution kernel

$$V(x,y) = \frac{x}{y}\,\theta(x<y) + \frac{\bar{x}}{\bar{y}}\,\theta(x>y)$$

- TMDA generated in $p^2 = 0$ limit (using $\alpha_g \equiv g^2/16\pi^2$)

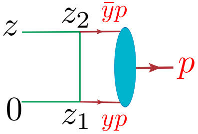
$$\Psi^{\text{exch}}(x,k_\perp;y) = \frac{\alpha_g}{\pi}\,\frac{V(x,y)}{(k_\perp^2 + m^2)^2}$$

Convolution model, scalar case

- Superpose $yp, \bar{y}p$ states the weight $\varphi_0(y)$ = "primordial" DA
- TMDA is given by a convolution

$$\Psi^{\rm conv}(x,k_\perp) = \frac{\alpha_g}{\pi}\,\frac{1}{(k_\perp^2+m^2)^2}\int_0^1 V(x,y)\,\varphi_0(y)\,dy$$

- Use "primordial" soft TMDA $\Psi_0(y,k_\perp) \equiv \psi_0(x,k_\perp^2)/\pi$ (and $m=0$)



$$\Psi^{B_0}(x,k_\perp) = \frac{\alpha_g}{\pi k_\perp^2}\int_0^1 dy \times \left[\int_0^1 d\xi\,\psi_0\left(y,\frac{\xi k_\perp^2}{V(x,y)}\right)\right]$$

- Term in square brackets may be written as

$$\left[\cdots\right] = \frac{V(x,y)}{k_\perp^2}\left\{\varphi_0(y) - \int_{k_\perp^2/V(x,y)}^{\infty}\psi_0(y,{k'_\perp}^2)\,d{k'_\perp}^2\right\}$$

- For large $k_\perp$, leading $1/k_\perp^4$ term is determined by DA $\varphi_0(y)$ only

Convolution model, spinor quarks

- For spin-1/2 quarks interacting via (pseudo)scalar gluon field $\Rightarrow$ extra $k_\perp^2$ factor from numerator trace

$$\Psi^{B_0}(x, k_\perp) = \frac{\alpha_g}{\pi} \int_0^1 dy \int_0^1 d\xi \, \psi_0 \left(y, \frac{\xi k_\perp^2}{V(x,y)} \right)$$
$$= \frac{\alpha_g}{\pi} \frac{1}{k_\perp^2} [V \otimes \varphi_0](x) + \ldots$$

- $k_\perp \to 0$ limit is finite

$$\Psi_Y^{B_0}(x, k_\perp = 0) = \alpha_g \int_0^1 dy \, \Psi_0 \left(y, k_\perp = 0 \right)$$

- Using Gaussian model for B_0

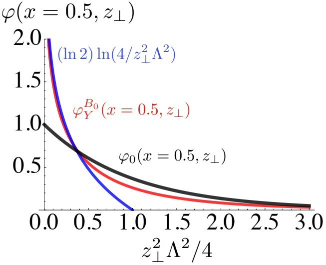
$$\Psi_Y^{B_0,G}(x, k_\perp = 0) = \alpha_g \frac{f_\pi}{\Lambda^2}$$

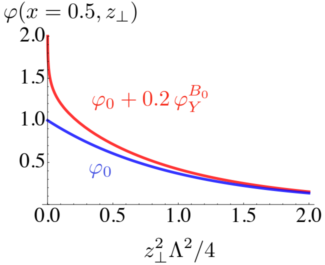
Evolution in impact parameter space

- In impact parameter space:

$$\varphi_Y^{B_0}(x, z_\perp^2) = \alpha_g \int_0^1 dy\, V(x,y) \int_1^\infty \frac{d\nu}{\nu}\, \varphi_0\Big(y, \nu\, z_\perp^2\, V(x,y)\Big)$$

- Integral over ν is cut at $\nu \sim 4/z_\perp^2 \Lambda^2 \Rightarrow \ln(z_\perp^2 \Lambda^2/4)$
- We can keep hard quarks massless
- Illustration for Gaussian model with flat DA $\varphi_0(x, z_\perp) = \exp[-z_\perp^2 \Lambda^2/4]$



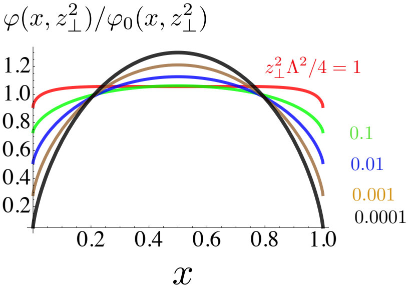


Adding UV divergent correction

- Adding self-energy part (with $\mu = \Lambda/2$ and Bessel form for log singularity)

$$\delta\varphi_Y(x, z_\perp^2) = \alpha_g \left[\int_0^1 dy\, V(x,y) \int_1^\infty \frac{d\nu}{\nu}\, \varphi_0\Big(y, \nu\, z_\perp^2\, V(x,y)\Big) - K_0(z_\perp \Lambda/2)\, \varphi_0(x, z_\perp^2) \right]$$

- Total IDA $\varphi(x, z_\perp^2) = \varphi_0(x, z_\perp^2) + \delta\varphi_Y(x, z_\perp^2)$
- Illustration for Gaussian model with flat DA $\varphi_0(x) = 1$ and $\alpha_g = 0.2$



Summary

- Outlined a new approach to transverse momentum dependence
- Introduced virtuality distribution $\Phi(x, \sigma)$
- Introduced transverse momentum distribution $\Psi(x, k_\perp)$ and wrote it in terms of $\Phi(x, \sigma)$
- Results of covariant calculations in terms of $\Phi(x, \sigma)$ converted into expressions involving $\Psi(x, k_\perp)$
- Proposed simple models for soft VDAs/TMDAs, and used them for comparison with experimental data on the pion transition form factor
- Described generation of hard tails from soft primordial TMDAs for scalar gluons

Outlook

- Future directions: building hard tail for QCD case
- Extension of VDA approach onto inclusive reactions, such as Drell-Yan and SIDIS processes
- Building VDA-based models for soft parts of TMDs that would have a non-Gaussian behavior at large $k_\perp$
- Generating hard tails from these soft TMDs