QCD Evolution Workshop

Phenomenological implementations of TMD evolution

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Outline

- ✔ *Brief summary of the CSS resummation scheme*
- ✔ *Matching procedures - the Y factor*
- ✔ *Non perturbative contributions to resummation in SIDIS*
  - *Real world: “ultra-high energy and Q^2”*
    - *HERA*
    - *COMPASS*
- ✔ *Is any “universal” matching possible in SIDIS ?*

Resummation of large logarithms

- *Calculating a cross section which describes a hadronic process over the whole q_T range is highly non-trivial*
- *Resummation of large logarithms in Semi-Inclusive DIS production, in the limit $q_T << Q$, arising from emission of soft and collinear gluons*
- ***Collins – Soper – Sterman (CSS) resummation*** *Nucl. Phys. B250, 199 (1985)*

$$\boxed{\frac{1}{\sigma_0}\frac{d\sigma}{dQ^2dydq_T^2} = \quad X_{div}(q_T) + Y_{reg}(q_T)}$$

Divergent part for $q_T \to 0$
(Asymptotic part)
Must be resummed!

Regular part for $q_T \to 0$
(contains at most logarithmic divergenses)

Resummation of large logarithms

- ✔ To ensure momentum conservation, consider the problem in the Fourier conjugate space

$$\delta^2(\boldsymbol{q}_T - \boldsymbol{k}_{1T} - \boldsymbol{k}_{2T} - \ldots\ldots - \boldsymbol{k}_{nT} + \ldots) = \int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2} e^{-i\boldsymbol{b}_T\cdot(\boldsymbol{q}_T - \boldsymbol{k}_{1T} - \boldsymbol{k}_{2T} - \ldots\ldots - \boldsymbol{k}_{nT} + \ldots)}$$

$$\boxed{\frac{1}{\sigma_0}\frac{d\sigma}{dQ^2dydq_T^2} = \left[\int \frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2} X_{div}(b_T)\right] + Y_{reg}(q_T)}$$

$$X_{div}(b_T) \longrightarrow W(b_T) = \exp\left[S(b_T)\right] \times (\text{PDFs and Hard coefficients})$$

$$\boxed{\frac{1}{\sigma_0}\frac{d\sigma}{dQ^2dydq_T^2} = \int \frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2} \sum_j e_j^2 W_j(x_1, x_2, b_T, Q) + Y(x_1, x_2, q_T, Q)}$$

Resummed part

Regular part

CSS in Drell Yan

$$\frac{1}{\sigma_0}\frac{d\sigma}{dQ^2\,dy\,dq_T^2} = \int \frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2}\sum_j e_j^2 W_j(x_1,x_2,b_T,Q) + Y(x_1,x_2,q_T,Q)$$

Resummed part

Regular part

$$W_j(x_1,x_2,b_T,Q) = \exp\left[S_j(b_T,Q)\right]\sum_{i,k} C_{ji}\otimes f_i(x_1,C_1^2/b_T^2)\; C_{\bar{j}k}\otimes f_k(x_2,C_1^2/b_T^2)$$

PDFs convoluted with Wilson Coefficients

$$[C_{ji}\otimes f_i](x,\mu^2) = \int_x^1 \frac{dz}{z} C_{ji}(z,\alpha_s(\mu)) f_i(x/z,\mu)$$

$$C_{ji}(z,\alpha(\mu)) \quad = \quad \delta_{ij}\delta(1-z) + \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{2\pi}\right)^n C_{ij}^{(n)}(z)$$

CSS in Drell Yan

$$\frac{1}{\sigma_0}\frac{d\sigma}{dQ^2 dy dq_T^2} = \int \frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2}\sum_j e_j^2 W_j(x_1, x_2, b_T, Q) + Y(x_1, x_2, q_T, Q)$$

Resummed part

Regular part

$$W_j(x_1, x_2, b_T, Q) = \exp\left[S_j(b_T, Q)\right]\sum_{i,k} C_{ji}\otimes f_i(x_1, C_1^2/b_T^2)\; C_{\bar{j}k}\otimes f_k(x_2, C_1^2/b_T^2)$$

Sudakov factor

$$S_j(b_T, Q) = \int_{C_1^2/b_T^2}^{Q^2}\frac{d\kappa^2}{\kappa^2}\left[A_j(\alpha_s(\kappa))\ln\left(\frac{Q^2}{\kappa^2}\right) + B_j(\alpha_s(\kappa))\right]$$

$$\begin{aligned} A_j(\alpha(\mu)) &= \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{2\pi}\right)^n A_j^{(n)} \\ B_j(\alpha(\mu)) &= \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{2\pi}\right)^n B_j^{(n)} \end{aligned}$$

Leading Log (LL) : $A^{(1)}$;
Next to LL (NLL) : $A^{(2)}, B^{(1)}, C^{(1)}$;
Next to NLL (NNLL) : $A^{(3)}, B^{(2)}, C^{(2)}$;
Fixed order $\alpha_s(FXO)$: $A^{(1)}, B^{(1)}, C^{(1)}$;

CSS in SIDIS

$$\frac{d\sigma}{dx\,dz\,dQ^2\,d^2q_T} = \sigma_0^{SIDIS}\left\{\int \frac{d^2\boldsymbol{b}_T\, e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2}\sum_j e_j^2 W_j^{SIDIS}(x,z,b_T,Q) + Y^{SIDIS}\right\}$$

$$W_j(x_1,x_2,b_T,Q) = \sum_{i,k}\exp\left[S_j^{SIDIS}(b_T,Q)\right]\left[C_{ji}\otimes f_i\right]\left(x,\frac{C_1^2}{b_T^2}\right)\left[C_{kj}^{out}\otimes D_k\right]\left(z,\frac{C_1^2}{b_T^2}\right)$$

- The resummed cross section, W, does not describe the whole P_T range.
  It sums all known logarithmic terms dominating the low P_T region, but does not take into account the full fixed order (NLO) corrections, which are important at large P_T values.

Warning: here NLO means first order in α_s of the collinear QCD cross section

- Because of the oscillatory nature of the Fourier integrand, W may become **negative** (i.e. unphysical) at large P_T
- For a consistent description over the whole P_T range we need to **MATCH** the resummed cross section with the NLO (fixed order) cross section

The Y factor and the asymptotic part

$$\frac{d\sigma}{dx\,dz\,dQ^2\,d^2q_T} = \sigma_0^{SIDIS}\left\{\int \frac{d^2\boldsymbol{b}_T\, e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2}\sum_j e_j^2 W_j^{SIDIS}(x,z,b_T,Q) + Y^{SIDIS}\right\}$$

$$\frac{d^5\sigma^{\text{NLO}}}{dQ^2\,dx_{bj}\,dz_f\,dq_T^2\,d\phi} = \frac{d^5\sigma^{\text{ASY}}}{dQ^2\,dx_{bj}\,dz_f\,dq_T^2\,d\phi} + Y$$

Warning: here NLO means first order in α_s of the collinear QCD cross section

Y = NLO - ASY

$$\begin{aligned}
&\frac{d^5\sigma^{\text{asymp}}}{dQ^2\,dx_{bj}\,dz_f\,dq_T^2\,d\phi}\\
&= \frac{\alpha_{em}^2\alpha_s}{8\pi x_{bj}^2 S_{ep}^2 Q^2} A_1 \frac{2Q^2}{q_T^2}\sum_{q,\bar q} e_q^2\Bigg[2 f_q(x_{bj},\mu) D_q(z_f,\mu)\left(C_F \ln\left(\frac{Q^2}{q_T^2}\right) - \frac{3}{2}C_F\right)\\
&\quad + \left\{f_q(x_{bj},\mu)\otimes P_{qq}^{\text{in},(0)} + f_g(x_{bj},\mu)\otimes P_{qg}^{\text{in},(0)}\right\} D_q(z_f,\mu)\\
&\quad + f_q(x_{bj},\mu)\left\{P_{qq}^{\text{out},(0)}\otimes D_q(z_f,\mu) + P_{gq}^{\text{out},(0)}\otimes D_g(z_f,\mu)\right\}\Bigg],
\end{aligned}$$

ASY = Q^2/q_T^2 [A Ln(Q^2/q_T^2) + B + C]

The Y factor and the asymptotic part

$$\frac{d\sigma}{dx\,dz\,dQ^2\,d^2q_T} = \sigma_0^{SIDIS}\left\{\int \frac{d^2\boldsymbol{b}_T\, e^{i\boldsymbol{q}_T\cdot \boldsymbol{b}_T}}{(2\pi)^2}\sum_j e_j^2 W_j^{SIDIS}(x, z, b_T, Q) + Y^{SIDIS}\right\}$$

$$\frac{d^5\sigma^{\text{NLO}}}{dQ^2\,dx_{bj}\,dz_f\,dq_T^2\,d\phi} = \frac{d^5\sigma^{\text{ASY}}}{dQ^2\,dx_{bj}\,dz_f\,dq_T^2\,d\phi} + Y$$

Warning: here NLO means first order in α_s of the collinear QCD cross section

Y = NLO - ASY

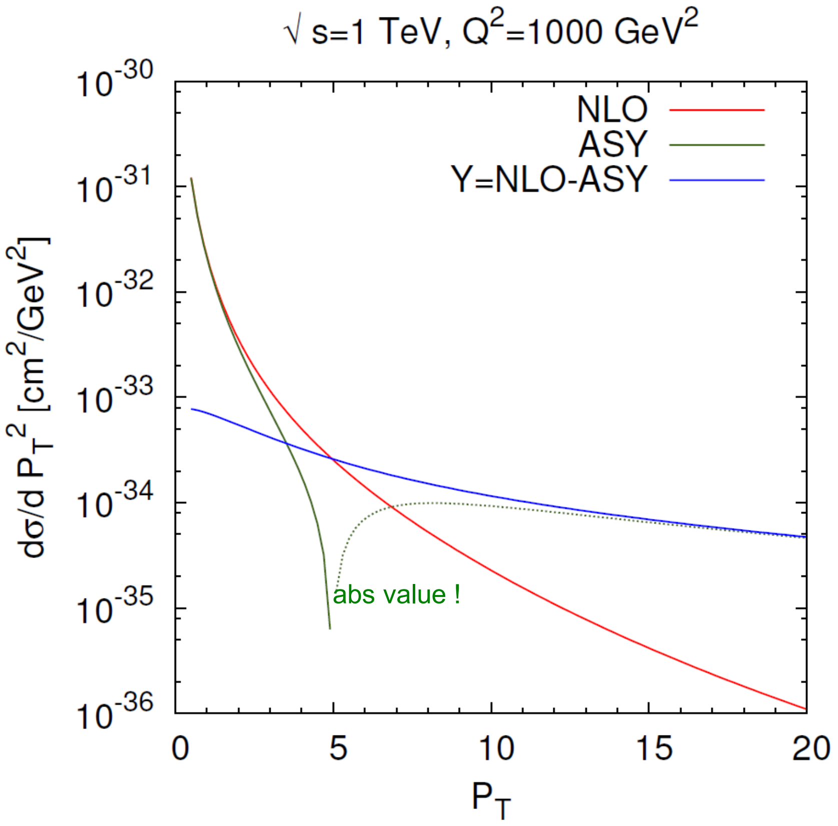
ASY = Q^2/q_T^2 [A Ln(Q^2/q_T^2) + B + C]

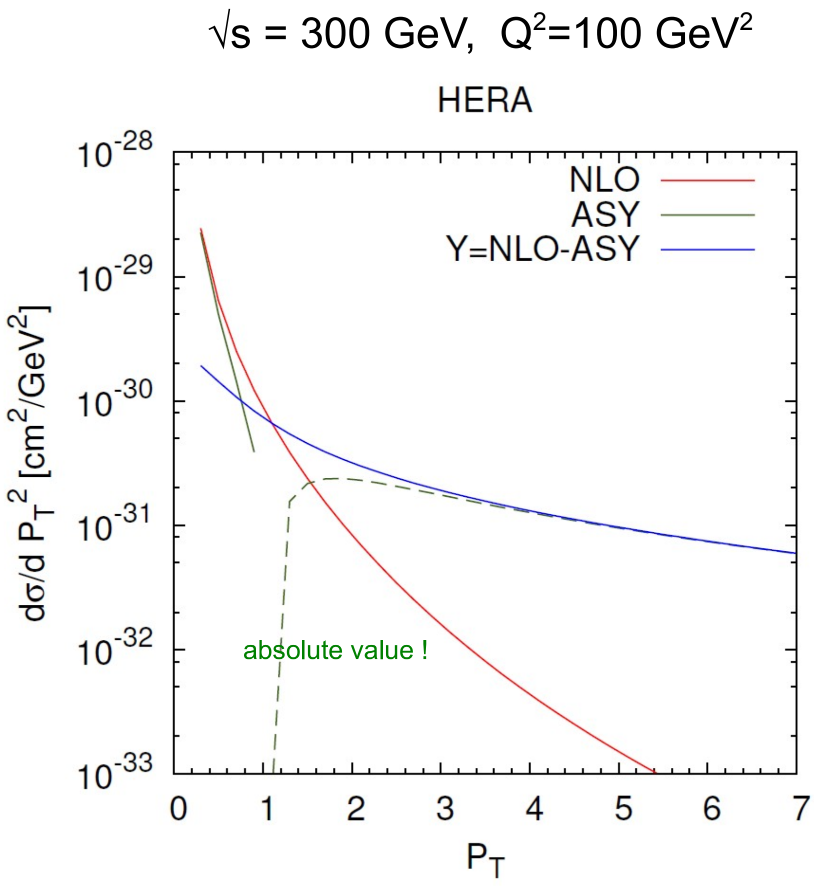
At P_T ~ Q,
if W → ASY then
W + Y → ASY + NLO – ASY = **NLO**

MATCHING PRESCRIPTION at P_T ~ Q

This matching prescription only works if and when W → ASY

The Y factor and the asymptotic part





Notice that, as ASY becomes negative at large P_T, Y can become much larger than NLO in that region

Y can still diverge at extremely small P_T , due to non exact cancellations of logs between NLO and ASY

- ✓ **Does a kinematical range in which W ~ ASY exist ?**
- ✓ Before we can answer this question we should worry about the non-perturbative contributions to the Sudakov factor

Non perturbative contributions

$$\frac{d\sigma}{dx\,dz\,dQ^2d^2q_T} = \sigma_0^{SIDIS}\left\{\int \frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2}\sum_j e_j^2 W_j^{SIDIS}(x,z,b_T,Q) + Y^{SIDIS}\right\}$$

- The CSS formalism relies on a Fourier integral which runs from 0 to ∞

  No prediction can be made without an ansatz prescription for the non–perturbative region, where b_T is large and p_T is small
- The Sudakov factor diverges at large b_T

$$S_j(b_T,Q) = \int_{\boxed{C_1^2/b_T^2}}^{Q^2}\frac{d\kappa^2}{\kappa^2}\left[A_j(\alpha_s(\kappa))\boxed{\ln\left(\frac{Q^2}{\kappa^2}\right)} + B_j(\alpha_s(\kappa))\right]$$

- In the CCS scheme a freezing prescription is used, such that

$$\boxed{b_* = \frac{b_T}{\sqrt{1+b_T^2/b_{max}^2}} \qquad \mu_b = C_1/b_*}$$

Non perturbative contributions

$$\frac{d\sigma}{dx\,dz\,dQ^2\,d^2q_T} = \sigma_0^{SIDIS}\left\{\int \frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2}\sum_j e_j^2 W_j^{SIDIS}(x,z,b_T,Q) + Y^{SIDIS}\right\}$$

$$= \sigma_0\left\{\int \frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2}\sum_j e_j^2 W_j^{SIDIS}(x,z,b_*,Q)\,F_{NP}(x,z,b_T,Q) + Y^{SIDIS}\right\}$$

- ✔ W, the perturbative part of the Sudakov factor, is a function of b*

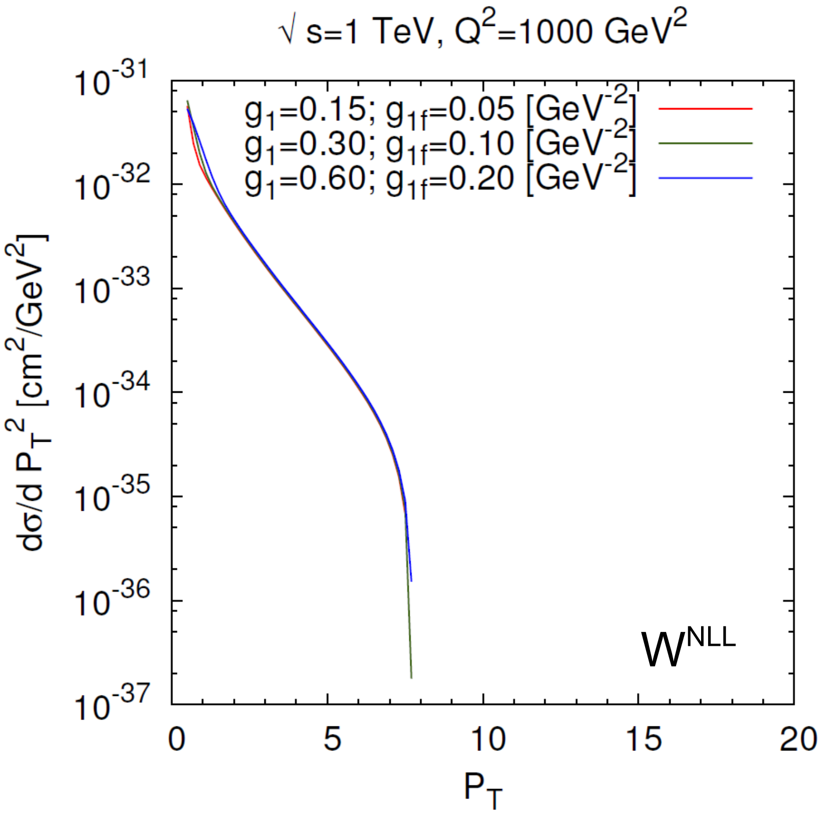
$$b_* = \frac{b_T}{\sqrt{1+b_T^2/b_{max}^2}} \qquad \mu_b = C_1/b_*$$

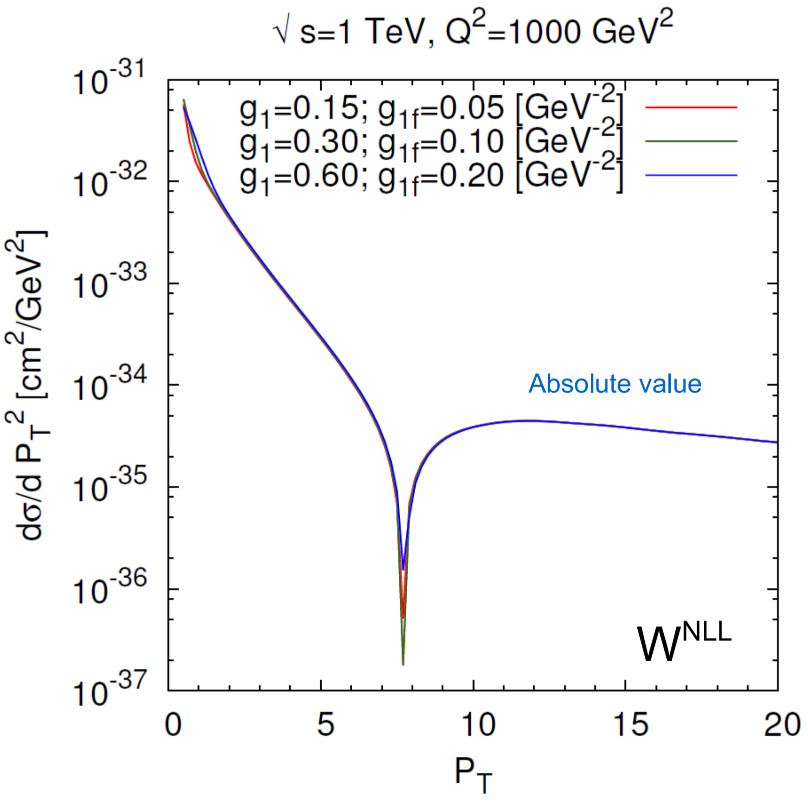
- ✔ F_{NP}, the non-perturbative part of the Sudakov factor, accounts for the **non-perturbative** behavior at large b_T (i.e. small P_T)

- ✔ Just for illustration, let's consider a simple (Gaussian) model for F_{NP}

$$F_{NP} \rightarrow \exp\left[\,(-\,g_1 - g_{1f}\,/\,z^2\, - g_2\ \ln(Q/Q_0)\,)\ b^2\,\right]$$

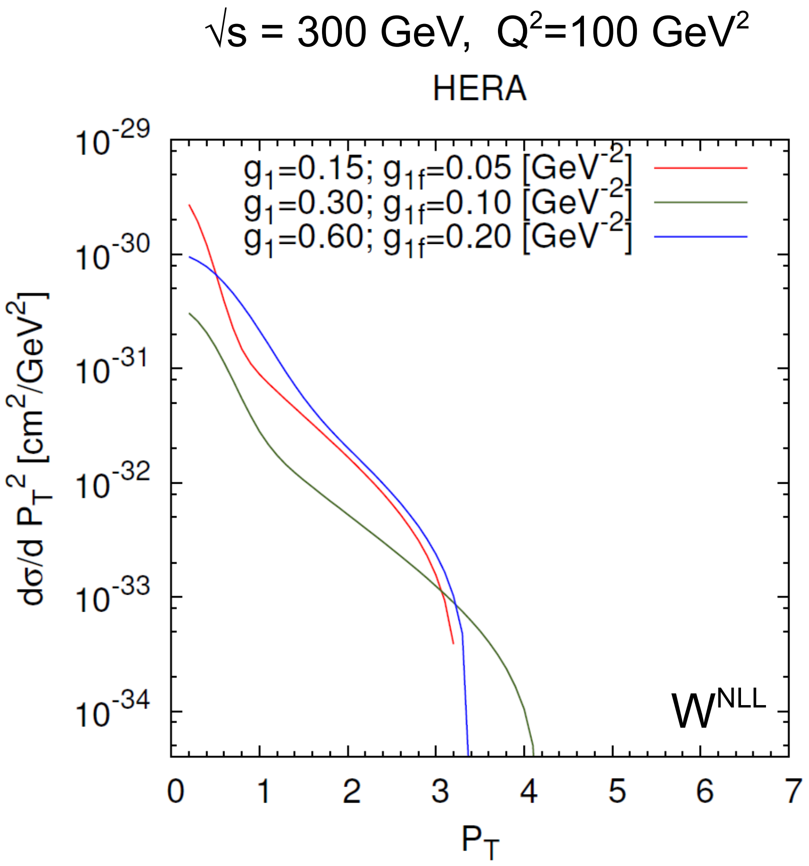
Non perturbative contributions – very high energy SIDIS

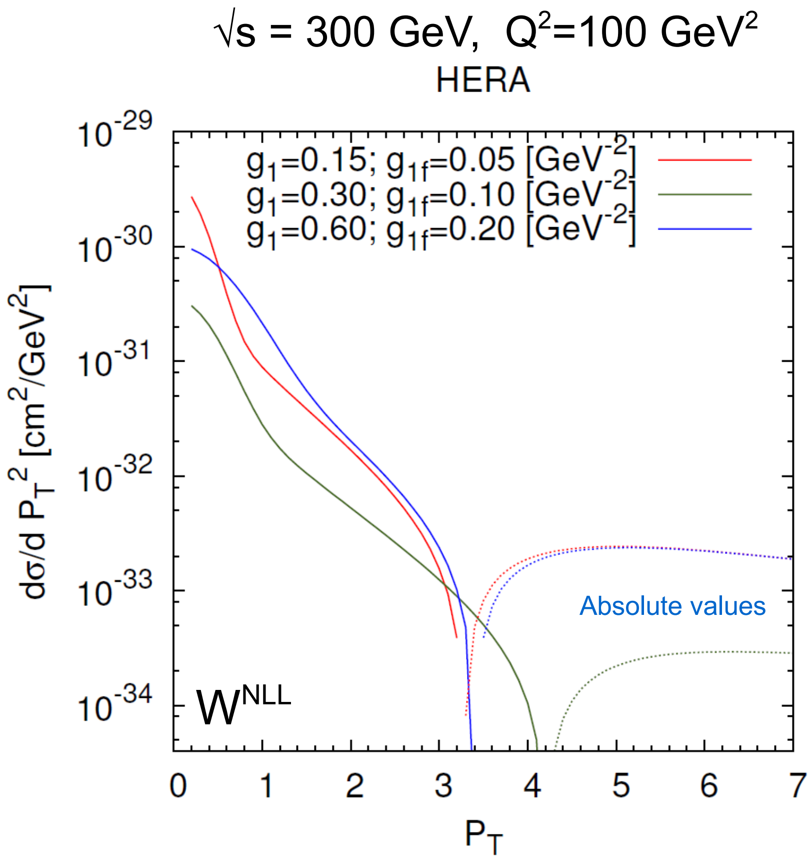




- F_{NP} takes into account the **non-perturbative** behavior at large b_T (i.e. small P_T)
- F_{NP} induces a (very mild) dependence on the parameters of the non-perturbative model at small P_T
- The three curves change sign at the same P_T

Non perturbative contributions - HERA

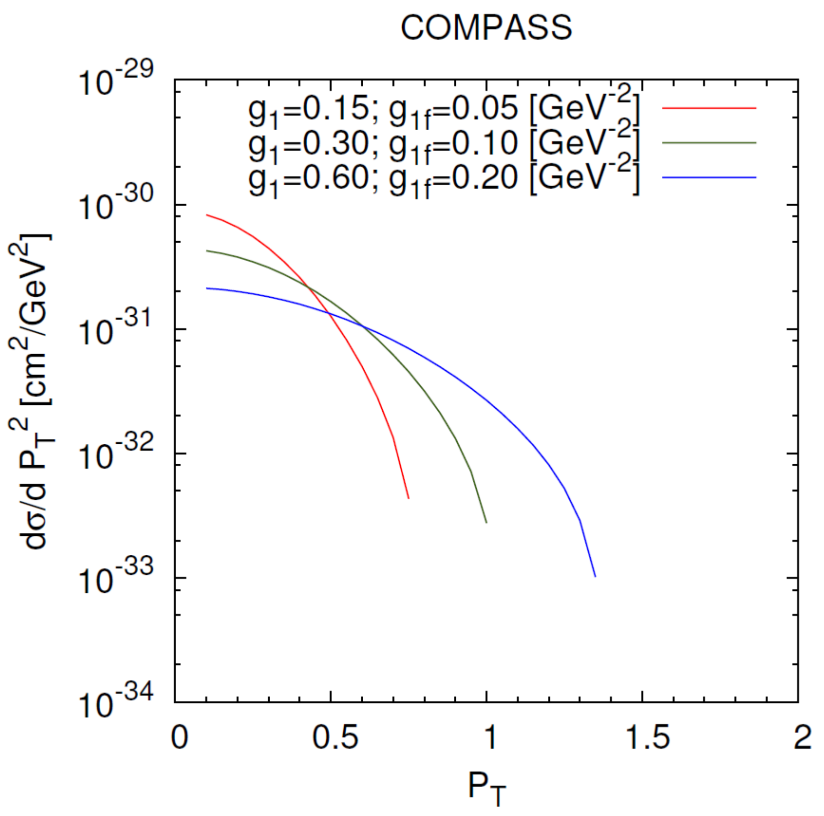




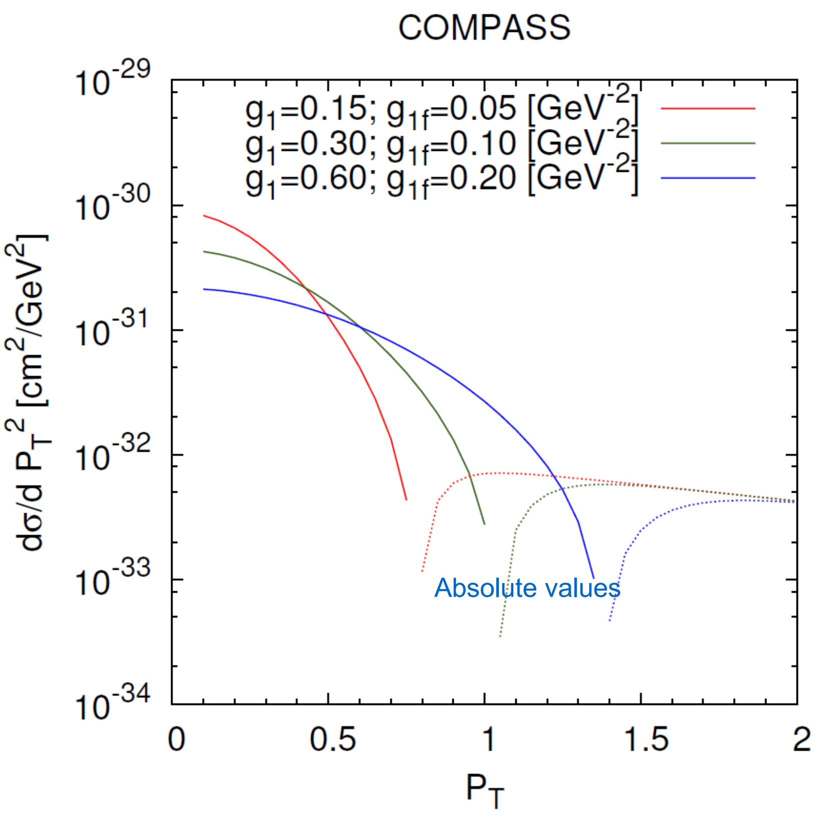
- F_{NP} takes into account the **non-perturbative** behavior at large b_T (i.e. small P_T)
- F_{NP} induces a (more visible) dependence on the parameters of the non-perturbative model at small P_T
- The three curves change sign at very similar values of P_T

Non perturbative contributions - COMPASS

$\sqrt{s}$ = 17 GeV, Q^2=10 GeV^2

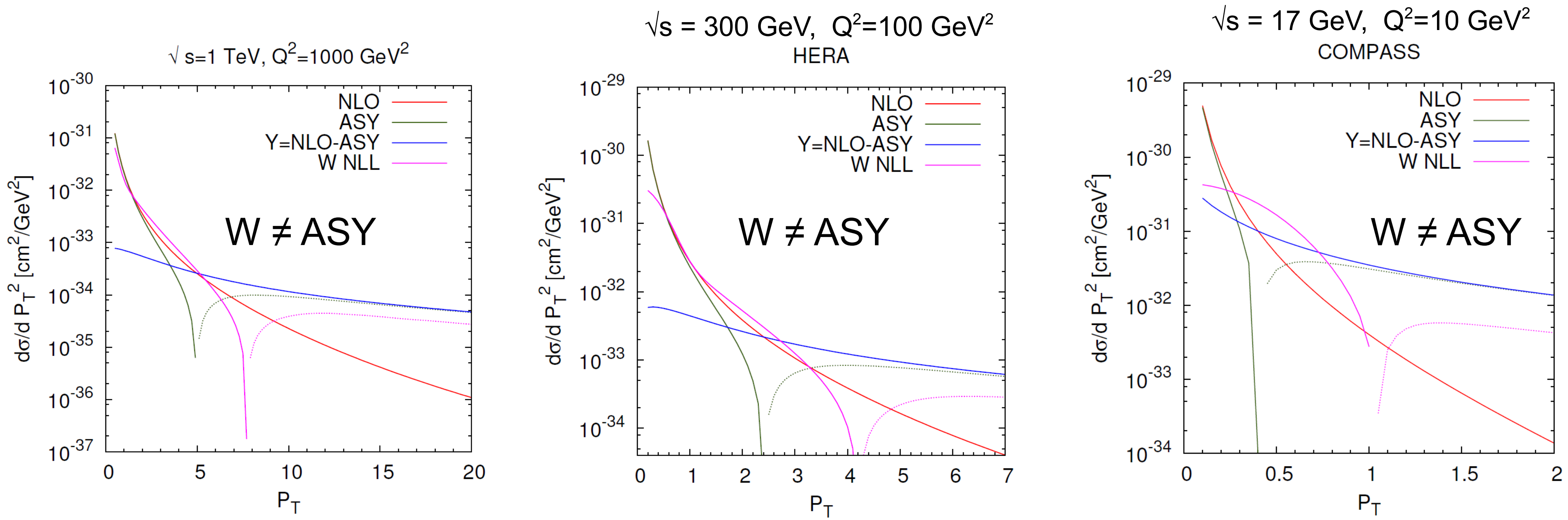


$\sqrt{s}$ = 17 GeV, Q^2=10 GeV^2



- F_{NP} takes into account the **non-perturbative** behavior at large b_T (i.e. small P_T)
- F_{NP} induces a **VERY STRONG** dependence on the parameters of the non-pert. model at small/moderate P_T
- The three curves change sign at **very different** values of P_T

Interplay between perturbative and non-perturbative contributions



- Notice that ASY and W become negative at different values of P_T
- Y can become very large
- The P_T values at which ASY and W become negative depend strongly on the considered kinematics

At $P_T \sim Q$, if $W \rightarrow$ ASY then $W+Y \rightarrow$ NLO

IS ANY MATCHING POSSIBLE ???

Fixed order cross section

We saw that W never approaches ASY
This is partly due to non-perturbative contributions
Therefore, instead of setting dσ = W + Y, let's try a different matching prescription

$$d\sigma = W^{NLL} - W^{FXO} + NLO$$

- ✓ W^{FXO} is the NLL resummed cross section approximated at first order in α_s, with a first order expansion of the Sudakov exponential

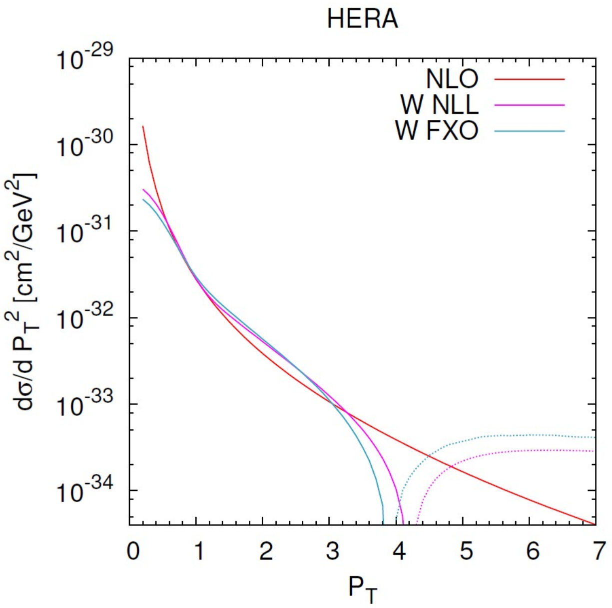
In principle, if there is no non-perturbative content and in the limit $b_T \rightarrow 0$ and $P_T \rightarrow$ infinity
Then one can show that $W^{FXO} \rightarrow$ ASY

- ✓ In general W^{FXO} contains the same non-perturbative content as that we give to W^{NLL}

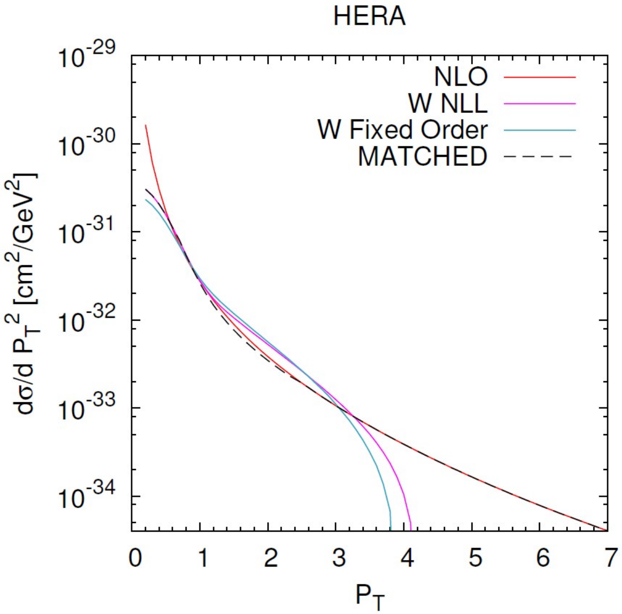
Therefore, with this prescription we might be able to find kinematical regions in which $W^{FXO} \sim W^{NLL}$

A case when this matching works ...

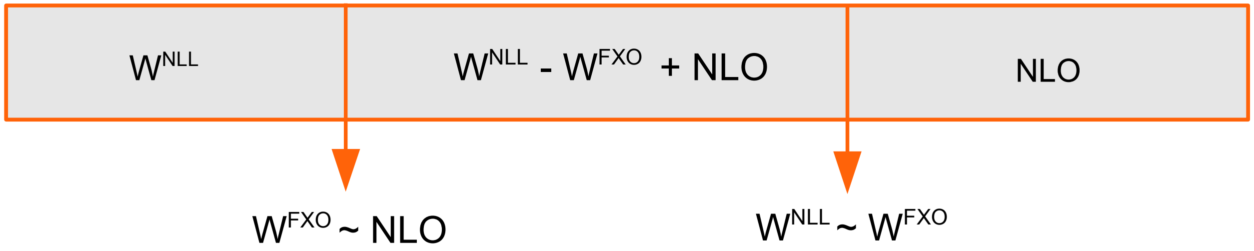
$\sqrt{s}$ = 300 GeV, Q^2=100 GeV^2



$\sqrt{s}$ = 300 GeV, Q^2=100 GeV^2



Here W^{NLL} and W^{FXO} are roughly the same over a range wide enough to allow for a safe matching



W^{NLL}

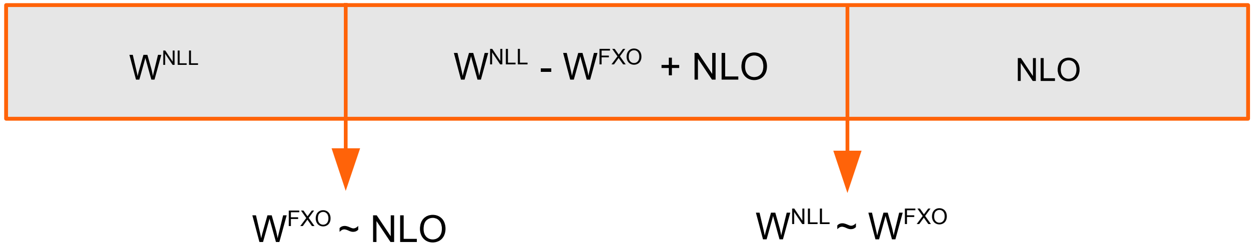
$W^{NLL} - W^{FXO}$ + NLO

NLO

$W^{FXO} \sim$ NLO

$W^{NLL} \sim W^{FXO}$

A case when this matching works …

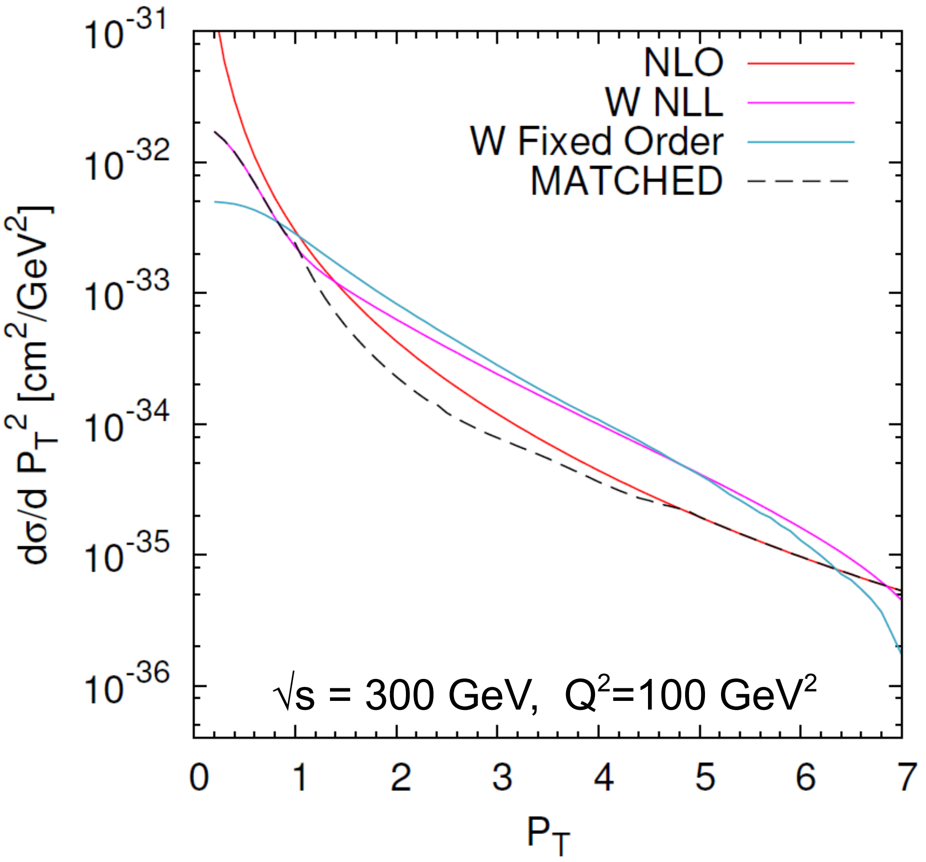


W^{NLL} | $W^{NLL} - W^{FXO} + NLO$ | NLO

$W^{FXO} \sim$ NLO

$W^{NLL} \sim W^{FXO}$

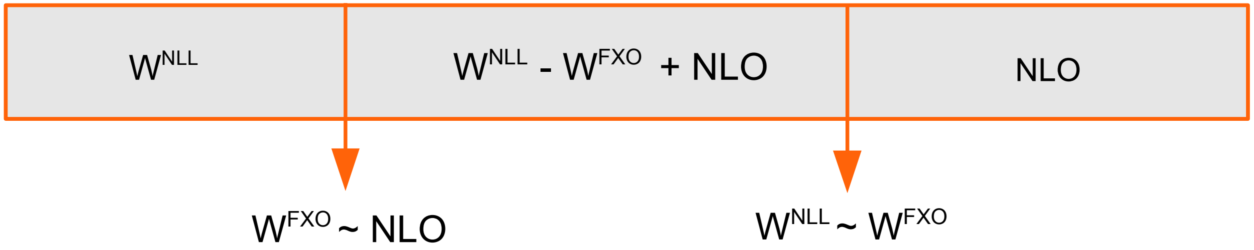
Matching OK !



Here all the curves are reasonably close to each other and they have roughly the same curvature.

- ✔ At “low” P_T there is a region where $W^{NLL} \sim W^{FXO}$
- ✔ At “large” P_T there is a region where $W^{FXO} \sim$ NLO

COMPASS ... a case when the matching does not work



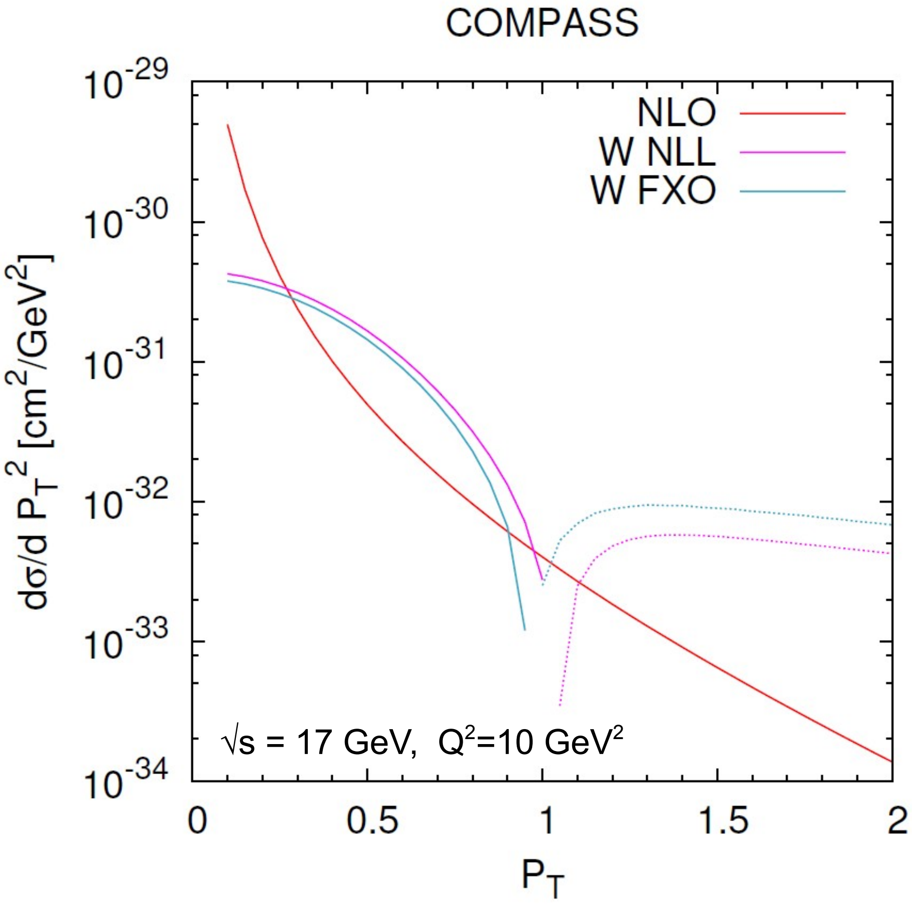
W^{NLL}

$W^{NLL} - W^{FXO} + NLO$

NLO

$W^{FXO} \sim NLO$

$W^{NLL} \sim W^{FXO}$



Here the curves are far from each other and they have different curvatures.

- ✗ The matching should be done over a very small P_T region
- ✗ No way to realize a smooth matching, without “cusps”.

Conclusions ?

- ✔ *Resummation in the impact parameter b_T space is a very powerful tool. However, it's successful implementation is affected by a number of practical difficulties (the kinematics of the process, the parameters used to model the non-perturbative content of the SIDIS cross section, etc ...).*
- ✔ *Performing phenomenological studies in the b_T space is rather difficult, as we loose the direct connection of our inputs to the exact outcome in the conjugate P_T space.*

  It becomes hard to define the boundaries of the three regions of interest:

  $$P_T \sim \lambda_{QCD} << Q\,,\ \lambda_{QCD} << P_T << Q\,,\ P_T \sim Q\,,\quad P_T > Q\,.$$

- ✔ *Matching prescriptions have to be applied to achieve a reliable description of the SIDIS process over the full P_T range, going smoothly from one region to the following.*
- ✔ *Matching procedures seem to be successful only in those cases where W^{NLL}, W^{FXO}, ASY and NLO are reasonably close to each other and have similar curvatures over reasonably wide regions (to allow us to switch smoothly from one the other) and when the effect of the non-perturbative Sudakov contribution, F_{NP}, does not stretch to the large P_T region.*
- ✔ *For COMPASS and HERMES data, these matching procedures cannot be reliably applied.*

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