# QED Radiative Corrections in DVCS and SIDIS: New Approaches and Recent Developments

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# Outline

- From Mo and Tsai Methods to Current RC Calculations: Brief Historic Review
  - Covariant approaches of Bardin and Shumeiko
  - POLRAD 2.0: Properties and opportunities
  - Other codes and other research groups dealing with RC
- Radiative Correction in DVCS
  - Approaches to approximate and exact calculations
  - Numeric illustration and available codes
  - Remaining uncertainties in RC calculation to DVCS
- Radiative Correction in SIDIS
  - Contributions to RC calculation and available codes
  - Hadronic tensor and exact calculation for SIDIS with polarized particles
  - Structure functions and RC procedure of experimental data

# Mo and Tsai Approach for Elastic and Inelastic Processes

- Mo and Tsai firstly elaborated a systemic approach to calculate radiative corrections in elastic and inelastic electron scattering.
- For inelastic and deep inelastic processes they showed that actual $Q^2$ and $W^2$ going to hadronic part cover a wide kinematic region including the resonance region.
- Also they proved that elastic processes with the radiated photon (so-called radiative tail from elastic peak or simply elastic radiative tail) has to be added as a contribution to the total RC.
- One assumption (and limitation) in their calculations was the approximate way to consider the soft-photon contribution
  - Specifically, they introduced a parameter $\Delta$ such that $\Delta \ll m_e, E, E'$.
  - Then they considered the region over photon energy $\omega$ and kept only the leading term $1/\omega$. This allowed them to calculate the term with the soft photons analytically (even with a photon mass $\lambda$) and extract infrared divergence in the form of $\log(m_e/\lambda)$. The infrared divergence is canceled with respective term obtained when calculating loop diagrams (i.e., the vertex function).
  - The correction from the region above $\Delta$ is evaluated numerically.

# Covariant Approach of Bardin and Shumeiko

- Bardin and Shumeiko improved the calculation approach in 5 aspects:
- They developed an approach for extraction and cancellation of the infrared divergence which is free of the artificial parameter $\Delta$.
- They presented all results in the covariant form, so the formulas can be directly applied in any coordinate system.
- They developed a code TERAD that calculates RC for unpolarized target including nuclear targets; in this case radiative tail from quasielastic peak have to be considered and added.
- They suggested the radiative correction procedure of experimental data or unfolding procedure.
- With collaboration with Kukhto they obtained the formulas for RC on polarized protons.

# RC to DIS of Polarized Particles: POLRAD 2.0

- We (IA, Shumeiko) essentially improved the calculation of RC to polarized targets.
  - The most essential improvement was the idea of using the basis in the four-dimensional space and of expansion polarization vectors over momenta such as momenta of initial and final electrons and initial proton. This allowed us to avoid a tedious and intricate procedure of tensor integration used before.
- We (IA, Ilyichev, Soroko, Shumeiko, Tolkachev) created the code POLRAD 2.0 that allows to calculations for:
  - RC in DIS on polarized targets of spin of $1/2$ and 1. All contributions including quasielastic radiative tail were implemented.
  - RC to quadruple asymmetry for spin-one targets.
  - RC to semi-inclusive DIS (including polarized targets) in the simple quark-parton model i.e., for the three-dimensional cross section $d\sigma/dxdydz$.
  - Approximate contribution of double bremsstrahlung
  - Electroweak effects
  - The iterative procedure of RC of experimental data

# Monte Carlo Generator RADGEN

We constructed the Monte Carlo generator RADGEN using POLRAD 2.0 (IA, Boettcher, Ryckbosch, hep-ph/9906408)

The cross section is represented in the sum of two positively definite contributions

$$\sigma_{obs} = \sigma_{non-rad} + \sigma_{rad}$$

where $\sigma_{non-rad}$ contains loop diagrams and soft photon emission and $\sigma_{rad}$ is the contribution of additional hard photon emission with energy larger than a minimal photon energy $\epsilon_{min}$ associated with resolution in calorimeter.

In spite of introducing the artificial parameter $\epsilon_{min}$ there is no loosing an accuracy and no acquired dependence of the cross section of this parameter

Two modes for generator operation

- integrals are calculated for each event and grid for a simulation is stored
- look-up table calculated in advance is used for interpolation of the grid

# RC Procedure of Experimental Data for polarized DIS

- Sometimes experimentalists calculate RC to spin asymmetry using a RC code (e.g., POLRAD 2.0) and then add the calculated effect to observed asymmetry in a bin. This is incorrect procedure because of bias resulted from the difference between the values of $g_1$ finally extracted in the bin and used for RC calculation.
- Example of correct procedure is in using

$$A_{extr}(x, Q^2) = A_{meas} - \alpha C \int dx_{true} dQ^2_{true} K A_{teor}(x, Q^2)$$

and solving this equation with the iteration procedure that provides equality of $A_{extr}(x, Q^2)$ and $A_{teor}(x, Q^2)$

- The similar procedure can be defined (and it is implemented in POLRAD 2.0) for the case when both $g_1$ and $g_2$ are measured in the same experiment and solving respective equations are required.
- These ideas have to be kept in mind when we define the procedure of RC of experimental data for other asymmetries, e.g., those measured in SIDIS.

# Exact Calculation of the RC of the lowest order

The complete RC of the lowest order (and multiple soft photon contributions) calculated suing the covariant technique (i.e., that used to create POLRAD, DIFFRAD, EXCLURAD, and other codes) is represented in the form

$$\sigma_{obs} = \sigma_0 \exp(\delta_{inf})(1 + \delta_{VR} + \delta_{vac}) + \sigma_F$$

Here the corrections $\delta_{inf}$ and $\delta_{vac}$ come from the radiation of soft photons and the effects of vacuum polarization, the correction $\delta_{VR}$ is infrared-free sum of factorized parts of real and virtual photon radiation, and $\sigma_F$ is an infrared free contribution from the process of emission of an additional real photon.

The contribution of hard photons $\sigma_F$ is represented in the form of three-dimensional integral over kinematic variables of an unobserved photon.

$$\sigma_F = \alpha^4 C_{kin} \int d\Omega_k \int_0^{v_m} dv \sum_n \left[ \frac{v f_{kin}}{\tilde{Q}^4} L^{(n)}_{\mu\nu,\mu'} T^{(n)}_{\mu\nu,\mu'} - \frac{f^0_{kin}}{vQ^4} L^{0(n)}_{\mu\nu,\mu'} T^{0(n)}_{\mu\nu,\mu'} \right]$$

The integrals need to be calculated numerically. This integral is finite for $v \to 0$ and not positively definite.

# Leading, Next-to-Leading, and Exact Contributions to RC

By “exactly” calculated RC we understand the estimation of the lowest order RC contribution with any predetermined accuracy.

The structure of the dependence on the electron mass in RC cross section:

$$\sigma_{RC} = A \log \frac{Q^2}{m^2} + B + O(m^2/Q^2)$$

where $A$ and $B$ do not depend on the electron mass.

$\log(\frac{Q^2}{m^2}) \sim 15$ for $Q^2 \sim$ 1GeV$^2$

- If only $A$ is kept, this is the leading log approximation.
- If both contributions are kept (i.e., contained $A$ and $B$), this is the calculation with the next-to-leading accuracy, practically equivalent to exact calculation.

# Other Codes for RC in $ep$-scattering

**POLRAD 2.0** *FORTRAN code for the RC procedure of experimental data in polarized inclusive and semi-inclusive DIS. The iteration procedure based on MINUIT fitting the data is included. Estimation of higher order and electroweak corrections is done.*

**RADGEN** *Monte Carlo generator of radiative events in the DIS on polarized and unpolarized targets. Can be applied for RC generation in inclusive, semi-inclusive and exclusive DIS processes. This version uses a look-up table for photonic angles which provides for fast event generation.*

**DIFFRAD** *FORTRAN code for RC calculation in the processes of electroproduction of vector mesons. Versions with Monte Carlo and numerical integrations are available. Monte Carlo code allows to estimate RC to the quasi-real photoproduction case (i.e., the final electron is not detected)*

**HAPRAD** *FORTRAN code for RC calculation in the processes of semi-inclusive hadron electroproduction. Versions with Monte Carlo and numerical integrations are available.*

**ESFRAD** *(Authors: A. Afanasev, I. Akushevich, N. Merenkov) FORTRAN code for RC calculation in the processes of elastic, inelastic and deep inelastic scattering using the method of electron structure functions.*

**ELARADGEN** *(Authors: A. Afanasev, I. Akushevich, A. Ilychev, B. Niczyporuk) Monte Carlo generator of radiative events in the kinematics of elastic ep-scattering measurements.*

**MASCARAD** *FORTRAN code for RC calculation in elastic electron-nucleon scattering with a polarized target and/or recoil polarization. The experimental acceptances are accounted for.*

**EXCLURAD** *(Authors: A. Afanasev, I. Akushevich, V. Burkert, K. Joo) FORTRAN code for RC calculation in the process of exclusive pi electroproduction on a nucleon.*

All these codes can be downloaded from https://www.jlab.org/RC/

# Other Groups Contributed to RC in $ep$-scattering

**Bohm, Hollik, Spiesberger:** *One-loop correction in electroweak physics; general theory of renormalization in electroweak theory. We compared POLRAD 2.0 with code HERACLES produced by Hubert Spiesberger.*

**Bardin et al.:** *We worked in parallel using the same approach of covariant calculation of RC. We compared POLRAD 2.0 with the code HECTOR produced by Dima Bardin with collaborators.*

**Eduard Kuraev:** *Produced multiple brilliant results in quantum electrodynamics. I was involved in his group. In our last conversations he advised me how to construct hadronic tensor for SIDIS. We directly used the results of his group in our work on RC to DVCS such as*

- *Asymptotic expressions for loop integrals in non–collinear kinematics, JINR E2-98-53, hep-ph/0703048*
- *Approach for the calculation in leading log approximation, shifted kinematics, Phys.Rev. C77, 055206 (2008)*

**Nikolai Merenkov:** Andrei Afanasev and me worked a lot together for calculating RC for elastic $ep$ scattering and producing the codes MASCARAD and ESFRAD

**Marc Vanderhaeghen:** One-loop correction and soft photon emission in DVCS, Phys.Rev. C62(2000)025501.

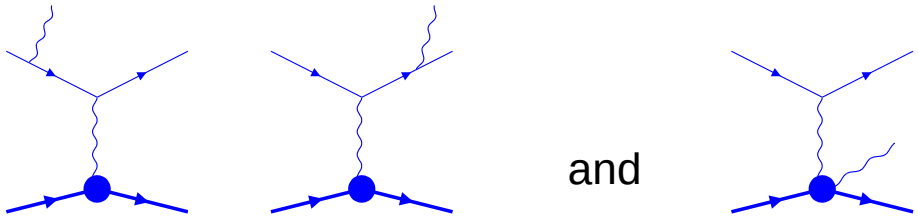
**Profs. Maximon and Tjon:** Phys.Rev. C62 (2000) 054320; Contributions of the box and crossed-box (two-photon exchange) diagrams.

# Part II:

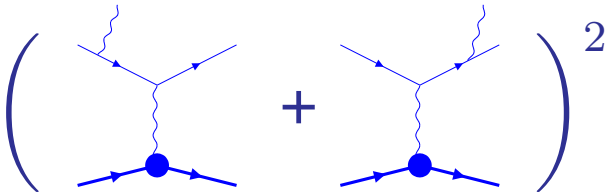
# RC to DVCS

# Base processes: BH and DVCS

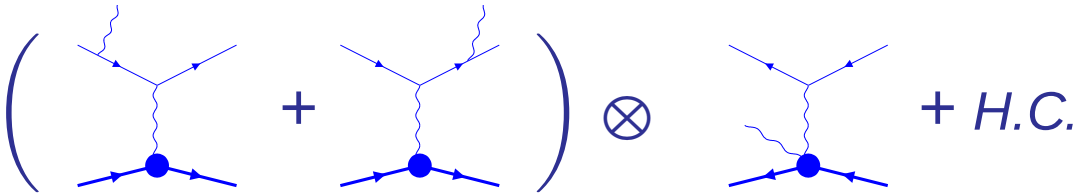
- BH and DVCS amplitudes



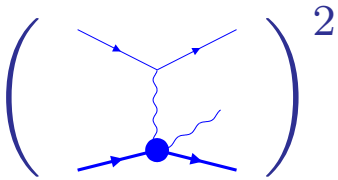

- Three respective contributions to the cross section of the process $e + p \to e' + p' + \gamma$
  - BH process



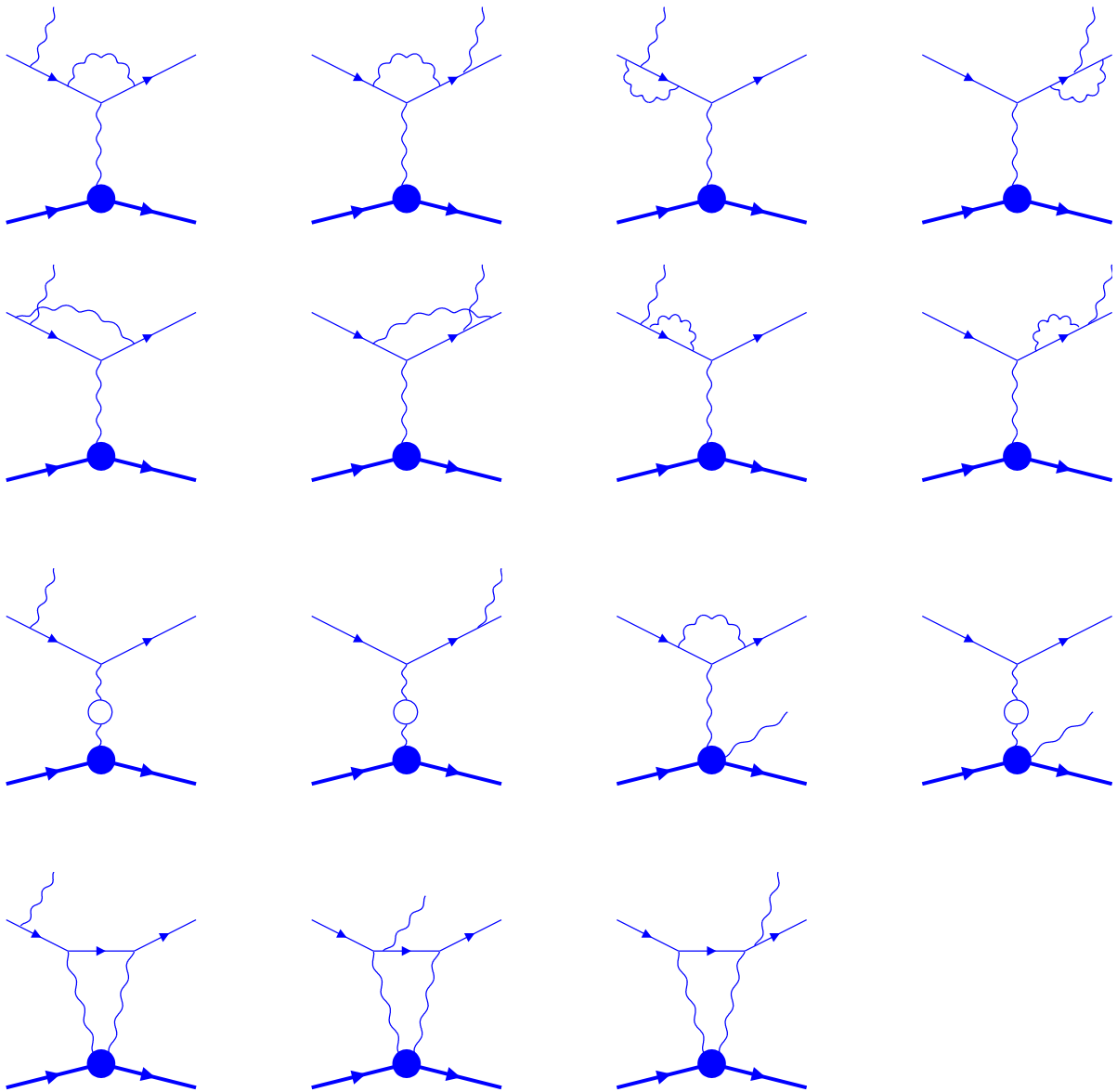

  - Interference of BH and DVCS amplitudes




  - Pure DVCS process



# Structure of RC: One-loop contribution



One-loop Correction: Emission of real and additional virtual photons from leptonic line

Correction due to vacuum polarization and One-loop correction with real photon emission from hadron line

Box-diagram contribution.

# Steps in RC calculation

- Matrix element squared.
- Integration over loops and taking care on ultraviolet divergence (i.e., making the electron charge and mass renormalization).
- Phase space parametrization and integration over a part of kinematical variables of an additional photon
  - BH cross section is defined by four kinematical variables: $x$, $Q^2$, $t$ and $\phi$.
  - The cross section with two photon emitted is defined by seven kinematical variables:
    - *the same four variables:* $x$, $Q^2$, $t$ *and* $\phi$ *and*
    - *three additional variables: two-photon invariant mass* $V^2$ *and two angles of the photon pair.*
- Extract and cancel the infrared divergence without making new assumptions.
- Add a contribution of higher order corrections (calculated approximately).
- Code the results to have
  - A program for RC calculation in a kinematical point defined by $x$, $Q^2$, $t$ and $\phi$.
  - Monte Carlo Generator with inclusion of RC contributions.
- Analyze uncertainties in RC calculation.

# RC in the Leading Log Approximation

$$\sigma_{obs}(S,x,Q^2,t,\phi) = (1+2\Pi(t))\sigma_{BH}(S,x,Q^2,t,\phi) + \frac{\alpha}{2\pi}L\Bigg[$$

$$\int_0^1 dz_1 \left(\frac{1+z_1^2}{1-z_1}\right)\left(\frac{\sin\theta_s'}{\mathcal{D}_{0s}^{1/2}}\theta(z-z_1^m)\left(\frac{x_s}{x}\right)^2\sigma_{BH}(z_1S,x_s,z_1Q^2,t,\bar{\phi}_s) - \sigma_{BH}(S,x,Q^2,t,\phi)\right)$$

$$+\int_0^1 dz_2 \left(\frac{1+z_2^2}{1-z_2}\right)\left(\frac{\sin\theta_p'}{\mathcal{D}_{0p}^{1/2}}\theta(z-z_2^m)\frac{1}{z_2}\left(\frac{x_p}{x}\right)^2\sigma_{BH}(S,x_p,z_2^{-1}Q^2,t,\bar{\phi}_p) - \sigma_{BH}(S,x,Q^2,t,\phi)\right)\Bigg]$$

where $x_s = z_1Q^2/(z_1S-X)$ and $x_p = Q^2/(z_2S-X)$ are Bjorken x in shifted kinematics
Integration limits do not depend on $\phi$ ($\xi^2 = \lambda_t/\lambda_Y$):

$$z_{1,2}^m = \frac{Xt \mp 2M^2t + \xi(XS_x - 2M^2Q^2)}{St \mp 2M^2Q^2 + \xi(SS_x + 2M^2Q^2)}$$

**This formula is also valid for interference of BH and DVCS amplitudes and for pure DVCS contribution.**

# RC in the Next-to-Leading Approximation

Combining all contributions we have

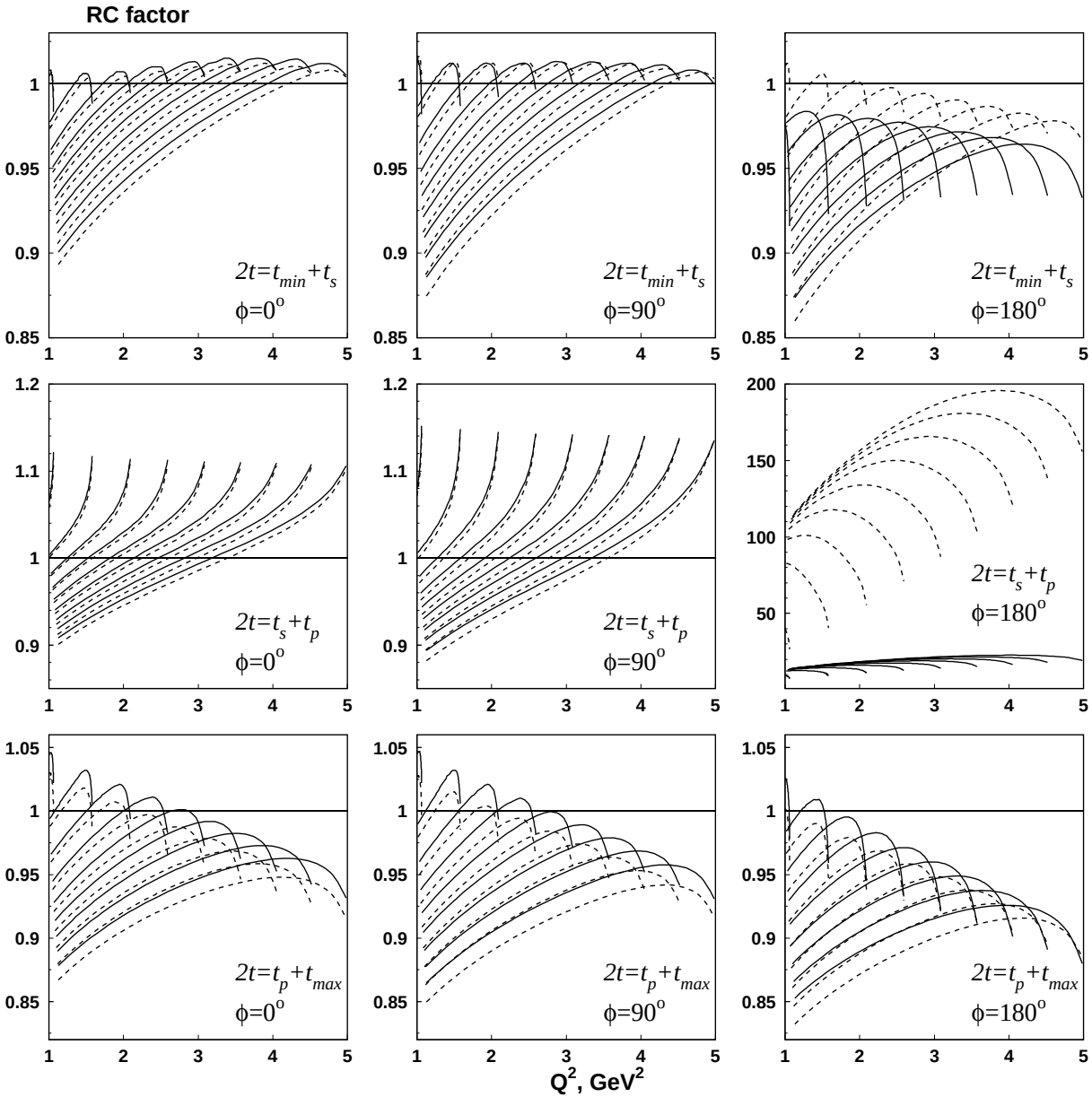
$$\frac{d\sigma}{d\Gamma_0} = A \log \frac{Q^2}{m^2} + B + \sum_{i=1}^{4} T_i^v \mathcal{F}_i + \int_0 \frac{dV^2}{V^2} \sum_{i=1}^{4} \Big( T_i^F(V^2) - T_i^F(0) \Big) \mathcal{F}_i$$

- $A$ and $B$ are resulted from the sum of (and infrared divergent) terms of the loop and two-gamma constitutions. They do not depend on the lepton mass.
- $\mathcal{F}_i$ are squared combinations of formfactors.
- $T_i^v$ came from nonfactorized (and lepton mass independent) part of the loop cross section.
- $T_i^F(V^2)$ came from nonfactorized (can have mass-dependence) part of the contribution of two-gamma contribution:

$$T_i^F = \frac{T_{i1}^F}{w} \log \frac{w^2}{m^2 V^2} + \frac{T_{i2}^F}{u} \log \frac{u^2}{m^2 V^2} + T_{i3}^F \log \frac{V_{\rm I}}{m^2 V^2} + T_{i4}^F \log \frac{Q^2}{m^2} + T_{i5}^F$$

Quantities $T_{ij}^F$ are rational (mass independent) functions of $Q^2$, $u$, $w$, and $V^2$.

# Leading and Next-to-Leading Contributions



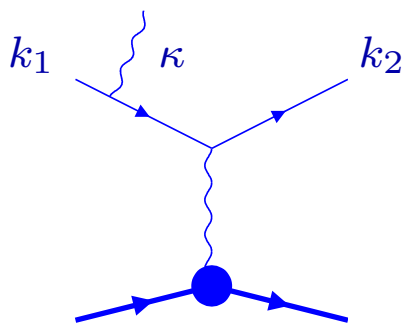

The $Q^2$ dependence of LO (dashed) and NLO (solid) RC factor for several $x$, $t$ and $\phi$ calculated for beam energy equaling 5.75 GeV and without any cuts on the the invariant mass of two photons.

Nine curves at each plot correspond (from the left to the right) to nine values of $x$: 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, and 0.5. $Q^2_{max}$ is defined by kinematics.

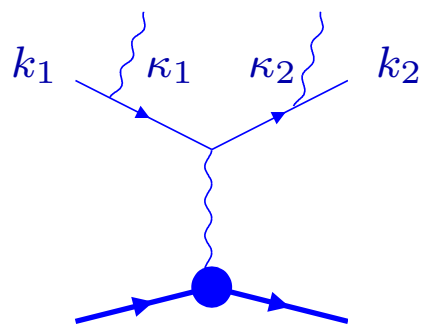
# Large effect for $\phi$ =180$^o$ and $t = (t_s + t_p)/2$

- The large effect comes from the two-photon emission process when both two irradiated photons are collinear: one is collinear to the initial electron and and another is collinear to the final electron.
- The corresponding BH process (i.e., one photon emission process) is the process with the emitted photon with 4-momentum corresponding to the sum of momenta of the two collinear photons. This photon is not collinear and therefore the respective cross section of the BH process is not large.

Base process:

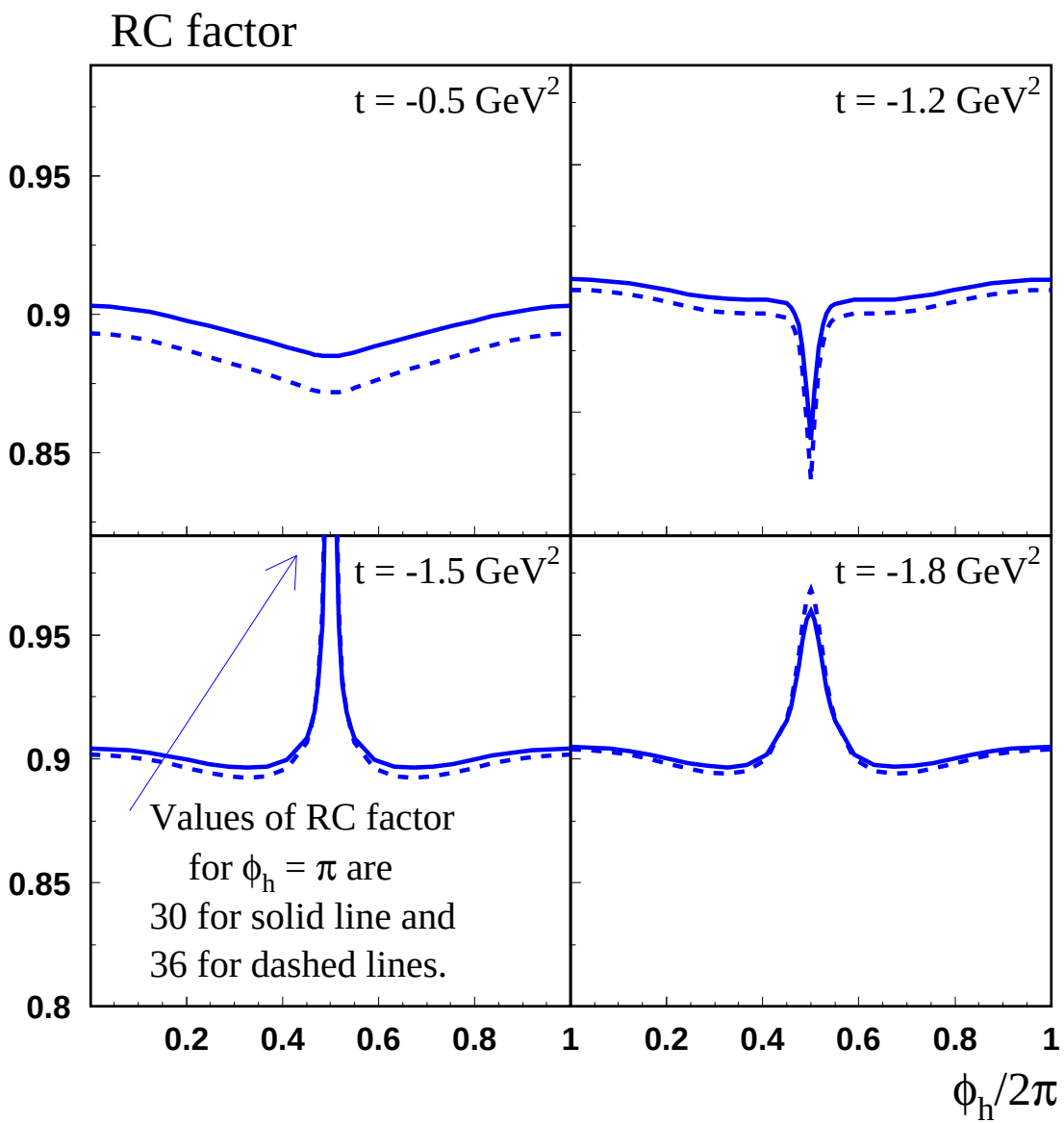



Correction:



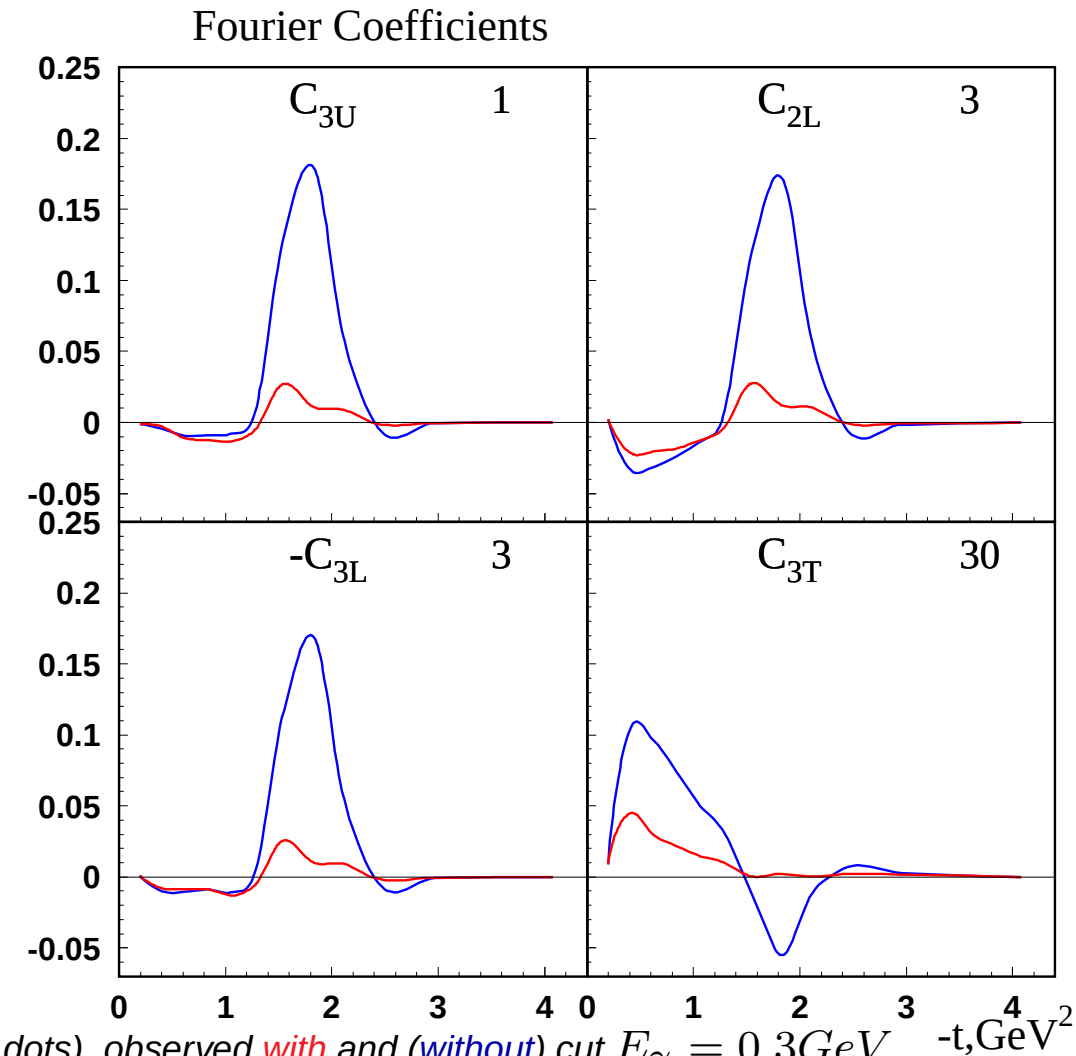

When $\vec{\kappa}_1 \| \vec{k}_1$ and $\vec{\kappa}_2 \| \vec{k}_2$ the correction is large, but respective cross section of the base process is not large because $\vec{\kappa} = \vec{\kappa}_1 + \vec{\kappa}_2$ is not colinear neither $\vec{k}_1$ nor $\vec{k}_2$

# Numerical results: $\phi$ dependence of RC factor



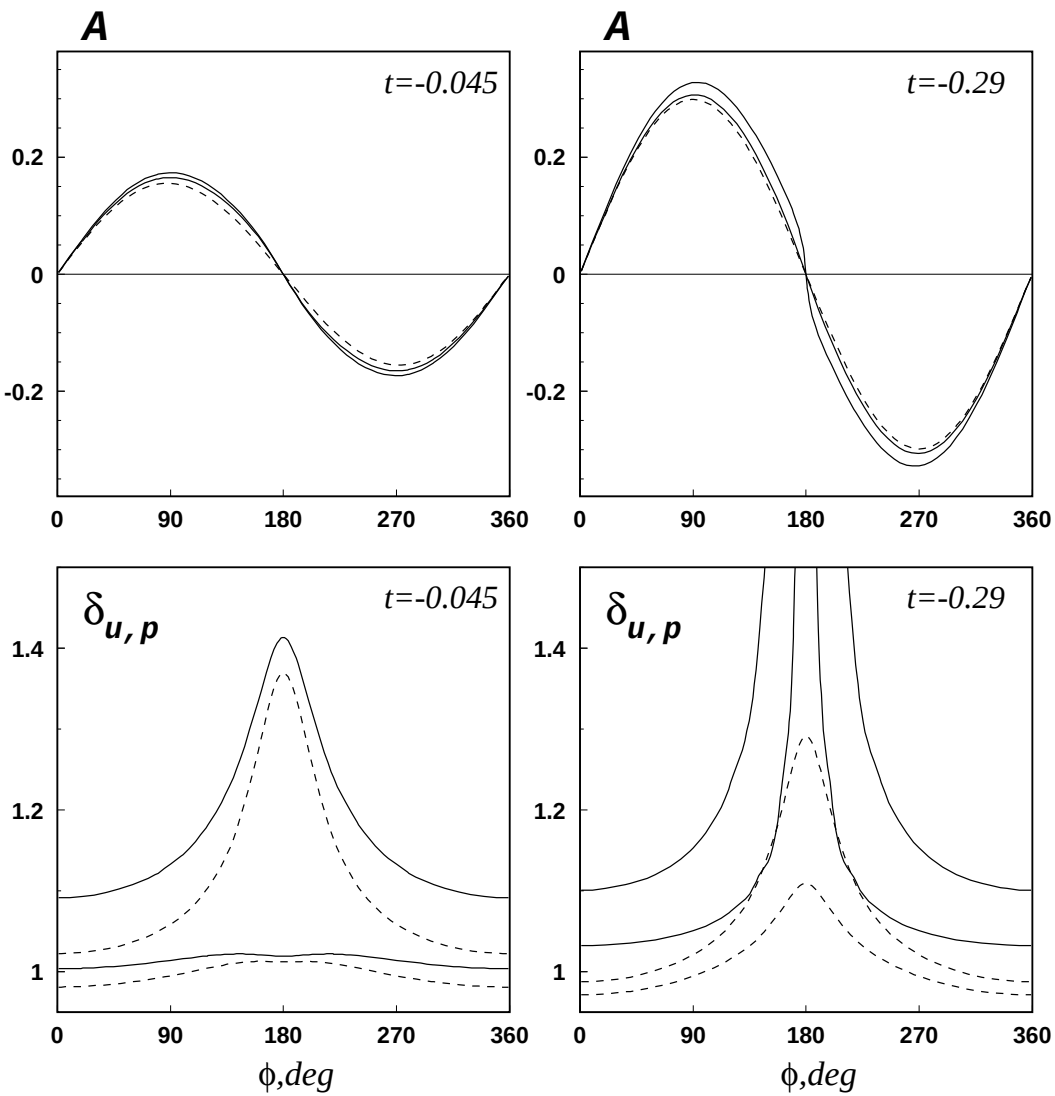
# Numerical results: Fourier Coefficients

$$C_{nPol} = \frac{1}{2\pi f}\int_0^{2\pi} d\phi \cos(n\phi)\ \ \mathcal{P}_1\mathcal{P}_2\ \ \sigma_{BH,Pol}, \qquad Pol = U, L, P$$




*Notation: BH (black with dots), observed with and (without) cut* $E_\gamma = 0.3GeV.$

# The $\phi$-dependence of the asymmetry and RC factors

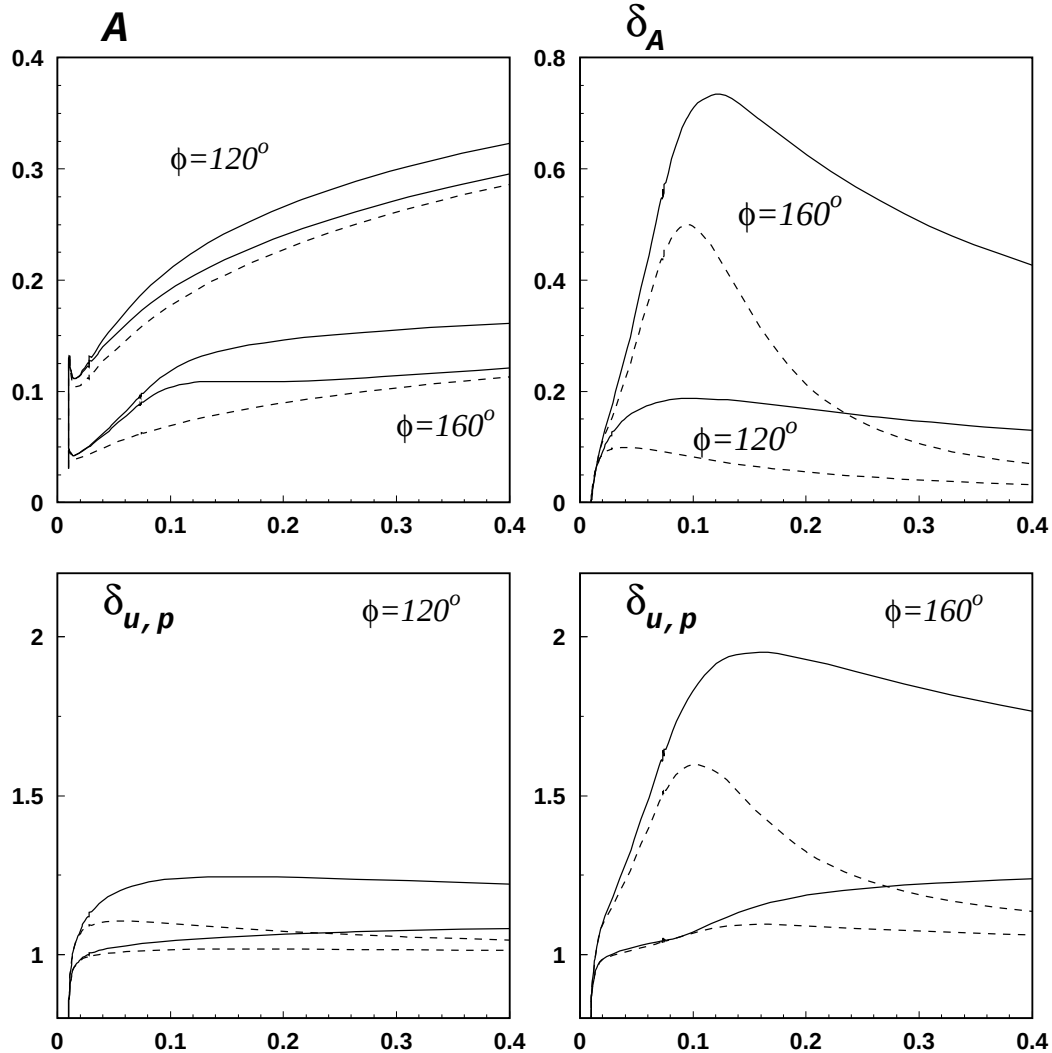



The $\phi$-dependence of the asymmetry (upper) and RC factors (lower plots). Dashed curve at the upper plots gives the $A_{1\gamma}$ and solid curved show the observed asymmetry with $V^2_{cut}$=0.3 GeV$^2$ (the curve closer to dashed curve) and without cuts. Dashed and solid curves at bottom plots show $\delta_{u,p}$ with and without the cut, respectively. The curves with higher values corresponds to $\delta_p$, i.e., $\delta_p > \delta_u$. Kinematical variables used for this example were $x$=0.1, $Q^2$=1GeV$^2$, and $E_{beam}$=5.75GeV.

The observed asymmetry can be represented as

$$A = A_{1\gamma}\frac{\delta_p}{\delta_u}$$

# The $t$-dependence of the asymmetry and RC factors




The $t$-dependence of the asymmetry (upper left), RC to asymmetry (upper right)

$$\delta_A = \frac{A - A_{1\gamma}}{A_{1\gamma}}$$

and RC factors (lower plots). Dashed curve for $A$ gives the $\sigma_{1\gamma}$ and solid curved show the observed cross sections with $V^2_{cut}$=0.3 GeV$^2$ (the curve closer to dashed curve) and without cuts. Dashed and solid curves at the other three plots show $\delta_{A,u,p}$ with and without the cut, respectively. Kinematical variables used for this example were $x$=0.1, $Q^2$=1GeV$^2$, and $E_{beam}$=5.75GeV.

The observed asymmetry can be represented as

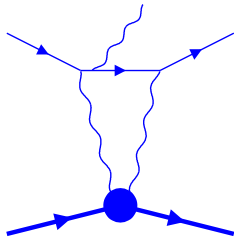
$$A = A_{1\gamma}\frac{\delta_p}{\delta_u}$$

# Codes for Numerical Calculation of RC in a kinematical point

- **DVCSLL** is the code to calculate RC to BH and DVCS process in leading approximation:
  - *Cases of longitudinal and transverse target polarization are included.*
  - *Higher order correction are included in terms of electron structure functions.*
  - *Cut on missing energy is implemented.*
  - *Both numeric and Monte Carlo integration methods are implemented*
  - *Integration over $\phi$ is implemented.*
  - *BMK approximation is used to describe hadronic structure for DVCS.*
- **BHexact** is the code to calculate RC to BH with the next-to-leading accuracy:
  - *Calculate leading log and next-to-leading contribution separately.*
  - *Numeric integration is used.*
  - *Cases of longitudinal and transverse target polarization are included.*
- **BHRadgen** is the Monte Carlo Generator “radiated” (two photons in final state) or “non-radiated” (one photon in final state):
  - *Based on DVCSLL.*
  - *Three additional kinematical variables (to describe an additional photon) generated for “radiated” event.*
  - *Only BH is implemented. Contributions of DVCS can be added (in the BMK approximation)*

# Uncertainties in RC calculation

- Accuracy of theoretical calculation
  - *Leading log (DVCSLL) and next-to-leading (BHexact) accuracy of the calculation of the additional photon emission (deserve careful consideration)*
  - *Higher order corrections through exponentiation procedure (not so high effect is expected)*
  - *Accuracy of numeric integration (under control)*
  - *Approximations made when experimental cuts are implemented (could be tested (and finally resolved) using Monte Carlo generators)*
- Model dependence
  - *Model for nucleon formfactors (essential effect is not expected, but needed to be checked for each specific data analysis)*
  - *BMK approximation (could be important for RC to DVCS)*
  - *Models for GPDs in RC to DVCS (not investigated; could be very important)*
- Physical contributions not taken into account yet
  - *Pentagon (or 5-point) diagrams (importance is not known), e.g.,*

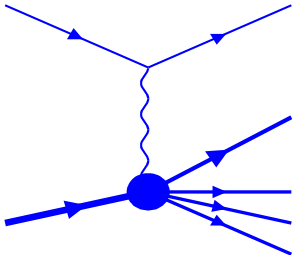


# Part III:

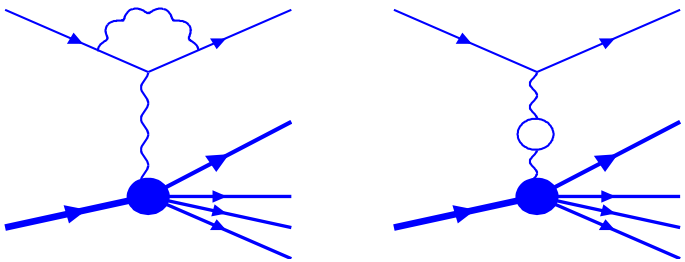
# RC to SIDIS

# Contribution to RC in SIDIS

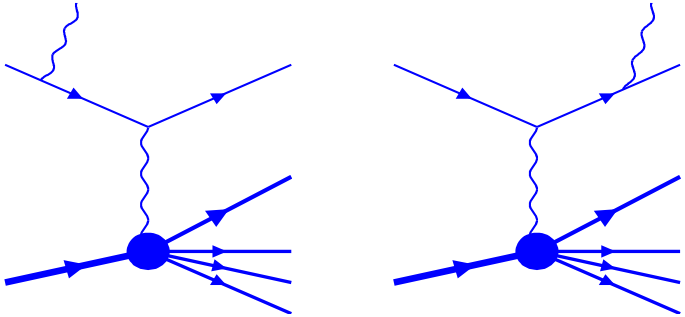
The Born cross section



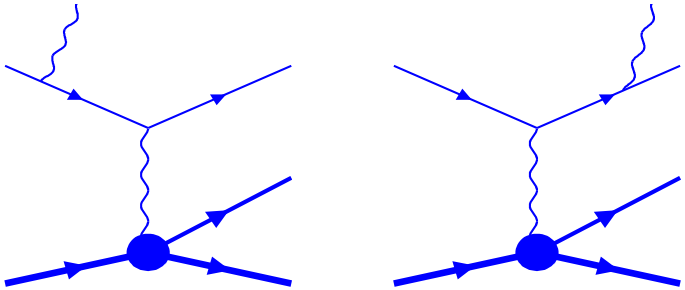
Loop diagrams



Emission of a radiated photon (semi-inclusive processes)



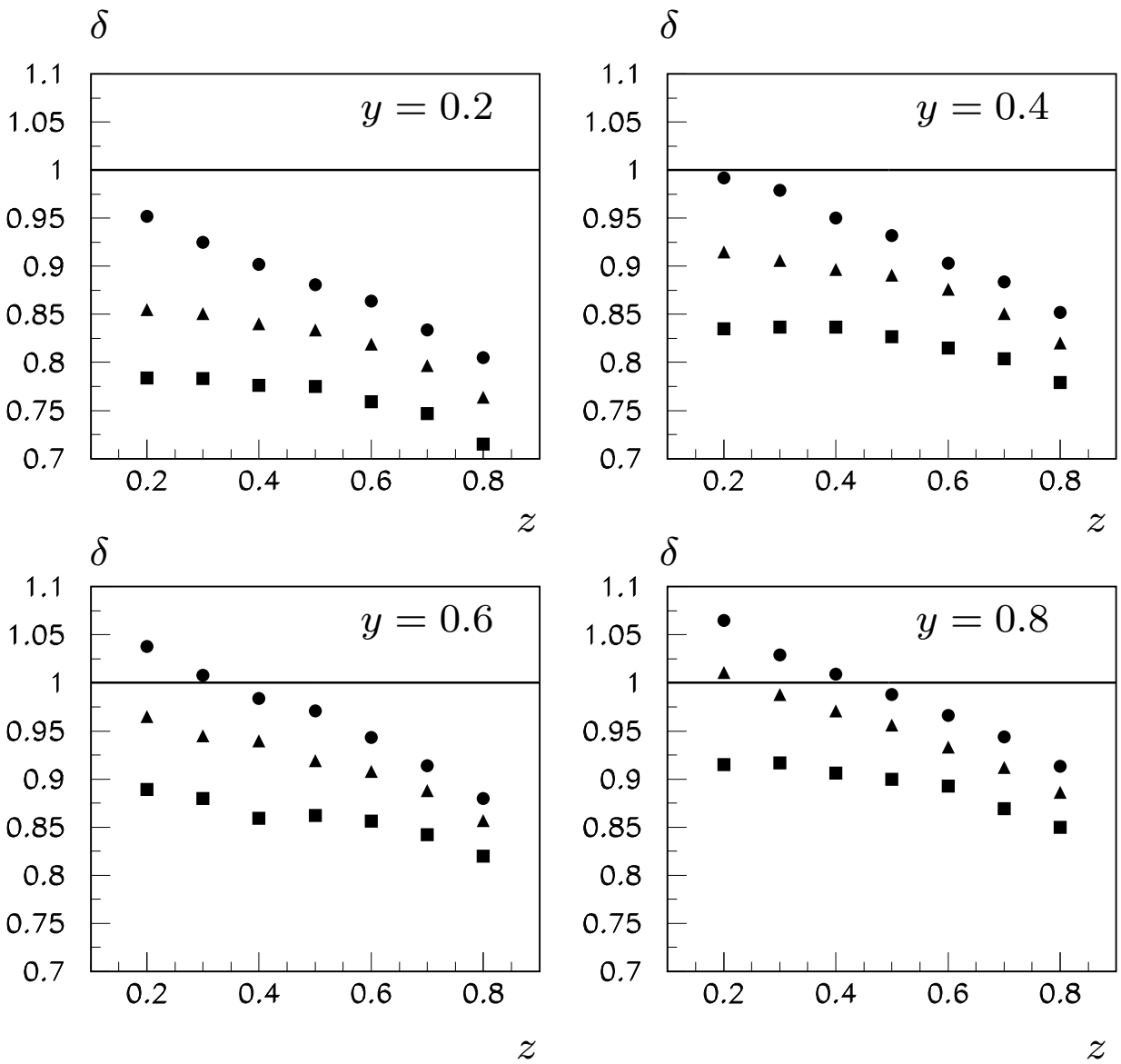
Emission of a radiated photon (exclusive processes)



# Calculation of RC to SIDIS using POLRAD 2.0

- Option SIRAD has to be used.
- Simple QPM is assumed. The RC cross section is calculated in terms of parton distributions $f(x)$ and fragmentation functions $D(z)$.
- RC for unpolarized cross section and polarized part of the cross section are calculated.
- No exclusive radiative tail is separately calculated.
- Several models for parton distributions and fragmentation functions are implemented.
- RC has traditional form: factorized part representing the contributions of loops and soft photon emission and the term in the form of two-dimensional integral representing the hard photon emission.
- Integration over $p_t$ and $\phi_h$ is assumed, i.e., the RC to the three-fold cross section is calculated: $d\sigma/dxdydz$.

# Numerical results for pions



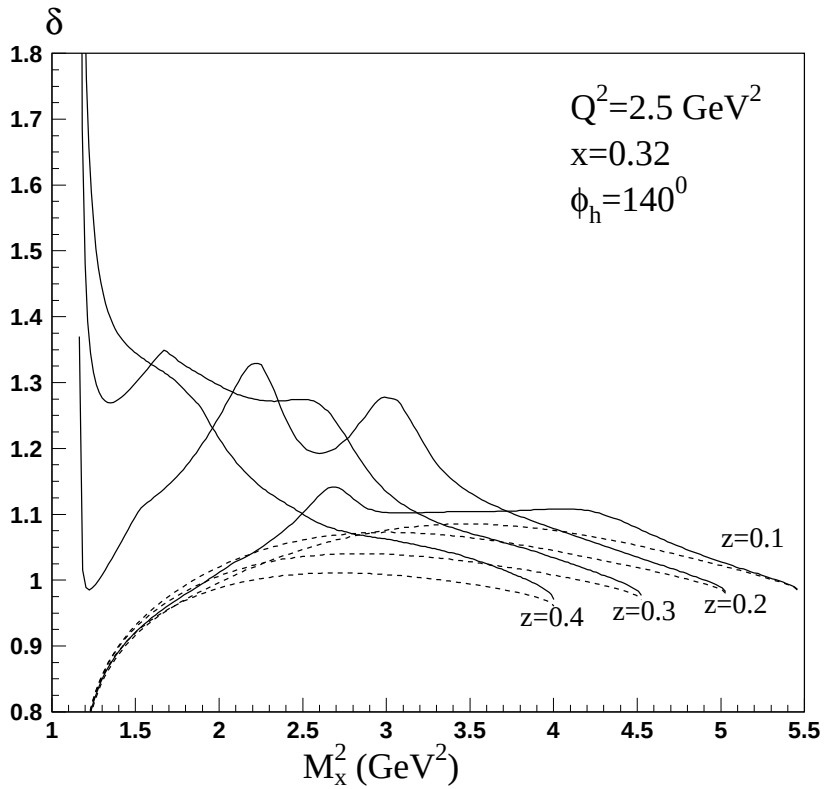

*Radiative correction to the semi-inclusive cross section for kinematics of HERMES; $\sqrt{S}$=7.19 GeV. Symbols from top to bottom correspond to the $x$=0.05, 0.45 and 0.7. The results for $x$=0.15 are skipped, because they practically coincide with ones for $x$=0.05.*

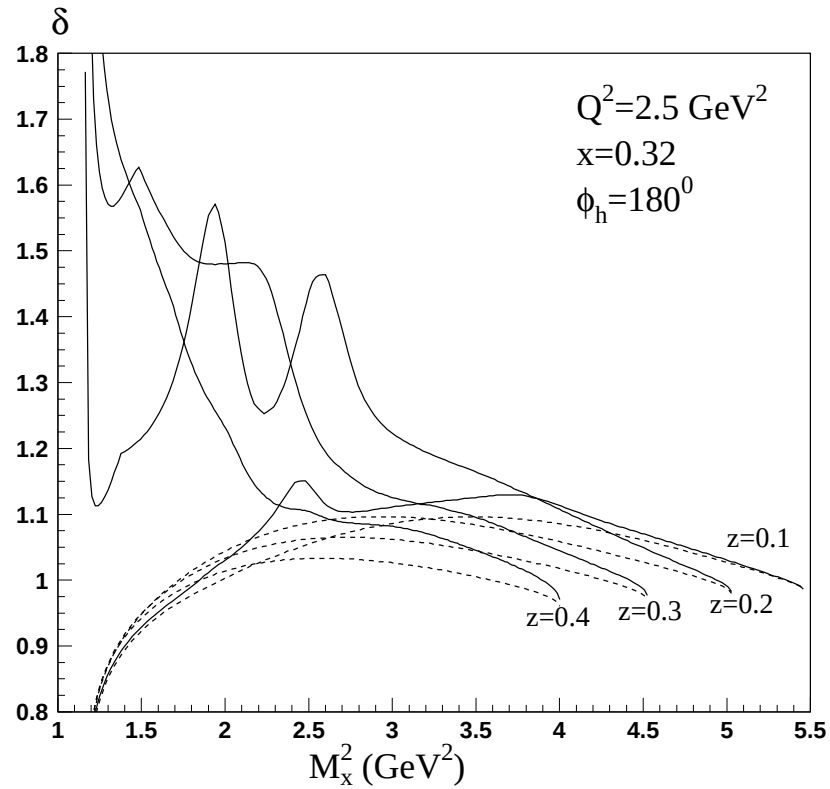
# Calculation of RC to SIDIS using HAPRAD 2.0

- Original version is based on the calculation in Akushevich, Soroko, Shumeiko EPJ C10(1999)681. In this paper an approach to calculate radiative corrections to unpolarized cross section of semi-inclusive electroproduction is developed. Then the contribution of the exclusive radiative tail was calculated in (Akushevich, Ilyichev, Osipenko, Phys.Lett. B672(2009)35)
- calculate the RC to five-dimensional cross section $d^5\sigma/dxdydzdp_t^2d\theta_h$ as well as to four- and three-dimensional cross sections (i.e., $d^4\sigma/dxdydzdp_t^2d$ and $d^3\sigma/dxdydz$).
- For the three-dimensional cross section the results for RC are close to that given by POLRAD 2.0 (SIRAD)
- The correction is of standard form:

$$\sigma_{obs} = \sigma_0 e^{\delta_{inf}}(1 + \delta_{VR} + \delta_{vac}) + \sigma_F + \sigma_{exl}.$$

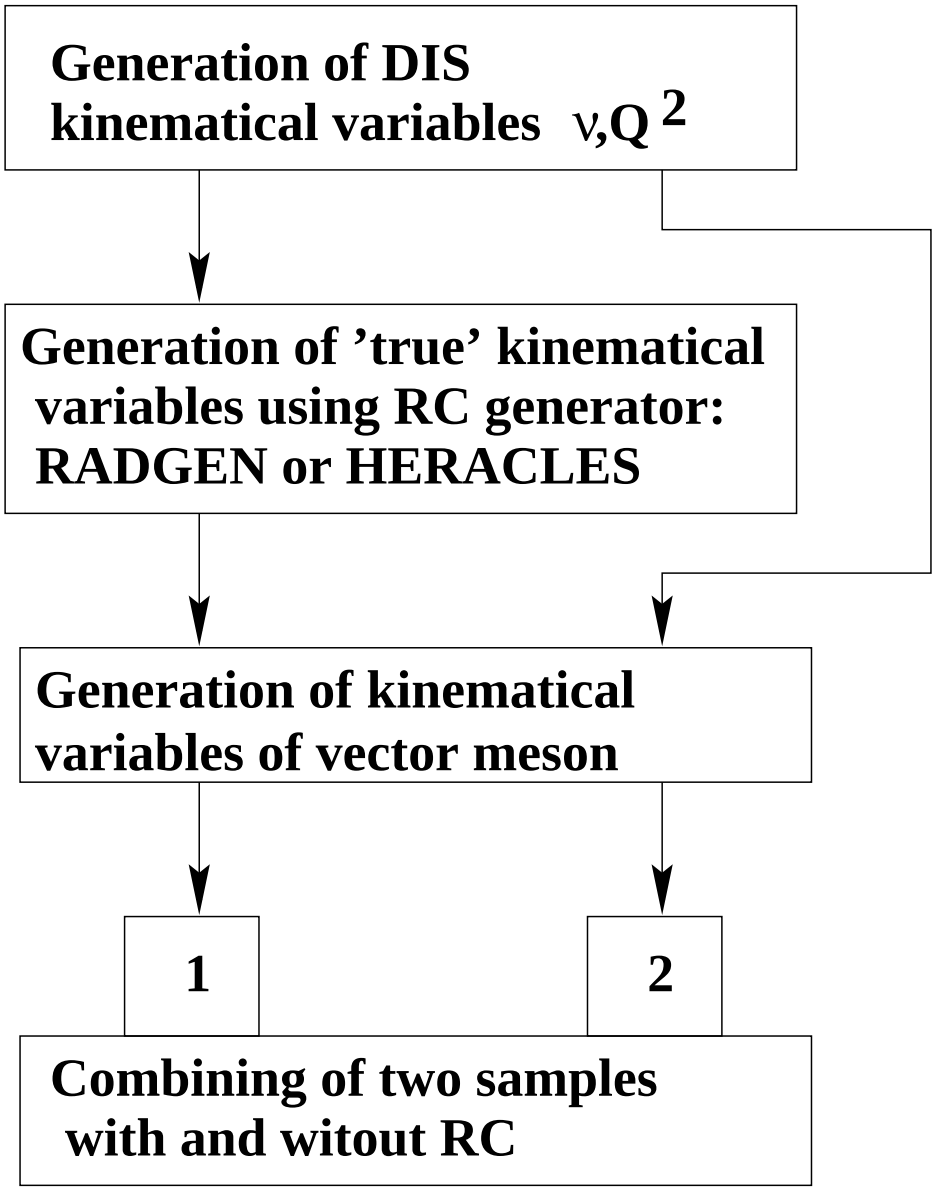
# Importance of exclusive radiative tail



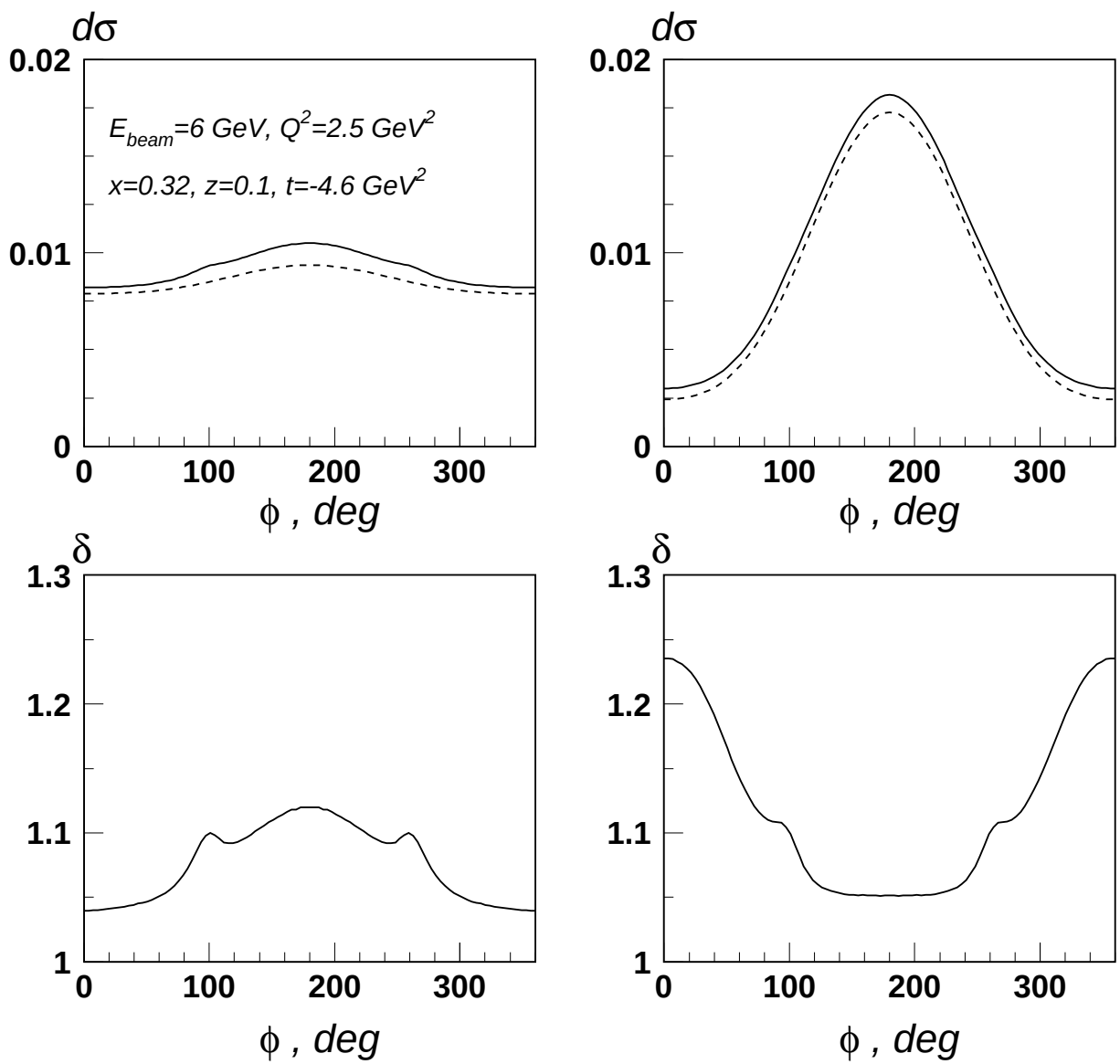

$M_X^2$-dependence of the RC factor for the semi-inclusive $\pi^+$ electroproduction at fixed proton for lepton beam energy 6 GeV: solid lines show the total correction, dashed lines represent the correction excluding the exclusive radiative tail (Akushevich, Ilyichev, Osipenko, Phys.Lett. B672(2009)35)

# Calculation of RC to SIDIS using Monte Carlo generator



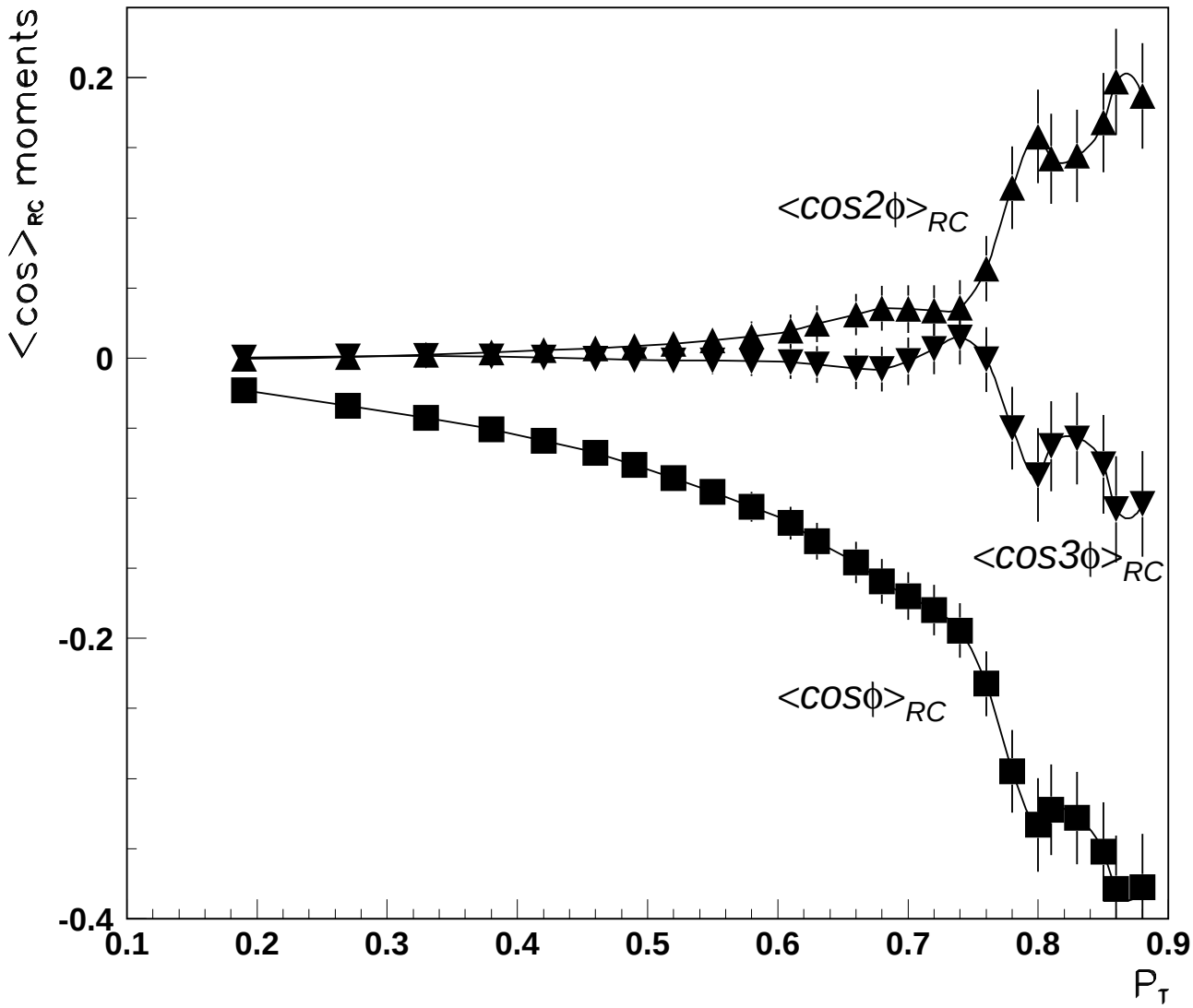

Possible scheme of Monte Carlo calculation of the RC factor (Akushevich, hep-ph/9906410)

# The $\phi$-dependence of the cross section




Azimuthal modulations due to RC in SIDIS: Different input of SFs produce different corrections (even by sign)

# The $p_T$-dependence of the cross section




This plot illustrates that the RC may be very significant at large PT. Also the plot illustrates occurrence of the effects not observed at the level of the Born cross section (i.e., $< cos(3\phi) >$.

# General task for RC to SIDIS

- Ultimate purpose of this task is to create the code for exact (i.e., leading+next-to-leading) calculation of RC to the SIDIS cross section of electron scattering by the proton target with both particles arbitrary polarized and to develop a Monte Carlo generator based on this code.
- Stages of the theoretical calculation and practical implementation include the steps:
  - *Elaborate the covariant hadronic tensor of SIDIS based on the developments of the theoretical groups of Aram Kotzinian (Nuclear Physics B 441 (1995) 234-256), Peter Mulders (Phys.Rev. D49 (1994) 96-113) and Alessandro Bacchetta (JHEP 0702:093,2007).*
  - *Calculate the SIDIS cross sections using the elaborated hadronic tensor, compare analytic expressions for the Born cross section between the results obtained by researchers of these groups and understand possible discrepancies.*
  - *Calculate the coefficients from contraction of leptonic tensor involving RC with tensor structures from the hadronic tensor.*
  - *Code these coefficients and complete the code for RC due to DIS radiative tail.*
  - *Obtain the hadronic tensor of exclusive process, calculate the contractions for the kinematics of exclusive process and implement this part of RC to the code.*
  - *Implement set of available four-dimensional DIS and three-dimensional exclusive SFs, investigate the model dependence of the RC, and identify the kinematical regions where uncertainty in SFs could result in significant effect on RC.*
  - *Develop a Monte Carlo generator that will generate the channel of scattering (non-radiative, radiative DIS, and radiative exclusive scattering) and the kinematical variables of the additionally radiated photon.*

# Explicit Expression for SIDIS Hadronic Tensor

$$W_{\mu\nu} = n_\mu n_\nu T_1 + p_\mu p_\nu T_2 + p_{h\mu} p_{h\nu} T_3 + (p_\mu p_{h\nu} + p_{h\mu} p_\nu) T_4 + (p_\mu p_{h\nu} - p_{h\mu} p_\nu) T_5$$

$$+ (p_\mu n_\nu + n_\mu p_\nu) T_6 + (p_\mu n_\nu - n_\mu p_\nu) T_7 + (p_{h\mu} n_\nu + n_\mu p_{h\nu}) T_8 + (p_{h\mu} n_\nu - n_\mu p_{h\nu}) T_9$$

where $n_\mu = \frac{1}{M}\epsilon_{\mu p q p_h}$, $S_x = \frac{Q^2}{x}$, $\lambda_y = S_x^2 + 4M^2Q^2$, $Z = z\lambda_Y^{1/2} - (z^2 S_x^2 - 4M^2(p_t^2 + m_h^2))^{1/2}$

$$T_1 = \frac{4M^2}{\lambda_Y p_t^2} H_{22}^{(0)} - \frac{8(nS)M^3}{\lambda_Y^{3/2} p_t^3} H_{222}^{(S)}$$

$$T_2 = \frac{1}{4M^2 \lambda_Y p_t^2} (Z^2 S_x^2 H_{11}^{(0)} - 8(ZQS_x \Re H_{01}^{(0)} - 2M^2 p_t Q^2 H_{00}^{(0)}) M^2 p_t)$$

$$+ \frac{(nS)}{2M^3 \lambda_Y^{3/2} p_t^3} (-Z^2 H_{112}^{(S)} S_x^2 + 8(ZQS_x \Re H_{012}^{(S)} - 2M^2 p_t Q^2 H_{002}^{(S)}) M^2 p_t)$$

$$T_3 = \frac{H_{11}^{(0)}}{p_t^2} - \frac{2(nS)M}{\lambda_Y^{1/2} p_t^3} H_{112}^{(S)}$$

$$T_4 = \frac{1}{2M^2 \sqrt{\lambda_Y} p_t} (-ZS_x H_{11}^{(0)} + 4M^2 p_t Q \Re H_{01}^{(0)}) + \frac{(nS)}{M \lambda_Y p_t^3} (ZS_x H_{112}^{(S)} - 4M^2 Q p_t H_{012}^{(S)})$$

$$T_5 = i \frac{2Q}{\sqrt{\lambda_Y} p_t} \Im H_{01}^{(0)} - i \frac{4(nS)MQ}{\lambda_Y p_t^2} \Im H_{012}^{(S)}$$

# Explicit Expression for SIDIS Hadronic Tensor

$$W_{\mu\nu} = n_\mu n_\nu T_1 + p_\mu p_\nu T_2 + p_{h\mu} p_{h\nu} T_3 + (p_\mu p_{h\nu} + p_{h\mu} p_\nu) T_4 + (p_\mu p_{h\nu} - p_{h\mu} p_\nu) T_5$$

$$+ (p_\mu n_\nu + n_\mu p_\nu) T_6 + (p_\mu n_\nu - n_\mu p_\nu) T_7 + (p_{h\mu} n_\nu + n_\mu p_{h\nu}) T_8 + (p_{h\mu} n_\nu - n_\mu p_{h\nu}) T_9$$

where $n_\mu = \frac{1}{M}\epsilon_{\mu p q p_h}$, $S_x = \frac{Q^2}{x}$, $\lambda_y = S_x^2 + 4M^2Q^2$, $Z = z\lambda_Y^{1/2} - (z^2 S_x^2 - 4M^2(p_t^2 + m_h^2))^{1/2}$

$$\begin{aligned}
T_6 &= -\frac{1}{M{\lambda_Y}^{3/2} p_t^3}\big(4p_t M^2 Q(2Mp_t \Re H_{023}^{(S)} - \sqrt{\lambda_z}\Re H_{021}^{(S)}) - \sqrt{\lambda_z} S_x (2Mp_t \Re H_{123}^{(S)} \\
&\quad - \sqrt{\lambda_z}\Re H_{121}^{(S)}))(qS) + (4M^2 Q p_t \Re H_{021}^{(S)} - S_x\sqrt{\lambda_z}\Re H_{121}^{(S)})\sqrt{\lambda_Y}(p_h S)\big) \\
T_7 &= -i\frac{1}{M{\lambda_Y}^{3/2} p_t^3}\big(4p_t M^2 Q(2Mp_t \Im H_{023}^{(S)} - \sqrt{\lambda_z}\Im H_{021}^{(S)}) - \sqrt{\lambda_z} S_x (2Mp_t \Im H_{123}^{(S)} \\
&\quad - \sqrt{\lambda_z}\Im H_{121}^{(S)}))(qS) + (4M^2 Q p_t \Im H_{021}^{(S)} - S_x\sqrt{\lambda_z}\Im H_{121}^{(S)})\sqrt{\lambda_Y}(p_h S)\big) \\
T_8 &= -\frac{2M}{\lambda_Y p_t^3}\big((qS)(-\sqrt{\lambda_z}\Re H_{121}^{(S)} + 2Mp_t \Re H_{123}^{(S)}) - (p_h S)\sqrt{\lambda_Y}\Re H_{121}^{(S)}\big) \\
T_9 &= i\frac{2M}{\lambda_Y p_t^3}\big((qS)(\sqrt{\lambda_z}\Im H_{121}^{(S)} - 2Mp_t \Im H_{123}^{(S)}) - (p_h S)\sqrt{\lambda_Y}\Im H_{121}^{(S)}\big)
\end{aligned}$$

where $\lambda_z = z^2 S_x^2 - 4M^2(p_t^2 + m_h^2)$, so $Z = z\lambda_Y^{1/2} - \lambda_z^{1/2}$.

# Current Status and Further steps

- We calculated the born cross section using the hadronic tensor

$$\frac{d^5\sigma}{dxdydzdp_t^2 d\phi_h} = \frac{\alpha^2 \pi y}{4zQ^2} L_{\mu\nu} 2MW_{\mu\nu} = \frac{\alpha^2 \pi y}{4zQ^2} \sum_i K_i^0 H_i(x, Q^2, z, p_t^2)$$

  and found that this expression exactly reproduces the cross section obtained by Alessandro Bacchetta (JHEP 0702:093,2007). We believe that this is complete the test of our analytical expressions for the hadronic tensor in SIDIS.

- We calculated the cross section with an additional photon radiated.

$$\frac{d^5\sigma_r ad}{dxdydzdp_t^2 d\phi_h} = \frac{\alpha^3 y}{8zQ^2} L_{\mu\nu}^{rad} 2MW_{\mu\nu} = \frac{\alpha^3 y}{8zQ^2} \int \frac{d^3k}{k_0} \sum_i K_i^{rad} H_i(x_{tr}, Q_{tr}^2, z_{tr}, p_{t,tr}^2)$$

- Thus we are able to write total (observed) cross section in the standard form:

$$\sigma_{obs} = \sigma_0 e^{\delta_{inf}} (1 + \delta_{VR} + \delta_{vac}) + \sigma_F + \sigma_{exl}.$$

- One interesting effect we found for $H_5$. The theorem states that terms 1/squared propagator cannot appear in expression for the contractions $K_i^{rad}$ without $m^2$ or terms canceling one propagator. The theorem is satisfied for $H_5$ but the ways of cancellation is a little exotic. We will need to further investigate the term numerically.

# RC procedure of experimental data in SIDIS

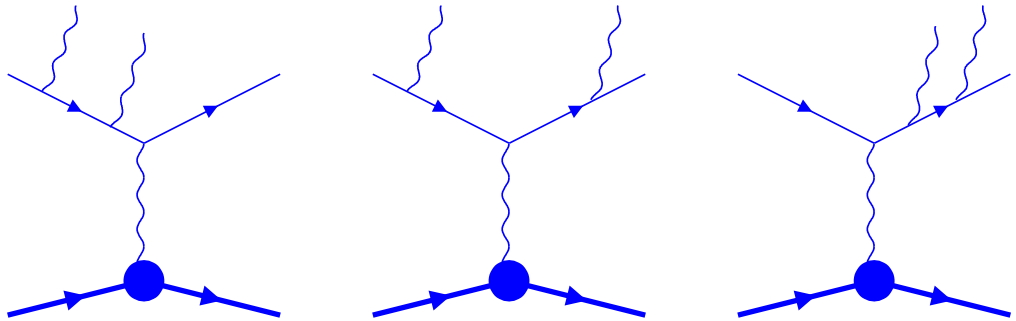
The possible (successful) strategy of RC could be developed using our experience in the modeling for DIS. The RC procedure of experimental data should involve an iteration procedure in which the fits of SFs of interest are re-estimated at each step of this iteration procedure.

- *The fit of SFs are constructed to have the model in the region covered by the experiment*
- *Use experimental data or theoretical models to construct the models in the regions of softer processes, resonance region, and exclusive scattering*
- *Check that the constructed models provide correct asymptotic behavior when we go to the kinematical bounds (Regge limit, QCD limit)*
- *Joint all the models to have continuous function of all four variables in all kinematical regions necessary for RC calculation*
- *Implement this scheme in a computer code and define the iteration procedure*
- *If several SFs are measured in an experiment, implement the procedure of their separation in data and model each of them.*
- *If other SFs are necessary (e.g., unpolarized SFs when spin asymmetries are measured), construct the models for them as well.*
- *Pay specific attention to exclusive SFs, because the radiative tail from exclusive peak is important (or even dominate) in certain kinematical regions.*
- *Pay specific attention to $p_T$ dependence because RC is too sensitive for $p_T$ model choice.*

# Conclusion

- Newly achieved accuracies in Jlab and new physics studied at Jlab require paying renewed attention to RC calculations and their implementation in data analysis software.
- For DVCS:
  - *The calculation of RC to BH and DVCS with the leading accuracy was completed and respective codes (both for numeric estimates of RC and for Monte Carlo generating events) were created.*
  - *Exact calculation of RC to BH is completed and the code for estimating RC in the leading and next-to-leading accuracies was created.*
  - *Exact calculation of DVCS (i.e., the correction to interference of diagrams with emission from lepton and hadron lines) is in progress.*
  - *There are kinematical configurations when the high RCs (More than 100%) are possible.*
  - *The calculation of the effects not taken into account yet (e.g., the Pentagon diagrams) is of interest for both theoreticians and experimentalists.*
- for SIDIS:
  - *Hadronic tensor for the SIDIS cross sections in the covariant form is constructed and tested.*
  - *Exact calculation of RC for the complete SIDIS cross section containing 18 SFs is completed and coding is in progress.*
  - *We expect sensitivity of the results for RC to specific assumptions used for constructing SIDIS SFs:*
    - *Broad discussion and efforts of theoreticians and experimentalists are required to complete the evaluation of all SIDIS SFs as well as SFs in resonance region and exclusive SFs.*
    - *Iteration procedure with fitting of measured SFs and joining with models beyond SIDIS measurements at each iteration step looks the better solution.*

# Two-photon emission: Matrix elements



Six matrix elements of the process are denoted $\mathcal{M}_{1-6} = e^4 t^{-1} J^h_\mu J_{1-6,\mu}$, where

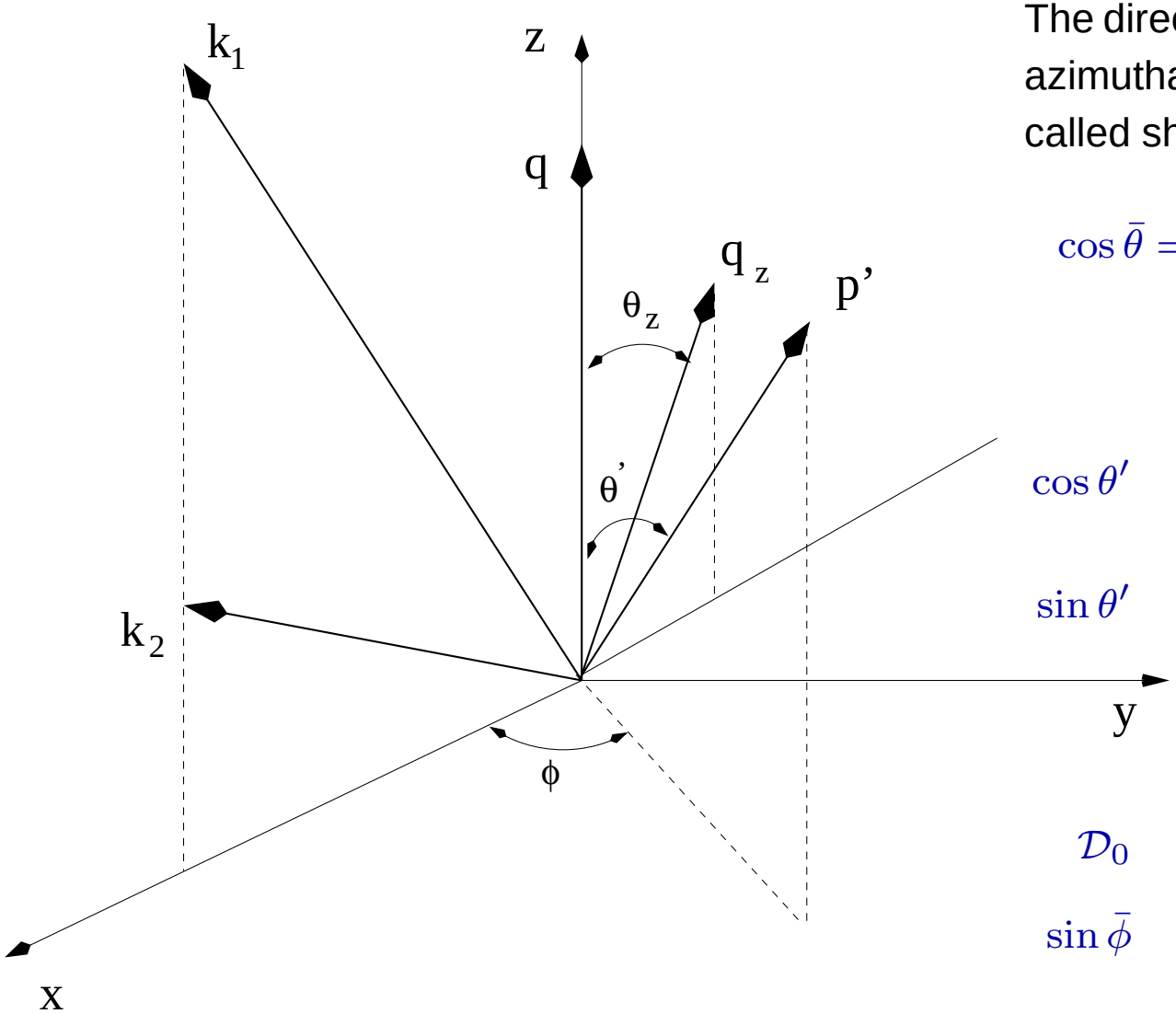
$$J_{1\mu} = \bar{u}_2 \gamma_\mu \frac{\hat{k}_1 - \hat{\kappa} + m}{-2\kappa k_1 + V^2} \hat{\epsilon}_2 \frac{\hat{k}_1 - \hat{\kappa}_1 + m}{-2k_1\kappa_1} \hat{\epsilon}_1 u_1 \qquad J_{2\mu} = \bar{u}_2 \gamma_\mu \frac{\hat{k}_1 - \hat{\kappa} + m}{-2\kappa k_1 + V^2} \hat{\epsilon}_1 \frac{\hat{k}_1 - \hat{\kappa}_2 + m}{-2k_1\kappa_2} \hat{\epsilon}_2 u_1$$

$$J_{3\mu} = \bar{u}_2 \hat{\epsilon}_2 \frac{\hat{k}_2 + \hat{\kappa}_2 + m}{2k_2\kappa_2} \hat{\epsilon}_1 \frac{\hat{k}_2 + \hat{\kappa} + m}{2\kappa k_2 + V^2} \gamma_\mu u_1 \qquad J_{4\mu} = \bar{u}_2 \hat{\epsilon}_1 \frac{\hat{k}_2 + \hat{\kappa}_1 + m}{2k_2\kappa_1} \hat{\epsilon}_2 \frac{\hat{k}_2 + \hat{\kappa} + m}{2\kappa k_2 + V^2} \gamma_\mu u_1$$

$$J_{5\mu} = \bar{u}_2 \hat{\epsilon}_1 \frac{\hat{k}_2 + \hat{\kappa}_1 + m}{2k_2\kappa_1} \gamma_\mu \frac{\hat{k}_1 - \hat{\kappa}_2 + m}{-2k_1\kappa_2} \hat{\epsilon}_2 u_1 \qquad J_{6\mu} = \bar{u}_2 \hat{\epsilon}_2 \frac{\hat{k}_2 + \hat{\kappa}_2 + m}{2k_2\kappa_2} \gamma_\mu \frac{\hat{k}_1 - \hat{\kappa}_1 + m}{-2k_1\kappa_1} \hat{\epsilon}_1 u_1$$

where $V^2 = \kappa^2 = (\kappa_1 + \kappa_2)^2$.

# Definitions of vectors and angles in the Lab. frame




The direction of $q_z$ defines new polar ($\bar{\theta}$) and azimuthal ($\bar{\phi}$) angles of the final proton (so-called shifted kinematics):

$$\cos\bar{\theta} = \cos\theta' \cos\theta_z - \sin\theta' \sin\theta_z \cos\phi$$

$$\cos\theta' = \frac{A\cos\theta_z + \sqrt{\mathcal{D}_0}\sin\theta_z\cos\phi}{\cos^2\theta_z + \sin^2\theta_z\cos^2\phi}$$

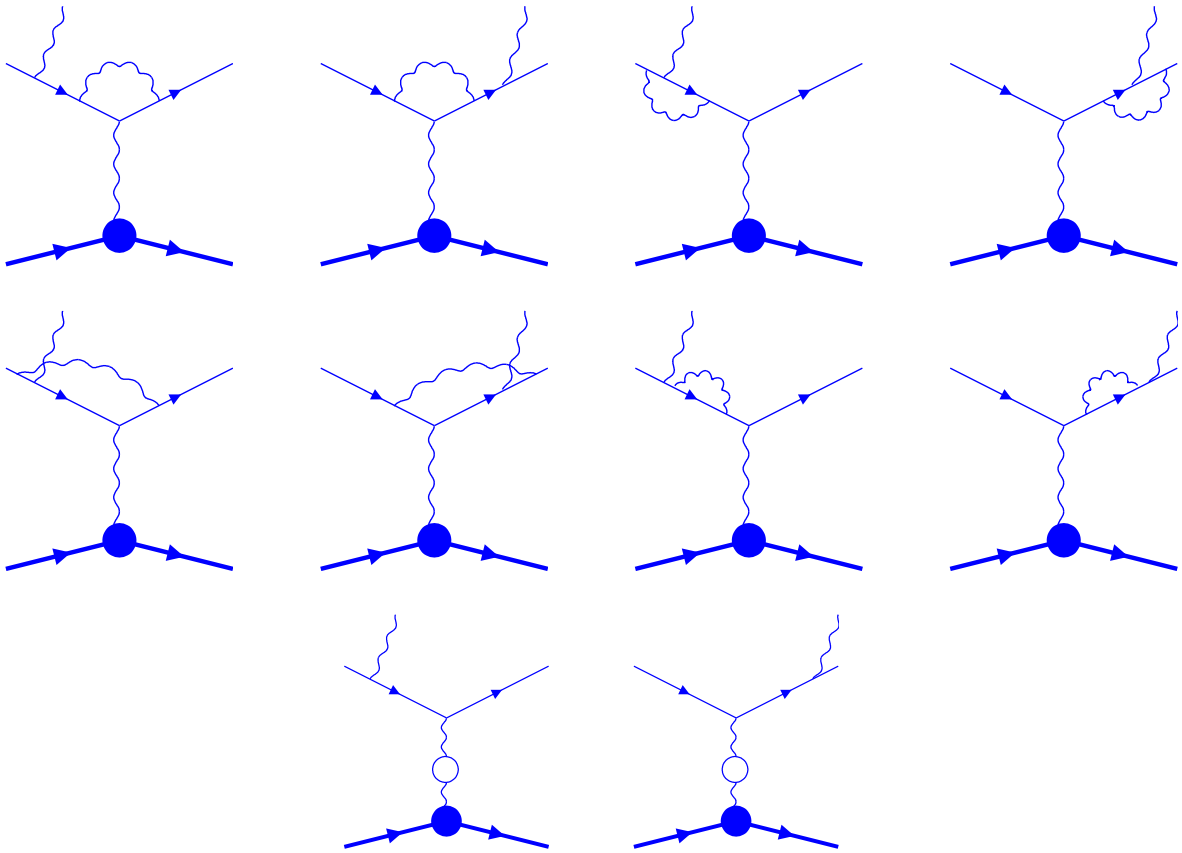
$$\sin\theta' = \frac{\cos\theta_z\sqrt{\mathcal{D}_0} - A\sin\theta_z\cos\phi}{\cos^2\theta_z + \sin^2\theta_z\cos^2\phi}$$

$$\mathcal{D}_0 = \cos^2\theta_z + \sin^2\theta_z\cos^2\phi - A^2$$

$$\sin\bar{\phi} = \frac{\sin\theta'\sin\phi}{\sin\bar{\theta}}$$

# Loop diagrams

Feynman graphs of one-loop effects for the BH cross section



$$\sigma_V = \frac{\alpha}{\pi}\left(\log\frac{4M^2\omega_{min}^2}{SX} + \frac{3}{2}\right)L\sigma_{BH} = -\frac{\alpha L}{2\pi}\sigma_{BH}\left(\int_0^{1-\Delta_1} dz_1\,\frac{1+z_1^2}{1-z_1} + \int_0^{1-\Delta_2} dz_2\,\frac{1+z_2^2}{1-z_2}\right)$$

# Two-photon emission: Phase Space

The phase space for the cross section $d\sigma_r = \frac{M_r^2}{2S(2\pi)^8} d\Gamma$ of the process $e + p \to e' + p' + \gamma_1 + \gamma_2$ is parametrized as

$$d\Gamma = \frac{dp_2}{2E_2}\frac{dk_1}{2\omega_1}\frac{dk_2}{2\omega_2}\frac{dp'}{2E'}\delta(p + p_1 - p_2 - p' - k_1 - k_2) = d\Gamma_0 dV^2 d\Gamma_{2\gamma}$$

where $V$ is invariant mass of two photons and

$$d\Gamma_{2\gamma} = \frac{dk_1}{2\omega_1}\frac{dk_2}{2\omega_2}\delta(p + p_1 - p_2 - p' - k_1 - k_2) = \frac{1}{8}d\Omega_R = \frac{1}{8}\cos(\theta_R)d\phi_R$$

The four kinematical variables to describe the kinematics (and phase space) of one-photon emission process are usual:

$$Q^2, t, x_B, \phi_h$$

Three additional variables to describe the kinematics of the additional photon is

$$V^2, \phi_R, \theta_R.$$

# Analytical Integration

- Integration of the matrix element squared over $d\Gamma_{2\gamma}$ (or over $\phi_R$ and $\theta_R$) can be performed analytically.
- 69 specific integrals including vector and tensor integrals were calculated and combined in the table of integrals.
- All integrals were calculated analytically and the results of analytical integration were tested numerically.
- Note, the integration performed exactly: even the approximation of the small lepton mass is not required.
- The integrals were calculated using the system of center-mass of two photons. The result of the integration is represented in covariant form, and therefore can be presented in Lab. system.

# Example of Analytical Integration

$$\mathrm{J}[A] = \frac{2}{\pi}\int d\Gamma_{2\gamma}\ A = \frac{1}{4\pi}\int d\Omega_R\ A = \frac{1}{4\pi}\int d\cos\theta_R\phi_R\ A.$$

the angles $\theta_R$ and $\phi_R$ define the orientation of momenta of photons in the system where $\vec{k}=0$, i.e., in the two-photon central mass system.

$$\mathrm{J}[1] = 1; \qquad \mathrm{J}\Big[\frac{1}{w_1}\Big] = \mathrm{J}\Big[\frac{1}{w_2}\Big] = L_1 = \frac{1}{\sqrt{\lambda_1}}\log\frac{w+\sqrt{\lambda_1}}{w-\sqrt{\lambda_1}}; \qquad \mathrm{J}\Big[\frac{1}{w_1^2}\Big] = \mathrm{J}\Big[\frac{1}{w_2^2}\Big] = \frac{1}{m^2V^2}$$

$$\mathrm{J}\Big[\frac{1}{u_1^2w_2}\Big] = \mathrm{J}\Big[\frac{1}{u_2^2w_1}\Big] = \frac{1}{\lambda_I}\Big(\frac{u_I}{m^2V^2}+w_IL_I\Big); \quad \mathrm{J}\Big[\frac{D}{u_1^2w_2}\Big] = -\mathrm{J}\Big[\frac{D}{u_2^2w_1}\Big] = \frac{1}{2\lambda_I}\left(\frac{\Phi_{I2}}{m^2V^2}-\Phi_{I1}L_I\right)$$

The new variables are functions of kinematical variables, e.g.,

$$w_I = w(wu - V^2Q^2) - 2m^2V^2(w+u), \quad V_I = wu - V^2Y_m, \quad \lambda_I = V_I^2 - 4m^4V^4,$$

$$W_{Ip} = w^2u - V^2w_Y, \quad U_{Ip} = u^2w - V^2u_Y, \quad L_I = \frac{1}{\sqrt{\lambda_I}}\log\frac{V_I+\sqrt{\lambda_I}}{V_I-\sqrt{\lambda_I}},$$

$$\Phi_{I1} = S_{xt}W_{Ip} - 2V^2(SV_I + 2m^2V^2X), \quad \Phi_{I2} = S_{xt}U_{Ip} - 2V^2(XV_I + 2m^2V^2S), \quad \ldots$$

# Infrared Divergence

The cross section containing IR is represented in the form:

$$\frac{d\sigma_{IR}}{d\Gamma_0} = \frac{\alpha}{\pi}\delta_R^{IR}\frac{d\sigma_0}{d\Gamma_0}, \quad \delta_R^{IR} = \frac{1}{4\pi}\int\limits_0^{V_m^2} dV^2 \int d\Gamma_{2\gamma} 4(F_1^{IR} + F_2^{IR})$$

where $F_{1,2}^{IR} = \left(\frac{k_2}{u_{1,2}} - \frac{k_1}{w_{1,2}}\right)^2 = \frac{Q^2+2m^2}{u_{1,2}w_{1,2}} - \frac{m^2}{u_{1,2}^2} - \frac{m^2}{w_{1,2}^2}$, $w_{1,2} = 2k_1\kappa_{1,2}$, and $u_{1,2} = 2k_2\kappa_{1,2}$.

The integration over the 3-momentum of one of photons and then to over $V^2$ is performed using the $\delta$-function from phase space. The integration region over the momentum of remaining photon (denoted by $\kappa_{cm}$) in the two-photon center-mass system can be split into two parts by an infinitesimal parameter $\bar{\kappa}$ resulting in $\delta_R^{IR} = \delta_1 + \delta_2$ with

$$\delta_1 = \frac{1}{4\pi}\int\limits_0^{V_m^2} dV^2 \int d\Gamma_{2\gamma} 4(F_1^{IR}+F_2^{IR})\theta(\bar{\kappa}-\kappa_{cm}), \quad \delta_2 = \frac{1}{4\pi}\int\limits_0^{V_m^2} dV^2 \int d\Gamma_{2\gamma} 4(F_1^{IR}+F_2^{IR})\theta(\kappa_{cm}-\bar{\kappa})$$

- The second term does not contain infrared divergence and is calculated straightforwardly

$$\delta_2 = 2\log\left(\frac{V_m^2}{4\bar{\kappa}^2}\right)(L_m - 1)$$

# Infrared Divergence (cont.)

- The calculation of the first term is performed in the dimensional regularization. The phase space of remaining photon (after integration using the $\delta$-function) is rewritten in $d$-dimensional space as :

$$\delta_1 = \frac{(2\sqrt{\pi}\mu)^{4-d}}{\Gamma(d/2-1)} \int_0^1 d\alpha \int_0^{\bar{\kappa}} \frac{d\kappa_{cm}}{\kappa_{cm}^{5-d}} \int_{-1}^{1} d\zeta (1-\zeta^2)^{\frac{n}{2}-2} \left( \frac{Q^2+2m^2}{(E_\alpha - p_\alpha \zeta)^2} - \frac{m^2}{(E_1 - p_1\zeta)^2} - \frac{m^2}{(E_2 - p_2\zeta)^2} \right)$$

Energies are taken in the system of center mass of two photons:

$$E_1 = \frac{w}{4k_{cm}}, \quad E_2 = \frac{u}{4k_{cm}}, \quad E_\alpha = \frac{w\alpha + u(1-\alpha)}{4k_{cm}}$$

and $p_\alpha^2 = E_\alpha^2 - m_\alpha^2$, $m_\alpha^2 = m^2 + \alpha(1-\alpha)Q^2$.

- The first step in the calculation is the integration over $\zeta$. The result of this integration involves the hyperheometric function, however allows for expansion over $k_{cm}$. The forthcoming integration over $k_{cm}$, extraction of IRD terms, integration over $\alpha$, and expansion over $m$ keeping only leading and next-to-leading terms result in

$$\delta_R^{IR} = \left( 2P_{IR} + \log\left( \frac{V_m^4}{u_0 w_0} \right) \right) (L_m - 1) + \frac{1}{2} L_m^2 - \frac{\pi^2}{6} - \frac{1}{2} \log^2 \frac{u_0}{w_0}$$

# Loop effects: Details of Calculation

- For all of two-, three-, and four-denominator integrals we use Feynman parametrization, e.g.,

$$\int d^4 l \frac{1}{A(l)B(l)C(l)} = \int_0^1 d\alpha \int_0^1 d\beta \int_0^1 d\gamma \int d^4 l \frac{2\delta(1-\alpha-\beta-\gamma)}{(\alpha A(l)+\beta B(l)+\gamma C(l))^3}$$

- Vector and tensor integration is performed for integrals containing $l_\mu$ and $l_\mu l_\nu$ in numerator.
- All integrals are calculated with next-to-leading accuracy within dimensional regularization and the Table of calculated integrals is created. For example

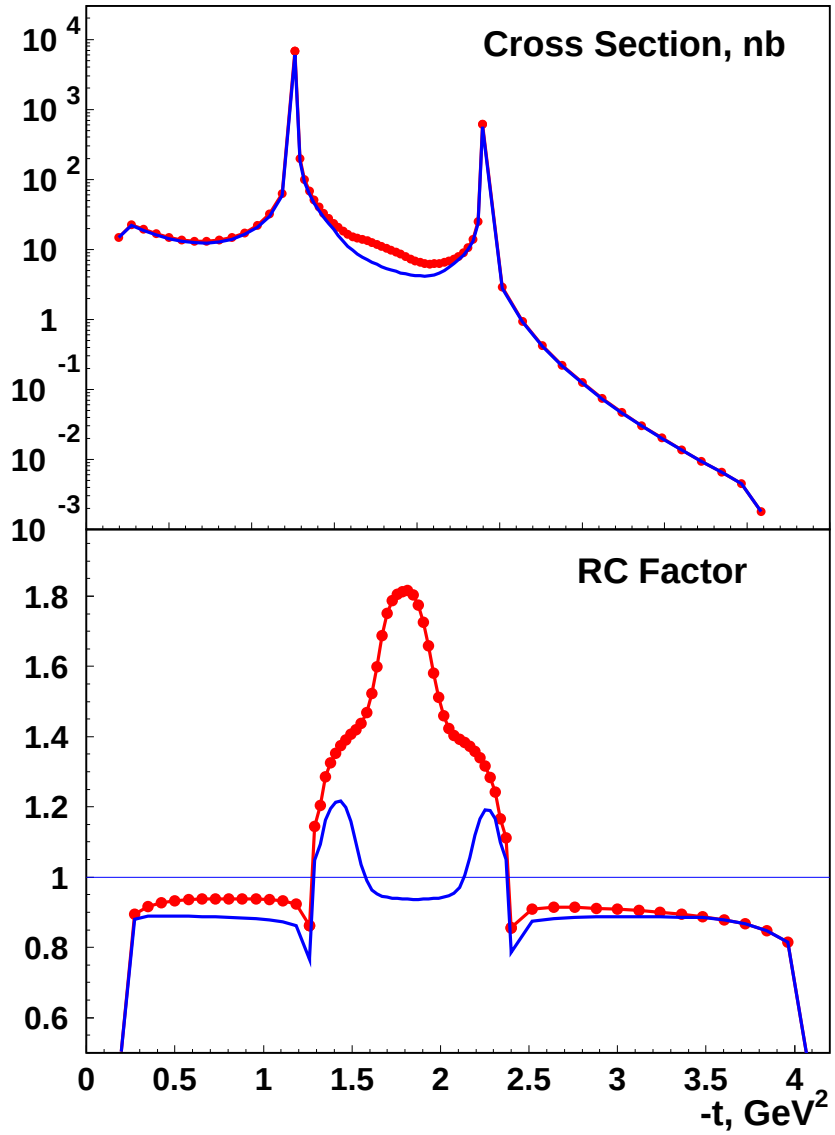
$$J_{012q} = -\frac{P_{IR} L_m}{w} + \frac{1}{wQ^2}\left(2L_m L_w - L_t^2 - \Phi\left(1-\frac{t}{Q^2}\right) - \frac{\pi^2}{6}\right)$$

$$J^\mu_{0q} = \left(-P + 1 - \frac{1}{2}L_w\right)(k_{1\mu} - k_\mu)$$

where $L_m = \log(Q^2/m^2)$, $L_w = \log(w/m^2)$, $L_t = \log(-t/m^2)$.

- Important is that all integrals can be calculated analytically.

# Numerical results: Cross section and RC factor



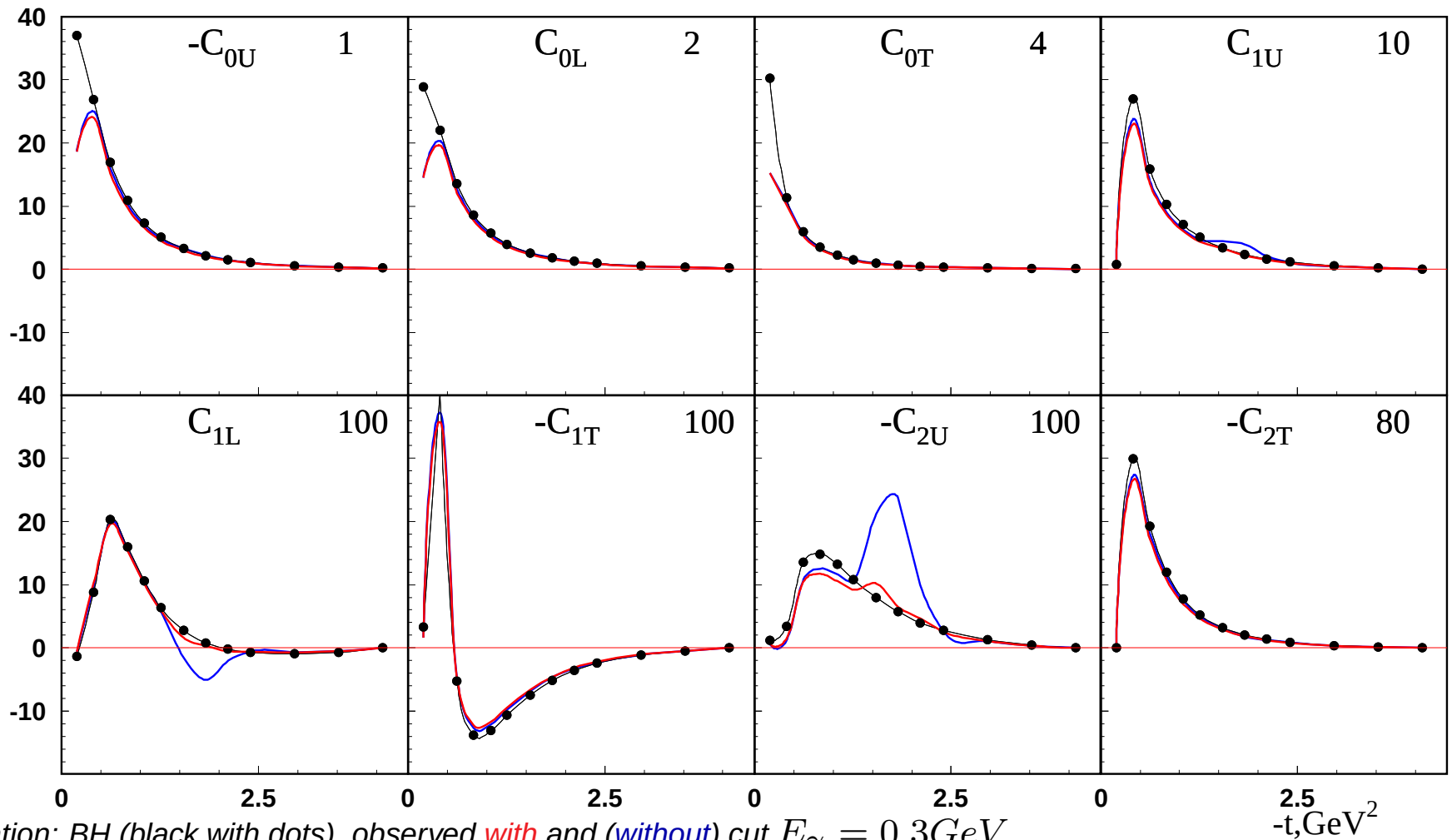

The observed cross sections of the BH process (upper plot) and respective RC factors (lower plot) for beam energy 5.77 GeV, $x$=0.4, and $Q^2$=1.8GeV$^2$.

The red (blue) line shows the results of calculation without (with) the cut on missing energy ($E_\gamma$ <0.3 GeV).

$$\mathrm{RCfactor} = \frac{\sigma_{observed}}{\sigma_{BH}}$$

# Numerical results: Fourier Coefficients

$$C_{nPol} = \frac{1}{2\pi f}\int_0^{2\pi} d\phi \cos(n\phi)\ \ \mathcal{P}_1\mathcal{P}_2\ \ \sigma_{BH,Pol}, \qquad Pol = U, L, P$$




*Notation: BH (black with dots), observed with and (without) cut* $E_\gamma = 0.3GeV$.

# Numerical results: $\phi$ dependence