Single Spin Asymmetry in Electroproduction of J/ψ and QCD-evolved TMD's

Anuradha Misra

University of Mumbai

May 16, 2014

- Based on
  Rohini Godbole, Abiram Kaushik, AM and Vaibhav Rawoot, arXiv:1405.3560

- Earlier work
  Rohini Godbole, Asmita Mukherjee, AM and Vaibhav Rawoot, Phys. Rev. D **85**, 094013(2012)
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Heavy Quarkonium production

Heavy Quarkonium systems provide a unique lab for studying the interplay between perturbative and non-perturbative physics

Important to understand the production mechanism

Models for J/ψ Production

- Color Singlet Model (CSM)
- Color Evaporation Model (CEM)
- NRQCD factorization approach

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- $Q\bar{Q}$ pair is formed in the short-distance process in color-singlet state and has the same spin and angular momentum quantum numbers as the quarkonium
- Amplitude to create Quarkonium is product of amplitude to create the corresponding heavy quark pair, a spin projector and the radial wave function at the origin obtained from leptonic width
- Recent studies show the NLO and NNLO corrections to CSM improve the fits at TEVATRON and RHIC
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- The cross-section for a quarkonium state H is some fraction F_H of the cross-section for producing $Q\bar{Q}$ pair with invariant mass below the $M\bar{M}$ threshold
  where M is the lowest mass meson containing the heavy quark Q

$$\sigma_{CEM}[h_A h_B \to H + X] = F_H \sum_{i,j} \int_{4m^2}^{4m_M^2} d\hat{s} \int dx_1 dx_2 f_i(x_1,\mu) f_j(x_2,\mu) \times \hat{\sigma}_{ij}(\hat{s}) \delta(\hat{s} - x_1 x_2 s)$$

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- effective theory based on a systematic expansion in both α_s and v, which is heavy quark velocity within the bound state

$$\sigma[H] = \sum_n \sigma_n(\Lambda)\langle \mathcal{O}_n^H(\Lambda)\rangle$$

- σ_n are short-distance coefficients.
- $\langle \mathcal{O}_n^H(\Lambda)\rangle$ are long distance matrix elements that are formulated in terms of the effective field theory NRQCD.
- The NRQCD factorization approach to heavy-quarkonium production is by far the most sound theoretically and most successful phenomenologically. *Butenschon and Kniehl, Phys. Rev. Lett.* **106**, *022003 (2011)*

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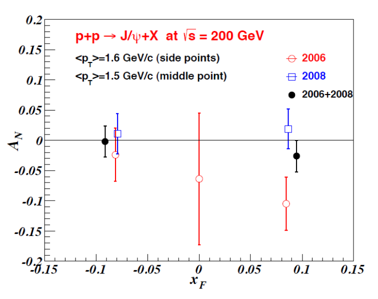
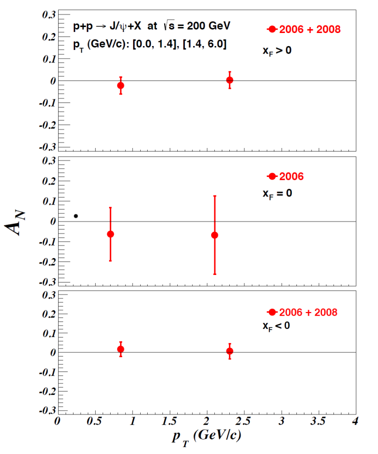
SSA at PHENIX experiment

- First measurement of transverse SSA in J/ψ production from polarized p p collisions at $\sqrt{200}$ GeV : PHENIX experiment 2006, 2008



- Modified Analysis → PHENIX experiment 2006 and 2008



No sizable asymmetry

Need to study SSAs in J/ψ production

Transverse Single Spin Asymmetry in $e + p^\uparrow \to J/\psi + X$

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- First estimate of SSA in photoproduction (i.e. low virtuality electroproduction) of charmonium in scattering of electrons off transversely polarized protons using Color Evaporation Model
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Cross section for J/ψ production using CEM

- Generalization of CEM expression for electroproduction of J/ψ by taking into account the transverse momentum dependence of the WW function and gluon distribution function

$$\sigma^{e+p^{\uparrow}\to e+J/\psi+X} = \int_{4m_c^2}^{4m_D^2} dM_{c\bar{c}}^2\, dx_{\gamma}\, dx_g\, [d^2\mathbf{k}_{\perp\gamma} d^2\mathbf{k}_{\perp g}]\, f_{g/p^{\uparrow}}(x_g, \mathbf{k}_{\perp g})\, f_{\gamma/e}(x_{\gamma}, \mathbf{k}_{\perp\gamma})\, \frac{d\hat{\sigma}^{\gamma g\to c\bar{c}}}{dM_{c\bar{c}}^2}$$

- Distribution function of the photon in the electron given by William Weizsacker approximation (Kniehl 1991)

$$f_{\gamma/e}(y,E) = \frac{\alpha}{\pi}\{\frac{1+(1-y)^2}{y}\left(ln\frac{E}{m} - \frac{1}{2}\right) + \frac{y}{2}\left[ln\left(\frac{2}{y}-2\right)+1\right] + \frac{(2-y)^2}{2y}ln\left(\frac{2-2y}{2-y}\right)\}.$$

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- We assume $k_\perp$ dependence of pdf's to be factorized in gaussian form (Anselmino etal. Eur. Phys. J. A 39, 89 (2009))

$$f(x, k_\perp) = f(x)\frac{1}{\pi\langle k_\perp^2\rangle} e^{-k_\perp^2/\langle k_\perp^2\rangle} \qquad \langle k_\perp^2\rangle = 0.25 GeV^2$$

- For the $k_\perp$ dependent WW function Gaussian form

$$f_{\gamma/e}(x_\gamma, k_{\perp\gamma}) = f_{\gamma/e}(x_\gamma)\frac{1}{\pi\langle k_{\perp\gamma}^2\rangle} e^{-k_{\perp\gamma}^2/\langle k_{\perp\gamma}^2\rangle}.$$

Single Spin Asymmetry

- Expression for the numerator of the asymmetry

$$\frac{d^4\sigma^\uparrow}{dy\, d^2\mathbf{q}_T} - \frac{d^4\sigma^\downarrow}{dy\, d^2\mathbf{q}_T} = \frac{1}{2}\int_{4m_c^2}^{4m_D^2}[dM^2]\int[dx_\gamma\, dx_g\, d^2\mathbf{k}_{\perp\gamma}\, d^2\mathbf{k}_{\perp g}]\,\Delta^N f_{g/p^\uparrow}(x_g,\mathbf{k}_{\perp g})$$
$$\times f_{\gamma/e}(x_\gamma,\mathbf{k}_{\perp\gamma})\,\delta^4(p_g + p_\gamma - q)\,\hat{\sigma}_0^{\gamma g\to c\bar{c}}(M^2).$$

where $q = p_c + p_{\bar{c}}$

Partonic cross section

$$\hat{\sigma_0}^{\gamma g\to c\bar{c}}(M^2) = \frac{1}{2}e_c^2\frac{4\pi\alpha\alpha_s}{M^2}[(1+\gamma-\frac{1}{2}\gamma^2)\ln\frac{1+\sqrt{1-\gamma}}{1-\sqrt{1-\gamma}} - (1+\gamma)\sqrt{1-\gamma}].$$

$\gamma = 4\, m_c^2/M^2$

Gluon Sivers function

Parameterization for Gluon Sivers Function

$$\Delta^N f_{g/p^\uparrow}(x, k_\perp) = 2\,\mathcal{N}_g(x)\, h(k_\perp)\, f_{g/p}(x) \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle} \mathbf{S} \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{k}}_\perp)$$

(Anselmino etal. Eur. Phys. J. A 39, 89 (2009))

Two choices for $\mathcal{N}_g(x)$ *(Boer and Vogelsang Phys. Rev. D 69, 094025 (2004))*

(a) $\mathcal{N}_g(x) = (\mathcal{N}_u(x) + \mathcal{N}_d(x))/2$,

(b) $\mathcal{N}_g(x) = \mathcal{N}_d(x)$,

For u and d quarks,

$$\mathcal{N}_f(x) = N_f x^{a_f} (1-x)^{b_f} \frac{(a_f + b_f)^{(a_f+b_f)}}{{a_f}^{a_f}\, {b_f}^{b_f}}$$

a_f, b_f and N_f are best fit parameters.

Models for Gluon Sivers function

We have used two models proposed by Anselmino etal.
(Anselmino etal. Eur. Phys. J. A 39, 89 (2009), Phys. Rev. D **70***, 074025 (2004))*
Parameterization for Gluon Sivers Function

$$\Delta^N f_{g/p^{\uparrow}}(x, k_\perp) = 2\,\mathcal{N}_g(x)\, h(k_\perp)\, f_{g/p}(x) \frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle} \mathbf{S}\cdot(\hat{\mathbf{p}} \times \hat{\mathbf{k}}_\perp)$$

Two functional forms for $h(k_\perp)$

Model I

$$h(k_\perp) = \sqrt{2e}\,\frac{k_\perp}{M_1}\, e^{-{k_\perp}^2/M_1^2}$$

Model II

$$h(k_\perp) = \frac{2k_\perp M_0}{{k_\perp}^2 + M_0^2}$$

$M_0 = \sqrt{\langle k_\perp^2\rangle}$ and M_1 are best fit parameters.

The transverse momenta q_T and $k_\perp$ have azimuthal angles ϕ_q and $\phi_{k_\perp}$

$$\mathbf{q}_T = q_T(\cos\phi_q, \sin\phi_q, 0) \qquad \mathbf{k}_\perp = k_\perp(\cos\phi_{k_\perp}, \sin\phi_{k_\perp}, 0)$$

$\phi_{k_\perp}$ is the angle that transverse momentum of the parton $k_\perp$ makes with x axis.

The mixed product $\mathbf{S} \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{k}}_\perp)$ gives an azimuthal dependence

$$\mathbf{S} \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{k}}_\perp) = \cos\phi_{k_\perp}$$

Expression for Asymmetry

Taking $sin(\phi_q - \phi_S)$ as a weight, the asymmetry integrated over the azimuthal angle of J/ψ is
(*J. C. Collins et al., Phys. Rev. D 73, 094023 (2006).*)

$$A_N^{sin(\phi_q - \phi_S)} = \frac{\int d\phi_q [d\sigma^{\uparrow} - d\sigma^{\downarrow}] sin(\phi_q - \phi_S)}{\int d\phi_q [d\sigma^{\uparrow} + d\sigma^{\downarrow}]}$$

$$A_N = \frac{\int d\phi_q [\int_{4m_c^2}^{4m_D^2} [dM^2] \int [d^2\mathbf{k}_{\perp g}] \Delta^N f_{g/p^{\uparrow}}(x_g, \mathbf{k}_{\perp g}) f_{\gamma/e}(x_{\gamma}, \mathbf{q}_T - \mathbf{k}_{\perp g}) \hat{\sigma}_0] sin(\phi_q - \phi_S)}{2 \int d\phi_q [\int_{4m_c^2}^{4m_D^2} [dM^2] \int [d^2\mathbf{k}_{\perp g}] f_{g/P}(x_g, \mathbf{k}_{\perp g}) f_{\gamma/e}(x_{\gamma}, \mathbf{q}_T - \mathbf{k}_{\perp g}) \hat{\sigma}_0]}$$

where

$$d\sigma = \frac{d^3\sigma}{dy\, d^2\mathbf{q}_T}, \qquad x_{g,\gamma} = \frac{M}{\sqrt{s}}\, e^{\pm y}$$

QCD evolution of TMDs

- DGLAP equations describe the evolution of densities as function of Q^2 at given energy scale or rapidity.
- How does one evolve the TMDs?
- Early phenomenological fits of Sivers function were performed either neglecting QCD evolution or applying DGLAP evolution only to the collinear part of TMD parametrization

$$f_{g/p}(x, k_\perp; Q) = f_{g/p}(x; Q) \frac{1}{\pi \langle k_\perp^2 \rangle} e^{-k_\perp^2/\langle k_\perp^2 \rangle} \qquad \langle k_\perp^2 \rangle = 0.25\, GeV^2$$

$$\Delta^N f_{g/p^\uparrow}(x, k_\perp; Q) = 2\mathcal{N}_g(x) f_{g/p}(x; Q) \sqrt{2e} \frac{k_\perp}{M_1} \frac{1}{\pi \langle k_\perp^2 \rangle} e^{-k_\perp^2/\langle k_\perp^2 \rangle_S}$$

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Q^2 Evolution of TMD's

- The energy evolution of a general transverse momentum dependent distribution(TMD) $F(x, k_\perp, Q)$ is more naturally described in b-space.

$$F(x, b; Q) = \int d^2k_\perp e^{-ik_\perp.b} F(x, k_\perp; Q)$$

- $$F(x, b, Q_f) = F(x, b, Q_i) R_{pert}(Q_f, Q_i, b_*) R_{NP}(Q_f, Q_i, b)$$

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The perturbative part is given by

$$R(Q_f, Q_i, b) = \exp\left\{ -\int_{Q_i}^{Q_f} \frac{d\mu}{\mu}\left(A \ln \frac{Q_f^2}{\mu^2} + B \right) \right\} \left(\frac{Q_f^2}{Q_i^2} \right)^{-D(b;Q_i)}$$

where $\frac{dD}{d\ln\mu} = \Gamma_{\text{cusp}}$

The non-perturbative exponential part contains a Q-dependent factor universal to all TMDs and a factor which gives the gaussian width in b-space of the particular TMD

$$R_{NP} = \exp\left\{ -b^2 \left(g_1^{\text{TMD}} + \frac{g_2}{2} \ln \frac{Q_f}{Q_i} \right) \right\}$$

Q^2-dependent TMD's in momentum space obtained by Fourier transforming $F(x, b, Q_f)$.

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TMD Evolution of Sivers Function

- TMD evolved Sivers function is

$$\Delta^N \widehat{f}_{q/p^\uparrow}(x, k_\perp; Q) = \frac{k_\perp}{M_1}\sqrt{2e}\,\frac{\langle k_S^2\rangle^2}{\langle k_\perp^2\rangle}\,\Delta^N f_{q/p^\uparrow}(x, Q_0)\,R(Q, Q_0)\,\frac{e^{-k_\perp^2/w_S^2}}{\pi w_S^4},$$

- Width of the Gaussian function evolves as

$$w_S^2(Q, Q_0) = \langle k_S^2\rangle + 2g_2 \ln\frac{Q}{Q_0} \ .$$

$$\frac{1}{\langle k_S^2\rangle} = \frac{1}{M_1^2} + \frac{1}{\langle k_{\perp g}^2\rangle} \ .$$

$\langle k_{\perp g}^2\rangle = 0.25\ GeV^2$ $\quad g_2 = 0.68$ $\quad b_{max} = 0.5\ GeV^{-1}$

Exact Solution of TMD Evolution Equations

- Exact solution can be obtained numerically: Anselmino 2012
- TMD evolution of TMDs is driven by

$$R(Q, Q_0, b) \equiv \exp\left\{ \ln \frac{Q}{Q_0} \int_{Q_0}^{\mu_b} \frac{d\mu'}{\mu'} A(\mu') + \int_{Q_0}^{Q} \frac{d\mu}{\mu} B\left(\mu, \frac{Q^2}{\mu^2}\right) \right\} .$$

where b is the parton impact parameter,

$$b_*(b) \equiv \frac{b}{\sqrt{1 + b^2/b_{\rm max}^2}}, \qquad \mu_b = \frac{C_1}{b_*(b)} .$$

with $C_1 = 2e^{-\gamma_E}$ where $\gamma_E = -0.577$, $b_* \to b_{max}$

- A and B are anomalous dimensions, which are given at $O(\alpha_s)$ by

$$B(\mu; \frac{Q^2}{\mu^2}) = \alpha_s(\mu) \frac{C_F}{\pi} \left(\frac{3}{2} - \ln \frac{Q^2}{\mu^2} \right)$$

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Parameter sets for analytical and exact formalisms

Parameters used to estimate the asymmetry using the formulation provided by Anselmino *et al.*

TMD-e1 : extracted at $Q_0 = 1.0$ GeV for the exact solution of TMD evolution equations

$$N_u = 0.77,\ a_u = .68,\ b_u = 3.1\ ,$$
$$N_d = -1.00,\ a_d = 1.11,\ b_d = 3.1\ ,$$
$$M_1^2 = 0.40\ \text{GeV}^2,\ \langle k_\perp^2 \rangle = 0.25\ \text{GeV}^2\ ,$$
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TMD-a : Parameters fitted to analytical solution

$$N_u = 0.75,\ a_u = .82,\ b_u = 4.0\ ,$$
$$N_d = -1.00,\ a_d = 1.36,\ b_d = 4.0\ ,$$
$$M_1^2 = 0..34\ \text{GeV}^2,\ \langle k_\perp^2 \rangle = 0.25\ \text{GeV}^2,\ ,$$
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TMD-e1 : extracted at $Q_0 = 1.0$ GeV for the exact solution of TMD evolution equations

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$$N_d = -1.00,\ a_d = 1.11,\ b_d = 3.1\ ,$$
$$M_1^2 = 0.40\ \text{GeV}^2,\ \langle k_\perp^2 \rangle = 0.25\ \text{GeV}^2\ ,$$
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TMD-a : Parameters fitted to analytical solution

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CSS Evolution at NLL

Echevarria, Idilbi, Kang Vitev, Phys.Rev. D89 (2014) 074013

The perturbative part is given by

$$R(Q_f, Q_i, b) = \exp\left\{ -\int_{Q_i}^{Q_f} \frac{d\mu}{\mu} \left(A \ln \frac{Q_f^2}{\mu^2} + B \right) \right\} \left(\frac{Q_f^2}{Q_i^2} \right)^{-D(b;Q_i)}$$

A=Γ_{cusp} , B=γ^V, $\frac{dD}{d\ln\mu} = \Gamma_{\text{cusp}}$

$$f_{q/H}(x, b; Q_f) = f_{q/H}(x, Q_i) \exp\left\{ -\int_{Q_i}^{Q_f} \frac{d\mu}{\mu} \left(A \ln \frac{Q^2}{\mu^2} + B \right) \right\} \left(\frac{Q_f^2}{Q_i^2} \right)^{-D(b^*;Q_i)}$$

$$\times \exp\left\{ -b^2 \left(g_1^{\text{pdf}} + \frac{g_2}{2} \ln \frac{Q_f}{Q_i} \right) \right\}$$

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Sivers function

$$f_{1T}^{\prime\perp g}(x_g, b; Q_f) = -2g_1^{\text{sivers}} f_{1T}^{\perp g}(x_g; Q_i)\, b \exp\left\{-\int_{Q_i}^{Q_f} \frac{d\mu}{\mu}\left(A \ln\frac{Q^2}{\mu^2} + B\right)\right\}$$

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The expansion coefficients with the appropriate gluon anomalous dimensions at NLL are

$$A^{(1)} = C_A$$

$$\mathrm{A}^{(2)} = \frac{1}{2} C_F \left(C_A\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{5}{9} C_A N_f\right)$$

$$\mathrm{B}^{(1)} = -\frac{1}{2}\left(\frac{11}{3} C_A - \frac{2}{3} N_f\right)$$

$$\mathrm{D}^{(1)} = \frac{C_A}{2} \ln\frac{Q_i^2 b^2}{c^2}$$

Choosing the initial scale $Q_i = c/b$, the D term vanishes at NLL.

Most recent parameters: Echevarria, Idilbi, Kang Vitev PRD89(2014)

Obtained by performing a global fit of all experimental data on Sivers asymmetry in SIDIS from HERMES, COMPASS and JLAb

We call this set TMD-e2

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\begin{aligned}
N_u &= 0.106,\ a_u = 1.051,\ b_u = 4.857\ ,\\
N_d &= -0.163,\ a_d = 1.552,\ b_d = 4.857\ ,\\
\langle k_{s\perp}^2\rangle &= 0.282\ \text{GeV}^2,\ \langle k_{\perp}^2\rangle = 0.38\ \text{GeV}^2\ ,\\
g_2 &= 0.16\ \text{GeV}^2,\ b_{max} = 1.5 GeV^{-1}
\end{aligned}
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This set was fitted at $Q_0 = \sqrt{2.4}$ GeV.

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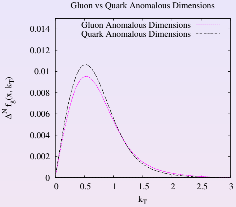
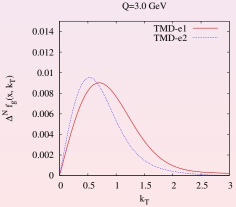
Gluon Sivers functions in the TMD-e1 at $Q = 3.0$ obtained using gluon and quark anomalous dimensions respectively, in the evolution kernel.

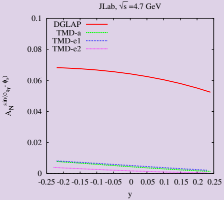


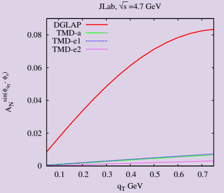
Gluon Sivers functions at Q=3.0 obtained using two different fits- TMD-e1 and TMD-e2.



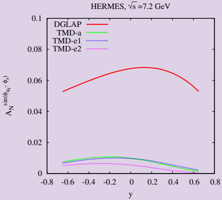
JLab and HERMES with parameterizations TMD Exact-1, TMD Exact -2 and TMD a

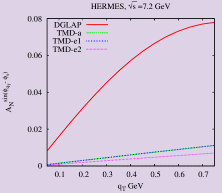
Jlab, $\sqrt{s}$=4.7 GeV





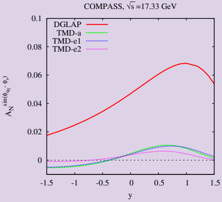
HERMES, $\sqrt{s}$=7.2 GeV

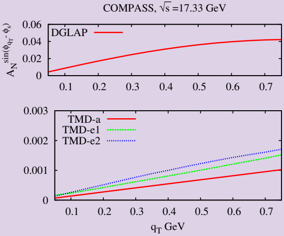




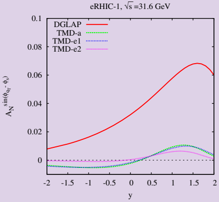
COMPASS and eRHIC I with parameterizations TMD Exact-1, TMD Exact -2 and TMD a

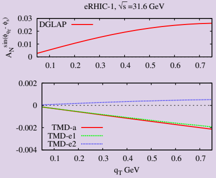
COMPASS, $\sqrt{s}$=17.33 GeV





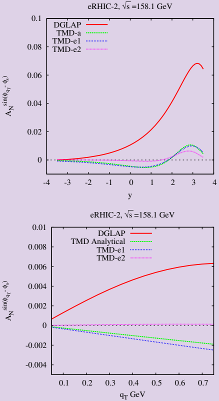
eRHIC, $\sqrt{s}$=31.6 GeV



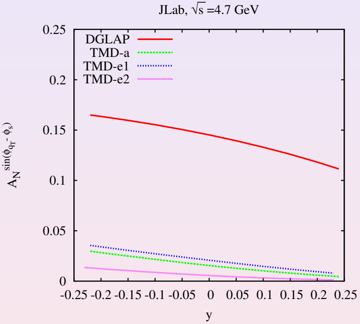


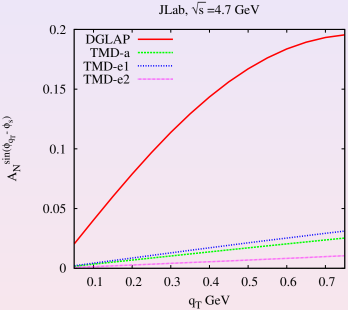
eRHIC II with parameterizations TMD Exact-1, TMD Exact -2 and TMD a

eRHIC, $\sqrt{s}$=158.1 GeV



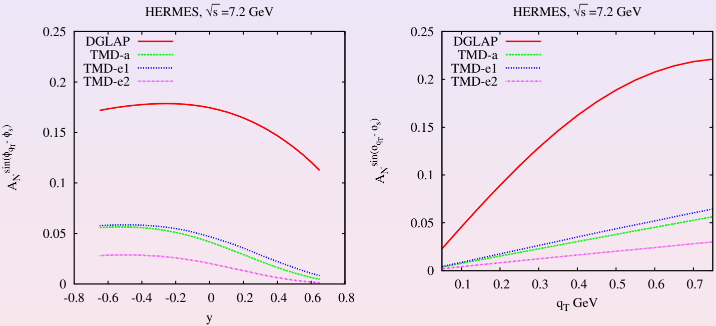
Asymmetries with parametrization (b) : JLab ($\sqrt{s} = 4.7$ GeV)





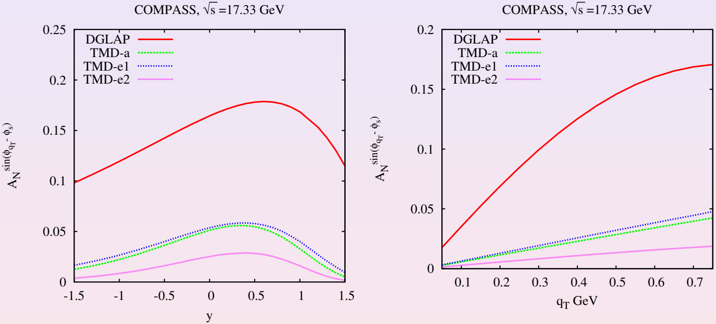
The integration ranges are $(0 \leq q_T \leq 1)$ GeV and $(-0.25 \leq y \leq 0.25)$

Asymmetries with parametrization (b) :HERMES energy ($\sqrt{s} = 7.2$ GeV)



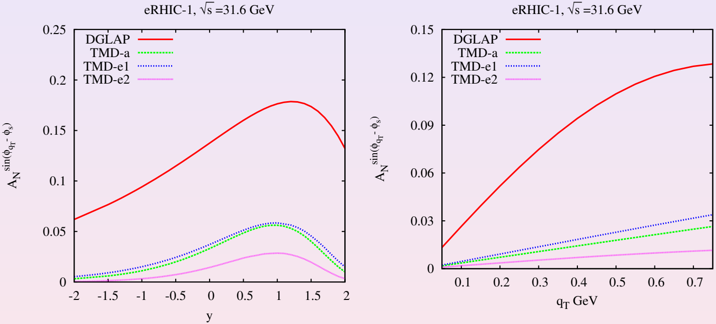
The integration ranges are $(0 \leq q_T \leq 1)$ GeV and $(-0.6 \leq y \leq 0.6)$

Asymmetries with parametrization (b) :COMPASS energy ($\sqrt{s} = 17.33$ GeV)



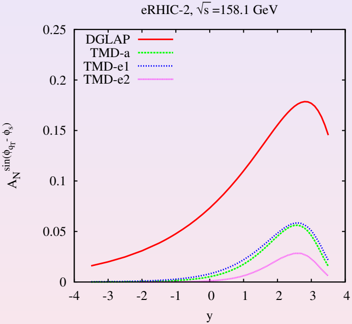
The integration ranges are $(0 \leq q_T \leq 1)$ GeV and $(-1.5 \leq y \leq 1.5)$

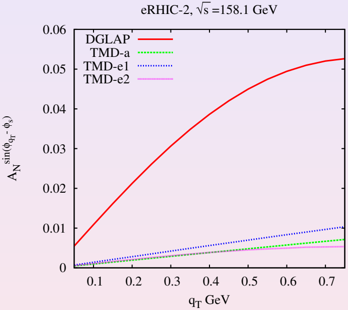
Asymmetries with parametrization (b) :eRHIC energy ($\sqrt{s} = 31.6$ GeV)



The integration ranges are $(0 \leq q_T \leq 1)$ GeV and $(-2.1 \leq y \leq 2.1)$

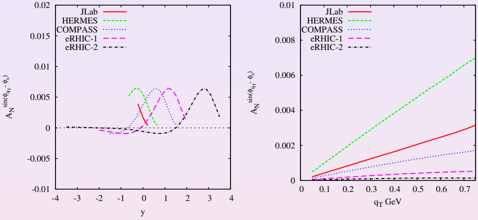
Asymmetries with parametrization (b) :eRHIC energy ($\sqrt{s} = 158.1$ GeV)





The integration ranges are $(0 \leq q_T \leq 1)$ GeV and $(-3.7 \leq y \leq 3.7)$

y and q_T distribution for all c.o.m energies using the TMD-e2 fit and parametrization (a) of the gluon Sivers function.



- drift of the asymmetry peak towards higher values of rapidity
- general decrease of asymmetry values with increasing c.o.m energy

Summary

- Transverse SSA in electroprduction of J/ψ using Color Evaporation Model for J/ψ production is estimated.
- Sizable asymmetry is predicted at energies of JLab, HERMES, COMPASS and eRHIC experiments when TMD's are evolved using DGAP evolution
- Substantial reduction in asymmetry when TMD evolution is taken into account
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