Generalized TMDs

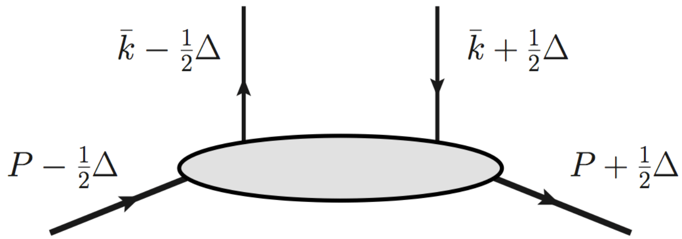
(A. Metz, Temple University, Philadelphia)

- Introduction
- Parameterization of GTMDs
- In what regard can GTMDs be interesting/useful ?
- Comments on recent criticism by
  Courtoy, Goldstein, Gonzalez, Liuti, Rajan, arXiv:1309.7029, arXiv:1310.5157
- Lessons from explicit calculations of GTMDs
- Summary

Talk mainly based on
Goeke, Meißner, Metz, Schlegel, arXiv:0805.3165, arXiv:0906.5323
Kanazawa, Lorcé, Metz, Pasquini, Schlegel, arXiv:1403.5226

Definition of GPDs and TMDs

- GPDs
  - Appear in QCD-description of hard exclusive reactions (DVCS, HEMP)
  - Kinematics (symmetric frame)



$$P = \frac{p+p'}{2} \qquad \Delta = p'-p = k'-k$$

  - GPD-correlator

$$F^{[\gamma^+]} = \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{i\bar{k}\cdot z}\, \langle p' \,|\, \bar{\psi}\Big(-\frac{z}{2}\Big)\, \gamma^+\, \mathcal{W}_{GPD}\, \psi\Big(\frac{z}{2}\Big)\, |\, p\rangle \Big|_{z^+=z_\perp=0}$$

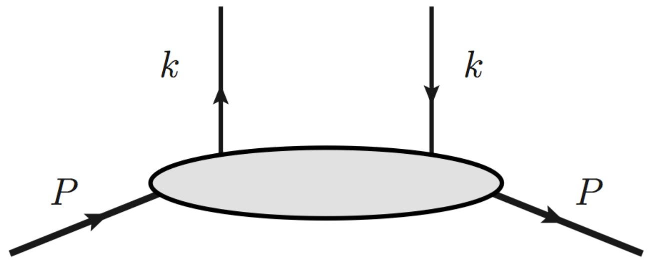
$$= \frac{1}{2P^+}\, \bar{u}(p')\, \Big(\gamma^+\, H(x,\xi,t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M}\, E(x,\xi,t)\Big)\, u(p)$$

$$x = \frac{\bar{k}^+}{P^+} \qquad \xi = -\frac{\Delta^+}{2P^+} \qquad t = \Delta^2$$

  - Leading twist: (proper twist expansion and $\mathcal{W}_{GPD}$ require light-cone vector n)

$$\bar{\psi}\,\gamma^+\,\psi = \bar{\psi}\, n\cdot\gamma\,\psi \qquad \bar{\psi}\,\gamma^+\gamma_5\,\psi \qquad \bar{\psi}\, i\sigma^{j+}\gamma_5\,\psi$$

- TMDs
  - Appear in QCD-description of hard semi-inclusive reactions (SIDIS, DY, etc.)
  - Kinematics



  - TMD-correlator

$$\Phi^{[\gamma^+]} = \frac{1}{2}\int \frac{dz^-}{2\pi}\,\frac{d^2\vec{z}_\perp}{(2\pi)^2}\,e^{ik\cdot z}\,\langle p\,|\,\bar{\psi}\Big(-\frac{z}{2}\Big)\,\gamma^+\,\mathcal{W}_{TMD}\,\psi\Big(\frac{z}{2}\Big)\,|\,p\rangle\Big|_{z^+=0}$$

$$= f_1(x,\vec{k}_\perp^{\,2}) - \frac{\epsilon_\perp^{ij}k_\perp^i S_\perp^j}{M}\,f_{1T}^\perp(x,\vec{k}_\perp^{\,2})$$

Note: $\epsilon_\perp^{ij} = P_\alpha n_\beta\,\epsilon^{\alpha\beta ij}/(P\cdot n)$

  - Leading twist:

$$\bar{\psi}\,\gamma^+\,\psi \qquad \bar{\psi}\,\gamma^+\gamma_5\,\psi \qquad \bar{\psi}\,i\sigma^{j+}\gamma_5\,\psi$$

Definition of GTMDs

- GTMD-correlator

$$W^{[\Gamma]} = \frac{1}{2} \int \frac{dz^-}{2\pi} \frac{d^2\vec{z}_\perp}{(2\pi)^2} \, e^{i\bar{k}\cdot z} \, \langle p' \, | \, \bar{\psi}\Big(-\frac{z}{2}\Big) \, \Gamma \, \mathcal{W}_{GTMD} \, \psi\Big(\frac{z}{2}\Big) \, | \, p \rangle \, \Big|_{z^+=0}$$

$\rightarrow$ $W^{[\Gamma]}$ appears, e.g., in handbag diagram of DVCS (before kin. approximations)

- Projection onto GPDs and TMDs

$$\begin{aligned} F^{[\Gamma]} &= \frac{1}{2} \int \frac{dz^-}{2\pi} \, e^{i\bar{k}\cdot z} \, \langle p' \, | \, \bar{\psi}\Big(-\frac{z}{2}\Big) \, \Gamma \, \mathcal{W}_{GPD} \, \psi\Big(\frac{z}{2}\Big) \, | \, p \rangle \, \Big|_{z^+=z_\perp=0} \\ &= \int d^2\vec{k}_\perp \, W^{[\Gamma]} \end{aligned}$$

$$\begin{aligned} \Phi^{[\Gamma]} &= \frac{1}{2} \int \frac{dz^-}{2\pi} \frac{d^2\vec{z}_\perp}{(2\pi)^2} \, e^{ik\cdot z} \, \langle p \, | \, \bar{\psi}\Big(-\frac{z}{2}\Big) \, \Gamma \, \mathcal{W}_{TMD} \, \psi\Big(\frac{z}{2}\Big) \, | \, p \rangle \, \Big|_{z^+=0} \\ &= W^{[\Gamma]}\Big|_{\Delta=0} \end{aligned}$$

$\rightarrow$ GPDs and TMDs appear as certain limits of GTMDs ("mother distributions")

Parameterization of GTMDs

(Meißner, Metz, Schlegel, 2009)

- Follow corresponding treatment for GPDs (Diehl, 2001)
- Use constraints on $W^{[\Gamma]}(P, \Delta, \bar{k}, n)$ from hermiticity and parity
- Eliminate redundant terms by means of Gordon identities, and

$$\det \begin{pmatrix} g^{\alpha\mu} & g^{\beta\mu} & g^{\gamma\mu} & g^{\delta\mu} & g^{\varepsilon\mu} \\ g^{\alpha\nu} & g^{\beta\nu} & g^{\gamma\nu} & g^{\delta\nu} & g^{\varepsilon\nu} \\ g^{\alpha\rho} & g^{\beta\rho} & g^{\gamma\rho} & g^{\delta\rho} & g^{\varepsilon\rho} \\ g^{\alpha\sigma} & g^{\beta\sigma} & g^{\gamma\sigma} & g^{\delta\sigma} & g^{\varepsilon\sigma} \\ g^{\alpha\tau} & g^{\beta\tau} & g^{\gamma\tau} & g^{\delta\tau} & g^{\varepsilon\tau} \end{pmatrix} = 0$$

- Examples (twist-2, chiral even sector)

$$W^{[\gamma^+]} = \frac{1}{2M}\, \bar{u}(p') \left[F_{1,1} + \frac{i\sigma^{i+}\bar{k}_\perp^i}{P^+} F_{1,2} + \frac{i\sigma^{i+}\Delta_\perp^i}{P^+} F_{1,3} + \frac{i\sigma^{ij}\bar{k}_\perp^i \Delta_\perp^j}{M^2} F_{1,4} \right] u(p)$$

$$W^{[\gamma^+\gamma_5]} = \frac{1}{2M}\, \bar{u}(p') \left[-\frac{i\epsilon_\perp^{ij}\bar{k}_\perp^i \Delta_\perp^j}{M^2} G_{1,1} + 3 \text{ more terms} \right] u(p)$$

- Full set of variables: $F_{1,1}(x, \xi, \vec{k}_\perp^{\,2}, \vec{k}_\perp \cdot \vec{\Delta}_\perp, \vec{\Delta}_\perp^2)$

- GTMDs are complex-valued functions: $F_{1,1} = F_{1,1}^{\mathrm{e}} + i F_{1,1}^{\mathrm{o}}$

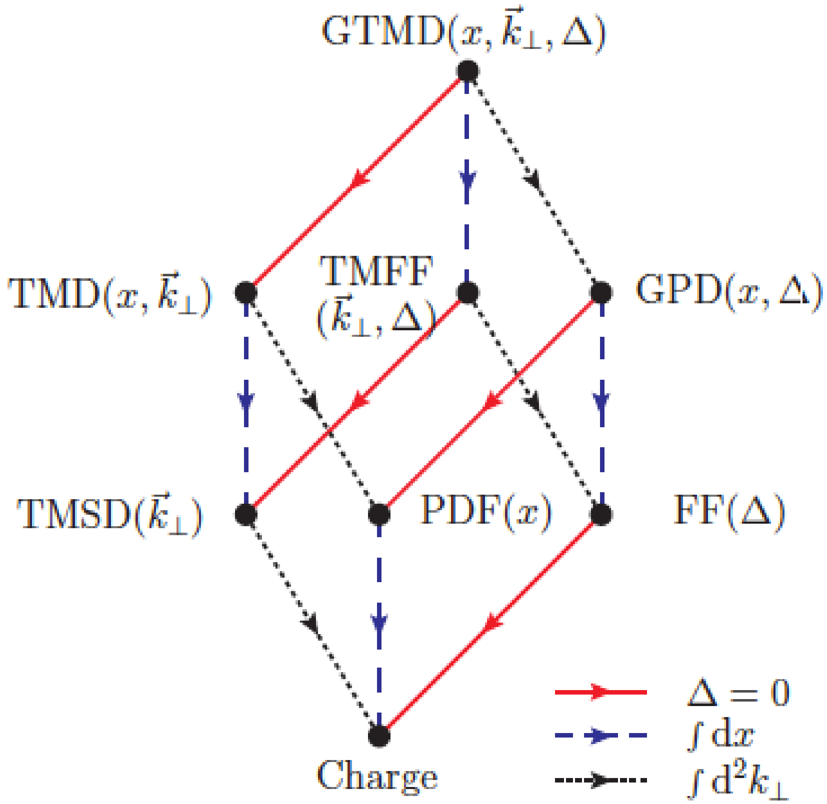
- GTMDs are mother functions (examples for $\xi = 0$)

$$
\begin{aligned}
H(x, 0, \vec{\Delta}_\perp^2) &= \int d^2\vec{k}_\perp \left[F_{1,1}^{\mathrm{e}} \right] \\
E(x, 0, \vec{\Delta}_\perp^2) &= \int d^2\vec{k}_\perp \left[- F_{1,1}^{\mathrm{e}} + 2 \left(\frac{\vec{k}_\perp \cdot \vec{\Delta}_\perp}{\vec{\Delta}_\perp^2} F_{1,2}^{\mathrm{e}} + F_{1,3}^{\mathrm{e}} \right) \right] \\
f_1(x, \vec{k}_\perp^2) &= F_{1,1}^{\mathrm{e}} \Big|_{\Delta=0} \\
f_{1T}^{\perp}(x, \vec{k}_\perp^2) &= - F_{1,2}^{\mathrm{o}} \Big|_{\Delta=0}
\end{aligned}
$$

- Some GTMDs contain very unique information, e.g., $F_{1,4}$ $G_{1,1}$

- Parameterization confirmed by independent treatment and extended to gluon sector (Lorcé, Pasquini, 2013)

GTMDs as Mother Functions



(from Lorcé, Pasquini, Vanderhaeghen, 2011)

- GTMDs describe the most general (2-parton) structure of hadrons
- In particular, modeling GTMDs is very useful

GTMDs and Nontrivial GPD-TMD Relations

- Several nontrivial relations between GPDs and TMDs found
(Burkardt, 2002, ... / Burkardt, Hwang, 2003 / Meißner, Metz, Goeke 2007 / ...)

- Sample quantitative relations in spectator models
  - relation between E and $f_{1T}^{\perp}$ (Burkardt, Hwang, 2003)

$$\begin{aligned} \langle k_{\perp}^{i}(x) \rangle_{UT} &= -\int d^2\vec{k}_{\perp}\, k_{\perp}^{i}\, \frac{\epsilon_{\perp}^{jk} k_{\perp}^{j} S_{\perp}^{k}}{M}\, f_{1T}^{\perp}(x, \vec{k}_{\perp}^{\,2}) \\ &= \int d^2\vec{b}_{\perp}\, \mathcal{I}^{i}(x, \vec{b}_{\perp})\, \frac{\epsilon_{\perp}^{jk} b_{\perp}^{j} S_{\perp}^{k}}{M} \Big(\mathcal{E}(x, \vec{b}_{\perp}^{\,2}) \Big)' \end{aligned}$$

  - relation between $\tilde{H}_T$ and $h_{1T}^{\perp}$ (Meißner, Metz, Goeke, 2007)

$$\int d^2\vec{k}_{\perp}\, h_{1T}^{\perp}(x, \vec{k}_{\perp}^{\,2}) = \frac{3}{(1-x)^2}\, \tilde{H}_T(x, 0, 0)$$

- Can any of those relations have a model-independent status ?

- Results using GTMDs

$$E(x,0,\vec{\Delta}_\perp^2) = \int d^2\vec{k}_\perp \Big[-F^{\rm e}_{1,1} + 2\Big(\frac{\vec{k}_\perp\cdot\vec{\Delta}_\perp}{\vec{\Delta}_\perp^2} F^{\rm e}_{1,2} + F^{\rm e}_{1,3}\Big)\Big]$$

$$f_{1T}^\perp(x,\vec{k}_\perp^2) = -F^{\rm o}_{1,2}\Big|_{\Delta=0}$$

$$\tilde{H}_T(x,0,\vec{\Delta}_\perp^2) = \int d^2\vec{k}_\perp \Big[\Big(\frac{\vec{k}_\perp\cdot\vec{\Delta}_\perp}{\vec{\Delta}_\perp^2} H^{\rm e}_{1,1} + H^{\rm e}_{1,2}\Big)$$

$$-2\Big(\frac{2(\vec{k}_\perp\cdot\vec{\Delta}_\perp)^2 - \vec{k}_\perp^2\vec{\Delta}_\perp^2}{(\vec{\Delta}_\perp^2)^2} H^{\rm e}_{1,4} + \frac{\vec{k}_\perp\cdot\vec{\Delta}_\perp}{\vec{\Delta}_\perp^2} H^{\rm e}_{1,5} + H^{\rm e}_{1,6}\Big)\Big]$$

$$h_{1T}^\perp(x,\vec{k}_\perp^2) = H^{\rm e}_{1,4}\Big|_{\Delta=0}$$

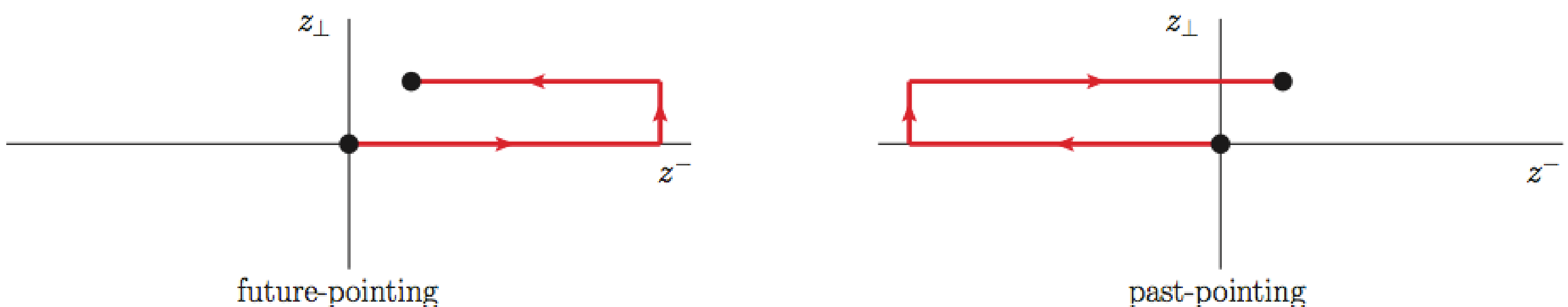
- Lessons
  - no model-independent nontrivial relations between GPDs and TMDs
  - relations in spectator models due to simplicity of the models
    (Meißner, Metz, Goeke, 2007 / Gamberg, Schlegel, 2009)
  - no information on numerical violation of relations
  - for instance, so far phenomenology of relation between E and $f_{1T}^\perp$ successful

GTMDs and Orbital Angular Momentum

- Parton OAM in longitudinally polarized nucleon (Lorcé, Pasquini, 2011)

$$L = -\int dx\, d^2\vec{k}_\perp \, \frac{\vec{k}_\perp^2}{M^2}\, F_{1,4}(x, 0, \vec{k}_\perp^2, 0, 0)$$

- Extension to gauge theory (QCD)
  - staple-like gauge link (Hatta, 2011)



$$L = L_{\rm JM}$$

$\longrightarrow$ $L_{\rm JM}$ could be computed in Lattice QCD

  - straight/direct gauge link (Ji, Xiong, Yuan, 2012 / Lorcé, 2013)

$$L = L_{\rm Ji}$$

$\longrightarrow$ same equation for both $L_{\rm JM}$ and $L_{\rm Ji}$

Further Aspects/Applications of GTMDs

- Spin-orbit couplings (Lorcé, Pasquini, 2011 / Lorcé, 2014)

$$F_{1,4} \quad \longleftrightarrow \quad \vec{S}_N \cdot \vec{L}_q$$
$$G_{1,1} \quad \longleftrightarrow \quad \vec{S}_q \cdot \vec{L}_q$$

- Relation to Wigner distributions
  (Ji, 2003 / Belitsky, Ji, Yuan, 2003 / Lorcé, Pasquini, 2011 / ...)
  - Fourier transform of GTMDs for $(\xi = 0)$

$$\mathrm{WD}(x, \vec{k}_\perp, \vec{b}_\perp) \simeq \int d^2\vec{\Delta}_\perp \, e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp} \, \mathrm{GTMD}(x, \vec{k}_\perp, \vec{\Delta}_\perp)$$

- Observables: Gluon GTMDs have been used to describe exclusive diffractive processes
  (Martin et al, 1999 / Khoze, Martin, Ryskin, 2000 / Martin, Ryskin, 2001 / ...)
  - no general principle forbids observability of GTMDs
  - however, in practice, how much information on GTMDs can be obtained from experiment ?

Comments on Recent Criticism

(Kanazawa, Lorcé, Metz, Pasquini, Schlegel, 2014)

- Criticism by Courtoy, Goldstein, Gonzalez, Liuti, Rajan, 2013

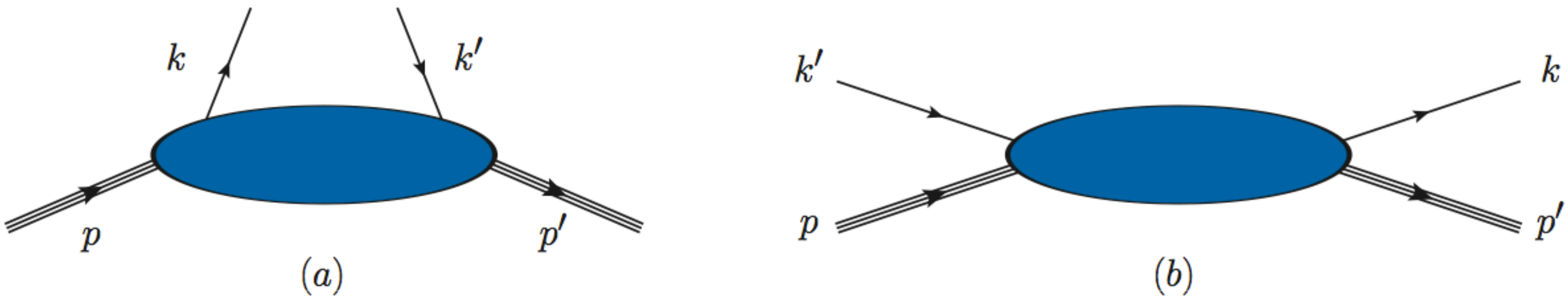
- Here focus on only part of the discussion

- Claim 1: The GTMDs $F_{1,4}$ and $G_{1,1}$ are accompanied by parity-odd structures
  - did both papers deriving the GTMD parameterization make a mistake ?
  - if claim correct, $F_{1,4}$ and $G_{1,1}$ must vanish in models having no parity-violating interaction $\rightarrow$ in contrast to nonzero results that were on the market (Goeke, Meißner, Metz, Schlegel, 2008, 2009 / Lorcé, Pasquini, 2011 / Lorcé, Pasquini, Xiong, Yuan, 2012)
  - if claim correct, then e.g. relation between OAM and F_{14} would be meaningless
  - actually, $F_{1,4}$ and $G_{1,1}$ are accompanied by parity-even structures

$$\bar{u}(p', \Lambda') \, \frac{i\sigma^{ij} \bar{k}_\perp^i \Delta_\perp^j}{M^2} \, u(p, \Lambda) \;\propto\; \vec{S}_L \cdot (\vec{\bar{k}}_\perp \times \vec{\Delta}_\perp)$$

note: both $\vec{S}_L$ and $\vec{\bar{k}}_\perp \times \vec{\Delta}_\perp$ are axial vectors

- Claim 2: Elastic 2-particle scattering picture leads to fewer GTMDs
  - parton correlators have close analogy to elastic quark-nucleon scattering



  - in 2-particle scattering only three independent 4-vectors
    $\rightarrow$ only one independent transverse vector (in cm frame)
    $\rightarrow$ structure $\vec{S}_L \cdot (\vec{\bar{k}}_\perp \times \vec{\Delta}_\perp)$ would be impossible (or redundant)

  - actually, parameterization of GTMD correlator requires four independent 4-vectors: $P, \Delta, \bar{k}, n$
    $\rightarrow$ two independent transverse vectors
    $\rightarrow$ 2-particle scattering is too restrictive for GTMD counting
    $\rightarrow$ vector n played also important role in correct parameterization of GPDs (Diehl, 2001)

Lessons from Explicit Calculations of GTMDs

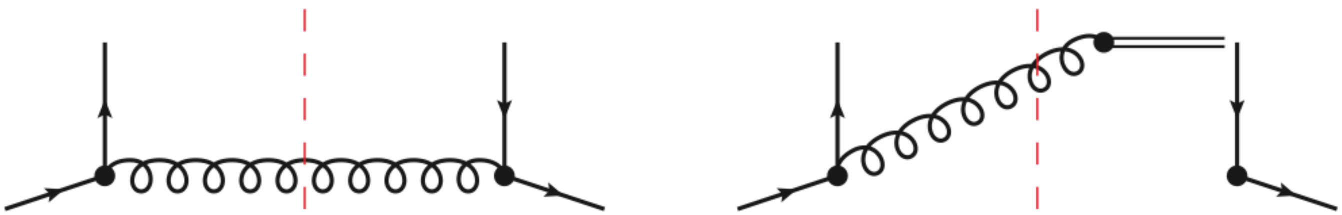
(Kanazawa, Lorcé, Metz, Pasquini, Schlegel, 2014)

- GTMDs in scalar diquark model
  - some results for $\xi = 0$

$$F^q_{1,4} = G^q_{1,1} = -\frac{g_s^2}{2(2\pi)^3} \frac{(1-x)^2 M^2}{\left[\vec{k}'^2_\perp + \mathcal{M}^2(x)\right] \left[\vec{k}^2_\perp + \mathcal{M}^2(x)\right]} + \mathcal{O}(g_s^4)$$

  - confirms nonzero result obtained earlier (Meißner, Metz, Schlegel, 2009)

- GTMDs in quark target model

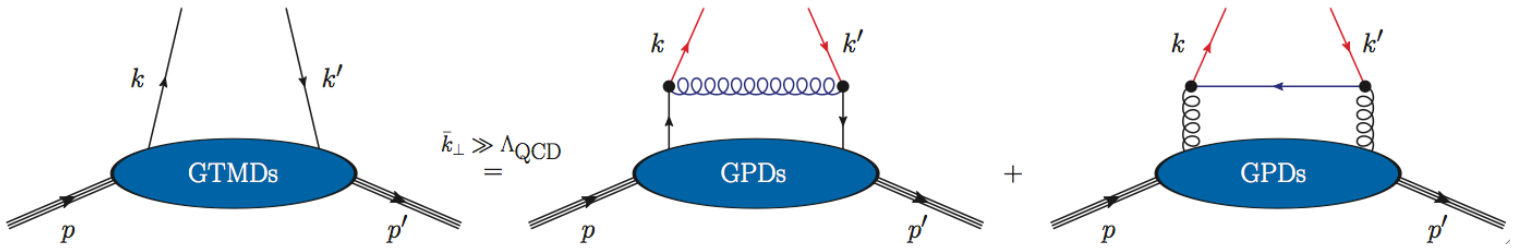


  - results for $\xi = 0$

$$F^q_{1,4} = -G^q_{1,1} \neq 0 \qquad F^g_{1,4} \neq 0 \qquad G^g_{1,1} \neq 0$$

  - results for quarks confirmed by Mukherjee, Nair, Ojha, 2014

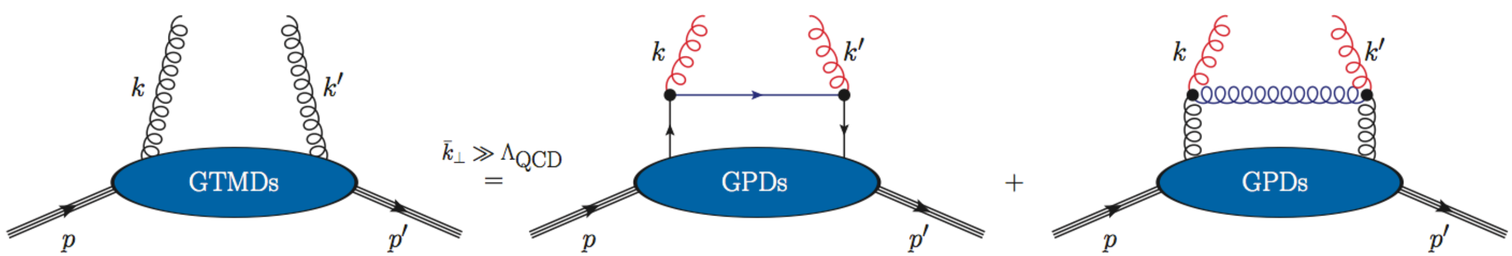
- GTMDs at large transverse momenta in perturbative QCD ($\xi = 0$)
  - calculation for quark GTMDs



$$F^q_{1,4} \;=\; \frac{\alpha_S}{2\pi^2}\int_x^1 \frac{dz}{z}\,\frac{M^2\left[C_F\,\tilde{H}^q(\frac{x}{z},0,-\vec{\Delta}^2_\perp)-T_R\,(1-z)^2\,\tilde{H}^g(\frac{x}{z},0,-\vec{\Delta}^2_\perp)\right]}{\left[\vec{k}'^2_\perp + z(1-z)\frac{\vec{\Delta}^2_\perp}{4}\right]\left[\vec{k}^2_\perp + z(1-z)\frac{\vec{\Delta}^2_\perp}{4}\right]}$$

$$G^q_{1,1} \;\neq\; 0$$

  - calculation for gluon GTMDs



$$F^g_{1,4} \neq 0 \qquad G^g_{1,1} \neq 0$$

- $F_{1,4}$ and OAM L_{JM}
  - definition of L^q_{JM} (Jaffe, Manohar, 1990 / Hägler, Mukherjee, Schäfer, 2003 / ...)

$$\hat{\mathcal{O}}^q(x, r^-, \vec{r}_\perp) \equiv \int \frac{dz^-}{4\pi}\, e^{ixP^+z^-} \left[\bar{\psi}(r^- - \frac{z^-}{2}, \vec{r}_\perp)\, \gamma^+ (\vec{r}_\perp \times i\vec{\partial}_\perp)_z\, \psi(r^- + \frac{z^-}{2}, \vec{r}_\perp)\right]$$

$$L^q_{\text{JM}}(x) \equiv \lim_{\Delta \to 0} \frac{P^+ \int dr^- \int d^2\vec{r}_\perp \, \langle p', \Lambda | \hat{\mathcal{O}}^q(x, r^-, \vec{r}_\perp) | p, \Lambda \rangle}{\langle p', \Lambda | p, \Lambda \rangle}$$

  - corresponding definitions exist for scalar partons and gluons
  - in scalar diquark model and quark target model we confirmed

$$L^{s,q,g}_{\text{JM}} = -\int dx\, d^2\vec{k}_\perp\, \frac{\vec{k}^2_\perp}{M^2}\, F^{s,q,g}_{1,4}(x, 0, \vec{k}^2_\perp, 0, 0)$$

  - no reason to doubt relation between $F_{1,4}$ and OAM
  - side-remark: in quark target model we confirmed that $F^q_{1,4}$, and therefore also L^q_{JM}, does not depend on direction of Wilson line (c.f. Hatta, 2011)

Summary

- Parameterization of GTMDs for spin-$\frac{1}{2}$ hadron exists (for quarks and gluons)

- In contrast to recent claim, (twist-2) GTMD correlator not over-parameterized
  - 2-particle elastic scattering not suitable for counting of GTMDs
  - $F_{1,4}$ and $G_{1,1}$ are nonzero and independent

- GTMDs are useful in several respects
  - describe the most general parton structure of hadrons
  - relation between GTMDs and parton OAM
    $\rightarrow$ in particular, L_{JM} could be computed in Lattice QCD
  - relation between GTMDs and spin-orbit interactions
  - etc.

- Non-vanishing of $F_{1,4}$ and $G_{1,1}$, and relation to OAM, confirmed by explicit calculations