Transverse single-spin asymmetries in proton-proton collisions within collinear factorization

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QCD Evolution Workshop
Santa Fe, NM
May 15, 2014
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  - Collinear twist-3 formalism

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Motivation

- TSSAs in proton-proton collisions

\[ p^\uparrow p \rightarrow \pi X \]

\[ A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} = \frac{d\sigma_L - d\sigma_R}{d\sigma_L + d\sigma_R} \]

Data available from RHIC (BRAHMS, PHENIX, STAR) and FNAL (E704)

(Figure thanks to K. Kanazawa)
Collinear twist-3 formalism

\[
d\sigma = H \otimes f_{a/A}(3) \otimes f_{b/B}(2) \otimes D_{c/C}(2) \\
+ H' \otimes f_{a/A}(2) \otimes f_{b/B}(3) \otimes D_{c/C}(2) \\
+ H'' \otimes f_{a/A}(2) \otimes f_{b/B}(2) \otimes D_{c/C}(3)
\]

Collinear twist-3 approach
(Efremov and Teryaev (1982, 1985); Qiu and Sterman (1992, 1999))

\[
P_{hT} >> \Lambda_{QCD}
\]
Collinear twist-3 formalism

\[ d\sigma = H \otimes f_{a/A}(3) \otimes f_{b/B}(2) \otimes D_{c/C}(2) + H' \otimes f_{a/A}(2) \otimes f_{b/B}(3) \otimes D_{c/C}(2) + H'' \otimes f_{a/A}(2) \otimes f_{b/B}(2) \otimes D_{c/C}(3) \]

- T-odd effect
  → need to generate an imaginary part
  → soft-gluon pole (SGP) or soft-fermion pole (SFP)
  → internal particle goes on-shell

- One can also have SGPs with tri-gluon correlations

Collinear twist-3 approach
(Efremov and Teryaev (1982, 1985); Qiu and Sterman (1992, 1999))
\[ P_{hT} \gg \Lambda_{QCD} \]
• SGP term (Qiu and Sterman (1999), Kouvaris, et al. (2006)):

\[
E_\ell \frac{d^3 \Delta \sigma (s_T)}{d^3 \ell} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int_{z_{\text{min}}}^{1} \frac{dz}{z^2} D_{c \rightarrow h}(z) \int_{x'_{\text{min}}}^{1} \frac{dx'}{x'} \frac{1}{x'S + T/z} \phi_{b/B}(x') \\
\times \sqrt{4\pi \alpha_s} \left( \frac{\epsilon_{\ell s T \mu} n}{z \bar{u}} \right) \frac{1}{x} \left[ T_{a,F}(x,x) - x \left( \frac{d}{dx} T_{a,F}(x,x) \right) \right] H_{ab \rightarrow c}(s, \tilde{t}, \bar{u})
\]

• SFP term (Koike and Tomita (2009); Kanazawa and Koike (2011)):

\[
E_h \frac{d^3 \Delta \sigma_{\text{SFP}}}{dP^3_h} = \frac{\alpha_s^2 M N \pi}{2} e^{m_p S_{\perp}} \int_{z_{\text{min}}}^{1} \frac{dz}{z^3} \int_{x'_{\text{min}}}^{1} \frac{dx'}{x'} \int \frac{dx}{x} \frac{1}{x'S + T/z} \delta \left( x - \frac{-x'U/z}{x'S + T/z} \right) \\
\times \left[ \sum_{a,b,c} \left( G^a_F(0, x) + \tilde{G}^a_F(0, x) \right) \left\{ q^b(x') (D^c(z) \delta_{ab \rightarrow c} + D^c(z) \delta_{ab \rightarrow \bar{c}}) \\
+ q^\bar{b}(x') (D^c(z) \delta_{ab \rightarrow c} + D^c(z) \delta_{ab \rightarrow \bar{c}}) \right\} \\
+ \sum_{a,b} \left( G^a_F(0, x) + \tilde{G}^a_F(0, x) \right) \left( q^b(x') D^g(z) \delta_{ab \rightarrow g} + q^\bar{b}(x') D^g(z) \delta_{ab \rightarrow \bar{g}} \right) \\
+ \sum_{a,c} \left( G^a_F(0, x) + \tilde{G}^a_F(0, x) \right) G(x') (D^c(z) \delta_{ag \rightarrow c} + D^c(z) \delta_{ag \rightarrow \bar{c}}) \\
+ \sum_{a} \left( G^a_F(0, x) + \tilde{G}^a_F(0, x) \right) G(x') D^g(z) \delta_{ag \rightarrow g} \right] 
\]

\[ T_F \sim G_F \sim F_{FT} \]

\[ \tilde{T}_F \sim \tilde{G}_F \sim G_{FT} \]
• Tri-gluon correlators (Beppu, Kanazawa, Koike, Yoshida (2013)):

$$E_{P_h} \frac{d^3 \Delta \sigma}{d^3 P_h} = \frac{2\pi M_N \alpha_s^2}{S} \sum_{i,j} \int \frac{dx}{x} \int \frac{dx'}{x'} f_i(x') \int \frac{dz}{z^2} D_j(z) \delta(\hat{s} + \hat{t} + \hat{u}) \frac{1}{z\hat{u}}$$

$$\times \left[ \zeta_{ij} \left( \frac{d}{dx} O(x) - \frac{2O(x)}{x} \right) \hat{\sigma}_{g_i \rightarrow j}^{(O)} + \left( \frac{d}{dx} N(x) - \frac{2N(x)}{x} \right) \hat{\sigma}_{g_i \rightarrow j}^{(N)} \right]$$

For many years the SGP term involving the Qiu-Sterman function was thought to be the dominant contribution to TSSAs in $p^\uparrow p \rightarrow hX$
A puzzle with TSSAs (the “sign mismatch” issue)

\[ p^+ p \rightarrow h X \]

RHIC, STAR (2012)

\[ \ell N^\uparrow \rightarrow \ell' h X \]

CERN, COMPASS (2013)

\[ \pi F_{FT}(x, x) = f_{1T}^{(1)}(x) \]

\[ F_{FT} \sim T_F \]
A puzzle with TSSAs (the “sign mismatch” issue)

$ \pi^+ p \rightarrow h X $  

RHIC, STAR (2012)

CERN, COMPASS (2013)

$ \ell N^+ \rightarrow \ell^+ h X $  

$ F_{FT}(x, x) = f_{1T}^{(1)}(x) $  

$ F_{FT} \sim T_F $  

“sign mismatch”  (Kang, Qiu, Vogelsang, Yuan (2011))
Sivers input agrees reasonably well with the JLab data

- Node in $k_T$ for the Sivers function can be ruled out/Also node in $x$ is disfavored from proton data from HERMES (see also Kang and Prokudin (2012))
- FIRST INDICATION that the Sivers effect is intimately connected to the re-scattering of the active parton with the target remnants (PROCESS DEPENDENT)

KQVY input gives the wrong sign SGP contribution on the side of the transversely polarized incoming proton cannot be the main cause of the large TSSAs seen in pion production (i.e., $T_F(x,x)$ term)
\[ d\sigma = H \otimes f_{a/A}(3) \otimes f_{b/B}(2) \otimes D_{c/C}(2) + H' \otimes f_{a/A}(2) \otimes f_{b/B}(3) \otimes D_{c/C}(2) + H'' \otimes f_{a/A}(2) \otimes f_{b/B}(2) \otimes D_{c/c}(3) \]

Negligible (Kanazawa and Koike (2000))
• Collinear twist-3 fragmentation term:

\[
\hat{H}^{h/q}(z) = z^2 \int d^2 \vec{k}_\perp \frac{\vec{k}_\perp^2}{2M_h^2} H_{1}^{h/q}(z, z^2 \vec{k}_\perp^2)
\]

Collins-type function

\[
2z^3 \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{1 - \frac{1}{z_1}} \hat{H}_{FU}^{h/q}(z, z_1) = H^{h/q}(z) + 2z\hat{H}^{h/q}(z)
\]

3-parton correlator

There are 2 independent (unpolarized) collinear twist-3 FFs

Collinear twist-3 fragmentation structure is richer than that for the TMD formalism
Calculation of twist-3 fragmentation term (Metz and DP, PLB 723 (2013))

\[
\frac{P_{h}^{0}d\sigma_{\text{pol}}}{d^{3}\vec{P}_{h}} = -\frac{2\alpha_{s}^{2}M_{h}}{S} \epsilon_{\perp \mu \nu} S_{\perp}^{\mu} P_{h}^{\nu} \sum_{i} \sum_{a,b,c} \int_{z_{\text{min}}}^{1} dz \int_{x_{\text{min}}}^{1} dx' \frac{1}{x' S + T/z - x' t} \\
\times \frac{1}{x} h_{1}^{a}(x) f_{1}^{b}(x') \left\{ \left( \hat{H}_{C/c}^{c}(z) - z \frac{d\hat{H}_{C/c}^{c}(z)}{dz} \right) S_{\hat{H}}^{i} + \frac{1}{z} H_{C/c}^{c}(z) S_{H}^{i} \right\} \\
+ 2z^{2} \int \frac{dz_{1}}{z_{1}^{2}} PV \frac{1}{z - \frac{1}{z_{1}}} \hat{H}_{FU}^{c,c;S}(z, z_{1}) \frac{1}{\xi} S_{\hat{H}_{FU}}^{i}
\]

First time we have a complete pQCD result for this term in pp within the collinear twist-3 approach

Also has been studied for TSSA in SIDIS (Kanazawa and Koike (2013))

“Derivative term” has been calculated previously (Kang, Yuan, Zhou (2010))

Derivative and non-derivative piece combine into a “compact” form as on the distribution side

Must determine numerical significance of 3-parton fragmentation correlator
\[
E_\ell \frac{d^3 \Delta \sigma(\vec{s}_T)}{d^3 \ell} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int_{z_{\text{min}}}^{1} \frac{dz}{z^2} D_{c\rightarrow h}(z) \int_{x'_{\text{min}}}^{1} \frac{dx'}{x'} x' S + T/z \phi_{b/B}(x') \\
\quad \times \sqrt{4\pi \alpha_s} \left( \frac{\epsilon_{\ell s T n \bar{n}}}{z \hat{u}} \right) \frac{1}{x} \left[ T_{a,F}(x,x) - x \left( \frac{d}{dx} T_{a,F}(x,x) \right) \right] H_{ab\rightarrow c}(\hat{s}, \hat{t}, \hat{u})
\]

\[
\frac{P_h^0 d\sigma_{\text{pol}}}{d^3 P_h} = -\frac{2\alpha_s^2 M_h}{S} \sum_{\rho, \nu} S_{\rho, \nu} \sum_{a,b,c} \int_{z_{\text{min}}}^{1} \frac{dz}{z^3} \int_{x'_{\text{min}}}^{1} \frac{dx'}{x'} x' S + T/z - x \hat{u} - x' \hat{t} \\
\quad \times \frac{1}{x} h_1^a(x) f_1^b(x') \left\{ \left( \hat{H}_{\rho, \nu}^{C/c}(z) - z \frac{d \hat{H}_{\rho, \nu}^{C/c}(z)}{dz} \right) S_{\hat{H}}^i + \frac{1}{2} \hat{H}_{\rho, \nu}^{C/c}(z) S_{\hat{S}}^i \right\} \\
\quad + 2z^2 \int \frac{dz_1}{z_1^2} PV \frac{1}{z - \frac{1}{z_1}} \hat{H}_{FU}^{C/c, \Xi}(z, z_1) S_{\hat{S}}^i 
\]

Recall: \( H^{h/q}(z) = -2z \hat{H}^{h/q}(z) + 2z^3 \int_{z}^{\infty} \frac{dz_1}{z_1^2} \frac{1}{z - \frac{1}{z_1}} \hat{H}_{FU}^{h/q, \Xi}(z, z_1) \)
The role of twist-3 fragmentation in TSSAs


- Numerical study (Note: we only use $\sqrt{S} = 200$ GeV data $\Rightarrow$ higher $P_T$ values)

\[ F_{FT} \sim T_F \]

\begin{itemize}
  \item SGP: $\pi F_{FT}(x, x) = f_{1T}^{(1)}(x)$, Sivers function taken from Torino group (2009/2013)
  \item SFP/Tri-gluon: neglect for now
\end{itemize}

\begin{itemize}
  \item Transversity: taken from Torino group (2013), but allow $\beta$ parameters to be free
  \item $\hat{H}^{h/q}(z)$: use Collins function extracted by the Torino group (2013)
  \[ \hat{H}^{h/q}(z) = z^2 \int d^2\vec{k}_\perp \frac{\vec{k}^2_{\perp}}{2M^2_h} H^{1h/q}_{1}(z, z^2\vec{k}^2_{\perp}) \]
  \item $\hat{H}^{h/q, S}_{FU}(z, z_1)$ $\Rightarrow$ use the following ansatz:
  \[ \frac{\hat{H}^{\pi^+/(u,\bar{d}), S}_{FU}(z, z_1)}{D^\pi^+/(u,\bar{d})(z) D^\pi^+/(u,\bar{d})(z/z_1)} = \frac{N_{fav}}{2I_{fav} J_{fav}} z^{\alpha_{fav}} (z/z_1)^{\alpha'_{fav}} (1 - z)^{\beta_{fav}} (1 - z/z_1)^{\beta'_{fav}} \]
  \[ (\text{similar for disfavored, } \pi^- \text{ defined through } \text{c.c.}, \pi^0 \text{ defined as average of } \pi^+ \text{ and } \pi^-) \]  
\end{itemize}
8 free parameters: \( N_{fav}, \alpha_{fav} = \alpha'_{fav}, \beta_{fav}, \beta'_{fav} = \beta'_{dis} \)

\( N_{dis}, \alpha_{dis} = \alpha'_{dis}, \beta_{dis}, \beta^T_u = \beta^T_d \)

\[
\begin{array}{c}
\chi^2 / \text{d.o.f.} = 1.03 \\
N_{fav} = -0.0338 & N_{dis} = 0.216 \\
\alpha_{fav} = \alpha'_{fav} = -0.198 & \beta_{fav} = 0.0 \\
\beta'_{fav} = \beta'_{dis} = -0.180 & \alpha_{dis} = \alpha'_{dis} = 3.99 \\
\beta_{dis} = 3.34 & \beta^T_u = \beta^T_d = 1.10 \\
\end{array}
\]

Above parameters are from using 2009 Sivers function (SV1). Using 2013 Sivers function (SV2) gives similar values and \( \chi^2 / \text{d.o.f.} = 1.10 \)
Including the (total) fragmentation term leads to very good agreement with the RHIC data, especially with its characteristic rise towards large $x_F$.

Without the 3-parton FF, one has difficulty describing the RHIC data.
- SV2 – 2013 Sivers function from Torino group \(\Rightarrow\) flavor-dependent large-\(x\) behavior and slower decrease at large-\(x\) than SV1
  - Including 3-parton FF, one can accommodate such a Sivers function through the \(H\) term
  - Without the 3-parton FF, one would have serious issues handling such a (negative) SGP contribution to obtain a (large) positive \(A_N\)
Favored and disfavored (chiral-odd) collinear twist-3 FFs are roughly equal in magnitude but opposite in sign \( \Rightarrow \) similar to Collins FF

\( A_N \) for \( \pi^+ (\pi^-) \) dominated by favored (disfavored) fragmentation
Flat $P_T$ dependence thought to be an issue for collinear twist-3 approach $\Rightarrow A_N \sim 1/P_T$

First shown by Kanazawa and Koike (2011) that this does not have to be the case

Our analysis also shows a flat $P_T$ dependence for $A_N$ seen so far at RHIC $\Rightarrow$ remains flat even to larger $P_T$ values
Summary and outlook

• For many years it was unclear what mechanism causes large TSSAs in hadron production from $pp$ collisions

• Twist-3 fragmentation could finally give us an explanation
  
  ➡️ Full analytical pQCD result now available
  ➡️ Including this term allows for a very good description of the RHIC data, in particular the rise in $A_N$ towards large $x_F$ and flat $P_T$ dependence
  ➡️ Our analysis provides a consistency between spin/azimuthal asymmetries in $pp$ (collinear) and SIDIS, $e^+e^-$ (TMD)
  ➡️ Future work: include SFPs (can help with charged pions) and proper evolution of the 3-parton FF
• Global analysis involving several reactions will be needed in order to extract all the collinear twist-3 distribution and fragmentation functions in \( p^\uparrow p \rightarrow hX \)

  ➡️ Measurement of \( p^\uparrow p \rightarrow \text{jet } X \) by the AnDY Collaboration (Bland, et al. (2013))

  ➡️ Measurements of Drell-Yan in \( p^\uparrow p \) and \( p^\uparrow p \rightarrow \gamma X \) at RHIC (also DY experiment planned at COMPASS for \( \pi p^\uparrow \))

  ➡️ Large \( P_{h\perp} \) measurement of Sivers and Collins asymmetries in SIDIS should also be possible at JLab12, COMPASS, or a future EIC

  ➡️ HERMES (Airapetian, et al. (2013)) / JLab (Allada, et al. (2013)) have recently published data on \( ep^\uparrow \rightarrow hX / en^\uparrow \rightarrow hX \)

  ➡️ Can one consistently describe all of these reactions?
Backup slides
• Large TSSAs observed in the mid-1970s in the detection of hyperons from proton-beryllium collisions (Bunce, et al. (1976))

• Initially thought to contradict pQCD (Kane, Pumplin, Repko (1978)) – within the naïve collinear parton model:

\[ A_N \sim \alpha_s m_q / P_{h\perp} \]

• Higher-twist approach to calculating TSSAs in pp collisions introduced in the 1980s (Efremov and Teryaev (1982, 1985))

• Benchmark calculations performed starting in the early 1990s (Qiu and Sterman (1992, 1999); Kouvaris, et al. (2006); Koike and Tomita (2009), etc.)

• RHIC (BRAHMS, STAR, PHENIX) has provided the most recent experimental data on proton-proton TSSAs (also FNAL (E704) in the 1990s)
Experimental data

RHIC, STAR (2012) ($\sqrt{s} = 200$ GeV)

RHIC, PHENIX (2013) ($\sqrt{s} = 62.4$ GeV)

RHIC, BRAHMS (2008) ($\sqrt{s} = 62.4$ GeV)

Also preliminary data from BRAHMS at $\sqrt{s} = 200$ GeV

$$x_F = \frac{2p_z}{\sqrt{s}}$$
Data tells us (if fragmentation mechanism dominates) that the pions care about the transverse spin of the fragmenting quark \( \rightarrow \) fragment in a particular direction (left or right).

Small and negative \( x_F \) \( \rightarrow \) probe sea quarks and gluons in \( p^\uparrow \)

- \( gg \rightarrow gg \) channel gives large contribution to unpolarized cross section, but NO gluon “transversity” \( \rightarrow \) no such channel in spin-dependent cross section

- Little information on sea quark “transversity” \( \rightarrow \) might speculate sea quarks, on average, are less likely to emerge from \( p^\uparrow \) with a transverse spin in a certain direction

Large \( x_F \) \( \rightarrow \) probe valence quarks in \( p^\uparrow \)

- From SIDIS we know \( u \) quarks (\( d \) quarks) are more likely emerge from \( p^\uparrow \) with their transverse spin aligned (anti-aligned) with \( p^\uparrow \) \( \rightarrow \) pions more likely to fragment in a particular direction (left or right)

- \( gg \rightarrow gg \) channel dies out in this region \( \rightarrow \) unpolarized cross section becomes smaller

\[
A_N = \frac{d\sigma_L - d\sigma_R}{d\sigma_L + \sigma_R}
\]
An aside: TSSAs in SIDIS and the TMD formalism

\[ A_{UT}^{\sin(\phi_h - \phi_s)} = \frac{\int d\phi_h d\phi_S \sin(\phi_h - \phi_S) d\sigma}{\int d\phi_h d\phi_S d\sigma} \]

(Figure from Bacchetta, et al. (2007))
\[ A_{UT}^{\sin(\phi_h - \phi_s)} = \frac{w(k_\perp) f_{1T}^q(x, \vec{k}_\perp) \otimes D_1^{h/q}(z, \vec{p}_\perp)}{f_1^q(x, \vec{k}_\perp) \otimes D_1^{h/q}(z, \vec{p}_\perp)} \]

TMD approach
(Sivers (1990, 1991); Collins (1993))

\[ Q >> P_{hT} \geq \Lambda_{QCD} \]

- T-odd effect \( \Rightarrow \) imaginary phase is generated by “Wilson line”
  \( \Rightarrow \) multiple re-interactions of the quark with the target remnants
- Process dependence: \( f_{1T}^{\perp}(x, \vec{k}_\perp^2)|_{SIDIS} = -f_{1T}^{\perp}(x, \vec{k}_\perp^2)|_{DY} \) (Collins (2002))

\[
k^0 \frac{d \sigma_{pol}^N}{d^3 k^I} = \frac{8 \pi \alpha_{em}^2 x y^2 M}{Q^8} \left( \frac{\hat{s}^2 + \hat{t}^2}{\hat{u}^2} \right) \left( 2 + \frac{\hat{u}}{t} \right) \epsilon^{S_N P k k'} \sum_q e_q^2 x \tilde{F}_{FT}^{q/N} (x, x)
\]

with \[
\tilde{F}_{FT} (x, x) = F_{FT} (x, x) - x \frac{d}{dx} F_{FT} (x, x)
\]

(Work has also been done on both photons coupling to the same quark: Metz, Schlegel, Goeke (2006); Afanasev, Strikman, Weiss (2007); Schlegel (2012))
• A note on the TMD approach to TSSAs in $pp$ collisions

→ Only a phenomenological model, since there is no proof such a formalism holds in processes with only one (large) scale

→ Use Sivers function extracted from SIDIS $\Rightarrow$ large uncertainties due to unknown large $x$ behavior $\Rightarrow$ cannot draw any definite conclusions

→ NO sign mismatch problem, but if one takes the re-scattering picture seriously then the issue cannot be avoided
\[ d\sigma = H \otimes f_{a/A(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)} \]

\[ + \quad H' \otimes f_{a/A(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)} \]

- **Collinear twist-3 fragmentation term:**

  - Negligible (Kanazawa and Koike (2000))

- Anselmino, et al. (2012) – TMD approach
  - Uses Collins function extracted from $e^+e^-$ and SIDIS
  - Only looks at “derivative term” using simple parameterization

- Kang, Yuan, Zhou (2010) – CT3 approach
  - Could at the very least give a contribution comparable to SGP term
There are 2 independent (unpolarized) collinear twist-3 FFs

Collinear twist-3 fragmentation structure is richer than that for the TMD formalism
Theoretical description: collinear twist-3 formalism

Twist-3 collinear PDFs for a transversely polarized $p$

(c) gives a twist-4 contribution

Rewrite in terms of $F$ or $D$ (see, e.g., Zhou, Yuan, Liang (2010))
• Symmetry properties

\[ F^q_{FT}(x, x_1) = F^q_{FT}(x_1, x) \quad \text{and} \quad G^q_{FT}(x, x_1) = -G^q_{FT}(x_1, x) \]

\[ F^q_{DT}(x, x_1) = -F^q_{DT}(x_1, x) \quad \text{and} \quad G^q_{DT}(x, x_1) = G^q_{DT}(x_1, x) \]

• Relations between F-type and D-type functions (see, e.g., Eguchi, et al. (2006))

\[ F^q_{DT}(x, x_1) = PV \frac{1}{x - x_1} F^q_{FT}(x, x_1) \]

\[ G^q_{DT}(x, x_1) = PV \frac{1}{x - x_1} G^q_{FT}(x, x_1) + \delta(x - x_1) \tilde{g}^q(x) \]

• \( g_T \) can be related to D-type functions through the EOM (see, e.g., Efremov and Teryaev (1985); Jaffe and Ji (1992); Boer, Mulders, Teryaev (1998)):

\[ x g_T^q(x) = \int dx_1 \left[ G^q_{DT}(x, x_1) - F^q_{DT}(x, x_1) \right] \]

There are 3 independent collinear twist-3 functions relevant for a transversely polarized \( p \):

\( \tilde{g}, F_{FT}, G_{FT} \) or \( \tilde{g}, F_{DT}, G_{DT} \)
(c) gives a twist-4 contribution

Rewrite in terms of $F$ or $D$
• Relations between F-type and D-type function

\[ \hat{H}_{DU}^{h/q, \Re}(z, z_1) = \text{PV} \frac{1}{z} - \frac{1}{z_1} \hat{H}_{FU}^{h/q, \Re}(z, z_1) - \frac{1}{z^2} \hat{H}^{h/q}(z) \delta \left( \frac{1}{z} - \frac{1}{z_1} \right) \]

\[ \hat{H}_{DU}^{h/q, \Im}(z, z_1) = \text{PV} \frac{1}{z} - \frac{1}{z_1} \hat{H}_{FU}^{h/q, \Im}(z, z_1) \]

• \( H(E) \) can be related to the imaginary (real) part of the D-type function through the EOM:

\[ H^{h/q}(z) = 2z^3 \int \frac{dz_1}{z_1^2} \hat{H}_{DU}^{h/q, \Im}(z, z_1) \]

\[ E^{h/q}(z) = -2z^3 \int \frac{dz_1}{z_1^2} \hat{H}_{DU}^{h/q, \Re}(z, z_1) \]

There are 2 independent collinear twist-3 functions relevant for the fragmentation of a quark into an unpolarized \( h \)
Involves $F_{FT}$ in a QED process ($q\gamma q$ correlator) relate to $F_{FT}$ in a QCD process ($qgq$ correlator) through a diquark model.

\[
\begin{align*}
(F_{FT}^{u/p})_{QED} &= \frac{\alpha_{em}}{3C_F\alpha_s}(F_{FT}^{u/p})_{QCD} \\
(F_{FT}^{d/p})_{QED} &= \frac{4\alpha_{em}}{3C_F\alpha_s}(F_{FT}^{d/p})_{QCD} \\
(F_{FT}^{u/n})_{QED} &= -\frac{2\alpha_{em}}{3C_F\alpha_s}(F_{FT}^{d/p})_{QCD} \\
(F_{FT}^{d/n})_{QED} &= \frac{\alpha_{em}}{3C_F\alpha_s}(F_{FT}^{u/p})_{QCD}
\end{align*}
\]

Use 3 different inputs for $F_{FT}$ in a QCD process:

1) **Sivers**: fit from Anselmino, et al. (2008) of Sivers asymmetry from SIDIS data
2) **KQVY**: fit from Kouvaris, et al. (2006) for SSAs in $pp$ collisions
3) **KP**: simultaneous fit from Kang and Prokudin (2012) of $pp$ and SIDIS data
Proton SSA:

- **Sivers** input agrees exactly with the HERMES data (Airapetian, et al. (2009))

- **KP** input appears to become too large at large $x$ (result of the node in $x$ for the up quark Sivers function)

  ![Graph](image)

  Node in $x$ in the Sivers function is not preferred, although it cannot be definitively excluded by the current data → need more accurate data at larger $x$

- **KQVY** input also appears to become too large at large $x$ and actually diverges as $x \to 1$
Node in $x$ or $k_T$ in the Sivers function:

- Attempt to simultaneously fit SIDIS and $pp$ data (Kang and Prokudin (2012))

SIDIS data from HERMES (left) and COMPASS (right)

Proton-proton data from STAR at $y = 3.3$ (left) and $y = 3.7$ (right)

Proton-proton data from BRAHMS for $\pi^+$ (left) and $\pi^-$ (right)