Joint resummation for pion wave function and pion transition form factor

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Talk based on:
Hsiang-nan Li, Yue-Long Shen, YMW, JHEP 01 (2014) 004.

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What is pion wave function?

- TMD wave function in k_T factorization (Collins, 2003):

$$\Phi(x, k_T, \zeta^2, \mu_f) = \int \frac{dy^+}{2\pi} \frac{d^2 y_T}{(2\pi)^2} e^{-ixP^- y^+ + i\mathbf{k}_T \cdot \mathbf{y}_T} \times \langle 0 | \bar{q}(y) W_y(u)^\dagger I_{u;y,0} W_0(u) \not{n}_+ \gamma_5 q(0) | \pi(P) \rangle .$$

  - Light-cone divergence regularized by the rapidity parameter $\zeta^2 = 4(n_- \cdot u)^2 / u^2$.
  - Transverse gauge link $I_{u;y,0}$ to ensure a strict gauge invariance. Does not contribute in covariant gauge, but contributes in light-cone gauge (Belitsky, Ji and Yuan, 2003).

- Light-cone distribution amplitude in collinear factorization:

$$\langle 0 | \bar{q}(0) \, [0, y] \not{y} \, \gamma_5 \, q(y) | \pi^+(p) \rangle \overset{y^2=0}{=} i f_\pi \, p \cdot y \int_0^1 dx \, e^{-ixp\cdot y} \, \phi_\pi(x, \mu) .$$

- ER-BL evolution implies expansion in Gegenbauer-polynomials:

$$\phi_\pi(x, \mu) = 6x(1-x) \sum_{n=0}^{+\infty} \left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right]^{\gamma_n^{(0)}/2\beta_0} a_n(\mu_0) \, C_n^{3/2}(2x-1) .$$

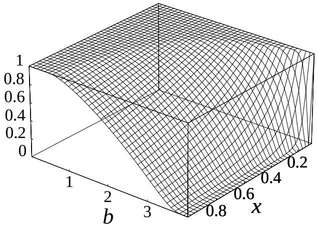
Why pion wave function?

- Key nonperturbative quantity in k_T factorization for many exclusive processes.

- NLO k_T factorization for the $\gamma^* \pi^0 \to \gamma$ form factor (Nandi and Li, 2007):
[For a different scheme, see Brodsky and Lepage (1981), and Musatov and Radyushkin (1997)]

$$F(Q^2) = \Phi \otimes H \otimes S \otimes J \,.$$

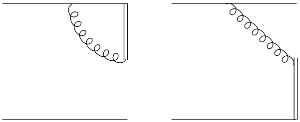
  - Soft contribution suppressed by the Sudakov mechanism (Botts and Sterman, 1989; Li and Sterman, 1992).
  - Transverse momentum dependence becomes important at the end-points.
  - Threshold resummation can suppress the end-point contribution further.



- NLO k_T factorization for the pion e.m. form factor (Li, Shen, YMW and Zou, 2011) and the $B \to \pi \ell \nu$ form factors (Li, Shen and YMW 2012).

Structure of Pion wavefunction at NLO

- Quark-Wilson-line vertex diagrams (Nandi and Li, 2007):



(d) (e)

$$\Phi_d^{(1)} \otimes H^{(0)} = \frac{\alpha_s C_F}{4\pi} \left[\frac{1}{\varepsilon} + \ln \frac{4\pi\mu_f^2}{k_T^2 e^{\gamma_E}} - \ln^2 \frac{\zeta^2}{k_T^2} + \ln \frac{\zeta^2}{k_T^2} + \ldots \right] H^{(0)},$$

$$\Phi_e^{(1)} \otimes H^{(0)} = \frac{\alpha_s C_F}{4\pi} \left[\ln^2 \frac{x\zeta^2}{k_T^2} + \ldots \right] H^{(0)}.$$

- The double rapidity logarithm $\ln^2 \zeta^2$ in the B meson case is absent here.

- Mixed logarithm $\ln x \ln(\zeta^2/k_T^2)$ in the sum $(\Phi_d^{(1)} + \Phi_e^{(1)}) \otimes H^{(0)}$.

- Unification of rapidity, k_T and threshold resummation: joint resummation (Li, 1998; Laenen, Sterman, Vogelsang, 2000, 2001).

Construction of evolution equation

- Rapidity derivative (Collins, Soper, 1981; Li, 1998):

$$\zeta^2 \frac{d}{d\zeta^2}\Phi = -\frac{u^2}{n_- \cdot u}\frac{n_-^\alpha}{2}\frac{d}{du^\alpha}\Phi .$$

Advantage: u dependence appears only through the Wilson line interactions.

- Rapidity derivative of the Feynman rule associated with the Wilson line:

$$\zeta^2 \frac{d}{d\zeta^2}\frac{u^\beta}{u \cdot l + i\varepsilon} = \frac{\hat{u}^\beta}{2u \cdot l},$$

$$\hat{u}^\beta = \frac{u^2}{n_- \cdot u}\left(\frac{n_- \cdot l}{u \cdot l}u^\beta - n_-^\beta\right).$$

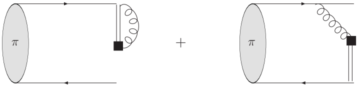
- The rapidity evolution equation:

$$\zeta^2 \frac{d}{d\zeta^2}\Phi(x, k_T, \zeta^2, \mu_f) = \Gamma(x, k_T, \zeta^2) \otimes \Phi(x, k_T, \zeta^2, \mu_f).$$

- The vertex $\hat{u}^\beta$ contracted to the vertex in Φ.
  Suppression of collinear dynamics associated with the Wilson link.
  Soft and hard gluon radiations are dominant in the kernel $\Gamma(x, k_T, \zeta^2)$.

Soft function

- Soft gluon radiations:



- The reducible diagram:

$$K_1 = -\frac{ig^2 C_F}{2} \int \frac{d^4 l}{(2\pi)^4} \frac{\hat{u}\cdot n_-}{(u\cdot l + i\varepsilon)(l^2 + i\varepsilon)(n_- \cdot l + i\varepsilon)} = -\frac{\alpha_s C_F}{4\pi} \left(\frac{1}{\varepsilon} - \gamma_E + \ln \frac{4\pi\mu^2}{\lambda^2} \right).$$

IR divergence regularized by the gluon mass λ.

- The irreducible diagram:

$$K_2 \otimes \Phi = \frac{ig^2 C_F}{2} \int \frac{d^4 l}{(2\pi)^4} \frac{\hat{u}\cdot n_-}{(u\cdot l + i\varepsilon)(l^2 + i\varepsilon)(n_- \cdot l + i\varepsilon)} \times \Phi(x - l^-/P^-, |\mathbf{k}_T - \mathbf{l}_T|, \zeta^2, \mu_f).$$

Fourier and Mellin transformations of $K_2 \otimes \Phi$:

$$\tilde{K}_2 = \frac{\alpha_s C_F}{2\pi} \left[K_0(\lambda b) - K_0\left(\frac{\zeta P^- b}{N}\right) \right] + O\left(1/\zeta^2\right).$$

- Unrenormalized soft function in the large N limit:

$$\tilde{K}^{(b)} = K_1 + \tilde{K}_2 = -\frac{\alpha_s C_F}{4\pi} \left(\frac{1}{\varepsilon} - \gamma_E + \ln \frac{4\pi\mu^2 N^2}{\zeta^2 P^{-2}} \right).$$

Cancelation of IR divergence!

Hard function

- Hard gluon radiations:

Subtraction to avoid the double counting of soft contribution.

- Analytical expressions:

$$G_1 = -\frac{ig^2 C_F}{2}\int \frac{d^4 l}{(2\pi)^4}\frac{(\bar{x}\not{P}+\not{l})\,\tilde{\not{u}}}{(u\cdot l+i\varepsilon)(l^2+i\varepsilon)[(\bar{x}P+l)^2+i\varepsilon]}\,,$$

$$G_2 = K_1\,.$$

- Unrenormalized hard function:

$$G^{(b)} = G_1 - G_2 = \frac{\alpha_s C_F}{4\pi}\left[\frac{1}{\varepsilon}-\gamma_E+\ln\frac{4\pi\mu^2}{\zeta^2(\bar{x}P^-)^2}-4\right].$$

Infrared finite due to the cancelation of soft divergence!

- Factorization scale dependence cancels between soft and hard functions.
$\Leftarrow$ μ independence of the mixed logarithm to be resummed.

RG improved evolution kernel

- Renormalized soft and hard functions:

$$\tilde{K}^{(r)}(\mu) = -\frac{\alpha_s C_F}{2\pi} \ln \frac{\mu N}{\zeta P^-}, \qquad G^{(r)}(\mu) = \frac{\alpha_s C_F}{2\pi}\left(\ln \frac{\mu}{\zeta P^-} - 2\right).$$

- RGE of soft and hard functions:

$$\mu \frac{d\tilde{K}^{(r)}}{d\mu} = -\frac{\alpha_s(\mu) C_F}{2\pi}, \qquad \mu \frac{dG^{(r)}}{d\mu} = \frac{\alpha_s(\mu) C_F}{2\pi}.$$

- RG improvement:

$$\tilde{K}^{(r)}(\mu) + G^{(r)}(\mu) = \tilde{K}^{(r)}(\mu_0) + G^{(r)}(\mu_1) - \int_{\mu_0}^{\mu_1} \frac{d\tilde{\mu}}{\tilde{\mu}} \frac{\alpha_s(\tilde{\mu}) C_F}{2\pi},$$

$$\mu_0 := \mu_0(\zeta) = \frac{\zeta P^-}{N}, \qquad \mu_1 := \mu_1(\zeta) = e^2 \zeta P^-.$$

Chosen to diminish the initial conditions $\tilde{K}^{(r)}(\mu_0)$ and $G^{(r)}(\mu_1)$.

- Gluon radiations from the spectator quark also contribute.
  Only contribute to the kernel Γ at NLL level.
  $\Rightarrow$ Do not generate the mixed logarithm $\ln x \ln(\zeta^2 P^{-2}/k_T^2)$.

Solution in Mellin and impact-parameter spaces

- Evolution equation in N and b spaces:

$$\zeta^2 \frac{d}{d\zeta^2} \tilde{\Phi}(N,b,\zeta^2,\mu_f) = \tilde{\Gamma}(N,b,\zeta^2)\ \tilde{\Phi}(N,b,\zeta^2,\mu_f)\,.$$

$$\tilde{\Gamma}(N,b,\zeta^2) = \tilde{K}^{(r)}(\mu) + G^{(r)}(\mu) = -\int_{\mu_0(\zeta)}^{\mu_1(\zeta)} \frac{d\tilde{\mu}}{\tilde{\mu}}\, \frac{\alpha_s(\tilde{\mu})\, C_F}{2\pi}\,.$$

- Solution:

$$\tilde{\Phi}(N,b,\zeta^2,\mu_f) = \exp\left\{ -\int_{\zeta_0^2}^{\zeta^2} \frac{d\tilde{\zeta}^2}{\tilde{\zeta}^2} \left[\int_{\mu_0(\tilde{\zeta})}^{\mu_1(\tilde{\zeta})} \frac{d\tilde{\mu}}{\tilde{\mu}}\, \frac{\alpha_s(\tilde{\mu})\, C_F}{2\pi} \right] \right\} \times \tilde{\Phi}(N,b,\zeta_0^2,\mu_f)\,.$$

- RGE for μ_f evolution:

$$\mu_f \frac{d}{d\mu_f} \tilde{\Phi}(N,b,\zeta^2,\mu_f) = \frac{3}{2}\, \frac{\alpha_s(\mu_f)\, C_F}{\pi}\, \tilde{\Phi}(N,b,\zeta^2,\mu_f)\,.$$

- Combined evolution:

$$\tilde{\Phi}(N,b,\zeta^2,\mu_f) = \exp\left\{ -\int_{\zeta_0^2}^{\zeta^2} \frac{d\tilde{\zeta}^2}{\tilde{\zeta}^2} \left[\int_{\mu_0(\tilde{\zeta})}^{\mu_1(\tilde{\zeta})} \frac{d\tilde{\mu}}{\tilde{\mu}}\, \frac{\alpha_s(\tilde{\mu})\, C_F}{2\pi} \right] + \frac{3}{2} \int_{\mu_i}^{\mu_f} \frac{d\tilde{\mu}}{\tilde{\mu}}\, \frac{\alpha_s(\tilde{\mu})\, C_F}{\pi} \right\}\ \tilde{\Phi}(N,b,\zeta_0^2,\mu_i)\,.$$

Evolution of the hard kernel

- Rapidity evolution:

$$\zeta^2 \frac{d}{d\zeta^2} \tilde{H}(N,b,\zeta^2,Q^2,\mu_f) = -\tilde{\Gamma}(N,b,\zeta^2)\, \tilde{H}(N,b,\zeta^2,Q^2,\mu_f)\,.$$

- Factorization scale evolution:

$$\mu_f \frac{d}{d\mu_f} \tilde{H}(N,b,\zeta^2,Q^2,\mu_f) = -\frac{3}{2}\,\frac{\alpha_s(\mu_f)\,C_F}{\pi}\, \tilde{H}(N,b,\zeta^2,Q^2,\mu_f)\,.$$

- Combined evolution:

$$\tilde{H}(N,b,\zeta^2,Q^2,\mu_f) = \exp\left\{ \int_{\zeta^2}^{\zeta_1^2} \frac{d\tilde{\zeta}^2}{\tilde{\zeta}^2} \left[\int_{\mu_0(\tilde{\zeta})}^{\mu_1(\tilde{\zeta})} \frac{d\tilde{\mu}}{\tilde{\mu}}\, \frac{\alpha_s(\tilde{\mu})\,C_F}{2\pi} \right] - \frac{3}{2} \int_t^{\mu_f} \frac{d\tilde{\mu}}{\tilde{\mu}}\, \frac{\alpha_s(\tilde{\mu})\,C_F}{\pi} \right\}\, \tilde{H}(N,b,\zeta_1^2,Q^2,t)\,.$$

  Depends on the final rapidity parameter ζ_1 and the characteristic hard scale t.

- Choices of boundary conditions:

$$\zeta_0^2 = \left(\frac{a\,N^{1/4}}{P^- b} \right)^2, \qquad \zeta_1^2 = \tilde{a}\,N^{1/2}\,.$$

  Eliminate the logarithmic enhancements in $\tilde{\Phi}(N,b,\zeta_0^2,\mu_i)$ and $\tilde{H}(N,b,\zeta_1^2,Q^2,t)$.

Joint resummation improved k_T factorization

- Factorization formula of $\gamma^* \pi^0 \to \gamma$ form factor:

$$
\begin{aligned}
F(Q^2) = \exp\left\{ - \int_{\zeta_0^2}^{\zeta_1^2} \frac{d\tilde{\zeta}^2}{\tilde{\zeta}^2} \left[\int_{\mu_0(\tilde{\zeta})}^{\mu_1(\tilde{\zeta})} \frac{d\tilde{\mu}}{\tilde{\mu}} \frac{\alpha_s(\tilde{\mu})\, C_F}{2\pi} \right] + \frac{3}{2} \int_{\mu_i}^{t} \frac{d\tilde{\mu}}{\tilde{\mu}} \frac{\alpha_s(\tilde{\mu})\, C_F}{\pi} \right\} \\
\tilde{\Phi}(N, b, \zeta_0^2, \mu_i) \otimes \tilde{H}(N, b, \zeta_1^2, Q^2, t)\,, \\
\equiv \tilde{\Phi}(N, b, \zeta_1^2, t) \otimes \tilde{H}(N, b, \zeta_1^2, Q^2, t)\,.
\end{aligned}
$$

- Have confirmed that the expansion of the exponential factor reproduces the mixed logarithm and the single logarithm $\ln(1/N)$ in the NLO pion transition form factor.

- A complete treatment of the logarithmic enhancement for an arbitrary rapidity parameter in the pion transition form factor.
  The conventional k_T factorization formula with the Sudakov and threshold resummations is not factorization-scheme independent.

Resummation improved wave functions

- Inverse Mellin transformation:

$$\overline{\Phi}(x,b,\zeta_1^2,t) = \int_{c-i\infty}^{c+i\infty} \frac{dN}{2\pi i}\,(1-x)^{-N}\,\tilde{\Phi}(N,b,\zeta_1^2,t).$$

  No Fourier transformation for the comparison of Sudakov resummation.

- Factorized model for the initial condition:

$$\Phi(x,k_T,\zeta_0^2,\mu_i) = \phi(x,\zeta_0^2,\mu_i)\,\Sigma(k_T^2)\,, \qquad \Sigma(k_T^2) = 4\pi\beta^2\exp(-\beta^2\,k_T^2)\,.$$

  Translated into the Mellin and impact-parameter spaces.
  See Radyushkin's talk for more discussion.

- Three different models of $\phi(x,\zeta_0^2,\mu_i)$:

$$\begin{aligned}
\phi^{\rm I}(x,\zeta_0^2,\mu_i) &= 6x(1-x) \Rightarrow \frac{6}{(N+1)(N+2)}\,, \\
\phi^{\rm II}(x,\zeta_0^2,\mu_i) &= 1 \Rightarrow \frac{1}{N}\,, \\
\phi^{\rm III}(x,\zeta_0^2,\mu_i) &= 6x(1-x)\left[1+a_2\,C_2^{3/2}(2x-1)\right] \\
&\Rightarrow \frac{6}{(N+1)(N+2)}\left[1+6a_2\,\frac{(N-1)(N-2)}{(N+3)(N+4)}\right].
\end{aligned}$$

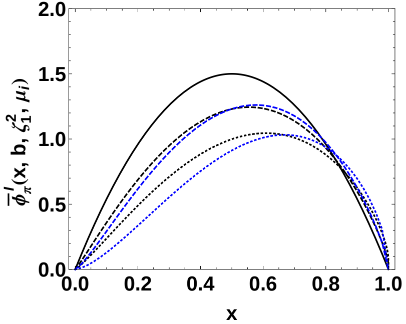
- Can include the higher-order Gegenbauer moments, but (Agaev et al, 2011; Kroll, 2011.)
  - The contributions from higher moments suppressed by the soft corrections.

Joint Resummation improved wave functions

- Inverse Mellin transformation with both frozen and running α_s.
- Analytical parametrization of α_s (Solovtsov and Shirkov, 1999):

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0}\left[\frac{1}{\ln(\mu^2/\Lambda_{\rm QCD}^2)} - \frac{\Lambda_{\rm QCD}^2}{\mu^2 - \Lambda_{\rm QCD}^2}\right].$$

- Resummation effect in pion wave function $\overline{\Phi}(x, b, \zeta_1^2, \mu_i)$:



solid: initial condition $\phi^{\rm I}(x, \zeta_0^2, \mu_i)$;

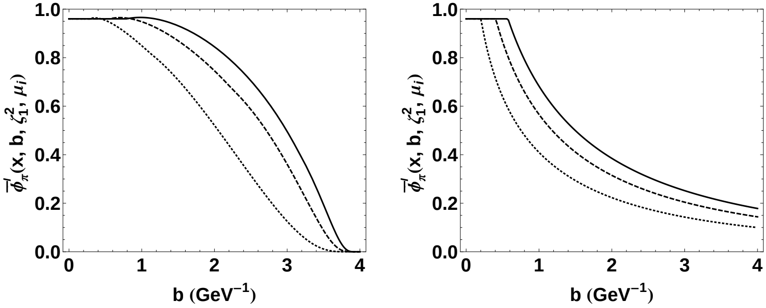
(blue) dashed: $b = \frac{2\,\tilde{a}}{aP^-}$ for a frozen $\alpha_s = 0.3$ (running α_s);

(blue) dotted: $b = \frac{4\,\tilde{a}}{aP^-}$ for a frozen $\alpha_s = 0.3$ (running α_s).

  - Stronger suppression of the small x region compared to the moderate x region.
  - The suppression strengthens with the transverse separation b at a given x.

Comparison of Joint and Sudakov resummations

- Joint vs Sudakov resummation:



Left: Sudakov resummation, solid, dashed, and dotted curves for $Q^2 = 5$ GeV^2, 10 GeV^2, and 40 GeV^2 at $x = 0.2$. Right: The same for the joint resummation.

- Large b region suppressed in both cases, but stronger with the joint resummation.

- Different phenomenological consequences with the two resummation techniques.

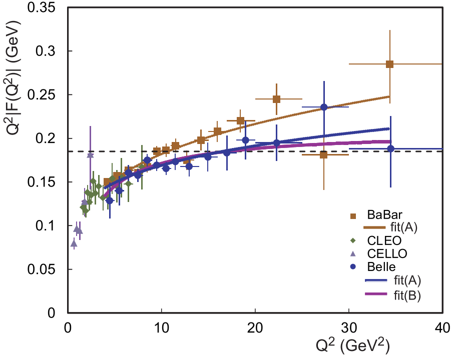
$\gamma^* \pi^0 \to \gamma$ form factor

- Transition matrix element:

$$\langle \gamma(P', \varepsilon) | j_\mu^{em}(q) | \pi^0(P) \rangle = i\, g_{em}^2\, \varepsilon_{\mu\nu\alpha\beta}\, \varepsilon^{*\nu}\, P^\alpha\, P'^\beta\, F(Q^2)\,.$$

- Different approaches to compute the pion form factor:
  - Direct approaches: Collinear (k_T) factorization formulae.
  - Indirect approaches: (Light-cone) QCD Sum rules.

- Status of experimental measurements:



Scaling violation?

Shape of pion wave function?

The onset of QCD factorization?

Some popular explanations:

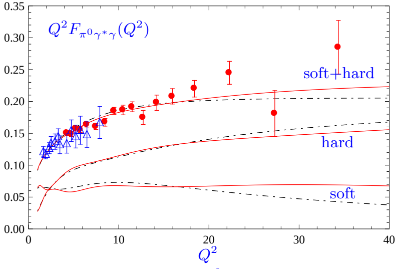
- Non-vanishing pion wave function at the end points (Radyushkin, 2009; Polyakov, 2009).

$$F(Q^2) \quad = \quad \frac{\sqrt{2}}{3}\int_0^1 \frac{\varphi_\pi(x)}{xQ^2}\left[1-\underbrace{\exp\left(-\frac{xQ^2}{2\bar{x}\sigma}\right)}\right].$$

$\Uparrow$

from k_T dependence of pion wave function

- Large soft (Feynman) corrections at moderate Q^2 (Agaev, Braun, Offen, Porkert, 2011).



The "hard" and "soft" contributions to the $\pi^0\gamma^*\gamma$ form factor for model I (solid curves) and model III (dash-dotted curves). The experimental data are from BaBar (full circles) and CLEO (open triangles).

- Threshold resummation generates power-like $[x(1-x)]^{c(Q^2)}$ distribution (Li and Mishima 2009). $c(Q^2)$ is around 1 for low Q^2, but small for high Q^2.

Factorization formula of pion form factor

- k_T factorization with the conventional resummation (Nandi and Li, 2007):

$$F(Q^2) = \frac{\sqrt{2} f_\pi}{3} \int_0^1 dx \int_0^\infty b\, db\, \overline{\Phi}(x,b,t)\, e^{-S(x,b,Q,t)}\, S_t(x,Q) \times K_0(\sqrt{x} Q b) \left[1 - \frac{\alpha_s(t) C_F}{4\pi} \left(3 \ln \frac{t^2 b}{2\sqrt{x} Q} + \gamma_E + 2 \ln x + 3 - \frac{\pi^2}{3} \right) \right].$$

  The rapidity parameter fixed as $\zeta^2 = 2$.

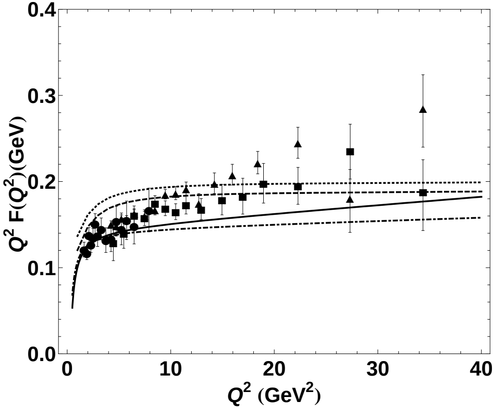
- k_T factorization with the joint resummation:

$$F(Q^2) = \frac{\sqrt{2} f_\pi}{3} \int_0^1 dx \int_0^\infty b\, db\, \overline{\Phi}(x,b,\zeta_1^2,t) K_0(\sqrt{x} Q b) \times \left[1 - \frac{\alpha_s(t) C_F}{4\pi} \left(3 \ln \frac{t^2 b}{2\sqrt{x} Q} + \ln 2 + 2 \right) \right].$$

- Choices of the factorization scale t:
  - $t^2 = \sqrt{x} Q / b$ to eliminate the remaining logarithm.
  - Typical scale of the hard scattering $t = \max(\sqrt{x} Q, 1/b)$ [default choice].
  - Two scenarios do not generate practical difference in our formalism. $\Rightarrow$ The joint resummation has suppressed the contribution from the nonperturbative region effectively.

Confronting the data I

- Results with the asymptotic pion wave function:



The experimental data from CLEO (dots), BaBar (triangles), and Belle (squares).

The dashed and dotted (dot-dashed and solid) curves from LO and NLO predictions with the conventional resummations (joint resummation).

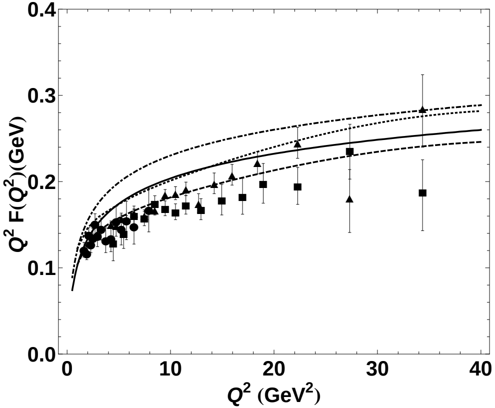
- The predicted $Q^2 F(Q^2)$ with the conventional resummations saturates as $Q^2 > 5\,\mathrm{GeV}^2$.
  Can accommodate the Belle data except the first two bins.

  Fail to describe the BaBar data.
- The joint-resummation effect decreases the predictions in the conventional approach.

  Due to the stronger suppression at small x from joint resummation.

Confronting the data II

- Results with the flat pion wave function:



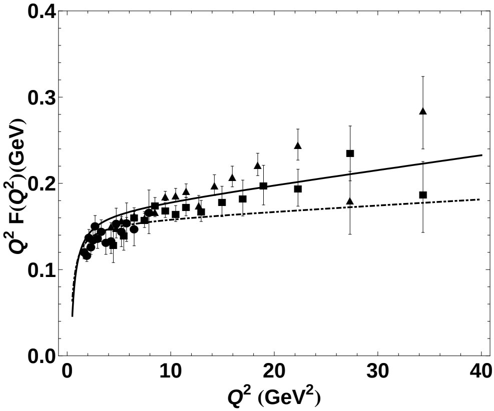
The experimental data from CLEO (dots), BaBar (triangles), and Belle (squares).

The dashed and dotted (dot-dashed and solid) curves from LO and NLO predictions with the conventional resummations (joint resummation).

- The form factor $Q^2 F(Q^2)$ grows steadily with Q^2 in both resummation formalisms. Tree-level k_T factorization formula $\Rightarrow Q^2 F(Q^2) \sim \ln(Q^2/k_T^2)$.
- The NLO curves from the conventional resummations and from the joint resummation turn out to be similar.

Confronting the data III

- Results with the non-asymptotic pion wave function:



The experimental data from CLEO (dots), BaBar (triangles), and Belle (squares).

The second Gegenbauer moment: $a_2 = 0.05$.

- Pion wave function with a small a_2 can describe the data better.
- The Chernyak-Zhitnitsky model or the Bakulev-Mikhailov-Stefanis model, which involve a large a_2, overshoot the data in our formalism.

Conclusion and outlook

- Constructed an evolution equation to resum the mixed logarithm $\ln x \ln(\zeta^2/k_T^2)$.
- The moderate x and small b regions more highlighted with joint resummation.
- The predictions for the pion transition form factor confronted with the data.
  A small a_2 favored in joint-resummation improved k_T factorization.
- More efforts are in demand on theory side:
  - Better control on the pion wave function.
  - Include the soft contribution in k_T factorization.
- Have checked that our joint-resummation formalism can be extend to k_T factorization of pion e.m. form factor.
  - The same ζ_0^2 diminishes the large logarithms in the amplitudes of effective diagrams at NLO.
  - Can find ζ_1^2 and ζ_2^2 to eliminate the large logarithms in the hard kernel.