Overview of TMD Factorization and Evolution (corrected)

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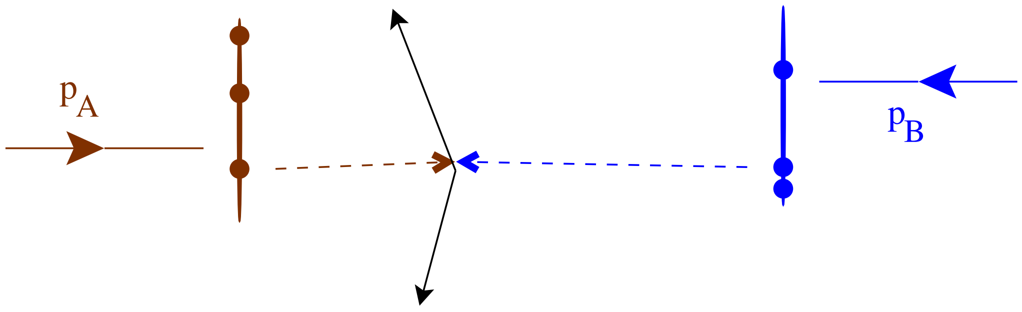
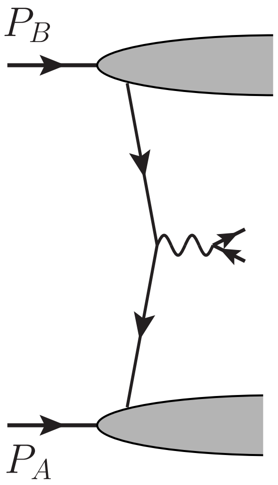
- TMD factorization/evolution
- How should non-perturbative part of evolution kernel behave?
- Tool for diagnosis and comparison of formalisms/fits:
  - Introduce scheme-independent $L(b_{\mathrm{T}})$ function
  - Examples

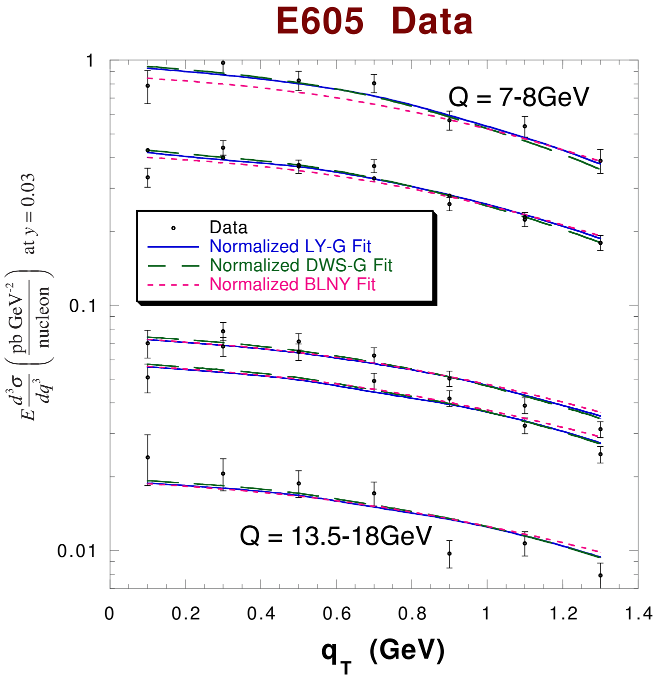
Basic parton model inspiration: Case of Drell-Yan at $q_{\mathsf{T}} \ll Q$

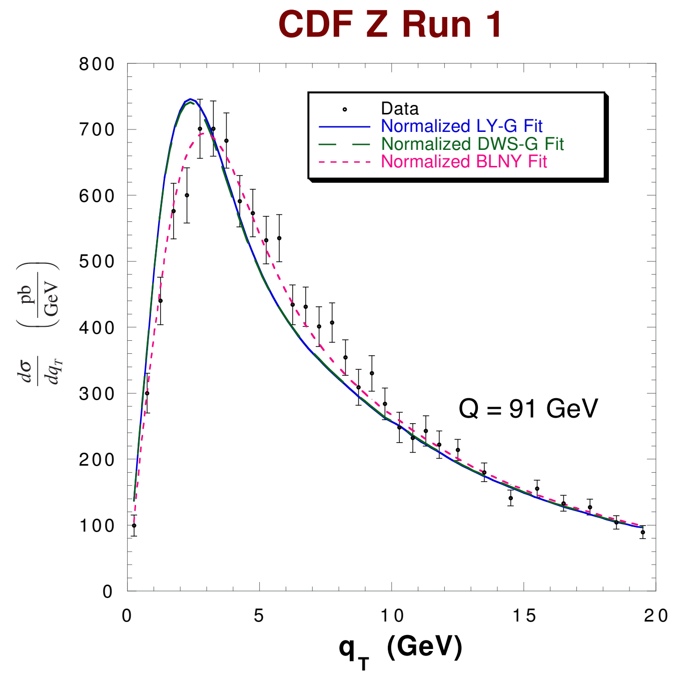




- Lorentz contracted high-energy hadrons
- $\boldsymbol{q}_{\mathsf{T}}(\text{leptons}) = \sum \boldsymbol{k}_{\mathsf{T}}(\text{quarks})$
- Use parton distribution in x and $\boldsymbol{k}_{\mathsf{T}}$
- *But* parton model needs to be substantially modified in QCD

Symptom of the QCD complications: q_T distribution broadens





Q: 7–18 GeV, $\sqrt{s} = 38.8\,\mathrm{GeV}$

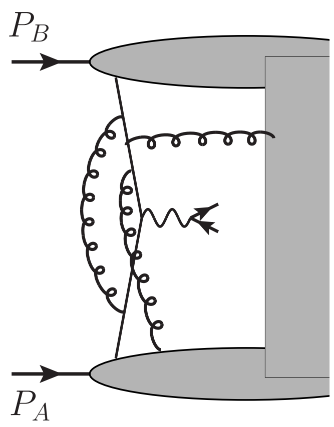
(Plot of $E\,\mathrm{d}\sigma/\mathrm{d}^3\mathbf{q}$)

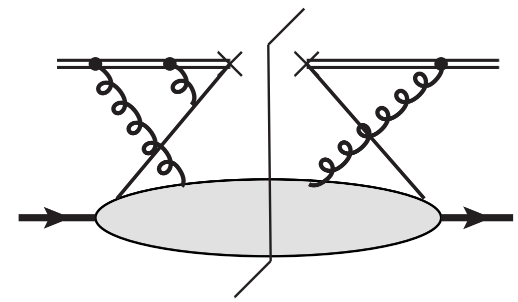
$Q = m_Z$, $\sqrt{s} = 1800\,\mathrm{GeV}$

(Plot of $\mathrm{d}\sigma/\mathrm{d}q_T$: has q_T factor.)

(Adapted from Landry et al., PRD 67,073016 (2003))

Steps to derive factorization, given typical structure of graphs + momentum regions:





Fourier trans. of $\langle p|\bar{\psi}$ WL $\psi|p\rangle$

- Approximations at leading power
- + Ward identities $\Longrightarrow$ Wilson lines for "misattached" glue (Feynman gauge)
- + contour deformation + "unitarity cancellation", etc
- $\Longrightarrow$ *initial*-state Wilson lines for DY
- Factorization of contributions of different regions, including central/soft
- Reorganize: Construct subtractions, define TMD pdfs, with glue restricted to correct hemisphere, etc. Soft factors somewhere.

$\Longrightarrow$ Broadening from pert. and non-pert. glue into increasing rapidity range.

Full TMD factorization (modernized Collins-Soper form)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4 q\,\mathrm{d}\Omega} = \frac{2}{s}\sum_j \frac{\mathrm{d}\hat{\sigma}_{j\bar{j}}(Q,\mu)}{\mathrm{d}\Omega}\int e^{i\boldsymbol{q}_\mathsf{T}\cdot\boldsymbol{b}_\mathsf{T}}\ \tilde{f}_{j/A}(x_A,\boldsymbol{b}_\mathsf{T};\zeta_A,\mu)\ \tilde{f}_{\bar{j}/B}(x_B,\boldsymbol{b}_\mathsf{T};\zeta_B,\mu)\,\mathrm{d}^2\boldsymbol{b}_\mathsf{T}$$

$+$ poln. terms $+$ high-q_T term $+$ power-suppressed

where can set $\zeta_A = \zeta_B = Q^2$, $\mu = Q$.

Evolution:

$$\frac{\partial \ln \tilde{f}_{f/H}(x,b_\mathsf{T};\zeta;\mu)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_\mathsf{T};\mu)$$

$$\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\ln\mu} = -\gamma_K(\alpha_s(\mu))$$

$$\frac{\mathrm{d}\ln \tilde{f}_{f/H}(x,b_\mathsf{T};\zeta;\mu)}{\mathrm{d}\ln\mu} = \gamma_f(\alpha_s(\mu);1) - \frac{1}{2}\gamma_K(\alpha_s(\mu))\ln\frac{\zeta}{\mu^2}$$

Small-b_T:

$$\tilde{f}_{f/H}(x,b_\mathsf{T};\zeta;\mu) = \sum_j \int_{x-}^{1+} \tilde{C}_{f/j}(x/\hat{x},b_\mathsf{T};\zeta,\mu,\alpha_s(\mu))\ f_{j/H}(\hat{x};\mu)\frac{\mathrm{d}\hat{x}}{\hat{x}}\ +\ O[(mb_\mathsf{T})^p]$$

Key to predictivity: Universality etc derived from QCD

- All process use the same TMD pdfs (and fragmentation functions)
- Except:
  - Scale dependence: Use evolution
  - Reversed Wilson lines in TMD pdfs between DY and SIDIS
  - Hence sign reversal for Sivers function etc
- Same evolution kernel $\tilde{K}$ (color triplet) in all cases, including all polarized cases (Sivers, Boer-Mulders, etc)
- But breakdown of TMD factorization in $HH \to HH + X$
- Non-perturbative information:
  - Ordinary pdfs
  - Large b_T TMD pdfs: "intrinsic transverse momentum"
  - Large b_T of evolution kernel $\tilde{K}(b_\mathrm{T}, \mu)$: recoil against radiation per unit rapidity

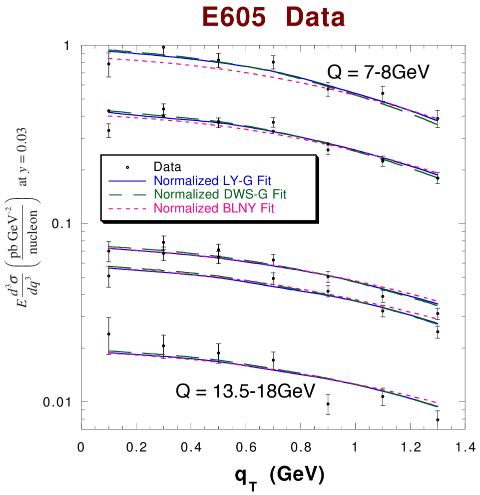
Formalisms used: They don't all appear compatible

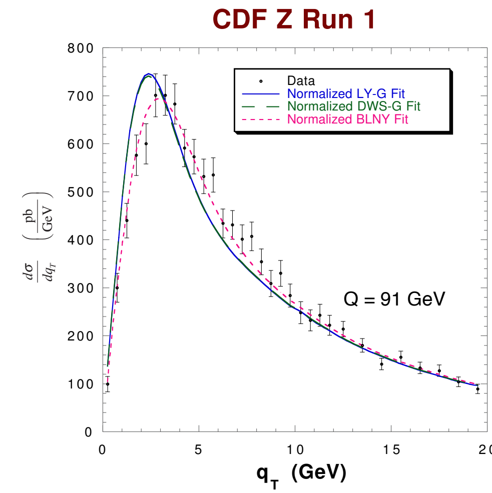
Parton model:	QCD complications ignored
Original CSS:	non-light-like axial gauge; soft factor
Ji–Ma–Yuan:	non-light-like Wilson lines; soft factor; parameter ρ
New CSS:	clean up, Wilson lines mostly light-like;
	absorb (square roots of) soft factor in TMD pdfs
Becher–Neubert:	SCET, but without actual finite TMD pdfs
Echevarría–Idilbi–Scimemi:	SCET
Mantry–Petriello:	SCET
Boer, Sun-Yuan:	Approximations on CSS

Disagreement on non-perturbative contribution to evolution ($\tilde{K}(b_{\mathrm{T}})$ at large b_{T}), or even whether it exists.

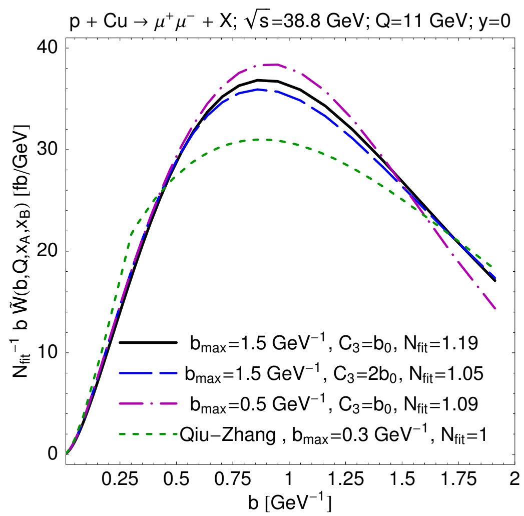
Geography of evolution of cross section

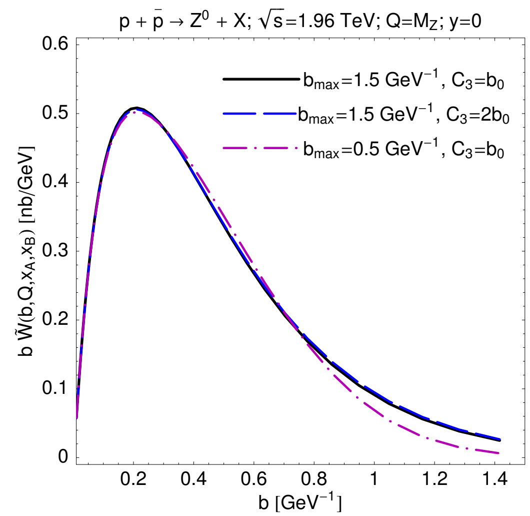
q_T





b_T

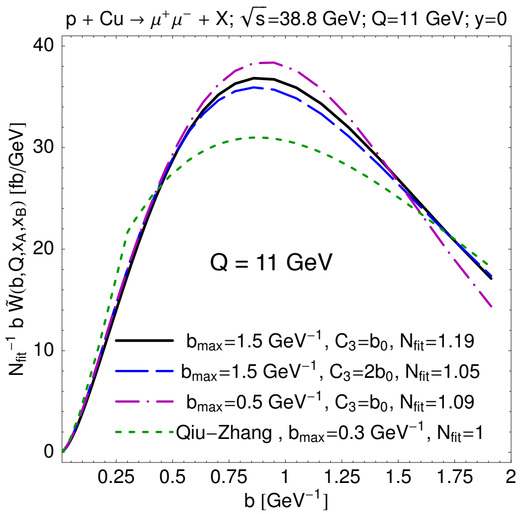


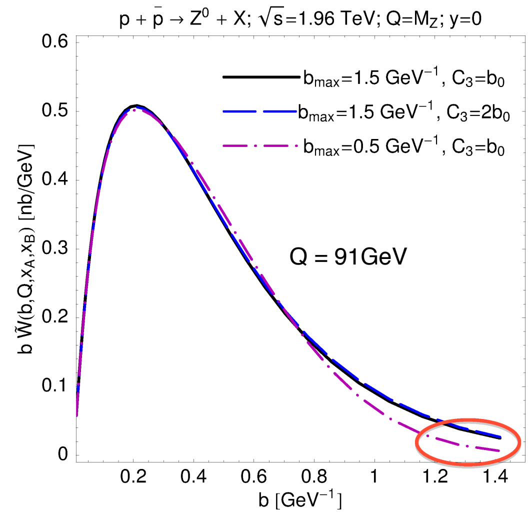


Q: 7–18 GeV, $\sqrt{s} = 38.8\,\mathrm{GeV}$ $\quad Q = m_Z,\ \sqrt{s} = 1800\,\mathrm{GeV}$

(Adapted from Landry et al., PRD 67,073016 (2003), Konychev & Nadolsky, PLB 633, 710 (2006))

Evolution of cross section (à la CSS)





$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4 q} = \text{norm.} \times \int e^{i\boldsymbol{q}_\mathsf{T}\cdot\boldsymbol{b}_\mathsf{T}} \widetilde{W}(b_\mathsf{T}, s, x_A, x_B)\, \mathrm{d}^2\boldsymbol{b}_\mathsf{T}$$

$$\left.\frac{\partial \widetilde{W}}{\partial \ln Q^2}\right|_{\text{fixed } x_A,\ x_B} = \left.\frac{\partial \widetilde{W}}{\partial \ln s}\right|_{\text{fixed } x_A,\ x_B} = \left(\tilde{K}(b_\mathsf{T}, \mu) + G(Q, \mu)\right) \widetilde{W}$$

- Universal $\tilde{K}$
- Perturbative: G, $\tilde{K}$ at small b_T, with RG
- Non-perturbative $\tilde{K}$ at large b_T

Different results for evolution at large b_T

With CSS prescription: $\tilde{K}(b_\mathsf{T},\mu) = \tilde{K}(b_*,\mu) - g_K(b_\mathsf{T}; b_{\max})$ fits at Q up to m_Z give:

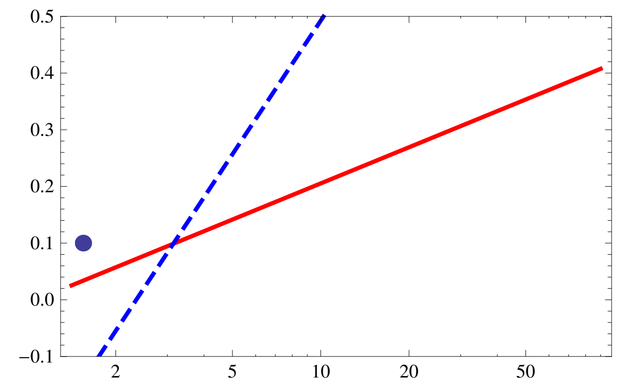
$$g_K(b_\mathsf{T}) = \frac{0.68^{+0.01}_{-0.02}}{2} b_\mathsf{T}^2 \qquad \text{(BLNY, } b_{\max} = 0.5\,\mathrm{GeV}^{-1} = 0.1\,\mathrm{fm})$$

$$g_K(b_\mathsf{T}) = \frac{0.158 \pm 0.023}{2} b_\mathsf{T}^2 \qquad \text{(KN, } b_{\max} = 1.5\,\mathrm{GeV}^{-1} = 0.3\,\mathrm{fm})$$

But this implies wrong behavior at large b_T, smaller Q:

With this parameterization

$$\tilde{W} = \ldots e^{-b^2[\text{coeff}(x) + \text{const}\,\ln(Q^2/Q_0^2)]}$$



Blue: BLNY, Red: KN

(Sun & Yuan, PRD 88, 114012 (2013))

Tool to compare different methods: The L function

(JCC & Rogers, in preparation)

- Shape change of transverse momentum distribution comes only from b_{T}-dependence of $\tilde{K}$
- So define scheme independent

$$L(b_{\mathsf{T}}) = -\frac{\partial}{\partial \ln b_{\mathsf{T}}^2} \frac{\partial}{\partial \ln Q^2} \ln \tilde{W}(b_{\mathsf{T}}, Q, x_A, x_B) \stackrel{\text{CSS}}{=} -\frac{\partial}{\partial \ln b_{\mathsf{T}}^2} \tilde{K}(b_{\mathsf{T}}, \mu)$$

- QCD predicts it is
  - independent of Q, x_A, x_B
  - independent of light-quark flavor
  - RG invariant
  - perturbatively calculable at small b_{T}
  - non-perturbative at large b_{T}

Relation of L function to properties of cross section

If L were constant, then

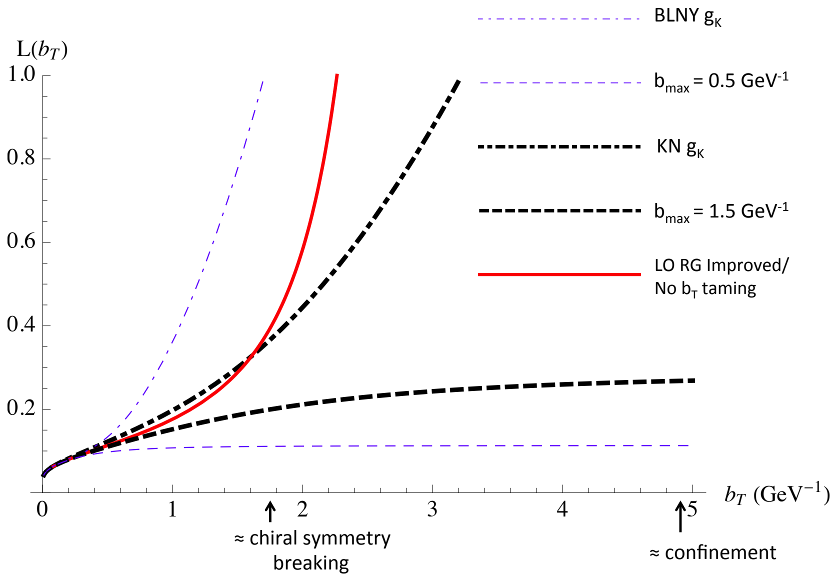
$$\tilde{W}(b_\mathsf{T}, Q) = \text{normalization} \times \tilde{W}(b_\mathsf{T}, Q_0) \times \left(\frac{1}{b_\mathsf{T}^2}\right)^{L \ln(Q^2/Q_0^2)}$$

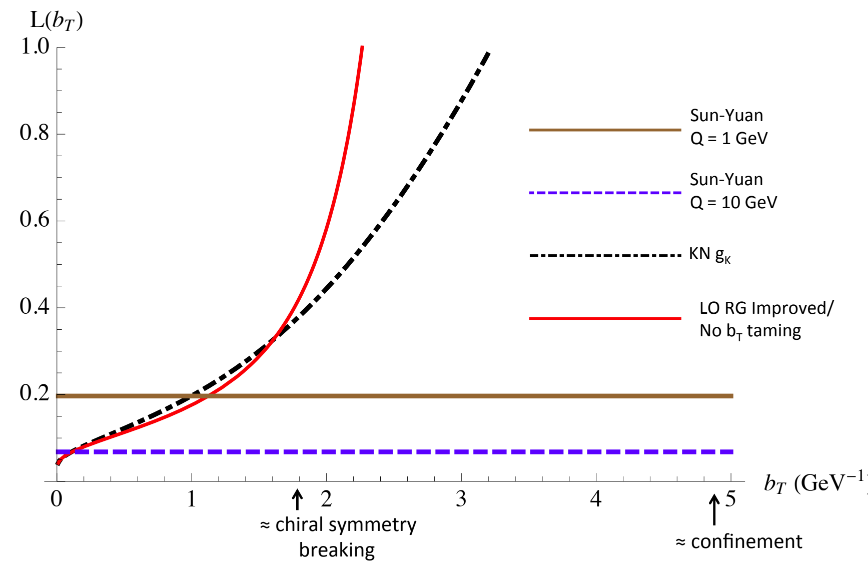
L is positive: $\tilde{W}$ decreases at large b_T and increases at small b_T when Q increases.

Of course, L is not actually constant.

Comparing different results using the L function

(Preliminary)





Q	Typical b_T
$2\,\mathrm{GeV}$	$3\,\mathrm{GeV}^{-1}$
$10\,\mathrm{GeV}$	$1.2\,\mathrm{GeV}^{-1}$
m_Z	$0.5\,\mathrm{GeV}^{-1}$

SY = Sun & Yuan (PRD 88, 114012 (2013)):

$$L_{\mathrm{SY}} = C_F \frac{\alpha_s(Q)}{\pi}$$

Depends on Q: contrary to QCD

Implications

- Important to determine actual non-perturbative part of TMD evolution (i.e., $\tilde{K}(b_\mathsf{T})$ at large b_T).
- Older fits (e.g., KN) OK for b_T up to around $3\,\mathrm{GeV}^{-1} = 0.6\,\mathrm{fm}$.
- But their extrapolation to larger b_T makes $\tilde{K}(b_\mathsf{T})$ too large.
- Use $L(b_\mathsf{T})$ to diagnose the issues: It's a universal scheme independent function in QCD.

- What does this mean physically? . . .

Physical meaning of non-perturbative $\tilde{K}(b_{\mathsf{T}})$

- Overall principle: Emission of glue is uniform in rapidity

- Old idea/intuition:
  - In one unit of rapidity emit Gaussian (??) distribution of k_{T}:
$$e^{-k_{\mathsf{T}}^2/k_{0\,\mathsf{T}}^2}\frac{1}{\pi k_{0\,\mathsf{T}}^2}$$
  - Exponential convolution $\Longrightarrow$ $\tilde{W}(b_{\mathsf{T}}, Q) = \tilde{W}(b_{\mathsf{T}}, Q_0) e^{-b_{\mathsf{T}}^2 \ln(Q^2/Q_0^2) k_{0\,\mathsf{T}}^2/4}$
  - Gives $\tilde{K}(b_{\mathsf{T}})_{\rm NP} = -b_{\mathsf{T}}^2 k_{0\,\mathsf{T}}^2/4$

- New proposal
  - Per unit rapidity: a probability of *no* emission, and a probability of emitting a particle (or more)
  - So
$$\tilde{K}(b_{\mathsf{T}})_{\rm NP} = \text{FT of } c\left[-\delta^{(2)}(\boldsymbol{k}_{\mathsf{T}}) + e^{-k_{\mathsf{T}}^2/k_{0\,\mathsf{T}}^2}/(\pi k_{0\,\mathsf{T}}^2)\right] = c\left[-1 + e^{-b_{\mathsf{T}}^2 k_{0\,\mathsf{T}}^2/4}\right]$$
  - ¿Change to exponential at large b_{T} instead of Gaussian? (Normal for Euclidean correlation function)