Sivers Function in the Quasi-Classical Approximation

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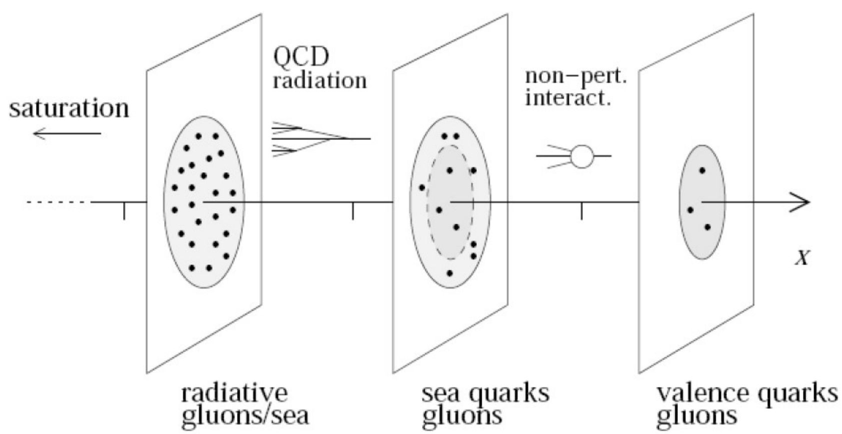

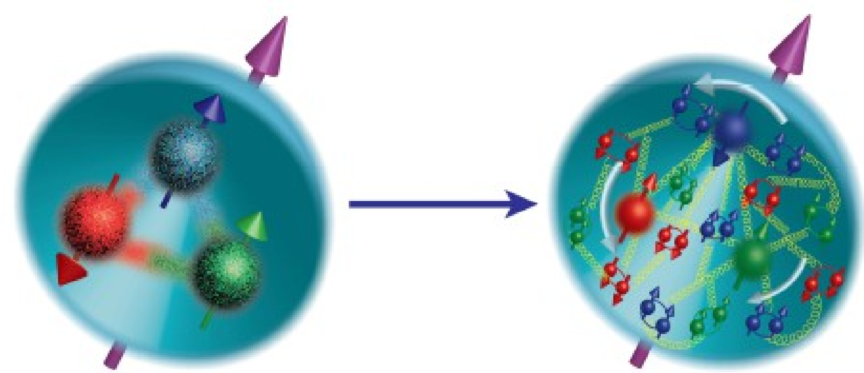

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QCD Evolution 2014

Outline

Review: the Sivers Function

- TMD parton distributions and factorization
- The Sivers function, time reversal, and initial- / final-state interactions
- The physical picture: QCD lensing

The Emergence of Saturation at High Energy

- The small-x limit: Regge kinematics
- The small-x gluon cascade and saturation
- DIS on a heavy nucleus: QCD shadowing
- The quasi-classical limit

Sivers Function in the Quasi-Classical Bjorken Limit

- A common regime: a heavy nucleus at Bjorken kinematics
- Quasi-classical factorization of the SIDIS cross-section and TMD's
- The Sivers function in the classical limit and orbital angular momentum
- Nuclear shadowing: the new importance of initial and final state interactions

Review: The Sivers Function

The Transverse-Momentum Paradigm

- **TMD Factorization** makes it possible to relate the intrinsic transverse-momentum distribution of partons within a hadron to external observables.

- The quark transverse-momentum structure of a hadron is described by the quark-quark correlation function

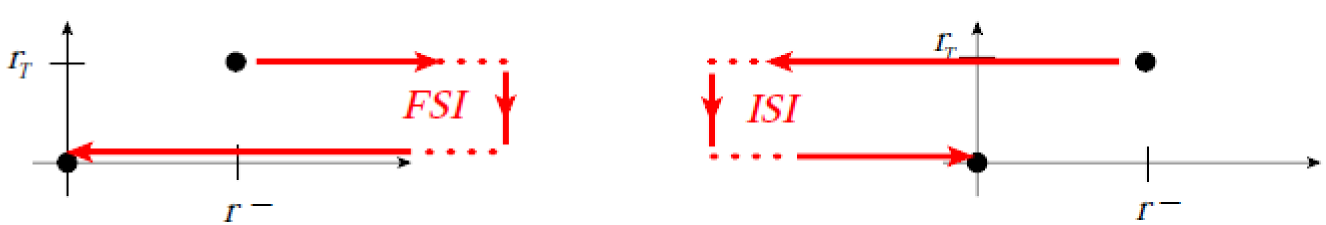
$$\Phi^C_{ij}(x,\underline{k}) \equiv \frac{1}{(2\pi)^3}\int d^{2-}r\, e^{ik\cdot r}\, \langle pS|\, \overline{\psi}_j(0)\, U_C[0,r]\, \psi_i(r)\, |pS\rangle_{r^+=0}$$

  - At lowest order in the coupling, Φ_{ij} possesses a quark density interpretation:

$$\Phi_{ij}(x,\underline{k}) = \frac{1}{4(2\pi)^3\mathcal{V}^-}\frac{1}{(k^+)^2}\sum_{\sigma\tau}\langle pS|\, b^\dagger_{k\tau} b_{k\sigma}\, |pS\rangle \left[\overline{U}^j_\tau(k) U^i_\sigma(k)\right]$$

  - The gauge link $U_C[0,r]$ makes the definition gauge-invariant, but mixes the parton density with the associated gluon field.

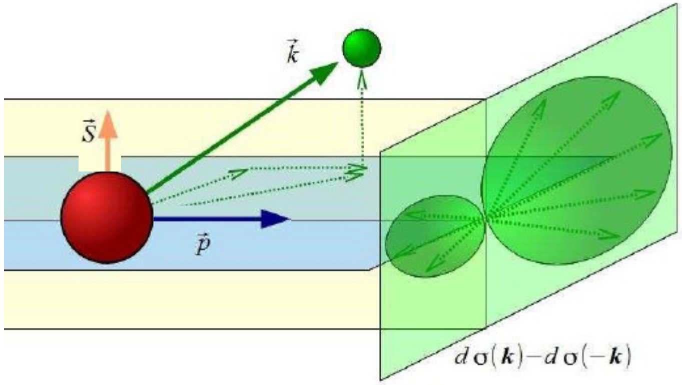
- Initial- and final-state interactions modify the parton distributions accessed in physical processes:



The Sivers Function: A Lesson In Time Reversal

- The Sivers function measures the intrinsic orbital asymmetry of quarks in a transversely-polarized hadron:

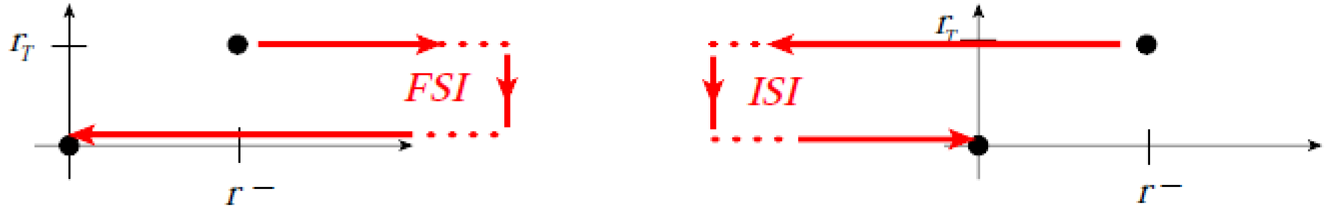
$$\frac{(\underline{S} \times \underline{k})}{m_N} f_{1T}^{\perp q}(x, k_T) = \frac{1}{4} \mathrm{Tr}\left[\Phi^C(x, \underline{k}; p, \underline{S})\, \gamma^+\right] - (\underline{S} \to -\underline{S})$$



- At the level of a pure parton density, the Sivers function vanishes due to the (PT) invariance of QCD:

$$f_{1T}^{\perp q} = 0 \quad + \mathcal{O}(\alpha_s)$$

- But the Sivers function is allowed at the level of quark/gluon/quark correlations due to nontrivial initial-, final-state interactions



- The relation due to (PT) symmetry between initial- and final-state interactions implies a precise sign-flip relation between the Sivers function for semi-inclusive deep inelastic scattering (SIDIS) and the Drell-Yan process (DY):

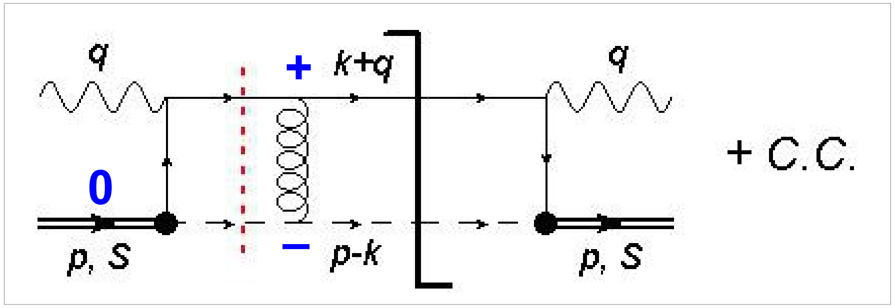
$$f_{1T}^{\perp}\big|_{SIDIS} = -\, f_{1T}^{\perp}\big|_{DY}$$

The Physical Picture: QCD Lensing

- The final state interactions of SIDIS permit a T-odd imaginary phase that occurs when an intermediate state goes on-shell:

$$\frac{1}{\ell^2 - m_\ell^2 + i\epsilon} = \text{P.V.}\left[\frac{1}{\ell^2 - m_\ell^2}\right] - i\pi\delta\left(\ell^2 - m_\ell^2\right)$$

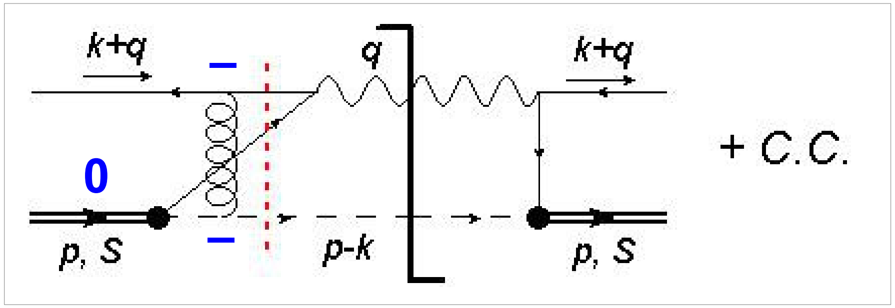
SIDIS (FSI)



- Pointlike proton (colorless)
- Pointlike scalar (antiquark)
- Yukawa coupling: proton/quark/scalar

- Color conservation indicates that the FSI in SIDIS yields a coherent attractive force on quark experiences as it escapes the hadron: **QCD lensing**

DY (ISI)



- In DY, the ISI yields a coherent repulsive force on the antiquark, resulting in the expected sign-flip due to lensing:

$$f_{1T}^{\perp}\big|_{SIDIS} = -\, f_{1T}^{\perp}\big|_{DY}$$

- But lensing due to color consevation cannot be the whole story, because the hadron remnants are not fully transparent to a colored probe.
    - **Shadowing** due to locally incoherent colors will compete with color-coherent lensing.

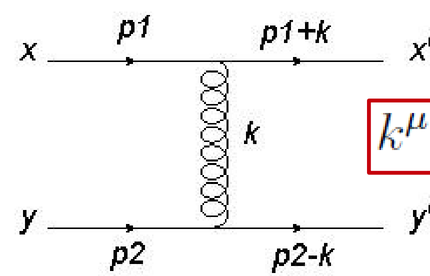
The Emergence of Saturation At High Energy

Regge Kinematics

- Small-x physics is motivated by the high-energy Regge limit, in which center-of-mass energy is the dominant scale.

$$s \gg t \gg \Lambda^2 \qquad x \approx \frac{Q^2}{s} \ll 1$$
$$Q^2 \sim t = const.$$

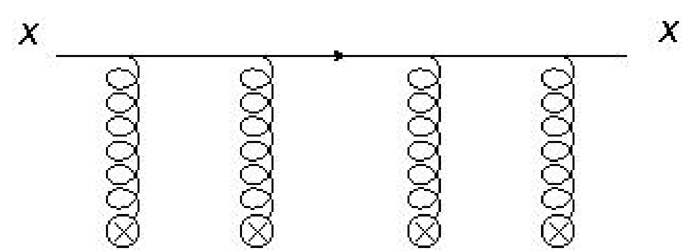
- Interactions in the Regge limit occur through eikonal scattering, by the exchange of **Glauber gluons.**



$$k^\mu \approx (0, 0, \underline{k})$$

- The interactions occur instantaneously and are naturally ordered along the light cone. This eikonal propagation of a projectile through the field of the target is re-summed into a Wilson line:

$$V_{\underline{x}} \equiv \mathcal{P} \exp \left[ig \int dx^- A^+(0, x^-, \underline{x}) \right]$$



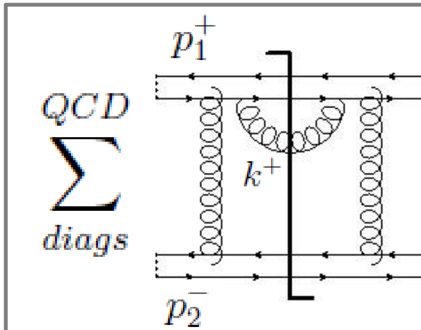
The Small-x Gluon Cascade

- When the energy becomes so large that its logarithm can compete with the coupling, it reorders the perturbation series:

$$s \sim \perp^2 e^{1/\alpha_s} \gg \perp^2 \gg \Lambda^2$$

$$\alpha_s \ln \frac{s}{\perp^2} \sim 1$$

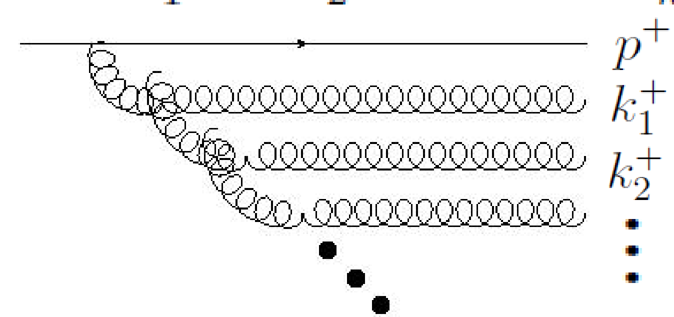
- Emission of an extra longitudinally-soft (small-x) gluon is suppressed by the coupling, but is systematically enhanced by its large phase space.



$$\sim \sigma_{LO} \times \alpha_s \ln \frac{1}{x}$$

- These emissions must be re-summed to give the small-x gluon cascade which dominates the physics of high energies.

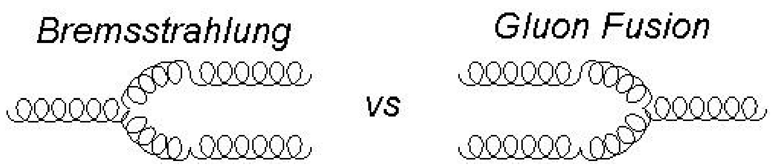
$$p^+ \gg k_1^+ \gg k_2^+ \gg \cdots \gg k_n^+$$



- This increase in gluon bremsstrahlung with energy gives the BFKL equation, which drives up the gluon density at small-x.

Gluon Saturation

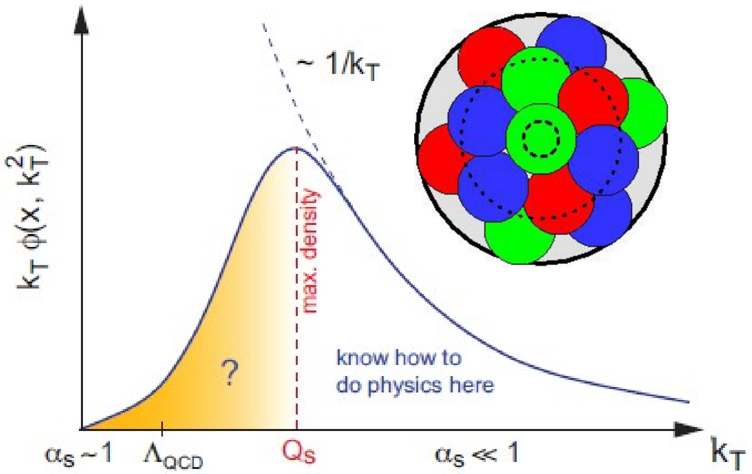
- At high enough densities, nonlinear gluon fusion begins to compete with bremsstrahlung, saturating the growth of the gluon density.



- Including these recombination processes yields a nonlinear evolution equation (ie, BK, JIMWLK):

- The proliferation of incoherent color sources generates a correlation length, described by the saturation scale Q_s , which cuts off the gluon distributions in the IR.

$$\phi \sim \frac{1}{r_T^2}\left(1 - e^{-\frac{1}{4}r_T^2 Q_s^2}\right)$$



- Nonlinear evolution yields a saturation scale that grows with energy, so eventually, it cuts off the IR while still in the perturbative domain.

$$Q_s(x) \sim Q_{s0}\left(\frac{1}{x}\right)^{\#}$$

Heavy Nuclei: A Resummation Parameter

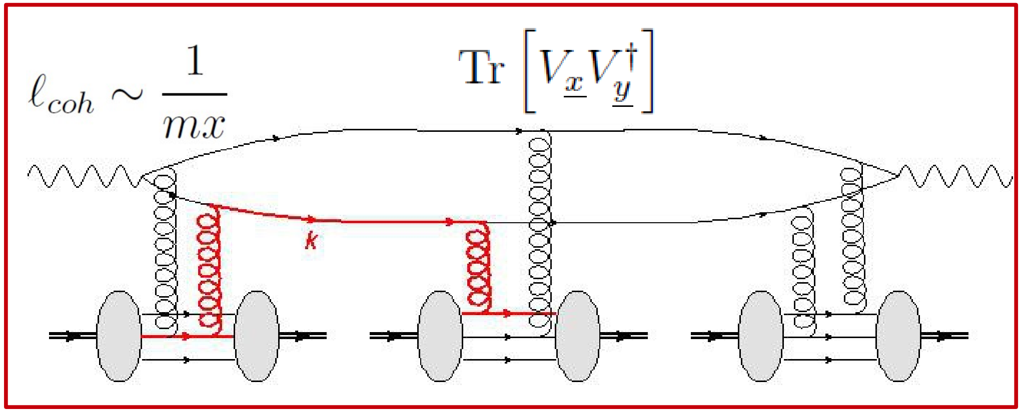
- Nature provides us with another way to approach the limit of dense color charges: using a heavy nucleus with a large number A of nucleons.

$$Q_s^2 \sim \alpha_s^2 A^{1/3}$$

- The nucleus provides a well-defined regime to re-sum the high-density corrections, without the need for quantum evolution.

$$\alpha_s \ll 1, \quad A \gg 1, \quad \alpha_s^2 A^{1/3} \sim 1$$

- **DIS on a heavy nucleus in the Regge Limit:** the virtual photon has a long coherence length and fluctuates into a quark/antiquark pair, undergoing eikonal rescattering from the $A^{1/3}$ nucleons.



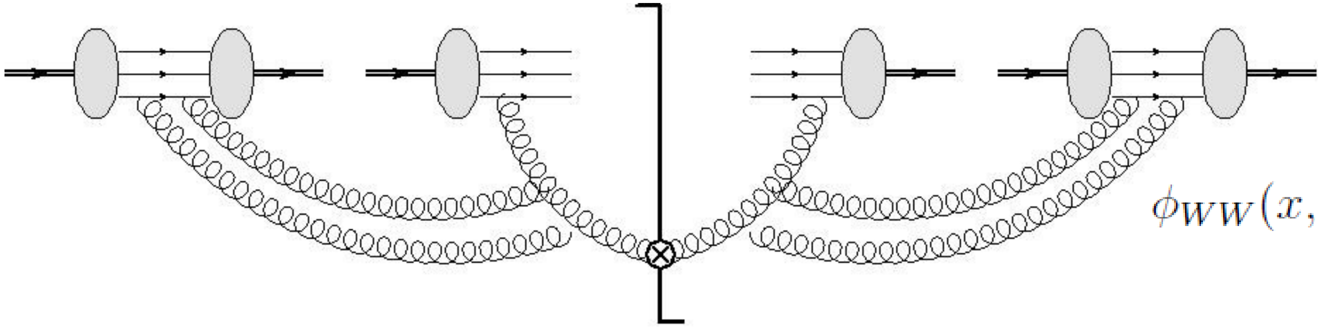
- The leading order effect in $A^{1/3}$ is a combinatoric enhancement that prefers each rescattering to occur on a different nucleon.

- Integrating over the momentum transfer **k** between nucleons puts the intermediate propagators on-shell and factorizes into a product of scattering on independent nucleons.
  - QCD shadowing!

$$\frac{1}{N_c} \mathrm{Tr}\left[V_{\underline{x}} V_{\underline{y}}^\dagger\right] \approx \exp\left[-\frac{1}{4}|x-y|_T^2 Q_s^2 \ln\frac{1}{|x-y|_T \Lambda}\right]$$

The Dense Limit is the Quasi-Classical Limit

- The 2-gluon/nucleon resummation parameter $\alpha_s^2 A^{1/3} \sim 1$ corresponds to interacting with the classical Weizsacker-Williams gluon field of the target.



$$\phi_{WW}(x, k_T^2) \equiv \frac{\pi}{(2\pi)^3} \sum_{\lambda, a} \langle A | \, \hat{a}_{k\lambda}^{a\dagger} a_{k\lambda}^{a} \, | A \rangle$$

- The gluon fields of the nucleus are characterized by high occupation numbers, reducing to their classical limit.

$$\phi_{WW}(x, k_T^2) = \frac{C_F}{\pi^3 \alpha_s} \int d^2b \, d^2r \, e^{i\underline{k}\cdot\underline{r}} \, \frac{1}{r_T^2} \left[1 - \exp\left(-\frac{1}{4} r_T^2 Q_s^2(\underline{b}) \, \ln \frac{1}{r_T \Lambda} \right) \right]$$

- Equivalently, one can solve the classical Yang-Mills equations for a heavy nucleus moving along the light-cone and recover the same formulas (McLerran – Venugopalan model).

$$\mathcal{D}_\mu F^{\mu\nu} = \delta^{\nu +} \rho(x^-, \underline{x})$$

- The high energy limit of QCD is the limit of high-density classical gluon fields.
- The resummation parameter $\alpha_s^2 A^{1/3} \sim 1$ embeds the perturbation series in a classical background field.

Sivers Function in the Quasi-Classical Bjorken Limit

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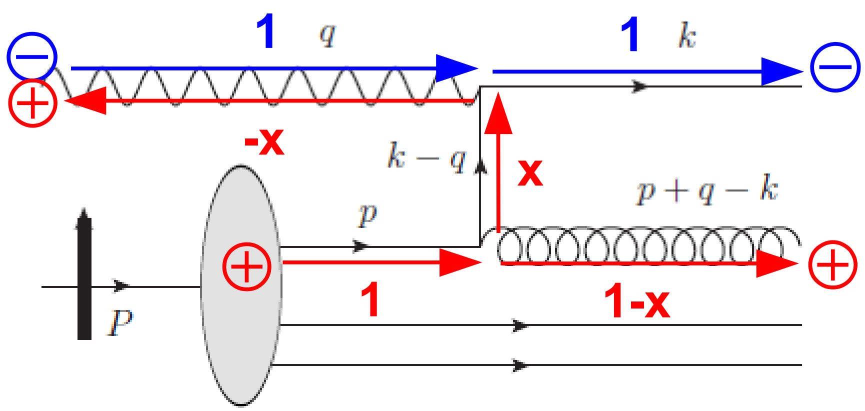
A Common Regime for Spin and Saturation

- There is a common limit that employs both the kinematics necessary for spin physics and the high densities necessary for saturation: **Bjorken kinematics in a heavy nucleus**

Large-x and Large-A

$$s_A \gg s, Q^2 \gg \perp^2 \gg \Lambda^2$$
$$x_A \ll 1\,, \quad x \sim \mathcal{O}(1)$$
$$\alpha_s \ll 1, \quad A \gg 1, \quad \alpha_s^2 A^{1/3} \sim 1$$



$$\ell_\gamma^- \sim \frac{1}{|q^+|} \ll R^-$$
$$\ell_q^- \sim \frac{1}{k^+} \gg R^-$$

- In SIDIS, the virtual photon has a short coherence length and interacts via a local "knockout" process due to the "large-x" Bjorken kinematics.

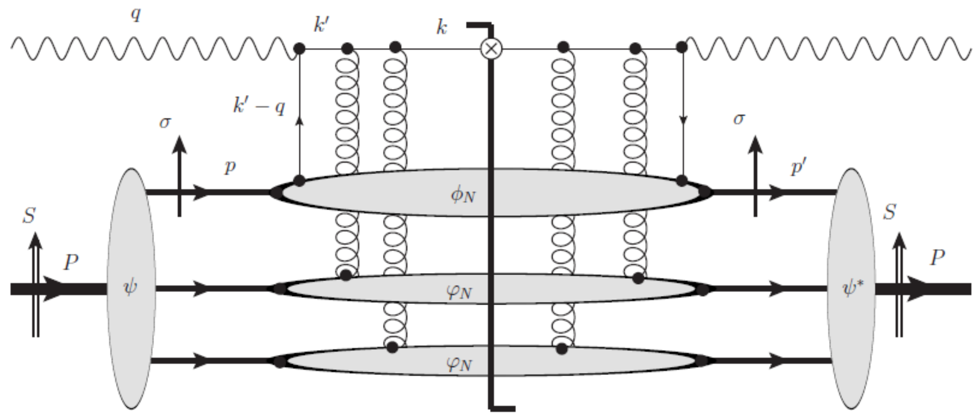
- But the struck quark has a long coherence length and can undergo eikonal final-state rescattering on the spectator nucleons.

- Since $Q^2 \gg \Lambda^2$, TMD factorization holds, and we can express the nuclear TMD's in terms of nucleons and Wilson lines.

- Since $Q_s^2 \gg \Lambda^2$, we can do the calculations perturbatively, using the machinery of saturation physics.

The SIDIS Cross Section (1)

$$\frac{d\sigma^{\gamma^* + A \to q + X}}{d^2k \, dy} =$$



- We can relate the scattering amplitude on the nucleus to the light-cone wave functions of the nucleons and the rest of the amplitude (knockout + rescattering)

$$M_{tot} \sim \int d^{2+}p_1 d^{2+}p_2 \psi(p_1)\psi(p_2)\, M(p_1, p_2) \qquad M^*_{tot} \sim \int d^{2+}p'_1 d^{2+}p'_2 \psi^*(p'_1)\psi^*(p'_2)\, M^*(p'_1, p'_2)$$

- By Fourier transforming the momentum difference between the amplitude and C.C., we obtain the Wigner distribution of nucleons in the nucleus:

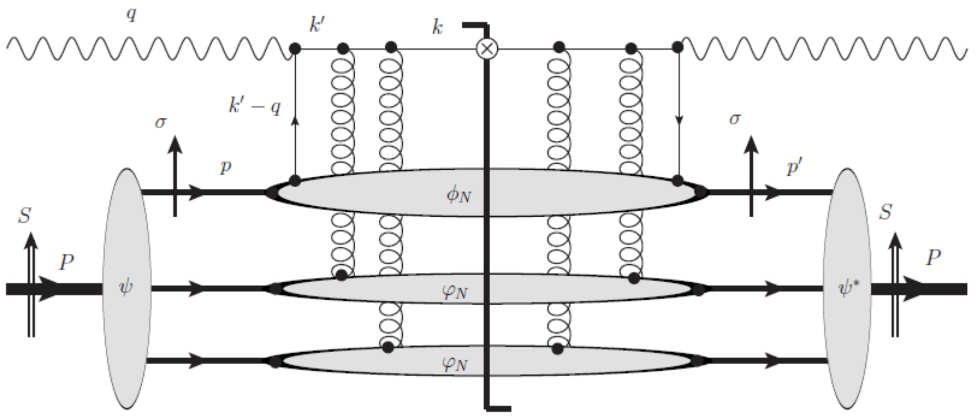
$$W(p,b) \equiv \int \frac{d^{2+}(\delta p)}{(2\pi)^3}\, e^{-i\,(\delta p)\cdot b} \psi(p + \tfrac{1}{2}\delta p)\, \psi^*(p - \tfrac{1}{2}\delta p)$$

- After introducing the Wigner functions, the knockout + rescattering amplitudes are still off-forward.

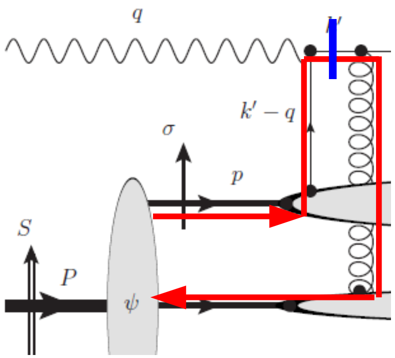
- But, using the fact that W(p,b) varies with impact parameter only over macroscopic scales ~ $A^{1/3}$, we can neglect the off-forwardness in the transverse momentum.

The SIDIS Cross Section (2)

$$\frac{d\sigma^{\gamma^*+A\to q+X}}{d^2k\,dy} =$$



- At this point, the knockout + rescattering amplitudes are still off-forward in the longitudinal momenta.



- But, as we did in the Regge limit, we can integrate over the longitudinal momentum carried between nucleons to put the intermediate propagator on shell.

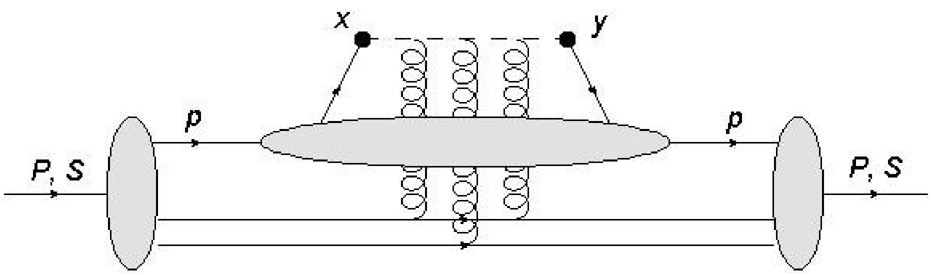
$$\int \frac{dr^+}{2\pi}\, e^{-ir^+(b_2^- - b_1^-)}\, M_{K+R}(p_1 - r, p_2 + r) = \frac{1}{k^-}\theta(b_2^- - b_1^-) M_K(p_1 - r) M_R(p_2 + r)$$

- This enforces path ordering and fully factorizes the knockout of a quark from a nucleon from the rescattering of that quark on all the other spectators. The result is a "quasi-classical factorization" of the SIDIS cross-section:

$$\frac{d\sigma^{\gamma^*+A\to q+X}}{d^2k\,dy} = A\int \frac{dp^+\,d^2p\,db^-}{2(2\pi)^3}\int d^2x\,d^2y\, W\left(p, b^-, \frac{\underline{x}+\underline{y}}{2}\right) \times \int \frac{d^2k'}{(2\pi)^2}\, e^{-i\,(\underline{k}-\underline{k}')\cdot(\underline{x}-\underline{y})}\, \frac{d\hat{\sigma}^{\gamma^*+N\to q+X}}{d^2k'\,dy}(p,q)\, D_{\underline{x}\,\underline{y}}[+\infty, b^-]$$

TMD's in the Quasi-Classical Limit

- We can perform the same analysis using the definition of the TMD distribution functions.



$$\Phi^A_{ij}(x, \underline{k}; P, S) \equiv \frac{1}{(2\pi)^3} \int d^{2-}(x-y)\, e^{ik\cdot(x-y)} \, \langle A(P,S)|\, \overline{\psi}_j(x) U[x,y] \psi_i(y)\, |A(P,S)\rangle$$

- Decompose the nuclear state into a superposition of nucleons using the light-cone wave functions:

$$|A(P,S)\rangle = \int d^{2+}p_1 d^{2+}p_2 \cdots \Psi(p_1)\Psi(p_2)\cdots |N(p_1)\rangle \otimes |N(p_2)\rangle \otimes \cdots$$

- The kinematics of the knockout and rescattering proceed in the same way, reducing the matrix element of the nucleus down to the matrix element of the nucleon:

$$\langle A|\, \overline{\psi} U \psi \,|A\rangle \sim W(p,b) \otimes \langle N|\, \overline{\psi} U \psi \,|N\rangle \otimes D_{xy}$$

- This expresses the TMD's of the nucleus in terms of the TMD's of the nucleons, their Wigner distributions, and the eikonal rescattering factor.

$$\Phi_A(\bar{x}, \underline{k}; P, J) = A \int \frac{dp^+\, d^2p\, db^-}{2(2\pi)^3}\, d^2x\, d^2y \sum_\sigma W_N^\sigma\left(p, b^-, \frac{\underline{x}+\underline{y}}{2}\right) \int \frac{d^2k'}{(2\pi)^2}\, e^{-i\,(\underline{k}-\underline{k}')\cdot(\underline{x}-\underline{y})}$$
$$\times\, \phi_N(x, \underline{k}' - x\,\underline{p}; p, \sigma)\, D_{\underline{xy}}[+\infty, b^-] + \mathcal{O}\left(\frac{\perp^2}{Q^2}\right) + \mathcal{O}(A^{-1/3})$$

Sivers Fuction of a Heavy Nucleus

$$\Phi_A(\bar{x}, \underline{k}; P, J) = A \int \frac{dp^+ \, d^2p \, db^-}{2(2\pi)^3} \, d^2x \, d^2y \sum_\sigma W_N^\sigma \left(p, b^-, \frac{\underline{x}+\underline{y}}{2} \right) \int \frac{d^2k'}{(2\pi)^2} \, e^{-i(\underline{k}-\underline{k}')\cdot(\underline{x}-\underline{y})} \times \phi_N(x, \underline{k}' - x\,\underline{p}; p, \sigma) \, D_{\underline{xy}}[+\infty, b^-] + \mathcal{O}\left(\frac{\perp^2}{Q^2}\right) + \mathcal{O}(A^{-1/3})$$

- Using our quasi-classical factorization formula, we can directly compute the Sivers function of the nucleus:

$$\frac{(\underline{S} \times \underline{k})}{m} f_{1T}^{\perp q}(x, k_T) = \frac{1}{4} \mathrm{Tr}\left[\Phi(x, \underline{k}; P, S)\gamma^+\right] - (\underline{k} \to -\underline{k})$$

- By splitting each factor into symmetric and antisymmetric parts, we can use the symmetry properties of the Sivers function to identify which channels can contribute

$$\hat{z}\cdot(\underline{J} \times \underline{k}) \, f_{1T}^{\perp A}(\bar{x}, k_T) = M_A \int \frac{dp^+ \, d^2p \, db^-}{2(2\pi)^3} \, d^2x \, d^2y \, \frac{d^2k'}{(2\pi)^2} \, e^{-i(\underline{k}-\underline{k}')\cdot(\underline{x}-\underline{y})} \times \left\{ i\, x\, \underline{p}\cdot(\underline{x}-\underline{y}) \, A \, W_{unp}^{OAM}\left(p^+, \underline{p}, b^-, \frac{\underline{x}+\underline{y}}{2}\right) f_1^N(x, k_T') + \frac{1}{m_N} \hat{z}\cdot(\underline{S} \times \underline{k}') \, W_{trans}^{symm}\left(p^+, \underline{p}, b^-, \frac{\underline{x}+\underline{y}}{2}\right) f_{1T}^{\perp N}(x, k_T') \right\} S_{\underline{xy}}[+\infty, b^-]$$

Orbital Angular Momentum:	$W^{OAM}(p, b) \equiv \frac{1}{2}\left[W(p, b) - (\vec{p}_\perp \to -\vec{p}_\perp)\right]$
Intrinsic Sivers function of the nucleons:	$f_{1T}^{\perp N}(x, k_T')$
The Odderon (T-odd rescattering):	$iO_{\underline{xy}} \equiv \frac{1}{2}\mathrm{Tr}\left[V_{\underline{x}} V_{\underline{y}}^\dagger - V_{\underline{y}} V_{\underline{x}}^\dagger\right]$

$\mathcal{O}(A^{-1/3})$

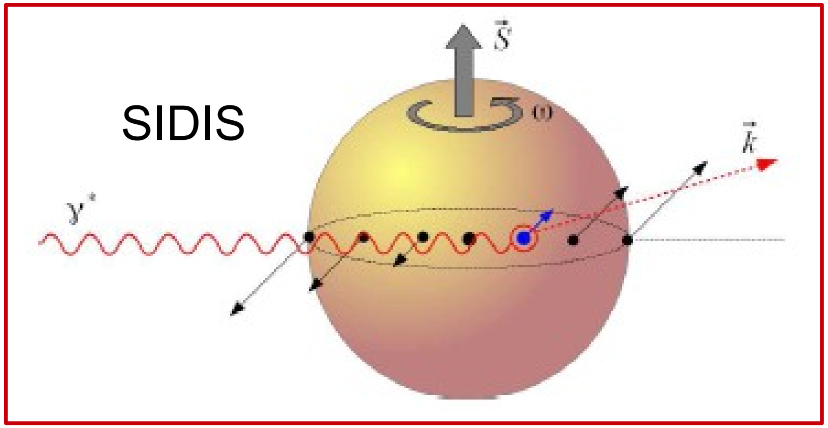
Orbital Angular Momentum and Shadowing

- We find a new mechanism that can generate the Sivers function: (orbital angular momentum of nucleons) x (their quark TMD's) x (symmetric rescattering).

- The essential spin-orbit effect is the presence of OAM, but OAM alone is not enough to generate the Sivers function. When we impose PT symmetry on the Wigner distribution, we find that the OAM part integrates to zero:

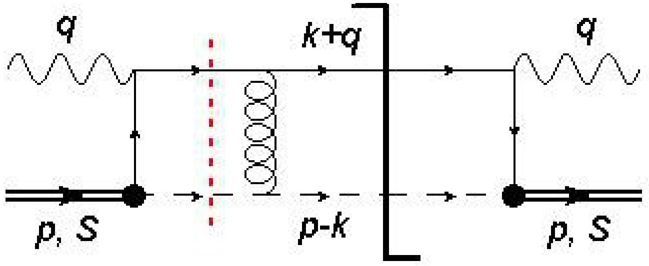
$$W^{OAM}(p, \vec{b}_\perp, b_z) = -W^{OAM}(p, \vec{b}_\perp, -b_z) \longrightarrow \boxed{\int db_z W^{OAM}(p, b) = 0}$$

- If we neglect the role of multiple rescattering, then it is equally likely to eject the quark from the front of the nucleus as from the back, and the net asymmetry integrates to zero.



- Final state interactions break this front-back symmetry through nuclear shadowing, making the quark more likely to be produced near the back of the nucleus.

- This mechanism is quite different from the "lensing" mechanism of the Sivers function, in which the rescattering is color-correlated and generates the preferred direction.



Comparing the Channels

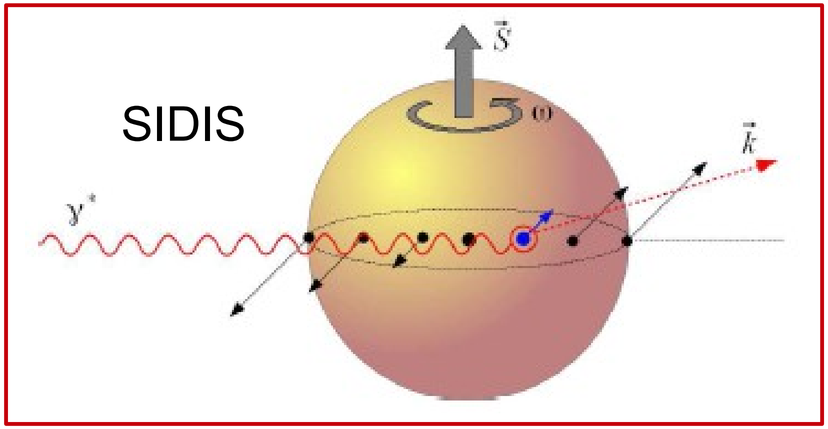
$$f_{1T}^{\perp A}\big|_{OAM} \sim W_{unp}^{OAM} \otimes f_1^N \otimes S_{xy} \qquad f_{1T}^{\perp A}\big|_{trans} \sim W_{trans}^{symm} \otimes f_{1T}^{\perp N} \otimes S_{xy}$$

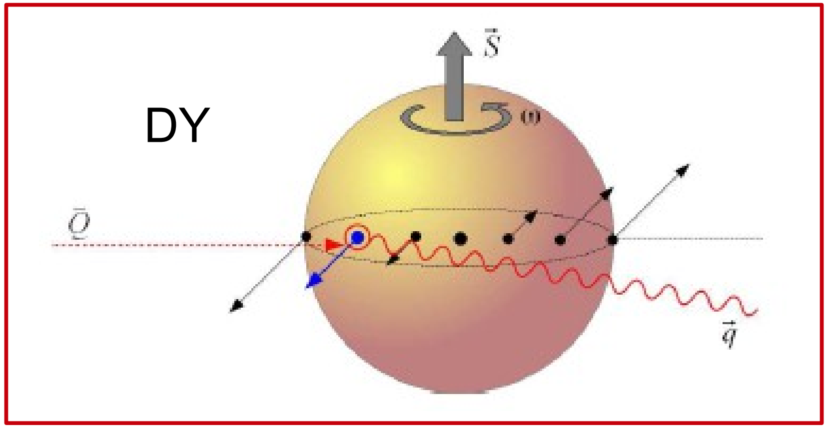
- The OAM channel uses the unpolarized quark TMD, which enters at leading order in α_s. But it requires at least one rescattering to be nonzero, bringing in a suppression by Q_s^2/k_T^2 at large k_T.
- The transversity channel uses the Sivers function, which enters at order α_s^2, but does not rely on nuclear shadowing.
- Thus OAM dominates for much of the range, with the transversity channel only taking over at very large k_T.

OAM dominates for

$$k_T < \frac{Q_s}{\sqrt{\alpha_s}}$$

- The OAM channel has a particularly simple interpretation of the SIDIS / DY sign flip: shadowing from the front vs. back of the nucleus.





Summary

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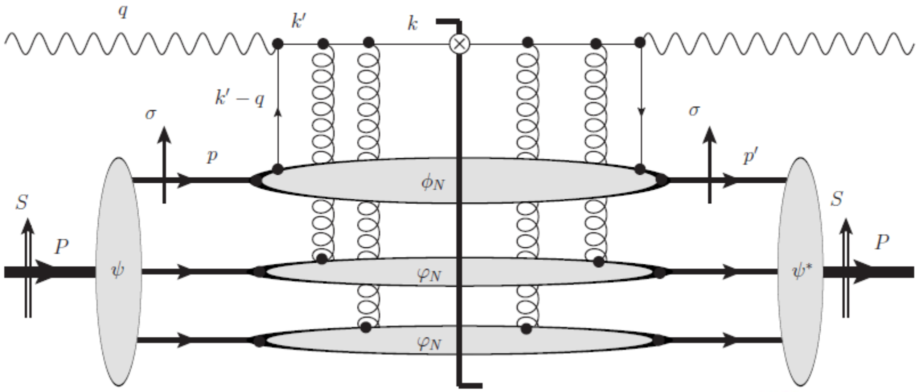
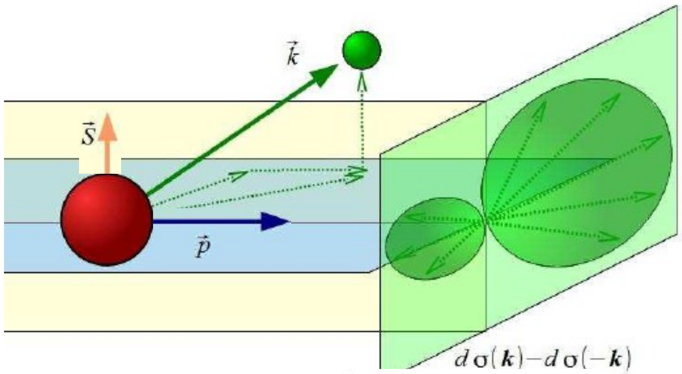
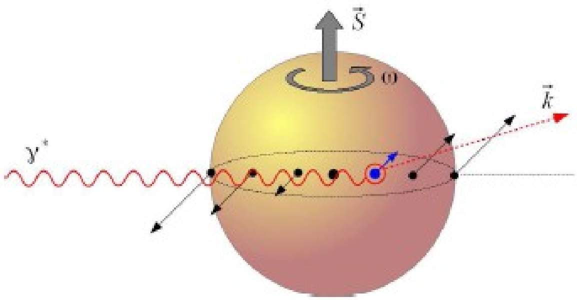
- Working in the quasi-classical large-x, large-A regime, we have derived a relation between the TMD's of a heavy nucleus and the TMD's of its nucleons.



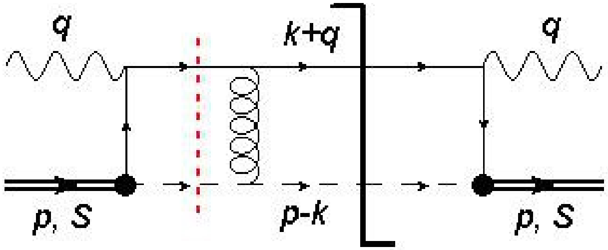
- We analyzed the T-odd Sivers function, identifying two contributing channels: an orbital angular momentum channel and a transversity channel.



- The OAM channel is a new mechanism, distinct from the usual "lensing" mechanism. It uses the inherent orbital motion, together with nuclear shadowing, to generate the preferred direction.



- This machinery is quite general and can be used to calculate the any of the nuclear TMD's. This is an important step as we move toward the intersection of spin and saturation in the coming EIC era.



Outlook and Future Prospects

The TMD Landscape in the Dense Limit

- The machinery we have developed is a robust method of calculating TMD's in the dense limit and can be applied to any of the interesting spin correlations.
- The one key modeling assumption about the target is the existence of a large parameter $A^{1/3}$ that controls the charge density.
- The system under consideration need not be a real nucleus; our approach is valid for any composite particle being decomposed into a large number of constituents (ie, partons in a high-energy proton).
- Our approach allows a clear separation into the "wave function part" which must be P, T even, and the "interaction part" which may contain a T-odd part.
- We can now proceed to evaluate other TMD's in the quasi-classical limit. A good baseline for comparison would be $f_1, g_1, f_{1T}^{\perp}$ for quarks and gluons.
- Full calculations of quark target TMD's exist in the literature, so we can in principle use these to do detailed calculations of the nuclear TMD's.
- Maybe this can shed some light on the zoo of spin correlations and on the distribution of the proton spin.

Toward the Regge Limit and Quantum Evolution

- TMD factorization seems to only require that $Q^2 \gg \Lambda^2$ and $Q^2 \gg \perp^2$, so this approach should also be valid if we consider the small-x limit $s \gg Q^2 \gg \perp^2 \gg \Lambda^2$.

- Such a limit may be useful for studying the overlap and transition between the TMD formalism, small-x saturation formalism, and twist-3 collinear formalism.

- What do the relevant evolution equations look like from this perspective? The nonlocal, semi-infinite Wilson lines are unusual quantities; will their small-x evolution be linear or nonlinear? Can we see the connections to DGLAP evolution, CSS evolution, and BK/JIMWLK evolution?

- Formulating, understanding, and solving the relevant evolution equations will be essential for linking these low-order calculations to future EIC phenomenology.

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Extra Slides

Toy Model: A Rigid Rotator

- Assumptions: Rigid Rotator

$$W_{unp}(p, b) \approx \frac{2\,(2\,\pi)^3}{A}\,\rho(\underline{b}, b^-)\,\delta^2\left(\underline{p} - \hat{y}\,p_{max}(b_x)\,\frac{b^-}{R^-(b_x)}\right)\,\delta\left(p^+ - \frac{P^+}{A}\right)$$

$$W_{trans}(p, b) = \beta W_{unp}(p, b)$$

- Result:

$$f_{1T}^{\perp A}(\bar{x}, k_T) = \frac{M_A\,N_c}{4\pi\,\alpha_s\,J\,C_F}\,\frac{1}{k_T^2}\int d^2b\left\{4\,\bar{x}\,p_{max}(\underline{b})\,C_1\left[e^{-k_T^2/Q_s^2(\underline{b})} + 2\,\frac{k_T^2}{Q_s^2(\underline{b})}\,Ei\left(-\frac{k_T^2}{Q_s^2(\underline{b})}\right)\right]\right.$$
$$\left. + \alpha_s\,\beta\,m_N\,C_2\,e^{-k_T^2/Q_s^2(\underline{b})}\right\},$$

- Asymptotics:

$$f_{1T}^{\perp A}(\bar{x}, k_T)\Big|_{k_T \gg Q_s} = \frac{S}{J}\left[-\frac{4\,\alpha_s\,m_N\,\bar{x}\,C_1}{3\beta\,k_T^6}\,\ln\frac{k_T^2}{\Lambda^2}\int d^2b\,T(\underline{b})\,p_{max}(\underline{b})\,Q_s^2(\underline{b}) + A\,f_{1T}^{\perp N}(\bar{x}, k_T)\right]$$

P, T – Invariance of the Wigner Distribution

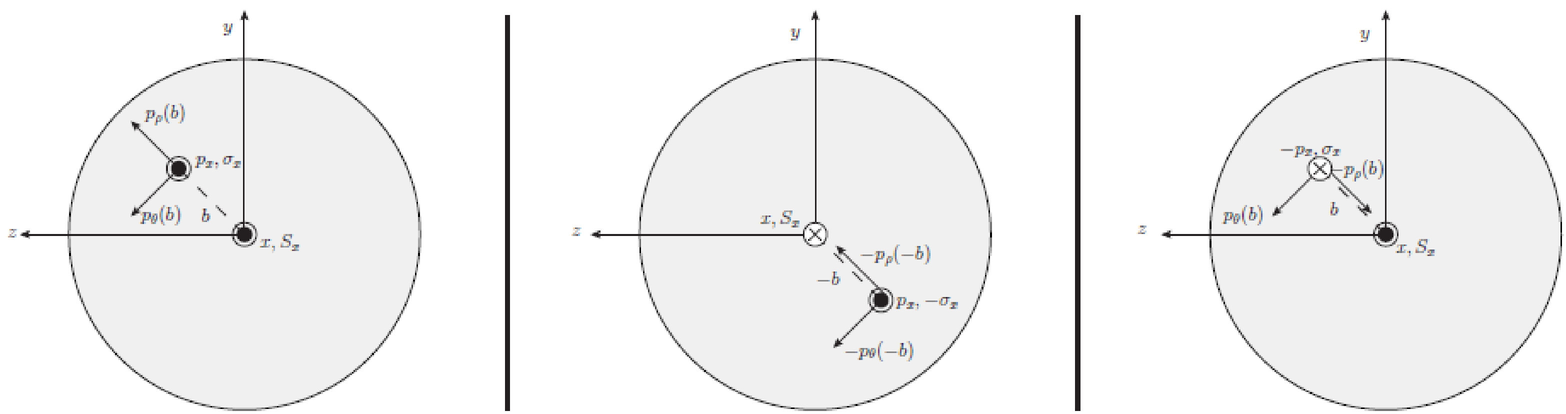

Figure 14: Illustration of the PT transformation and rotational symmetry in the rest frame used in (B4). Left panel: illustration of the momentum flow represented by $W_\sigma(p, b)$. Center panel: under a PT transformation, the spins S, σ and coordinate b are reversed, but the momentum p is invariant. Right panel: rotation of the center panel by π about the vector $\vec{S} \times \vec{b}$ returns the distribution to its original position b, with p_ρ and p_x having been reversed.

$$W_\sigma\left(p_\rho(b), p_\theta(b), p_x; b; S_x\right) = W_\sigma\left(-p_\rho(b), p_\theta(b), -p_x; b; S_x\right)$$

$$W_\sigma(-p_x, -p_y, p_z; b) = W_\sigma(p_x, p_y, p_z; \overline{b})$$

$$\overline{b} \equiv (b_x, b_y, -b_z)$$