Angular Momentum and Polarization in Hadron Collisions up to LHC Energies

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Abstract
Longstanding puzzles in spin physics can be confronted at the high energies of the LHC. Heavy quarks will be produced with significant polarization, both as single spin asymmetries and through polarization correlations. Lower energy proton accelerator and leptoproduction data suggest various mechanisms within QCD for polarization phenomena that can be tested at higher energies. Observation of strange and charm hadron polarization reveals important aspects of QCD spin physics. Top quark polarization is predicted to be significant, and polarization correlations will reveal important aspects of the gluon distributions of the hadrons.
Collaborators

- Work in preparation done mostly in collaboration with Simonetta Liuti
- Experimental analysis also with Pasquale Di Nezza and Liliet Calero Diaz - ALICE
- Related Aurore Courtoy, Osvaldo Gonzalez Hernandez, Kunal Kathuria, Abha Rajan
- Tracy McAskill, Jon Poage
Angular Momentum and Polarization in Hadron Collisions up to LHC Energies:
Polarization as a probe of non-perturbative QCD

1. Longstanding puzzles in spin physics can be confronted at the high energies of the LHC.
2. Will heavy quarks be produced with significant polarization, both as single spin asymmetries and through polarization correlations.
3. Lower energy proton accelerator and leptoproduction data suggest various mechanisms within QCD for polarization phenomena that can be tested at higher energies – LHC & EIC &/or LHeC.
4. Observation of strange and charm hadron polarization reveals important aspects of QCD spin physics.
5. Top quark polarization (SSA) is predicted to be significant.
6. Top polarization correlations will reveal important aspects of the gluon distributions of the hadrons.
Outline

Λ_{s,c,b} polarization Puzzles and Uses

I. Large polarization in hadron processes
   I. Very large p+p → A_N, A_{NN}, p’s
   II. Very large Pol’zn for inclusive Λ & Σ
   III. Intriguing Systematics
   IV. Explanations? Basic evidence for non-perturbative systematics of hadron structure & formation.
   V. Charmed & heavy hyperons (Fermilab fixed target)
   VI. Will hyperons maintain large Pol’zn?? Need understanding of NPQCD mechanism
   VII. If we do not understand large SSA’s we do not understand NPQCD!
Outline

Λ_{s,c,b} polarization Puzzles and Uses

I. Large polarization in hadron processes

II. Leptoproduction of Λ_s & Σ_s

   Not outstanding Single Spin Asymmetries yet

I. Large double correlations at small Q^2

II. Analysis more tractable

III. Which formalism is most useful? TMDs, GPDs, Generalized Fracture Functions?
Outline (cont’d)

I. Large polarization in hadron processes
II. Leptoproduction of $\Lambda_s$ & $S_s$

III. Tool to get into transversity –
   I. Chen, GG, Jaffe, Ji (e$^+e^-$ $\rightarrow \Lambda_s$ anti$\Lambda_s$ X)
   II. “off-diagonal” SIDIS via Transversity odd distributions (intrinsic charm?)
   III. Target fragmentation: GPDs, Fragmentation functions, Fracture Functions (many authors: D.Boer; M. Anselmino. et al.; A. Kotzinian; . . .)
   IV. Collider production – target or central region (e.g. D. Sivers)
   V. $\Pi^0$, $\eta$, K electroproduction $\rightarrow$ Chiral odd GPDs & Transversity: Liuti, GG, et al.

IV. TMDs, GPDs, Generalized Fracture Functions
   I. Why GPDs? Phases and transversity - - -
   II. Preliminary results & relations
Predicts small back to back transverse spin correlations
ALEPH measurement at Z mass (ave. over $\Lambda \Lambda \bar{\Lambda}$):

\[
P_T^\Lambda = 0.016 \pm 0.007 \quad \text{for } p_T > 0.3 \text{ GeV/c},
\]
\[
P_T^\Lambda = 0.019 \pm 0.007 \quad \text{for } p_T > 0.6 \text{ GeV/c},
\]
Large polarization in hadron+hadron

\[ p+p \rightarrow \Lambda + X \text{ Polzn}(x_F, p_T) \]
compiled by K.Heller (1997)
Curves Dharmaratna & GG

Fig. 4. Lambda polarization versus production transverse momentum \((p_T)\). For comparison, data for 400 GeV production (Ref. 10) are also shown.

Evolving Ideas about Source of Λ Polarization in Hadrons

- Semi-classical: Lund; Thomas precession; SU(6); Soffer, et al.
- Q Field Th: Single polarization requires interference => Real x Im part & helicity flip
- Kane, Pumplin, Repko: PQCD (PRL41,1689(1978) → \( P_L \sim \frac{\alpha(\hat{s})m_q}{\sqrt{\hat{s}}} \))
Evolving Ideas about Source of $\Lambda$ Polarization in Hadrons

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- Q Field Th: Single polarization requires interference $\Rightarrow$ Real x Im part & helicity flip
- Kane, Pumplin, Repko: PQCD (PRL41,1689(1978) $\Rightarrow P_L \sim \alpha(\hat{s})m_q / \sqrt{\hat{s}}$
- Complete order $\alpha_s$ calculation of quark, antiquark, gluon 2-body scattering $\Rightarrow s \uparrow + s\bar{\text{bar}}$ imbedded in hadron+hadron pdf’s (but small $m_S$) (Dharmaratna & GG 1990,1996) How does $s \uparrow$ get translated to $\Lambda \uparrow$ & enhanced?
- NPQCD must play a significant role in our understanding of orbital angular momentum & hadron formation.
Contributions to order $\alpha_S$ Imaginary Part
(Dharmaratna & GG 1990,1996)

quark+antiquark

quark+gluon

gluon+gluon

QCDIII GR.Goldstein
How to get to hyperon Polz’n?

s quark accelerates toward (ud) remnant of proton

Box represents loop contributions to Im part. Seen as GPD already have Im part!
Model of hyperon polarization

1. $p+p \rightarrow \Lambda + X$ has large negative $P_\Lambda$ with flat $s$ dependence & growth with $p_T$ (see Heller . . .)

2. Clues: $K^- p \rightarrow \Lambda + X$ at 176 GeV/c or $\sqrt{s}=18$GeV
Polzn even larger - need $s$-quark?

3. Simple factorization expectation
Kane, Pumplin, Repko
$P_\Lambda \sim \alpha(\hat{s}) m_q / \sqrt{\hat{s}}$

helicity flip $\sim m_q$/hard energy scale
Soft phenomenon?

Dharmaratna & GRG (1990,96,99)

Dharmaratna & GRG: 1. Gluon fusion
  dominant mechanism for producing
  polarized
  massive quark pair
2. Low $p_T$ phenomenon
3. Acceleration mechanism

compiled by K.Heller (1997)
$P_{\text{quark}}$ vs. flavor from gluon fusion grows with flavor

Does this give larger $P_{\text{hadron}}$ for heavier flavor?

What sets scales? quark “mass” or hyperon mass

g+g→Q+ X  
Polzn(Q)~ $m_Q/\sqrt{s}$

(Dharmaratna & GG 1990,1996)
Evolving Ideas about Source of $\Lambda$ Polarization in Hadrons (p+p)

- Semi-classical: Lund; Thomas precession; SU(6) re $\Lambda, \Sigma, \Xi$
- Q Field Th: Single Spin Asymmetry requires amplitude interference $\Rightarrow$ Real x Im part & helicity flip
- Kane, Pumplin, Repko: PQCD $\Rightarrow P_L \sim \alpha(\hat{s})m_q / \sqrt{\hat{s}}$
- Complete order $\alpha_s$ calculation of quark, antiquark, gluon 2-body scattering $\Rightarrow s \uparrow + \bar{s} \downarrow$ imbedded in hadron+hadron pdf’s (but small $m_S$) (Dharmaratna & GG 1990,1996) How does $s \uparrow$ get translated to $\Lambda \uparrow$ & enhanced?
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• Complete order $\alpha_s$ calculation of quark, antiquark, gluon 2-body scattering → $s\uparrow+s\bar{\text{bar}}$ imbedded in hadron+hadron pdf’s (but small $m_S$) (Dharmaratna & GG 1990,1996) How does $s\uparrow$ get translated to $\Lambda\uparrow$ & enhanced?
• Burkardt: impact parameter $b$ distortion of quarks in spinning hadron from FT-GPDs, production overlap region & anomalous moments $\kappa$ for hyperons
Evolving Ideas about Source of $\Lambda$ Polarization in Hadrons (p+p)

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- Complete order $\alpha_s$ calculation of quark, antiquark, gluon 2-body scattering $\rightarrow \uparrow s + \downarrow s$ bar imbedded in hadron+hadron pdf’s (but small $m_S$) (Dharmaratna & GG 1990,1996) How does $s \uparrow$ get translated to $\Lambda \uparrow$ & enhanced?
- Burkardt: impact parameter $b$ distortion of quarks in spinning hadron from FT-GPDs, production overlap region & anomalous moments $\kappa$ for hyperons
- S.Liuti, K. Kathuria & GG: Vorticity & $b$ in production (overlap)
  $\rightarrow G_2$: OAM & distortion $\rightarrow$ polarized hyperons
Charmed Hyperon Polarization

\[ P(Λ_c) \text{ vs. } p_T \text{ (GeV) for several } x_F \text{ values} \]


\[ P(Λ_c) \text{ does not fall off with } p_T \text{ Trend to be tested?} \]

GG – hep-ph/990757 proceedings FNAL workshop on Charmed hyperons
Vorticity picture

- High Energy $p + p$ at fixed $b$ has large relative OAM

- Fluid picture with laminar flow in viscous medium

\[ J = \int d^3x \nabla \times \pi = \int d^3x \nabla \frac{x^2}{2} \times \pi \]
\[ = \int d^3x \nabla \times \frac{x^2}{2} \pi - \int d^3x \frac{x^2}{2} \nabla \times \pi \]
\[ = -\int d^3x x^2 \gamma m_q \omega - \cdots \]
\[ \omega = \frac{1}{2} \nabla \times \pi / \gamma m_q \]

\( J \) contains $\omega = \) quark field “vorticity” which transfers to hadrons.

see Becattini, et al. PRC77, 024906 (2008) for heavy ion collisions
OAM in hyperon production

Roiling sea of vorticity $\rightarrow$ emerging heavy flavor quark with fraction of OAM represented by $G_2(x,p_T^2,0)$ ($\gamma^\perp$ term in quark correlator at twist 3) $\rightarrow$ polarized heavy flavor hadron

Work in progress
See S.Liuti talk re OAM
How are quark polarizations measured?
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Hadronization of polarized or unpolarized heavy flavor quark mixes Fragmentation Functions (Anselmino, Boer, et al.) with small $p_T$ production mechanisms – factorization? Initial or Final state interactions?
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Unpolarized quark $\rightarrow$ hadron $\uparrow$ + $X$ for larger $p_T$?
How are quark polarizations measured?

Hadronization of polarized or unpolarized heavy flavor quark mixes Fragmentation Functions (Anselmino, Boer, et al.) with small $p_T$ production mechanisms – factorization? Initial or Final state interactions?

Unpolarized quark $\rightarrow$ hadron $\uparrow\downarrow + X$ for larger $p_T$?

Top quarks decay before hadronizing $\Rightarrow$ decays are “self analyzing”
Unique feature of heavy flavors $\Rightarrow$ provide window into heavy flavor QCD
Direct measure of hard process - top polarization
Top decays weakly before hadronizing ⇒ decay “self-analyzing”

Analyze $t \rightarrow W^+ b$
Evolving Ideas about Source of $\Lambda$ Polarization in Hadrons

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- How does $s \uparrow$ get translated to $\Lambda \uparrow$ & enhanced?
- Acceleration of $s$ to (ud) remnant of N.
- NPQCD must play a significant role in our understanding of orbital angular momentum & hadron formation.
Evolving Ideas about Source of L Polarization in Hadrons

- Semi-classical: Lund; Thomas precession; SU(6)
- Q Field Th: Single polarization requires interference => Real x Im part & helicity flip
- Kane, Pumplin, Repko: PQCD $\rightarrow P_L \sim \alpha(\hat{s})m_q / \sqrt{s}$
- Complete order $\alpha_s$ calculation of quark, antiquark, gluon 2-body scattering $\rightarrow s \uparrow + \bar{s} \bar{\uparrow}$ imbedded in hadron+hadron pdf’s (Dharmaratna & GG 1990,1996)
- How does $s \uparrow$ get translated to $\Lambda \uparrow$?
- Consider electroproduction of $\Lambda$’s. Prelude to hadron production. QCD more under control.
  - Soft matrix elements from TMDs & SIDIS or GPDs &/or Fracture Functions
Electroproduction of \( \Lambda \)

Simple tree level model for extended fracture function (Trentadue & Veneziano)
Diquark spectator & fragmentation
“d\(\sigma\)” squares & sums over \(X\) states with anti-s flavor
Diquark\(\rightarrow\)\(\Lambda\)+s-bar simple vertex

\[
z, p_T
\]

\[
\Lambda = uds
\]

\[
x, k_T
\]

\[
G_{p,h}^i(x, z, p_T; Q^2)
\]

\[
z = E_L / (1-x) E_{gP_{CM}}
\]

or \(P_L^+ = z(1-x)P^+\) for target fragment
Dipole form factors dampen $P \rightarrow u + \text{diquark vertex } \Lambda$, diquark, struck quark all on shell

\[
(P - k)^2 = m_s^2 + \frac{M^2_\Lambda}{z} + \frac{1 - z}{z} \left( \frac{\vec{P}_{\Lambda T}}{z} - \frac{z}{1 - z} \frac{\vec{P}_{XT}}{z} \right)^2
\]

\[
k^2 = xM^2 - \frac{k_T^2}{(1 - x)} - \frac{x}{(1 - x)} (P - k)^2
\]

$d((k+q)^2) \rightarrow x = x_{Bj}$
diquark model extended fracture function

\[(P - k)^2 = m_s^2 + \frac{M^2_{\Lambda}}{z} + \frac{1 - z}{z} \left( \overrightarrow{P}_{\Lambda T} - \frac{z}{1 - z} \overrightarrow{P}_{\Lambda T} \right)^2 \]

\[k^2 = xM^2 - \frac{\overrightarrow{k}_T^2}{(1 - x)} - \frac{x}{(1 - x)}(P - k)^2\]

\[\mathcal{F}^{\lambda_q}_{\Lambda_N, \Lambda_A} (x, k_T, z, p_T, Q^2) = \sum_{\Lambda_X} \int \frac{d^3 P_X}{(2\pi)^3 2E_X} \int \frac{d^4 \xi}{(2\pi)^4} e^{i k \cdot \xi} \langle P | \bar{\psi}(\xi) | P_h; X \rangle \langle P_h; X | \psi(0) | P \rangle.\]

quark correlator for Extended Fracture Functions
helicity labels \(<P, \Lambda_N| \; \& \; |P_h, \Lambda_A ; X >\)
For unpolarized d\sigma, sum over all helicity labels.
For polarized \Lambda, keep floating
In the spectator model

\[ F_{\lambda_q, \Lambda_N}^{\Lambda, \Lambda'} (x, k_T, z, p_T, Q^2) = A_{\Lambda_N, \lambda_q} \sum_{\Lambda_X} B_{\Lambda_X}^{\Lambda, \Lambda'} \]

where

\[ A_{\Lambda_N, \lambda_q} = | \phi_{\lambda, \Lambda} (k, P) |^2, \]

with

\[ \phi_{\Lambda, \lambda} (k, P) = \Gamma (k) \frac{\bar{u} (k, \lambda_q) U (P, \Lambda_N)}{k^2 - m^2}, \]

and

\[ k = P - P_x - P_\Lambda \Rightarrow k^2 = k^2 (x, k_T, z, p_T) \]

whereas

\[ B_{\Lambda_X}^{\Lambda, \Lambda'} = \tilde{\phi}_{\Lambda_X, \Lambda'}^* (P_X, P_h) \tilde{\phi}_{\Lambda_X, \Lambda} (P_X, P_h), \]

with

\[ \tilde{\phi}_{\Lambda_X, \Lambda} (P_X, P_h) = \Gamma (P_X) \bar{v} (P_X, \Lambda_X) U (P_h, \Lambda_\Lambda) \]
\[ \sum_{\Lambda_X} B^{\Lambda,\Lambda'}_{\Lambda_X} = (1-x)^2 \left( -z M_X + (1-z) M_\Lambda \right)^2 + p_T^2 \delta_{\Lambda,\Lambda} \]

\[ A_{\Lambda,\Lambda} \] is squared & summed over

\[ \rightarrow f(x, k_T) \left( k^2 - m_{\text{dipole}}^2 \right)^4 \left|_{P_X^2=(P-k-P_\Lambda)^2=m_s^2} \right. \]

x=0.2, Q^2=Q_0^2

QCDIII  GR.Goldstein
Spin dependence?
a. non-trivial quark or proton-$\Lambda$ spin correlation $\Rightarrow$ axial diquark
b. SSA need phase $\Rightarrow$ beyond tree
Figure shows final state interaction contribution to $\Lambda$
Figure shows final state interaction contribution to $\Lambda$. [Diagram]
Final State Interactions or gauge links . . . .

Recall f.s.i. (e.g. Brodsky, Hwang & Schmidt; Gamberg & Goldstein, etc.)

\[ P_y = C_F \alpha_s (m^2) \frac{(xM + m)k_x}{[(xM + m)^2 + \vec{k}_\perp^2]} \frac{\Lambda(\vec{k}_\perp^2)}{\Lambda(0)} \ln \left( \frac{\Lambda(\vec{k}_\perp^2)}{\Lambda(0)} \right) \]

\[ \Lambda(\vec{k}_\perp^2) = \vec{k}_\perp^2 + (1-x) \left( -M^2 + \frac{m^2}{x} + \frac{\lambda^2}{1-x} \right) \]

For Frac.Fn. model replace denom with \([[-z M_x + (1-z)M_\Lambda]^2 + p_T^2]\)

\[ \Omega(k_T^2, p_T^2) = x M^2 - \{ k_T^2 + x[m_s^2 + \frac{M_\Lambda^2}{z} + \frac{1-z}{z}(\vec{p}_T - \frac{z}{1-z} \vec{p}_{XT})^2] \} / (1-x) \]

numerator with \((1-x)^2 k_T^2 (z M_x - (1-z)M_\Lambda^2)\) from Im flip × non-flip
Formally, for a quark-nucleon FF,

\[ \mathcal{F}(x, k_T, z, p_T, Q^2) = \sum_X \int \frac{d^3 P_X}{(2\pi)^3 2E_X} \int \frac{d^4 \xi}{(2\pi)^4} e^{i k \cdot \xi} \times \langle P \mid \bar{\psi}(\xi) \mid P_{\text{hadron}}; X \rangle \times \langle P_{\text{hadron}}; X \mid \psi(0) \mid P \rangle. \]  

(6)
Generalized Fracture Function $\Rightarrow$ extended GPDs (flavor changing)

$$\langle p | \psi | X \rangle$$

$$\langle X | \bar{\psi} | \Lambda \ h \rangle$$

$\Lambda = uds$

NPQCD loop with GPD (quarks) (Fig.2a):

\[
\int dx_1 dx_2 \ [\mathcal{H}_{N \rightarrow Y}^* \mathcal{H}_{N \rightarrow Y}] (x_1, \zeta, t, Q^2) \\
\times f(x_2, Q^2) \hat{\sigma}_{12 \rightarrow sX}^{LO} (x_1, x_2, x_F, p_T)
\]
Gluon fusion is largest source of polarized quarks & gluons . . .
Gluons will be plentiful at LHC. Move toward Gluon GPDs
GPD source of $\Lambda$ ↑

CFF $\Rightarrow$ Re & Im parts

Each line has helicity summed over except $\Lambda_L \& \Lambda'_L$

Real & Im CFF multiplied for Polzn

QCDIII  GR.Goldstein
3. Polarized top quark production and spin correlations
Top decays vs. mass

$P_{\text{quark}}$ vs. flavor from gluon fusion grows with flavor

Does this give larger $P_{\text{hadron}}$ for heavier flavor?

What sets scales? quark “mass” or hyperon mass

$g + g \rightarrow Q + X$

$\text{Polzn}(Q) \sim m_Q / \sqrt{s}$
Direct measure of hard process - top polarization
preliminary predictions of D&G PQCD


Analyze $t \rightarrow W^+ b$

preliminary - do not quote!
How is actual top polarization determined? Its decay is good analyzer.

\[ U_{t,\bar{t}} = \sum_{\lambda_b} B_{\lambda_b,\bar{t}} B_{\lambda_b, t} \]
\[ \propto (I + \vec{p}_t \cdot \vec{\sigma}_t / p_t)_{t,\bar{t}} (p_b \cdot p_\nu), \]
Polarized top pair production and polarized gluon distributions

Double spin asymmetries are “naïve-T” even → get “tree-level”
QCD contributions ⇒ good test of BSM
Dilepton events

Tree-level QCD
q+q (Tevatron)

QCDIII G.R.Goldstein
Polarized top pair production in p+p and polarized gluon distributions

\[ p \rightarrow g(\text{polzn x or y}, p_{1T}) : f_1^g(p_1) \text{ or } h_{1T}^\perp g(p_1) \]

\[ p \rightarrow g(\text{polzn x or y}, p_{2T}) : f_1^g(p_2) \text{ or } h_{1T}^\perp g(p_2) \]

\[ g_1(p_1) \uparrow \downarrow + g_2(p_2) \downarrow \rightarrow t(k_1) + \overline{t} \text{ (k}_2) \]

\[ A_{\Lambda_{g1}, \Lambda_{g2}; t, \overline{t}} |_{(b, c)} = \left( \frac{(\lambda^b \lambda^c)_{j,k}}{(m_t^2 - \hat{t})} a_{\Lambda_{g1}, \Lambda_{g2}; t, \overline{t}}^t + \frac{(\lambda^c \lambda^b)_{j,k}}{(m_t^2 - \hat{u})} a_{\Lambda_{g1}, \Lambda_{g2}; t, \overline{t}}^u \right) \]
At LHC:

Gluon fusion tree level mechanism
(Color gauge invariance)

\( g_1, g_2 \) carry helicity \( \Lambda_1 \Lambda_2 = \pm 1 \)
\( t, \bar{t} \) carry helicity \( \lambda_t \lambda_{\bar{t}} = \pm \frac{1}{2} \)

Introduced in:
Amplitudes for $p_1 + p_2 \rightarrow t + X_1 + \bar{t} + X_2$

Implicitly convoluted over $k_{T1}$ & $k_{T2}$

Differential cross section for inclusive $p_1 + p_2 \rightarrow t + X_1 + \bar{t} + X_2$

Regrouping terms gives gluon distributions
Polarized hard gluon fusion forms $t+\bar{t}$ amplitudes & density matrices in terms of helicities

$G^{(1)}_{\Lambda N_1, \Lambda g_1, \Lambda' g_1} = \sum_{\Lambda X_1} \int_{X_1} g^{(1)*}_{\Lambda N_1, \Lambda X_1, \Lambda' g_1} g^{(1)}_{\Lambda N_1, \Lambda X_1, \Lambda g_1}$

$(off)$diagonal gluon distributions

$\rho^{UP,UP}_{t',\bar{t};t,\bar{t}} = \sum_{\Lambda N_1, \Lambda N_2} \{ G^{(2)}_{\Lambda N_2, UP} \rho^{UP,UP}_{t',\bar{t};t,\bar{t}} G^{(1)}_{\Lambda N_1, UP} + G^{(2)}_{\Lambda N_2, UP} \rho^{UP,LP}_{t',\bar{t};t,\bar{t}} G^{(1)}_{\Lambda N_1, LP}$

$+ G^{(2)}_{\Lambda N_2, LP} \rho^{LP,UP}_{t',\bar{t};t,\bar{t}} G^{(1)}_{\Lambda N_1, UP} + G^{(2)}_{\Lambda N_2, LP} \rho^{LP,LP}_{t',\bar{t};t,\bar{t}} G^{(1)}_{\Lambda N_1, LP} \}$

$\rho^{UP,UP}_{t',\bar{t};t,\bar{t}} = [A^{*}_{RR,t',\bar{t}} A_{RR,t,\bar{t}} + A^{*}_{RL,t',\bar{t}} A_{RL,t,\bar{t}} + A^{*}_{LR,t',\bar{t}} A_{LR,t,\bar{t}} + A^{*}_{LL,t',\bar{t}} A_{LL,t,\bar{t}}]$
Combine $g$ distributions with hard gluon fusion $t+t\bar{t}$ amplitudes

$$
G_{\Lambda N_1, R, R}^{(1)} + G_{\Lambda N_1, L, L}^{(1)} = G_{\Lambda N_1, XX}^{(1)} + G_{\Lambda N_1, YY}^{(1)} = G_{\Lambda N_1, UP}^{(1)} \\
G_{\Lambda N_1, R, L}^{(1)} + G_{\Lambda N_1, L, R}^{(1)} = G_{\Lambda N_1, YY}^{(1)} - G_{\Lambda N_1, XX}^{(1)} = G_{\Lambda N_1, LP}^{(1)}
$$

Linearly polarized gluon distributions arise naturally in the heavy pair production
Simple spin structure

$$
G_{\Lambda'; \Lambda, g, \Lambda}(x, k_T^2) = \Gamma(k_T^2) \bar{U}_{\Lambda'}(p') \epsilon^i_{\Lambda g}(k) \gamma_j U_{\Lambda}(p) \\
\rho_{\Lambda g, \Lambda g} = \sum_{\Lambda', \Lambda} G^*_{\Lambda'; \Lambda, g, \Lambda} G_{\Lambda'; \Lambda, g, \Lambda}$$
$q + q-bar \rightarrow t + t-bar$

- The quark spin correlations are transmitted to the decay products.
- The correlations between the lepton directions and the parent top spin (in the top rest frame) produce correlations between the lepton directions.
- Correlations expressed as a weighting factor.
How is actual top polarization determined? Its decay is good analyzer.

\[ U_{t,\bar{t}} = \sum_{\lambda_b} B^*_{\lambda_b,t} B_{\lambda_b,\bar{t}} \]

\[ \propto (I + \vec{p}_t \cdot \vec{\sigma}_t / p_t)_{t,\bar{t}}(p_b \cdot p_\nu), \]

Calculated in top rest frame
The light quark-antiquark annihilation mechanism gives rise to the angular distribution between opposite charge lepton pairs,

\[
W(\theta, p, p_t, p_l) = \frac{1}{4} \left\{ 1 + [\sin^2\theta([p^2 + m^2](\hat{p}_t)_x(\hat{p}_t)_x + [p^2 - m^2](\hat{p}_t)_y(\hat{p}_t)_y) - 2mpc\cos\theta\sin\theta((\hat{p}_l)_x(\hat{p}_l)_z + (\hat{p}_l)_z(\hat{p}_l)_x) + ([p^2 - m^2] + [p^2 + m^2]\cos^2\theta)(\hat{p}_l)_z(\hat{p}_l)_z)/([p^2 + m^2] + (p^2 - m^2)\cos^2\theta) \right\}
\]

\(m\) = top mass, \(\theta\) = t production angle in q+q-bar CM
\(p\) = light quark 3-momentum in CM
Unit vectors \(\hat{p}\)-hat are anti-lepton\(^+\) and lepton\(^-\) 3-momenta directions in the top and anti-top rest frames.
$g_1 + g_2 \rightarrow t + \bar{t}$

**Spin correlations**

Correlations expressed as a weighting factor *for unpolarized gluons*.

- The *gluon fusion mechanism* gives rise to a higher order angular distribution due to the combination of two spin 1 gluons.

\[
W(\theta, p, \vec{p}_l, \vec{p}_t) = \frac{1}{4} - \frac{1}{4} \left\{ \left[ p^4 \sin^4 \theta + m^4 \right] (\hat{p}_l)_x (\hat{p}_l)_x + \left[ p^2 (p^2 - 2m^2) \sin^4 \theta - m^4 \right] (\hat{p}_l)_y (\hat{p}_l)_y \\
+ \left[ p^4 \sin^4 \theta - 2p^2 (p^2 - m^2) \sin^2 \theta + m^2 (2p^2 - m^2) \right] (\hat{p}_l)_z (\hat{p}_l)_z \\
+ 2mp^2 \sqrt{p^2 - m^2} \cos \theta \sin^3 \theta \left[ (\hat{p}_l)_x (\hat{p}_l)_z - (\hat{p}_l)_y (\hat{p}_l)_z \right] \right\} \\
/ \left[ p^2 (2m^2 - p^2) \sin^4 \theta + 2p^2 (p^2 - m^2) \sin^2 \theta + m^2 (2p^2 - m^2) \right]
\]

Use these to test SM vs. BSM – Integrated version agrees – with big errors

GG – also Mahlon & Parke
The gluon spin correlations are transmitted to (determine the spin of) the decay products.

The correlations between the lepton directions and the parent top spin (in the top rest frame) produce correlations between the lepton directions.

The **gluon fusion mechanism** gives rise to a higher order (wrt quark antiquark) angular distribution due to the combination of two spin 1 gluons.

Helicity structure of polarized top-anti-top production

$$\sum_{\text{all--helicities--not--tops}} \bar{G}_{\Lambda N \bar{g} \bar{g}'} A^{*}_{\Lambda g \bar{g}':t',\bar{t}'} A_{\Lambda g \bar{g}';t,\bar{t}} G_{\Lambda N \Lambda g \Lambda g'}$$

$$\propto \rho_{t',\bar{t}';t,\bar{t}}$$

\[QCDIII \text{ G.R.Goldstein} 5/10/14\]
Observable cross section with polarized $t + t$-bar

Use top pair polarization to determine the polarized gluon distributions
Gluon linear polarization
with like and unlike t-\(\bar{t}\) helicities
(work in progress S.Liuti & GG)

\[ \mathbf{F} \sim \mathbf{G}_{XX} + \mathbf{G}_{YY} , \mathbf{H} \sim \mathbf{G}_{XX} - \mathbf{G}_{YY} \]
or linear polarization

\[
\begin{array}{c|c|c}
\rho_{t', \overline{t}'; t, \overline{t}} & \bar{F} F & \bar{H} H \\
++;++ & \gamma^{-2} (1 + \beta^2 (1 + \sin^4 \theta)) & \gamma^{-2} (-1 + \beta^2 (1 + \sin^4 \theta)) \\
+-;+- & \beta^2 \sin^2 \theta (2 - \sin^2 \theta) & -\beta^2 \sin^4 \theta \\
\end{array}
\]

\[
\begin{array}{c|c|c}
& \bar{F} H & \bar{H} F \\
& -2 \frac{\beta^2}{\gamma} \sin^2 \theta & -2 \frac{\beta^2}{\gamma} \sin^2 \theta \\
& 0 & 0 \\
\end{array}
\]
Top spin correlations & gluon polarizations

<table>
<thead>
<tr>
<th>$\rho_{t',\bar{t}';t,\bar{t}}$</th>
<th>UP,UP</th>
<th>LP,LP</th>
<th>UP,LP + LP,UP</th>
</tr>
</thead>
<tbody>
<tr>
<td>++, ++</td>
<td>$\gamma^{-2}(1 + \beta^2(1 + \sin^4\theta))$</td>
<td>$\gamma^{-2}(-1 + \beta^2(1 + \sin^4\theta))$</td>
<td>$-4\gamma^{-2}\beta^2\sin^2\theta$</td>
</tr>
<tr>
<td>++, --</td>
<td>$\beta^2\sin^2\theta(2 - \sin^2\theta)$</td>
<td>$-\beta^2\sin^4\theta$</td>
<td>0</td>
</tr>
<tr>
<td>+, --</td>
<td>$\gamma^{-2}(-1 + \beta^2(1 + \sin^4\theta))$</td>
<td>$\gamma^{-2}(1 + \beta^2(1 + \sin^4\theta))$</td>
<td>$+4\gamma^{-2}\beta^2\sin^2\theta$</td>
</tr>
<tr>
<td>+, +</td>
<td>$\beta^2\sin^4\theta$</td>
<td>$-\beta^2\sin^2\theta(2 - \sin^2\theta)$</td>
<td>0</td>
</tr>
<tr>
<td>+, --</td>
<td>$-2\gamma^{-1}\beta^2\sin^3\theta$</td>
<td>$-2\gamma^{-1}\beta^2\sin^3\theta$</td>
<td>$-4\gamma^{-1}\beta^2\sin^2\theta\cos\theta$</td>
</tr>
<tr>
<td>+, +</td>
<td>$2\gamma^{-1}\beta^2\sin^3\theta\cos\theta$</td>
<td>$2\gamma^{-1}\beta^2\sin^3\theta\cos\theta$</td>
<td>$4\gamma^{-1}\beta^2\sin^2\theta\cos\theta$</td>
</tr>
</tbody>
</table>

TABLE I: Values of double density matrix elements $\rho$ for combinations in Eq. 10 using values of helicity amplitudes from Eq. 12

UP = unpolarized, LP = Linearly polarized gluon distributions assuming $g+g \rightarrow t + t$-bar in single plane CM

Taking X-Z plane for $p+p \rightarrow (t+t\bar{t})_{CM} + X$ gives $\phi$ dependence to $t+t\bar{t}$ plane for opposite helicities: $\text{Re}(e^{\pm(1\text{or}2)i\phi} \cdot e^{\pm(-i(1\text{or}2)\phi)})$ leading to $\cos 2\phi$ for UP,LP and LP,UP and $\cos 4\phi$ modulations for LP,LP.
Azimuthal dependence

Evaluated $g+g \rightarrow t+t\bar{t}$ in CM X-Z plane

Rotate 2-spinors
Matrix element rotates
Or gluon x & y rotate.
Amplitude phases

$\rightarrow \cos 2\phi$ & $\cos 4\phi$
modulations of $t+t\bar{t}$ angular distributions, depending on helicities
Multiply each configuration of $t$ & $t\bar{t}$ by actual top polarization decay as a good analyzer.

\[ U_{t,\bar{t}} = \sum_{\lambda_b} B_{\lambda_b,t}^* B_{\lambda_b,\bar{t}} \]

\[ \propto (I + \vec{p}_t \cdot \vec{t}/p_t)_{t,\bar{t}} (p_b \cdot p_\nu), \]

Provides unique method to decompose gluon distributions.
Each row in table of top-antitop density matrix elements gets distinct kinematic variation.
...also.... Boer, Brodsky, C. Pisano (PRL 2012, JHEP2013) have considered the determination of polarized gluon distributions from azimuthal distributions of unpolarized $Q\bar{Q}$ production in SIDIS and $p+p$. This amounts to summing the columns of the table.
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However, we consider the polarizations of the $t$-$t\bar{t}$ as levers to differentiate between like pairs of gluon polarizations and unlike pairs. Separating rows AND columns.
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However, we consider the polarizations of the $t-\overline{t}$ as levers to differentiate between like pairs of gluon polarizations and unlike pairs
Separating rows AND columns

Our method of separating rows is new. Mahlon and Parke , (PRD81, 074024 (2010)) took integrated spin correlation measure which was confirmed by D0 & now at ATLAS. We single out gluon distributions by separating different $t+t\overline{t}$ helicities (instead of just overall polarizations)

Work being completed . . . .
Summary

• Hyperon polarization is touchstone for understanding transversity & hence NPQCD

• Several ways to begin to explain phenomena
  ▪ “Upside down” TMDs with f.s.i.
  ▪ Extended GPDs ➔ Extended Fracture Functions
  ▪ Vorticity & OAM: Work in progress

• How to see quark or gluon polarization?
• top quarks as a probe of SSA sources
• & t+tbar spin correlations - window into linearly polarized gluon distributions