QCD evolution of nuclear quark-gluon correlation function

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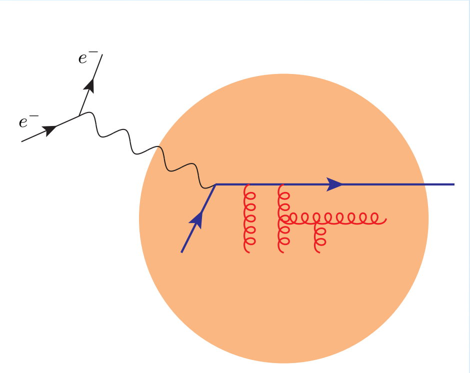
Outline

* Introduction
* NLO at leading twist → DGLAP evolution of PDF (FF)
* NLO at twist-four (TMB effect)

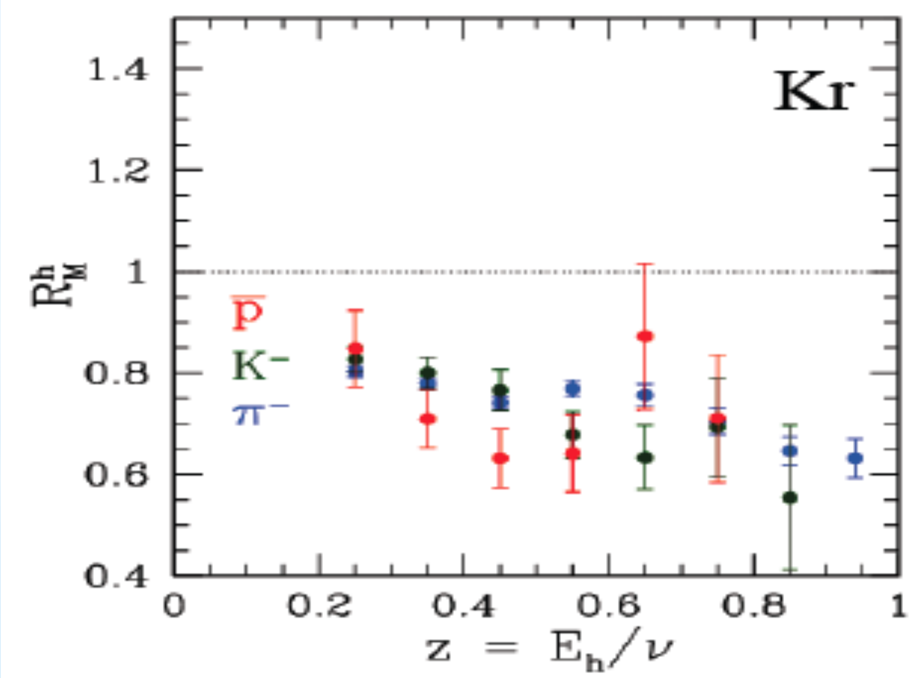
  → QCD evolution of nuclear q-g correlation function
* Summary

Introduction

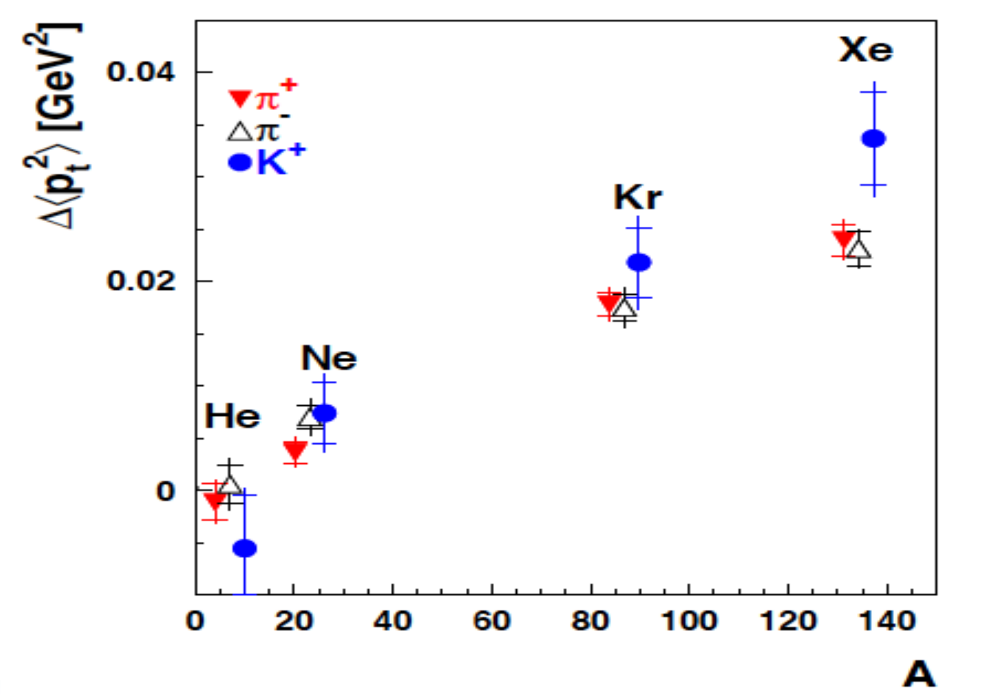
- Multiple scattering in SIDIS



$$\Delta\langle \ell_T^2 \rangle = \langle \ell_T^2 \rangle^{eA} - \langle \ell_T^2 \rangle^{eN}$$



HERMES: Phys.Lett.B577 (2003) 37



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TMB (Transverse momentum broadening)

$$\Delta\langle \ell_T^2\rangle = \langle \ell_T^2\rangle^{eA} - \langle \ell_T^2\rangle^{eN}$$

$$= \frac{\int d\ell_T^2 \ell_T^2 \frac{d\sigma_{eA}}{dQ^2 d\ell_T^2}}{\frac{d\sigma_{eA}}{dQ^2}} - \frac{\int d\ell_T^2 \ell_T^2 \frac{d\sigma_{eN}}{dQ^2 d\ell_T^2}}{\frac{d\sigma_{eN}}{dQ^2}}$$

$$= \frac{\int d\ell_T^2 \ell_T^2 \frac{d\sigma_{eA}^S}{dQ^2 d\ell_T^2} + \int d\ell_T^2 \ell_T^2 \frac{d\sigma_{eA}^D}{dQ^2 d\ell_T^2} + \cdots}{\frac{d\sigma_{eA}}{dQ^2}} - \frac{\int d\ell_T^2 \ell_T^2 \frac{d\sigma_{eN}^S}{dQ^2 d\ell_T^2}}{\frac{d\sigma_{eN}}{dQ^2}}$$

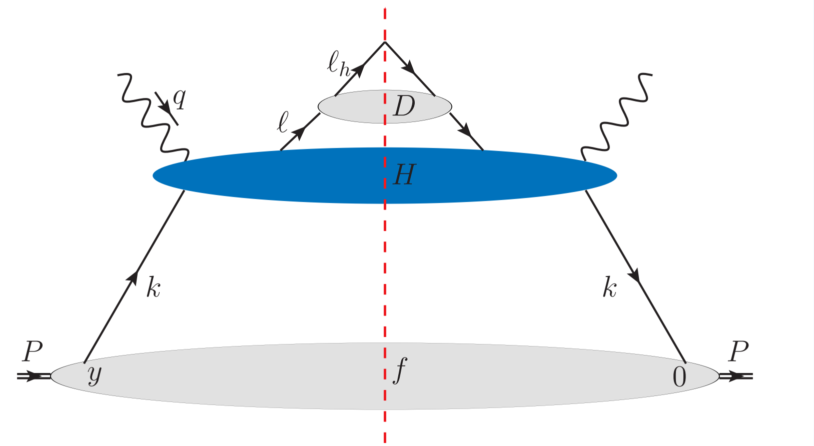
$$\approx \frac{\int d\ell_T^2 \ell_T^2 \frac{d\sigma_{eA}^D}{dQ^2 d\ell_T^2}}{\frac{d\sigma_{eA}}{dQ^2}}$$

Single scattering --> localized in space

Double scattering --> leading contribution to transverse Momentum broadening

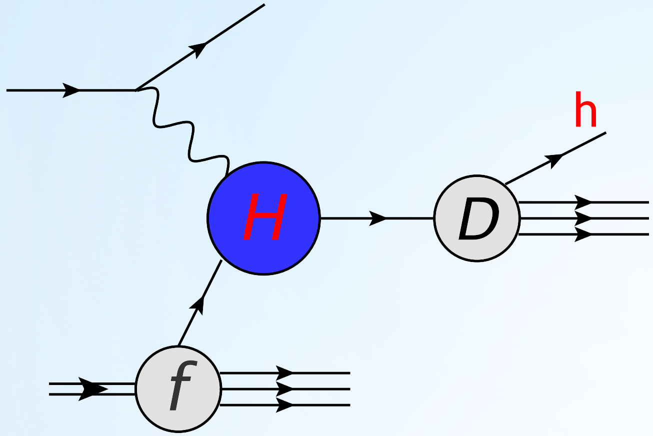
TMB – QCD dynamics beyond the single scattering picture

Single scattering in SIDIS



(a)

SIDIS at leading twist



$$e(L_1) + A(P) \to e(L_2) + h(\ell_h) + X$$

$$x_B = \frac{Q^2}{2p \cdot q}, \qquad y = \frac{p \cdot q}{p \cdot L_1}, \qquad z_h = \frac{p \cdot \ell_h}{p \cdot q},$$

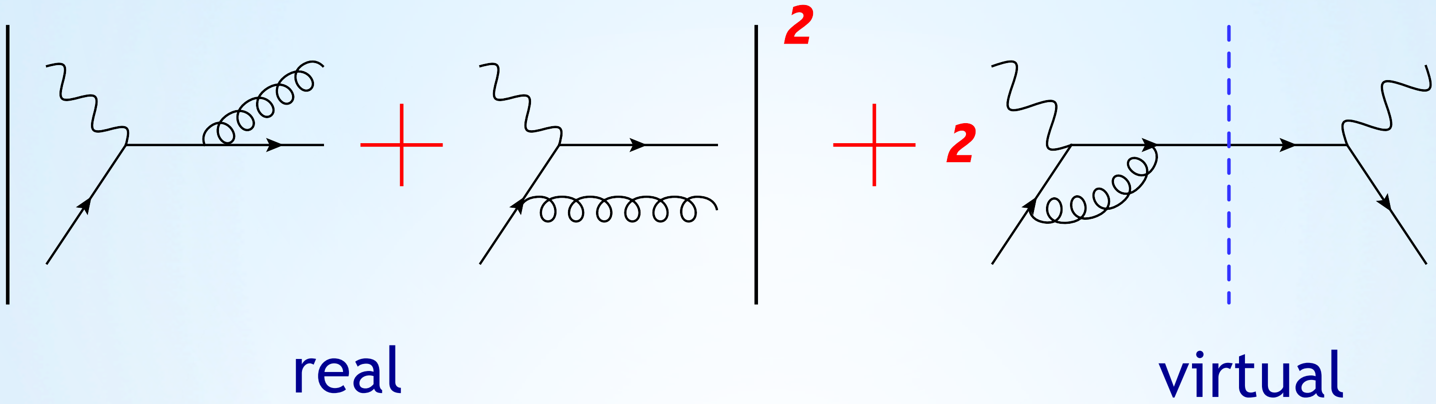
$$\hat{x} = \frac{x_B}{x}, \qquad \hat{z} = \frac{z_h}{z}, \qquad z = \frac{\ell_h}{\ell}$$

- SIDIS Cross section - LO

$$\frac{d\sigma^{LO}}{dx_B dy dz_h} = \sigma_0 \sum_q e_q^2 \int \frac{dz}{z} D_{h/q}(z) \int \frac{dx}{x} f_q(x) \delta(1 - \hat{x}) \delta(1 - \hat{z})$$

$$\sigma_0 = \frac{2\pi\alpha_{\rm em}^2}{Q^2} \frac{1 + (1 - y)^2}{y} (1 - \epsilon) \qquad\qquad n = 4 - 2\epsilon$$

- NLO – SIDIS



Four possible regions of gluon momentum:

1. collinear to initial quark (PDF)
2. collinear to final quark (FF)
3. soft (IR singularities cancel)
4. hard (NLO correction)

- In MSbar scheme ($\frac{1}{\hat{\epsilon}} = \boxed{\frac{1}{\epsilon}} + \ln(4\pi) - \gamma_E$) $\quad n = 4 - 2\epsilon$

FF: $D_q(z_h,\mu^2) = D_q^0(z_h) + \frac{\alpha_s}{2\pi}\int \frac{dz}{z}\left(-\frac{1}{\hat{\epsilon}}\right) P_{qq}(\hat{z}) D_q(z)$

PDF: $f_q(x_B,\mu^2) = f_q^0(x_B) + \frac{\alpha_s}{2\pi}\int \frac{dx}{x}\left(-\frac{1}{\hat{\epsilon}}\right) P_{qq}(\hat{x}) f_q(x)$

- Cross section for SIDIS at twist-2

$$\frac{d\sigma}{dx_B dy dz_h} = \sigma_0 \sum_q e_q^2 \int \frac{dx}{x}\frac{dz}{z} f_q(x,\mu^2) D_q(z,\mu^2)\delta(1-\hat{x})\delta(1-\hat{z}) \longrightarrow \text{LO}$$

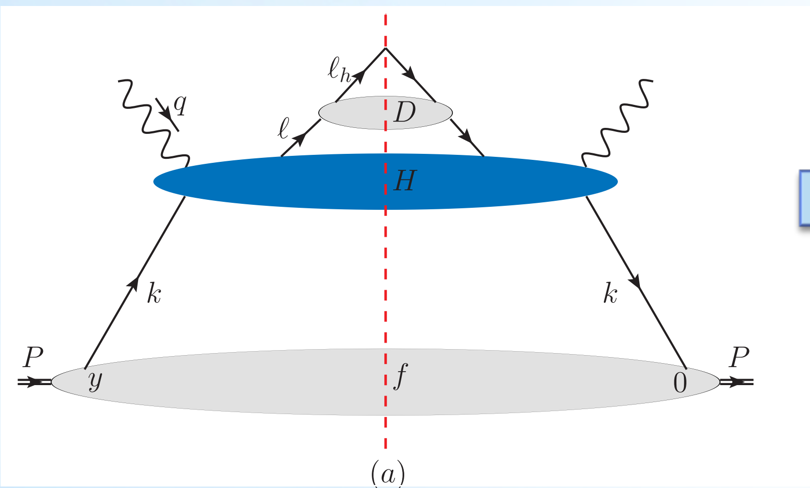
$$+ \sigma_0 \frac{\alpha_s}{2\pi} \sum_q e_q^2 \int \frac{dx}{x}\frac{dz}{z} f_q(x,\mu^2) D_q(z,\mu^2) H^{NLO}(x,z,\mu^2) \longrightarrow \text{NLO}$$

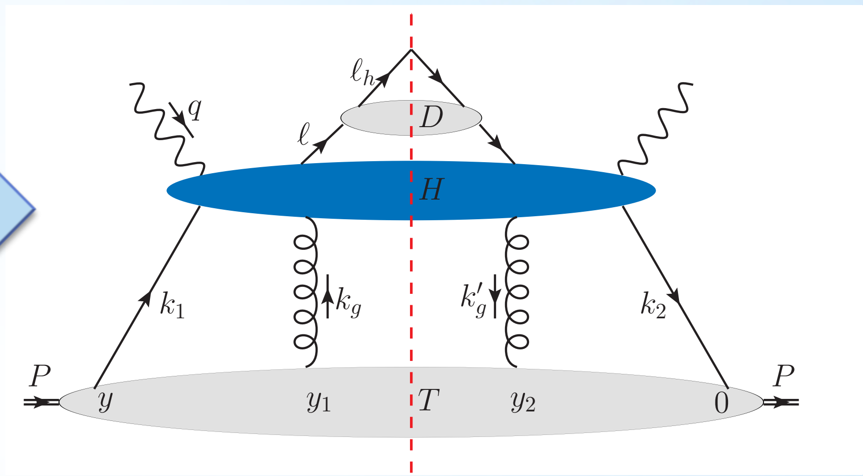
- DGLAP evolution (scaling violation)

$$\mu^2 \frac{d}{d\mu^2}\begin{bmatrix} f_q(x_B,\mu^2) \\ f_g(x_B,\mu^2) \end{bmatrix} = \frac{\alpha_s}{2\pi}\int_{x_B}^1 \frac{dx}{x}\begin{bmatrix} P_{qq}(\hat{x}) & P_{qg}(\hat{x}) \\ P_{gq}(\hat{x}) & P_{gg}(\hat{x}) \end{bmatrix}\begin{bmatrix} f_q\left(x,\mu^2\right) \\ f_g\left(x,\mu^2\right) \end{bmatrix} \longrightarrow \text{PDF}$$

$$\mu^2 \frac{d}{d\mu^2}\begin{bmatrix} D_q(z_h,\mu^2) \\ D_g(z_h,\mu^2) \end{bmatrix} = \frac{\alpha_s}{2\pi}\int_{z_h}^1 \frac{dz}{z}\begin{bmatrix} P_{qq}(\hat{z}) & P_{gq}(\hat{z}) \\ P_{qg}(\hat{z}) & P_{gg}(\hat{z}) \end{bmatrix}\begin{bmatrix} D_q\left(z,\mu^2\right) \\ D_g\left(z,\mu^2\right) \end{bmatrix} \longrightarrow \text{FF}$$

Double scattering in SIDIS (Twist-4)



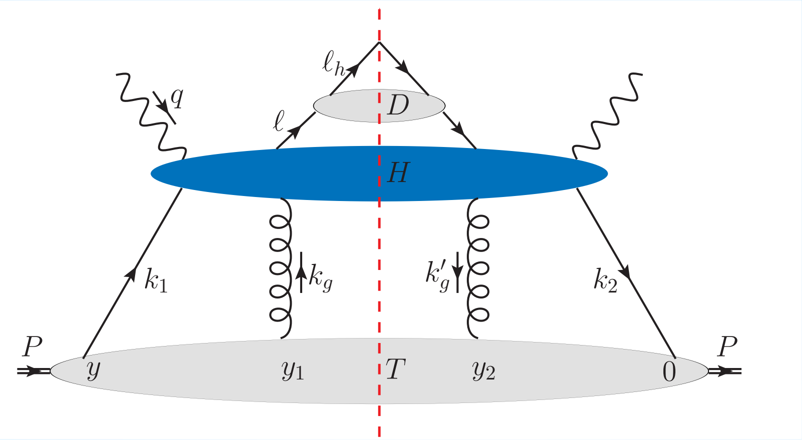


$$\sigma \sim \sigma_0 \left[1 + \frac{1}{Q^2} + \cdots\right]$$

Large nuclei

$$\sigma \sim \sigma_0 \left[1 + \boxed{\frac{A^{1/3}}{Q^2}} + \cdots\right]$$

General formalism



$$k_1 = x_1 p$$
$$k_2 = (x_1 + x_3) p$$
$$k_g = x_2 p + k_T$$
$$k'_g = (x_2 - x_3) p + k_T$$

- Twist-4 contribution

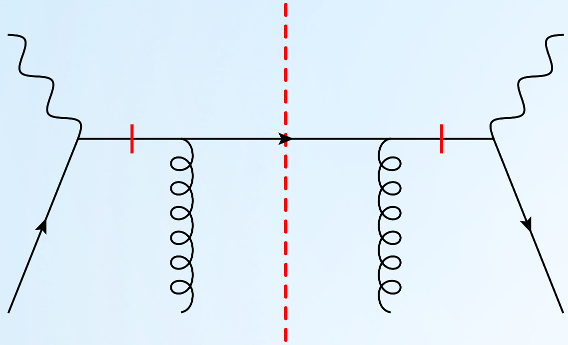
Qiu, Sterman

$$\frac{dW^D_{\mu\nu}}{dz_h} = \sum_q \int_{z_h}^1 \frac{dz}{z} D_{q\to h}(z) \int \frac{dy^-}{2\pi} \frac{dy_1^-}{2\pi} \frac{dy_2^-}{2\pi} \frac{1}{2} \langle A|\bar{\psi}_q(0)\,\gamma^+\, F_\sigma{}^+(y_2^-)\, F^{+\sigma}(y_1^-)\,\psi_q(y^-)|A\rangle$$
$$\times \left(-\frac{1}{2} g^{\alpha\beta}\right) \left[\frac{1}{2}\,\frac{\partial^2}{\partial k_T^\alpha \partial k_T^\beta}\,\overline{H}^D_{\mu\nu}(y^-, y_1^-, y_2^-, k_T, p, q, \hat{z})\right]_{k_T=0}$$

Color gauge invariant!

Twist-4 leading order

- LO contribution to weighted cross section



Guo, 1998
Guo, Qiu 2000

$$\sigma_h = (4\pi^2\alpha_s z_h^2/N_c)\sigma_0$$

$$\frac{d\langle \ell_{hT}^2 \sigma^D\rangle}{dx_B dy dz_h} = \sigma_h \sum_q e_q^2 \int \frac{dz}{z} D_{h/q}(z) \int \frac{dx}{x} T_{qg}(x,0,0)\delta(1-\hat{x})\delta(1-\hat{z})$$

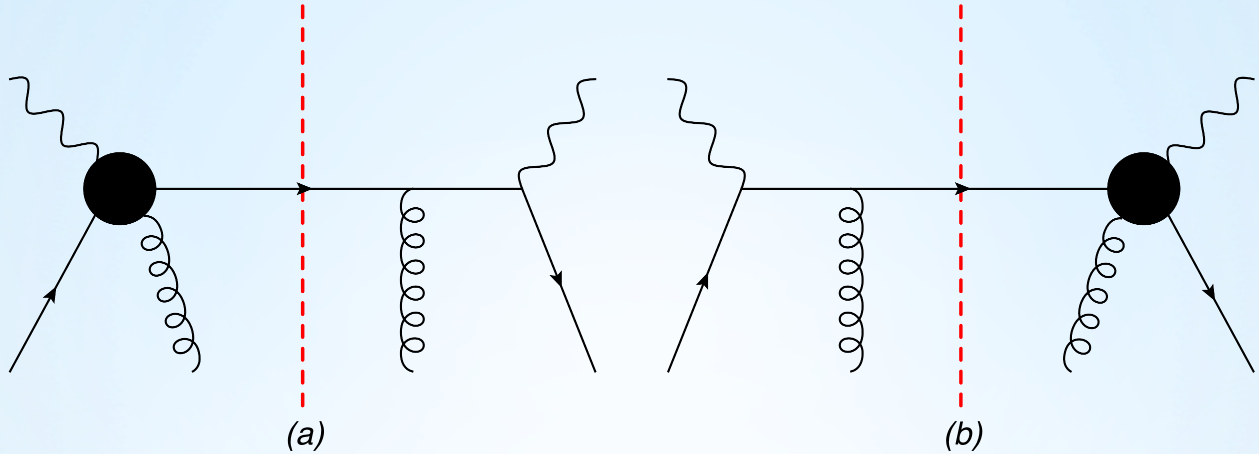
q-g correlation: $$T_{qg}(x_1,x_2,x_3) = \int \frac{dy^-}{2\pi} e^{ix_1p^+y^-} \int \frac{dy_1^- dy_2^-}{4\pi} e^{ix_2p^+(y_1^- - y_2^-)} e^{ix_3p^+y_2^-} \times \langle A|\bar{\psi}_q(0)\gamma^+ F_\sigma^+(y_2^-)F^{\sigma+}(y_1^-)\psi_q(y^-)|A\rangle \theta(y_2^-)\theta(y_1^- - y^-)$$

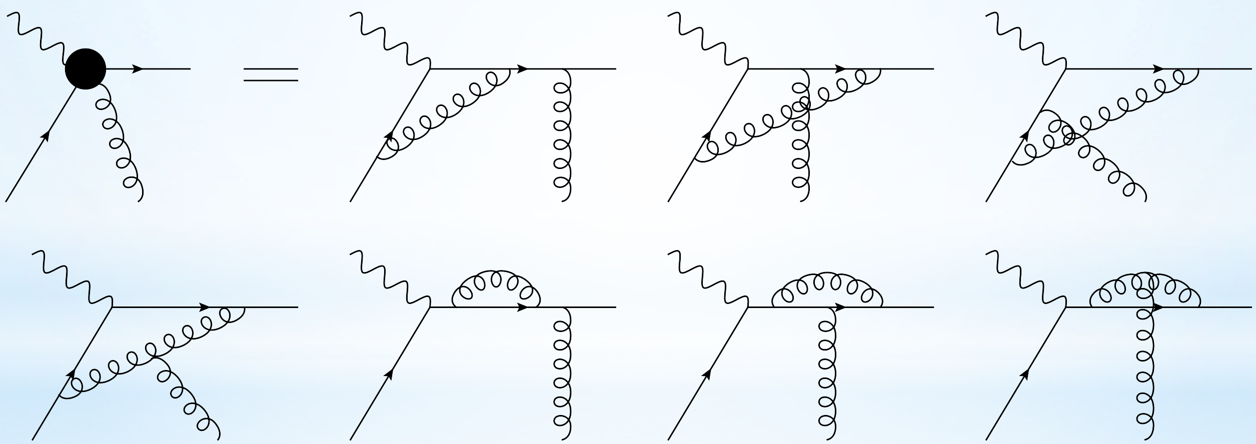
- Leading contribution to broadening of hadron

$$\Delta\langle \ell_{hT}^2\rangle = \left(\frac{4\pi^2\alpha_s}{N_c} z_h^2\right) \frac{\sum_q e_q^2 T_{qg}(x_B,0,0) D_{h/q}(z_h)}{\sum_q e_q^2 f_{q/A}(x_B) D_{h/q}(z_h)}$$

Provide a way to measure the fundamental T-4 quark-gluon correlation function.

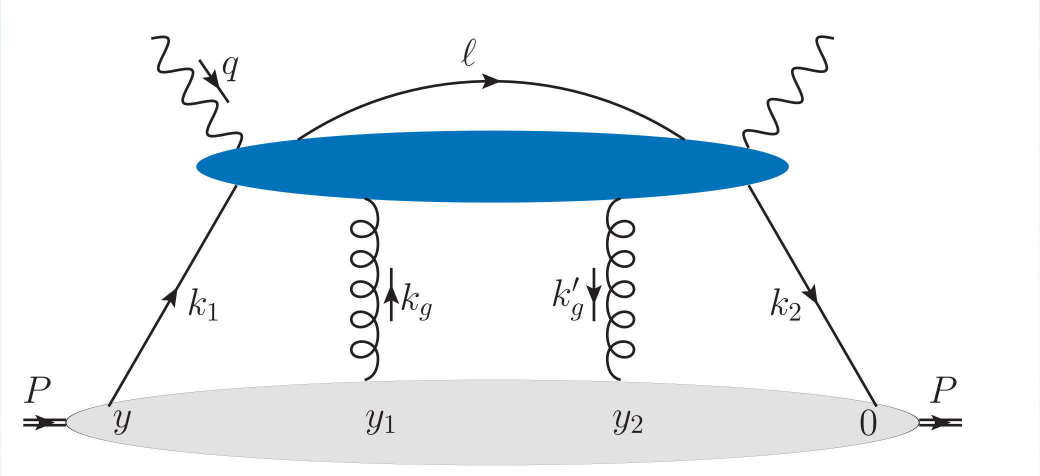
Twist-4 NLO – Virtual correction



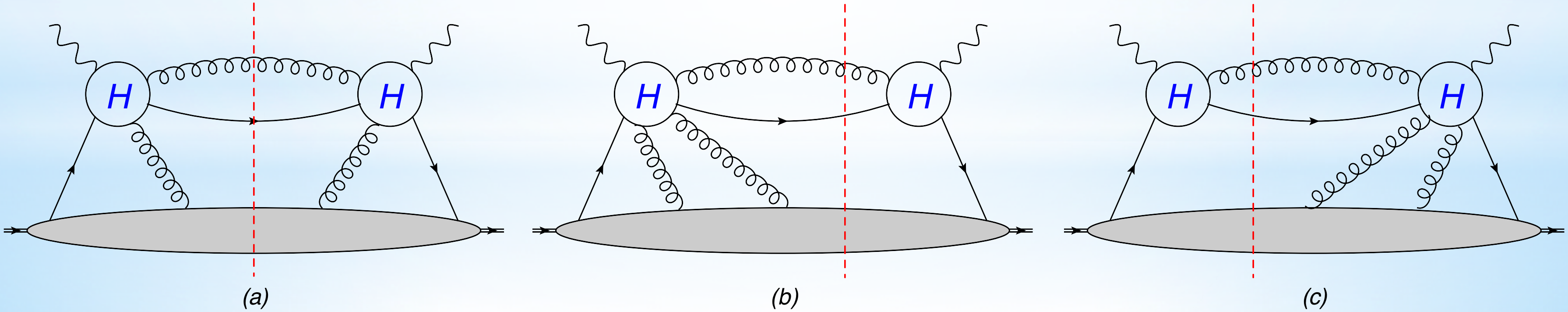


All additional diagrams are just self-energies for external on-shell lines and do not contribute. Only central-cut contribute to TMB.

Twist-4 NLO – Real correction



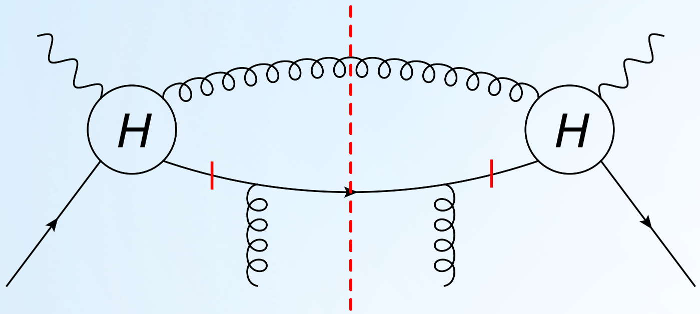
- Organized in terms of cuts



central-cut right-cut left-cut

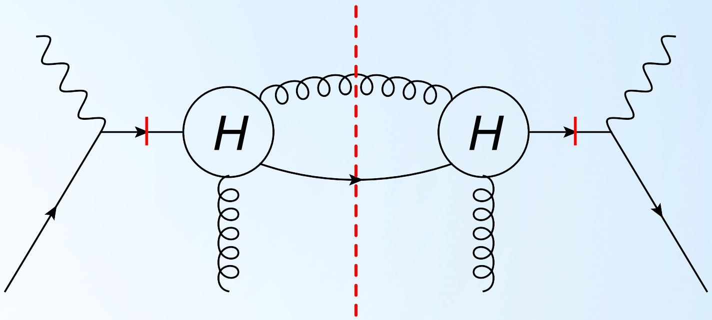
Central-cut

- Organized in terms of soft- and hard-rescatterings



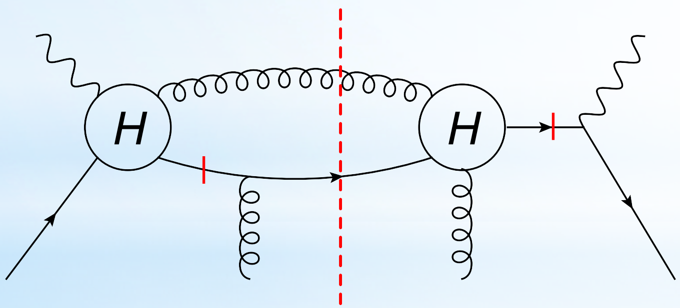
(a)

soft-soft



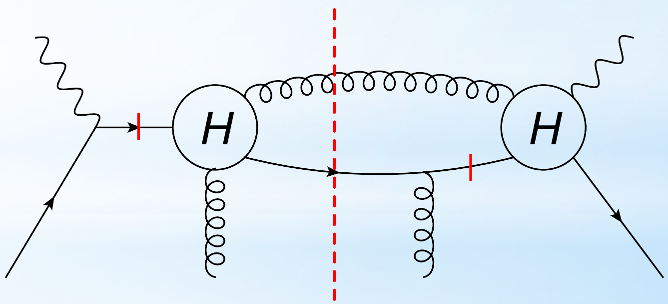
(b)

hard-hard



(c)

soft-hard



(d)

hard-soft

- Soft-collinear divergence (double pole)

Virtual: $-\frac{2}{\epsilon^2}C_F\delta(1-\hat{x})\delta(1-\hat{z})T_F(x,0,0)$

S-S: $\frac{2}{\epsilon^2}C_F\delta(1-\hat{x})\delta(1-\hat{z})T_F(x,0,0)$

} 0

H-H: $\frac{2}{\epsilon^2}C_A\delta(1-\hat{x})\delta(1-\hat{z})T_F(x_B,x-x_B,0)$

S-H: $-\frac{1}{\epsilon^2}C_A\delta(1-\hat{x})\delta(1-\hat{z})T_F(x,0,x_B-x)$ $\hat{x}=\frac{x_B}{x}$

H-S: $-\frac{1}{\epsilon^2}C_A\delta(1-\hat{x})\delta(1-\hat{z})T_F(x_B,x-x_B,x-x_B)$

} 0

Soft-collinear divergence disappears!

Total contribution

- Combine all the results

$$\frac{d\langle \ell_{hT}^2 \sigma\rangle}{dz_h} = \sigma_h \int \frac{dx}{x} \int_{z_h}^1 \frac{dz}{z} D_{q/h}(z) \frac{\alpha_s}{2\pi} \left(\frac{4\pi\mu^2}{Q^2}\right)^\epsilon \frac{1}{\Gamma(1-\epsilon)}$$

Divergences come from central-cut

$$\left\{ -\frac{1}{\epsilon}\delta(1-\hat{x}) T_F(x,0,0) P_{qq}(\hat{z}) - \frac{1}{\epsilon}\delta(1-\hat{z}) \left[T_F(x,0,0)P_{qq}(\hat{x}) + P_{qg\to qg}(\hat{x}) \otimes T_F(x,x,x_B)\right] + \cdots \right\}$$

New splitting function

$$P_{qg\to qg}(\hat{x}) \otimes T_F(x,x,x_B)$$

$$= C_A \left[\frac{2}{(1-\hat{x})_+} T(x_B, x-x_B, 0) - \frac{1}{2} \frac{1+\hat{x}}{(1-\hat{x})_+} \left(T(x,0,x_B-x) + T(x_B, x-x_B, x-x_B)\right)\right]$$

- Redefinition of FF (MSbar) – well known

$$D_q(z_h,\mu^2) = D_q^0(z_h) + \frac{\alpha_s}{2\pi} \int \frac{dz}{z} \left(-\frac{1}{\hat{\epsilon}}\right) P_{qq}(\hat{z}) D_q(z)$$ → DGLAP

- Redefinition of T4 (MSbar) - NEW

$$T_F(x_B,0,0,\mu^2) = T_F^{(0)}(x_B,0,0) - \frac{\alpha_s}{2\pi}\frac{1}{\hat{\epsilon}} \int_{x_B}^1 \frac{dx}{x} \left[P_{qq}(\hat{x}) T_F(x,0,0) + P_{qg\to qg}(\hat{x}) \otimes T_F(x,x,x_B)\right]$$

Factorization at twist-4

- Transverse momentum square weighted cross section

$$\frac{d\langle \ell_{hT}^2 \sigma\rangle}{dz_h} = \sigma_0 \int_{z_h}^1 \frac{dz}{z} D_{q/h}(z,\mu^2) \int_{x_B}^1 \frac{dx}{x} T_F(x,0,0,\mu^2)\delta(1-\hat{x})\delta(1-\hat{z})$$

→ T-4 LO

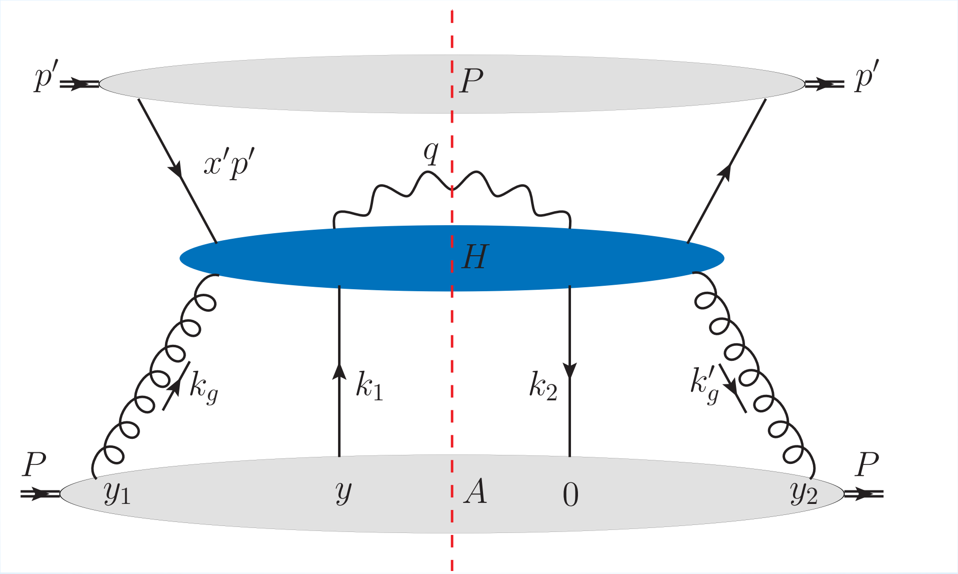
$$+ \sigma_0 \frac{\alpha_s}{2\pi} \int_{z_h}^1 \frac{dz}{z} D_{q/h}(z,\mu^2) \int_{x_B}^1 \frac{dx}{x} \left\{ \ln\left(\frac{Q^2}{\mu^2}\right) \left[\left(\delta(1-\hat{x}) P_{qq}(\hat{z}) + \delta(1-\hat{z}) P_{qq}(\hat{x})\right) T_F(x,0,0,\mu^2) \right.\right.$$
$$\left.\left. + \ \delta(1-\hat{z}) P_{qg\to qg}(\hat{x}) \otimes T_F(x,x,x_B,\mu^2)\right] + \left(F^C(\hat{x},\hat{z}) + F^A(\hat{x},\hat{z})\right) \otimes T_F(x,x,x_B,\mu^2) \right\}$$

T-4 NLO

- Evolution equation for T4 - NEW

$$\mu^2 \frac{\partial}{\partial \mu^2} T_F(x_B,0,0,\mu^2) = \frac{\alpha_s}{2\pi} \int_{x_B}^1 \frac{dx}{x} \left[P_{qq}(\hat{x}) T_F(x,0,0,\mu^2) + P_{qg\to qg}(\hat{x}) \otimes T_F(x,x,x_B,\mu^2) \right]$$

Verification in Drell-Yan



Real + Virtual → soft divergence cancel

Factorization in Drell-Yan at twist-4

$$\frac{d\langle q_T^2\sigma\rangle^{DY}}{dQ^2} = \sigma_0^{DY}\int\frac{dx'}{x'}f_{\bar{q}}(x')\int\frac{dx}{x}\frac{\alpha_s}{2\pi}\left(\frac{4\pi\mu^2}{Q^2}\right)^{\epsilon}\frac{1}{\Gamma(1-\epsilon)}\left\{-\frac{2}{\epsilon}T_F(x,0,0)P_{qq}(z)\right.$$
$$\left.-\frac{1}{\epsilon}\left[P_{qg\to qg}(z)\otimes T_F(x,x,x_B)\right]+\cdots\right\}$$

- Redefinition of beam PDF

$$f_q(x',\mu^2) = f_q^0(x') + \frac{\alpha_s}{2\pi}\int\frac{d\xi}{\xi}\left(-\frac{1}{\hat{\epsilon}}\right)P_{qq}(z)f_q(\xi) \qquad z = \frac{x'}{\xi}$$

- Redefinition of nuclear T-4 gluon-quark correlation function

$$T_F(x_B,0,0,\mu^2) = T_F^{(0)}(x_B,0,0) - \frac{\alpha_s}{2\pi}\frac{1}{\hat{\epsilon}}\int_{x_B}^{1}\frac{dx}{x}\left[P_{qq}(\hat{x})T_F(x,0,0) + P_{qg\to qg}(\hat{x})\otimes T_F(x,x,x_B)\right]$$

Exactly the same as that in SIDIS, it is universal!

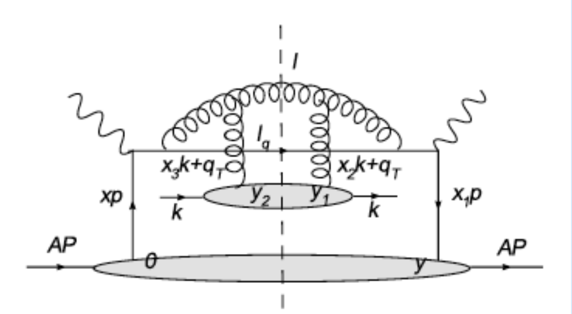
- Transverse momentum weighted cross section

$$\frac{d\langle q_T^2\sigma\rangle^{DY}}{dQ^2} = \sigma_0^{DY}\int\frac{dx'}{x'}f_{\bar{q}}(x',\mu^2)\int\frac{dx}{x}T_F(x,0,0,\mu^2)\delta(1-z)$$
$$+\sigma_0^{DY}\frac{\alpha_s}{2\pi}\int\frac{dx'}{x'}f_{\bar{q}}(x',\mu^2)\int\frac{dx}{x}H^{NLO}(z,x)\otimes T_F(x,x,x_B,\mu^2)$$

Discussion - Evolution of jet transport parameter

- Related to jet transport parameter

J. Casalderrey-Solana and X.-N. Wang (2008)



$$T_{qg}(x_B,0,0,\mu^2) \approx \frac{N_c}{4\pi^2\alpha_s} f_{q/A}(x_B,\mu^2) \int dy^- \hat{q}(\mu^2,y^-)$$

$$\hat{q}(\mu^2,y^-) = \frac{4\pi^2\alpha_s C_R}{N_c^2-1} \rho_N^A(y^-) x f_{g/N}(x)$$

- Evolution equation of jet transport parameter

$$\mu^2 \frac{\partial}{\partial\mu^2} T_F(x_B,0,0,\mu^2) = \frac{\alpha_s}{2\pi}\int_{x_B}^1 \frac{dx}{x}\left[P_{qq}(\hat{x})T_F(x,0,0,\mu^2) + P_{qg\to qg}(\hat{x})\otimes T_F(x,x,x_B,\mu^2)\right]$$

$$P_{qg\to qg}(\hat{x})\otimes T_F(x,x,x_B)$$
$$= C_A\left[\frac{2}{(1-\hat{x})_+}T(x_B,x-x_B,0) - \frac{1}{2}\frac{1+\hat{x}}{(1-\hat{x})_+}\left(T(x,0,x_B-x)+T(x_B,x-x_B,x-x_B)\right)\right]$$

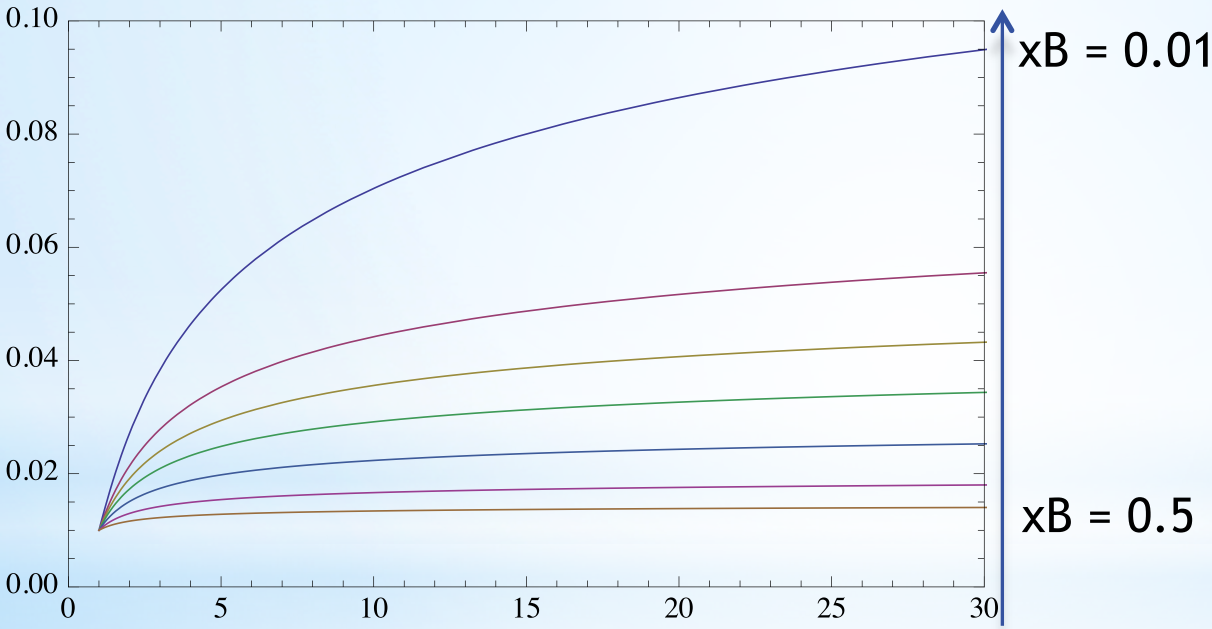
1. Large-xB limit (xB→1, LPM interference regime):

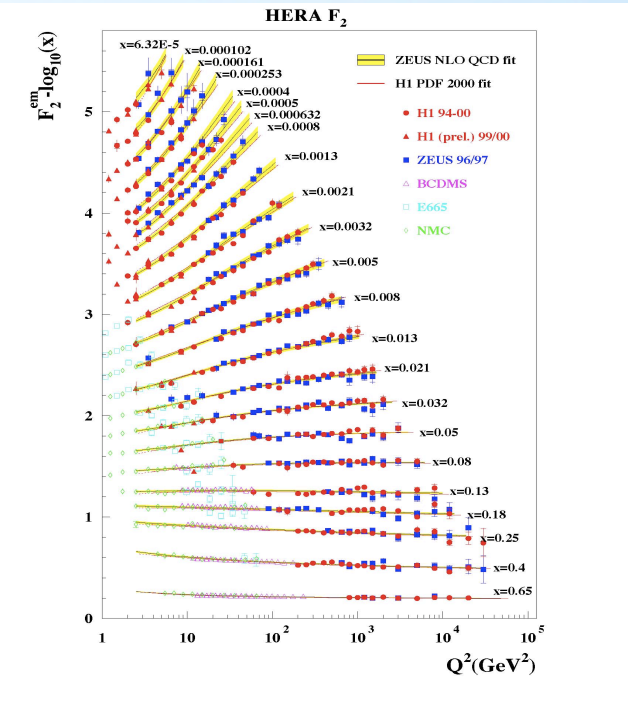
$$\mu^2 \frac{\partial \hat{q}(\mu^2)}{\partial\mu^2} = 0$$

Scaling in large Bjorken-x

2. Evolution of qhat in intermediate-x

$$\frac{\partial \hat{q}(\mu^2)}{\partial \ln \mu^2} = \frac{\alpha_s}{2\pi} C_A \ln(1/x_B)\hat{q}(\mu^2) \quad \Longrightarrow \quad \hat{q}(\mu^2) = \hat{q}(\mu_0^2) Exp\left[\frac{\alpha_s}{2\pi} C_A \ln(1/x_B)\ln(\mu^2/\mu_0^2)\right]$$

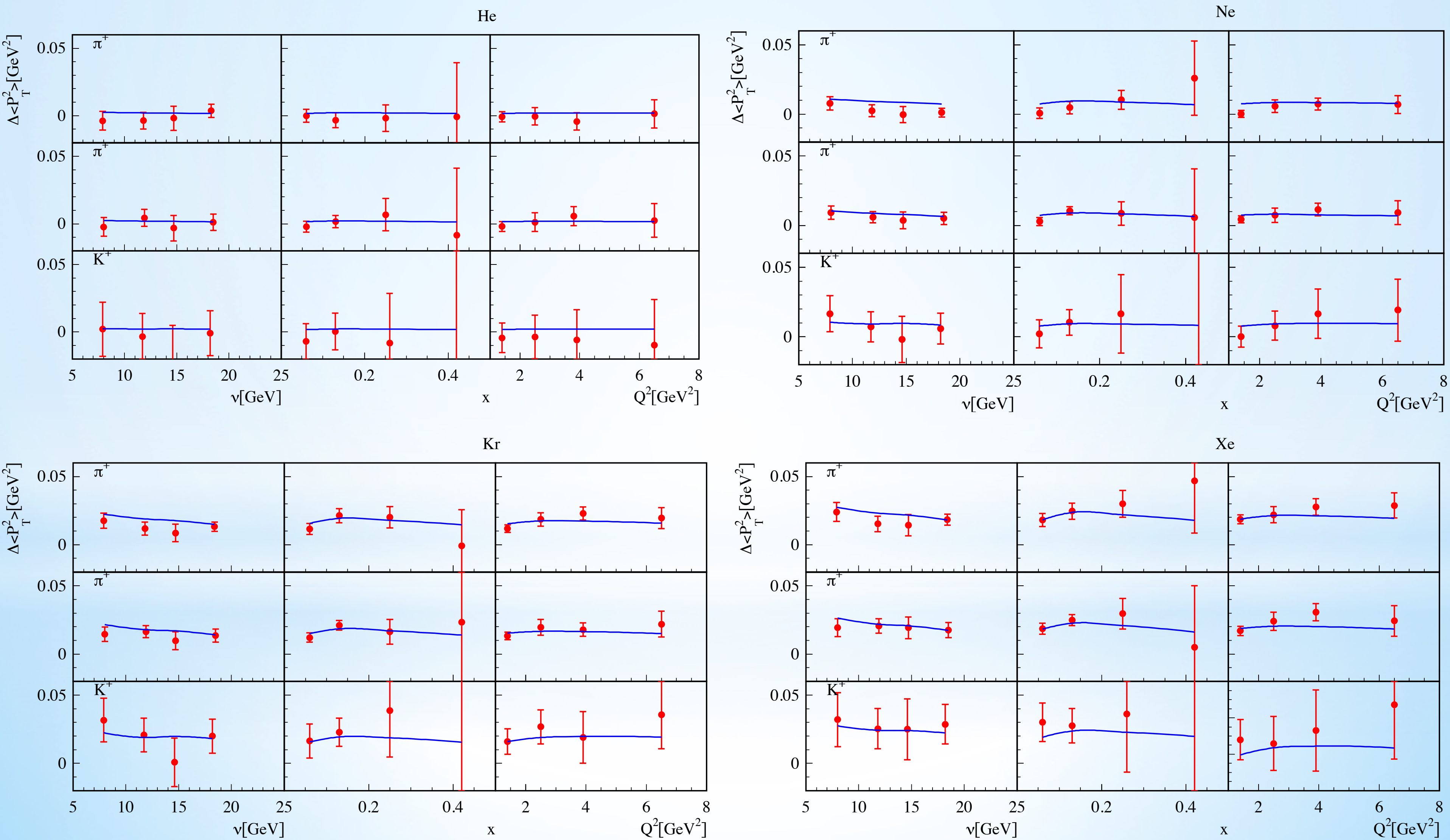




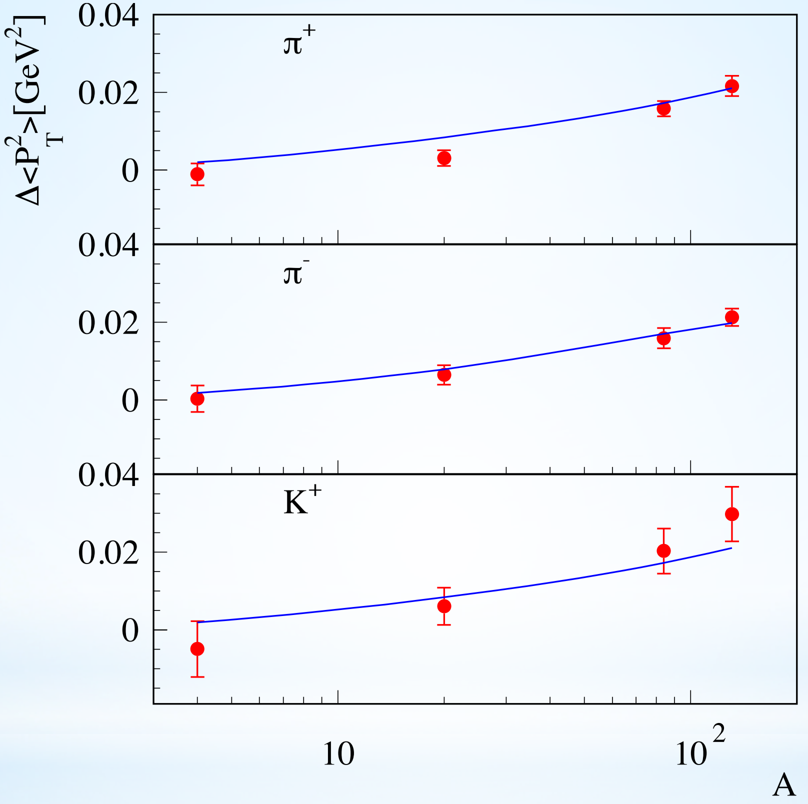
Scaling violation!

- Compare to HERMES data (LO with scale dependent q-g correlation function)

$$\hat{q}(\mu_0 = 1) = 0.015 GeV^2/fm$$



- Compare to HERMES data (LO with scale dependent q-g correlation function)



Summary

- We have calculated single inclusive hadron transverse momentum broadening in SIDIS as well as DY dilepton production at one loop order at twist-4.

- We verified QCD factorization beyond leading twist (twist-4).

- We derived the QCD evolution equation for twist-4 q-g correlation function.

- Our numerical results describe the TMB data at HERMES quite well for all kinematic dependences.