GPDs and fitting procedures for DVCS

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Outline

Introduction to GPDs

Local fits

Global fits (small $x_B$)

Global fits (all DVCS data)

Neural networks approach

Outcomes
Attractiveness of GPDs (1/3)

1. Well-defined within the QCD

\[ F^q(x, \eta, t = \Delta^2) = \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P_2 | \bar{q}(-z)\gamma^+ q(z) | P_1 \rangle \bigg|_{z^+ = 0, z_\perp = 0} \]

\[ P = P_1 + P_2 ; \quad \Delta = P_2 - P_1 ; \quad \eta = -\frac{\Delta^+}{P^+} \text{ (skewedness)} \]
Attractiveness of GPDs (2/3)

- Decomposition into nucleon helicity conserving and non-conserving parts:

\[
F^a = \frac{\bar{u}(P_2)\gamma^+ u(P_1)}{P^+} H^a + \frac{\bar{u}(P_2)i\sigma^{+\nu} u(P_1)\Delta^\nu}{2MP^+} E^a \quad a = q, g
\]

\[
H^q(x, 0, 0)\big|_{x \geq 0} = q(x) \quad \int_{-1}^{1} dx \ H^q(x, \eta, t) = F_1^q(t)
\]

- 2. Close contact to 3D quark-gluon hadron structure

\[
\frac{1}{2} \int_{-1}^{1} dx \ x \left[ H^q(x, \eta, t) + E^q(x, \eta, t) \right] = J^q(t) \quad [\text{Ji '97}]
\]

\[
q(x, b_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{ib_\perp \cdot \Delta_\perp} H^q(x, 0, -\Delta_\perp^2) \quad [\text{Burkardt '00}]
\]
Attractiveness of GPDs (3/3)

- **3. Accessible to experiments**
- **Deeply virtual Compton scattering (DVCS)**

\[
P = P_1 + P_2, \quad t = (P_2 - P_1)^2
\]
\[
q = (q_1 + q_2)/2
\]

Generalized Bjorken limit:
\[
-q^2 \sim Q^2/2 \to \infty
\]
\[
\xi = \frac{-q^2}{2P \cdot q} = \frac{x_B}{2 - x_B} \to \text{const}
\]

- We work at leading order accuracy where cross-section can be expressed in terms of **four Compton form factors (CFFs)**

\[
\mathcal{F} \in \{ \mathcal{H}(\xi, t, Q^2), \mathcal{E}(\xi, t, Q^2), \tilde{\mathcal{H}}(\xi, t, Q^2), \tilde{\mathcal{E}}(\xi, t, Q^2) \}
\]
Factorization of DVCS $\rightarrow$ GPDs

- [Collins et al. '98]

\[-q_1^2 = Q^2\]
\[q_2^2 = 0\]

\[\gamma\]

Compton form factor is a convolution:

\[^aH(x, t, Q^2) = \int dx \ C^a(x, \xi, Q^2/Q_0^2) \ H^a(x, \eta = \xi, t, Q_0^2)\]

\[a=NS,S(\Sigma,G)\]
Dispersion-relation access to GPDs at LO

[Teryaev '05; K.K., Müller and Passek-K. '07, '08; Diehl and Ivanov '07]

• LO perturbative prediction is “handbag” amplitude

\[ \mathcal{H}(\xi, t, Q^2) \overset{\text{LO}}{=} \int_{-1}^{1} dx \left( \frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right) H(x, \xi, t, Q^2) \]

• giving access to GPD on the “cross-over” line \( \eta = x \)

\[ \frac{1}{\pi} \text{Im} \mathcal{H}(\xi = x, t, Q^2) \overset{\text{LO}}{=} H(x, x, t, Q^2) - H(-x, x, t, Q^2) \]

• while dispersion relation connects it to \( \Re \mathcal{H} \)

\[ \Re \mathcal{H}(\xi, t, Q^2) = \]
\[ \frac{1}{\pi} \text{PV} \int_{0}^{1} d\xi' \left( \frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'} \right) \text{Im} \mathcal{H}(\xi', t, Q^2) + C_{\mathcal{H}}(t, Q^2) \]
Curse of dimensionality

- It is relatively easy to find a coin lying somewhere on 100 meter string. It is very difficult to find it on a football field.
Curse of dimensionality

- *It is relatively easy to find a coin lying somewhere on 100 meter string. It is very difficult to find it on a football field.*
- When the dimensionality increases, the volume of the space increases so fast that the available data becomes sparse.
- Analogously, in contrast to $PDFs(x)$, it is very difficult to perform truly model independent extraction of $GPDs(x, \xi, t)$
- Known GPD constraints don’t decrease the dimensionality of the GPD domain space.
- As an intermediate step, one can attempt extraction of $CFFs(\xi, t)$

- (Dependence on additional variable, photon virtuality $Q^2$, is in principle known — given by evolution equations.)
## Fit types

<table>
<thead>
<tr>
<th>Fit type</th>
<th>Data used</th>
<th>pQCD order</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Local fits</td>
<td>fixed target</td>
<td>LO</td>
<td>CFFs</td>
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<tr>
<td>2. Global fits</td>
<td>collider/fixed target</td>
<td>((N)N)LO</td>
<td>CFFs/GPDs</td>
</tr>
<tr>
<td>3. Neural nets</td>
<td>fixed target</td>
<td>LO</td>
<td>CFFs</td>
</tr>
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**Intro to GPDs**

**Local fits**

**Global fits (small $x_B$)**

**Global fits (all DVCS data)**

**Neural networks**

**Outcomes**

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*Krešimir Kumerički: GPDs and fitting procedures for DVCS*
## Local fits to HERMES data

- Most complete set of asymmetries (14) is measured in 12 bins:

<table>
<thead>
<tr>
<th>bin no.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-t$ [GeV$^2$]</td>
<td>0.03</td>
<td>0.1</td>
<td>0.2</td>
<td>0.42</td>
<td>0.1</td>
<td>0.1</td>
<td>0.13</td>
<td>0.2</td>
<td>0.08</td>
<td>0.1</td>
<td>0.13</td>
<td>0.19</td>
</tr>
<tr>
<td>$x_B$</td>
<td>0.08</td>
<td>0.1</td>
<td>0.11</td>
<td>0.12</td>
<td>0.05</td>
<td>0.08</td>
<td>0.12</td>
<td>0.2</td>
<td>0.06</td>
<td>0.08</td>
<td>0.11</td>
<td>0.17</td>
</tr>
<tr>
<td>$Q^2$ [GeV$^2$]</td>
<td>1.9</td>
<td>2.5</td>
<td>2.9</td>
<td>3.5</td>
<td>1.5</td>
<td>2.2</td>
<td>3.1</td>
<td>5.0</td>
<td>1.2</td>
<td>1.9</td>
<td>2.8</td>
<td>4.9</td>
</tr>
</tbody>
</table>

\[ A_C \equiv \frac{d\sigma_{e^+} - d\sigma_{e^-}}{d\sigma_{e^+} + d\sigma_{e^-}} = \frac{A_{\text{Interference}}(F)}{|A_{\text{DVCS}}|^2(F^2) + |A_{\text{BH}}|^2} \]

\[ A_C^{\cos(1\phi)} \propto \left[ F_1 \text{Re} H - \frac{t}{4M_p^2} F_2 \text{Re} E + \frac{x_B}{2}(F_1 + F_2) \text{Re} \tilde{H} \right] \]

- To express asymmetries in terms of CFFs we use formulas from [Belitsky, Müller, Kirchner '01, Belitsky, Müller '10].
Method of stepwise regression

- Constraints from data are too weak to constrain simultaneously all eight \{\text{Im } \mathcal{F}, \text{Re } \mathcal{F}\} CFFs
- Let us take smaller number of CFFs, choosing only those which are reliably extracted. **Stepwise regression** algorithm:
  1. Perform single-CFF fit with each of 8 CFFs and see which one alone describes data best (it is \text{Im } \mathcal{H}, by far).
  2. Combine \text{Im } \mathcal{H} with each of other seven CFFs and see which pair describes data best.
  3. Proceed until there is either no improvement in data description or new CFFs are not extracted with any statistical significance
Method of stepwise regression

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  3. Proceed until there is either no improvement in data description or new CFFs are not extracted with any statistical significance.
- It turns out that already 2nd step is the final one, and there are two equally good pairs of CFFs:
  1. \( (\text{Im } \mathcal{H}, \text{Re } \mathcal{H}) \) with \( \chi^2/\text{n.d.o.f.} = 102.3/120 \), and
  2. \( (\text{Im } \mathcal{H}, \text{Re } \mathcal{E}) \) with \( \chi^2/\text{n.d.o.f.} = 103.0/120 \).
Stepwise regression — results

- Scenario 1: Fit of $\text{Im } \mathcal{H}$, $\text{Re } \mathcal{H}$ and $\text{Im } \tilde{\mathcal{H}}$. $\chi^2 / n_{\text{d.o.f.}} = 148.8 / 144$. (In good agreement with [Guidal ’10])

- Scenario 2: Fit of $\text{Im } \mathcal{H}$ and $\text{Re } \mathcal{E}$. $\chi^2 / n_{\text{d.o.f.}} = 134.2 / 144$. 
Modelling GPDs in moment space

- Instead of considering momentum fraction dependence $H(x, \ldots)$
- ... it is convenient to make a transform into complementary space of conformal moments $j$:

$$H_j^q(\eta, \ldots) \equiv \frac{\Gamma(3/2)\Gamma(j+1)}{2^{j+1}\Gamma(j+3/2)} \int_{-1}^{1} dx \ \eta^j \ C_j^{3/2}(x/\eta) \ H^q(x, \eta, \ldots)$$

- They are analogous to Mellin moments in DIS: $x^j \rightarrow C_j^{3/2}(x)$
- $C_j^{3/2}(x)$ — Gegenbauer polynomials
Advantages of conformal moments

1. The evolution equations are most simple: There is **no mixing** among moments at LO, and in special ($CS$) scheme not even at NLO

2. Stable and fast **computer code** for evolution and fitting

3. Moments are equal to matrix elements of **local** operators and are thus directly accessible on the **lattice**
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- To model \( \eta \)-dependence we use SO(3) partial wave expansion of crossed process \( \gamma^* \gamma \rightarrow p \bar{p} \) where scattering angle \( \theta \) corresponds to \( -\frac{1}{\eta} \).
I-PW model — only leading SO(3) partial wave

\[ H_j(\xi, t, \mu_0^2) = \left( \begin{array}{c} \frac{N'_\Sigma F_\Sigma(t) B(1 + j - \alpha_\Sigma(0), 8)}{N'_G F_G(t) B(1 + j - \alpha_G(0), 6)} \end{array} \right) \]

\[ \alpha_a(t) = \alpha_a(0) + 0.15t \quad F_a(t) = \frac{j + 1 - \alpha(0)}{j + 1 - \alpha(t)} \left( 1 - \frac{t}{M_0^a \pm} \right)^{-p_a} \]

...corresponding in forward case to PDFs of form

\[ \Sigma(x) = N'_\Sigma x^{-\alpha_\Sigma(0)} (1 - x)^7; \quad G(x) = N'_G x^{-\alpha_G(0)} (1 - x)^5 \]

- \( M_0^G = \sqrt{0.7} \text{ GeV} \) is fixed by the \( J/\psi \) production data
- Free parameter (for DVCS): \( M_0^\Sigma \)

For small \( \xi \) (small \( x_{Bj} \)) valence quarks are less important \( \Rightarrow \Sigma \approx \text{sea} \)
Inclusion of subleading PW — flexible models

\[ H_j(\eta, t) = \left( \frac{N'_{\text{sea}} F_{\text{sea}}(t) B(1 + j - \alpha_{\text{sea}}(0), 8)}{N'_G F_G(t) B(1 + j - \alpha_G(0), 6)} \right) + \left( \begin{array}{c}
s_{\text{sea}} \\ s_G \end{array} \right) \left( \text{subleading partial waves, } \eta^- \text{-dependence!} \right) \]

- nl-PW — addition of second PW needed for good fits
- two new parameters: \( s^{(2)}_{\text{sea}} \) and \( s^{(2)}_G \)
Example of fit result

- **Intro to GPDs**
- **Local fits**
- **Global fits (small $x_B$)**
- **Global fits (all DVCS data)**
- **Neural networks**
- **Outcomes**

---

Krešimir Kumerički: GPDs and fitting procedures for DVCS
Resulting small-\(x\) \(H(x, x, t)\)

\[
x H_{\text{sea}}(x, x, t, Q^2) \text{ at } \mathcal{N}P\text{ LO}
\]

\[
H_{G}(x, x, t, Q^2) \text{ at } \mathcal{N}P\text{ LO}
\]

- \(P=0\): LO; \(P=1\): NLO; \(P=2\): NNLO
- The whole procedure is extended to meson production [Müller, Lautenschlager, Passek-Kumerički, Schäfer '13]
Extending global analysis to fixed target data

- **Hybrid models** at LO
- **Sea quarks and gluons** modelled like just described (conformal moments + SO(3) partial wave expansion + $Q^2$ evolution).
- **Valence quarks** model (ignoring $Q^2$ evolution):

\[
\text{Im} \mathcal{H}(\xi, t) = \pi \left[ \frac{4}{9} H^{u_{\text{val}}}(\xi, \xi, t) + \frac{1}{9} H^{d_{\text{val}}}(\xi, \xi, t) + \frac{2}{9} H^{\text{sea}}(\xi, \xi, t) \right]
\]

\[
H(x, x, t) = n r 2^\alpha \left( \frac{2x}{1+x} \right)^{-\alpha(t)} \left( \frac{1-x}{1+x} \right)^b \frac{1}{\left( 1 - \frac{1-x}{1+x} \frac{t}{M^2} \right)^p}.
\]

- Fixed: $n$ (from PDFs), $\alpha(t)$ (eff. Regge), $p$ (counting rules)

\[
\alpha^{\text{val}}(t) = 0.43 + 0.85 \frac{t}{\text{GeV}^2} \quad (\rho, \omega)
\]
• $\Re \mathcal{H}$ determined by dispersion relations

\[
\Re \mathcal{H}(\xi, t) = \frac{1}{\pi} \text{PV} \int_{0}^{1} d\xi' \left( \frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'} \right) \Im \mathcal{H}(\xi', t) - \frac{C}{\left(1 - \frac{t}{M_C^2}\right)^2}
\]

• Typical set of free parameters:

\[
\begin{align*}
&M_{\text{sea}}^{(2,4)}, s_{\text{sea}}^{(2,4)}, s_G^{(2,4)} & \text{sea* quarks and gluons } H \\
r_{\text{val}}, M_{\text{val}}, b_{\text{val}} & \text{valence } H \\
\tilde{r}_{\text{val}}, \tilde{M}_{\text{val}}, \tilde{b}_{\text{val}} & \text{valence } \tilde{H} \\
C, M_C & \text{subtraction constant } (H, E) \\
r_\pi, M_\pi & \"pion pole\" \tilde{E}
\end{align*}
\]

• Global fit to 175 data points turns out fine:

KM10 model: $\chi^2/d.o.f. = 135.9/160.$

* $s_{\text{sea}}, G = \text{strengths of subleading partial waves. LO evolution is included.}$
HERMES (2008)

\[ BCA \equiv \frac{d\sigma_{e^+} - d\sigma_{e^-}}{d\sigma_{e^+} + d\sigma_{e^-}} \sim A_C^{\cos 0\phi} + A_C^{\cos 1\phi} \cos \phi \sim \Re \mathcal{H} \]

\[ BSA \equiv \frac{d\sigma_{e^\uparrow} - d\sigma_{e^\downarrow}}{d\sigma_{e^\uparrow} + d\sigma_{e^\downarrow}} \sim A_L^{\sin 1\phi} \sin \phi \sim \Im \mathcal{H} \]
**CLAS (2007)**

- BSA. (Only data with $|t| \leq 0.3 \text{GeV}^2$ used for fits.)
Hall A (2006)

- Fit to unpolarized cross section $d\sigma/(dx_B dQ^2 dt d\phi) \sim \Re H$

- KM10 fit needs unusually large $\Re \tilde{H}$. 

![Graphs showing fit results for different $Q^2$ and $t$ values]

Krešimir Kumerički: GPDs and fitting procedures for DVCS
Including data with polarized target

- KMM12: $\chi^2 / n_{\text{d.o.f.}} = 124.1/80$, strictly speaking not a good fit, but best what we have at the moment
Polarized target (II)

- Surprisingly large \( \sin(2\phi) \) harmonic of \( A_{UL} \) cannot be described within this leading twist framework

Krešimir Kumerički: GPDs and fitting procedures for DVCS
Comparison to others

\[ t = -0.28 \text{ GeV}^2 \]
\[ Q^2 \approx 2 \text{ GeV}^2 \]

\[ \text{Im} \, H(x_B,t,Q^2)/x_B \]

[Guidal '08, Guidal and Moutarde '09], seven CFF fit (blue squares), [Guidal '10] \( \mathcal{H}, \tilde{\mathcal{H}} \) CFF fit (green diamonds), [Moutarde '09] \( H \) GPD fit (red circles)
Essentially a least-squares fit of a complicated many-parameter function. $f(x) = \tanh(\sum w_i \tanh(\sum w_j \cdots))$

$\Rightarrow$ no theory bias
Preliminary neural Net HERMES fit

- Fit to all HERMES DVCS data with two types of neural nets
  - \((x_B, t) - (7 \text{ neurons}) - (\Im H, \Re H, \Im \tilde{H})\): \(\chi^2/n_{\text{pts}} = 135.4/144\)
  - \((x_B, t) - (7 \text{ neurons}) - (\Im H, \Re E)\): \(\chi^2/n_{\text{pts}} = 120.2/144\)
Neural Net HERMES fit - BSA/BCA

![Graphs showing neural network fits for HERMES data with BSA/BCA.]
Neural Net HERMES fit - CFFs

Neural Net HERMES fit - CFFs

Krešimir Kumerički: GPDs and fitting procedures for DVCS
Prediction for COMPASS II BCSA

\[ BCSA = \frac{d\sigma_{\mu^{\downarrow+}} - d\sigma_{\mu^{\uparrow-}}}{d\sigma_{\mu^{\downarrow+}} + d\sigma_{\mu^{\uparrow-}}} \quad (E_\mu = 160 \text{ GeV}) \]
Simulation of EIC capabilities

- Parton densities from combined fit to HERA collider and EIC pseudo-data [Aschenauer, Fazio, K.K and Müller ’13]
KM models are available at WWW

- Google for "gpd page" — get binary code for cross sections

% xs.exe

xs.exe ModelID Charge Polarization Ee Ep xB Q2 t phi

returns cross section (in nb) for scattering of lepton of energy Ee on unpolarized proton of energy Ep. Charge=-1 is for electron.

ModelID is one of
  0 debug, always returns 42,
  1 KM09a - arXiv:0904.0458 fit without Hall A,
  2 KM09b - arXiv:0904.0458 fit with Hall A,
  3 KM10 - preliminary hybrid fit with LO sea evolution, from Trento presentation,
  4 KM10a - preliminary hybrid fit with LO sea evolution, without Hall A data
  5 KM10b - preliminary hybrid fit with LO sea evolution, with Hall A data

xB Q2 t phi -- usual kinematics (phi is in Trento convention)

% xs.exe 1 -1 1 27.6 0.938 0.111 3. -0.3 0

0.18584386497251
GPD page and server

- Durham-like CFF/GPD server page
New directions (instead of Summary)

- Improving GPD models
- Adding deeply virtual meson production data and going to NLO [Müller, Lautenschlager, Schäfer '13]
- Including higher twists [Braun, Manashov et al.]
- Global neural network fits
New directions (instead of Summary)

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The end.
Mapping asymmetries to CFFs

- Inverting these relations gives mapping from the set of observables...

\[
\begin{align*}
A_{\sin(1\phi)}^{\text{LU},I} & & A_{\cos(1\phi)}^{C} & & A_{\cos(0\phi)}^{C} & & A_{\sin(1\phi)}^{\text{UL},+} \\
A_{\cos(1\phi)}^{\text{LL},+} & & A_{\cos(0\phi)}^{\text{LL},+} & & A_{\sin(\varphi)\cos(1\phi)}^{\text{UT},I} & & A_{\cos(\varphi)\sin(1\phi)}^{\text{UT},I} \\
A_{\sin(\varphi)\cos(0\phi)}^{\text{UT},\text{DVCS}} & & A_{\sin(\varphi)\cos(0\phi)}^{\text{UT},I} & & A_{\sin(\varphi)\sin(1\phi)}^{\text{LT},I} & & A_{\cos(\varphi)\cos(1\phi)}^{\text{LT},I} \\
A_{\cos(\varphi)\cos(0\phi)}^{\text{LT},\text{BH}+\text{DVCS}} & & A_{\cos(\varphi)\cos(0\phi)}^{\text{LT},I} & & & & & \\
\end{align*}
\]

- ...to the set of eight real and imaginary parts of CFFs (sometimes called sub-CFFs or just CFFs)...

\[
\mathcal{F} = \left( \text{Im } H, \text{Re } H, \text{Im } E, \text{Re } E, \text{Im } \tilde{H}, \text{Re } \tilde{H}, \text{Im } \tilde{E}, \text{Re } \tilde{E} \right)
\]

- ...where error propagation is straightforward.
Mapping — results

- (Here compared also to standard local least-squares fit)

- Only $\text{Im } \mathcal{H}$, $\text{Re } \mathcal{H}$ and $\text{Im } \tilde{\mathcal{H}}$ are reliably constrained.
Function fitting by a neural net

- **Theorem:** Given enough neurons, any smooth function \( f(x_1, x_2, \cdots) \) can be approximated to any desired accuracy. Single hidden layer is sufficient (but not always most efficient).
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- **With simple training of neural nets to data there is a danger of overfitting** (a.k.a. overtraining).

- **Solution:** Divide data (randomly) into two sets: *training sample* and *validation sample*. Stop training when error of validation sample starts increasing.

\[ f(x), \quad x \quad \text{GOOD!} \]
Toy fitting example

- Fit to data generated according to function (which we pretend not to know).

- Fit with
  1. Standard Minuit fit with ansatz \( f(x) = x^a(1 - x)^b \)
  2. Neural network fit
Toy fitting example

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  1. Standard Minuit fit with ansatz \( f(x) = x^a(1 - x)^b \)
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Modelling conformal moments of GPDs (I)

- How to model \( \eta \)-dependence of GPD’s \( H_j(\eta, t) \)?
- Idea: consider crossed \( t \)-channel process \( \gamma^* \gamma \rightarrow p\bar{p} \)

When crossing back to DVCS channel we have:

\[
\cos \theta_{cm} \rightarrow -\frac{1}{\eta}
\]
Modelling conformal moments of GPDs (I)

• How to model $\eta$-dependence of GPD’s $H_j(\eta, t)$?
• Idea: consider crossed $t$-channel process $\gamma^*\gamma \rightarrow p\bar{p}$

When crossing back to DVCS channel we have:

$$\cos \theta_{cm} \rightarrow -\frac{1}{\eta}$$

• ... and dependence on $\theta_{cm}$ in $t$-channel is given by SO(3) partial wave decomposition of $\gamma^*\gamma$ scattering

$$\mathcal{H}(\eta, \ldots) = \mathcal{H}^{(t)}(\cos \theta_{cm} = -\frac{1}{\eta}, \ldots) = \sum_J (2J+1) f_J(\ldots) d^J_{0,\nu}(\cos \theta)$$

• $d^J_{0,\nu}$ — Wigner SO(3) functions (Legendre, Gegenbauer, ...)
  $\nu = 0, \pm 1$ — depending on hadron helicities
Modelling conformal moments of GPDs (II)

• OPE expansion of both $\mathcal{H}$ and $\mathcal{H}^{(t)}$ leads to

$$H_j(\eta, t) = \eta^{j+1} H_j^{(t)}(\cos \theta = -\frac{1}{\eta}, s(t) = t)$$

• and $t$-channel partial waves are modelled as:

$$H_j(\eta, t) = \sum_{J} h_{J,j} \frac{1}{(1-t/M^2)^p} \frac{1}{J - \alpha(t)}$$

• Similar to “dual” parametrization [Polyakov, Shuvaev ’02]
Fit results - LO

- For consistency, we don’t take standard PDFs, but fit GPDs to DIS data. This determines $N_{\text{sea}}$, $N_G$, $\alpha_{\text{sea}}(0)$ and $\alpha_G(0)$, leaving only $M_0^{\text{sea}}$, $s_{\text{sea}}$ and $s_G$ for DVCS data

- $\chi^2$ values:

<table>
<thead>
<tr>
<th>model</th>
<th>$\alpha_s$</th>
<th>$\chi^2$/d.o.f. DIS</th>
<th>$\chi^2$/d.o.f. DVCS</th>
<th>$\chi^2_{\text{f}}$/n.o.p.</th>
<th>$\chi^2_{\text{W}}$/n.o.p.</th>
<th>$\chi^2_{Q^2}$/n.o.p.</th>
</tr>
</thead>
<tbody>
<tr>
<td>l, dipole</td>
<td>LO</td>
<td>49.7/82</td>
<td>280./100</td>
<td>181./56</td>
<td>63.6/29</td>
<td>36.2/16</td>
</tr>
<tr>
<td>l, exp.</td>
<td>LO</td>
<td>49.7/82</td>
<td>316./100</td>
<td>192./56</td>
<td>79./29</td>
<td>44.9/16</td>
</tr>
<tr>
<td>nl, dipole</td>
<td>LO</td>
<td>49.7/82</td>
<td>95.9/98</td>
<td>53.2/56</td>
<td>27./29</td>
<td>15.8/16</td>
</tr>
<tr>
<td>nl, exp.</td>
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<td>49.7/82</td>
<td>97.9/98</td>
<td>49.1/56</td>
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<td>17.7/16</td>
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<tr>
<td>$\Sigma$, dipole</td>
<td>LO</td>
<td>49.7/82</td>
<td>101./98</td>
<td>57.7/56</td>
<td>27.4/29</td>
<td>16./16</td>
</tr>
<tr>
<td>$\Sigma$, exp.</td>
<td>LO</td>
<td>49.7/82</td>
<td>102./98</td>
<td>51./56</td>
<td>32.3/29</td>
<td>18.6/16</td>
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<tr>
<td>l, dipole</td>
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<td>189./56</td>
<td>51.1/29</td>
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- Parameter values:

<table>
<thead>
<tr>
<th>model</th>
<th>$\alpha_s$</th>
<th>$N_{\text{sea}}$</th>
<th>$\alpha_{\text{sea}}(0)$</th>
<th>$(M_{\text{sea}}^2)^2$</th>
<th>$s_{\text{sea}}$</th>
<th>$\alpha_G(0)$</th>
<th>$s_G$</th>
<th>$B_{\text{sea}}$</th>
<th>$b_{\text{eff}}$</th>
<th>BCA</th>
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<tbody>
<tr>
<td>l, dipole</td>
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<td>0.152</td>
<td>1.158</td>
<td>0.062</td>
<td>1.247</td>
<td></td>
<td></td>
<td>33.</td>
<td>5.7</td>
<td>0.19</td>
</tr>
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<td>l, exp.</td>
<td>LO</td>
<td>0.152</td>
<td>1.158</td>
<td></td>
<td></td>
<td>1.247</td>
<td></td>
<td>29.</td>
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<td>0.23</td>
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<td>0.152</td>
<td>1.158</td>
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<td>nl, exp.</td>
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<td>1.158</td>
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<td>$\Sigma$, exp.</td>
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<td>1.158</td>
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<td>1.247</td>
<td>-34</td>
<td></td>
<td>3.1</td>
<td>5.8</td>
<td>0.15</td>
</tr>
</tbody>
</table>

(boldface numbers = bad fits)
Fit results - NLO

- \( \chi^2 \) values:

<table>
<thead>
<tr>
<th>model</th>
<th>( \alpha_s )</th>
<th>( \chi^2 / \text{d.o.f DIS} )</th>
<th>( \chi^2 / \text{d.o.f DVCS} )</th>
<th>( \chi^2 / \text{n.o.p} )</th>
<th>( \chi^2 / \text{n.o.p} )</th>
<th>( \chi^2 / \text{n.o.p} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>l NLO(MS)</td>
<td>71.6/82</td>
<td>148./100</td>
<td>77.6/56</td>
<td>36.8/29</td>
<td>33.9/16</td>
<td></td>
</tr>
<tr>
<td>l NLO(CS)</td>
<td>71.6/82</td>
<td>105./100</td>
<td>62.9/56</td>
<td>25.1/29</td>
<td>17.1/16</td>
<td></td>
</tr>
<tr>
<td>nl NLO(MS)</td>
<td>71.6/82</td>
<td>102./98</td>
<td>60.2/56</td>
<td>23.9/29</td>
<td>17.5/16</td>
<td></td>
</tr>
<tr>
<td>nl NLO(CS)</td>
<td>71.6/82</td>
<td>104./98</td>
<td>61.4/56</td>
<td>24.9/29</td>
<td>18.1/16</td>
<td></td>
</tr>
<tr>
<td>( \Sigma ) NLO(MS)</td>
<td>71.6/82</td>
<td>101./98</td>
<td>60.5/56</td>
<td>23.9/29</td>
<td>17.5/16</td>
<td></td>
</tr>
<tr>
<td>( \Sigma ) NLO(CS)</td>
<td>71.6/82</td>
<td>104./98</td>
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<td>24.9/29</td>
<td>18.1/16</td>
<td></td>
</tr>
</tbody>
</table>

- Parameter values:

<table>
<thead>
<tr>
<th>model</th>
<th>( \alpha_s )</th>
<th>( N_{\text{sea}} )</th>
<th>( \alpha_{\text{sea}}(0) )</th>
<th>( (M_{\text{sea}})^2 )</th>
<th>( s_{\text{sea}} )</th>
<th>( \alpha_{G}(0) )</th>
<th>( s_{G} )</th>
<th>( B_{\text{sea}} )</th>
<th>( b_{\text{eff}} )</th>
<th>BCA</th>
</tr>
</thead>
<tbody>
<tr>
<td>l NLO(MS)</td>
<td>0.168</td>
<td>1.128</td>
<td>0.71</td>
<td>1.099</td>
<td></td>
<td></td>
<td></td>
<td>3.5</td>
<td>5.0</td>
<td>0.10</td>
</tr>
<tr>
<td>l NLO(CS)</td>
<td>0.168</td>
<td>1.128</td>
<td>0.57</td>
<td>1.099</td>
<td></td>
<td></td>
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<td>4.2</td>
<td>5.7</td>
<td>0.09</td>
</tr>
<tr>
<td>nl NLO(MS)</td>
<td>0.168</td>
<td>1.128</td>
<td>0.59</td>
<td>0.04</td>
<td>1.099</td>
<td>0.02</td>
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<td>4.0</td>
<td>5.6</td>
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<tr>
<td>nl NLO(CS)</td>
<td>0.168</td>
<td>1.128</td>
<td>0.58</td>
<td>-0.01</td>
<td>1.099</td>
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<tr>
<td>( \Sigma ) NLO(MS)</td>
<td>0.168</td>
<td>1.128</td>
<td>0.60</td>
<td>3.10</td>
<td>1.099</td>
<td>1.10</td>
<td></td>
<td>4.0</td>
<td>5.7</td>
<td>0.09</td>
</tr>
<tr>
<td>( \Sigma ) NLO(CS)</td>
<td>0.168</td>
<td>1.128</td>
<td>0.58</td>
<td>-0.42</td>
<td>1.099</td>
<td>-0.58</td>
<td></td>
<td>4.1</td>
<td>5.6</td>
<td>0.09</td>
</tr>
</tbody>
</table>

(boldface numbers = bad fits)

- \( s_{\text{sea},G} \) small \( \rightarrow \) skewness ratio \( r \sim 1.5 \)
### Parameter values

<table>
<thead>
<tr>
<th></th>
<th>KMM12</th>
<th>KM10</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M_v)</td>
<td>0.951 ± 0.282</td>
<td>4.00 ± 3.33</td>
</tr>
<tr>
<td>(r_v)</td>
<td>1.121 ± 0.099</td>
<td>0.62 ± 0.06</td>
</tr>
<tr>
<td>(b_v)</td>
<td>0.400 ± 0.000</td>
<td>0.40 ± 0.67</td>
</tr>
<tr>
<td>(C)</td>
<td>1.003 ± 0.565</td>
<td>8.78 ± 0.98</td>
</tr>
<tr>
<td>(M_C)</td>
<td>2.080 ± 3.754</td>
<td>0.97 ± 0.11</td>
</tr>
<tr>
<td>(t_{M_v})</td>
<td>3.523 ± 13.17</td>
<td>0.88 ± 0.24</td>
</tr>
<tr>
<td>(t_{r_v})</td>
<td>1.302 ± 0.206</td>
<td>7.76 ± 1.39</td>
</tr>
<tr>
<td>(t_{b_v})</td>
<td>0.400 ± 0.001</td>
<td>2.05 ± 0.40</td>
</tr>
<tr>
<td>(r_{pi})</td>
<td>3.837 ± 0.141</td>
<td>3.54 ± 1.77</td>
</tr>
<tr>
<td>(M_{pi})</td>
<td>4.000 ± 0.036</td>
<td>0.73 ± 0.37</td>
</tr>
<tr>
<td>(M_{02S})</td>
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<td>0.51 ± 0.02</td>
</tr>
<tr>
<td>SECS</td>
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<td>0.28 ± 0.02</td>
</tr>
<tr>
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<td>-0.13 ± 0.01</td>
</tr>
<tr>
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<td>-2.79 ± 0.12</td>
</tr>
<tr>
<td>THIG</td>
<td>0.945 ± 0.107</td>
<td>0.90 ± 0.05</td>
</tr>
</tbody>
</table>