Multiple hard scattering and parton correlations in the proton

M. Diehl

Deutsches Elektronen-Synchroton DESY

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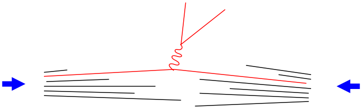
Hadron-hadron collisions

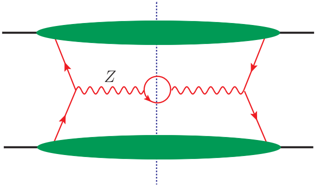
- standard description based on factorization formulae

  cross sect = parton distributions × parton-level cross sect

  example: Z production

  $pp \to Z + X \to \ell^+\ell^- + X$





- factorization formulae are for inclusive cross sections $pp \to Y + X$
  where Y = produced in parton-level scattering, specified in detail
  X = summed over, no details

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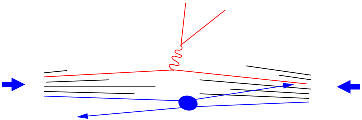
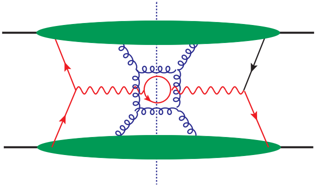
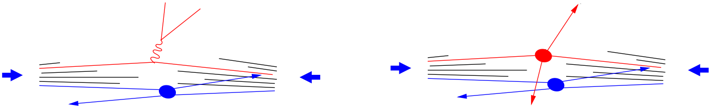



- factorization formulae are for inclusive cross sections $pp \to Y + X$
  where Y = produced in parton-level scattering, specified in detail
  X = summed over, no details
- also have interactions between "spectator" partons
  their effects cancel in inclusive cross sections thanks to unitarity
  but they affect the final state (namely X)

Multiparton interactions (MPI)



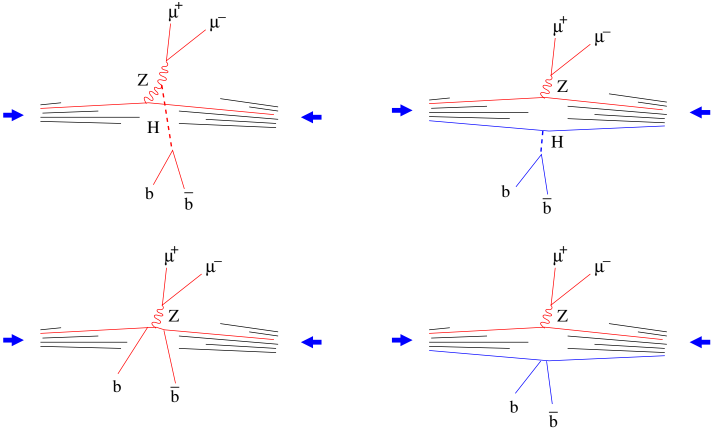
- secondary (and tertiary etc.) interactions generically take place in hadron-hadron collisions
- predominantly low-p_T scattering
  $\rightsquigarrow$ underlying event (UE)
- at high collision energy (esp. at LHC) can be hard
  $\rightsquigarrow$ multiple hard scattering
- many studies:
  theory: phenomenology, theoretical foundations (1980s, recent activity)
  experiment: ISR, SPS, HERA (photoproduction), Tevatron, LHC
  Monte Carlo generators: Pythia, Herwig++, Sherpa

  and ongoing activity: see e.g. the MPI@LHC workshop series
  https://indico.cern.ch/event/231843

Relevance for LHC

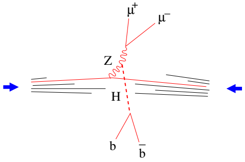
example: $pp \to H + Z \to b\bar{b} + Z$ Del Fabbro, Treleani 1999

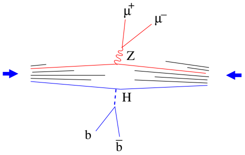
- multiple interactions contribute to signal and background



same for $pp \to H + W \to b\bar{b} + W$ study for Tevatron: Bandurin et al, 2010

Double vs. single hard scattering





- double hard scattering:
  net $\boldsymbol{p}_T$ of produced system (Z or $b\bar{b}$ pair) $\ll$ hard scale Q (e.g. M_Z)
- single hard scattering:
  $\boldsymbol{p}_T$ distribution up to values $\sim Q$
- no generic suppression for transv. mom. $\ll Q$:

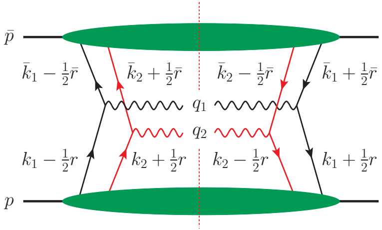
$$\frac{d\sigma_{\text{single}}}{d^2\boldsymbol{p}_Z\, d^2\boldsymbol{p}_{b\bar{b}}} \sim \frac{d\sigma_{\text{double}}}{d^2\boldsymbol{p}_Z\, d^2\boldsymbol{p}_{b\bar{b}}} \sim \frac{\Lambda^2}{Q^2}$$

  but since single scattering populates larger phase space :

$$\sigma_{\text{single}} \sim \frac{1}{Q^2} \gg \sigma_{\text{double}} \sim \frac{\Lambda^2}{Q^4}$$

- however: double hard scattering enhanced at small x

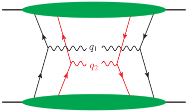
Space-time structure



momentum conservation $\Rightarrow r + \bar{r} = 0$

- transverse parton momenta not the same in amplitude $\mathcal{A}$ and in $\mathcal{A}^*$
  cross section $\propto \int d^2\boldsymbol{r}\, F(x_i, \boldsymbol{k}_i, \boldsymbol{r}) F(\bar{x}_i, \bar{\boldsymbol{k}}_i, -\boldsymbol{r})$
- Fourier trf. to impact parameter: $F(x_i, \boldsymbol{k}_i, \boldsymbol{r}) \to F(x_i, \boldsymbol{k}_i, \boldsymbol{y})$
  cross section $\propto \int d^2\boldsymbol{y}\, F(x_i, \boldsymbol{k}_i, \boldsymbol{y}) F(\bar{x}_i, \bar{\boldsymbol{k}}_i, \boldsymbol{y})$
- interpretation: $\boldsymbol{y}$ = transv. dist. between two scattering partons
  = equal in both colliding protons

Cross section formula



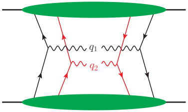
$$\frac{d\sigma_{\text{double}}}{dx_1\, d\bar{x}_1\;\, dx_2\, d\bar{x}_2} = \frac{1}{C}\, \hat{\sigma}_1\, \hat{\sigma}_2 \int d^2\boldsymbol{y}\; F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$$

$C =$ combinatorial factor
$\hat{\sigma}_i =$ parton-level cross sections
$F(x_1, x_2, \boldsymbol{y}) =$ double parton distribution (DPD)
$\boldsymbol{y} =$ transv. distance between partons

- follows from Feynman graphs and hard-scattering approximation
  no semi-classical approximation required
- can make $\hat{\sigma}_i$ differential in further variables (e.g. for jet pairs)
- can extend $\hat{\sigma}_i$ to higher orders in α_s
  get usual convolution integrals over x_i in $\hat{\sigma}_i$ and F

Paver, Treleani 1982, 1984; Mekhfi 1985, . . . , MD, Ostermeier, Schäfer 2012

Cross section formula



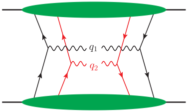
- for measured transv. momenta

$$\frac{d\sigma_{\text{double}}}{dx_1\, d\bar{x}_1\, d^2\boldsymbol{q}_1\;\, dx_2\, d\bar{x}_2\, d^2\boldsymbol{q}_2} = \frac{1}{C}\,\hat{\sigma}_1\,\hat{\sigma}_2$$

$$\times \Bigg[\prod_{i=1}^{2}\int d^2\boldsymbol{k}_i\, d^2\bar{\boldsymbol{k}}_i\, \delta(\boldsymbol{q}_i - \boldsymbol{k}_i - \bar{\boldsymbol{k}}_i)\Bigg] \int d^2\boldsymbol{y}\, F(x_i, \boldsymbol{k}_i, \boldsymbol{y})\, F(\bar{x}_i, \bar{\boldsymbol{k}}_i, \boldsymbol{y})$$

- $F(x_i, \boldsymbol{k}_i, \boldsymbol{y}) = k_T$ dependent two-parton distribution
- has structure of a Wigner function:
  $\boldsymbol{k}_1, \boldsymbol{k}_2 =$ transv. parton momenta averaged over $\mathcal{A}$ and $\mathcal{A}^*$
  $\boldsymbol{y} =$ transv. distance between partons averaged over $\mathcal{A}$ and $\mathcal{A}^*$

Cross section formula



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$$\times \left[\prod_{i=1}^{2} \int d^2\boldsymbol{k}_i\, d^2\bar{\boldsymbol{k}}_i\, \delta(\boldsymbol{q}_i - \boldsymbol{k}_i - \bar{\boldsymbol{k}}_i) \right] \int d^2\boldsymbol{y}\, F(x_i, \boldsymbol{k}_i, \boldsymbol{y})\, F(\bar{x}_i, \bar{\boldsymbol{k}}_i, \boldsymbol{y})$$

- $F(x_i, \boldsymbol{k}_i, \boldsymbol{y}) = k_T$ dependent two-parton distribution
- operator definition as for TMDs

$$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}) = \underset{z_i \to (x_i, \boldsymbol{k}_i)}{\mathcal{FT}} \langle p| \bar{q}\big(-\tfrac{1}{2} z_2\big) \Gamma_2\, q\big(\tfrac{1}{2} z_2\big)\, \bar{q}\big(y - \tfrac{1}{2} z_1\big) \Gamma_1\, q\big(y + \tfrac{1}{2} z_1\big) |p\rangle$$

  - essential for studying factorization, scale dependence, etc.
  - similar def for collinear distributions $F(x_i, \boldsymbol{y})$
    bilinear op's $\bar{q}\, \Gamma_i\, q$ at different transv. positions
    $\Rightarrow$ not a twist-four operator but product of two twist-two operators

Pocket formula

- if two-parton density factorizes as

$$F(x_1, x_2, \boldsymbol{y}) = f(x_1)\, f(x_2)\, G(\boldsymbol{y})$$

  where $f(x_i) =$ usual PDF

- if assume same $G(\boldsymbol{y})$ for all parton types
  then cross sect. formula turns into

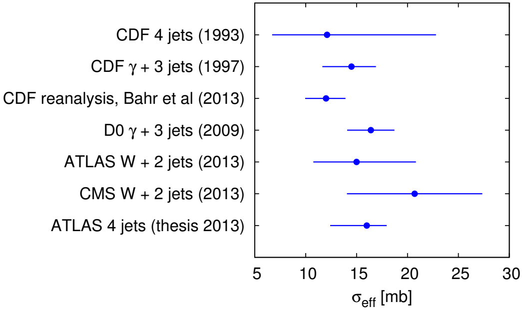
$$\frac{d\sigma_{\text{double}}}{dx_1\, d\bar{x}_1\;\, dx_2\, d\bar{x}_2} = \frac{1}{C}\, \frac{d\sigma_1}{dx_1\, d\bar{x}_1}\; \frac{d\sigma_2}{x_2\, \bar{x}_2}\; \frac{1}{\sigma_{\text{eff}}}$$

  with $1/\sigma_{\text{eff}} = \int d^2\boldsymbol{y}\; G(\boldsymbol{y})^2$

  $\rightsquigarrow$ scatters are completely independent

- underlies bulk of phenomenological estimates
- pocket formula fails if any of the above assumptions is invalid
  and if further terms must be added to original expression of cross sect.

Experimental investigations (only a sketch)



- double charm production ($c\bar{c}c\bar{c}$) at LHCb (2011, 2012):
  $J/\Psi + J/\Psi$, $J/\Psi + C$, $C + C$ with $C = D^0, D^+, D_s^+, \Lambda_c^+$

Parton correlations

- if neglect correlations between two partons

$$F(x_1, x_2, \boldsymbol{y}) = \int d^2\boldsymbol{y}'\ f(x_1, \boldsymbol{y}' + \boldsymbol{y})\, f(x_2, \boldsymbol{y}')$$

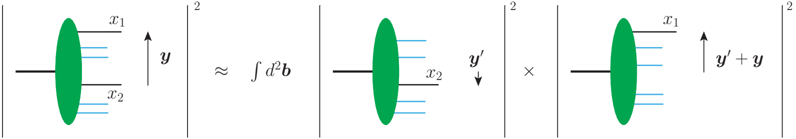
where $f(x_i, \boldsymbol{y})$ = impact parameter dependent single-parton density

and if neglect correlations between x and $\boldsymbol{y}$ of single parton

$$f(x_i, \boldsymbol{y}) = f(x_i) F(\boldsymbol{y})$$

with same $F(\boldsymbol{y})$ for all partons

then $G(\boldsymbol{y}) = \int d^2\boldsymbol{y}'\ F(\boldsymbol{y}')\, F(\boldsymbol{y}' + \boldsymbol{y})$



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- for Gaussian $F(\boldsymbol{y})$ with average $\langle \boldsymbol{y}^2 \rangle$

$$\sigma_{\text{eff}} = 4\pi \langle \boldsymbol{y}^2 \rangle = 41\,\text{mb}\ \times \langle \boldsymbol{y}^2 \rangle / (0.57\,\text{fm})^2$$

determinations of $\langle \boldsymbol{y}^2 \rangle$ from GPDs and form factors: $(0.57\,\text{fm} - 0.67\,\text{fm})^2$

is $\gg \sigma_{\text{eff}} \sim 10$ to $20\,\text{mb}$ from experimental extractions

if $F(\boldsymbol{y})$ is Fourier trf. of dipole then $41\,\text{mb} \to 36\,\text{mb}$

complete independence between two partons is disfavored
or something is seriously wrong with σ_{eff}

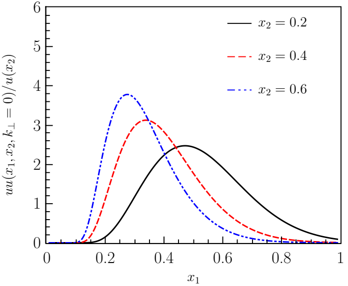
cf. Calucci, Treleani 1999; Frankfurt, Strikman, Weiss 2003-04; Blok et al 2013

Correlations involving x

- $F(x_1, x_2, \boldsymbol{y}) = f(x_1)\, f(x_2)\, G(\boldsymbol{y})$ cannot hold for all x_1, x_2
- most obvious: energy conservation $\Rightarrow\; x_1 + x_2 \le 1$

  often used to suppress region of large $x_1 + x_2$:
  $F(x_1, x_2, \boldsymbol{y}) = (1 - x_1 - x_2)^n\, f(x_1)\, f(x_2)\, G(\boldsymbol{y})$
- significant x_1 – x_2 correlations found in constituent quark model

Rinaldi, Scopetta, Vento: arXiv:1302.6462



plot shows $\int d^2\boldsymbol{y}\; F_{uu}(x_1, x_2, \boldsymbol{y})/f_u(x_2)$
is x_2 independent if factorization holds

- unknown: size of correlations when one or both of x_1, x_2 small

Correlations involving x and $\boldsymbol{y}$

- single-parton distribution $f(x, \boldsymbol{y})$ is Fourier trf. of generalized parton distributions(GPDs) at zero skewness
  - $\rightsquigarrow$ information from exclusive processes and theory
    - HERA results on $\gamma p \to J/\Psi\, p$ give
      $$\langle \boldsymbol{y}^2 \rangle \propto \text{const} + 4\alpha' \log(1/x)$$
      with $\alpha' \approx 0.15\,\text{GeV}^{-2} = (0.08\,\text{fm})^2$ for gluons at $x \sim 10^{-3}$
    - lattice simulations → strong decrease of $\langle \boldsymbol{y}^2 \rangle$ with x above ~ 0.1 seen by comparing moments $\int dx\, x^{n-1} f(x, \boldsymbol{y})$ for $n = 0, 1, 2$
- expect similar correlations between x_i and $\boldsymbol{y}$ in two-parton dist's even if two partons are not independent
- in double parton scattering $\boldsymbol{y}$ unobservable:
  $$d\sigma \propto \int d^2\boldsymbol{y}\, F(x_i, \boldsymbol{y})\, F(\bar{x}_i, \boldsymbol{y})$$
  in $f(x, \boldsymbol{y})$ impact parameter is Fourier conjugate to measurable momentum transfer

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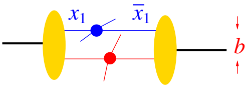
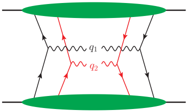
Consequence for multiple interactions:

- if interaction 1 produces high-mass system
  → have large $x_1, \bar{x}_1$
  → smaller $\boldsymbol{y}$ → more central collision
  → secondary interactions enhanced



Frankfurt, Strikman, Weiss 2003, study in Pythia: Corke, Sjöstrand 2011

Spin correlations



- polarizations of two partons can be correlated even in unpolarized target — already pointed out by Mekhfi (1985)
  - quarks: longitudinal and transverse pol.
  - gluons: longitudinal and linear pol.
- in general not suppressed in hard scattering
  - for $q\bar{q} \to \ell^+\ell^-$ have $d\hat{\sigma}_{\Delta q \Delta \bar{q}}/d\Omega = -\, d\hat{\sigma}_{q\bar{q}}/d\Omega$
  - for many channels parton pol. also changes angular distribution

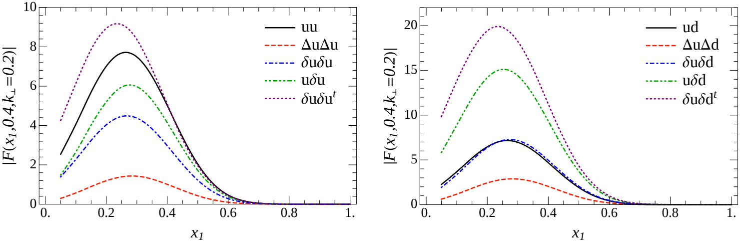
consequences for double scattering rate and differential distributions

Manohar, Waalewijn 2012; Kasemets, MD 2012

Spin correlations

- how important are spin correlations?
  large effects expected in valence quark region

  study in bag model: Chang, Manohar, Waalewijn: arXiv:1211:3132

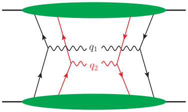


plots show $F(x_1, x_2 = 0.4, k_\perp)$ for different pol. combinations

$k_\perp$ = Fourier conjugate to $\boldsymbol{y}$

- unknown: size of correlations when one or both of x_1, x_2 small
- change with scale → talk by Tomas Kasemets on Thursday

Color structure



- quark lines in amplitude and its conjugate can couple to color singlet or octet:

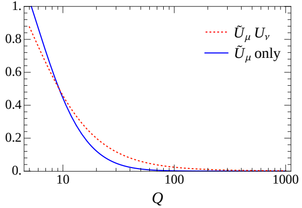
$$^{1}F \longrightarrow (\bar{q}_2 \mathbb{1} q_2)\ (\bar{q}_1 \mathbb{1} q_1) \qquad\qquad ^{8}F \longrightarrow (\bar{q}_2 t^a q_2)\ (\bar{q}_1 t^a q_1)$$

- ^{8}F describes color correlation between quarks 1 and 2 is essentially unknown (no probability interpretation as a guide)
- for two-gluon dist's more color structures: $1,\ 8_S,\ 8_A,\ 10,\ \overline{10},\ 27$
- for k_T integrated distributions: color correlations suppressed by Sudakov logarithms Mekhfi 1988

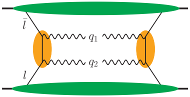
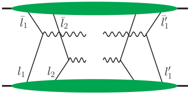
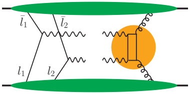
. . . but not necessarily negligible for moderately hard scales

Manohar, Waalewijn arXiv:1202:3794 used SCET methods

U = Sudakov factor, Q = hard scale



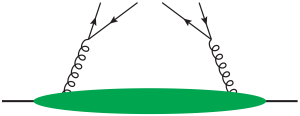
Scale and energy dependence

	$\dfrac{s\,d\sigma}{\prod_{i=1}^{2} dx_i\, d\bar{x}_i\, d^2\boldsymbol{q}_i}$	$\dfrac{s\,d\sigma}{\prod_{i=1}^{2} dx_i\, d\bar{x}_i}$
	$\dfrac{1}{\Lambda^2 Q^2}$	1
	$\dfrac{1}{\Lambda^2 Q^2}$	$\dfrac{\Lambda^2}{Q^2}$
	$\dfrac{1}{\Lambda^2 Q^2}$	$\dfrac{\Lambda^2}{Q^2}$

- interference between single and double scattering:
  - leading power when differential in $\boldsymbol{q}_i$
  - power suppressed when $\int d^2\boldsymbol{q}_i$, twist-three parton distributions
- at small $x_1 \sim x_2 \sim x$ expect
  - single scattering $\propto x^{-\lambda}$ with $xf(x) \sim x^{-\lambda}$
  - double scattering $\propto x^{-2\lambda}$
  - interference ? how do three-particle correlators behave for small x ?

Behavior at small interparton distance

- for $\boldsymbol{y} \ll 1/\Lambda$ in perturbative region $F(x_1, x_2, \boldsymbol{y})$ dominated by graphs with splitting of single parton



- find strong correlations in x_1, x_2, spin and color between two partons
  e.g. 100% correlation for longitudinal pol. of q and $\bar{q}$

- can compute short-distance behavior:

$$F(x_1, x_2, \boldsymbol{y}) \sim \frac{1}{\boldsymbol{y}^2} \text{ splitting fct} \otimes \text{usual PDF}$$

Scale evolution for collinear distributions without color correlation

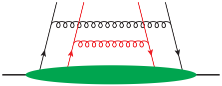
- if define two-parton distributions as operator matrix elements in analogy with usual PDFs

$$F(x_1, x_2, \boldsymbol{y}; \mu) \sim \langle p|\mathcal{O}_1(\boldsymbol{0};\mu)\, \mathcal{O}_2(\boldsymbol{y};\mu)|p\rangle \qquad f(x;\mu) \sim \langle p|\mathcal{O}(\boldsymbol{0};\mu)|p\rangle$$

where $\mathcal{O}(\boldsymbol{y};\mu)$ = twist-two operator renormalized at scale μ

- $F(x_i, \boldsymbol{y})$ for $\boldsymbol{y} \neq \boldsymbol{0}$:

  separate DGLAP evolution for partons 1 and 2

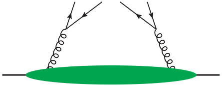


$$\frac{d}{d\log\mu} F(x_i, \boldsymbol{y}) = P \otimes_{x_1} F + P \otimes_{x_2} F$$

  two independent parton cascades

- $\int d^2\boldsymbol{y}\, F(x_i, \boldsymbol{y})$:

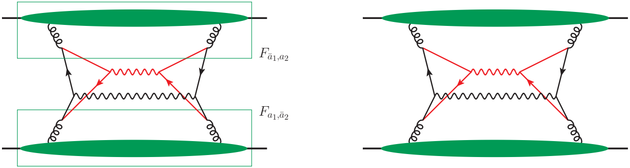
  extra term from $2 \to 4$ parton transition since $F(x_i, \boldsymbol{y}) \sim 1/\boldsymbol{y}^2$



  Kirschner 1979; Shelest, Snigirev, Zinovev 1982
  Gaunt, Stirling 2009; Ceccopieri 2011

- which evolution eq. is relevant for double hard scattering?

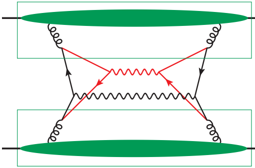
Deeper problems with the splitting graphs

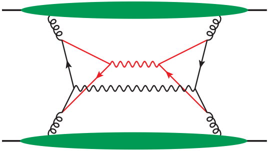


- contribution from splitting graphs in cross section
  gives divergent integrals $\int d^2\boldsymbol{y}\, F(x_1,x_2,\boldsymbol{y})\, F(\bar{x}_1,\bar{x}_2,\boldsymbol{y}) \sim \int d\boldsymbol{y}^2/\boldsymbol{y}^4$
- double counting problem between double scattering with splitting
  and single scattering at loop level

  MD, Ostermeier, Schäfer 2011; Gaunt, Stirling 2011; Gaunt 2012
  Blok, Dokshitzer, Frankfurt, Strikman 2011; Ryskin, Snigirev 2011, 2012
  same problem for jets: Cacciari, Salam, Sapeta 2009
- possible solution:
  subtract splitting contribution from two-parton dist's when $\boldsymbol{y}$ is small
  will also modify their scale evolution; remains to be worked out

Deeper problems with the splitting graphs





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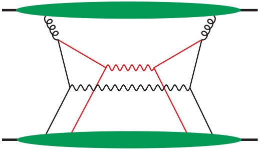
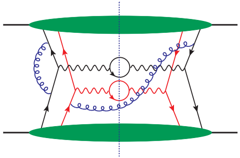
- also have graphs with single PDF for one and double PDF for other proton

What is double parton scattering?



Blok et al, 2011-13; Gaunt 2012

Sudakov factors

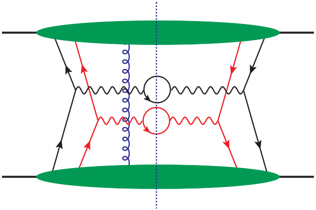


- for k_T dependent distributions, i.e. measured $\boldsymbol{q}_i$:
  Sudakov logarithms for all color channels
  close relation with physics of parton showers
- for double Drell-Yan process
  can adapt Collins-Soper-Sterman formalism for single Drell-Yan
  $\rightsquigarrow$ include and resum Sudakov logs in k_T dependent parton dist's
  MD, D Ostermeier, A Schäfer 2011

  for jet production inherit problems of usual TMD factorization
- at leading double log accuracy: singlet and octet dist's ^1F and ^8F
  have same Sudakov factor as in single scattering
- beyond double log: Sudakov factors mix singlet and octet dist's

Factorization?

- open problem (for TMD and collinear formulations):
  exchange of gluons in Glauber region



Not discussed in this talk:

- multiparton interactions in pA collisions
- small-x approach
  connection with diffraction, AGK rules — Bartels, Salvadore, Vacca 2008
  ridge effect in pp and pA — Dumitru et al 2011; ...

Conclusions

- multiple hard scattering is not generically suppressed in sufficiently differential cross sections
- current phenomenology relies on strong simplifications
- have several elements for a formulation of factorization but important open questions still unsolved
  - crosstalk with single hard scattering at small distances closely related with evolution equations ($1 \to 2$ parton splitting)
  - Glauber gluon exchange
- double hard scattering depends on detailed hadron structure including correlation and interference effects
  - corresponding nucleon matrix elements largely unknown theoretical activity only started
  - transverse distance between partons essential
- subject remains of high interest for
  - understanding high-multiplicity final states at LHC
  - study of hadron structure in its own right