Renormalization of Twist Four Operators in Light Cone Gauge

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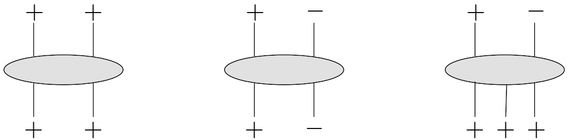
Based on the work of
Yao Ji, A. V. Belitsky, arXiv:1405.2828v1

Introduction



Quasipartonic operators:

Bukhvostov, Frolov, Lipatov, Kuraev, 1985 (BFLK)

—Operators involving only good fields.

Non-quasipartonic operators:

V.M. Braun, A.N. Manashov, J. Rohrwild, 2009

—Bad fields appear, Mix under renormalization at 1-loop.

Goal of study

We investigate the one-loop evolution kernels of the flavor nonsinglet sector for $2 \to 2$ and $2 \to 3$ transitions through computing Feynman diagrams in light cone gauge.

We also provide results of the singlet sector for certain channels.

Light cone formalism

Introduce light cone representation
Define light-cone vectors:

$$n_\mu = (1,0,0,1)/\sqrt{2}\,, \quad \bar{n}_\mu = n^\mu\,, \quad e_\perp^\mu = (0,-1,-i,0)/\sqrt{2}, \quad \bar{e} = e^*.$$

The four-vector x^μ is then decomposed as

$$x^\mu = n^\mu x^- + \bar{n}^\mu x^+ - \bar{e}_\perp^\mu x_\perp - e_\perp^\mu \bar{x}_\perp$$

while the Dirac spinors Ψ are written as

$$\Psi = \frac{1}{2}\gamma^-\gamma^+\Psi + \frac{1}{2}\gamma^+\gamma^-\Psi \equiv \Psi_+ + \Psi_-$$

Light cone gauge condition:

$$A^+ = 0$$

Operator basis in light cone gauge

$$X = \{X_+, X_-, D_\perp X_+, \cdots\}$$

Operator Basis

Composite operators built up by fields localized on a light-cone ray

$$\mathbb{O}(z_1, \ldots, z_N) = C_{I_1 I_2 \ldots I_N} [z_0^-, z_1^-]_{I_1 J_1} X_1^{J_1}(z_1^-) [z_0^-, z_2^-]_{I_2 J_2} X^{J_2}(z_2^-) \ldots$$
$$[z_0^-, z_N^-]_{I_N J_N} X_N^{J_N}(z_N^-) \,,$$

$[z_0^-, z_i^-]$ is the Wilson line connecting the primary quark/gluon fields,

$$[z_0^-, z_k^-] = P \exp \left(ig \int_{z_k^-}^{z_0^-} dz^- \, A^+(z^-) \right)$$

Light cone gauge condition $A^+ = 0$, then the Wilson line is simply

$$[z_0^-, z_k^-] = 1$$

This means we could focus our attention on the interactions between the quark and gluon fields and not worry about the effects of Wilson lines.

Twistor representation

In this representation, the four-vectors are contracted with $\sigma^\mu = (1, \vec{\sigma})$.

$$x_{\alpha\dot{\alpha}} = x_\mu \sigma^\mu_{\alpha\dot{\alpha}} \,.$$

The light-cone vectors n and $\bar{n}$ are factorized into two twistors λ_α and $\mu_{\dot{\alpha}}$

$$n_{\alpha\dot{\alpha}} = \lambda_\alpha \bar{\lambda}_{\dot{\alpha}} \,, \qquad \bar{n}_{\alpha\dot{\alpha}} = \mu_\alpha \bar{\mu}_{\dot{\alpha}} \,,$$

The choice of the twistors is not unique. We take $\lambda^\alpha = (0, \sqrt[4]{2})$ and $\mu^\alpha = (\sqrt[4]{2}, 0)$ for our analysis.
The fields are rewritten as:

$$\Psi = \begin{pmatrix} \psi_\alpha \\ \chi^{\dot{\beta}} \end{pmatrix} , \qquad \bar{\Psi} = (\chi^\beta, \bar{\psi}_{\dot{\alpha}})$$

$$F_{\alpha\beta,\dot{\alpha}\dot{\beta}} = \sigma^\mu_{\alpha\dot{\alpha}} \sigma^\nu_{\beta\dot{\beta}} F_{\mu\nu} = 2(\varepsilon_{\dot{\alpha}\dot{\beta}} f_{\alpha\beta} - 2\varepsilon_{\alpha\beta} \bar{f}_{\dot{\alpha}\dot{\beta}})$$

Twistor representation

The "Plus" and "Minus" components are projected out by

$$\psi_+ = \lambda^\alpha \psi_\alpha, \qquad \chi_+ = \lambda^\alpha \chi_\alpha, \qquad f_{++} = \lambda^\alpha \lambda^\beta f_{\alpha\beta}$$

$$\bar{\psi}_+ = \bar{\lambda}^{\dot{\alpha}} \bar{\psi}_{\dot{\alpha}}, \qquad \bar{\chi}_+ = \bar{\lambda}^{\dot{\alpha}} \bar{\chi}_{\dot{\alpha}}, \qquad \bar{f}_{++} = \bar{\lambda}^{\dot{\alpha}} \bar{\lambda}^{\dot{\beta}} \bar{f}_{\dot{\alpha}\dot{\beta}}$$

$$\psi_- = \mu^\alpha \psi_\alpha, \qquad \chi_- = \mu^\alpha \chi_\alpha, \qquad f_{+-} = \lambda^\alpha \mu^\beta f_{\alpha\beta}$$

The covariant derivative is decomposed as

$$D_{++} = \lambda^\alpha \bar{\lambda}^{\dot{\alpha}} D_{\alpha\dot{\alpha}}, \qquad D_{+-} = \lambda^\alpha \bar{\mu}^{\dot{\alpha}} D_{\alpha\dot{\alpha}}, \qquad D_{--} = \mu^\alpha \bar{\mu}^{\dot{\alpha}} D_{\alpha\dot{\alpha}}$$

The fields can be classified as

$$\Phi_+ = \{\psi_+, \chi_+, f_{++}, \cdots\}, \qquad \Phi_- = \{\psi_-, \chi_-, f_{+-}, \cdots\},$$

Building blocks: conformal primary fields.

$$X_+ = \{\Phi_+, \bar{\Phi}_+\}, \qquad X_- = \{\Phi_-, \bar{\Phi}_-, D_{+-}\Phi_+, D_{-+}\bar{\Phi}_+\},$$

Our study focus on the composite operators with generic form

$$\mathbb{O}_4 = X_+ X_+ X_+ X_+ , \qquad \mathbb{O}_3 = X_- X_+ X_+ .$$

Bridging light cone and twistor representations

In order to study the renormalization of the conformal operators in the light cone gauge, we find the following relations

$$\psi_+ = \frac{\sqrt[4]{2}}{4}(1+\gamma_5)\gamma^-\gamma^+\Psi\,, \qquad \psi_- = \frac{\sqrt[4]{2}}{4}(1+\gamma_5)\gamma^+\gamma^-\Psi\,,$$
$$\chi_+ = \frac{\sqrt[4]{2}}{4}\bar{\Psi}(1+\gamma_5)\gamma^+\gamma^-\,, \qquad \chi_- = -\frac{\sqrt[4]{2}}{4}\bar{\Psi}(1+\gamma_5)\gamma^-\gamma^+\,,$$

where we use $\gamma_5 = \mathrm{diag}(1,-1)$ and

$$f_{++} = \sqrt{2}\partial^+ A_\perp$$
$$f^a_{+-} = -\frac{1}{2\sqrt{2}}\left((\partial^+ A^-)^a + (\bar{D}_\perp A_\perp)^a - (D_\perp \bar{A}_\perp)^a - gf^{abc}\bar{A}^b_\perp A^c_\perp\right)\,,$$

Also

$$D_{-+} = \bar{D}_{+-} = 2\bar{D}_\perp\,, \qquad D_{+-} = \bar{D}_{-+} = 2D_\perp\,,$$
$$D_{++} = \bar{D}_{++} = 2D^+\,, \qquad D_{--} = \bar{D}_{--} = 2D^-\,.$$

Evolution equation

At one-loop order:

$$\frac{d}{d\ln\mu}\begin{pmatrix}\mathbb{O}_3\\ \mathbb{O}_4\end{pmatrix} = -\frac{\alpha_s}{2\pi}\begin{pmatrix}\mathbb{H}^{(3\to 3)} & \mathbb{H}^{(3\to 4)}\\ 0 & \mathbb{H}^{(4\to 4)}\end{pmatrix}\begin{pmatrix}\mathbb{O}_3\\ \mathbb{O}_4\end{pmatrix} + O(\alpha_s^2)\,.$$

$$\mathbb{H}^{(N\to N)} = \sum_{j<k}\mathbb{H}_{jk}^{(2\to 2)}\,,$$

$$\mathbb{H}^{(3\to 4)} = \sum_{j<k}\left(\mathbb{H}_{jk}^{(2\to 2)} + \mathbb{H}_{jk}^{(2\to 3)}\right)\,.$$

Field number changes due to contribution of non-quasipartonic operator arising from twist-four level.

Renormalization in momentum space

The momentum fraction dependence of the operators are extracted by Dirac-delta function

$$\mathcal{O}(x_1, \ldots, x_N) = \int \prod_{n=1}^{N} \frac{d^4 k_n}{(2\pi)^4} \delta(k_n^+ - x_n)\, \mathcal{O}(k_1, \ldots, k_N)\,.$$

The evolution kernels arise from the N to M-particle transition amplitude

$$\mathcal{O}(x_1, \ldots, x_N) = \int \prod_{m=1}^{M} dy_m \int \prod_{m=1}^{M} \frac{d^4 p_m}{(2\pi)^4} \delta(p_m^+ - y_m) \mathcal{O}(p_1, \ldots, p_M)$$
$$\times \int \prod_{n=1}^{N} \frac{d^4 k_n}{(2\pi)^4} \delta(k_n^+ - x_n) \mathcal{G}(k_1, \ldots, k_n | p_1, \ldots, p_M)$$

$\mathcal{G}(k_1, \ldots, k_n | p_1, \ldots, p_M)$ is a sum of corresponding Feynman graphs.

Light cone gauge method

Gluon propagator

$$G^{ab}_{\mu\nu}(k) = -\frac{id_{\mu\nu}(k)}{k^2 + i0}\,, \qquad d_{\mu\nu}(k) = g_{\mu\nu} - \frac{k^\mu n^\nu + k^\nu n^\mu}{k^+}\,.$$

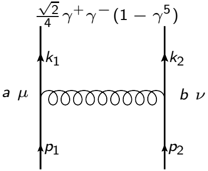
Loop integral

$$\int \frac{d^4 k}{(2\pi)^4} = \int_{-\infty}^{\infty} \frac{dk^+}{2\pi} \int_{-\infty}^{\infty} \frac{dk^-}{2\pi} \int_{-\mu}^{\mu} \frac{d^2 k_\perp}{(2\pi)^2}$$
$$= \int_{-\infty}^{\infty} \frac{dk^+}{2\pi} \int_{-\infty}^{\infty} \frac{d\beta}{4\pi} \int_{-\mu}^{\mu} \frac{d^2 k_\perp}{(2\pi)^2} k_\perp^2$$

Integration over β is readily worked out

$$\vartheta^k_{\alpha_1,\ldots,\alpha_n}(x_1,\ldots,x_n) = \int_{-\infty}^{\infty} \frac{d\beta}{2\pi i} \beta^k \prod_{\ell=1}^{n} (x_\ell \beta - 1 + i0)^{-\alpha_\ell}\,.$$

Comuting Feynman diagram: a simple example



This diagram corresponds to the transition of $\chi_+^i\psi_-^j \to \chi_+^{i'}\psi_-^{j'} + \chi_-^{i'}\psi_+^{j'}$. The evolution kernel is then calculated by

$$\mathcal{G} = -\frac{i\sqrt{2}}{4}g^2 t^a \otimes t^a \bar{\psi}(p_2)[\gamma^\nu(\not{k} - \not{p}_1 - \not{p}_2)\gamma^+\gamma^-\not{k}\gamma^\mu](1+\gamma^5)\psi(p_1)$$

$$\times\left(g_{\mu\nu} + \frac{(k-p_1)_\mu n_\nu + (k-p_1)_\nu n_\mu}{(p_1-k)^+}\right)\frac{1}{k^2(k-p_1)^2(k-p)^2}$$

Comuting Feynman diagram: a simple example

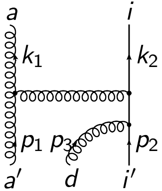
The result has form

$$
\begin{aligned}
\mathcal{G} = \bar{\psi}(p_1)\frac{\sqrt{2}(1+\gamma_5)}{4}\Bigg\{ & 2\vartheta^0_{11}(x_1, x_1 - y_1) - \frac{2y_2}{y_1 - x_1}\vartheta^0_{111}(x_1, x_1 - y_1, -x_2) \\
& + \gamma^+\gamma^-\Bigg[\frac{y_2}{y_1 - x_1}\vartheta^0_{111}(x_1, x_1 - y_1, -x_2) \\
& \qquad - \frac{x_2}{y_1 - x_1}\vartheta^0_{11}(x_1, x_1 - y_1 - y_2)\Bigg] \\
& + \frac{\gamma^+ \not{p}_1^\perp}{y_1 - x_1}[\vartheta^0_{12}(x_1, x_1 - y_1) - \vartheta^0_{11}(x_1, x_1 - y_1) \\
& - \vartheta^0_{111}(x_1, x_1 - y_1, -x_2)] + \frac{\gamma^+ \not{p}_2^\perp}{y_1 - x_1}\vartheta^0_{111}(x_1, x_1 - y_1, -x_2)\Bigg\}\psi(p_2)
\end{aligned}
$$

$\gamma^+ \not{p}_1^\perp$ and $\gamma^+ \not{p}_2^\perp$ require use of equation of motion.

Equation-of-Motion graph

The use of equation of motion $\not{p}\psi(p) = -g \int d^4p' \not{A}(p')\psi(p-p')$ result in the graph of the type



The way to make sense of this graph is to extract terms proportional to $\not{p}$ from the results.

Properties of the evolution kernels

Since the composite operators under our study are all built up by the conformal primaries, we should expect the one-loop evolution kernels to be conformally invariant.
Namely

$$[\mathcal{K}, \tilde{S}^{\pm,0}]\mathcal{O}(x_1, \ldots x_N) = 0\,,$$

This is explicitly confirmed by our analysis.
Moreover, we Fourier transformed our results and compared our results against the known results obtained by conformal analysis. We find complete agreement, up to the use of exchange symmetries for certain channels.

Conclusion

- We calculated evolution kernels for twist four operators built by conformal primary fields for the non-singlet sector and singlet sector of certain transitions.
- We confirmed the conformal invariance of the evolution kernels explicitly from our analysis.
- We compared our results against the previous results of the same kind and find total agreement.

The End